Computer algebra independent integration tests

4-Trig-functions/4.1-Sine/4.1.3.1-a+b-sin-^m-c+d-sin-^n-A+B-sin-

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [358]. This is test number [76].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (358)	% 0.00 (0)
Mathematica	% 97.21 (348)	% 2.79 (10)
Maple	% 81.01 (290)	% 18.99 (68)
Maxima	% 37.15 (133)	% 62.85 (225)
Fricas	% 76.82 (275)	% 23.18 (83)
Sympy	% 25.14 (90)	% 74.86 (268)
Giac	% 36.31 (130)	% 63.69 (228)
Mupad	% 49.72 (178)	% 50.28 (180)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

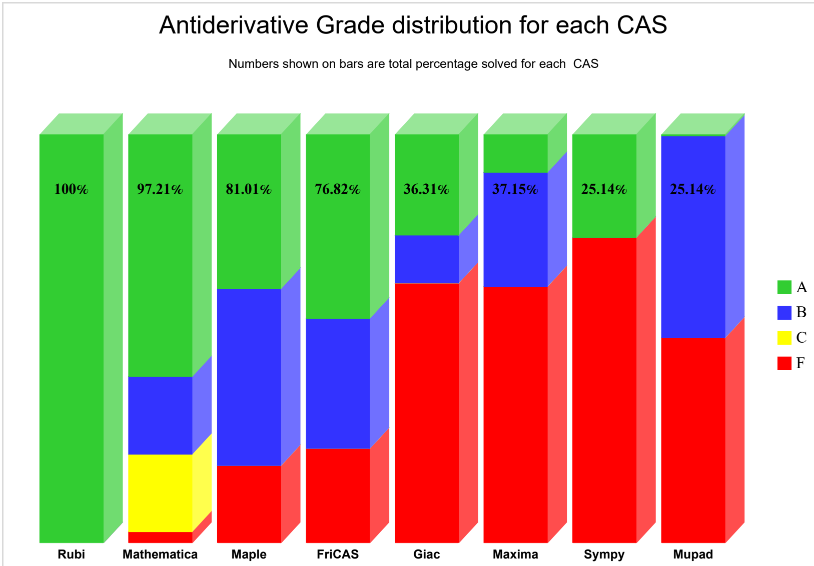
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

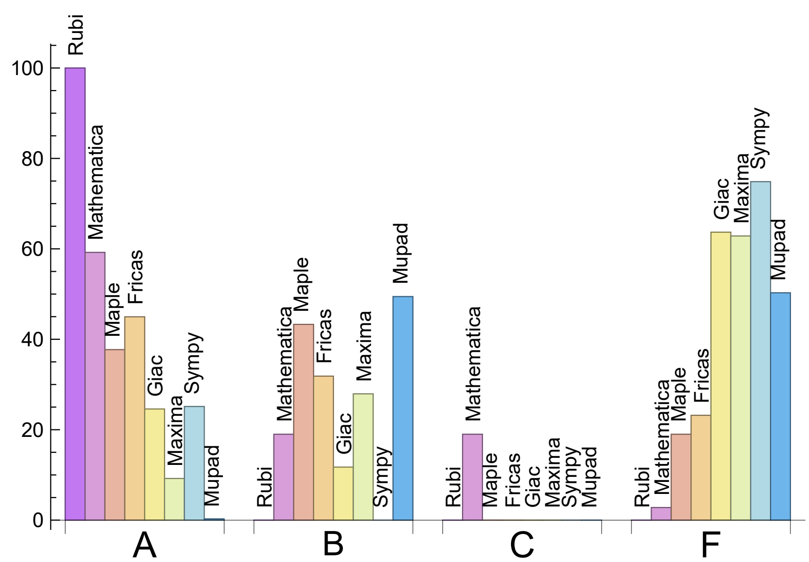
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	59.22	18.99	18.99	2.79
Maple	37.71	43.30	0.00	18.99
Maxima	9.22	27.93	0.00	62.85
Fricas	44.97	31.84	0.00	23.18
Sympy	25.14	0.00	0.00	74.86
Giac	24.58	11.73	0.00	63.69
Mupad	0.28	49.44	0.00	50.28

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	10	80.00 %	20.00 %	0.00 %
Maple	68	100.00 %	0.00 %	0.00 %
Maxima	225	83.11 %	8.00 %	8.89 %
Fricas	83	98.80 %	0.00 %	1.20 %
Sympy	268	30.60 %	69.03 %	0.37 %
Giac	228	33.33 %	27.19 %	39.47 %
Mupad	180	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

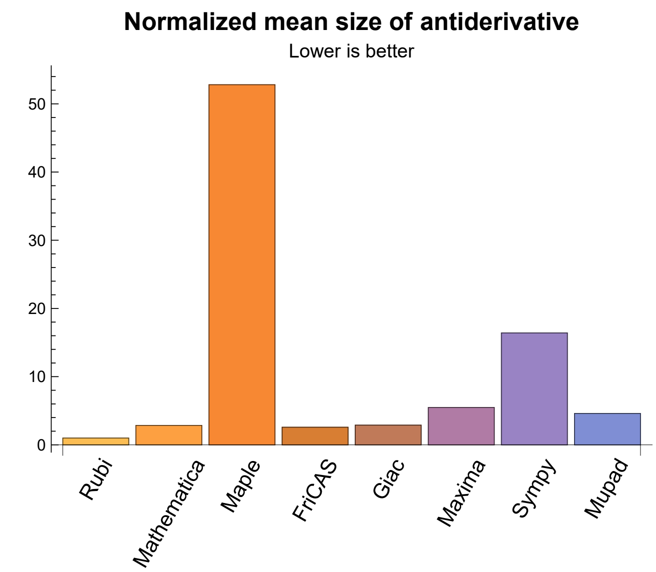
1.3 Performance

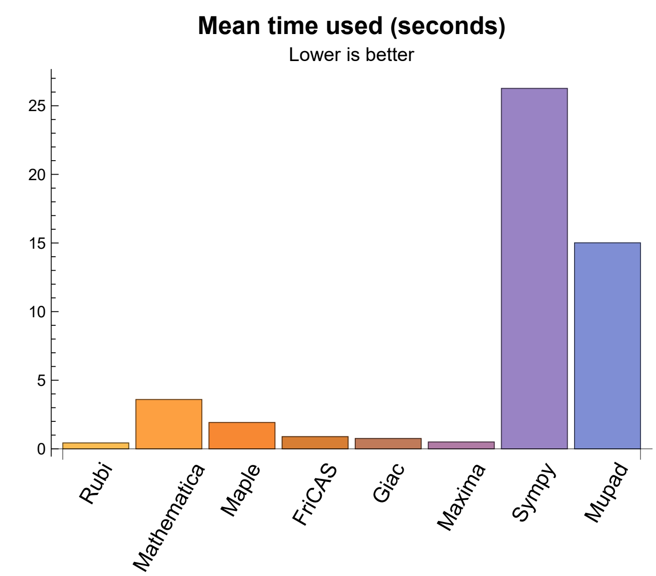
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.43	179.38	1.00	156.00	1.00
Mathematica	3.59	496.05	2.85	223.00	1.47
Maple	1.92	38573.66	52.81	274.00	1.97
Maxima	0.50	782.17	5.48	506.00	4.15
Fricas	0.89	527.67	2.58	234.00	1.77
Sympy	26.27	2372.78	16.41	1443.00	12.12
Giac	0.75	311.58	2.89	215.00	1.50
Mupad	15.01	833.30	4.60	298.50	2.09

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{358}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {7, 8, 10, 11, 12, 88, 195, 198, 199, 200, 201, 202, 209, 210, 211, 212, 213, 214, 215, 216, 217, 243, 304, 305, 306, 313, 320, 326, 327, 334, 335, 336, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://`

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

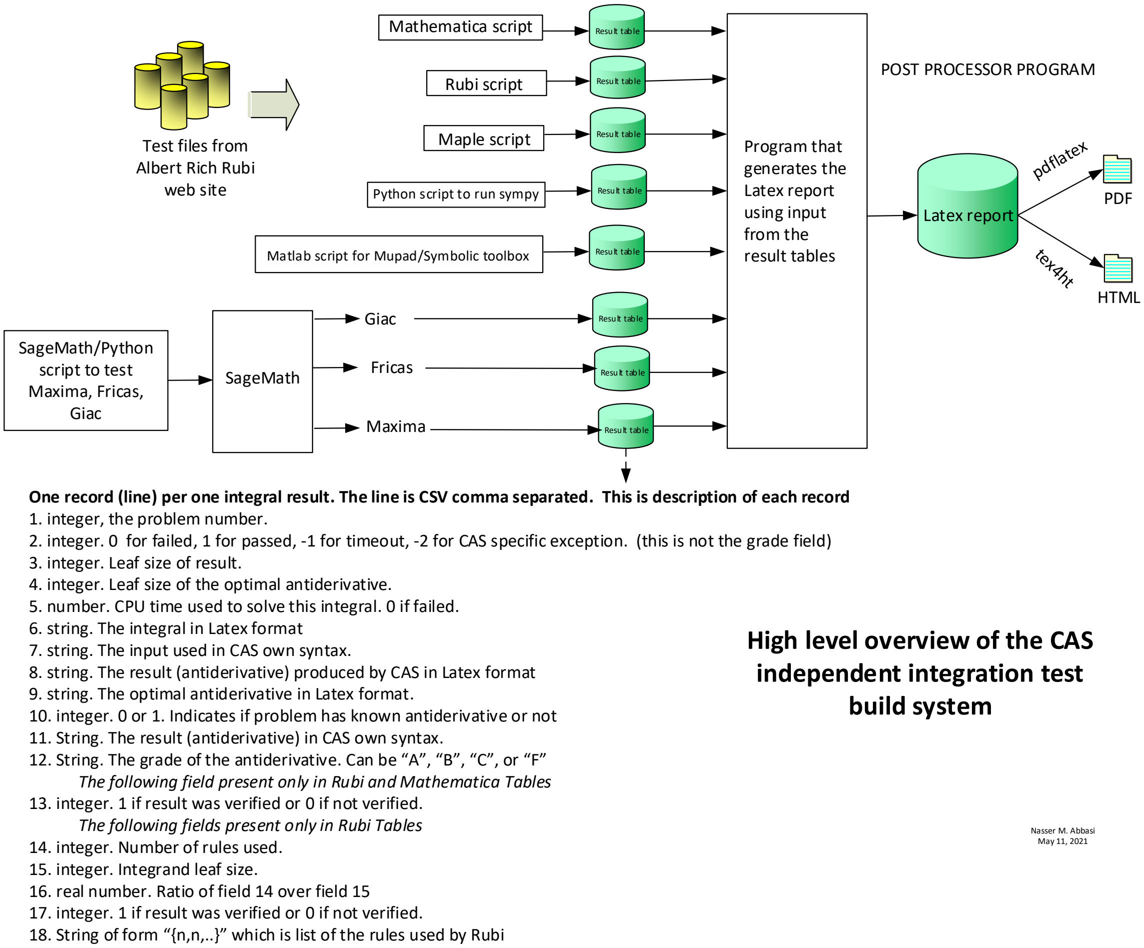
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 4, 5, 6, 7, 10, 15, 16, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 65, 66, 69, 70, 71, 74, 75, 77, 80, 81, 82, 83, 84, 85, 86, 87, 88, 91, 92, 101, 108, 109, 110, 111, 116, 117, 118, 119, 123, 124, 125, 126, 127, 131, 132, 133, 134, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 173, 174, 175, 177, 178, 179, 180, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 203, 204, 206, 207, 208, 211, 212, 213, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 235, 238, 239, 241, 242, 244, 245, 246, 247, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 266, 267, 269, 270, 271, 275, 276, 277, 279, 281, 282, 285, 286, 287, 288, 289, 293, 294, 295, 296, 298, 299, 302, 303, 305, 306, 332, 334, 336, 337, 341, 350, 351, 352, 358 }

B grade: { 8, 11, 12, 14, 21, 22, 32, 33, 34, 35, 46, 47, 48, 49, 50, 55, 63, 64, 67, 68, 72, 73, 76, 78, 79, 89, 90, 98, 99, 100, 115, 170, 171, 172, 232, 233, 234, 236, 237, 240, 243, 264, 265, 268, 272, 273, 274, 278, 280, 283, 284, 297, 301, 304, 335, 339, 340, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

C grade: { 3, 9, 93, 94, 95, 96, 97, 102, 103, 104, 105, 106, 107, 112, 113, 114, 120, 121, 122, 128, 129, 130, 135, 136, 176, 183, 195, 198, 199, 200, 201, 202, 205, 209, 210, 214, 215, 216, 217, 248, 249, 250, 290, 291, 292, 300, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 338 }

F grade: { 13, 196, 197, 328, 329, 330, 331, 333, 348, 349 }

2.1.3 Maple

A grade: { 18, 20, 21, 23, 24, 25, 32, 35, 36, 37, 43, 50, 51, 55, 56, 57, 58, 59, 63, 65, 68, 69, 74, 75, 80, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 94, 98, 99, 100, 101, 102, 103, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 134, 137, 139, 140, 141, 142, 147, 149, 150, 151, 152, 160, 161, 162, 163, 177, 185, 191, 194, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 251, 252, 253, 254, 258, 259, 261, 268, 269, 275, 281, 282, 286, 287, 288, 289, 290, 293, 294, 295, 296, 300, 301, 302, 303, 310, 311, 358 }

B grade: { 16, 17, 19, 22, 26, 27, 28, 29, 30, 31, 33, 34, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 52, 53, 54, 60, 61, 62, 64, 66, 67, 70, 71, 72, 73, 76, 77, 78, 79, 86, 87, 88, 95, 96, 97, 104, 105, 106, 107, 114, 122, 131, 135, 136, 138, 143, 144, 145, 146, 148, 153, 154, 155, 156, 157, 158, 159, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 235, 248, 249, 250, 255, 256, 257, 260, 262, 263, 264, 265, 266, 267, 270, 271, 272, 273, 274, 276, 277, 278, 279, 280, 283, 284, 285, 291, 292, 297, 298, 299, 304, 305, 306, 307, 308, 309, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 352, 353, 354, 355, 356, 357 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351 }

2.1.4 Maxima

A grade: { 17, 18, 20, 30, 43, 56, 66, 77, 135, 176, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 244, 245, 246, 247, 251, 252, 253, 254, 258, 259, 261, 358 }

B grade: { 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 108, 109, 110, 111, 115, 116, 117, 118, 119, 123, 124, 125, 126, 127, 144, 155, 168, 182, 189, 205, 206, 207, 235, 236, 237, 238, 239, 240, 241, 242, 243, 260, 265, 266, 267, 268, 272, 273, 274, 275, 279, 280, 281, 282 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 112, 113, 114, 120, 121, 122, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 248, 249, 250, 255, 256, 257, 262, 263, 264, 269, 270, 271, 276, 277, 278, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357 }

2.1.5 FriCAS

A grade: { 14, 15, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 89, 102, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 137, 138, 139, 140, 141, 142, 146, 147, 148, 149, 150, 151, 152, 157, 158, 159, 160, 161, 162, 163, 164, 171, 172, 173, 178, 179, 184, 185, 186, 191, 192, 193, 194, 206, 207, 211, 212, 213, 222, 223, 224, 225, 226, 227, 228, 229, 244, 245, 246, 247, 248, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 268, 275, 282, 286, 287, 288, 289, 290, 293, 294, 295, 296, 300, 301, 302, 303, 350, 351, 358 }

B grade: { 16, 21, 22, 32, 33, 34, 35, 36, 37, 46, 47, 48, 49, 50, 51, 55, 63, 64, 71, 72, 73, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 170, 205, 218, 219, 220, 221, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 257, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 276, 277, 278, 279, 280, 281, 283, 284, 285, 291, 292, 297, 298, 299, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 352 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 135, 136, 143, 144, 145, 153, 154, 155, 156, 165, 166, 167, 168, 169, 174, 175, 176, 177, 180, 181, 182, 183, 187, 188, 189, 190, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 208, 209, 210, 214, 215, 216, 217, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 353, 354, 355, 356, 357 }

2.1.6 Sympy

A grade: { 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 224, 225, 226, 227, 235, 236, 237, 238, 239, 244, 245, 246, 247, 251, 252, 253, 254, 258, 259, 260, 261, 266, 267, 268, 273, 274, 275, 279, 280, 281, 282 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 37, 49, 50, 51, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 228, 229, 230, 231, 232, 233, 234, 240, 241, 242, 243, 248, 249, 250, 255, 256, 257, 262, 263, 264, 265, 269, 270, 271, 272, 276, 277, 278, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358 }

2.1.7 Giac

A grade: { 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 52, 53, 54, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 70, 71, 72, 73, 74, 75, 77, 224, 225, 226, 227, 228, 230, 231, 233, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 251, 252, 253, 254, 255, 258, 259, 260, 261, 266, 268, 269, 274, 275, 276, 277, 281, 282, 358 }

B grade: { 15, 16, 21, 34, 35, 36, 37, 48, 49, 50, 51, 55, 61, 67, 68, 69, 76, 78, 79, 80, 229, 232, 234, 250, 256, 257, 262, 263, 264, 265, 267, 270, 271, 272, 273, 278, 279, 280, 283, 284, 285, 352 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 354, 355, 356, 357 }

2.1.8 Mupad

A grade: { 358 }

B grade: { 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 111, 118, 119, 125, 126, 127, 131, 132, 133, 134, 138, 139, 140, 141, 142, 147, 148, 149, 150, 151, 152, 158, 159, 160, 161, 162, 163, 164, 171, 172, 173, 191, 205, 206, 207, 211, 212, 213, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 310, 350, 351, 352 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 120, 121, 122, 123, 124, 128, 129, 130, 135, 136, 137, 143, 144, 145, 146, 153, 154, 155, 156, 157, 165, 166, 167, 168, 169, 170, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 208, 209, 210, 214, 215, 216, 217, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 353, 354, 355, 356, 357 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	373	248	0	0	0	0	0	-1
normalized size	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.841	2.349	8.248	0.000	0.447	0.000	0.000	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	204	0	0	0	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.492	1.548	7.543	0.000	0.441	0.000	0.000	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	392	0	0	0	0	0	-1
normalized size	1	1.00	2.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.217	3.846	5.395	0.000	0.427	0.000	0.000	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	157	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.224	0.889	1.973	0.000	0.448	0.000	0.000	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	212	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.488	1.327	11.460	0.000	0.446	0.000	0.000	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	260	0	0	0	0	0	-1
normalized size	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.846	4.574	11.615	0.000	0.446	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	596	0	0	0	0	0	-1
normalized size	1	1.00	1.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.872	18.383	1.087	0.000	0.485	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	478	0	0	0	0	0	-1
normalized size	1	1.00	2.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.494	15.404	1.051	0.000	0.478	0.000	0.000	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	409	0	0	0	0	0	-1
normalized size	1	1.00	2.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.213	67.653	1.055	0.000	0.467	0.000	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	250	0	0	0	0	0	-1
normalized size	1	1.00	1.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.396	4.857	0.895	0.000	0.462	0.000	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	523	0	0	0	0	0	-1
normalized size	1	1.00	2.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.675	21.903	0.893	0.000	0.455	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	5918	0	0	0	0	0	-1
normalized size	1	1.00	26.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.453	22.178	6.039	0.000	0.451	0.000	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.156	11.252	5.908	0.000	0.457	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	107	0	0	41	0	0	61
normalized size	1	1.00	2.89	0.00	0.00	1.11	0.00	0.00	1.65
time (sec)	N/A	0.119	1.635	4.153	0.000	0.448	0.000	0.000	12.982

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	0	0	41	0	5502	38
normalized size	1	1.00	1.00	0.00	0.00	1.17	0.00	157.20	1.09
time (sec)	N/A	0.094	0.386	1.659	0.000	0.446	0.000	64.011	12.848

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	147	493	0	804	0	371	3718
normalized size	1	1.00	0.96	3.22	0.00	5.25	0.00	2.42	24.30
time (sec)	N/A	0.393	0.879	0.372	0.000	0.522	0.000	0.177	17.182

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	131	342	336	123	853	184	454
normalized size	1	1.00	0.72	1.88	1.85	0.68	4.69	1.01	2.49
time (sec)	N/A	0.295	0.983	0.586	0.602	0.439	7.882	0.172	14.833

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	95	208	200	102	486	145	389
normalized size	1	1.00	0.67	1.46	1.41	0.72	3.42	1.02	2.74
time (sec)	N/A	0.250	0.843	0.485	0.328	0.436	4.093	0.179	13.800

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	105	74	185	179	82	396	114	345
normalized size	1	1.08	0.76	1.91	1.85	0.85	4.08	1.18	3.56
time (sec)	N/A	0.186	0.675	0.391	0.334	0.447	2.126	0.143	13.381

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	48	74	73	43	138	58	122
normalized size	1	1.00	0.98	1.51	1.49	0.88	2.82	1.18	2.49
time (sec)	N/A	0.083	0.156	0.287	0.383	0.449	0.788	0.145	14.329

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	125	113	265	116	828	124	111
normalized size	1	1.00	2.23	2.02	4.73	2.07	14.79	2.21	1.98
time (sec)	N/A	0.169	0.928	0.407	0.588	0.424	4.034	0.154	12.636

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	160	160	456	162	700	92	132
normalized size	1	1.00	2.22	2.22	6.33	2.25	9.72	1.28	1.83
time (sec)	N/A	0.225	0.625	0.438	0.435	0.440	8.108	0.149	12.506

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	147	115	737	183	1035	139	172
normalized size	1	1.00	1.41	1.11	7.09	1.76	9.95	1.34	1.65
time (sec)	N/A	0.237	0.736	0.481	0.533	0.414	15.781	0.183	12.981

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	174	159	1080	251	1831	187	228
normalized size	1	1.00	1.23	1.12	7.61	1.77	12.89	1.32	1.61
time (sec)	N/A	0.285	0.902	0.467	0.386	0.423	29.472	0.199	13.223

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	200	203	1425	305	3232	267	310
normalized size	1	1.00	1.14	1.15	8.10	1.73	18.36	1.52	1.76
time (sec)	N/A	0.307	0.888	0.513	0.398	0.429	52.384	0.211	13.345

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	219	569	571	158	1586	278	661
normalized size	1	1.00	0.96	2.48	2.49	0.69	6.93	1.21	2.89
time (sec)	N/A	0.368	2.043	0.670	0.543	0.462	25.002	0.225	15.132

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	163	463	460	135	1210	244	553
normalized size	1	1.00	0.86	2.45	2.43	0.71	6.40	1.29	2.93
time (sec)	N/A	0.296	1.587	0.574	0.341	0.449	15.016	0.214	14.888

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	137	365	360	114	910	208	542
normalized size	1	1.00	0.93	2.48	2.45	0.78	6.19	1.41	3.69
time (sec)	N/A	0.216	1.085	0.571	0.468	0.442	9.139	0.183	14.154

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	54	166	164	75	372	118	238
normalized size	1	1.00	0.61	1.87	1.84	0.84	4.18	1.33	2.67
time (sec)	N/A	0.137	0.149	0.480	0.490	0.424	3.697	0.153	14.205

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	67	186	179	77	396	111	339
normalized size	1	1.00	0.68	1.90	1.83	0.79	4.04	1.13	3.46
time (sec)	N/A	0.149	0.796	0.388	0.365	0.433	2.434	0.147	13.705

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	191	299	624	177	2365	163	244
normalized size	1	1.00	1.63	2.56	5.33	1.51	20.21	1.39	2.09
time (sec)	N/A	0.289	1.299	0.431	0.484	0.429	8.092	0.177	14.701

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	238	198	839	237	2474	135	246
normalized size	1	1.00	2.18	1.82	7.70	2.17	22.70	1.24	2.26
time (sec)	N/A	0.284	0.631	0.460	0.441	0.441	16.123	0.160	14.150

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	278	249	1139	277	1647	159	233
normalized size	1	1.00	2.48	2.22	10.17	2.47	14.71	1.42	2.08
time (sec)	N/A	0.277	0.736	0.505	0.628	0.443	26.923	0.181	14.679

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	191	161	1571	263	2008	229	269
normalized size	1	1.00	2.55	2.15	20.95	3.51	26.77	3.05	3.59
time (sec)	N/A	0.229	0.950	0.474	0.612	0.428	45.068	0.193	13.351

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	261	205	2087	335	3262	301	331
normalized size	1	1.00	2.27	1.78	18.15	2.91	28.37	2.62	2.88
time (sec)	N/A	0.286	1.242	0.526	0.422	0.414	78.088	0.231	13.358

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	285	249	2604	407	4816	373	423
normalized size	1	1.00	1.83	1.60	16.69	2.61	30.87	2.39	2.71
time (sec)	N/A	0.374	1.599	0.513	0.455	0.422	125.915	0.250	13.544

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	313	293	3120	475	0	445	500
normalized size	1	1.00	1.59	1.49	15.84	2.41	0.00	2.26	2.54
time (sec)	N/A	0.465	3.646	0.573	0.689	0.444	0.000	0.325	14.053

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	255	651	661	181	1948	347	812
normalized size	1	1.00	0.96	2.46	2.49	0.68	7.35	1.31	3.06
time (sec)	N/A	0.391	4.308	0.853	0.359	0.502	52.762	0.274	14.869

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	232	611	617	158	1753	301	705
normalized size	1	1.00	1.05	2.75	2.78	0.71	7.90	1.36	3.18
time (sec)	N/A	0.322	2.571	0.763	0.347	0.503	37.384	0.254	14.929

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	209	568	571	137	1579	273	661
normalized size	1	1.00	1.15	3.14	3.15	0.76	8.72	1.51	3.65
time (sec)	N/A	0.234	1.951	0.674	0.356	0.467	23.432	0.225	14.782

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	64	263	264	92	682	162	325
normalized size	1	1.00	0.55	2.25	2.26	0.79	5.83	1.38	2.78
time (sec)	N/A	0.148	0.229	0.666	0.383	0.454	10.351	0.186	14.288

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	133	364	360	106	910	204	536
normalized size	1	1.00	0.96	2.64	2.61	0.77	6.59	1.48	3.88
time (sec)	N/A	0.200	1.086	0.576	0.344	0.439	8.101	0.190	14.166

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	95	208	200	100	486	145	390
normalized size	1	1.00	0.68	1.49	1.43	0.71	3.47	1.04	2.79
time (sec)	N/A	0.222	0.854	0.481	0.524	0.433	4.211	0.212	13.664

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	223	449	1139	218	4255	234	323
normalized size	1	1.00	1.43	2.88	7.30	1.40	27.28	1.50	2.07
time (sec)	N/A	0.310	1.424	0.443	0.469	0.437	15.993	0.219	14.120

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	280	399	1386	286	4665	233	341
normalized size	1	1.00	1.72	2.45	8.50	1.75	28.62	1.43	2.09
time (sec)	N/A	0.348	0.877	0.470	0.608	0.451	29.894	0.181	14.005

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	316	323	1685	337	4665	226	336
normalized size	1	1.00	2.07	2.11	11.01	2.20	30.49	1.48	2.20
time (sec)	N/A	0.342	1.105	0.509	0.702	0.445	48.977	0.201	14.002

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	356	374	2118	363	2951	213	316
normalized size	1	1.00	2.36	2.48	14.03	2.40	19.54	1.41	2.09
time (sec)	N/A	0.330	1.212	0.485	0.581	0.442	76.726	0.239	15.978

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	283	205	2701	331	3262	301	346
normalized size	1	1.00	3.68	2.66	35.08	4.30	42.36	3.91	4.49
time (sec)	N/A	0.236	2.516	0.527	0.537	0.426	116.395	0.269	13.392

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	313	249	3390	405	0	373	408
normalized size	1	1.00	2.65	2.11	28.73	3.43	0.00	3.16	3.46
time (sec)	N/A	0.289	2.939	0.519	0.529	0.448	0.000	0.312	13.544

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	339	293	4078	475	0	445	500
normalized size	1	1.00	2.17	1.88	26.14	3.04	0.00	2.85	3.21
time (sec)	N/A	0.375	5.285	0.571	0.572	0.452	0.000	0.364	13.794

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	378	337	4765	541	0	517	577
normalized size	1	1.00	1.92	1.71	24.19	2.75	0.00	2.62	2.93
time (sec)	N/A	0.444	6.694	0.576	0.683	0.455	0.000	0.388	13.963

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	274	678	1796	261	6690	343	397
normalized size	1	1.00	1.44	3.57	9.45	1.37	35.21	1.81	2.09
time (sec)	N/A	0.363	2.362	0.429	0.644	0.461	26.866	0.202	14.778

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	220	449	1120	218	4255	235	319
normalized size	1	1.00	1.40	2.86	7.13	1.39	27.10	1.50	2.03
time (sec)	N/A	0.318	1.398	0.410	0.456	0.458	16.310	0.164	14.012

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	188	299	608	179	2365	164	241
normalized size	1	1.00	1.59	2.53	5.15	1.52	20.04	1.39	2.04
time (sec)	N/A	0.278	1.330	0.396	0.428	0.445	8.105	0.157	14.577

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	127	113	256	117	828	122	110
normalized size	1	1.00	2.23	1.98	4.49	2.05	14.53	2.14	1.93
time (sec)	N/A	0.155	0.586	0.379	0.549	0.434	3.945	0.145	12.941

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	57	35	28	83	41	39
normalized size	1	1.00	1.00	1.63	1.00	0.80	2.37	1.17	1.11
time (sec)	N/A	0.136	0.028	0.369	0.321	0.414	2.532	0.176	12.554

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	108	93	266	73	578	102	118
normalized size	1	1.00	1.71	1.48	4.22	1.16	9.17	1.62	1.87
time (sec)	N/A	0.202	0.592	0.448	0.627	0.416	7.815	0.287	12.323

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	157	145	423	107	1236	177	178
normalized size	1	1.00	1.54	1.42	4.15	1.05	12.12	1.74	1.75
time (sec)	N/A	0.257	0.890	0.546	0.638	0.420	15.149	0.260	12.469

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	240	189	619	141	2468	237	239
normalized size	1	1.00	1.69	1.33	4.36	0.99	17.38	1.67	1.68
time (sec)	N/A	0.307	1.141	0.490	0.350	0.425	28.583	0.200	13.233

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	354	778	2982	370	10608	412	500
normalized size	1	1.00	1.48	3.24	12.42	1.54	44.20	1.72	2.08
time (sec)	N/A	0.410	2.066	0.515	0.758	0.464	68.938	0.259	14.825

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	311	549	2094	322	7337	369	414
normalized size	1	1.00	1.73	3.05	11.63	1.79	40.76	2.05	2.30
time (sec)	N/A	0.362	1.319	0.464	0.522	0.436	44.148	0.225	14.952

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	274	399	1378	291	4665	233	336
normalized size	1	1.00	1.69	2.46	8.51	1.80	28.80	1.44	2.07
time (sec)	N/A	0.332	0.883	0.453	0.482	0.447	25.785	0.202	14.341

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	234	198	833	242	2474	136	242
normalized size	1	1.00	2.17	1.83	7.71	2.24	22.91	1.26	2.24
time (sec)	N/A	0.277	0.596	0.426	0.461	0.447	15.057	0.192	14.527

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	156	160	452	166	702	92	133
normalized size	1	1.00	2.17	2.22	6.28	2.31	9.75	1.28	1.85
time (sec)	N/A	0.207	0.584	0.415	0.457	0.436	7.386	0.160	12.826

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	110	97	265	69	578	102	117
normalized size	1	1.00	1.77	1.56	4.27	1.11	9.32	1.65	1.89
time (sec)	N/A	0.197	0.512	0.444	0.349	0.408	7.228	0.176	12.286

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	53	145	47	41	469	87	82
normalized size	1	1.00	0.85	2.34	0.76	0.66	7.56	1.40	1.32
time (sec)	N/A	0.140	0.125	0.352	0.386	0.420	7.631	0.196	12.383

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	237	183	651	116	2674	235	183
normalized size	1	1.00	2.55	1.97	7.00	1.25	28.75	2.53	1.97
time (sec)	N/A	0.220	1.015	0.498	0.472	0.424	27.731	0.198	12.450

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	285	233	835	151	4228	295	197
normalized size	1	1.00	2.11	1.73	6.19	1.12	31.32	2.19	1.46
time (sec)	N/A	0.270	0.964	0.502	0.368	0.434	49.831	0.230	12.749

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	329	277	998	187	5868	355	337
normalized size	1	1.00	1.88	1.58	5.70	1.07	33.53	2.03	1.93
time (sec)	N/A	0.325	1.155	0.533	0.571	0.434	90.732	0.249	12.883

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	388	649	3282	429	10608	376	501
normalized size	1	1.00	1.60	2.67	13.51	1.77	43.65	1.55	2.06
time (sec)	N/A	0.412	2.502	0.521	0.566	0.461	112.095	0.284	14.754

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	348	474	2394	392	7337	305	419
normalized size	1	1.00	1.73	2.36	11.91	1.95	36.50	1.52	2.08
time (sec)	N/A	0.392	1.606	0.495	0.520	0.456	74.016	0.259	14.711

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	308	323	1679	338	4665	226	333
normalized size	1	1.00	2.01	2.11	10.97	2.21	30.49	1.48	2.18
time (sec)	N/A	0.331	1.092	0.481	0.522	0.441	46.582	0.210	14.365

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	272	249	1134	279	1647	159	230
normalized size	1	1.00	2.47	2.26	10.31	2.54	14.97	1.45	2.09
time (sec)	N/A	0.265	0.698	0.467	0.468	0.422	26.026	0.209	15.081

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	139	115	733	191	1035	138	172
normalized size	1	1.00	1.35	1.12	7.12	1.85	10.05	1.34	1.67
time (sec)	N/A	0.225	0.824	0.453	0.354	0.418	13.789	0.169	13.026

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	156	145	423	106	1236	175	178
normalized size	1	1.00	1.53	1.42	4.15	1.04	12.12	1.72	1.75
time (sec)	N/A	0.249	0.839	0.515	0.420	0.410	14.445	0.178	12.429

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	237	185	650	108	2674	235	183
normalized size	1	1.00	2.63	2.06	7.22	1.20	29.71	2.61	2.03
time (sec)	N/A	0.204	1.051	0.463	0.374	0.424	27.389	0.203	12.469

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	65	227	60	56	1098	134	126
normalized size	1	1.00	0.77	2.70	0.71	0.67	13.07	1.60	1.50
time (sec)	N/A	0.152	0.206	0.421	0.363	0.410	20.934	0.354	14.421

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	325	271	1019	150	6135	355	217
normalized size	1	1.00	2.69	2.24	8.42	1.24	50.70	2.93	1.79
time (sec)	N/A	0.223	1.142	0.511	0.408	0.435	92.701	0.269	13.158

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	373	321	1201	185	8396	415	231
normalized size	1	1.00	2.30	1.98	7.41	1.14	51.83	2.56	1.43
time (sec)	N/A	0.289	1.414	0.562	0.566	0.437	155.471	0.268	13.139

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	401	365	1387	221	0	475	474
normalized size	1	1.00	1.96	1.78	6.77	1.08	0.00	2.32	2.31
time (sec)	N/A	0.345	3.548	0.509	0.577	0.457	0.000	0.304	14.522

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	149	119	0	287	0	0	-1
normalized size	1	1.00	0.75	0.60	0.00	1.45	0.00	0.00	-0.01
time (sec)	N/A	0.487	2.988	1.301	0.000	0.436	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	123	103	0	243	0	0	-1
normalized size	1	1.00	0.78	0.66	0.00	1.55	0.00	0.00	-0.01
time (sec)	N/A	0.412	1.518	1.230	0.000	0.432	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	104	81	0	184	0	0	-1
normalized size	1	1.00	0.90	0.70	0.00	1.59	0.00	0.00	-0.01
time (sec)	N/A	0.320	0.958	1.109	0.000	0.424	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	87	63	0	130	0	0	-1
normalized size	1	1.00	1.19	0.86	0.00	1.78	0.00	0.00	-0.01
time (sec)	N/A	0.240	0.422	1.310	0.000	0.422	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	166	159	0	254	0	0	-1
normalized size	1	1.00	1.36	1.30	0.00	2.08	0.00	0.00	-0.01
time (sec)	N/A	0.336	1.264	1.486	0.000	0.442	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	157	227	0	318	0	0	-1
normalized size	1	1.00	1.37	1.97	0.00	2.77	0.00	0.00	-0.01
time (sec)	N/A	0.318	1.606	1.239	0.000	0.449	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	199	268	0	394	0	0	-1
normalized size	1	1.00	1.58	2.13	0.00	3.13	0.00	0.00	-0.01
time (sec)	N/A	0.335	2.240	1.607	0.000	0.461	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	217	352	0	490	0	0	-1
normalized size	1	1.00	1.33	2.16	0.00	3.01	0.00	0.00	-0.01
time (sec)	N/A	0.373	3.451	1.733	0.000	0.455	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	1355	121	0	358	0	0	-1
normalized size	1	1.00	6.45	0.58	0.00	1.70	0.00	0.00	-0.00
time (sec)	N/A	0.554	6.642	1.157	0.000	0.449	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	1173	105	0	313	0	0	-1
normalized size	1	1.00	7.02	0.63	0.00	1.87	0.00	0.00	-0.01
time (sec)	N/A	0.453	6.515	1.258	0.000	0.444	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	106	83	0	228	0	0	-1
normalized size	1	1.00	0.88	0.69	0.00	1.90	0.00	0.00	-0.01
time (sec)	N/A	0.387	4.594	1.269	0.000	0.430	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	89	65	0	193	0	0	-1
normalized size	1	1.00	1.10	0.80	0.00	2.38	0.00	0.00	-0.01
time (sec)	N/A	0.330	0.582	1.295	0.000	0.440	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	175	197	0	310	0	0	-1
normalized size	1	1.00	1.09	1.22	0.00	1.93	0.00	0.00	-0.01
time (sec)	N/A	0.442	1.182	1.489	0.000	0.446	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	355	282	0	385	0	0	-1
normalized size	1	1.00	2.02	1.60	0.00	2.19	0.00	0.00	-0.01
time (sec)	N/A	0.481	0.896	1.465	0.000	0.451	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	344	386	0	449	0	0	-1
normalized size	1	1.00	1.97	2.21	0.00	2.57	0.00	0.00	-0.01
time (sec)	N/A	0.478	1.170	1.599	0.000	0.453	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	342	354	0	521	0	0	-1
normalized size	1	1.00	1.95	2.02	0.00	2.98	0.00	0.00	-0.01
time (sec)	N/A	0.491	1.771	1.690	0.000	0.464	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	357	440	0	654	0	0	-1
normalized size	1	1.00	1.61	1.98	0.00	2.95	0.00	0.00	-0.00
time (sec)	N/A	0.511	2.687	1.697	0.000	0.489	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	1569	121	0	405	0	0	-1
normalized size	1	1.00	7.47	0.58	0.00	1.93	0.00	0.00	-0.00
time (sec)	N/A	0.534	6.834	1.369	0.000	0.465	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	1351	105	0	334	0	0	-1
normalized size	1	1.00	8.39	0.65	0.00	2.07	0.00	0.00	-0.01
time (sec)	N/A	0.472	6.714	1.341	0.000	0.440	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	1157	83	0	287	0	0	-1
normalized size	1	1.00	9.33	0.67	0.00	2.31	0.00	0.00	-0.01
time (sec)	N/A	0.407	6.505	1.290	0.000	0.455	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	89	65	0	232	0	0	-1
normalized size	1	1.00	1.10	0.80	0.00	2.86	0.00	0.00	-0.01
time (sec)	N/A	0.305	1.014	1.240	0.000	0.441	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	193	233	0	353	0	0	-1
normalized size	1	1.00	0.96	1.16	0.00	1.76	0.00	0.00	-0.00
time (sec)	N/A	0.521	1.348	1.556	0.000	0.449	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	444	354	0	430	0	0	-1
normalized size	1	1.00	2.04	1.62	0.00	1.97	0.00	0.00	-0.00
time (sec)	N/A	0.546	1.690	1.470	0.000	0.455	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	434	434	0	505	0	0	-1
normalized size	1	1.00	1.93	1.93	0.00	2.24	0.00	0.00	-0.00
time (sec)	N/A	0.549	2.209	1.986	0.000	0.461	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	422	524	0	554	0	0	-1
normalized size	1	1.00	1.94	2.41	0.00	2.55	0.00	0.00	-0.00
time (sec)	N/A	0.549	3.093	1.985	0.000	0.471	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	355	432	0	633	0	0	-1
normalized size	1	1.00	1.64	1.99	0.00	2.92	0.00	0.00	-0.00
time (sec)	N/A	0.557	4.377	1.882	0.000	0.485	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	485	526	0	760	0	0	-1
normalized size	1	1.00	1.82	1.98	0.00	2.86	0.00	0.00	-0.00
time (sec)	N/A	0.587	6.833	2.047	0.000	0.489	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	157	111	478	115	0	0	-1
normalized size	1	1.00	0.78	0.56	2.39	0.58	0.00	0.00	-0.00
time (sec)	N/A	0.385	5.590	1.168	0.497	0.440	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	134	95	386	95	0	0	-1
normalized size	1	1.00	0.84	0.60	2.43	0.60	0.00	0.00	-0.01
time (sec)	N/A	0.350	1.758	1.100	0.463	0.432	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	113	73	294	67	0	0	-1
normalized size	1	1.00	0.96	0.62	2.49	0.57	0.00	0.00	-0.01
time (sec)	N/A	0.317	0.601	1.120	0.463	0.427	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	44	53	174	44	0	0	128
normalized size	1	1.00	0.60	0.73	2.38	0.60	0.00	0.00	1.75
time (sec)	N/A	0.275	0.203	1.019	0.452	0.426	0.000	0.000	13.150

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	140	130	0	162	0	0	-1
normalized size	1	1.00	1.54	1.43	0.00	1.78	0.00	0.00	-0.01
time (sec)	N/A	0.283	0.450	1.294	0.000	0.433	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	284	225	0	231	0	0	-1
normalized size	1	1.00	2.09	1.65	0.00	1.70	0.00	0.00	-0.01
time (sec)	N/A	0.330	0.542	1.280	0.000	0.452	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	404	350	0	282	0	0	-1
normalized size	1	1.00	2.24	1.94	0.00	1.57	0.00	0.00	-0.01
time (sec)	N/A	0.425	0.853	1.783	0.000	0.442	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	953	143	762	153	0	0	-1
normalized size	1	1.00	3.94	0.59	3.15	0.63	0.00	0.00	-0.00
time (sec)	N/A	0.651	6.819	1.360	0.462	0.436	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	159	121	670	133	0	0	-1
normalized size	1	1.00	0.79	0.60	3.33	0.66	0.00	0.00	-0.00
time (sec)	N/A	0.558	2.827	1.507	0.529	0.432	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	130	105	577	110	0	0	-1
normalized size	1	1.00	0.84	0.68	3.75	0.71	0.00	0.00	-0.01
time (sec)	N/A	0.479	1.162	1.267	0.461	0.439	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	113	81	482	80	0	0	492
normalized size	1	1.00	0.98	0.70	4.19	0.70	0.00	0.00	4.28
time (sec)	N/A	0.408	0.657	1.416	0.581	0.430	0.000	0.000	17.178

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	87	63	343	60	0	0	137
normalized size	1	1.00	1.12	0.81	4.40	0.77	0.00	0.00	1.76
time (sec)	N/A	0.313	0.282	1.308	0.457	0.426	0.000	0.000	17.508

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	176	168	0	217	0	0	-1
normalized size	1	1.00	1.30	1.24	0.00	1.61	0.00	0.00	-0.01
time (sec)	N/A	0.354	0.509	1.780	0.000	0.445	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	300	258	0	206	0	0	-1
normalized size	1	1.00	1.71	1.47	0.00	1.18	0.00	0.00	-0.01
time (sec)	N/A	0.392	0.860	1.555	0.000	0.446	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	430	426	0	279	0	0	-1
normalized size	1	1.00	1.91	1.89	0.00	1.24	0.00	0.00	-0.00
time (sec)	N/A	0.484	1.411	1.735	0.000	0.462	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	176	143	945	166	0	0	-1
normalized size	1	1.00	0.73	0.59	3.90	0.69	0.00	0.00	-0.00
time (sec)	N/A	0.647	4.186	1.471	0.493	0.448	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	158	121	854	148	0	0	-1
normalized size	1	1.00	0.76	0.58	4.09	0.71	0.00	0.00	-0.00
time (sec)	N/A	0.567	2.632	1.600	1.255	0.441	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	132	105	761	125	0	0	904
normalized size	1	1.00	0.82	0.66	4.76	0.78	0.00	0.00	5.65
time (sec)	N/A	0.480	1.215	1.602	0.483	0.441	0.000	0.000	22.785

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	113	83	663	95	0	0	683
normalized size	1	1.00	0.93	0.69	5.48	0.79	0.00	0.00	5.64
time (sec)	N/A	0.413	0.696	1.464	0.476	0.428	0.000	0.000	19.157

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	89	65	505	77	0	0	479
normalized size	1	1.00	1.05	0.76	5.94	0.91	0.00	0.00	5.64
time (sec)	N/A	0.315	0.303	1.407	0.508	0.445	0.000	0.000	17.535

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	204	200	0	262	0	0	-1
normalized size	1	1.00	1.17	1.15	0.00	1.51	0.00	0.00	-0.01
time (sec)	N/A	0.438	0.772	1.941	0.000	0.442	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	357	308	0	277	0	0	-1
normalized size	1	1.00	1.59	1.38	0.00	1.24	0.00	0.00	-0.00
time (sec)	N/A	0.480	1.362	1.689	0.000	0.457	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	479	410	0	238	0	0	-1
normalized size	1	1.00	1.86	1.59	0.00	0.92	0.00	0.00	-0.00
time (sec)	N/A	0.555	2.286	2.058	0.000	0.486	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	118	174	0	140	0	0	173
normalized size	1	1.00	1.26	1.85	0.00	1.49	0.00	0.00	1.84
time (sec)	N/A	0.339	1.002	0.873	0.000	0.455	0.000	0.000	16.166

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	102	129	0	121	0	0	149
normalized size	1	1.00	1.09	1.37	0.00	1.29	0.00	0.00	1.59
time (sec)	N/A	0.337	0.821	0.853	0.000	0.438	0.000	0.000	14.959

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	84	91	0	92	0	0	122
normalized size	1	1.00	0.89	0.97	0.00	0.98	0.00	0.00	1.30
time (sec)	N/A	0.333	0.556	0.825	0.000	0.445	0.000	0.000	1.787

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	63	57	0	61	0	0	75
normalized size	1	1.00	0.68	0.62	0.00	0.66	0.00	0.00	0.82
time (sec)	N/A	0.309	0.175	0.803	0.000	0.436	0.000	0.000	0.938

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	120	395	175	0	0	0	-1
normalized size	1	1.00	1.20	3.95	1.75	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.337	1.133	0.728	0.551	0.834	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	147	403	0	0	0	0	-1
normalized size	1	1.00	1.48	4.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.358	1.157	0.689	0.000	1.174	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	101	137	0	87	0	0	-1
normalized size	1	1.00	1.10	1.49	0.00	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.345	0.539	0.710	0.000	0.428	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	103	205	0	106	0	0	153
normalized size	1	1.00	1.10	2.18	0.00	1.13	0.00	0.00	1.63
time (sec)	N/A	0.337	0.596	0.723	0.000	0.438	0.000	0.000	17.626

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	205	185	0	148	0	0	323
normalized size	1	1.00	1.40	1.27	0.00	1.01	0.00	0.00	2.21
time (sec)	N/A	0.358	1.486	0.842	0.000	0.450	0.000	0.000	17.532

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	172	147	0	126	0	0	174
normalized size	1	1.00	1.18	1.01	0.00	0.86	0.00	0.00	1.19
time (sec)	N/A	0.361	1.612	0.790	0.000	0.454	0.000	0.000	16.567

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	96	86	0	83	0	0	103
normalized size	1	1.00	0.72	0.64	0.00	0.62	0.00	0.00	0.77
time (sec)	N/A	0.348	0.748	0.748	0.000	0.443	0.000	0.000	1.842

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	81	91	0	87	0	0	122
normalized size	1	1.00	0.84	0.95	0.00	0.91	0.00	0.00	1.27
time (sec)	N/A	0.325	0.556	0.798	0.000	0.447	0.000	0.000	14.292

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	136	494	0	0	0	0	-1
normalized size	1	1.00	0.94	3.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.382	0.685	0.699	0.000	2.818	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	210	748	366	0	0	0	-1
normalized size	1	1.00	1.33	4.73	2.32	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.385	0.861	0.672	0.571	6.007	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	198	594	0	0	0	0	-1
normalized size	1	1.00	1.33	3.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.387	0.945	0.662	0.000	2.949	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	125	223	0	125	0	0	-1
normalized size	1	1.00	1.30	2.32	0.00	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.275	0.989	0.659	0.000	0.447	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	123	217	0	134	0	0	245
normalized size	1	1.00	0.84	1.49	0.00	0.92	0.00	0.00	1.68
time (sec)	N/A	0.376	1.352	0.678	0.000	0.469	0.000	0.000	18.994

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	126	339	0	155	0	0	279
normalized size	1	1.00	0.82	2.20	0.00	1.01	0.00	0.00	1.81
time (sec)	N/A	0.373	1.954	0.689	0.000	0.458	0.000	0.000	20.227

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	223	203	0	160	0	0	383
normalized size	1	1.00	1.13	1.03	0.00	0.81	0.00	0.00	1.93
time (sec)	N/A	0.476	2.649	0.901	0.000	0.477	0.000	0.000	18.173

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	113	114	0	117	0	0	131
normalized size	1	1.00	0.63	0.63	0.00	0.65	0.00	0.00	0.73
time (sec)	N/A	0.466	0.799	0.769	0.000	0.460	0.000	0.000	16.003

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	165	147	0	119	0	0	174
normalized size	1	1.00	1.16	1.04	0.00	0.84	0.00	0.00	1.23
time (sec)	N/A	0.362	1.802	0.794	0.000	0.458	0.000	0.000	16.324

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	102	129	0	117	0	0	149
normalized size	1	1.00	1.06	1.34	0.00	1.22	0.00	0.00	1.55
time (sec)	N/A	0.318	0.832	0.804	0.000	0.445	0.000	0.000	2.701

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	177	590	0	0	0	0	-1
normalized size	1	1.00	0.92	3.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.463	1.498	0.724	0.000	4.601	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	231	846	0	0	0	0	-1
normalized size	1	1.00	1.10	4.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.485	1.645	0.638	0.000	10.487	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	207	1092	506	0	0	0	-1
normalized size	1	1.00	0.98	5.15	2.39	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.490	1.165	0.648	0.480	1.107	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	204	832	0	0	0	0	-1
normalized size	1	1.00	1.04	4.24	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.488	1.191	0.688	0.000	6.120	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	145	309	0	165	0	0	-1
normalized size	1	1.00	1.51	3.22	0.00	1.72	0.00	0.00	-0.01
time (sec)	N/A	0.276	2.964	0.678	0.000	0.452	0.000	0.000	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	146	368	0	182	0	0	341
normalized size	1	1.00	1.00	2.52	0.00	1.25	0.00	0.00	2.34
time (sec)	N/A	0.376	4.080	0.697	0.000	0.458	0.000	0.000	20.756

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	144	423	0	196	0	0	357
normalized size	1	1.00	0.73	2.16	0.00	1.00	0.00	0.00	1.82
time (sec)	N/A	0.483	5.616	0.718	0.000	0.472	0.000	0.000	20.715

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	269	259	0	183	0	0	482
normalized size	1	1.00	1.08	1.04	0.00	0.73	0.00	0.00	1.93
time (sec)	N/A	0.567	6.850	0.826	0.000	0.528	0.000	0.000	20.031

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	135	142	0	134	0	0	384
normalized size	1	1.00	0.60	0.63	0.00	0.59	0.00	0.00	1.70
time (sec)	N/A	0.559	1.583	0.839	0.000	0.484	0.000	0.000	17.653

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	232	203	0	150	0	0	383
normalized size	1	1.00	1.21	1.06	0.00	0.78	0.00	0.00	1.99
time (sec)	N/A	0.460	2.073	0.850	0.000	0.484	0.000	0.000	18.428

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	212	185	0	142	0	0	321
normalized size	1	1.00	1.49	1.30	0.00	1.00	0.00	0.00	2.26
time (sec)	N/A	0.359	1.852	0.803	0.000	0.460	0.000	0.000	18.169

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	121	174	0	139	0	0	173
normalized size	1	1.00	1.26	1.81	0.00	1.45	0.00	0.00	1.80
time (sec)	N/A	0.325	0.965	0.824	0.000	0.453	0.000	0.000	16.679

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	183	671	0	0	0	0	-1
normalized size	1	1.00	0.77	2.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.569	2.836	0.688	0.000	0.961	0.000	0.000	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	292	927	0	0	0	0	-1
normalized size	1	1.00	1.08	3.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.593	3.496	0.646	0.000	1.168	0.000	0.000	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	251	1191	0	0	0	0	-1
normalized size	1	1.00	0.95	4.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.595	2.573	0.646	0.000	25.359	0.000	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	244	1455	749	0	0	0	-1
normalized size	1	1.00	0.92	5.51	2.84	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.609	2.900	0.661	1.185	44.324	0.000	0.000	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	238	1019	0	0	0	0	-1
normalized size	1	1.00	0.96	4.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.599	2.685	0.717	0.000	0.967	0.000	0.000	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	434	389	0	199	0	0	-1
normalized size	1	1.00	4.52	4.05	0.00	2.07	0.00	0.00	-0.01
time (sec)	N/A	0.272	6.945	0.701	0.000	0.484	0.000	0.000	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	442	393	0	214	0	0	406
normalized size	1	1.00	3.03	2.69	0.00	1.47	0.00	0.00	2.78
time (sec)	N/A	0.379	6.989	0.730	0.000	0.499	0.000	0.000	22.848

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	442	505	0	234	0	0	827
normalized size	1	1.00	2.19	2.50	0.00	1.16	0.00	0.00	4.09
time (sec)	N/A	0.491	7.106	0.765	0.000	0.537	0.000	0.000	25.256

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	436	560	0	243	0	0	841
normalized size	1	1.00	1.77	2.28	0.00	0.99	0.00	0.00	3.42
time (sec)	N/A	0.591	7.143	0.810	0.000	0.541	0.000	0.000	28.344

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	185	595	0	0	0	0	-1
normalized size	1	1.00	0.94	3.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.463	1.306	0.717	0.000	4.083	0.000	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	146	501	0	0	0	0	-1
normalized size	1	1.00	1.00	3.43	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.365	0.664	0.676	0.000	2.823	0.000	0.000	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	119	399	176	0	0	0	-1
normalized size	1	1.00	1.24	4.16	1.83	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.322	1.130	0.680	0.795	1.362	0.000	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	97	165	0	0	0	0	-1
normalized size	1	1.00	0.86	1.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.364	0.337	0.592	0.000	1.412	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	191	302	0	337	0	0	-1
normalized size	1	1.00	1.85	2.93	0.00	3.27	0.00	0.00	-0.01
time (sec)	N/A	0.252	0.543	0.595	0.000	0.518	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	222	465	0	424	0	0	-1
normalized size	1	1.00	1.45	3.04	0.00	2.77	0.00	0.00	-0.01
time (sec)	N/A	0.355	0.631	0.618	0.000	0.536	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	271	939	0	0	0	0	-1
normalized size	1	1.00	1.00	3.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.577	3.475	0.657	0.000	14.903	0.000	0.000	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	212	853	0	0	0	0	-1
normalized size	1	1.00	1.01	4.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.478	1.604	0.627	0.000	8.938	0.000	0.000	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	190	759	367	0	0	0	-1
normalized size	1	1.00	1.19	4.77	2.31	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.386	0.864	0.642	0.548	0.945	0.000	0.000	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	143	408	0	0	0	0	-1
normalized size	1	1.00	1.43	4.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.346	1.176	0.651	0.000	1.068	0.000	0.000	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	186	303	0	329	0	0	-1
normalized size	1	1.00	1.81	2.94	0.00	3.19	0.00	0.00	-0.01
time (sec)	N/A	0.256	0.562	0.604	0.000	0.515	0.000	0.000	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	178	130	0	272	0	0	-1
normalized size	1	1.00	1.19	0.87	0.00	1.81	0.00	0.00	-0.01
time (sec)	N/A	0.373	0.671	0.563	0.000	0.506	0.000	0.000	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	306	429	0	437	0	0	-1
normalized size	1	1.00	1.41	1.98	0.00	2.01	0.00	0.00	-0.00
time (sec)	N/A	0.479	0.947	0.553	0.000	0.533	0.000	0.000	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	573	1287	0	0	0	0	-1
normalized size	1	1.00	1.77	3.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.713	7.012	0.675	0.000	45.496	0.000	0.000	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	243	1206	0	0	0	0	-1
normalized size	1	1.00	0.92	4.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.607	2.540	0.640	0.000	1.412	0.000	0.000	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	199	1106	504	0	0	0	-1
normalized size	1	1.00	0.94	5.24	2.39	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.494	1.155	0.631	0.493	1.125	0.000	0.000	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	179	606	0	0	0	0	-1
normalized size	1	1.00	1.20	4.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.392	0.979	0.644	0.000	2.705	0.000	0.000	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	99	135	0	85	0	0	156
normalized size	1	1.00	1.05	1.44	0.00	0.90	0.00	0.00	1.66
time (sec)	N/A	0.333	0.509	0.661	0.000	0.431	0.000	0.000	14.861

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	214	466	0	408	0	0	-1
normalized size	1	1.00	1.42	3.09	0.00	2.70	0.00	0.00	-0.01
time (sec)	N/A	0.354	0.673	0.626	0.000	0.527	0.000	0.000	0.000

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	305	431	0	419	0	0	-1
normalized size	1	1.00	1.47	2.07	0.00	2.01	0.00	0.00	-0.00
time (sec)	N/A	0.480	0.961	0.569	0.000	0.546	0.000	0.000	0.000

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	246	151	0	306	0	0	-1
normalized size	1	1.00	1.00	0.62	0.00	1.25	0.00	0.00	-0.00
time (sec)	N/A	0.570	0.937	0.602	0.000	0.560	0.000	0.000	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	2903	0	0	0	0	0	-1
normalized size	1	1.00	16.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.315	14.238	5.774	0.000	0.483	0.000	0.000	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.342	180.061	8.099	0.000	0.482	0.000	0.000	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.335	180.081	8.309	0.000	0.477	0.000	0.000	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	462	0	0	0	0	0	-1
normalized size	1	1.00	3.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.289	4.220	4.976	0.000	0.458	0.000	0.000	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	275	0	0	0	0	0	-1
normalized size	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	1.832	2.533	0.000	0.435	0.000	0.000	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	7409	0	0	0	0	0	-1
normalized size	1	1.00	60.24	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.303	25.568	1.087	0.000	0.452	0.000	0.000	0.000

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	9240	0	0	0	0	0	-1
normalized size	1	1.00	62.43	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.330	6.934	8.469	0.000	0.454	0.000	0.000	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	12580	0	0	0	0	0	-1
normalized size	1	1.00	85.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.332	7.178	9.297	0.000	0.490	0.000	0.000	0.000

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	200	0	0	0	0	0	-1
normalized size	1	1.00	1.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.294	4.870	0.983	0.000	0.458	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	200	0	0	0	0	0	-1
normalized size	1	1.00	1.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.288	2.383	1.115	0.000	0.462	0.000	0.000	0.000

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	667	0	725	562	0	0	749
normalized size	1	1.00	2.43	0.00	2.64	2.04	0.00	0.00	2.72
time (sec)	N/A	0.503	6.819	1.342	0.546	0.487	0.000	0.000	21.060

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	192	174	0	498	313	0	0	480
normalized size	1	1.16	1.05	0.00	3.00	1.89	0.00	0.00	2.89
time (sec)	N/A	0.354	1.766	1.282	0.533	0.499	0.000	0.000	19.203

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	116	0	323	165	0	0	105
normalized size	1	1.00	1.12	0.00	3.11	1.59	0.00	0.00	1.01
time (sec)	N/A	0.282	0.442	1.280	0.536	0.468	0.000	0.000	1.442

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	200	0	0	0	0	0	-1
normalized size	1	1.00	1.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.282	2.229	0.016	0.000	0.467	0.000	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	3178	0	0	0	0	0	-1
normalized size	1	1.00	23.72	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.300	6.760	1.006	0.000	0.498	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	8147	0	0	0	0	0	-1
normalized size	1	1.00	60.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.332	6.872	1.038	0.000	0.492	0.000	0.000	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	353	0	0	205	0	0	368
normalized size	1	1.00	1.32	0.00	0.00	0.77	0.00	0.00	1.38
time (sec)	N/A	0.427	14.131	5.935	0.000	0.488	0.000	0.000	22.233

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	269	0	0	137	0	0	239
normalized size	1	1.00	1.41	0.00	0.00	0.72	0.00	0.00	1.25
time (sec)	N/A	0.310	10.734	6.181	0.000	0.485	0.000	0.000	15.484

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	211	0	0	89	0	0	134
normalized size	1	1.00	1.85	0.00	0.00	0.78	0.00	0.00	1.18
time (sec)	N/A	0.222	8.790	5.410	0.000	0.480	0.000	0.000	14.158

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	675	0	0	0	0	0	-1
normalized size	1	1.00	4.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.309	11.616	2.870	0.000	0.448	0.000	0.000	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	2552	0	0	0	0	0	-1
normalized size	1	1.00	16.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.268	17.204	4.367	0.000	0.479	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	3601	0	0	0	0	0	-1
normalized size	1	1.00	21.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.329	92.471	3.809	0.000	0.478	0.000	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	5163	0	0	0	0	0	-1
normalized size	1	1.00	29.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.336	49.215	6.070	0.000	0.471	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	63	0	0	78	0	0	64
normalized size	1	1.00	1.85	0.00	0.00	2.29	0.00	0.00	1.88
time (sec)	N/A	0.274	0.540	8.038	0.000	0.480	0.000	0.000	14.447

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	67	0	0	77	0	0	61
normalized size	1	1.00	1.97	0.00	0.00	2.26	0.00	0.00	1.79
time (sec)	N/A	0.238	1.142	8.063	0.000	0.469	0.000	0.000	14.457

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	66	0	0	78	0	0	61
normalized size	1	1.00	2.00	0.00	0.00	2.36	0.00	0.00	1.85
time (sec)	N/A	0.237	1.109	7.862	0.000	0.486	0.000	0.000	14.378

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	61	0	0	77	0	0	64
normalized size	1	1.00	1.74	0.00	0.00	2.20	0.00	0.00	1.83
time (sec)	N/A	0.238	0.540	8.121	0.000	0.479	0.000	0.000	14.363

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	0	0	36	0	0	36
normalized size	1	1.00	1.00	0.00	0.00	1.00	0.00	0.00	1.00
time (sec)	N/A	0.132	0.487	7.658	0.000	0.487	0.000	0.000	13.542

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	37	0	0	37
normalized size	1	1.00	1.00	0.00	0.00	1.00	0.00	0.00	1.00
time (sec)	N/A	0.123	0.475	8.398	0.000	0.496	0.000	0.000	13.497

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	87	158	157	105	440	131	300
normalized size	1	1.00	0.62	1.13	1.12	0.75	3.14	0.94	2.14
time (sec)	N/A	0.186	0.140	0.597	0.429	0.462	8.486	0.171	15.304

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	77	136	138	91	359	113	256
normalized size	1	1.00	0.64	1.12	1.14	0.75	2.97	0.93	2.12
time (sec)	N/A	0.168	0.102	0.497	0.404	0.458	5.693	0.175	15.286

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	55	117	112	77	267	77	292
normalized size	1	1.00	0.57	1.22	1.17	0.80	2.78	0.80	3.04
time (sec)	N/A	0.116	0.476	0.401	0.372	0.442	3.647	0.162	15.030

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	54	89	86	63	196	77	250
normalized size	1	1.00	0.66	1.09	1.05	0.77	2.39	0.94	3.05
time (sec)	N/A	0.106	0.347	0.406	0.389	0.449	1.373	0.168	15.188

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	74	99	85	92	0	107	212
normalized size	1	1.00	0.97	1.30	1.12	1.21	0.00	1.41	2.79
time (sec)	N/A	0.104	0.147	0.433	0.391	0.481	0.000	0.161	13.227

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	77	95	83	111	0	153	226
normalized size	1	1.00	0.97	1.20	1.05	1.41	0.00	1.94	2.86
time (sec)	N/A	0.179	0.190	0.361	0.548	0.493	0.000	0.187	13.194

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	142	94	90	152	0	137	220
normalized size	1	1.00	1.82	1.21	1.15	1.95	0.00	1.76	2.82
time (sec)	N/A	0.121	0.038	0.555	0.405	0.481	0.000	0.195	13.500

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	141	103	117	175	0	150	245
normalized size	1	1.00	1.81	1.32	1.50	2.24	0.00	1.92	3.14
time (sec)	N/A	0.131	0.490	0.526	0.447	0.462	0.000	0.180	13.316

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	210	109	145	166	0	174	244
normalized size	1	1.00	2.44	1.27	1.69	1.93	0.00	2.02	2.84
time (sec)	N/A	0.148	0.069	0.621	0.426	0.447	0.000	0.196	13.116

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	268	132	175	201	0	174	244
normalized size	1	1.00	2.55	1.26	1.67	1.91	0.00	1.66	2.32
time (sec)	N/A	0.234	0.074	0.715	0.440	0.428	0.000	0.185	13.155

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	306	155	207	240	0	242	340
normalized size	1	1.00	2.35	1.19	1.59	1.85	0.00	1.86	2.62
time (sec)	N/A	0.197	0.080	0.732	0.598	0.465	0.000	0.205	13.365

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	254	257	715	248	3614	156	326
normalized size	1	1.00	1.97	1.99	5.54	1.92	28.02	1.21	2.53
time (sec)	N/A	0.208	0.945	0.446	0.589	0.458	78.444	0.214	17.076

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	228	155	543	225	2290	113	261
normalized size	1	1.00	2.21	1.50	5.27	2.18	22.23	1.10	2.53
time (sec)	N/A	0.188	0.801	0.433	0.561	0.449	45.407	0.182	16.929

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	189	131	392	204	1268	93	178
normalized size	1	1.00	2.12	1.47	4.40	2.29	14.25	1.04	2.00
time (sec)	N/A	0.173	0.756	0.428	0.639	0.445	25.572	0.192	14.924

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	107	71	348	156	461	63	110
normalized size	1	1.00	1.30	0.87	4.24	1.90	5.62	0.77	1.34
time (sec)	N/A	0.138	0.462	0.415	0.624	0.425	14.110	0.188	13.448

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	92	86	387	154	573	79	134
normalized size	1	1.00	1.59	1.48	6.67	2.66	9.88	1.36	2.31
time (sec)	N/A	0.114	0.239	0.414	0.532	0.407	9.114	0.171	13.309

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	313	130	433	310	0	99	199
normalized size	1	1.00	3.19	1.33	4.42	3.16	0.00	1.01	2.03
time (sec)	N/A	0.164	1.006	0.729	0.586	0.440	0.000	0.194	14.771

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	167	169	519	406	0	146	210
normalized size	1	1.00	1.48	1.50	4.59	3.59	0.00	1.29	1.86
time (sec)	N/A	0.398	2.974	0.732	0.686	0.451	0.000	0.221	15.774

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	245	209	622	498	0	180	288
normalized size	1	1.00	1.78	1.51	4.51	3.61	0.00	1.30	2.09
time (sec)	N/A	0.225	3.888	0.870	0.410	0.449	0.000	0.299	15.741

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	348	249	706	594	0	213	314
normalized size	1	1.00	2.27	1.63	4.61	3.88	0.00	1.39	2.05
time (sec)	N/A	0.246	6.229	0.826	0.458	0.465	0.000	0.302	15.280

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	267	422	406	239	996	314	830
normalized size	1	1.00	0.82	1.29	1.24	0.73	3.05	0.96	2.54
time (sec)	N/A	0.579	2.000	0.525	0.363	0.469	5.370	0.182	15.576

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	185	274	264	160	571	198	547
normalized size	1	1.00	0.87	1.29	1.24	0.75	2.68	0.93	2.57
time (sec)	N/A	0.360	1.123	0.408	0.371	0.469	2.308	0.154	15.413

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	104	147	143	84	277	101	134
normalized size	1	1.00	0.94	1.32	1.29	0.76	2.50	0.91	1.21
time (sec)	N/A	0.157	0.439	0.304	0.374	0.471	1.002	0.151	13.314

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	45	59	57	43	94	48	100
normalized size	1	1.00	0.94	1.23	1.19	0.90	1.96	1.00	2.08
time (sec)	N/A	0.023	0.098	0.157	0.446	0.435	0.351	0.128	13.263

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	196	294	0	292	0	141	3074
normalized size	1	1.00	2.00	3.00	0.00	2.98	0.00	1.44	31.37
time (sec)	N/A	0.273	0.653	0.404	0.000	0.488	0.000	0.154	16.580

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	217	434	0	655	0	204	5102
normalized size	1	1.00	1.75	3.50	0.00	5.28	0.00	1.65	41.15
time (sec)	N/A	0.325	1.271	0.493	0.000	0.487	0.000	0.196	20.316

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	345	2021	0	967	0	594	554
normalized size	1	1.00	1.96	11.48	0.00	5.49	0.00	3.38	3.15
time (sec)	N/A	0.421	2.711	0.515	0.000	0.528	0.000	0.232	15.589

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	464	464	437	745	724	364	1865	474	1291
normalized size	1	1.00	0.94	1.61	1.56	0.78	4.02	1.02	2.78
time (sec)	N/A	0.952	3.128	0.639	0.465	0.496	11.952	0.217	15.987

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	296	496	478	245	1129	311	765
normalized size	1	1.00	0.88	1.48	1.42	0.73	3.36	0.93	2.28
time (sec)	N/A	0.703	1.567	0.515	0.440	0.456	6.002	0.184	15.705

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	160	278	268	144	571	172	492
normalized size	1	1.00	0.96	1.67	1.61	0.87	3.44	1.04	2.96
time (sec)	N/A	0.271	0.755	0.409	0.420	0.448	2.760	0.173	14.603

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	106	117	114	70	199	88	91
normalized size	1	1.00	1.13	1.24	1.21	0.74	2.12	0.94	0.97
time (sec)	N/A	0.061	0.325	0.297	0.439	0.463	0.949	0.158	13.161

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	177	713	0	452	0	314	7371
normalized size	1	1.00	1.04	4.17	0.00	2.64	0.00	1.84	43.11
time (sec)	N/A	0.522	0.623	0.430	0.000	0.512	0.000	0.193	20.069

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	192	848	0	731	0	498	8706
normalized size	1	1.00	0.97	4.28	0.00	3.69	0.00	2.52	43.97
time (sec)	N/A	0.581	0.996	0.512	0.000	0.575	0.000	0.201	21.578

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	226	1916	0	1483	0	703	8632
normalized size	1	1.00	1.05	8.91	0.00	6.90	0.00	3.27	40.15
time (sec)	N/A	0.623	1.366	0.544	0.000	0.553	0.000	0.242	22.497

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	604	604	528	1077	1056	432	2878	566	1395
normalized size	1	1.00	0.87	1.78	1.75	0.72	4.76	0.94	2.31
time (sec)	N/A	1.486	4.763	0.653	0.627	0.507	22.093	0.252	16.176

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	355	725	704	299	1804	380	976
normalized size	1	1.00	0.77	1.57	1.52	0.65	3.90	0.82	2.11
time (sec)	N/A	1.128	2.425	0.625	0.461	0.491	10.854	0.213	15.626

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	156	414	398	178	960	217	550
normalized size	1	1.00	0.78	2.06	1.98	0.89	4.78	1.08	2.74
time (sec)	N/A	0.335	0.913	0.512	0.430	0.473	5.283	0.177	14.623

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	117	120	178	171	93	371	116	330
normalized size	1	0.92	0.94	1.40	1.35	0.73	2.92	0.91	2.60
time (sec)	N/A	0.101	0.497	0.410	0.381	0.453	2.085	0.165	14.256

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	233	1357	0	627	0	617	10256
normalized size	1	1.00	0.95	5.52	0.00	2.55	0.00	2.51	41.69
time (sec)	N/A	0.895	0.981	0.479	0.000	0.549	0.000	0.211	21.761

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	244	1534	0	1027	0	588	11993
normalized size	1	1.00	0.86	5.42	0.00	3.63	0.00	2.08	42.38
time (sec)	N/A	0.941	1.497	0.555	0.000	0.594	0.000	0.213	23.883

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	830	2906	0	1670	0	986	13891
normalized size	1	1.00	2.72	9.53	0.00	5.48	0.00	3.23	45.54
time (sec)	N/A	0.934	3.145	0.564	0.000	0.609	0.000	0.303	25.413

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	788	1110	1124	470	0	479	839
normalized size	1	1.00	3.58	5.05	5.11	2.14	0.00	2.18	3.81
time (sec)	N/A	0.361	1.272	0.437	0.727	0.483	0.000	0.198	14.054

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	141	200	524	606	303	5763	222	297
normalized size	1	0.99	1.40	3.66	4.24	2.12	40.30	1.55	2.08
time (sec)	N/A	0.206	0.465	0.416	0.725	0.471	9.445	0.156	16.666

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	126	179	256	154	1307	160	122
normalized size	1	1.00	1.88	2.67	3.82	2.30	19.51	2.39	1.82
time (sec)	N/A	0.203	0.469	0.360	0.492	0.470	4.181	0.141	13.681

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	79	65	78	66	109	40	35
normalized size	1	1.00	2.26	1.86	2.23	1.89	3.11	1.14	1.00
time (sec)	N/A	0.049	0.159	0.227	0.518	0.439	1.832	0.146	12.920

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	148	176	0	595	0	113	154
normalized size	1	1.00	1.47	1.74	0.00	5.89	0.00	1.12	1.52
time (sec)	N/A	0.170	0.327	0.545	0.000	0.482	0.000	0.168	13.322

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	209	615	0	1538	0	443	437
normalized size	1	1.00	1.15	3.40	0.00	8.50	0.00	2.45	2.41
time (sec)	N/A	0.349	1.222	0.576	0.000	0.506	0.000	0.193	15.531

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	313	2482	0	3303	0	753	1076
normalized size	1	1.00	1.11	8.77	0.00	11.67	0.00	2.66	3.80
time (sec)	N/A	0.550	1.606	0.559	0.000	0.614	0.000	0.298	17.439

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	547	946	1382	584	0	494	663
normalized size	1	1.00	2.40	4.15	6.06	2.56	0.00	2.17	2.91
time (sec)	N/A	0.523	3.571	0.427	0.570	0.462	0.000	0.217	16.346

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	338	489	831	375	5358	277	365
normalized size	1	1.00	2.56	3.70	6.30	2.84	40.59	2.10	2.77
time (sec)	N/A	0.510	1.675	0.414	0.575	0.458	17.795	0.193	16.038

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	180	252	454	208	1062	141	94
normalized size	1	1.00	2.12	2.96	5.34	2.45	12.49	1.66	1.11
time (sec)	N/A	0.211	0.339	0.431	0.502	0.432	8.343	0.154	13.738

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	43	70	214	117	372	68	97
normalized size	1	1.00	0.66	1.08	3.29	1.80	5.72	1.05	1.49
time (sec)	N/A	0.052	0.057	0.330	0.350	0.415	4.585	0.185	13.388

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	229	327	0	1285	0	259	302
normalized size	1	1.00	1.51	2.15	0.00	8.45	0.00	1.70	1.99
time (sec)	N/A	0.419	0.628	0.561	0.000	0.500	0.000	0.216	14.783

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	313	770	0	3123	0	425	844
normalized size	1	1.00	1.14	2.80	0.00	11.36	0.00	1.55	3.07
time (sec)	N/A	0.672	2.830	0.554	0.000	0.593	0.000	0.232	16.763

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	1522	2641	0	4997	0	944	1686
normalized size	1	1.00	3.94	6.84	0.00	12.95	0.00	2.45	4.37
time (sec)	N/A	0.961	6.366	0.586	0.000	0.695	0.000	0.306	17.693

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	366	936	1682	649	11456	596	593
normalized size	1	1.00	1.63	4.16	7.48	2.88	50.92	2.65	2.64
time (sec)	N/A	0.809	2.971	0.434	0.584	0.481	56.435	0.222	15.695

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	514	617	1132	432	3468	382	286
normalized size	1	1.00	3.13	3.76	6.90	2.63	21.15	2.33	1.74
time (sec)	N/A	0.461	0.898	0.418	0.549	0.459	33.403	0.193	16.544

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	176	151	733	271	1819	223	245
normalized size	1	1.00	1.39	1.19	5.77	2.13	14.32	1.76	1.93
time (sec)	N/A	0.227	0.689	0.439	0.456	0.428	15.102	0.178	14.259

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	63	114	387	190	1015	130	150
normalized size	1	1.00	0.62	1.12	3.79	1.86	9.95	1.27	1.47
time (sec)	N/A	0.075	0.093	0.324	0.394	0.416	8.342	0.166	13.804

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	502	606	0	2292	0	578	591
normalized size	1	1.00	2.19	2.65	0.00	10.01	0.00	2.52	2.58
time (sec)	N/A	0.724	1.251	0.540	0.000	0.543	0.000	0.261	17.184

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	1253	1049	0	4486	0	772	1349
normalized size	1	1.00	3.29	2.75	0.00	11.77	0.00	2.03	3.54
time (sec)	N/A	1.081	6.384	0.570	0.000	0.660	0.000	0.303	17.669

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	508	508	548	2918	0	7283	0	1272	2387
normalized size	1	1.00	1.08	5.74	0.00	14.34	0.00	2.50	4.70
time (sec)	N/A	1.447	4.960	0.650	0.000	0.849	0.000	0.593	19.882

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	305	242	0	467	0	0	-1
normalized size	1	1.00	1.19	0.95	0.00	1.82	0.00	0.00	-0.00
time (sec)	N/A	0.460	1.248	1.400	0.000	0.450	0.000	0.000	0.000

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	176	161	0	306	0	0	-1
normalized size	1	1.00	0.92	0.84	0.00	1.59	0.00	0.00	-0.01
time (sec)	N/A	0.339	0.743	1.437	0.000	0.428	0.000	0.000	0.000

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	117	102	0	175	0	0	-1
normalized size	1	1.00	0.99	0.86	0.00	1.48	0.00	0.00	-0.01
time (sec)	N/A	0.249	0.359	1.362	0.000	0.449	0.000	0.000	0.000

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	82	58	0	85	0	0	-1
normalized size	1	1.00	1.32	0.94	0.00	1.37	0.00	0.00	-0.02
time (sec)	N/A	0.058	0.123	1.019	0.000	0.439	0.000	0.000	0.000

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	903	139	0	651	0	0	-1
normalized size	1	1.00	9.03	1.39	0.00	6.51	0.00	0.00	-0.01
time (sec)	N/A	0.246	9.035	1.787	0.000	1.259	0.000	0.000	0.000

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	901	274	0	1012	0	0	-1
normalized size	1	1.00	7.15	2.17	0.00	8.03	0.00	0.00	-0.01
time (sec)	N/A	0.262	8.797	2.019	0.000	1.447	0.000	0.000	0.000

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	967	628	0	1750	0	0	-1
normalized size	1	1.00	5.04	3.27	0.00	9.11	0.00	0.00	-0.01
time (sec)	N/A	0.370	10.267	2.426	0.000	2.280	0.000	0.000	0.000

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	390	312	0	637	0	0	-1
normalized size	1	1.00	1.04	0.83	0.00	1.70	0.00	0.00	-0.00
time (sec)	N/A	0.920	4.607	1.369	0.000	0.474	0.000	0.000	0.000

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	267	207	0	430	0	0	-1
normalized size	1	1.00	0.91	0.70	0.00	1.46	0.00	0.00	-0.00
time (sec)	N/A	0.712	2.265	1.435	0.000	0.451	0.000	0.000	0.000

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	144	150	0	257	0	0	-1
normalized size	1	1.00	0.87	0.91	0.00	1.56	0.00	0.00	-0.01
time (sec)	N/A	0.316	1.075	1.283	0.000	0.448	0.000	0.000	0.000

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	101	77	0	137	0	0	-1
normalized size	1	1.00	1.00	0.76	0.00	1.36	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.397	1.085	0.000	0.431	0.000	0.000	0.000

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	356	291	0	880	0	0	-1
normalized size	1	1.00	2.33	1.90	0.00	5.75	0.00	0.00	-0.01
time (sec)	N/A	0.503	3.535	1.734	0.000	1.474	0.000	0.000	0.000

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	381	592	0	1428	0	0	-1
normalized size	1	1.00	1.99	3.10	0.00	7.48	0.00	0.00	-0.01
time (sec)	N/A	0.552	5.013	2.315	0.000	1.677	0.000	0.000	0.000

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	416	895	0	2208	0	0	-1
normalized size	1	1.00	1.88	4.05	0.00	9.99	0.00	0.00	-0.00
time (sec)	N/A	0.610	5.341	2.825	0.000	2.593	0.000	0.000	0.000

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	1565	374	0	863	0	0	-1
normalized size	1	1.00	2.93	0.70	0.00	1.62	0.00	0.00	-0.00
time (sec)	N/A	1.203	6.869	1.395	0.000	0.505	0.000	0.000	0.000

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	891	257	0	593	0	0	-1
normalized size	1	1.00	2.08	0.60	0.00	1.38	0.00	0.00	-0.00
time (sec)	N/A	1.066	6.608	1.427	0.000	0.474	0.000	0.000	0.000

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	202	152	0	361	0	0	-1
normalized size	1	1.00	0.95	0.72	0.00	1.70	0.00	0.00	-0.00
time (sec)	N/A	0.368	4.243	1.394	0.000	0.464	0.000	0.000	0.000

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	119	99	0	191	0	0	-1
normalized size	1	1.00	0.86	0.72	0.00	1.38	0.00	0.00	-0.01
time (sec)	N/A	0.112	1.512	1.241	0.000	0.445	0.000	0.000	0.000

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	450	543	0	1314	0	0	-1
normalized size	1	1.00	2.06	2.49	0.00	6.03	0.00	0.00	-0.00
time (sec)	N/A	0.885	5.822	2.052	0.000	2.273	0.000	0.000	0.000

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	460	932	0	2046	0	0	-1
normalized size	1	1.00	1.74	3.52	0.00	7.72	0.00	0.00	-0.00
time (sec)	N/A	0.938	5.966	2.545	0.000	2.594	0.000	0.000	0.000

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	504	1587	0	3046	0	0	-1
normalized size	1	1.00	1.64	5.15	0.00	9.89	0.00	0.00	-0.00
time (sec)	N/A	0.972	8.240	2.970	0.000	2.979	0.000	0.000	0.000

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	375	610	0	629	0	0	-1
normalized size	1	1.00	1.32	2.15	0.00	2.21	0.00	0.00	-0.00
time (sec)	N/A	1.001	0.863	1.949	0.000	0.489	0.000	0.000	0.000

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	246	396	0	448	0	0	-1
normalized size	1	1.00	1.23	1.98	0.00	2.24	0.00	0.00	-0.00
time (sec)	N/A	0.585	0.538	1.756	0.000	0.482	0.000	0.000	0.000

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	135	232	0	303	0	0	-1
normalized size	1	1.00	1.04	1.78	0.00	2.33	0.00	0.00	-0.01
time (sec)	N/A	0.270	0.480	1.527	0.000	0.468	0.000	0.000	0.000

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	106	128	0	210	0	0	151
normalized size	1	1.00	1.34	1.62	0.00	2.66	0.00	0.00	1.91
time (sec)	N/A	0.070	0.218	1.292	0.000	0.448	0.000	0.000	1.056

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	238	199	0	744	0	0	-1
normalized size	1	1.00	1.75	1.46	0.00	5.47	0.00	0.00	-0.01
time (sec)	N/A	0.283	3.343	2.073	0.000	1.492	0.000	0.000	0.000

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	374	899	0	2159	0	0	-1
normalized size	1	1.00	1.81	4.34	0.00	10.43	0.00	0.00	-0.00
time (sec)	N/A	0.617	6.970	2.719	0.000	3.773	0.000	0.000	0.000

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	847	2277	0	4180	0	0	-1
normalized size	1	1.00	2.74	7.37	0.00	13.53	0.00	0.00	-0.00
time (sec)	N/A	1.054	10.778	3.580	0.000	7.777	0.000	0.000	0.000

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	684	1030	0	784	0	0	-1
normalized size	1	1.00	2.42	3.64	0.00	2.77	0.00	0.00	-0.00
time (sec)	N/A	1.000	1.067	1.868	0.000	0.494	0.000	0.000	0.000

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	357	694	0	584	0	0	-1
normalized size	1	1.00	1.76	3.42	0.00	2.88	0.00	0.00	-0.00
time (sec)	N/A	0.575	0.742	1.590	0.000	0.491	0.000	0.000	0.000

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	246	389	0	407	0	0	-1
normalized size	1	1.00	1.85	2.92	0.00	3.06	0.00	0.00	-0.01
time (sec)	N/A	0.279	0.456	1.321	0.000	0.484	0.000	0.000	0.000

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	150	176	0	293	0	0	-1
normalized size	1	1.00	1.72	2.02	0.00	3.37	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.205	1.091	0.000	0.435	0.000	0.000	0.000

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	419	624	0	1561	0	0	-1
normalized size	1	1.00	2.24	3.34	0.00	8.35	0.00	0.00	-0.01
time (sec)	N/A	0.590	3.137	2.007	0.000	3.290	0.000	0.000	0.000

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	542	2049	0	3403	0	0	-1
normalized size	1	1.00	1.86	7.02	0.00	11.65	0.00	0.00	-0.00
time (sec)	N/A	1.018	9.543	3.069	0.000	7.667	0.000	0.000	0.000

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	1395	4707	0	5864	0	0	-1
normalized size	1	1.00	3.47	11.71	0.00	14.59	0.00	0.00	-0.00
time (sec)	N/A	1.562	13.484	3.972	0.000	15.273	0.000	0.000	0.000

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	523	1438	0	980	0	0	-1
normalized size	1	1.00	1.70	4.67	0.00	3.18	0.00	0.00	-0.00
time (sec)	N/A	1.059	1.791	2.466	0.000	0.503	0.000	0.000	0.000

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	544	982	0	744	0	0	-1
normalized size	1	1.00	2.48	4.48	0.00	3.40	0.00	0.00	-0.00
time (sec)	N/A	0.579	1.140	2.036	0.000	0.483	0.000	0.000	0.000

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	267	449	0	536	0	0	-1
normalized size	1	1.00	1.77	2.97	0.00	3.55	0.00	0.00	-0.01
time (sec)	N/A	0.287	0.770	1.639	0.000	0.479	0.000	0.000	0.000

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	227	279	0	392	0	0	-1
normalized size	1	1.00	1.80	2.21	0.00	3.11	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.372	1.699	0.000	0.457	0.000	0.000	0.000

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	550	1418	0	2577	0	0	-1
normalized size	1	1.00	2.11	5.43	0.00	9.87	0.00	0.00	-0.00
time (sec)	N/A	0.984	5.610	2.665	0.000	6.574	0.000	0.000	0.000

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	1318	4092	0	5151	0	0	-1
normalized size	1	1.00	3.34	10.36	0.00	13.04	0.00	0.00	-0.00
time (sec)	N/A	1.536	12.375	4.202	0.000	14.831	0.000	0.000	0.000

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	519	519	2103	7322	0	8555	0	0	-1
normalized size	1	1.00	4.05	14.11	0.00	16.48	0.00	0.00	-0.00
time (sec)	N/A	2.152	13.857	5.811	0.000	45.087	0.000	0.000	0.000

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.338	26.082	3.868	0.000	0.531	0.000	0.000	0.000

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.300	9.073	3.368	0.000	0.475	0.000	0.000	0.000

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.304	4.652	1.057	0.000	0.472	0.000	0.000	0.000

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.341	28.141	10.288	0.000	0.482	0.000	0.000	0.000

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	245	0	0	0	0	0	-1
normalized size	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.915	26.650	1.285	0.000	0.490	0.000	0.000	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.265	8.255	1.237	0.000	0.471	0.000	0.000	0.000

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	244	0	0	0	0	0	-1
normalized size	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.403	5.568	0.972	0.000	0.469	0.000	0.000	0.000

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	603	0	0	0	0	0	-1
normalized size	1	1.00	2.24	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.470	17.227	0.983	0.000	0.489	0.000	0.000	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	300	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.987	7.726	9.533	0.000	0.469	0.000	0.000	0.000

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	198	212	0	0	0	0	0	-1
normalized size	1	0.99	1.07	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.362	3.589	7.608	0.000	0.459	0.000	0.000	0.000

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	275	0	0	0	0	0	-1
normalized size	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	1.816	0.018	0.000	0.453	0.000	0.000	0.000

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	473	0	0	0	0	0	-1
normalized size	1	1.00	2.48	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.292	7.078	4.046	0.000	0.462	0.000	0.000	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	651	0	0	0	0	0	-1
normalized size	1	1.00	2.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.618	2.496	14.319	0.000	0.479	0.000	0.000	0.000

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	467	467	651	0	0	0	0	0	-1
normalized size	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.347	2.533	12.570	0.000	0.471	0.000	0.000	0.000

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	3281	0	0	0	0	0	-1
normalized size	1	1.00	11.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.628	8.136	1.204	0.000	0.515	0.000	0.000	0.000

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	672	0	0	0	0	0	-1
normalized size	1	1.00	2.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.555	11.894	1.134	0.000	0.481	0.000	0.000	0.000

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	672	0	0	0	0	0	-1
normalized size	1	1.00	2.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.544	5.714	0.891	0.000	0.477	0.000	0.000	0.000

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	672	0	0	0	0	0	-1
normalized size	1	1.00	2.33	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.545	6.224	0.917	0.000	0.507	0.000	0.000	0.000

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	682	0	0	0	0	0	-1
normalized size	1	1.00	2.53	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.430	6.082	1.351	0.000	0.484	0.000	0.000	0.000

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	573	0	0	0	0	0	-1
normalized size	1	1.00	2.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.483	6.936	3.466	0.000	0.486	0.000	0.000	0.000

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.200	10.115	1.431	0.000	0.494	0.000	0.000	0.000

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.230	4.874	2.750	0.000	0.480	0.000	0.000	0.000

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	0	0	56	0	0	98
normalized size	1	1.00	1.00	0.00	0.00	1.44	0.00	0.00	2.51
time (sec)	N/A	0.172	0.678	6.471	0.000	0.536	0.000	0.000	14.774

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	0	0	57	0	0	99
normalized size	1	1.00	1.00	0.00	0.00	1.42	0.00	0.00	2.48
time (sec)	N/A	0.172	0.743	6.558	0.000	0.548	0.000	0.000	14.621

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	188	1246	0	1308	0	776	16312
normalized size	1	1.00	0.94	6.26	0.00	6.57	0.00	3.90	81.97
time (sec)	N/A	0.580	1.596	0.461	0.000	0.610	0.000	6.894	27.619

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	840	840	2042	6776580	0	0	0	0	-1
normalized size	1	1.00	2.43	8067.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.156	6.750	100.983	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	630	630	1901	3150101	0	0	0	0	-1
normalized size	1	1.00	3.02	5000.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.886	10.186	159.526	0.000	2.233	0.000	0.000	0.000

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	1949	99082	0	0	0	0	-1
normalized size	1	1.00	4.67	237.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.537	6.563	1.846	0.000	0.522	0.000	0.000	0.000

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	544	544	2266	198381	0	0	0	0	-1
normalized size	1	1.00	4.17	364.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.377	7.426	3.294	0.000	0.643	0.000	0.000	0.000

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	858	858	2837	827062	0	0	0	0	-1
normalized size	1	1.00	3.31	963.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.616	8.770	19.791	0.000	0.906	0.000	0.000	0.000

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.093	17.694	1.284	0.000	0.873	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [11] had the largest ratio of [.2857]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	5	1.00	33	0.152
2	A	6	5	1.00	33	0.152
3	A	5	4	1.00	31	0.129
4	A	4	3	1.00	33	0.091
5	A	5	3	1.00	33	0.091
6	A	6	3	1.00	33	0.091
7	A	6	5	1.00	35	0.143
8	A	5	5	1.00	35	0.143
9	A	4	4	1.00	35	0.114
10	A	9	9	1.00	35	0.257
11	A	10	10	1.00	35	0.286
12	A	9	5	1.00	33	0.152
13	A	4	3	1.00	34	0.088
14	A	1	1	1.00	43	0.023
15	A	1	1	1.00	37	0.027

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	6	6	1.00	31	0.194
17	A	7	6	1.00	34	0.176
18	A	6	6	1.00	34	0.176
19	A	5	5	1.08	34	0.147
20	A	4	4	1.00	32	0.125
21	A	4	3	1.00	34	0.088
22	A	4	4	1.00	34	0.118
23	A	4	4	1.00	34	0.118
24	A	5	5	1.00	34	0.147
25	A	6	5	1.00	34	0.147
26	A	8	6	1.00	36	0.167
27	A	7	6	1.00	36	0.167
28	A	6	5	1.00	36	0.139
29	A	5	4	1.00	36	0.111
30	A	5	5	1.00	34	0.147
31	A	5	5	1.00	36	0.139
32	A	5	5	1.00	36	0.139
33	A	5	4	1.00	36	0.111
34	A	3	3	1.00	36	0.083
35	A	4	4	1.00	36	0.111
36	A	5	4	1.00	36	0.111
37	A	6	4	1.00	36	0.111
38	A	9	6	1.00	36	0.167
39	A	8	6	1.00	36	0.167
40	A	7	5	1.00	36	0.139
41	A	6	4	1.00	36	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
42	A	6	5	1.00	36	0.139
43	A	6	6	1.00	34	0.176
44	A	6	6	1.00	36	0.167
45	A	6	6	1.00	36	0.167
46	A	6	5	1.00	36	0.139
47	A	6	4	1.00	36	0.111
48	A	3	3	1.00	36	0.083
49	A	4	4	1.00	36	0.111
50	A	5	4	1.00	36	0.111
51	A	6	4	1.00	36	0.111
52	A	7	7	1.00	36	0.194
53	A	6	6	1.00	36	0.167
54	A	5	5	1.00	36	0.139
55	A	4	3	1.00	34	0.088
56	A	4	4	1.00	36	0.111
57	A	4	4	1.00	36	0.111
58	A	5	5	1.00	36	0.139
59	A	6	5	1.00	36	0.139
60	A	8	7	1.00	36	0.194
61	A	7	6	1.00	36	0.167
62	A	6	6	1.00	36	0.167
63	A	5	5	1.00	36	0.139
64	A	4	4	1.00	34	0.118
65	A	4	4	1.00	36	0.111
66	A	4	3	1.00	36	0.083
67	A	4	3	1.00	36	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
68	A	5	4	1.00	36	0.111
69	A	6	4	1.00	36	0.111
70	A	8	6	1.00	36	0.167
71	A	7	6	1.00	36	0.167
72	A	6	5	1.00	36	0.139
73	A	5	4	1.00	36	0.111
74	A	4	4	1.00	34	0.118
75	A	5	5	1.00	36	0.139
76	A	4	3	1.00	36	0.083
77	A	4	3	1.00	36	0.083
78	A	4	3	1.00	36	0.083
79	A	5	4	1.00	36	0.111
80	A	6	4	1.00	36	0.111
81	A	6	4	1.00	36	0.111
82	A	5	4	1.00	36	0.111
83	A	4	4	1.00	36	0.111
84	A	3	3	1.00	36	0.083
85	A	5	5	1.00	36	0.139
86	A	5	5	1.00	36	0.139
87	A	5	5	1.00	36	0.139
88	A	6	6	1.00	36	0.167
89	A	6	4	1.00	38	0.105
90	A	5	4	1.00	38	0.105
91	A	4	4	1.00	38	0.105
92	A	3	3	1.00	38	0.079
93	A	6	5	1.00	38	0.132

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
94	A	6	5	1.00	38	0.132
95	A	6	6	1.00	38	0.158
96	A	6	5	1.00	38	0.132
97	A	7	6	1.00	38	0.158
98	A	6	4	1.00	38	0.105
99	A	5	4	1.00	38	0.105
100	A	4	4	1.00	38	0.105
101	A	3	3	1.00	38	0.079
102	A	7	5	1.00	38	0.132
103	A	7	5	1.00	38	0.132
104	A	7	6	1.00	38	0.158
105	A	7	6	1.00	38	0.158
106	A	7	5	1.00	38	0.132
107	A	8	6	1.00	38	0.158
108	A	6	4	1.00	38	0.105
109	A	5	4	1.00	38	0.105
110	A	4	4	1.00	38	0.105
111	A	3	3	1.00	38	0.079
112	A	4	4	1.00	38	0.105
113	A	5	5	1.00	38	0.132
114	A	6	6	1.00	38	0.158
115	A	7	4	1.00	38	0.105
116	A	6	4	1.00	38	0.105
117	A	5	4	1.00	38	0.105
118	A	4	4	1.00	38	0.105
119	A	3	3	1.00	38	0.079

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
120	A	5	5	1.00	38	0.132
121	A	6	6	1.00	38	0.158
122	A	7	7	1.00	38	0.184
123	A	7	4	1.00	38	0.105
124	A	6	4	1.00	38	0.105
125	A	5	4	1.00	38	0.105
126	A	4	4	1.00	38	0.105
127	A	3	3	1.00	38	0.079
128	A	6	5	1.00	38	0.132
129	A	7	7	1.00	38	0.184
130	A	8	7	1.00	38	0.184
131	A	3	2	1.00	40	0.050
132	A	3	2	1.00	40	0.050
133	A	3	2	1.00	40	0.050
134	A	3	2	1.00	40	0.050
135	A	5	5	1.00	40	0.125
136	A	5	5	1.00	40	0.125
137	A	3	2	1.00	40	0.050
138	A	3	2	1.00	40	0.050
139	A	3	3	1.00	40	0.075
140	A	3	3	1.00	40	0.075
141	A	3	3	1.00	40	0.075
142	A	3	2	1.00	40	0.050
143	A	5	5	1.00	40	0.125
144	A	5	5	1.00	40	0.125
145	A	5	5	1.00	40	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
146	A	2	2	1.00	40	0.050
147	A	3	3	1.00	40	0.075
148	A	3	3	1.00	40	0.075
149	A	4	3	1.00	40	0.075
150	A	4	3	1.00	40	0.075
151	A	3	3	1.00	40	0.075
152	A	3	2	1.00	40	0.050
153	A	6	5	1.00	40	0.125
154	A	6	5	1.00	40	0.125
155	A	6	6	1.00	40	0.150
156	A	6	5	1.00	40	0.125
157	A	2	2	1.00	40	0.050
158	A	3	3	1.00	40	0.075
159	A	4	3	1.00	40	0.075
160	A	5	3	1.00	40	0.075
161	A	5	3	1.00	40	0.075
162	A	4	3	1.00	40	0.075
163	A	3	3	1.00	40	0.075
164	A	3	2	1.00	40	0.050
165	A	7	5	1.00	40	0.125
166	A	7	5	1.00	40	0.125
167	A	7	6	1.00	40	0.150
168	A	7	6	1.00	40	0.150
169	A	7	5	1.00	40	0.125
170	A	2	2	1.00	40	0.050
171	A	3	3	1.00	40	0.075

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
172	A	4	3	1.00	40	0.075
173	A	5	3	1.00	40	0.075
174	A	6	5	1.00	40	0.125
175	A	5	5	1.00	40	0.125
176	A	5	5	1.00	40	0.125
177	A	7	4	1.00	40	0.100
178	A	3	3	1.00	40	0.075
179	A	4	4	1.00	40	0.100
180	A	7	5	1.00	40	0.125
181	A	6	5	1.00	40	0.125
182	A	5	5	1.00	40	0.125
183	A	5	5	1.00	40	0.125
184	A	3	3	1.00	40	0.075
185	A	4	4	1.00	40	0.100
186	A	5	4	1.00	40	0.100
187	A	8	6	1.00	40	0.150
188	A	7	6	1.00	40	0.150
189	A	6	6	1.00	40	0.150
190	A	5	5	1.00	40	0.125
191	A	3	2	1.00	40	0.050
192	A	4	4	1.00	40	0.100
193	A	5	4	1.00	40	0.100
194	A	6	4	1.00	40	0.100
195	A	5	5	1.00	36	0.139
196	A	5	5	1.00	36	0.139
197	A	5	5	1.00	36	0.139

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
198	A	5	5	1.00	34	0.147
199	A	3	3	1.00	23	0.130
200	A	5	5	1.00	36	0.139
201	A	5	5	1.00	36	0.139
202	A	5	5	1.00	36	0.139
203	A	4	4	1.00	38	0.105
204	A	4	4	1.00	38	0.105
205	A	4	3	1.00	38	0.079
206	A	3	3	1.16	38	0.079
207	A	3	2	1.00	38	0.053
208	A	4	4	1.00	38	0.105
209	A	4	4	1.00	38	0.105
210	A	4	4	1.00	38	0.105
211	A	4	3	1.00	40	0.075
212	A	3	3	1.00	40	0.075
213	A	2	2	1.00	40	0.050
214	A	5	5	1.00	40	0.125
215	A	5	5	1.00	38	0.132
216	A	5	5	1.00	40	0.125
217	A	5	5	1.00	40	0.125
218	A	2	2	1.00	46	0.043
219	A	2	2	1.00	45	0.044
220	A	2	2	1.00	44	0.045
221	A	2	2	1.00	43	0.047
222	A	1	1	1.00	47	0.021
223	A	1	1	1.00	46	0.022

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
224	A	13	4	1.00	32	0.125
225	A	12	4	1.00	32	0.125
226	A	10	5	1.00	30	0.167
227	A	5	5	1.00	24	0.208
228	A	7	5	1.00	30	0.167
229	A	9	7	1.00	32	0.219
230	A	7	6	1.00	32	0.188
231	A	7	4	1.00	32	0.125
232	A	10	5	1.00	32	0.156
233	A	12	6	1.00	32	0.188
234	A	12	4	1.00	32	0.125
235	A	11	6	1.00	32	0.188
236	A	9	4	1.00	32	0.125
237	A	8	3	1.00	32	0.094
238	A	8	3	1.00	30	0.100
239	A	3	3	1.00	24	0.125
240	A	9	4	1.00	30	0.133
241	A	15	9	1.00	32	0.281
242	A	13	7	1.00	32	0.219
243	A	15	7	1.00	32	0.219
244	A	5	4	1.00	33	0.121
245	A	4	4	1.00	33	0.121
246	A	3	3	1.00	31	0.097
247	A	1	1	1.00	21	0.048
248	A	6	6	1.00	33	0.182
249	A	6	6	1.00	33	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
250	A	7	7	1.00	33	0.212
251	A	6	5	1.00	35	0.143
252	A	5	5	1.00	35	0.143
253	A	4	4	1.00	33	0.121
254	A	2	2	1.00	23	0.087
255	A	7	7	1.00	35	0.200
256	A	7	7	1.00	35	0.200
257	A	7	7	1.00	35	0.200
258	A	7	5	1.00	35	0.143
259	A	6	5	1.00	35	0.143
260	A	10	8	1.00	33	0.242
261	A	8	6	0.92	23	0.261
262	A	8	7	1.00	35	0.200
263	A	8	8	1.00	35	0.229
264	A	8	7	1.00	35	0.200
265	A	3	3	1.00	35	0.086
266	A	2	2	0.99	35	0.057
267	A	4	4	1.00	33	0.121
268	A	2	2	1.00	23	0.087
269	A	5	5	1.00	35	0.143
270	A	6	6	1.00	35	0.171
271	A	7	6	1.00	35	0.171
272	A	3	2	1.00	35	0.057
273	A	5	5	1.00	35	0.143
274	A	4	4	1.00	33	0.121
275	A	2	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
276	A	6	5	1.00	35	0.143
277	A	7	6	1.00	35	0.171
278	A	8	6	1.00	35	0.171
279	A	6	5	1.00	35	0.143
280	A	5	5	1.00	35	0.143
281	A	4	4	1.00	33	0.121
282	A	3	3	1.00	23	0.130
283	A	7	5	1.00	35	0.143
284	A	8	6	1.00	35	0.171
285	A	9	6	1.00	35	0.171
286	A	5	5	1.00	37	0.135
287	A	4	4	1.00	37	0.108
288	A	4	4	1.00	35	0.114
289	A	2	2	1.00	25	0.080
290	A	3	3	1.00	37	0.081
291	A	3	3	1.00	37	0.081
292	A	4	4	1.00	37	0.108
293	A	6	6	1.00	37	0.162
294	A	5	5	1.00	37	0.135
295	A	5	5	1.00	35	0.143
296	A	3	3	1.00	25	0.120
297	A	4	4	1.00	37	0.108
298	A	4	4	1.00	37	0.108
299	A	4	4	1.00	37	0.108
300	A	7	6	1.00	37	0.162
301	A	6	5	1.00	37	0.135

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
302	A	6	5	1.00	35	0.143
303	A	4	3	1.00	25	0.120
304	A	5	4	1.00	37	0.108
305	A	5	5	1.00	37	0.135
306	A	5	4	1.00	37	0.108
307	A	7	6	1.00	37	0.162
308	A	6	6	1.00	37	0.162
309	A	5	5	1.00	35	0.143
310	A	3	3	1.00	25	0.120
311	A	5	5	1.00	37	0.135
312	A	6	6	1.00	37	0.162
313	A	7	6	1.00	37	0.162
314	A	7	7	1.00	37	0.189
315	A	6	6	1.00	37	0.162
316	A	5	5	1.00	35	0.143
317	A	3	3	1.00	25	0.120
318	A	6	6	1.00	37	0.162
319	A	7	7	1.00	37	0.189
320	A	8	7	1.00	37	0.189
321	A	7	6	1.00	37	0.162
322	A	6	6	1.00	37	0.162
323	A	5	5	1.00	35	0.143
324	A	4	4	1.00	25	0.160
325	A	7	6	1.00	37	0.162
326	A	8	7	1.00	37	0.189
327	A	9	7	1.00	37	0.189

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
328	A	7	4	1.00	35	0.114
329	A	8	6	1.00	33	0.182
330	A	7	5	1.00	35	0.143
331	A	7	4	1.00	35	0.114
332	A	11	7	1.00	37	0.189
333	A	4	4	1.00	37	0.108
334	A	7	7	1.00	37	0.189
335	A	7	4	1.00	37	0.108
336	A	6	6	1.00	35	0.171
337	A	5	5	0.99	33	0.152
338	A	3	3	1.00	23	0.130
339	A	6	6	1.00	35	0.171
340	A	7	7	1.00	35	0.200
341	A	8	7	1.00	35	0.200
342	A	9	5	1.00	37	0.135
343	A	9	5	1.00	37	0.135
344	A	9	5	1.00	37	0.135
345	A	9	5	1.00	37	0.135
346	A	9	5	1.00	35	0.143
347	A	7	6	1.00	39	0.154
348	A	4	4	1.00	36	0.111
349	A	4	4	1.00	40	0.100
350	A	1	1	1.00	55	0.018
351	A	1	1	1.00	51	0.020
352	A	6	6	1.00	35	0.171
353	A	7	7	1.00	39	0.180

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
354	A	5	5	1.00	39	0.128
355	A	3	3	1.00	39	0.077
356	A	4	4	1.00	39	0.103
357	A	5	5	1.00	39	0.128
358	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

3.1 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=373

$$\frac{a^3(A(4n+11)+B(4n+9))\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{d^2f(n+2)(n+3)\sqrt{\cos^2(e+fx)}}+\frac{a^3\left(A\left(4n^2+21n+2\right.\right.}{}$$

```
[Out] -a^3*(B*(2*n^2+14*n+27)+A*(2*n^2+15*n+28))*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/
d/f/(4+n)/(n^2+5*n+6)-a*B*cos(f*x+e)*(d*sin(f*x+e))^(1+n)*(a+a*sin(f*x+e))^
2/d/f/(4+n)-(A*(4+n)+B*(6+n))*cos(f*x+e)*(d*sin(f*x+e))^(1+n)*(a^3+a^3*sin(
f*x+e))/d/f/(3+n)/(4+n)+a^3*(B*(4*n^2+19*n+15)+A*(4*n^2+21*n+20))*cos(f*x+e
)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*(d*sin(f*x+e))^(1+n)
/d/f/(4+n)/(n^2+3*n+2)/(cos(f*x+e)^2)^(1/2)+a^3*(B*(9+4*n)+A*(11+4*n))*cos(
f*x+e)*hypergeom([1/2, 1+1/2*n],[1/2*n+2],sin(f*x+e)^2)*(d*sin(f*x+e))^(2+n
)/d^2/f/(2+n)/(3+n)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.84, antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2976, 2968, 3023, 2748, 2643}

$$\frac{a^3(A(4n+11)+B(4n+9))\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{d^2f(n+2)(n+3)\sqrt{\cos^2(e+fx)}}+\frac{a^3\left(A\left(4n^2+21n+2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((a^3*(B*(27 + 14*n + 2*n^2) + A*(28 + 15*n + 2*n^2))*Cos[e + f*x]*(d*Sin[
e + f*x])^(1 + n))/(d*f*(2 + n)*(3 + n)*(4 + n))) + (a^3*(B*(15 + 19*n + 4*
n^2) + A*(20 + 21*n + 4*n^2))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2
, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(d*f*(1 + n)*(2 + n)
*(4 + n)*Sqrt[Cos[e + f*x]^2]) + (a^3*(B*(9 + 4*n) + A*(11 + 4*n))*Cos[e +
f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2]*(d*Sin[e
+ f*x])^(2 + n))/(d^2*f*(2 + n)*(3 + n)*Sqrt[Cos[e + f*x]^2]) - (a*B*Cos[e
+ f*x]*(d*Sin[e + f*x])^(1 + n)*(a + a*Sin[e + f*x])^2)/(d*f*(4 + n)) - ((A
*(4 + n) + B*(6 + n))*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n)*(a^3 + a^3*Sin[
e + f*x]))/(d*f*(3 + n)*(4 + n))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
```

```
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rubi steps

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx = -\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\sin(e+}{df(4+n)}$$

$$= -\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\sin(e+}{df(4+n)}$$

$$= -\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\sin(e+}{df(4+n)}$$

$$= -\frac{a^3\left(B\left(27+14n+2n^2\right)+A\left(28+15n+2n^2\right)\right)}{df(2+n)(3+n)(4}$$

$$= -\frac{a^3\left(B\left(27+14n+2n^2\right)+A\left(28+15n+2n^2\right)\right)}{df(2+n)(3+n)(4}$$

$$= -\frac{a^3\left(B\left(27+14n+2n^2\right)+A\left(28+15n+2n^2\right)\right)}{df(2+n)(3+n)(4}$$

Mathematica [A] time = 2.35, size = 248, normalized size = 0.66

$$\frac{a^3\sin(e+fx)\cos(e+fx)(d\sin(e+fx))^n\left(\sin(e+fx)\left(\frac{(3A+B)\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{n+2}+\sin(e+fx)\left(\frac{3(A+B)\,{}_2F_1\left(\frac{1}{2},\right.}{}\right.\right.\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]),x]
```

```
[Out] (a^3*Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^n*((A*Hypergeometric2F1[1/2
, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2])/(1 + n) + Sin[e + f*x]*(((3*A + B)
*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2])/(2 + n) + Si
n[e + f*x]*((3*(A + B)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Sin[e +
 f*x]^2])/(3 + n) + Sin[e + f*x]*(((A + 3*B)*Hypergeometric2F1[1/2, (4 + n)
/2, (6 + n)/2, Sin[e + f*x]^2])/(4 + n) + (B*Hypergeometric2F1[1/2, (5 + n)
```

```
/2, (7 + n)/2, Sin[e + f*x]^2]*Sin[e + f*x])/(5 + n))))))/(f*Sqrt[Cos[e + f
*x]^2])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Ba^3\cos\left(fx+e\right)^4-(3A+5B)a^3\cos\left(fx+e\right)^2+4(A+B)a^3-\left((A+3B)a^3\cos\left(fx+e\right)^2-4(A+B\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B)
*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*(d*sin(
f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^3\left(d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(d*sin(f*x + e))^n, x
)
```

maple [F] time = 8.25, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)\right)^n\left(a+a\sin\left(fx+e\right)\right)^3\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^3\left(d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(d*sin(f*x + e))^n, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d\,\sin\left(e+f\,x\right)\right)^n\,\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3,x)
```

```
[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**3*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.2 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=277

$$\frac{a^2(2A(n+3)+B(2n+5))\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{d^2f(n+2)(n+3)\sqrt{\cos^2(e+fx)}}+\frac{a^2(A(2n+3)+2B(n+$$

```
[Out] -a^2*(A*(3+n)+B*(4+n))*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(2+n)/(3+n)-B*co
s(f*x+e)*(d*sin(f*x+e))^(1+n)*(a^2+a^2*sin(f*x+e))/d/f/(3+n)+a^2*(2*B*(1+n)
+A*(3+2*n))*cos(f*x+e)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)
*(d*sin(f*x+e))^(1+n)/d/f/(1+n)/(2+n)/(cos(f*x+e)^2)^(1/2)+a^2*(2*A*(3+n)+B
*(5+2*n))*cos(f*x+e)*hypergeom([1/2, 1+1/2*n],[1/2*n+2],sin(f*x+e)^2)*(d*si
n(f*x+e))^(2+n)/d^2/f/(2+n)/(3+n)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2976, 2968, 3023, 2748, 2643}

$$\frac{a^2(2A(n+3)+B(2n+5))\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{d^2f(n+2)(n+3)\sqrt{\cos^2(e+fx)}}+\frac{a^2(A(2n+3)+2B(n+$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((a^2*(A*(3 + n) + B*(4 + n))*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*
(2 + n)*(3 + n))) + (a^2*(2*B*(1 + n) + A*(3 + 2*n))*Cos[e + f*x]*Hypergeom
etric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n
))/(d*f*(1 + n)*(2 + n)*Sqrt[Cos[e + f*x]^2]) + (a^2*(2*A*(3 + n) + B*(5 +
2*n))*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x
]^2]*(d*Sin[e + f*x])^(2 + n))/(d^2*f*(2 + n)*(3 + n)*Sqrt[Cos[e + f*x]^2])
 - (B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n)*(a^2 + a^2*Sin[e + f*x]))/(d*f*
(3 + n))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx = -\frac{B\cos(e+fx)(d\sin(e+fx))^{1+n}\left(a^2+a^2\sin(e+\right.}{df(3+n)}$$

$$= -\frac{B\cos(e+fx)(d\sin(e+fx))^{1+n}\left(a^2+a^2\sin(e+\right.}{df(3+n)}$$

$$= -\frac{a^2(A(3+n)+B(4+n))\cos(e+fx)(d\sin(e+f}{df(2+n)(3+n)}$$

$$= -\frac{a^2(A(3+n)+B(4+n))\cos(e+fx)(d\sin(e+f}{df(2+n)(3+n)}$$

$$= -\frac{a^2(A(3+n)+B(4+n))\cos(e+fx)(d\sin(e+f}{df(2+n)(3+n)}$$

Mathematica [A] time = 1.55, size = 204, normalized size = 0.74

$$\frac{a^2\sin(e+fx)\cos(e+fx)(d\sin(e+fx))^n\left(\sin(e+fx)\left(\frac{(2A+B)\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{n+2}+\sin(e+fx)\left(\frac{(A+2B)\,{}_2F_1\left(\frac{1}{2},\frac{n+}{2}\right.}{n}\right.\right.\right.}{f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x]
```

```
[Out] (a^2*Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^n*((A*Hypergeometric2F1[1/2
, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2])/(1 + n) + Sin[e + f*x]*(((2*A + B)
*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2])/(2 + n) + Si
n[e + f*x]*(((A + 2*B)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Sin[e +
 f*x]^2])/(3 + n) + (B*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Sin[e +
 f*x]^2]*Sin[e + f*x])/(4 + n)))))/(f*Sqrt[Cos[e + f*x]^2])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left((A+2B)a^2\cos\left(fx+e\right)^2-2(A+B)a^2+\left(Ba^2\cos\left(fx+e\right)^2-2(A+B)a^2\right)\sin\left(fx+e\right)\right)\left(d\sin\left(fx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*(d*sin(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{2}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e))^n, x
)
```

maple [F] time = 7.54, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)\right)^{n}\left(a+a\sin\left(fx+e\right)\right)^{2}\left(A+B\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{2}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e))^n, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(d\,\sin\left(e+f\,x\right)\right)}^n\,\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2,x)
```

```
[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**2*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.3 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=191

$$\frac{a(A+B)\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{d^2f(n+2)\sqrt{\cos^2(e+fx)}}+\frac{a(A(n+2)+B(n+1))\cos(e+fx)(d\sin}{df(n+1)(n+}$$

```
[Out] -a*B*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(2+n)+a*(B*(1+n)+A*(2+n))*cos(f*x+
e)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*(d*sin(f*x+e))^(1+n
)/d/f/(1+n)/(2+n)/(cos(f*x+e)^2)^(1/2)+a*(A+B)*cos(f*x+e)*hypergeom([1/2, 1
+1/2*n],[1/2*n+2],sin(f*x+e)^2)*(d*sin(f*x+e))^(2+n)/d^2/f/(2+n)/(cos(f*x+e
)^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2968, 3023, 2748, 2643}

$$\frac{a(A+B)\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{d^2f(n+2)\sqrt{\cos^2(e+fx)}}+\frac{a(A(n+2)+B(n+1))\cos(e+fx)(d\sin}{df(n+1)(n+}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(2 + n))) + (a*(B*(1 + n
) + A*(2 + n))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Si
n[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(d*f*(1 + n)*(2 + n)*Sqrt[Cos[e + f
*x]^2]) + (a*(A + B)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)
/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(d^2*f*(2 + n)*Sqrt[Cos[e + f
*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx = \int (d\sin(e+fx))^n\left(aA+(aA+aB)\sin(e+fx)+a\right.$$

$$= -\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)} + \frac{\int (d\sin(e+f}{}$$

$$= -\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)} + \frac{(a(A+B))\int}{}$$

$$= -\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)} + \frac{a(B(1+n)+}{}$$

Mathematica [C] time = 3.85, size = 392, normalized size = 2.05

$$-\frac{a2^{-n-2}e^{ifnx}\left(1-e^{2i(e+fx)}\right)^{-n}\left(-ie^{-i(e+fx)}\left(-1+e^{2i(e+fx)}\right)\right)^n(\sin(e+fx)+1)\sin^{-n}(e+fx)(d\sin(e+fx))^n\left(\frac{2(A+B)e}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((2^(-2 - n)*a*E^(I*f*n*x)*(((-I)*(-1 + E^((2*I)*(e + f*x))))/E^(I*(e + f*
x)))^n*((2*(A + B)*Hypergeometric2F1[(-1 - n)/2, -n, (1 - n)/2, E^((2*I)*(e
 + f*x))])/(E^(I*(e + f*(1 + n)*x))*(1 + n)) - (2*(A + B)*E^(I*(e - f*(-1 +
 n)*x))*Hypergeometric2F1[(1 - n)/2, -n, (3 - n)/2, E^((2*I)*(e + f*x))])/(
-1 + n) + I*((B*Hypergeometric2F1[-1 - n/2, -n, -1/2*n, E^((2*I)*(e + f*x))
])/(E^(I*(2*e + f*(2 + n)*x))*(2 + n)) + (B*E^((2*I)*(e + f*x))*n*Hypergeom
etric2F1[1 - n/2, -n, 2 - n/2, E^((2*I)*(e + f*x))] - 2*(2*A + B)*(-2 + n)*
Hypergeometric2F1[-n, -1/2*n, 1 - n/2, E^((2*I)*(e + f*x))])/(E^(I*f*n*x)*(
-2 + n)*n)))*(d*Sin[e + f*x])^n*(1 + Sin[e + f*x]))/((1 - E^((2*I)*(e + f*x
)))^n*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sin[e + f*x]^n))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(Ba\cos\left(fx+e\right)^2-(A+B)a\sin\left(fx+e\right)-(A+B)a\right)\left(d\sin\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral(-(B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*(d*sin(
f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)
```

maple [F] time = 5.40, size = 0, normalized size = 0.00

$$\int\left(d\sin\left(fx+e\right)\right)^n\left(a+a\sin\left(fx+e\right)\right)\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d\,\sin\left(e+f\,x\right)\right)^n\,\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x)),x)
```

```
[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.4 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=202

$$\frac{(n+1)(A-B)\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{ad^2f(n+2)\sqrt{\cos^2(e+fx)}}+\frac{(-An+Bn+B)\cos(e+fx)(d\sin}{adf(n+}$$

```
[Out] (A-B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(a+a*sin(f*x+e))+(-A*n+B*n+B)*cos
(f*x+e)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*(d*sin(f*x+e))
^(1+n)/a/d/f/(1+n)/(cos(f*x+e)^2)^(1/2)+(A-B)*(1+n)*cos(f*x+e)*hypergeom([1
/2, 1+1/2*n],[1/2*n+2],sin(f*x+e)^2)*(d*sin(f*x+e))^(2+n)/a/d^2/f/(2+n)/(co
s(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2978, 2748, 2643}

$$\frac{(n+1)(A-B)\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{ad^2f(n+2)\sqrt{\cos^2(e+fx)}}+\frac{(-An+Bn+B)\cos(e+fx)(d\sin}{adf(n+}$$

Antiderivative was successfully verified.

```
[In] Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((B - A*n + B*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(a*d*f*(1 + n)*Sqrt[Cos[e + f*x]^
2]) + ((A - B)*(1 + n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 +
n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(a*d^2*f*(2 + n)*Sqrt[Cos[e
 + f*x]^2]) + ((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(a + a*S
in[e + f*x]))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(a+a\sin(e+fx))} + \frac{\int(d\sin(e+fx))^n(ad(B-A}{}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(a+a\sin(e+fx))} + \frac{((A-B)(1+n))\int(d\sin(e}{ad}$$

$$= \frac{(B-An+Bn)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(e+fx)\right)(d\sin(e+}{adf(1+n)\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 0.89, size = 157, normalized size = 0.78

$$\frac{\sin(e+fx)\cos(e+fx)(d\sin(e+fx))^n\left(\frac{(n+1)(A-B)\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{(n+2)\sqrt{\cos^2(e+fx)}}+\frac{(-An+Bn+B)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(e+f}{(n+1)\sqrt{\cos^2(e+fx)}}\right.}{af}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^n*(((B - A*n + B*n)*Hypergeomet
ric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2])/((1 + n)*Sqrt[Cos[e + f*
x]^2]) + ((A - B)*(1 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[
e + f*x]^2]*Sin[e + f*x])/((2 + n)*Sqrt[Cos[e + f*x]^2]) + (A - B)/(1 + Sin
[e + f*x])))/(a*f)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)\right)^{n}}{a\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)\right)^{n}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a), x)
```

maple [F] time = 1.97, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^{n}\left(A+B\sin\left(fx+e\right)\right)}{a+a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)\right)^{n}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,\sin\left(e+f\,x\right)\right)}^n\,\left(A+B\,\sin\left(e+f\,x\right)\right)}{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x)),x)
```

```
[Out] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.5 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=279

$$\frac{(n+1)(2A(1-n)+2Bn+B)\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{3a^2d^2f(n+2)\sqrt{\cos^2(e+fx)}}-\frac{n(-2An+A+2B(}{}$$

```
[Out] 1/3*(B+2*A*(1-n)+2*B*n)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/a^2/d/f/(1+sin(f*x+
e))+1/3*(A-B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(a+a*sin(f*x+e))^2-1/3*n*
(A-2*A*n+2*B*(1+n))*cos(f*x+e)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(f
*x+e)^2)*(d*sin(f*x+e))^(1+n)/a^2/d/f/(1+n)/(cos(f*x+e)^2)^(1/2)+1/3*(1+n)*
(B+2*A*(1-n)+2*B*n)*cos(f*x+e)*hypergeom([1/2, 1+1/2*n],[1/2*n+2],sin(f*x+e
)^2)*(d*sin(f*x+e))^(2+n)/a^2/d^2/f/(2+n)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2978, 2748, 2643}

$$\frac{(n+1)(2A(1-n)+2Bn+B)\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{3a^2d^2f(n+2)\sqrt{\cos^2(e+fx)}}-\frac{n(-2An+A+2B(}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -(n*(A - 2*A*n + 2*B*(1 + n))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2
, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(3*a^2*d*f*(1 + n)*S
qrt[Cos[e + f*x]^2]) + ((1 + n)*(B + 2*A*(1 - n) + 2*B*n)*Cos[e + f*x]*Hype
rgeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(
2 + n))/(3*a^2*d^2*f*(2 + n)*Sqrt[Cos[e + f*x]^2]) + ((B + 2*A*(1 - n) + 2*
B*n)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(3*a^2*d*f*(1 + Sin[e + f*x]))
 + ((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(3*d*f*(a + a*Sin[e + f*x
])^2)
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{3df(a+a\sin(e+fx))^2} + \frac{\int \frac{(d\sin(e+fx))^n(ad(2A+B-An+Bn}{a+a\sin(e+}}{3a^2d}$$

$$= \frac{(B+2A(1-n)+2Bn)\cos(e+fx)(d\sin(e+fx))^{1+n}}{3a^2df(1+\sin(e+fx))} + \frac{(A-B)\cos}{3df(}$$

$$= \frac{(B+2A(1-n)+2Bn)\cos(e+fx)(d\sin(e+fx))^{1+n}}{3a^2df(1+\sin(e+fx))} + \frac{(A-B)\cos}{3df(}$$

$$= -\frac{n(A-2An+2B(1+n))\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(e+fx)\right)}{3a^2df(1+n)\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 1.33, size = 212, normalized size = 0.76

$$\frac{\sin(e+fx)\cos(e+fx)(d\sin(e+fx))^n\left(-\frac{n(-2An+A+2B(n+1))\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(e+fx)\right)}{(n+1)\sqrt{\cos^2(e+fx)}} + \frac{(n+1)(-2A(n-1)+2Bn+B)\sin(e+fx)\,{}_2}{(n+2)\sqrt{\cos^2(e+}}\right.}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2,
x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^n*(-((n*(A - 2*A*n + 2*B*(1 + n
))*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2])/((1 + n)*S
qrt[Cos[e + f*x]^2])) + ((1 + n)*(B - 2*A*(-1 + n) + 2*B*n)*Hypergeometric2
F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2]*Sin[e + f*x])/((2 + n)*Sqrt[C
os[e + f*x]^2]) + (A - B)/(1 + Sin[e + f*x])^2 + ((-A + B)*n)/(1 + Sin[e +
f*x]) + (2*A + B - A*n + B*n)/(1 + Sin[e + f*x])))/(3*a^2*f)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)\right)^{n}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a^2*cos(f*x + e)^2 - 2*a
^2*sin(f*x + e) - 2*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^2, x
)
```

maple [F] time = 11.46, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^{n}\left(A+B\sin\left(fx+e\right)\right)}{\left(a+a\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^2, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,\sin\left(e+f\,x\right)\right)}^n\,\left(A+B\,\sin\left(e+f\,x\right)\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^2,x)
```

```
[Out] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.6 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=362

$$\frac{(1-n)(n+1)(-4An+7A+4Bn+3B)\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{15a^3d^2f(n+2)\sqrt{\cos^2(e+fx)}}-\frac{n\left(A\left(4n\right.\right.}{}$$

```
[Out] 1/5*(A-B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(a+a*sin(f*x+e))^3+1/15*(A*(5
-2*n)+2*B*n)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/a/d/f/(a+a*sin(f*x+e))^2+1/15*
(1-n)*(-4*A*n+4*B*n+7*A+3*B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(a^3+a^3*s
in(f*x+e))-1/15*n*(B*(-4*n^2-n+3)+A*(4*n^2-9*n+2))*cos(f*x+e)*hypergeom([1/
2, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*(d*sin(f*x+e))^(1+n)/a^3/d/f/(1+n)/
(cos(f*x+e)^2)^(1/2)+1/15*(1-n)*(1+n)*(-4*A*n+4*B*n+7*A+3*B)*cos(f*x+e)*hyp
ergeom([1/2, 1+1/2*n],[1/2*n+2],sin(f*x+e)^2)*(d*sin(f*x+e))^(2+n)/a^3/d^2/
f/(2+n)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.85, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2978, 2748, 2643}

$$\frac{(1-n)(n+1)(-4An+7A+4Bn+3B)\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{15a^3d^2f(n+2)\sqrt{\cos^2(e+fx)}}-\frac{n\left(A\left(4n\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -(n*(B*(3 - n - 4*n^2) + A*(2 - 9*n + 4*n^2))*Cos[e + f*x]*Hypergeometric2F
1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(15*
a^3*d*f*(1 + n)*Sqrt[Cos[e + f*x]^2]) + ((1 - n)*(1 + n)*(7*A + 3*B - 4*A*n
 + 4*B*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e +
 f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(15*a^3*d^2*f*(2 + n)*Sqrt[Cos[e + f*x]^
2]) + ((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(5*d*f*(a + a*Sin[e +
 f*x])^3) + ((A*(5 - 2*n) + 2*B*n)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(
15*a*d*f*(a + a*Sin[e + f*x])^2) + ((1 - n)*(7*A + 3*B - 4*A*n + 4*B*n)*Cos
[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(15*d*f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&&  !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{5df(a+a\sin(e+fx))^3} + \frac{\int \frac{(d\sin(e+fx))^n(ad(4A+B-An+Bn}{(a+a\sin(e}}{5a^2}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{5df(a+a\sin(e+fx))^3} + \frac{(A(5-2n)+2Bn)\cos(e+}{15adf(a+a\sin}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{5df(a+a\sin(e+fx))^3} + \frac{(A(5-2n)+2Bn)\cos(e+}{15adf(a+a\sin}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{5df(a+a\sin(e+fx))^3} + \frac{(A(5-2n)+2Bn)\cos(e+}{15adf(a+a\sin}$$

$$= -\frac{n\left(B\left(3-n-4n^2\right)+A\left(2-9n+4n^2\right)\right)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2}\right.}{15a^3df(1+n)\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 4.57, size = 260, normalized size = 0.72

$$\frac{(d\sin(e+fx))^n\left(\frac{2\sin(e+fx)\cos(e+fx)\left(n\left(A\left(-4n^2+9n-2\right)+B\left(4n^2+n-3\right)\right){}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(e+fx)\right)+\frac{(n-1)(n+1)^2(A(4n-7)-B(4n+3))\sin(e+fx)\,{}_2\cdots}{n+2}\cdots\right.}{(n+1)\sqrt{\cos^2(e+fx)}}\cdots\right.}{30a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3,
x]
```

```
[Out] ((d*Sin[e + f*x])^n*((2*Cos[e + f*x]*Sin[e + f*x]*(n*(A*(-2 + 9*n - 4*n^2)
+ B*(-3 + n + 4*n^2))*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e +
f*x]^2] + ((-1 + n)*(1 + n)^2*(A*(-7 + 4*n) - B*(3 + 4*n))*Hypergeometric2F
1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2]*Sin[e + f*x])/(2 + n)))/((1 +
n)*Sqrt[Cos[e + f*x]^2]) + (3*(A - B)*Sin[2*(e + f*x)])/(1 + Sin[e + f*x])^
3 + ((A*(5 - 2*n) + 2*B*n)*Sin[2*(e + f*x)])/(1 + Sin[e + f*x])^2 + ((-1 +
n)*(A*(-7 + 4*n) - B*(3 + 4*n))*Sin[2*(e + f*x)])/(1 + Sin[e + f*x])))/(30*
a^3*f)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)\right)^n}{3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(fx+e\right)^2-4\,a^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(3*a^3*cos(f*x + e)^2 - 4
*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)\right)^n}{\left(a\sin\left(fx+e\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^3, x
)
```

maple [F] time = 11.62, size = 0, normalized size = 0.00

$$\int \frac{\left(d \sin\left(f x + e\right)\right)^{n} \left(A + B \sin\left(f x + e\right)\right)}{\left(a + a \sin\left(f x + e\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)
```

```
[Out] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B \sin\left(f x + e\right) + A\right) \left(d \sin\left(f x + e\right)\right)^{n}}{\left(a \sin\left(f x + e\right) + a\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^3, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,\sin\left(e+f\,x\right)\right)}^n\,\left(A+B\,\sin\left(e+f\,x\right)\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^3,x)
```

```
[Out] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.7 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=336

$$-\frac{2a^3\left(A\left(32n^3+224n^2+478n+301\right)+2B\left(16n^3+104n^2+203n+115\right)\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+}{f(2n+3)(2n+5)(2n+7)\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -2*a*B*cos(f*x+e)*(d*sin(f*x+e))^(1+n)*(a+a*sin(f*x+e))^(3/2)/d/f/(7+2*n)-2
*a^3*(2*B*(16*n^3+104*n^2+203*n+115)+A*(32*n^3+224*n^2+478*n+301))*cos(f*x+
e)*hypergeom([1/2, -n],[3/2],1-sin(f*x+e))*(d*sin(f*x+e))^n/f/(3+2*n)/(5+2*
n)/(7+2*n)/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)-2*a^3*(2*B*(4*n^2+23*n+35)
+A*(8*n^2+50*n+77))*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(3+2*n)/(5+2*n)/(7+
2*n)/(a+a*sin(f*x+e))^(1/2)-2*a^2*(2*B*(5+n)+A*(7+2*n))*cos(f*x+e)*(d*sin(f
*x+e))^(1+n)*(a+a*sin(f*x+e))^(1/2)/d/f/(5+2*n)/(7+2*n)
```

Rubi [A] time = 0.87, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2976, 2981, 2776, 67, 65}

$$-\frac{2a^3\left(A\left(32n^3+224n^2+478n+301\right)+2B\left(16n^3+104n^2+203n+115\right)\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+}{f(2n+3)(2n+5)(2n+7)\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]),x]
```

```
[Out] (-2*a^3*(2*B*(115 + 203*n + 104*n^2 + 16*n^3) + A*(301 + 478*n + 224*n^2 +
32*n^3))*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*(d*
Sin[e + f*x])^n)/(f*(3 + 2*n)*(5 + 2*n)*(7 + 2*n)*Sin[e + f*x]^n*Sqrt[a + a
*Sin[e + f*x]]) - (2*a^3*(2*B*(35 + 23*n + 4*n^2) + A*(77 + 50*n + 8*n^2))*
Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*(5 + 2*n)*(7 + 2*n)*S
qrt[a + a*Sin[e + f*x]]) - (2*a^2*(2*B*(5 + n) + A*(7 + 2*n))*Cos[e + f*x]*
(d*Sin[e + f*x])^(1 + n)*Sqrt[a + a*Sin[e + f*x]])/(d*f*(5 + 2*n)*(7 + 2*n)
) - (2*a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n)*(a + a*Sin[e + f*x])^(3/2)
)/(d*f*(7 + 2*n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !IntegerQ[2*n]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = -\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\sin(e}{df(7+2n)}$$

$$= -\frac{2a^2(2B(5+n)+A(7+2n))\cos(e+fx)(d\sin}{df(5+2n)(7+}$$

$$= -\frac{2a^3\left(2B\left(35+23n+4n^2\right)+A\left(77+50n+8\right.\right.}{df(3+2n)(5+2n)(7+2n)\sqrt{}}$$

$$= -\frac{2a^3\left(2B\left(35+23n+4n^2\right)+A\left(77+50n+8\right.\right.}{df(3+2n)(5+2n)(7+2n)\sqrt{}}$$

$$= -\frac{2a^3\left(2B\left(35+23n+4n^2\right)+A\left(77+50n+8\right.\right.}{df(3+2n)(5+2n)(7+2n)\sqrt{}}$$

$$= -\frac{2a^3\left(2B\left(115+203n+104n^2+16n^3\right)+A\left(3\right.\right.}{}$$

Mathematica [A] time = 18.38, size = 596, normalized size = 1.77

$$\frac{2^{n+1}\tan\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)(a(\sin(e+fx)+1))^{5/2}\sin^{-n}(e+fx)\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\tan^2\left(\frac{1}{2}(e+fx)\right)+1}\right)^n\left(\tan^2\left(\frac{1}{2}(e+fx)\right)+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]
),x]
```

```
[Out] (2^(1 + n)*Sec[(e + f*x)/2]*(d*Sin[e + f*x])^n*(a*(1 + Sin[e + f*x]))^(5/2)
*Tan[(e + f*x)/2]*(Tan[(e + f*x)/2]/(1 + Tan[(e + f*x)/2]^2))^n*(1 + Tan[(e
 + f*x)/2]^2)^n*((A*Hypergeometric2F1[(1 + n)/2, 9/2 + n, (3 + n)/2, -Tan[(
e + f*x)/2]^2])/(1 + n) + (A*Hypergeometric2F1[4 + n/2, 9/2 + n, 5 + n/2, -
Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^7)/(8 + n) + Tan[(e + f*x)/2]*(((5*A +
 2*B)*Hypergeometric2F1[(2 + n)/2, 9/2 + n, (4 + n)/2, -Tan[(e + f*x)/2]^2]
)/(2 + n) + Tan[(e + f*x)/2]*(((11*A + 10*B)*Hypergeometric2F1[(3 + n)/2, 9
/2 + n, (5 + n)/2, -Tan[(e + f*x)/2]^2])/(3 + n) + Tan[(e + f*x)/2]*((5*(3*
A + 4*B)*Hypergeometric2F1[(4 + n)/2, 9/2 + n, (6 + n)/2, -Tan[(e + f*x)/2]
```

```
^2])/(4 + n) + Tan[(e + f*x)/2]*((5*(3*A + 4*B)*Hypergeometric2F1[9/2 + n,
(5 + n)/2, (7 + n)/2, -Tan[(e + f*x)/2]^2])/(5 + n) + Tan[(e + f*x)/2]*(((1
1*A + 10*B)*Hypergeometric2F1[9/2 + n, (6 + n)/2, (8 + n)/2, -Tan[(e + f*x)
/2]^2])/(6 + n) + ((5*A + 2*B)*Hypergeometric2F1[9/2 + n, (7 + n)/2, (9 + n
)/2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(7 + n)))))))/(f*Sqrt[Sec[(e +
 f*x)/2]^2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sin[e + f*x]^n)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left((A+2B)a^2\cos\left(fx+e\right)^2-2(A+B)a^2+\left(Ba^2\cos\left(fx+e\right)^2-2(A+B)a^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e
))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e))^
n, x)
```

maple [F] time = 1.09, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)\right)^n\left(a+a\sin\left(fx+e\right)\right)^{\frac{5}{2}}\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e))^
n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d\,\sin\left(e+f\,x\right)\right)^n\,\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.8 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=229

$$-\frac{2a^2\left(A\left(8n^2+30n+25\right)+2B\left(4n^2+13n+9\right)\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(2n+3)(2n+5)\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -2*a^2*(2*B*(4*n^2+13*n+9)+A*(8*n^2+30*n+25))*cos(f*x+e)*hypergeom([1/2, -n
],[3/2],1-sin(f*x+e))*(d*sin(f*x+e))^n/f/(3+2*n)/(5+2*n)/(sin(f*x+e)^n)/(a+
a*sin(f*x+e))^(1/2)-2*a^2*(2*B*(3+n)+A*(5+2*n))*cos(f*x+e)*(d*sin(f*x+e))^(
1+n)/d/f/(3+2*n)/(5+2*n)/(a+a*sin(f*x+e))^(1/2)-2*a*B*cos(f*x+e)*(d*sin(f*x
+e))^(1+n)*(a+a*sin(f*x+e))^(1/2)/d/f/(5+2*n)
```

Rubi [A] time = 0.49, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2976, 2981, 2776, 67, 65}

$$-\frac{2a^2\left(A\left(8n^2+30n+25\right)+2B\left(4n^2+13n+9\right)\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(2n+3)(2n+5)\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]),x]
```

```
[Out] (-2*a^2*(2*B*(9 + 13*n + 4*n^2) + A*(25 + 30*n + 8*n^2))*Cos[e + f*x]*Hyper
geometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*(d*Sin[e + f*x])^n)/(f*(3 + 2*
n)*(5 + 2*n)*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]]) - (2*a^2*(2*B*(3 + n)
 + A*(5 + 2*n))*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*(5 +
2*n)*Sqrt[a + a*Sin[e + f*x]]) - (2*a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 +
n)*Sqrt[a + a*Sin[e + f*x]])/(d*f*(5 + 2*n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
```

```
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = -\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}\sqrt{a+a\sin(e}{df(5+2n)}$$

$$= -\frac{2a^2(2B(3+n)+A(5+2n))\cos(e+fx)(d\sin(}{df(3+2n)(5+2n)\sqrt{a+a\sin(e+fx}$$

$$= -\frac{2a^2(2B(3+n)+A(5+2n))\cos(e+fx)(d\sin(}{df(3+2n)(5+2n)\sqrt{a+a\sin(e+fx}$$

$$= -\frac{2a^2(2B(3+n)+A(5+2n))\cos(e+fx)(d\sin(}{df(3+2n)(5+2n)\sqrt{a+a\sin(e+fx}$$

$$= -\frac{2a^2\left(2B\left(9+13n+4n^2\right)+A\left(25+30n+8n^2\right)}{f(3+}$$

Mathematica [B] time = 15.40, size = 478, normalized size = 2.09

$$2^{n+1}\tan\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)(a(\sin(e+fx)+1))^{3/2}\sin^{-n}(e+fx)\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\tan^2\left(\frac{1}{2}(e+fx)\right)+1}\right)^n\left(\tan^2\left(\frac{1}{2}(e+fx)\right)+1\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]
),x]
```

```
[Out] (2^(1 + n)*Sec[(e + f*x)/2]*(d*Sin[e + f*x])^n*(a*(1 + Sin[e + f*x]))^(3/2)
*Tan[(e + f*x)/2]*(Tan[(e + f*x)/2]/(1 + Tan[(e + f*x)/2]^2))^n*(1 + Tan[(e
 + f*x)/2]^2)^n*((A*Hypergeometric2F1[(1 + n)/2, 7/2 + n, (3 + n)/2, -Tan[(
e + f*x)/2]^2])/(1 + n) + Tan[(e + f*x)/2]*(((3*A + 2*B)*Hypergeometric2F1[
(2 + n)/2, 7/2 + n, (4 + n)/2, -Tan[(e + f*x)/2]^2])/(2 + n) + Tan[(e + f*x
)/2]*((2*(2*A + 3*B)*Hypergeometric2F1[(3 + n)/2, 7/2 + n, (5 + n)/2, -Tan[
(e + f*x)/2]^2])/(3 + n) + Tan[(e + f*x)/2]*((2*(2*A + 3*B)*Hypergeometric2
F1[7/2 + n, (4 + n)/2, (6 + n)/2, -Tan[(e + f*x)/2]^2])/(4 + n) + Tan[(e +
f*x)/2]*(((3*A + 2*B)*Hypergeometric2F1[7/2 + n, (5 + n)/2, (7 + n)/2, -Tan
[(e + f*x)/2]^2])/(5 + n) + (A*Hypergeometric2F1[7/2 + n, (6 + n)/2, (8 + n
)/2, -Tan[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(6 + n)))))))/(f*Sqrt[Sec[(e +
f*x)/2]^2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[e + f*x]^n)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(Ba\cos\left(fx+e\right)^{2}-(A+B)a\sin\left(fx+e\right)-(A+B)a\right)\sqrt{a\sin\left(fx+e\right)+a}\,\left(d\sin\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] integral(-(B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*
sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e))^
n, x)
```

maple [F] time = 1.05, size = 0, normalized size = 0.00

$$\int\left(d\sin\left(fx+e\right)\right)^{n}\left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e))^
n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d\,\sin\left(e+f\,x\right)\right)^n\,\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2),x)
[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e)),x)
[Out] Timed out
```

3.9 $\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=137

$$-\frac{2a(A(2n+3)+2B(n+1))\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2aB\cos(}{df(2n}$$

```
[Out] -2*a*(2*B*(1+n)+A*(3+2*n))*cos(f*x+e)*hypergeom([1/2, -n],[3/2],1-sin(f*x+e
))*(d*sin(f*x+e))^n/f/(3+2*n)/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)-2*a*B*c
os(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(3+2*n)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2981, 2776, 67, 65}

$$-\frac{2a(A(2n+3)+2B(n+1))\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2aB\cos(}{df(2n}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]),x]
```

```
[Out] (-2*a*(2*B*(1 + n) + A*(3 + 2*n))*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3
/2, 1 - Sin[e + f*x]]*(d*Sin[e + f*x])^n)/(f*(3 + 2*n)*Sin[e + f*x]^n*Sqrt[
a + a*Sin[e + f*x]]) - (2*a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(
3 + 2*n)*Sqrt[a + a*Sin[e + f*x]])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))\,dx = -\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \left(A+\frac{2B(1}{3+}\right.$$

$$= -\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{\left(a^2\left(A+\frac{2I}{}\right.\right.}{}$$

$$= -\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{\left(a^2\left(A+\frac{2I}{}\right.\right.}{}$$

$$= -\frac{2a\left(A+\frac{2B(1+n)}{3+2n}\right)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\mathrm{si}\right.}{f\sqrt{a+a\sin(e}}$$

Mathematica [C] time = 67.65, size = 409, normalized size = 2.99

$$-\frac{(1+i)2^{-n-2}e^{ifnx-\frac{3ie}{2}}\left(1-e^{2i(e+fx)}\right)^{-n}\left(-ie^{-i(e+fx)}\left(-1+e^{2i(e+fx)}\right)\right)^n\sqrt{a(\sin(e+fx)+1)}\,\sin^{-n}(e+fx)(d\sin(e+f}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^n*Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]),
x]
```

```
[Out] ((-1 - I)*2^(-2 - n)*E^(((-3*I)/2)*e + I*f*n*x)*(((-I)*(-1 + E^((2*I)*(e +
f*x))))/E^(I*(e + f*x)))^n*((2*B*Hypergeometric2F1[(-3 - 2*n)/4, -n, (1 - 2
*n)/4, E^((2*I)*(e + f*x))])/(E^((I/2)*f*(3 + 2*n)*x)*f*(3 + 2*n)) + 2*E^(I
*e)*(((-I)*(2*A + B)*Hypergeometric2F1[(-1 - 2*n)/4, -n, (3 - 2*n)/4, E^((2
*I)*(e + f*x))])/(E^((I/2)*f*(1 + 2*n)*x)*(f + 2*f*n)) + (E^((I/2)*(2*e + f
*(1 - 2*n)*x))*(-((2*A + B)*(-3 + 2*n)*Hypergeometric2F1[(1 - 2*n)/4, -n, (
5 - 2*n)/4, E^((2*I)*(e + f*x))]) + I*B*E^(I*(e + f*x))*(-1 + 2*n)*Hypergeo
metric2F1[(3 - 2*n)/4, -n, (7 - 2*n)/4, E^((2*I)*(e + f*x))]))/(f*(-3 + 2*n
)*(-1 + 2*n))))*(d*Sin[e + f*x])^n*Sqrt[a*(1 + Sin[e + f*x])])/((1 - E^((2*
I)*(e + f*x)))^n*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sin[e + f*x]^n)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\,\left(d\sin\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n,
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\,\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n,
 x)
```

maple [F] time = 1.06, size = 0, normalized size = 0.00

$$\int\left(d\sin\left(fx+e\right)\right)^{n}\sqrt{a+a\sin\left(fx+e\right)}\,\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\,\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n,
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\sin\left(e+f\,x\right)\right)}^n\,\left(A+B\,\sin\left(e+f\,x\right)\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\left(d\sin\left(e+fx\right)\right)^{n}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(d*sin(e + f*x))**n*(A + B*sin(e + f*x)
), x)
```

3.10 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=152

$$-\frac{(A-B)\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{f\sqrt{a\sin(e+fx)+a}}-2B\cos($$

```
[Out] -(A-B)*AppellF1(1/2,-n,1,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(d
*sin(f*x+e))^n/f/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)-2*B*cos(f*x+e)*hyper
geom([1/2, -n],[3/2],1-sin(f*x+e))*(d*sin(f*x+e))^n/f/(sin(f*x+e)^n)/(a+a*s
in(f*x+e))^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {2987, 2787, 2786, 2785, 130, 429, 2776, 67, 65}

$$-\frac{(A-B)\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{f\sqrt{a\sin(e+fx)+a}}-2B\cos($$

Antiderivative was successfully verified.

```
[In] Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -(((A - B)*AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2
]*Cos[e + f*x]*(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x
]])) - (2*B*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*
(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = (A-B)\int \frac{(d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx + \frac{B\int(d\sin(e+fx))^n\sqrt{a+a\sin}}{a}$$

$$= \frac{\left((A-B)\sqrt{1+\sin(e+fx)}\right)\int \frac{(d\sin(e+fx))^n}{\sqrt{1+\sin(e+fx)}}\,dx}{\sqrt{a+a\sin(e+fx)}} + \frac{(aB\cos(e+fx))\,\mathrm{S}}{f\sqrt{a-a\sin(}}$$

$$= \frac{\left((A-B)\sin^{-n}(e+fx)(d\sin(e+fx))^n\sqrt{1+\sin(e+fx)}\right)\int \frac{\sin^n(e}{\sqrt{1+\sin(}}}{\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2B\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)\sin^{-n}(e+fx)(d\sin}{f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2B\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)\sin^{-n}(e+fx)(d\sin}{f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{(A-B)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 4.86, size = 250, normalized size = 1.64

$$\frac{\cos(e+fx)\sqrt{a(\sin(e+fx)+1)}\ \sin^n(e+fx)\left(-\sin^2(e+fx)\right)^{-n}\left(1-\frac{1}{\sin(e+fx)+1}\right)^{-n}(d\sin(e+fx))^n\left(4(A-B)\sqrt{\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/Sqrt[a + a*Sin[e + f*x]
],x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]^n*(d*Sin[e + f*x])^n*Sqrt[a*(1 + Sin[e + f*x])]*
(4*(A - B)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 +
 Sin[e + f*x])^(-1)]*(-Sin[e + f*x])^n*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e
+ f*x])] - (A + B)*(1 + 2*n)*AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2,
1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*x]]*(1 - (1 + Sin[e + f*x])^(-1))^n)
)/(4*a*f*(1 + 2*n)*(-1 + Sin[e + f*x])*(-Sin[e + f*x]^2)^n*(1 - (1 + Sin[e
+ f*x])^(-1))^n)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)\right)^{n}}{\sqrt{a\sin\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/sqrt(a*sin(f*x + e) + a),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)\right)^{n}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/sqrt(a*sin(f*x + e) + a),
 x)
```

maple [F] time = 0.90, size = 0, normalized size = 0.00

$$\int\frac{\left(d\sin\left(fx+e\right)\right)^{n}\left(A+B\sin\left(fx+e\right)\right)}{\sqrt{a+a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)\right)^{n}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/sqrt(a*sin(f*x + e) + a),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,\sin\left(e+f\,x\right)\right)^{n}\,\left(A+B\,\sin\left(e+f\,x\right)\right)}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^(1/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(e+fx\right)\right)^{n}\left(A+B\sin\left(e+fx\right)\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((d*sin(e + f*x))**n*(A + B*sin(e + f*x))/sqrt(a*(sin(e + f*x) + 1)
), x)
```

3.11 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=226

$$-\frac{(-4An+A+B(4n+3))\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+}{4af\sqrt{a\sin(e+fx)+a}}$$

```
[Out] 1/2*(A-B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(a+a*sin(f*x+e))^(3/2)-1/4*(A
-4*A*n+B*(3+4*n))*AppellF1(1/2,-n,1,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*co
s(f*x+e)*(d*sin(f*x+e))^n/a/f/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)-1/2*(A-
B)*(1+2*n)*cos(f*x+e)*hypergeom([1/2, -n],[3/2],1-sin(f*x+e))*(d*sin(f*x+e)
)^n/a/f/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2978, 2987, 2787, 2786, 2785, 130, 429, 2776, 67, 65}

$$-\frac{(-4An+A+B(4n+3))\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+}{4af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(2*d*f*(a + a*Sin[e + f*x])
^(3/2)) - ((A - 4*A*n + B*(3 + 4*n))*AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e +
f*x], (1 - Sin[e + f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n)/(4*a*f*Sin[e +
 f*x]^n*Sqrt[a + a*Sin[e + f*x]]) - ((A - B)*(1 + 2*n)*Cos[e + f*x]*Hyperge
ometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*(d*Sin[e + f*x])^n)/(2*a*f*Sin[e
 + f*x]^n*Sqrt[a + a*Sin[e + f*x]])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
```

```
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
```

```
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{(d\sin(e+fx))^n\left(ad(A+B-An+B\right.}{\sqrt{a+a\,s}}}{2}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} + \frac{((A-B)(1+2n))\int(d\sin}{}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} + \frac{\left(\left(-\frac{1}{2}a^2(A-B)d(1+2n)\right.\right.}{}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} + \frac{\left(\left(-\frac{1}{2}a^2(A-B)d(1+2n)\right.\right.}{}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)(1+2n)\cos(e+f}{}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)(1+2n)\cos(e+f}{}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} - \frac{(A+3B-4An+4Bn)F_1}{}$$

Mathematica [B] time = 21.90, size = 523, normalized size = 2.31

$$\frac{\sec(e+fx)(d\sin(e+fx))^n\left(A\left(a^2\sqrt{2-2\sin(e+fx)}\,(\sin(e+fx)+1)^2(-\sin(e+fx))^{-n}F_1\left(1;\frac{1}{2},-n;2;\frac{1}{2}(\sin(e\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(3
/2),x]
```

```
[Out] (Sec[e + f*x]*(d*Sin[e + f*x])^n*(a*B*(1 + Sin[e + f*x])*((a*AppellF1[1, 1/
2, -n, 2, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*x]]*
(1 + Sin[e + f*x]))/(-Sin[e + f*x])^n - (4*Sqrt[(-1 + Sin[e + f*x])/(1 + Si
n[e + f*x])]*(-2*a*(1 + 2*n)*AppellF1[1/2 - n, -1/2, -n, 3/2 - n, 2/(1 + Si
n[e + f*x]), (1 + Sin[e + f*x])^(-1)] + a*(-1 + 2*n)*AppellF1[-1/2 - n, -1/
```

```
2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)]*(1 + Sin[e +
 f*x])))/((-1 + 4*n^2)*(1 - (1 + Sin[e + f*x])^(-1))^n)) + A*((a^2*AppellF1
[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e +
f*x]]*(1 + Sin[e + f*x])^2)/(-Sin[e + f*x])^n - (4*a*Sqrt[(-1 + Sin[e + f*x
])/(1 + Sin[e + f*x])]*(1 + Sin[e + f*x])*(2*a*(1 + 2*n)*AppellF1[1/2 - n,
-1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)] + a*(-1 +
 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[
e + f*x])^(-1)]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*(1 - (1 + Sin[e + f*x])^
(-1))^n))))/(8*a^3*f*Sqrt[a*(1 + Sin[e + f*x])])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\,\left(d\sin\left(fx+e\right)\right)^{n}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n/
(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^(3/2
), x)
```

maple [F] time = 0.89, size = 0, normalized size = 0.00

$$\int\frac{\left(d\sin\left(fx+e\right)\right)^{n}\left(A+B\sin\left(fx+e\right)\right)}{\left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^(3/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,\sin\left(e+f\,x\right)\right)}^n\,\left(A+B\,\sin\left(e+f\,x\right)\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^(3/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sin\left(e+fx\right)\right)^{n}\left(A+B\sin\left(e+fx\right)\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*sin(e + f*x))**n*(A + B*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(
3/2), x)
```

3.12 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=221

$$-\frac{2^{m+\frac{1}{2}}(A-B)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^nF_1\left(\frac{1}{2};-n,\frac{1}{2}-\right.}{f}$$

```
[Out] -2^(3/2+m)*B*AppellF1(1/2,-n,-1/2-m,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*co
s(f*x+e)*(d*sin(f*x+e))^n*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f/(sin
(f*x+e)^n)-2^(1/2+m)*(A-B)*AppellF1(1/2,-n,1/2-m,3/2,1-sin(f*x+e),1/2-1/2*s
in(f*x+e))*cos(f*x+e)*(d*sin(f*x+e))^n*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x
+e))^m/f/(sin(f*x+e)^n)
```

Rubi [A] time = 0.45, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2987, 2787, 2786, 2785, 133}

$$-\frac{2^{m+\frac{1}{2}}(A-B)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^nF_1\left(\frac{1}{2};-n,\frac{1}{2}-\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((2^(3/2 + m)*B*AppellF1[1/2, -n, -1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Si
n[e + f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^(-1/2 - m
)*(a + a*Sin[e + f*x])^m)/(f*Sin[e + f*x]^n)) - (2^(1/2 + m)*(A - B)*Appell
F1[1/2, -n, 1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e + f
*x]*(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m
)/(f*Sin[e + f*x]^n)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
```

```
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = (A-B)\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m\,dx$$

$$= \Big((A-B)(1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\Big)$$

$$= \Big((A-B)\sin^{-n}(e+fx)(d\sin(e+fx))^n(1+\sin(e$$

$$= -\frac{\Big((A-B)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx}{}$$

$$= -\frac{2^{\frac{3}{2}+m}BF_1\left(\frac{1}{2};-n,-\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1}{}$$

Mathematica [B] time = 22.18, size = 5918, normalized size = 26.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^m\left(d\sin\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^m\left(d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x
)
```

maple [F] time = 6.04, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)\right)^{n}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(d\,\sin\left(e+f\,x\right)\right)}^n\,\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
```

```
[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(d\sin\left(e+fx\right)\right)^{n}\left(A+B\sin\left(e+fx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(d*sin(e + f*x))**n*(A + B*sin(e + f*x))
, x)
```

3.13 $\int (d\sin(e+fx))^n (a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=114

$$\frac{\sec(e+fx)(a-a\sin(e+fx))(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m(d\sin(e+fx))^{n+1}F_1\left(n+1;-\frac{1}{2},\frac{1}{2}-m;n+\right.}{df(n+1)\sqrt{1-\sin(e+fx)}}$$

```
[Out] AppellF1(1+n,1/2-m,-1/2,2+n,-sin(f*x+e),sin(f*x+e))*sec(f*x+e)*(d*sin(f*x+e
))^(1+n)*(1+sin(f*x+e))^(1/2-m)*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m/d/f/(1+
n)/(1-sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3008, 135, 133}

$$\frac{\sec(e+fx)(a-a\sin(e+fx))(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m(d\sin(e+fx))^{n+1}F_1\left(n+1;-\frac{1}{2},\frac{1}{2}-m;n+\right.}{df(n+1)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (AppellF1[1 + n, -1/2, 1/2 - m, 2 + n, Sin[e + f*x], -Sin[e + f*x]]*Sec[e +
 f*x]*(d*Sin[e + f*x])^(1 + n)*(1 + Sin[e + f*x])^(1/2 - m)*(a - a*Sin[e +
f*x])*(a + a*Sin[e + f*x])^m)/(d*f*(1 + n)*Sqrt[1 - Sin[e + f*x]])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 3008

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(f*Cos[e + f*x]
), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, Si
n[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c +
 a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+f\ldots\right.}{\ldots}$$

$$= \frac{\left(\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx\ldots\right.}{f\sqrt{1\ldots}}$$

$$= \frac{\left(\sec(e+fx)(1+\sin(e+fx))^{\frac{1}{2}-m}(a-a\sin(e+f\ldots\right.}{\ldots}$$

$$= \frac{F_1\left(1+n;-\frac{1}{2},\frac{1}{2}-m;2+n;\sin(e+fx),-\sin(e+\ldots\right.}{\ldots}$$

Mathematica [F] time = 11.25, size = 0, normalized size = 0.00

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] Integrate[(d*Sin[e + f*x])^n*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m, x
]
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a\sin\left(fx+e\right)-a\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x, algorithm
="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(a\sin\left(fx+e\right)-a\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x, algorithm
="giac")
```

```
[Out] integrate(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n,
x)
```

maple [F] time = 5.91, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)\right)^{n}\left(a-a\sin\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(a\sin\left(fx+e\right)-a\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x, algorithm
="maxima")
```

```
[Out] -integrate((a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n,
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\sin\left(e+f\,x\right)\right)}^n\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(a-a\,\sin\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(a + a*sin(e + f*x))^m*(a - a*sin(e + f*x)),x)
```

```
[Out] int((d*sin(e + f*x))^n*(a + a*sin(e + f*x))^m*(a - a*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-a\left(\int\left(-\left(d\sin\left(e+fx\right)\right)^{n}\left(a\sin\left(e+fx\right)+a\right)^{m}\right)dx+\int\left(d\sin\left(e+fx\right)\right)^{n}\left(a\sin\left(e+fx\right)+a\right)^{m}\sin\left(e+fx\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))**m,x)
```

```
[Out] -a*(Integral(-(d*sin(e + f*x))**n*(a*sin(e + f*x) + a)**m, x) + Integral((d
*sin(e + f*x))**n*(a*sin(e + f*x) + a)**m*sin(e + f*x), x))
```

3.14 $\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx$

Optimal. Leaf size=37

$$-\frac{\cos(c+dx)\sin^{n+1}(c+dx)(a\sin(c+dx)+a)^{-n-2}}{d}$$

```
[Out] -cos(d*x+c)*sin(d*x+c)^(1+n)*(a+a*sin(d*x+c))^(-2-n)/d
```

Rubi [A] time = 0.12, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2974}

$$-\frac{\cos(c+dx)\sin^{n+1}(c+dx)(a\sin(c+dx)+a)^{-n-2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^n*(a + a*Sin[c + d*x])^(-2 - n)*(-1 - n - (-2 - n)*Sin[c +
 d*x]),x]
```

```
[Out] -((Cos[c + d*x]*Sin[c + d*x]^(1 + n)*(a + a*Sin[c + d*x])^(-2 - n))/d)
```

Rule 2974

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n},
x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[m
+ n + 2, 0] && EqQ[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]
```

Rubi steps

$$\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx = -\frac{\cos(c+dx)\sin^{1+n}(c+dx)(a+a\sin(}{d}$$

Mathematica [B] time = 1.63, size = 107, normalized size = 2.89

$$-\frac{2^n\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)(-\sin(c+dx)+\cos(c+dx)+1)(a(\sin(c+dx)+1))^{-n-2}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^n*(a + a*Sin[c + d*x])^(-2 - n)*(-1 - n - (-2 - n)*S
in[c + d*x]),x]
```

```
[Out] -((2^n*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Cos[(c + d*x
)/4]*(-Sin[(c + d*x)/4] + Sin[(3*(c + d*x))/4]))^n*(1 + Cos[c + d*x] - Sin[
c + d*x])*(a*(1 + Sin[c + d*x]))^(-2 - n))/d)
```

fricas [A] time = 0.45, size = 41, normalized size = 1.11

$$-\frac{(a \sin(dx+c)+a)^{-n-2} \sin(dx+c)^{n} \cos(dx+c) \sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x,
algorithm="fricas")
```

```
[Out] -(a*sin(d*x + c) + a)^(-n - 2)*sin(d*x + c)^n*cos(d*x + c)*sin(d*x + c)/d
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x,
algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 4.15, size = 0, normalized size = 0.00

$$\int (\sin^{n}(dx+c)) (a+a\sin(dx+c))^{-2-n} (-1-n-(-2-n)\sin(dx+c))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x)
```

```
[Out] int(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int ((n+2)\sin(dx+c)-n-1)(a\sin(dx+c)+a)^{-n-2} \sin(dx+c)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x,
algorithm="maxima")
```

```
[Out] integrate(((n + 2)*sin(d*x + c) - n - 1)*(a*sin(d*x + c) + a)^(-n - 2)*sin(
d*x + c)^n, x)
```

mupad [B] time = 12.98, size = 61, normalized size = 1.65

$$-\frac{\sin(c+d\,x)^n\,\sin(2\,c+2\,d\,x)}{a^2\,d\,(a\,(\sin(c+d\,x)+1))^n\,\left(2\sin(c+d\,x)^2+4\,\sin(c+d\,x)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(sin(c + d*x)^n*(n - sin(c + d*x)*(n + 2) + 1))/(a + a*sin(c + d*x))^(
n + 2),x)
```

```
[Out] -(sin(c + d*x)^n*sin(2*c + 2*d*x))/(a^2*d*(a*(sin(c + d*x) + 1))^n*(4*sin(c
 + d*x) + 2*sin(c + d*x)^2 + 2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**n*(a+a*sin(d*x+c))**(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x
)
```

```
[Out] Timed out
```

3.15 $\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx$

Optimal. Leaf size=35

$$-\frac{\cos(c+dx)\sin^{-m-1}(c+dx)(a\sin(c+dx)+a)^m}{d}$$

```
[Out] -cos(d*x+c)*sin(d*x+c)^(-1-m)*(a+a*sin(d*x+c))^m/d
```

Rubi [A] time = 0.09, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {2974}

$$-\frac{\cos(c+dx)\sin^{-m-1}(c+dx)(a\sin(c+dx)+a)^m}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^(-2 - m)*(a + a*Sin[c + d*x])^m*(1 + m - m*Sin[c + d*x]),x
]
```

```
[Out] -((Cos[c + d*x]*Sin[c + d*x]^(-1 - m)*(a + a*Sin[c + d*x])^m)/d)
```

Rule 2974

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n},
x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[m
+ n + 2, 0] && EqQ[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]
```

Rubi steps

$$\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx = -\frac{\cos(c+dx)\sin^{-1-m}(c+dx)(a+a\sin(c+$$

Mathematica [A] time = 0.39, size = 35, normalized size = 1.00

$$-\frac{\cos(c+dx)\sin^{-m-1}(c+dx)(a(\sin(c+dx)+1))^m}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^(-2 - m)*(a + a*Sin[c + d*x])^m*(1 + m - m*Sin[c + d
*x]),x]
```

```
[Out] -((Cos[c + d*x]*Sin[c + d*x]^(-1 - m)*(a*(1 + Sin[c + d*x]))^m)/d)
```

fricas [A]   time = 0.45, size = 41, normalized size = 1.17

$$-\frac{(a\sin(dx+c)+a)^m \sin(dx+c)^{-m-2}\cos(dx+c)\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x, algori
thm="fricas")
```

```
[Out] -(a*sin(d*x + c) + a)^m*sin(d*x + c)^(-m - 2)*cos(d*x + c)*sin(d*x + c)/d
```

giac [B]   time = 64.01, size = 5502, normalized size = 157.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x, algori
thm="giac")
```

```
[Out] -8*(cos(2*pi*m*floor(-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*t
an(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2
*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) + 1/4*pi*m*sgn(4*tan(d*x + c)^2*
tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x
+ c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) - 1/4*pi*m)
*e^(m*log(sqrt(2)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(
d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)
^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + ab
s(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x +
1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*
c) + 2)*tan(d*x + c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*ta
n(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*
c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x +
 c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*ta
n(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*abs(
a)/(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x + c)^2 + tan(1/2*d*x +
1/2*c)^2 + 1)) - m*log(4*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
+ 1)) - 2*log(4*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)))*ta
n(-1/2*pi + 1/4*pi*m*sgn(2*a*tan(1/2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2
```

```
*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*
a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/4*pi*m*sgn(tan(1/2
*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(4*a*tan(1/2*d
*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/
2*pi*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*pi*m +
 pi*floor(d*x/pi + c/pi + 1/2))^2*tan(1/2*d*x + 1/2*c)^3 - 2*e^(m*log(sqrt(
2)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(
1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*
d*x + 1/2*c) + 2)*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c
)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(
d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x
 + c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*ta
n(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/
2*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d
*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 +
2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*abs(a)/(tan(d*x + c
)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x + c)^2 + tan(1/2*d*x + 1/2*c)^2 + 1))
- m*log(4*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)) - 2*log(4
*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)))*sin(2*pi*m*floor(
-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2
*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x
 + 1/2*c) + 2) + 5/8) + 1/4*pi*m*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^
2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*
x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) - 1/4*pi*m)*tan(-1/2*pi + 1/4*pi
*m*sgn(2*a*tan(1/2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/
2*d*x + 1/2*c) - 2*a)*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/
2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 -
1)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*
a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/2*pi*sgn(tan(1/2*d
*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*pi*m + pi*floor(d*x/pi +
 c/pi + 1/2))*tan(1/2*d*x + 1/2*c)^3 - cos(2*pi*m*floor(-1/8*sgn(4*tan(d*x
+ c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*t
an(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/
8) + 1/4*pi*m*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^
2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*ta
n(1/2*d*x + 1/2*c) + 2) - 1/4*pi*m)*e^(m*log(sqrt(2)*sqrt(abs(4*tan(d*x + c
)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(
d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x
 + c)^2*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^
2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*
x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2 + abs(4*tan(d*x +
 c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*ta
n(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(1
/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x
 + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2
```

+ 8*tan(1/2*d*x + 1/2*c) + 2))*abs(a)/(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^
2 + tan(d*x + c)^2 + tan(1/2*d*x + 1/2*c)^2 + 1)) - m*log(4*abs(tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)) - 2*log(4*abs(tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 + 1)))*tan(-1/2*pi + 1/4*pi*m*sgn(2*a*tan(1/2*d*x
 + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*
sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d
*x + 1/2*c)) - 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1
/2*c)) + 1/4*pi*m*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)
^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/2*pi*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn
(tan(1/2*d*x + 1/2*c)) - 1/4*pi*m + pi*floor(d*x/pi + c/pi + 1/2))^2*tan(1/
2*d*x + 1/2*c) - cos(2*pi*m*floor(-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1
/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(
1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) + 1/4*pi*m*sgn(4*ta
n(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)
 + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2
) - 1/4*pi*m)*e^(m*log(sqrt(2)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*
c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2
*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2*tan(1/2*d*x +
1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*t
an(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1
/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/
2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1
/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1/2*c)^2 + ab
s(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x +
1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*
c) + 2))*abs(a)/(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x + c)^2 + t
an(1/2*d*x + 1/2*c)^2 + 1)) - m*log(4*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^2 + 1)) - 2*log(4*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c
)^2 + 1)))*tan(1/2*d*x + 1/2*c)^3 + 2*e^(m*log(sqrt(2)*sqrt(abs(4*tan(d*x +
 c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*ta
n(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d
*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c
)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*
d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2 + abs(4*tan(d*x
 + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*
tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan
(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d
*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^
2 + 8*tan(1/2*d*x + 1/2*c) + 2))*abs(a)/(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c
)^2 + tan(d*x + c)^2 + tan(1/2*d*x + 1/2*c)^2 + 1)) - m*log(4*abs(tan(1/2*d
*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)) - 2*log(4*abs(tan(1/2*d*x + 1/2*
c))/(tan(1/2*d*x + 1/2*c)^2 + 1)))*sin(2*pi*m*floor(-1/8*sgn(4*tan(d*x + c)
^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d
*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) +
 1/4*pi*m*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*ta

n(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) - 1/4*pi*m)*tan(-1/2*pi + 1/4*pi*m*sgn(2*a*tan(1/2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/2*pi*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*pi*m + pi*floor(d*x/pi + c/pi + 1/2))*tan(1/2*d*x + 1/2*c) + cos(2*pi*m*floor(-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) + 1/4*pi*m*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) - 1/4*pi*m)*e^(m*log(sqrt(2)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1/2*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*abs(a)/(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x + c)^2 + tan(1/2*d*x + 1/2*c)^2 + 1)) - m*log(4*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)) - 2*log(4*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)))*tan(1/2*d*x + 1/2*c))/(d*tan(-1/2*pi + 1/4*pi*m*sgn(2*a*tan(1/2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/2*pi*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*pi*m + pi*floor(d*x/pi + c/pi + 1/2))^2*tan(1/2*d*x + 1/2*c)^4 + 2*d*tan(-1/2*pi + 1/4*pi*m*sgn(2*a*tan(1/2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/2*pi*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*pi*m + pi*floor(d*x/pi + c/pi + 1/2))^2*tan(1/2*d*x + 1/2*c)^2 + d*tan(1/2*d*x + 1/2*c)^4 + d*tan(-1/2*pi + 1/4*pi*m*sgn(2*a*tan(1/2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1

```
/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/2*pi*sgn(tan(1/2*d*x + 1/2*c)^2 - 1
)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*pi*m + pi*floor(d*x/pi + c/pi + 1/2))^2 +
 2*d*tan(1/2*d*x + 1/2*c)^2 + d)
```

maple [F] time = 1.66, size = 0, normalized size = 0.00

$$\int \left(\sin^{-2-m}(dx+c)\right)(a+a\sin(dx+c))^{m}(1+m-m\sin(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x)
```

```
[Out] int(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int (m\sin(dx+c)-m-1)(a\sin(dx+c)+a)^{m}\sin(dx+c)^{-m-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x, algori
thm="maxima")
```

```
[Out] -integrate((m*sin(d*x + c) - m - 1)*(a*sin(d*x + c) + a)^m*sin(d*x + c)^(-m
 - 2), x)
```

mupad [B] time = 12.85, size = 38, normalized size = 1.09

$$-\frac{\sin(2\,c+2\,d\,x)\,\left(a\,\left(\sin(c+d\,x)+1\right)\right)^{m}}{2\,d\sin(c+d\,x)^{m+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(c + d*x))^m*(m - m*sin(c + d*x) + 1))/sin(c + d*x)^(m + 2),
x)
```

```
[Out] -(sin(2*c + 2*d*x)*(a*(sin(c + d*x) + 1))^m)/(2*d*sin(c + d*x)^(m + 2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**(-2-m)*(a+a*sin(d*x+c))**m*(1+m-m*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.16 $\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=153

$$\frac{a^2(Ab-aB)\cos(e+fx)}{b^2f\left(a^2-b^2\right)(a+b\sin(e+fx))}-\frac{2a\left(-2a^3B+a^2Ab+3ab^2B-2Ab^3\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3f\left(a^2-b^2\right)^{3/2}}+\frac{x(Ab-2aB)}{b^3}-\frac{B}{}$$

```
[Out] (A*b-2*B*a)*x/b^3-2*a*(A*a^2*b-2*A*b^3-2*B*a^3+3*B*a*b^2)*arctan((b+a*tan(1
/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/b^3/(a^2-b^2)^(3/2)/f-B*cos(f*x+e)/b^2/f+a^
2*(A*b-B*a)*cos(f*x+e)/b^2/(a^2-b^2)/f/(a+b*sin(f*x+e))
```

Rubi [A] time = 0.39, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2988, 3023, 2735, 2660, 618, 204}

$$-\frac{2a\left(a^2Ab-2a^3B+3ab^2B-2Ab^3\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3f\left(a^2-b^2\right)^{3/2}}+\frac{a^2(Ab-aB)\cos(e+fx)}{b^2f\left(a^2-b^2\right)(a+b\sin(e+fx))}+\frac{x(Ab-2aB)}{b^3}-\frac{B}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[e + f*x]^2*(A + B*Sin[e + f*x]))/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((A*b - 2*a*B)*x)/b^3 - (2*a*(a^2*A*b - 2*A*b^3 - 2*a^3*B + 3*a*b^2*B)*ArcT
an[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(b^3*(a^2 - b^2)^(3/2)*f) - (
B*Cos[e + f*x])/(b^2*f) + (a^2*(A*b - a*B)*Cos[e + f*x])/(b^2*(a^2 - b^2)*f
*(a + b*Sin[e + f*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx = \frac{a^2(Ab-aB)\cos(e+fx)}{b^2\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{\int \frac{ab(Ab-aB)+\left(a^2-b^2\right)(Ab-aB)\sin(e+fx)+b(\cdots}{a+b\sin(e+fx)}}{b^2\left(a^2-b^2\right)\cdots}$$

$$= -\frac{B\cos(e+fx)}{b^2f} + \frac{a^2(Ab-aB)\cos(e+fx)}{b^2\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{\int \frac{ab^2(Ab-aB)+b\left(a^2\cdots\right.}{a+b\cdots}}{b^3\left(\cdots\right.}$$

$$= \frac{(Ab-2aB)x}{b^3} - \frac{B\cos(e+fx)}{b^2f} + \frac{a^2(Ab-aB)\cos(e+fx)}{b^2\left(a^2-b^2\right)f(a+b\sin(e+fx))} - \frac{(a\cdots}{\cdots}$$

$$= \frac{(Ab-2aB)x}{b^3} - \frac{B\cos(e+fx)}{b^2f} + \frac{a^2(Ab-aB)\cos(e+fx)}{b^2\left(a^2-b^2\right)f(a+b\sin(e+fx))} - \frac{(2\cdots}{\cdots}$$

$$= \frac{(Ab-2aB)x}{b^3} - \frac{B\cos(e+fx)}{b^2f} + \frac{a^2(Ab-aB)\cos(e+fx)}{b^2\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{(4\cdots}{\cdots}$$

$$= \frac{(Ab-2aB)x}{b^3} - \frac{2a\left(a^2Ab-2Ab^3-2a^3B+3ab^2B\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+f\cdots\right.}{\sqrt{a^2-b^2}}\right.}{b^3\left(a^2-b^2\right)^{3/2}f}\cdots$$

Mathematica [A] time = 0.88, size = 147, normalized size = 0.96

$$\frac{\frac{a^2b(Ab-aB)\cos(e+fx)}{(a-b)(a+b)(a+b\sin(e+fx))} + \frac{2a\left(2a^3B-a^2Ab-3ab^2B+2Ab^3\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + (e+fx)(Ab-2aB) - bB\cos(e+fx)}{b^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[e + f*x]^2*(A + B*Sin[e + f*x]))/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((A*b - 2*a*B)*(e + f*x) + (2*a*(-(a^2*A*b) + 2*A*b^3 + 2*a^3*B - 3*a*b^2*B
)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - b*B
*Cos[e + f*x] + (a^2*b*(A*b - a*B)*Cos[e + f*x])/((a - b)*(a + b)*(a + b*Si
n[e + f*x])))/(b^3*f)
```

fricas [B] time = 0.52, size = 804, normalized size = 5.25

$$\left[-\frac{2\left(2\,Ba^{6}-Aa^{5}b-4\,Ba^{4}b^{2}+2\,Aa^{3}b^{3}+2\,Ba^{2}b^{4}-Aab^{5}\right)fx+\left(2\,Ba^{5}-Aa^{4}b-3\,Ba^{3}b^{2}+2\,Aa^{2}b^{3}+\left(2\,Ba^{4}b-\cdots\right.\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/2*(2*(2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A*a^3*b^3 + 2*B*a^2*b^4 - A*a
*b^5)*f*x + (2*B*a^5 - A*a^4*b - 3*B*a^3*b^2 + 2*A*a^2*b^3 + (2*B*a^4*b - A
*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(((2*
a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 + 2*(a*cos(f*x +
 e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 -
2*a*b*sin(f*x + e) - a^2 - b^2)) + 2*(2*B*a^5*b - A*a^4*b^2 - 3*B*a^3*b^3 +
 A*a^2*b^4 + B*a*b^5)*cos(f*x + e) + 2*((2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*b^
3 + 2*A*a^2*b^4 + 2*B*a*b^5 - A*b^6)*f*x + (B*a^4*b^2 - 2*B*a^2*b^4 + B*b^6
)*cos(f*x + e))*sin(f*x + e))/((a^4*b^4 - 2*a^2*b^6 + b^8)*f*sin(f*x + e) +
 (a^5*b^3 - 2*a^3*b^5 + a*b^7)*f), -((2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A
*a^3*b^3 + 2*B*a^2*b^4 - A*a*b^5)*f*x + (2*B*a^5 - A*a^4*b - 3*B*a^3*b^2 +
2*A*a^2*b^3 + (2*B*a^4*b - A*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)*sin(f*x + e
))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x +
e))) + (2*B*a^5*b - A*a^4*b^2 - 3*B*a^3*b^3 + A*a^2*b^4 + B*a*b^5)*cos(f*x
+ e) + ((2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*b^3 + 2*A*a^2*b^4 + 2*B*a*b^5 - A*
b^6)*f*x + (B*a^4*b^2 - 2*B*a^2*b^4 + B*b^6)*cos(f*x + e))*sin(f*x + e))/((
a^4*b^4 - 2*a^2*b^6 + b^8)*f*sin(f*x + e) + (a^5*b^3 - 2*a^3*b^5 + a*b^7)*f
)]
```

giac [B] time = 0.18, size = 371, normalized size = 2.42

$$\frac{\frac{2\left(2\,Ba^{4}-Aa^{3}b-3\,Ba^{2}b^{2}+2\,Aab^{3}\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{2}b^{3}-b^{5}\right)\sqrt{a^{2}-b^{2}}}-\frac{2\left(Ba^{2}b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-Aab^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+2\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\cdots\right.\right.}{\left(a\tan\left(\frac{1}{2}\cdots\right.\right.}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] (2*(2*B*a^4 - A*a^3*b - 3*B*a^2*b^2 + 2*A*a*b^3)*(pi*floor(1/2*(f*x + e)/pi
 + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^
```

```
2*b^3 - b^5)*sqrt(a^2 - b^2)) - 2*(B*a^2*b*tan(1/2*f*x + 1/2*e)^3 - A*a*b^2
*tan(1/2*f*x + 1/2*e)^3 + 2*B*a^3*tan(1/2*f*x + 1/2*e)^2 - A*a^2*b*tan(1/2*
f*x + 1/2*e)^2 - B*a*b^2*tan(1/2*f*x + 1/2*e)^2 + 3*B*a^2*b*tan(1/2*f*x + 1
/2*e) - A*a*b^2*tan(1/2*f*x + 1/2*e) - 2*B*b^3*tan(1/2*f*x + 1/2*e) + 2*B*a
^3 - A*a^2*b - B*a*b^2)/((a*tan(1/2*f*x + 1/2*e)^4 + 2*b*tan(1/2*f*x + 1/2*
e)^3 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)*(a^2*b^2
- b^4)) - (2*B*a - A*b)*(f*x + e)/b^3)/f
```

maple [B] time = 0.37, size = 493, normalized size = 3.22

$$-\frac{2B}{f\,b^2\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}+\frac{2A\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f\,b^2}-\frac{4B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a}{f\,b^3}+\frac{2a\tan\left(\ldots\right.}{f\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)a+2\tan\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x)
```

```
[Out] -2/f/b^2*B/(1+tan(1/2*f*x+1/2*e)^2)+2/f/b^2*A*arctan(tan(1/2*f*x+1/2*e))-4/
f/b^3*B*arctan(tan(1/2*f*x+1/2*e))*a+2/f*a/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/
2*f*x+1/2*e)*b+a)/(a^2-b^2)*tan(1/2*f*x+1/2*e)*A-2/f*a^2/b/(tan(1/2*f*x+1/2
*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*tan(1/2*f*x+1/2*e)*B+2/f*a^2/b/
(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*A-2/f*a^3/b^2/(
tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*B-2/f*a^3/b^2/(a
^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*A+4/
f*a/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2)
)*A+4/f*a^4/b^3/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^
2-b^2)^(1/2))*B-6/f*a^2/b/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e
)+2*b)/(a^2-b^2)^(1/2))*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 17.18, size = 3718, normalized size = 24.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x)^2*(A + B*sin(e + f*x)))/(a + b*sin(e + f*x))^2,x)
```

```
[Out] ((2*(A*a^2*b - 2*B*a^3 + B*a*b^2))/(b^2*(a^2 - b^2)) - (2*tan(e/2 + (f*x)/2
)^3*(B*a^2 - A*a*b))/(b*(a^2 - b^2)) + (2*tan(e/2 + (f*x)/2)*(2*B*b^2 - 3*B
*a^2 + A*a*b))/(b*(a^2 - b^2)) + (2*tan(e/2 + (f*x)/2)^2*(A*a^2*b - 2*B*a^3
 + B*a*b^2))/(b^2*(a^2 - b^2)))/(f*(a + 2*b*tan(e/2 + (f*x)/2) + 2*a*tan(e/
2 + (f*x)/2)^2 + a*tan(e/2 + (f*x)/2)^4 + 2*b*tan(e/2 + (f*x)/2)^3)) + (log
(tan(e/2 + (f*x)/2) + 1i)*(A*b - 2*B*a)*1i)/(b^3*f) - (log(tan(e/2 + (f*x)/
2) - 1i)*(A*b*1i - B*a*2i))/(b^3*f) - (a*atan(((a*(-(a + b)^3*(a - b)^3)^(1
/2)*((32*(A^2*a^2*b^8 - 2*A^2*a^4*b^6 + A^2*a^6*b^4 + 4*B^2*a^4*b^6 - 8*B^2
*a^6*b^4 + 4*B^2*a^8*b^2 - 4*A*B*a^3*b^7 + 8*A*B*a^5*b^5 - 4*A*B*a^7*b^3))/
(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)*(2*A^2*a*b^10 - 9*A^2*
a^3*b^8 + 8*A^2*a^5*b^6 - 2*A^2*a^7*b^4 + 8*B^2*a^3*b^8 - 29*B^2*a^5*b^6 +
28*B^2*a^7*b^4 - 8*B^2*a^9*b^2 - 8*A*B*a^2*b^9 + 32*A*B*a^4*b^7 - 30*A*B*a^
6*b^5 + 8*A*B*a^8*b^3))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (a*(-(a + b)^3*(a -
b)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(4*A*a^2*b^11 - 6*A*a^4*b^9 + 2*A*a^6*b
^7 - 6*B*a^3*b^10 + 10*B*a^5*b^8 - 4*B*a^7*b^6))/(b^10 - 2*a^2*b^8 + a^4*b^
6) - (32*(A*a^3*b^9 + 2*B*a^2*b^10 - 3*B*a^4*b^8 + B*a^6*b^6 - A*a*b^11))/(
b^9 - 2*a^2*b^7 + a^4*b^5) + (a*((32*(a^2*b^12 - 2*a^4*b^10 + a^6*b^8))/(b^
9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)*(3*a*b^14 - 8*a^3*b^12 +
7*a^5*b^10 - 2*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4*b^6))*(-(a + b)^3*(a - b)^
3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^
4*b^5 - a^6*b^3))*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b
^7 + 3*a^4*b^5 - a^6*b^3))*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2)*1i)/(b
^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3) + (a*(-(a + b)^3*(a - b)^3)^(1/2)*((3
2*(A^2*a^2*b^8 - 2*A^2*a^4*b^6 + A^2*a^6*b^4 + 4*B^2*a^4*b^6 - 8*B^2*a^6*b^
4 + 4*B^2*a^8*b^2 - 4*A*B*a^3*b^7 + 8*A*B*a^5*b^5 - 4*A*B*a^7*b^3))/(b^9 -
2*a^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)*(2*A^2*a*b^10 - 9*A^2*a^3*b^8
 + 8*A^2*a^5*b^6 - 2*A^2*a^7*b^4 + 8*B^2*a^3*b^8 - 29*B^2*a^5*b^6 + 28*B^2*
a^7*b^4 - 8*B^2*a^9*b^2 - 8*A*B*a^2*b^9 + 32*A*B*a^4*b^7 - 30*A*B*a^6*b^5 +
 8*A*B*a^8*b^3))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (a*(-(a + b)^3*(a - b)^3)^(
1/2)*((32*(A*a^3*b^9 + 2*B*a^2*b^10 - 3*B*a^4*b^8 + B*a^6*b^6 - A*a*b^11))/
(b^9 - 2*a^2*b^7 + a^4*b^5) - (32*tan(e/2 + (f*x)/2)*(4*A*a^2*b^11 - 6*A*a^
4*b^9 + 2*A*a^6*b^7 - 6*B*a^3*b^10 + 10*B*a^5*b^8 - 4*B*a^7*b^6))/(b^10 - 2
*a^2*b^8 + a^4*b^6) + (a*((32*(a^2*b^12 - 2*a^4*b^10 + a^6*b^8))/(b^9 - 2*a
^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)*(3*a*b^14 - 8*a^3*b^12 + 7*a^5*b
^10 - 2*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4*b^6))*(-(a + b)^3*(a - b)^3)^(1/2
)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 -
 a^6*b^3))*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*
a^4*b^5 - a^6*b^3))*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2)*1i)/(b^9 - 3*
a^2*b^7 + 3*a^4*b^5 - a^6*b^3))/((64*(4*B^3*a^8 + 2*A^3*a^3*b^5 - A^3*a^5*b
^3 - 6*B^3*a^6*b^2 - 8*A*B^2*a^7*b + 13*A*B^2*a^5*b^3 - 9*A^2*B*a^4*b^4 + 5
*A^2*B*a^6*b^2))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (64*tan(e/2 + (f*x)/2)*(16*B
```

```
^3*a^9 - 4*A^3*a^2*b^7 + 6*A^3*a^4*b^5 - 2*A^3*a^6*b^3 + 24*B^3*a^5*b^4 - 4
0*B^3*a^7*b^2 - 24*A*B^2*a^8*b - 40*A*B^2*a^4*b^5 + 64*A*B^2*a^6*b^3 + 22*A
^2*B*a^3*b^6 - 34*A^2*B*a^5*b^4 + 12*A^2*B*a^7*b^2))/(b^10 - 2*a^2*b^8 + a^
4*b^6) - (a*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(A^2*a^2*b^8 - 2*A^2*a^4*b^6
+ A^2*a^6*b^4 + 4*B^2*a^4*b^6 - 8*B^2*a^6*b^4 + 4*B^2*a^8*b^2 - 4*A*B*a^3*b
^7 + 8*A*B*a^5*b^5 - 4*A*B*a^7*b^3))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(
e/2 + (f*x)/2)*(2*A^2*a*b^10 - 9*A^2*a^3*b^8 + 8*A^2*a^5*b^6 - 2*A^2*a^7*b^
4 + 8*B^2*a^3*b^8 - 29*B^2*a^5*b^6 + 28*B^2*a^7*b^4 - 8*B^2*a^9*b^2 - 8*A*B
*a^2*b^9 + 32*A*B*a^4*b^7 - 30*A*B*a^6*b^5 + 8*A*B*a^8*b^3))/(b^10 - 2*a^2*
b^8 + a^4*b^6) + (a*(-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(4
*A*a^2*b^11 - 6*A*a^4*b^9 + 2*A*a^6*b^7 - 6*B*a^3*b^10 + 10*B*a^5*b^8 - 4*B
*a^7*b^6))/(b^10 - 2*a^2*b^8 + a^4*b^6) - (32*(A*a^3*b^9 + 2*B*a^2*b^10 - 3
*B*a^4*b^8 + B*a^6*b^6 - A*a*b^11))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (a*((32*(
a^2*b^12 - 2*a^4*b^10 + a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(e/2
 + (f*x)/2)*(3*a*b^14 - 8*a^3*b^12 + 7*a^5*b^10 - 2*a^7*b^8))/(b^10 - 2*a^2
*b^8 + a^4*b^6))*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b
- 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(2*A*b^3 + 2*B*a^3 -
 A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(2*A*b^3 +
2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3) + (
a*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(A^2*a^2*b^8 - 2*A^2*a^4*b^6 + A^2*a^6*
b^4 + 4*B^2*a^4*b^6 - 8*B^2*a^6*b^4 + 4*B^2*a^8*b^2 - 4*A*B*a^3*b^7 + 8*A*B
*a^5*b^5 - 4*A*B*a^7*b^3))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x
)/2)*(2*A^2*a*b^10 - 9*A^2*a^3*b^8 + 8*A^2*a^5*b^6 - 2*A^2*a^7*b^4 + 8*B^2*
a^3*b^8 - 29*B^2*a^5*b^6 + 28*B^2*a^7*b^4 - 8*B^2*a^9*b^2 - 8*A*B*a^2*b^9 +
 32*A*B*a^4*b^7 - 30*A*B*a^6*b^5 + 8*A*B*a^8*b^3))/(b^10 - 2*a^2*b^8 + a^4*
b^6) + (a*(-(a + b)^3*(a - b)^3)^(1/2)*((32*(A*a^3*b^9 + 2*B*a^2*b^10 - 3*B
*a^4*b^8 + B*a^6*b^6 - A*a*b^11))/(b^9 - 2*a^2*b^7 + a^4*b^5) - (32*tan(e/2
 + (f*x)/2)*(4*A*a^2*b^11 - 6*A*a^4*b^9 + 2*A*a^6*b^7 - 6*B*a^3*b^10 + 10*B
*a^5*b^8 - 4*B*a^7*b^6))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (a*((32*(a^2*b^12 -
 2*a^4*b^10 + a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2
)*(3*a*b^14 - 8*a^3*b^12 + 7*a^5*b^10 - 2*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4
*b^6))*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^
2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(2*A*b^3 + 2*B*a^3 - A*a^2*b -
 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(2*A*b^3 + 2*B*a^3 -
A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*(-(a + b)^3
*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2)*2i)/(f*(b^9 - 3
*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.17 $\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^4\,dx$

Optimal. Leaf size=182

$$\frac{7ac^4(2A-B)\cos^3(e+fx)}{24f}+\frac{7a(2A-B)\cos^3(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{40f}+\frac{7ac^4(2A-B)\sin(e+fx)\cos(e+f}{16f}$$

```
[Out] 7/16*a*(2*A-B)*c^4*x+7/24*a*(2*A-B)*c^4*cos(f*x+e)^3/f+7/16*a*(2*A-B)*c^4*c
os(f*x+e)*sin(f*x+e)/f-1/6*a*B*c*cos(f*x+e)^3*(c-c*sin(f*x+e))^3/f+1/10*a*(
2*A-B)*cos(f*x+e)^3*(c^2-c^2*sin(f*x+e))^2/f+7/40*a*(2*A-B)*cos(f*x+e)^3*(c
^4-c^4*sin(f*x+e))/f
```

Rubi [A] time = 0.30, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2967, 2860, 2678, 2669, 2635, 8}

$$\frac{7ac^4(2A-B)\cos^3(e+fx)}{24f}+\frac{a(2A-B)\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{10f}+\frac{7a(2A-B)\cos^3(e+fx)\left(c^4-c^4\sin\right.}{40f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (7*a*(2*A - B)*c^4*x)/16 + (7*a*(2*A - B)*c^4*Cos[e + f*x]^3)/(24*f) + (7*a
*(2*A - B)*c^4*Cos[e + f*x]*Sin[e + f*x])/(16*f) - (a*B*c*Cos[e + f*x]^3*(c
 - c*Sin[e + f*x])^3)/(6*f) + (a*(2*A - B)*Cos[e + f*x]^3*(c^2 - c^2*Sin[e
+ f*x])^2)/(10*f) + (7*a*(2*A - B)*Cos[e + f*x]^3*(c^4 - c^4*Sin[e + f*x]))
/(40*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
```

```
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx = (ac)\int \cos^2(e+fx)(A+B\sin(e+fx))(c-c\text{s}$$

$$= -\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^3}{6f} + \frac{1}{2}(a($$

$$= -\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^3}{6f} + \frac{a(2A}{}$$

$$= -\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^3}{6f} + \frac{a(2A}{}$$

$$= \frac{7a(2A-B)c^4\cos^3(e+fx)}{24f} - \frac{aBc\cos^3(e+fx}{}$$

$$= \frac{7a(2A-B)c^4\cos^3(e+fx)}{24f} + \frac{7a(2A-B)c^4\text{co}}{}$$

$$= \frac{7}{16}a(2A-B)c^4x + \frac{7a(2A-B)c^4\cos^3(e+fx)}{24f}$$

Mathematica [A] time = 0.98, size = 131, normalized size = 0.72

$$\frac{ac^4(120(7A-5B)\cos(e+fx)+20(13A-7B)\cos(3(e+fx))+240A\sin(2(e+fx))-90A\sin(4(e+fx))-12}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,
x]
```

```
[Out] (a*c^4*(840*A*f*x - 420*B*f*x + 120*(7*A - 5*B)*Cos[e + f*x] + 20*(13*A - 7
*B)*Cos[3*(e + f*x)] - 12*A*Cos[5*(e + f*x)] + 36*B*Cos[5*(e + f*x)] + 240*
A*Sin[2*(e + f*x)] + 15*B*Sin[2*(e + f*x)] - 90*A*Sin[4*(e + f*x)] + 105*B*
Sin[4*(e + f*x)] - 5*B*Sin[6*(e + f*x)]))/(960*f)
```

fricas [A] time = 0.44, size = 123, normalized size = 0.68

$$-\frac{48\,(A-3\,B)ac^4\cos\left(fx+e\right)^5-320\,(A-B)ac^4\cos\left(fx+e\right)^3-105\,(2\,A-B)ac^4fx+5\left(8\,Bac^4\cos\left(fx+e\right)^5\right.}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm
="fricas")
```

```
[Out] -1/240*(48*(A - 3*B)*a*c^4*cos(f*x + e)^5 - 320*(A - B)*a*c^4*cos(f*x + e)^
3 - 105*(2*A - B)*a*c^4*f*x + 5*(8*B*a*c^4*cos(f*x + e)^5 + 2*(18*A - 25*B)
*a*c^4*cos(f*x + e)^3 - 21*(2*A - B)*a*c^4*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.17, size = 184, normalized size = 1.01

$$-\frac{Bac^4\sin\left(6\,fx+6\,e\right)}{192\,f}+\frac{7}{16}\left(2\,Aac^4-Bac^4\right)x-\frac{\left(Aac^4-3\,Bac^4\right)\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{\left(13\,Aac^4-7\,Bac^4\right)\cos\left(3\,fx+3\right.}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm
="giac")
```

```
[Out] -1/192*B*a*c^4*sin(6*f*x + 6*e)/f + 7/16*(2*A*a*c^4 - B*a*c^4)*x - 1/80*(A*
a*c^4 - 3*B*a*c^4)*cos(5*f*x + 5*e)/f + 1/48*(13*A*a*c^4 - 7*B*a*c^4)*cos(3
*f*x + 3*e)/f + 1/8*(7*A*a*c^4 - 5*B*a*c^4)*cos(f*x + e)/f - 1/64*(6*A*a*c^
4 - 7*B*a*c^4)*sin(4*f*x + 4*e)/f + 1/64*(16*A*a*c^4 + B*a*c^4)*sin(2*f*x +
 2*e)/f
```

maple [B] time = 0.59, size = 342, normalized size = 1.88

$$-\frac{A\,c^4a\left(\frac{8}{3}+\sin^4\left(fx+e\right)+\frac{4\left(\sin^2\left(fx+e\right)\right)}{3}\right)\cos\left(fx+e\right)}{5}-3A\,c^4a\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2A\,c^4a\left(2+\sin^2\left(fx+e\right)\right)\cos}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x)
```

```
[Out] 1/f*(-1/5*A*c^4*a*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-3*A*c^4*a*
(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/3*A*c^4*a*(
2+sin(f*x+e)^2)*cos(f*x+e)+2*A*c^4*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/
2*e)+3*A*c^4*a*cos(f*x+e)+B*c^4*a*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8
*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+3/5*B*c^4*a*(8/3+sin(f*x+e)^4+4/3*
sin(f*x+e)^2)*cos(f*x+e)+2*B*c^4*a*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(
f*x+e)+3/8*f*x+3/8*e)-2/3*B*c^4*a*(2+sin(f*x+e)^2)*cos(f*x+e)-3*B*c^4*a*(-1
/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+A*c^4*a*(f*x+e)-B*c^4*a*cos(f*x+e))
```

maxima [A] time = 0.60, size = 336, normalized size = 1.85

$$-\frac{64\left(3\,\cos\left(fx+e\right)^5-10\,\cos\left(fx+e\right)^3+15\,\cos\left(fx+e\right)\right)Aac^4-640\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)Aac^4+9}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm
="maxima")
```

```
[Out] -1/960*(64*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a*c^4
 - 640*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*c^4 + 90*(12*f*x + 12*e + sin(
4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a*c^4 - 480*(2*f*x + 2*e - sin(2*f*x +
 2*e))*A*a*c^4 - 960*(f*x + e)*A*a*c^4 - 192*(3*cos(f*x + e)^5 - 10*cos(f*x
 + e)^3 + 15*cos(f*x + e))*B*a*c^4 - 640*(cos(f*x + e)^3 - 3*cos(f*x + e))*
B*a*c^4 - 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48
*sin(2*f*x + 2*e))*B*a*c^4 - 60*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2
*f*x + 2*e))*B*a*c^4 + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^4 - 2880*
A*a*c^4*cos(f*x + e) + 960*B*a*c^4*cos(f*x + e))/f
```

mupad [B] time = 14.83, size = 454, normalized size = 2.49

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{A\,a\,c^4}{4}+\frac{7\,B\,a\,c^4}{8}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}\,\left(6\,A\,a\,c^4-2\,B\,a\,c^4\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}\,\left(12\,A\,a\,c^4-4\,B\,a\,c^4\right)-\mathrm{tar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^4,x)
```

```
[Out] (tan(e/2 + (f*x)/2)*((A*a*c^4)/4 + (7*B*a*c^4)/8) + tan(e/2 + (f*x)/2)^10*(
6*A*a*c^4 - 2*B*a*c^4) + tan(e/2 + (f*x)/2)^4*(12*A*a*c^4 - 4*B*a*c^4) - ta
n(e/2 + (f*x)/2)^11*((A*a*c^4)/4 + (7*B*a*c^4)/8) + tan(e/2 + (f*x)/2)^8*(2
2*A*a*c^4 - 18*B*a*c^4) + tan(e/2 + (f*x)/2)^5*((13*A*a*c^4)/2 - (37*B*a*c^
4)/4) - tan(e/2 + (f*x)/2)^7*((13*A*a*c^4)/2 - (37*B*a*c^4)/4) + tan(e/2 +
(f*x)/2)^2*((38*A*a*c^4)/5 - (34*B*a*c^4)/5) + tan(e/2 + (f*x)/2)^6*((68*A*
a*c^4)/3 - (44*B*a*c^4)/3) + tan(e/2 + (f*x)/2)^3*((27*A*a*c^4)/4 - (73*B*a
*c^4)/24) - tan(e/2 + (f*x)/2)^9*((27*A*a*c^4)/4 - (73*B*a*c^4)/24) + (34*A
*a*c^4)/15 - (22*B*a*c^4)/15)/(f*(6*tan(e/2 + (f*x)/2)^2 + 15*tan(e/2 + (f*
x)/2)^4 + 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 + 6*tan(e/2 + (
f*x)/2)^10 + tan(e/2 + (f*x)/2)^12 + 1)) + (7*a*c^4*atan((7*a*c^4*tan(e/2 +
 (f*x)/2)*(2*A - B))/(8*((7*A*a*c^4)/4 - (7*B*a*c^4)/8)))*(2*A - B))/(8*f)
```

sympy [A] time = 7.88, size = 853, normalized size = 4.69

$$\begin{cases} -\frac{9Aac^4x\sin^4\left(e+fx\right)}{8}-\frac{9Aac^4x\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4}+Aac^4x\sin^2\left(e+fx\right)-\frac{9Aac^4x\cos^4\left(e+fx\right)}{8}+Aac^4x\cos^2\left(e+fx\right)+ \\ x\left(A+B\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right)\left(-c\sin\left(e\right)+c\right)^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-9*A*a*c**4*x*sin(e + f*x)**4/8 - 9*A*a*c**4*x*sin(e + f*x)**2*c
os(e + f*x)**2/4 + A*a*c**4*x*sin(e + f*x)**2 - 9*A*a*c**4*x*cos(e + f*x)**
4/8 + A*a*c**4*x*cos(e + f*x)**2 + A*a*c**4*x - A*a*c**4*sin(e + f*x)**4*co
s(e + f*x)/f + 15*A*a*c**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*A*a*c**4*
sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 2*A*a*c**4*sin(e + f*x)**2*cos(e +
f*x)/f + 9*A*a*c**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) - A*a*c**4*sin(e + f
*x)*cos(e + f*x)/f - 8*A*a*c**4*cos(e + f*x)**5/(15*f) - 4*A*a*c**4*cos(e +
 f*x)**3/(3*f) + 3*A*a*c**4*cos(e + f*x)/f + 5*B*a*c**4*x*sin(e + f*x)**6/1
6 + 15*B*a*c**4*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 3*B*a*c**4*x*sin(e +
 f*x)**4/4 + 15*B*a*c**4*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 3*B*a*c**4*
x*sin(e + f*x)**2*cos(e + f*x)**2/2 - 3*B*a*c**4*x*sin(e + f*x)**2/2 + 5*B*
a*c**4*x*cos(e + f*x)**6/16 + 3*B*a*c**4*x*cos(e + f*x)**4/4 - 3*B*a*c**4*x
*cos(e + f*x)**2/2 - 11*B*a*c**4*sin(e + f*x)**5*cos(e + f*x)/(16*f) + 3*B*
a*c**4*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a*c**4*sin(e + f*x)**3*cos(e +
f*x)**3/(6*f) - 5*B*a*c**4*sin(e + f*x)**3*cos(e + f*x)/(4*f) + 4*B*a*c**4*
sin(e + f*x)**2*cos(e + f*x)**3/f - 2*B*a*c**4*sin(e + f*x)**2*cos(e + f*x)
/f - 5*B*a*c**4*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 3*B*a*c**4*sin(e + f*
x)*cos(e + f*x)**3/(4*f) + 3*B*a*c**4*sin(e + f*x)*cos(e + f*x)/(2*f) + 8*B
*a*c**4*cos(e + f*x)**5/(5*f) - 4*B*a*c**4*cos(e + f*x)**3/(3*f) - B*a*c**4
*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)*(-c*sin(e) + c
)**4, True))
```

3.18 $\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^3 \, dx$

Optimal. Leaf size=142

$$\frac{ac^3(5A-2B)\cos^3(e+fx)}{12f}+\frac{a(5A-2B)\cos^3(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{20f}+\frac{ac^3(5A-2B)\sin(e+fx)\cos(e+f}{8f}$$

```
[Out] 1/8*a*(5*A-2*B)*c^3*x+1/12*a*(5*A-2*B)*c^3*cos(f*x+e)^3/f+1/8*a*(5*A-2*B)*c
^3*cos(f*x+e)*sin(f*x+e)/f-1/5*a*B*c*cos(f*x+e)^3*(c-c*sin(f*x+e))^2/f+1/20
*a*(5*A-2*B)*cos(f*x+e)^3*(c^3-c^3*sin(f*x+e))/f
```

Rubi [A] time = 0.25, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2967, 2860, 2678, 2669, 2635, 8}

$$\frac{ac^3(5A-2B)\cos^3(e+fx)}{12f}+\frac{a(5A-2B)\cos^3(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{20f}+\frac{ac^3(5A-2B)\sin(e+fx)\cos(e+f}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a*(5*A - 2*B)*c^3*x)/8 + (a*(5*A - 2*B)*c^3*Cos[e + f*x]^3)/(12*f) + (a*(5
*A - 2*B)*c^3*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (a*B*c*Cos[e + f*x]^3*(c -
 c*Sin[e + f*x])^2)/(5*f) + (a*(5*A - 2*B)*Cos[e + f*x]^3*(c^3 - c^3*Sin[e
+ f*x]))/(20*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
```

```
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^3\,dx = (ac)\int \cos^2(e + fx)(A + B\sin(e + fx))(c - c\,\mathrm{s}$$

$$= -\frac{aBc\cos^3(e + fx)(c - c\sin(e + fx))^2}{5f} + \frac{1}{5}(a(5$$

$$= -\frac{aBc\cos^3(e + fx)(c - c\sin(e + fx))^2}{5f} + \frac{a(5A}{}$$

$$= \frac{a(5A - 2B)c^3\cos^3(e + fx)}{12f} - \frac{aBc\cos^3(e + fx}{}$$

$$= \frac{a(5A - 2B)c^3\cos^3(e + fx)}{12f} + \frac{a(5A - 2B)c^3\,\mathrm{co}}{}$$

$$= \frac{1}{8}a(5A - 2B)c^3x + \frac{a(5A - 2B)c^3\cos^3(e + fx)}{12f}$$

Mathematica [A] time = 0.84, size = 95, normalized size = 0.67

$$\frac{ac^3(15(-(A - 2B)\sin(4(e + fx)) + 4fx(5A - 2B) + 8A\sin(2(e + fx))) + 60(4A - 3B)\cos(e + fx) + 10(8A - 5}{480f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,
x]
```

```
[Out] (a*c^3*(60*(4*A - 3*B)*Cos[e + f*x] + 10*(8*A - 5*B)*Cos[3*(e + f*x)] + 6*B
*Cos[5*(e + f*x)] + 15*(4*(5*A - 2*B)*f*x + 8*A*Sin[2*(e + f*x)] - (A - 2*B
)*Sin[4*(e + f*x)])))/(480*f)
```

fricas [A] time = 0.44, size = 102, normalized size = 0.72

$$\frac{24\,Bac^3\cos\left(fx + e\right)^5 + 80\,(A - B)ac^3\cos\left(fx + e\right)^3 + 15\,(5\,A - 2\,B)ac^3fx - 15\left(2\,(A - 2\,B)ac^3\cos\left(fx + e\right)^3 -\right.}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] 1/120*(24*B*a*c^3*cos(f*x + e)^5 + 80*(A - B)*a*c^3*cos(f*x + e)^3 + 15*(5*
A - 2*B)*a*c^3*f*x - 15*(2*(A - 2*B)*a*c^3*cos(f*x + e)^3 - (5*A - 2*B)*a*c
^3*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.18, size = 145, normalized size = 1.02

$$\frac{Bac^3\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{Aac^3\sin\left(2\,fx+2\,e\right)}{4\,f}+\frac{1}{8}\left(5\,Aac^3-2\,Bac^3\right)x+\frac{\left(8\,Aac^3-5\,Bac^3\right)\cos\left(3\,fx+3\,e\right)}{48\,f}+\frac{\left(4\,Aac^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] 1/80*B*a*c^3*cos(5*f*x + 5*e)/f + 1/4*A*a*c^3*sin(2*f*x + 2*e)/f + 1/8*(5*A
*a*c^3 - 2*B*a*c^3)*x + 1/48*(8*A*a*c^3 - 5*B*a*c^3)*cos(3*f*x + 3*e)/f + 1
/8*(4*A*a*c^3 - 3*B*a*c^3)*cos(f*x + e)/f - 1/32*(A*a*c^3 - 2*B*a*c^3)*sin(
4*f*x + 4*e)/f
```

maple [A] time = 0.48, size = 208, normalized size = 1.46

$$\frac{-A\,c^3a\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2A\,c^3a\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}+2A\,c^3a\cos\left(fx+e\right)+\frac{B\,c^3a\left(\frac{8}{3}+\sin\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(-A*c^3*a*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)
-2/3*A*c^3*a*(2+sin(f*x+e)^2)*cos(f*x+e)+2*A*c^3*a*cos(f*x+e)+1/5*B*c^3*a*(
8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+2*B*c^3*a*(-1/4*(sin(f*x+e)^3
+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2*B*c^3*a*(-1/2*sin(f*x+e)*cos(f
*x+e)+1/2*f*x+1/2*e)+A*c^3*a*(f*x+e)-B*c^3*a*cos(f*x+e))
```

maxima [A] time = 0.33, size = 200, normalized size = 1.41

$$\frac{320\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)Aac^3-15\left(12\,fx+12\,e+\sin\left(4\,fx+4\,e\right)-8\,\sin\left(2\,fx+2\,e\right)\right)Aac^3+480\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] 1/480*(320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*c^3 - 15*(12*f*x + 12*e +
sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a*c^3 + 480*(f*x + e)*A*a*c^3 + 32
```

```
*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a*c^3 + 30*(12*
f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a*c^3 - 240*(2*f*x +
2*e - sin(2*f*x + 2*e))*B*a*c^3 + 960*A*a*c^3*cos(f*x + e) - 480*B*a*c^3*co
s(f*x + e))/f
```

mupad [B] time = 13.80, size = 389, normalized size = 2.74

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{3\,A\,a\,c^3}{4}+\frac{B\,a\,c^3}{2}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8\,\left(4\,A\,a\,c^3-2\,B\,a\,c^3\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(\frac{7\,A\,a\,c^3}{2}-3\,B\,a\,c^3\right)-\tan\left(\frac{e}{2}\right.}{f\,\left(\tan\left(\frac{e}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^3,x)
```

```
[Out] (tan(e/2 + (f*x)/2)*((3*A*a*c^3)/4 + (B*a*c^3)/2) + tan(e/2 + (f*x)/2)^8*(4
*A*a*c^3 - 2*B*a*c^3) + tan(e/2 + (f*x)/2)^3*((7*A*a*c^3)/2 - 3*B*a*c^3) -
tan(e/2 + (f*x)/2)^7*((7*A*a*c^3)/2 - 3*B*a*c^3) - tan(e/2 + (f*x)/2)^9*((3
*A*a*c^3)/4 + (B*a*c^3)/2) + tan(e/2 + (f*x)/2)^6*(8*A*a*c^3 - 8*B*a*c^3) +
 tan(e/2 + (f*x)/2)^2*((8*A*a*c^3)/3 - (8*B*a*c^3)/3) + tan(e/2 + (f*x)/2)^
4*((16*A*a*c^3)/3 - (4*B*a*c^3)/3) + (4*A*a*c^3)/3 - (14*B*a*c^3)/15)/(f*(5
*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 +
 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 + 1)) + (a*c^3*atan((a*c^3*
tan(e/2 + (f*x)/2)*(5*A - 2*B))/(4*((5*A*a*c^3)/4 - (B*a*c^3)/2)))*(5*A - 2
*B))/(4*f) - (a*c^3*(5*A - 2*B)*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2))/(4*f)
```

sympy [A] time = 4.09, size = 486, normalized size = 3.42

$$\begin{cases} -\frac{3Aac^3x\sin^4\left(e+fx\right)}{8}-\frac{3Aac^3x\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4}-\frac{3Aac^3x\cos^4\left(e+fx\right)}{8}+Aac^3x+\frac{5Aac^3\sin^3\left(e+fx\right)\cos\left(e+fx\right)}{8f}-\frac{2Aac^3\sin^2\left(e\right.}{} \\ x\left(A+B\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right)\left(-c\sin\left(e\right)+c\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-3*A*a*c**3*x*sin(e + f*x)**4/8 - 3*A*a*c**3*x*sin(e + f*x)**2*c
os(e + f*x)**2/4 - 3*A*a*c**3*x*cos(e + f*x)**4/8 + A*a*c**3*x + 5*A*a*c**3
*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 2*A*a*c**3*sin(e + f*x)**2*cos(e + f*
x)/f + 3*A*a*c**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 4*A*a*c**3*cos(e + f
*x)**3/(3*f) + 2*A*a*c**3*cos(e + f*x)/f + 3*B*a*c**3*x*sin(e + f*x)**4/4 +
 3*B*a*c**3*x*sin(e + f*x)**2*cos(e + f*x)**2/2 - B*a*c**3*x*sin(e + f*x)**
```

```
2 + 3*B*a*c**3*x*cos(e + f*x)**4/4 - B*a*c**3*x*cos(e + f*x)**2 + B*a*c**3*
sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a*c**3*sin(e + f*x)**3*cos(e + f*x)/(4
*f) + 4*B*a*c**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 3*B*a*c**3*sin(e +
 f*x)*cos(e + f*x)**3/(4*f) + B*a*c**3*sin(e + f*x)*cos(e + f*x)/f + 8*B*a*
c**3*cos(e + f*x)**5/(15*f) - B*a*c**3*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B
*sin(e))*(a*sin(e) + a)*(-c*sin(e) + c)**3, True))
```

3.19 $\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^2\,dx$

Optimal. Leaf size=97

$$\frac{ac^2(A-B)\cos^3(e+fx)}{3f}+\frac{ac^2(4A-B)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}ac^2x(4A-B)+\frac{aBc^2\sin(e+fx)\cos^3(e+fx)}{4f}$$

```
[Out] 1/8*a*(4*A-B)*c^2*x+1/3*a*(A-B)*c^2*cos(f*x+e)^3/f+1/8*a*(4*A-B)*c^2*cos(f*
x+e)*sin(f*x+e)/f+1/4*a*B*c^2*cos(f*x+e)^3*sin(f*x+e)/f
```

Rubi [A] time = 0.19, antiderivative size = 105, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {2967, 2860, 2669, 2635, 8}

$$\frac{ac^2(4A-B)\cos^3(e+fx)}{12f}+\frac{ac^2(4A-B)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}ac^2x(4A-B)-\frac{aB\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a*(4*A - B)*c^2*x)/8 + (a*(4*A - B)*c^2*Cos[e + f*x]^3)/(12*f) + (a*(4*A -
 B)*c^2*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (a*B*Cos[e + f*x]^3*(c^2 - c^2*S
in[e + f*x]))/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = (ac)\int \cos^2(e+fx)(A+B\sin(e+fx))(c-c\,\mathrm{si}$$

$$= -\frac{aB\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f}+\frac{1}{4}(a(4$$

$$= \frac{a(4A-B)c^2\cos^3(e+fx)}{12f}-\frac{aB\cos^3(e+fx)\left(c^2\right.}{4}$$

$$= \frac{a(4A-B)c^2\cos^3(e+fx)}{12f}+\frac{a(4A-B)c^2\cos(e}{8}$$

$$= \frac{1}{8}a(4A-B)c^2x+\frac{a(4A-B)c^2\cos^3(e+fx)}{12f}+$$

Mathematica [A] time = 0.67, size = 74, normalized size = 0.76

$$\frac{ac^2(3(8A\sin(2(e+fx))+16Afx+B\sin(4(e+fx))-4Bfx)+24(A-B)\cos(e+fx)+8(A-B)\cos(3(e+fx)}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,
x]
```

```
[Out] (a*c^2*(24*(A - B)*Cos[e + f*x] + 8*(A - B)*Cos[3*(e + f*x)] + 3*(16*A*f*x
- 4*B*f*x + 8*A*Sin[2*(e + f*x)] + B*Sin[4*(e + f*x)])))/(96*f)
```

fricas [A] time = 0.45, size = 82, normalized size = 0.85

$$\frac{8(A-B)ac^2\cos\left(fx+e\right)^3+3(4A-B)ac^2fx+3\left(2Bac^2\cos\left(fx+e\right)^3+(4A-B)ac^2\cos\left(fx+e\right)\right)\sin\left(fx+\right.}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] 1/24*(8*(A - B)*a*c^2*cos(f*x + e)^3 + 3*(4*A - B)*a*c^2*f*x + 3*(2*B*a*c^2
*cos(f*x + e)^3 + (4*A - B)*a*c^2*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.14, size = 114, normalized size = 1.18

$$\frac{Bac^2\sin\left(4fx+4e\right)}{32f}+\frac{Aac^2\sin\left(2fx+2e\right)}{4f}+\frac{1}{8}\left(4Aac^2-Bac^2\right)x+\frac{\left(Aac^2-Bac^2\right)\cos\left(3fx+3e\right)}{12f}+\frac{\left(Aac^2-Ba\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] 1/32*B*a*c^2*sin(4*f*x + 4*e)/f + 1/4*A*a*c^2*sin(2*f*x + 2*e)/f + 1/8*(4*A
*a*c^2 - B*a*c^2)*x + 1/12*(A*a*c^2 - B*a*c^2)*cos(3*f*x + 3*e)/f + 1/4*(A*
a*c^2 - B*a*c^2)*cos(f*x + e)/f
```

maple [B] time = 0.39, size = 185, normalized size = 1.91

$$-\frac{Ac^2a\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}-Ac^2a\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+Ac^2a\cos\left(fx+e\right)+Bc^2a\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(\right.}{}\right.}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(-1/3*A*c^2*a*(2+sin(f*x+e)^2)*cos(f*x+e)-A*c^2*a*(-1/2*sin(f*x+e)*cos(
f*x+e)+1/2*f*x+1/2*e)+A*c^2*a*cos(f*x+e)+B*c^2*a*(-1/4*(sin(f*x+e)^3+3/2*si
n(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+1/3*B*c^2*a*(2+sin(f*x+e)^2)*cos(f*x+e)
-B*c^2*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+A*c^2*a*(f*x+e)-B*c^2*a
*cos(f*x+e))
```

maxima [B] time = 0.33, size = 179, normalized size = 1.85

$$\frac{32\left(\cos\left(fx+e\right)^{3}-3\cos\left(fx+e\right)\right)Aac^{2}-24\left(2fx+2e-\sin\left(2fx+2e\right)\right)Aac^{2}+96\left(fx+e\right)Aac^{2}-32\left(\cos\left(f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] 1/96*(32*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*c^2 - 24*(2*f*x + 2*e - sin(
2*f*x + 2*e))*A*a*c^2 + 96*(f*x + e)*A*a*c^2 - 32*(cos(f*x + e)^3 - 3*cos(f
*x + e))*B*a*c^2 + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e)
)*B*a*c^2 - 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^2 + 96*A*a*c^2*cos(f*
x + e) - 96*B*a*c^2*cos(f*x + e))/f
```

mupad [B] time = 13.38, size = 345, normalized size = 3.56

$$\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(Aac^{2}+\frac{Bac^{2}}{4}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}\left(2Aac^{2}-2Bac^{2}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{6}\left(2Aac^{2}-2Bac^{2}\right)+\tan\left(\frac{e}{2}\right.}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{8}+4\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^2,x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(A*a*c^2 + (B*a*c^2)/4) + tan(e/2 + (f*x)/2)^4*(2*A*a*c
^2 - 2*B*a*c^2) + tan(e/2 + (f*x)/2)^6*(2*A*a*c^2 - 2*B*a*c^2) + tan(e/2 +
(f*x)/2)^2*((2*A*a*c^2)/3 - (2*B*a*c^2)/3) - tan(e/2 + (f*x)/2)^7*(A*a*c^2
+ (B*a*c^2)/4) + tan(e/2 + (f*x)/2)^3*(A*a*c^2 - (7*B*a*c^2)/4) - tan(e/2 +
 (f*x)/2)^5*(A*a*c^2 - (7*B*a*c^2)/4) + (2*A*a*c^2)/3 - (2*B*a*c^2)/3)/(f*(
4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/2 + (f*x)/2)^6 +
tan(e/2 + (f*x)/2)^8 + 1)) + (a*c^2*atan((a*c^2*tan(e/2 + (f*x)/2)*(4*A - B
))/(4*(A*a*c^2 - (B*a*c^2)/4)))*(4*A - B))/(4*f) - (a*c^2*(4*A - B)*(atan(t
an(e/2 + (f*x)/2)) - (f*x)/2))/(4*f)
```

sympy [A] time = 2.13, size = 396, normalized size = 4.08

$$\begin{cases} -\frac{Aac^{2}x\sin^{2}\left(e+fx\right)}{2}-\frac{Aac^{2}x\cos^{2}\left(e+fx\right)}{2}+Aac^{2}x-\frac{Aac^{2}\sin^{2}\left(e+fx\right)\cos\left(e+fx\right)}{f}+\frac{Aac^{2}\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{2Aac^{2}\cos^{3}\left(e+fx\right)}{3f}+\cdots \\ x\left(A+B\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right)\left(-c\sin\left(e\right)+c\right)^{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-A*a*c**2*x*sin(e + f*x)**2/2 - A*a*c**2*x*cos(e + f*x)**2/2 + A
*a*c**2*x - A*a*c**2*sin(e + f*x)**2*cos(e + f*x)/f + A*a*c**2*sin(e + f*x)
*cos(e + f*x)/(2*f) - 2*A*a*c**2*cos(e + f*x)**3/(3*f) + A*a*c**2*cos(e + f
*x)/f + 3*B*a*c**2*x*sin(e + f*x)**4/8 + 3*B*a*c**2*x*sin(e + f*x)**2*cos(e
 + f*x)**2/4 - B*a*c**2*x*sin(e + f*x)**2/2 + 3*B*a*c**2*x*cos(e + f*x)**4/
8 - B*a*c**2*x*cos(e + f*x)**2/2 - 5*B*a*c**2*sin(e + f*x)**3*cos(e + f*x)/
(8*f) + B*a*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a*c**2*sin(e + f*x)*c
os(e + f*x)**3/(8*f) + B*a*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*B*a*c**
2*cos(e + f*x)**3/(3*f) - B*a*c**2*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin
(e))*(a*sin(e) + a)*(-c*sin(e) + c)**2, True))
```

3.20 $\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))\,dx$

Optimal. Leaf size=49

$$\frac{aAc\sin(e+fx)\cos(e+fx)}{2f} + \frac{1}{2}aAcx - \frac{aBc\cos^3(e+fx)}{3f}$$

```
[Out] 1/2*a*A*c*x-1/3*a*B*c*cos(f*x+e)^3/f+1/2*a*A*c*cos(f*x+e)*sin(f*x+e)/f
```

Rubi [A] time = 0.08, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2967, 2669, 2635, 8}

$$\frac{aAc\sin(e+fx)\cos(e+fx)}{2f} + \frac{1}{2}aAcx - \frac{aBc\cos^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a*A*c*x)/2 - (a*B*c*Cos[e + f*x]^3)/(3*f) + (a*A*c*Cos[e + f*x]*Sin[e + f*
x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx=(ac)\int\cos^2(e+fx)(A+B\sin(e+fx))\,dx$$
$$=-\frac{aBc\cos^3(e+fx)}{3f}+(aAc)\int\cos^2(e+fx)\,dx$$
$$=-\frac{aBc\cos^3(e+fx)}{3f}+\frac{aAc\cos(e+fx)\sin(e+$$
$$=\frac{1}{2}aAcx-\frac{aBc\cos^3(e+fx)}{3f}+\frac{aAc\cos(e+fx)}{2f}$$

Mathematica [A] time = 0.16, size = 48, normalized size = 0.98

$$-\frac{ac(-3A(\sin(2(e+fx))-2e+2fx)+3B\cos(e+fx)+B\cos(3(e+fx)))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

```
[Out] -1/12*(a*c*(3*B*Cos[e + f*x] + B*Cos[3*(e + f*x)] - 3*A*(-2*e + 2*f*x + Sin
[2*(e + f*x)])))/f
```

fricas [A] time = 0.45, size = 43, normalized size = 0.88

$$-\frac{2\,Bac\cos\left(fx+e\right)^3-3\,Aacfx-3\,Aac\cos\left(fx+e\right)\sin\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -1/6*(2*B*a*c*cos(f*x + e)^3 - 3*A*a*c*f*x - 3*A*a*c*cos(f*x + e)*sin(f*x +
 e))/f
```

giac [A] time = 0.14, size = 58, normalized size = 1.18

$$\frac{1}{2} Aacx - \frac{Bac\cos\left(3 fx + 3 e\right)}{12 f} - \frac{Bac\cos\left(fx + e\right)}{4 f} + \frac{Aac\sin\left(2 fx + 2 e\right)}{4 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] 1/2*A*a*c*x - 1/12*B*a*c*cos(3*f*x + 3*e)/f - 1/4*B*a*c*cos(f*x + e)/f + 1/
4*A*a*c*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.29, size = 74, normalized size = 1.51

$$\frac{\frac{Bac\left(2+\sin^{2}\left(fx+e\right)\right)\cos\left(fx+e\right)}{3} - Aac\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2} + \frac{fx}{2} + \frac{e}{2}\right) - Bac\cos\left(fx+e\right) + Aac\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] 1/f*(1/3*B*a*c*(2+sin(f*x+e)^2)*cos(f*x+e)-A*a*c*(-1/2*sin(f*x+e)*cos(f*x+e
)+1/2*f*x+1/2*e)-B*a*c*cos(f*x+e)+A*a*c*(f*x+e))
```

maxima [A] time = 0.38, size = 73, normalized size = 1.49

$$-\frac{3\left(2 fx + 2 e - \sin\left(2 fx + 2 e\right)\right)Aac - 12\left(fx + e\right)Aac + 4\left(\cos\left(fx + e\right)^{3} - 3\cos\left(fx + e\right)\right)Bac + 12 Bac\cos\left(fx\right.}{12 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -1/12*(3*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a*c - 12*(f*x + e)*A*a*c + 4*(c
os(f*x + e)^3 - 3*cos(f*x + e))*B*a*c + 12*B*a*c*cos(f*x + e))/f
```

mupad [B] time = 14.33, size = 122, normalized size = 2.49

$$\frac{A\,a\,c\,x}{2} - \frac{A\,a\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5} + \left(\frac{a\,c\left(12\,B-9\,A\left(e+f\,x\right)\right)}{6} + \frac{3\,A\,a\,c\left(e+f\,x\right)}{2}\right)\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4} - A\,a\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right) + \frac{a\,c\left(4\,B-3\,A\left(\right.\right.}{6}}{f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x)),x)
```

```
[Out] (A*a*c*x)/2 - (tan(e/2 + (f*x)/2)^4*((a*c*(12*B - 9*A*(e + f*x)))/6 + (3*A*
a*c*(e + f*x))/2) + (a*c*(4*B - 3*A*(e + f*x)))/6 - A*a*c*tan(e/2 + (f*x)/2
) + (A*a*c*(e + f*x))/2 + A*a*c*tan(e/2 + (f*x)/2)^5)/(f*(tan(e/2 + (f*x)/2
)^2 + 1)^3)
```

sympy [A] time = 0.79, size = 138, normalized size = 2.82

$$\begin{cases} -\frac{Aacx\sin^2(e+fx)}{2} - \frac{Aacx\cos^2(e+fx)}{2} + Aacx + \frac{Aac\sin(e+fx)\cos(e+fx)}{2f} + \frac{Bac\sin^2(e+fx)\cos(e+fx)}{f} + \frac{2Bac\cos^3(e+fx)}{3f} - \frac{Bac\,cc}{} \\ x(A + B\sin(e))(a\sin(e) + a)(-c\sin(e) + c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-A*a*c*x*sin(e + f*x)**2/2 - A*a*c*x*cos(e + f*x)**2/2 + A*a*c*x
 + A*a*c*sin(e + f*x)*cos(e + f*x)/(2*f) + B*a*c*sin(e + f*x)**2*cos(e + f*
x)/f + 2*B*a*c*cos(e + f*x)**3/(3*f) - B*a*c*cos(e + f*x)/f, Ne(f, 0)), (x*
(A + B*sin(e))*(a*sin(e) + a)*(-c*sin(e) + c), True))
```

3.21 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=56

$$\frac{2a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))}-\frac{ax(A+2B)}{c}+\frac{aB\cos(e+fx)}{cf}$$

```
[Out] -a*(A+2*B)*x/c+a*B*cos(f*x+e)/c/f+2*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))
```

Rubi [A] time = 0.17, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2967, 2857, 2638}

$$\frac{2a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))}-\frac{ax(A+2B)}{c}+\frac{aB\cos(e+fx)}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),x]
```

```
[Out] -((a*(A + 2*B)*x)/c) + (a*B*Cos[e + f*x])/(c*f) + (2*a*(A + B)*Cos[e + f*x]
)/(f*(c - c*Sin[e + f*x]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx &= (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx \\
&= \frac{2a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))} + \frac{a\int(-Ac-2Bc-Bc\sin(e+fx))\,dx}{c^2} \\
&= -\frac{a(A+2B)x}{c} + \frac{2a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))} - \frac{(aB)\int\sin(e+fx)\,dx}{c} \\
&= -\frac{a(A+2B)x}{c} + \frac{aB\cos(e+fx)}{cf} + \frac{2a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))}
\end{aligned}$$

Mathematica [B] time = 0.93, size = 125, normalized size = 2.23

$$\frac{a(\sin(e+fx)+1)\left(\frac{4(A+B)\sin\left(\frac{fx}{2}\right)}{f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}-(x(A+2B))-\frac{B\sin(e)\sin(fx)}{f}+\frac{B\cos(e)\cos(fx)}{f}\right)}{c\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),
x]
```

```
[Out] (a*(-((A + 2*B)*x) + (B*Cos[e]*Cos[f*x])/f - (B*Sin[e]*Sin[f*x])/f + (4*(A
+ B)*Sin[(f*x)/2])/(f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])))*(1 + Sin[e + f*x]))/(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)
```

fricas [B] time = 0.42, size = 116, normalized size = 2.07

$$-\frac{(A+2B)afx - Ba\cos\left(fx+e\right)^2 - 2(A+B)a + \left((A+2B)afx - (2A+3B)a\right)\cos\left(fx+e\right) - \left((A+2B)afx \cdots\right.}{cf\cos\left(fx+e\right) - cf\sin\left(fx+e\right) + cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -((A + 2*B)*a*f*x - B*a*cos(f*x + e)^2 - 2*(A + B)*a + ((A + 2*B)*a*f*x - (
2*A + 3*B)*a)*cos(f*x + e) - ((A + 2*B)*a*f*x - B*a*cos(f*x + e) + 2*(A + B
)*a)*sin(f*x + e))/(c*f*cos(f*x + e) - c*f*sin(f*x + e) + c*f)
```

giac [B] time = 0.15, size = 124, normalized size = 2.21

$$-\frac{\frac{(Aa+2\,Ba)(fx+e)}{c}+\frac{2\left(2\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+2\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,Aa+3\,Ba\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)c}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] -((A*a + 2*B*a)*(f*x + e)/c + 2*(2*A*a*tan(1/2*f*x + 1/2*e)^2 + 2*B*a*tan(1
/2*f*x + 1/2*e)^2 - B*a*tan(1/2*f*x + 1/2*e) + 2*A*a + 3*B*a)/((tan(1/2*f*x
 + 1/2*e)^3 - tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e) - 1)*c))/f
```

maple [A] time = 0.41, size = 113, normalized size = 2.02

$$-\frac{4aA}{fc\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{4aB}{fc\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{2aB}{fc\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}-\frac{2a\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)A}{fc}-\frac{4a\arctan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] -4/f*a/c/(tan(1/2*f*x+1/2*e)-1)*A-4/f*a/c/(tan(1/2*f*x+1/2*e)-1)*B+2/f*a/c*
B/(1+tan(1/2*f*x+1/2*e)^2)-2/f*a/c*arctan(tan(1/2*f*x+1/2*e))*A-4/f*a/c*arc
tan(tan(1/2*f*x+1/2*e))*B
```

maxima [B] time = 0.59, size = 265, normalized size = 4.73

$$-\frac{2\left(Ba\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-2}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}+\frac{c\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{c\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}\right)+Aa\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}-\frac{1}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}}\right)+Ba\left(\frac{\arctan\left(\right.}{}\right.\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -2*(B*a*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^2/(cos(f*x + e) +
1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)^2/(cos(f*
```

```
x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/c) + A*a*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c -
 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) + B*a*(arctan(sin(f*x + e)/(cos
(f*x + e) + 1))/c - 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) - A*a/(c - c
*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

mupad [B] time = 12.64, size = 111, normalized size = 1.98

$$\frac{(4\,A\,a+4\,B\,a)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-2\,B\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+4\,A\,a+6\,B\,a}{f\,\left(-c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+c\right)}-\frac{A\,a\,f\,x+2\,B\,a\,f\,x}{c\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x)),x)
```

```
[Out] (4*A*a + 6*B*a + tan(e/2 + (f*x)/2)^2*(4*A*a + 4*B*a) - 2*B*a*tan(e/2 + (f*
x)/2))/(f*(c - c*tan(e/2 + (f*x)/2) + c*tan(e/2 + (f*x)/2)^2 - c*tan(e/2 +
(f*x)/2)^3)) - (A*a*f*x + 2*B*a*f*x)/(c*f)
```

sympy [A] time = 4.03, size = 828, normalized size = 14.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-A*a*f*x*tan(e/2 + f*x/2)**3/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(
e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + A*a*f*x*tan(e/2 + f*x/2)**2
/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2)
- c*f) - A*a*f*x*tan(e/2 + f*x/2)/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 +
f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + A*a*f*x/(c*f*tan(e/2 + f*x/2)**3
- c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) - 4*A*a*tan(e/2 + f
*x/2)**2/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 +
 f*x/2) - c*f) - 4*A*a/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 +
 c*f*tan(e/2 + f*x/2) - c*f) - 2*B*a*f*x*tan(e/2 + f*x/2)**3/(c*f*tan(e/2 +
 f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + 2*B*a*
f*x*tan(e/2 + f*x/2)**2/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2
+ c*f*tan(e/2 + f*x/2) - c*f) - 2*B*a*f*x*tan(e/2 + f*x/2)/(c*f*tan(e/2 + f
*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + 2*B*a*f*
x/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2)
 - c*f) - 4*B*a*tan(e/2 + f*x/2)**2/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2
+ f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + 2*B*a*tan(e/2 + f*x/2)/(c*f*tan
(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) -
```

```
6*B*a/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*
x/2) - c*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)/(-c*sin(e) + c), T
rue))
```

3.22 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=72

$$-\frac{a(A+7B)\cos(e+fx)}{3c^2f(1-\sin(e+fx))}+\frac{2a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2}+\frac{aBx}{c^2}$$

```
[Out] a*B*x/c^2-1/3*a*(A+7*B)*cos(f*x+e)/c^2/f/(1-sin(f*x+e))+2/3*a*(A+B)*cos(f*x
+e)/f/(c-c*sin(f*x+e))^2
```

Rubi [A] time = 0.22, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2967, 2857, 2735, 2648}

$$-\frac{a(A+7B)\cos(e+fx)}{3c^2f(1-\sin(e+fx))}+\frac{2a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2}+\frac{aBx}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a*B*x)/c^2 - (a*(A + 7*B)*Cos[e + f*x])/(3*c^2*f*(1 - Sin[e + f*x])) + (2*
a*(A + B)*Cos[e + f*x])/(3*f*(c - c*Sin[e + f*x])^2)
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx &= (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx\\ &= \frac{2a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2}+\frac{a\int \frac{-Ac-4Bc-3Bc\sin(e+fx)}{c-c\sin(e+fx)}\,dx}{3c^2}\\ &= \frac{aBx}{c^2}+\frac{2a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2}-\frac{(a(A+7B))\int \frac{1}{c-c\sin(e+fx)}\,dx}{3c}\\ &= \frac{aBx}{c^2}+\frac{2a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2}-\frac{a(A+7B)\cos(e+fx)}{3f\left(c^2-c^2\sin(e+fx)\right)}\end{aligned}$$

Mathematica [B] time = 0.63, size = 160, normalized size = 2.22

$$-\frac{a\left(-6(A+3B)\cos\left(e+\frac{fx}{2}\right)+2A\cos\left(e+\frac{3fx}{2}\right)+9Bfx\sin\left(e+\frac{fx}{2}\right)+3Bfx\sin\left(e+\frac{3fx}{2}\right)+14B\cos\left(e+\frac{3fx}{2}\right)+\cdots\right.}{6c^2f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
2,x]
```

```
[Out] -1/6*(a*(-9*B*f*x*Cos[(f*x)/2] - 6*(A + 3*B)*Cos[e + (f*x)/2] + 2*A*Cos[e +
 (3*f*x)/2] + 14*B*Cos[e + (3*f*x)/2] + 3*B*f*x*Cos[2*e + (3*f*x)/2] + 24*B
*Sin[(f*x)/2] + 9*B*f*x*Sin[e + (f*x)/2] + 3*B*f*x*Sin[e + (3*f*x)/2]))/(c^
2*f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3)
```

fricas [B] time = 0.44, size = 162, normalized size = 2.25

$$-\frac{6\,Bafx-\left(3\,Bafx+(A+7\,B)a\right)\cos\left(fx+e\right)^2+2\,(A+B)a+\left(3\,Bafx+(A-5\,B)a\right)\cos\left(fx+e\right)-\left(6\,Bafx-\cdots\right.}{3\left(c^2f\cos\left(fx+e\right)^2-c^2f\cos\left(fx+e\right)-2\,c^2f+\left(c^2f\cos\left(fx+e\right)+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] -1/3*(6*B*a*f*x - (3*B*a*f*x + (A + 7*B)*a)*cos(f*x + e)^2 + 2*(A + B)*a +
(3*B*a*f*x + (A - 5*B)*a)*cos(f*x + e) - (6*B*a*f*x - 2*(A + B)*a + (3*B*a*
f*x - (A + 7*B)*a)*cos(f*x + e))*sin(f*x + e))/(c^2*f*cos(f*x + e)^2 - c^2*
f*cos(f*x + e) - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

giac [A] time = 0.15, size = 92, normalized size = 1.28

$$\frac{\frac{3\left(fx+e\right)Ba}{c^2}-\frac{2\left(3\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+12\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+Aa-5\,Ba\right)}{c^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] 1/3*(3*(f*x + e)*B*a/c^2 - 2*(3*A*a*tan(1/2*f*x + 1/2*e)^2 - 3*B*a*tan(1/2*
f*x + 1/2*e)^2 + 12*B*a*tan(1/2*f*x + 1/2*e) + A*a - 5*B*a)/(c^2*(tan(1/2*f
*x + 1/2*e) - 1)^3))/f
```

maple [B] time = 0.44, size = 160, normalized size = 2.22

$$-\frac{2aA}{c^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{2aB}{c^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{8aA}{3c^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{8aB}{3c^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{\quad}{c^2f\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

```
[Out] -2*a/c^2/f/(tan(1/2*f*x+1/2*e)-1)*A+2*a/c^2/f/(tan(1/2*f*x+1/2*e)-1)*B-8/3*
a/c^2/f/(tan(1/2*f*x+1/2*e)-1)^3*A-8/3*a/c^2/f/(tan(1/2*f*x+1/2*e)-1)^3*B-4
*a/c^2/f/(tan(1/2*f*x+1/2*e)-1)^2*A-4*a/c^2/f/(tan(1/2*f*x+1/2*e)-1)^2*B+2*
a/c^2/f*B*arctan(tan(1/2*f*x+1/2*e))
```

maxima [B] time = 0.44, size = 456, normalized size = 6.33

$$\frac{2\left(Ba\left(\frac{\frac{9\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-4}{c^2-\frac{3c^2\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3c^2\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{c^2\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}}+\frac{3\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{c^2}\right)-\frac{Aa\left(\frac{3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-2\right)}{c^2-\frac{3c^2\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3c^2\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{c^2\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}}+\frac{\quad}{c^2-\frac{3c}{co}}\right.}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] 2/3*(B*a*((9*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 - 4)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3
*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) - A*a*(3*sin(f*x + e)/(cos(f*
x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c^2 - 3*c^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + A*a*(3*sin(f*x + e)/(cos(f*x + e)
+ 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + B*a*(3
*sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x +
e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3))/f
```

mupad [B] time = 12.51, size = 132, normalized size = 1.83

$$\frac{B\,a\,x}{c^2}-\frac{\left(\frac{a\,\left(6\,A-6\,B+9\,B\,\left(e+f\,x\right)\right)}{3}-3\,B\,a\,\left(e+f\,x\right)\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+\left(\frac{a\,\left(24\,B-9\,B\,\left(e+f\,x\right)\right)}{3}+3\,B\,a\,\left(e+f\,x\right)\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+}{c^2\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^2,x)
```

```
[Out] (B*a*x)/c^2 - ((a*(2*A - 10*B + 3*B*(e + f*x)))/3 + tan(e/2 + (f*x)/2)^2*((
a*(6*A - 6*B + 9*B*(e + f*x)))/3 - 3*B*a*(e + f*x)) + tan(e/2 + (f*x)/2)*((
a*(24*B - 9*B*(e + f*x)))/3 + 3*B*a*(e + f*x)) - B*a*(e + f*x))/(c^2*f*(tan
(e/2 + (f*x)/2) - 1)^3)
```

sympy [A] time = 8.11, size = 700, normalized size = 9.72

$$\begin{cases} -\frac{6Aa\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2f}-\frac{2Aa}{3c^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-9c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9c^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3c^2f}+\frac{}{3c^2f\tan^3\left(\frac{e}{2}\right.} \\ \frac{x(A+B\sin(e))(a\sin(e)+a)}{(-c\sin(e)+c)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*A*a*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**
2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 2*A*a/(3*
c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/
2 + f*x/2) - 3*c**2*f) + 3*B*a*f*x*tan(e/2 + f*x/2)**3/(3*c**2*f*tan(e/2 +
f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c*
*2*f) - 9*B*a*f*x*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**
2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 9*B*a*f*x
*tan(e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)
**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 3*B*a*f*x/(3*c**2*f*tan(e/2 +
 f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c
**2*f) + 6*B*a*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f
*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 24*B*a*tan(e
/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 +
9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 10*B*a/(3*c**2*f*tan(e/2 + f*x/2)**
3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f), N
e(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)/(-c*sin(e) + c)**2, True))
```

3.23 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=104

$$-\frac{a(A-4B)\cos(e+fx)}{15f\left(c^3-c^3\sin(e+fx)\right)}-\frac{ac(A+11B)\cos(e+fx)}{15f\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{2a(A+B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3}$$

```
[Out] 2/5*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^3-1/15*a*(A+11*B)*c*cos(f*x+e)/f/
(c^2-c^2*sin(f*x+e))^2-1/15*a*(A-4*B)*cos(f*x+e)/f/(c^3-c^3*sin(f*x+e))
```

Rubi [A] time = 0.24, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2967, 2857, 2750, 2648}

$$-\frac{a(A-4B)\cos(e+fx)}{15f\left(c^3-c^3\sin(e+fx)\right)}-\frac{ac(A+11B)\cos(e+fx)}{15f\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{2a(A+B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (2*a*(A + B)*Cos[e + f*x])/(5*f*(c - c*Sin[e + f*x])^3) - (a*(A + 11*B)*c*C
os[e + f*x])/(15*f*(c^2 - c^2*Sin[e + f*x])^2) - (a*(A - 4*B)*Cos[e + f*x])
/(15*f*(c^3 - c^3*Sin[e + f*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
```

```
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$

$$= \frac{2a(A+B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3} + \frac{a\int \frac{-Ac-6Bc-5Bc\sin(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{5c^2}$$

$$= \frac{2a(A+B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3} - \frac{a(A+11B)\cos(e+fx)}{15cf(c-c\sin(e+fx))^2} - \frac{(a(A-4B)}{}$$

$$= \frac{2a(A+B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3} - \frac{a(A+11B)\cos(e+fx)}{15cf(c-c\sin(e+fx))^2} - \frac{a(A-4B)}{15f\left(c^3-c^3\right.}$$

Mathematica [A] time = 0.74, size = 147, normalized size = 1.41

$$\frac{a\left(15(A-B)\cos\left(e+\frac{fx}{2}\right)-5(A-B)\cos\left(e+\frac{3fx}{2}\right)+A\sin\left(2e+\frac{5fx}{2}\right)+5A\sin\left(\frac{fx}{2}\right)+15B\sin\left(2e+\frac{3fx}{2}\right)-4B\right.}{30c^3f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
3,x]
```

```
[Out] (a*(15*(A - B)*Cos[e + (f*x)/2] - 5*(A - B)*Cos[e + (3*f*x)/2] + 5*A*Sin[(f
*x)/2] + 25*B*Sin[(f*x)/2] + 15*B*Sin[2*e + (3*f*x)/2] + A*Sin[2*e + (5*f*x
)/2] - 4*B*Sin[2*e + (5*f*x)/2]))/(30*c^3*f*(Cos[e/2] - Sin[e/2])*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^5)
```

fricas [A] time = 0.41, size = 183, normalized size = 1.76

$$-\frac{(A-4\,B)a\cos\left(fx+e\right)^{3}-(2\,A+7\,B)a\cos\left(fx+e\right)^{2}+3\,(A+B)a\cos\left(fx+e\right)+6\,(A+B)a+\left((A-4\,B)a\cos\right.}{15\left(c^{3}f\cos\left(fx+e\right)^{3}+3\,c^{3}f\cos\left(fx+e\right)^{2}-2\,c^{3}f\cos\left(fx+e\right)-4\,c^{3}f-\left(c^{3}f\cos\left(fx+e\right)^{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -1/15*((A - 4*B)*a*cos(f*x + e)^3 - (2*A + 7*B)*a*cos(f*x + e)^2 + 3*(A + B
)*a*cos(f*x + e) + 6*(A + B)*a + ((A - 4*B)*a*cos(f*x + e)^2 + 3*(A + B)*a*
cos(f*x + e) + 6*(A + B)*a)*sin(f*x + e))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*c
os(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*
c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))
```

giac [A] time = 0.18, size = 139, normalized size = 1.34

$$-\frac{2\left(15\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-15\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+15\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+25\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+5\,Ba\right.}{15\,c^{3}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -2/15*(15*A*a*tan(1/2*f*x + 1/2*e)^4 - 15*A*a*tan(1/2*f*x + 1/2*e)^3 + 15*B
*a*tan(1/2*f*x + 1/2*e)^3 + 25*A*a*tan(1/2*f*x + 1/2*e)^2 + 5*B*a*tan(1/2*f
*x + 1/2*e)^2 - 5*A*a*tan(1/2*f*x + 1/2*e) + 5*B*a*tan(1/2*f*x + 1/2*e) + 4
*A*a - B*a)/(c^3*f*(tan(1/2*f*x + 1/2*e) - 1)^5)
```

maple [A] time = 0.48, size = 115, normalized size = 1.11

$$\frac{2a\left(-\frac{14A+10B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{8A+8B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{6A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}\right)}{f\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

[Out] 2/f*a/c^3*(-1/3*(14*A+10*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/5*(8*A+8*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/4*(16*A+16*B)/(tan(1/2*f*x+1/2*e)-1)^4-A/(tan(1/2*f*x+1/2*e)-1)-1/2*(6*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^2)

maxima [B] time = 0.53, size = 737, normalized size = 7.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(A*a*(20*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 3*A*a*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 3*B*a*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 2*B*a*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

mupad [B] time = 12.98, size = 172, normalized size = 1.65

$$\frac{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{11\,A\,a\,\cos\left(e+f\,x\right)}{2}-\frac{B\,a}{4}-\frac{41\,A\,a}{4}+\frac{B\,a\,\cos\left(e+f\,x\right)}{2}+5\,A\,a\,\sin\left(e+f\,x\right)-5\,B\,a\,\sin\left(e+f\,x\right)+\frac{3\,A\,a\,\cos(}{4}\right.}{15\,c^3\,f\left(\frac{5\,\sqrt{2}\,\cos\left(\frac{3\,e}{2}-\frac{\pi}{4}+\frac{3\,f\,x}{2}\right)}{4}-\frac{5\,\sqrt{2}\,\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{f\,x}{2}\right)}{2}+\frac{\sqrt{2}\,\cos\left(\frac{5\,e}{2}\right.}{4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^3,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((11*A*a*cos(e + f*x))/2 - (B*a)/4 - (41*A*a)/4 + (B*a*cos(e + f*x))/2 + 5*A*a*sin(e + f*x) - 5*B*a*sin(e + f*x) + (3*A*a*cos(2*
```

```
e + 2*f*x))/4 + (3*B*a*cos(2*e + 2*f*x))/4 - (5*A*a*sin(2*e + 2*f*x))/4 + (
5*B*a*sin(2*e + 2*f*x))/4))/(15*c^3*f*((5*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f
*x)/2))/4 - (5*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/2 + (2^(1/2)*cos((5*e)/2
+ pi/4 + (5*f*x)/2))/4))
```

sympy [A] time = 15.78, size = 1035, normalized size = 9.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*a*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*
c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*ta
n(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 30*A*a*tan(e/
2 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**
4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c*
*3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 50*A*a*tan(e/2 + f*x/2)**2/(15*c**3*f*
tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 +
f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 1
5*c**3*f) + 10*A*a*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**
3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e
/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 8*A*a/(15*c**3*f
*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 +
 f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) -
15*c**3*f) - 30*B*a*tan(e/2 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75
*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*t
an(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 10*B*a*tan(e
/2 + f*x/2)**2/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)*
*4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c
**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 10*B*a*tan(e/2 + f*x/2)/(15*c**3*f*ta
n(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*
x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*
c**3*f) + 2*B*a/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)
**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*
c**3*f*tan(e/2 + f*x/2) - 15*c**3*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e
) + a)/(-c*sin(e) + c)**3, True))
```

3.24 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=142

$$-\frac{a(2A-5B)\cos(e+fx)}{105f\left(c^4-c^4\sin(e+fx)\right)}-\frac{a(2A-5B)\cos(e+fx)}{105f\left(c^2-c^2\sin(e+fx)\right)^2}-\frac{a(A+15B)\cos(e+fx)}{35cf(c-c\sin(e+fx))^3}+\frac{2a(A+B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4}$$

```
[Out] 2/7*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^4-1/35*a*(A+15*B)*cos(f*x+e)/c/f/
(c-c*sin(f*x+e))^3-1/105*a*(2*A-5*B)*cos(f*x+e)/f/(c^2-c^2*sin(f*x+e))^2-1/
105*a*(2*A-5*B)*cos(f*x+e)/f/(c^4-c^4*sin(f*x+e))
```

Rubi [A] time = 0.29, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {2967, 2857, 2750, 2650, 2648}

$$-\frac{a(2A-5B)\cos(e+fx)}{105f\left(c^4-c^4\sin(e+fx)\right)}-\frac{a(2A-5B)\cos(e+fx)}{105f\left(c^2-c^2\sin(e+fx)\right)^2}-\frac{a(A+15B)\cos(e+fx)}{35cf(c-c\sin(e+fx))^3}+\frac{2a(A+B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (2*a*(A + B)*Cos[e + f*x])/(7*f*(c - c*Sin[e + f*x])^4) - (a*(A + 15*B)*Cos
[e + f*x])/(35*c*f*(c - c*Sin[e + f*x])^3) - (a*(2*A - 5*B)*Cos[e + f*x])/(
105*f*(c^2 - c^2*Sin[e + f*x])^2) - (a*(2*A - 5*B)*Cos[e + f*x])/(105*f*(c^
4 - c^4*Sin[e + f*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
```

```
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$$

$$= \frac{2a(A+B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4} + \frac{a\int \frac{-Ac-8Bc-7Bc\sin(e+fx)}{(c-c\sin(e+fx))^3}\,dx}{7c^2}$$

$$= \frac{2a(A+B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4} - \frac{a(A+15B)\cos(e+fx)}{35cf(c-c\sin(e+fx))^3} - \frac{(a(2A-5B)) \cdots}{\cdots}$$

$$= \frac{2a(A+B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4} - \frac{a(A+15B)\cos(e+fx)}{35cf(c-c\sin(e+fx))^3} - \frac{a(2A-5B) \cdots}{105f\left(c^2-c^2 \cdots\right.}$$

$$= \frac{2a(A+B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4} - \frac{a(A+15B)\cos(e+fx)}{35cf(c-c\sin(e+fx))^3} - \frac{a(2A-5B) \cdots}{105f\left(c^2-c^2 \cdots\right.}$$

Mathematica [A] time = 0.90, size = 174, normalized size = 1.23

$$\frac{a\left(35(4A-B)\cos\left(e+\frac{fx}{2}\right)+14A\sin\left(2e+\frac{5fx}{2}\right)-42A\cos\left(e+\frac{3fx}{2}\right)+2A\cos\left(3e+\frac{7fx}{2}\right)+70A\sin\left(\frac{fx}{2}\right)+105\right.}{420c^4f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
4,x]
```

```
[Out] (a*(35*(4*A - B)*Cos[e + (f*x)/2] - 42*A*Cos[e + (3*f*x)/2] + 2*A*Cos[3*e +
 (7*f*x)/2] - 5*B*Cos[3*e + (7*f*x)/2] + 70*A*Sin[(f*x)/2] + 140*B*Sin[(f*x
)/2] + 105*B*Sin[2*e + (3*f*x)/2] + 14*A*Sin[2*e + (5*f*x)/2] - 35*B*Sin[2*
e + (5*f*x)/2]))/(420*c^4*f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])^7)
```

fricas [A] time = 0.42, size = 251, normalized size = 1.77

$$\frac{(2A-5B)a\cos\left(fx+e\right)^4+4(2A-5B)a\cos\left(fx+e\right)^3-3(3A+10B)a\cos\left(fx+e\right)^2+15(A+B)a\cos\left(fx\right.}{105\left(c^4f\cos\left(fx+e\right)^4-3c^4f\cos\left(fx+e\right)^3-8c^4f\cos\left(fx+e\right)^2+4c^4f\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm
="fricas")
```

```
[Out] 1/105*((2*A - 5*B)*a*cos(f*x + e)^4 + 4*(2*A - 5*B)*a*cos(f*x + e)^3 - 3*(3
*A + 10*B)*a*cos(f*x + e)^2 + 15*(A + B)*a*cos(f*x + e) + 30*(A + B)*a - ((
2*A - 5*B)*a*cos(f*x + e)^3 - 3*(2*A - 5*B)*a*cos(f*x + e)^2 - 15*(A + B)*a
*cos(f*x + e) - 30*(A + B)*a)*sin(f*x + e))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f
*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f +
 (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*
c^4*f)*sin(f*x + e))
```

giac [A] time = 0.20, size = 187, normalized size = 1.32

$$-\frac{2\left(105Aa\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6-210Aa\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+105Ba\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+455Aa\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm
="giac")
```

```
[Out] -2/105*(105*A*a*tan(1/2*f*x + 1/2*e)^6 - 210*A*a*tan(1/2*f*x + 1/2*e)^5 + 1
05*B*a*tan(1/2*f*x + 1/2*e)^5 + 455*A*a*tan(1/2*f*x + 1/2*e)^4 - 35*B*a*tan
(1/2*f*x + 1/2*e)^4 - 350*A*a*tan(1/2*f*x + 1/2*e)^3 + 140*B*a*tan(1/2*f*x
+ 1/2*e)^3 + 273*A*a*tan(1/2*f*x + 1/2*e)^2 - 56*A*a*tan(1/2*f*x + 1/2*e) +
 35*B*a*tan(1/2*f*x + 1/2*e) + 23*A*a - 5*B*a)/(c^4*f*(tan(1/2*f*x + 1/2*e)
 - 1)^7)
```

maple [A] time = 0.47, size = 159, normalized size = 1.12

$$\frac{2a\left(-\frac{28A+14B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{68A+60B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{16A+16B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{8A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{48A+48B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{56A+}{4\left(\tan\left(\frac{fx}{2}\right.\right.}\right.}{f\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x)
```

```
[Out] 2/f*a/c^4*(-1/3*(28*A+14*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/5*(68*A+60*B)/(tan(1
/2*f*x+1/2*e)-1)^5-A/(tan(1/2*f*x+1/2*e)-1)-1/7*(16*A+16*B)/(tan(1/2*f*x+1/
2*e)-1)^7-1/2*(8*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^2-1/6*(48*A+48*B)/(tan(1/2*f
*x+1/2*e)-1)^6-1/4*(56*A+40*B)/(tan(1/2*f*x+1/2*e)-1)^4)
```

maxima [B] time = 0.39, size = 1080, normalized size = 7.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm
="maxima")
```

```
[Out] 2/105*(A*a*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 13)/(c^4
 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*
c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7) + B*a*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 13)/(
c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x
```

```
 + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x +
e) + 1)^7) - 3*A*a*(49*sin(f*x + e)/(cos(f*x + e) + 1) - 147*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 210*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 210*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 -
35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 12)/(c^4 - 7*c^4*sin(f*x + e)/(cos
(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
- 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*
x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) - 4*B*a*(14*sin(f*
x + e)/(cos(f*x + e) + 1) - 42*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
 2)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1
)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f
*x + e) + 1)^7))/f
```

mupad [B] time = 13.22, size = 228, normalized size = 1.61

$$-\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{15Ba}{4}-\frac{171Aa}{2}+\frac{353Aa\cos(e+fx)}{8}+\frac{5Ba\cos(e+fx)}{4}+\frac{595Aa\sin(e+fx)}{8}-35Ba\sin(e+fx)+\frac{43A\cdots}{\cdots}\right.}{105c^4f\left(\frac{35\sqrt{2}\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{fx}{2}\right)}{8}-\frac{21\sqrt{2}\cos\cdots}{\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^4,x)
```

```
[Out] -(2*cos(e/2 + (f*x)/2)*((15*B*a)/4 - (171*A*a)/2 + (353*A*a*cos(e + f*x))/8
 + (5*B*a*cos(e + f*x))/4 + (595*A*a*sin(e + f*x))/8 - 35*B*a*sin(e + f*x)
+ (43*A*a*cos(2*e + 2*f*x))/2 - (25*A*a*cos(3*e + 3*f*x))/8 - (5*B*a*cos(2*
e + 2*f*x))/4 + (5*B*a*cos(3*e + 3*f*x))/4 - (77*A*a*sin(2*e + 2*f*x))/4 -
(21*A*a*sin(3*e + 3*f*x))/8 + (35*B*a*sin(2*e + 2*f*x))/4))/(105*c^4*f*((35
*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/8 - (21*2^(1/2)*cos((3*e)/2 - pi/4 + (3
*f*x)/2))/8 - (7*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/8 + (2^(1/2)*cos(
(7*e)/2 - pi/4 + (7*f*x)/2))/8))
```

sympy [A] time = 29.47, size = 1831, normalized size = 12.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-210*A*a*tan(e/2 + f*x/2)**6/(105*c**4*f*tan(e/2 + f*x/2)**7 - 7
35*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4
*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(
e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 420*A*a*tan(e
/2 + f*x/2)**5/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2
)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 +
3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**
4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 910*A*a*tan(e/2 + f*x/2)**4/(105*c**4*
f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/
2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x
/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 10
5*c**4*f) + 700*A*a*tan(e/2 + f*x/2)**3/(105*c**4*f*tan(e/2 + f*x/2)**7 - 7
35*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4
*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(
e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 546*A*a*tan(e
/2 + f*x/2)**2/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2
)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 +
3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**
4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 112*A*a*tan(e/2 + f*x/2)/(105*c**4*f*t
an(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 +
 f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)
**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c
**4*f) - 46*A*a/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/
2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 +
 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c*
*4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 210*B*a*tan(e/2 + f*x/2)**5/(105*c**4
*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e
/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*
x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 1
05*c**4*f) + 70*B*a*tan(e/2 + f*x/2)**4/(105*c**4*f*tan(e/2 + f*x/2)**7 - 7
35*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4
*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(
e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 280*B*a*tan(e
/2 + f*x/2)**3/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2
)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 +
3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**
4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 70*B*a*tan(e/2 + f*x/2)/(105*c**4*f*ta
n(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 +
f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)*
*3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c*
*4*f) + 10*B*a/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2
)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 +
3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**
4*f*tan(e/2 + f*x/2) - 105*c**4*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e)
+ a)/(-c*sin(e) + c)**4, True))
```

3.25 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$

Optimal. Leaf size=176

$$-\frac{2a(A-2B)\cos(e+fx)}{315f\left(c^5-c^5\sin(e+fx)\right)}-\frac{2ac(A-2B)\cos(e+fx)}{315f\left(c^3-c^3\sin(e+fx)\right)^2}-\frac{ac(A-2B)\cos(e+fx)}{105f\left(c^2-c^2\sin(e+fx)\right)^3}-\frac{a(A+19B)\cos(e+fx)}{63cf(c-c\sin(e+fx))^4}$$

```
[Out] 2/9*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^5-1/63*a*(A+19*B)*cos(f*x+e)/c/f/
(c-c*sin(f*x+e))^4-1/105*a*(A-2*B)*c*cos(f*x+e)/f/(c^2-c^2*sin(f*x+e))^3-2/
315*a*(A-2*B)*c*cos(f*x+e)/f/(c^3-c^3*sin(f*x+e))^2-2/315*a*(A-2*B)*cos(f*x
+e)/f/(c^5-c^5*sin(f*x+e))
```

Rubi [A] time = 0.31, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {2967, 2857, 2750, 2650, 2648}

$$-\frac{2a(A-2B)\cos(e+fx)}{315f\left(c^5-c^5\sin(e+fx)\right)}-\frac{2ac(A-2B)\cos(e+fx)}{315f\left(c^3-c^3\sin(e+fx)\right)^2}-\frac{ac(A-2B)\cos(e+fx)}{105f\left(c^2-c^2\sin(e+fx)\right)^3}-\frac{a(A+19B)\cos(e+fx)}{63cf(c-c\sin(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (2*a*(A + B)*Cos[e + f*x])/(9*f*(c - c*Sin[e + f*x])^5) - (a*(A + 19*B)*Cos
[e + f*x])/(63*c*f*(c - c*Sin[e + f*x])^4) - (a*(A - 2*B)*c*Cos[e + f*x])/(
105*f*(c^2 - c^2*Sin[e + f*x])^3) - (2*a*(A - 2*B)*c*Cos[e + f*x])/(315*f*(
c^3 - c^3*Sin[e + f*x])^2) - (2*a*(A - 2*B)*Cos[e + f*x])/(315*f*(c^5 - c^5
*Sin[e + f*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$$

$$= \frac{2a(A+B)\cos(e+fx)}{9f(c-c\sin(e+fx))^5} + \frac{a\int \frac{-Ac-10Bc-9Bc\sin(e+fx)}{(c-c\sin(e+fx))^4}\,dx}{9c^2}$$

$$= \frac{2a(A+B)\cos(e+fx)}{9f(c-c\sin(e+fx))^5} - \frac{a(A+19B)\cos(e+fx)}{63cf(c-c\sin(e+fx))^4} - \frac{(a(A-2B))}{}$$

$$= \frac{2a(A+B)\cos(e+fx)}{9f(c-c\sin(e+fx))^5} - \frac{a(A+19B)\cos(e+fx)}{63cf(c-c\sin(e+fx))^4} - \frac{a(A-2B)}{105c^2f(c-c}$$

$$= \frac{2a(A+B)\cos(e+fx)}{9f(c-c\sin(e+fx))^5} - \frac{a(A+19B)\cos(e+fx)}{63cf(c-c\sin(e+fx))^4} - \frac{a(A-2B)}{105c^2f(c-c}$$

$$= \frac{2a(A+B)\cos(e+fx)}{9f(c-c\sin(e+fx))^5} - \frac{a(A+19B)\cos(e+fx)}{63cf(c-c\sin(e+fx))^4} - \frac{a(A-2B)}{105c^2f(c-c}$$

Mathematica [A] time = 0.89, size = 200, normalized size = 1.14

$$\frac{a\left(-42(2A+B)\cos\left(e+\frac{3fx}{2}\right)+36A\sin\left(2e+\frac{5fx}{2}\right)-A\sin\left(4e+\frac{9fx}{2}\right)+315A\cos\left(e+\frac{fx}{2}\right)+9A\cos\left(3e+\frac{7fx}{2}\right.\right.}{1260c^5f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
5,x]
```

```
[Out] (a*(315*A*Cos[e + (f*x)/2] - 42*(2*A + B)*Cos[e + (3*f*x)/2] + 9*A*Cos[3*e
+ (7*f*x)/2] - 18*B*Cos[3*e + (7*f*x)/2] + 189*A*Sin[(f*x)/2] + 252*B*Sin[(
f*x)/2] + 210*B*Sin[2*e + (3*f*x)/2] + 36*A*Sin[2*e + (5*f*x)/2] - 72*B*Sin
[2*e + (5*f*x)/2] - A*Sin[4*e + (9*f*x)/2] + 2*B*Sin[4*e + (9*f*x)/2]))/(12
60*c^5*f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9)
```

fricas [A] time = 0.43, size = 305, normalized size = 1.73

$$-\frac{2\,(A-2\,B)a\cos\left(fx+e\right)^{5}-8\,(A-2\,B)a\cos\left(fx+e\right)^{4}-25\,(A-2\,B)a\cos\left(fx+e\right)^{3}+5\,(4\,A+13\,B)a\cos\left(f\right.}{315\left(c^{5}f\cos\left(fx+e\right)^{5}+5\,c^{5}f\cos\left(fx+e\right)^{4}-8\,c^{5}f\cos\left(fx+e\right)^{3}-20\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm
="fricas")
```

```
[Out] -1/315*(2*(A - 2*B)*a*cos(f*x + e)^5 - 8*(A - 2*B)*a*cos(f*x + e)^4 - 25*(A
 - 2*B)*a*cos(f*x + e)^3 + 5*(4*A + 13*B)*a*cos(f*x + e)^2 - 35*(A + B)*a*c
os(f*x + e) - 70*(A + B)*a + (2*(A - 2*B)*a*cos(f*x + e)^4 + 10*(A - 2*B)*a
*cos(f*x + e)^3 - 15*(A - 2*B)*a*cos(f*x + e)^2 - 35*(A + B)*a*cos(f*x + e)
 - 70*(A + B)*a)*sin(f*x + e))/(c^5*f*cos(f*x + e)^5 + 5*c^5*f*cos(f*x + e)
^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e
) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 - 12*c^5*f*co
s(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f)*sin(f*x + e))
```

giac [A] time = 0.21, size = 267, normalized size = 1.52

$$-\frac{2\left(315\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-945\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+315\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+2625\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm
="giac")
```

```
[Out] -2/315*(315*A*a*tan(1/2*f*x + 1/2*e)^8 - 945*A*a*tan(1/2*f*x + 1/2*e)^7 + 3
15*B*a*tan(1/2*f*x + 1/2*e)^7 + 2625*A*a*tan(1/2*f*x + 1/2*e)^6 - 315*B*a*t
an(1/2*f*x + 1/2*e)^6 - 3465*A*a*tan(1/2*f*x + 1/2*e)^5 + 945*B*a*tan(1/2*f
*x + 1/2*e)^5 + 3843*A*a*tan(1/2*f*x + 1/2*e)^4 - 441*B*a*tan(1/2*f*x + 1/2
*e)^4 - 2247*A*a*tan(1/2*f*x + 1/2*e)^3 + 609*B*a*tan(1/2*f*x + 1/2*e)^3 +
1143*A*a*tan(1/2*f*x + 1/2*e)^2 - 81*B*a*tan(1/2*f*x + 1/2*e)^2 - 207*A*a*t
an(1/2*f*x + 1/2*e) + 99*B*a*tan(1/2*f*x + 1/2*e) + 58*A*a - 11*B*a)/(c^5*f
*(tan(1/2*f*x + 1/2*e) - 1)^9)
```

maple [A] time = 0.51, size = 203, normalized size = 1.15

$$\frac{2a\left(-\frac{46A+18B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{128A+128B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\frac{236A+168B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{32A+32B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}}-\frac{10A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{248A+}{7\left(\tan\left(\frac{fx}{2}\right.\right.}\right.}{f\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x)
```

```
[Out] 2/f*a/c^5*(-1/3*(46*A+18*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/8*(128*A+128*B)/(tan
(1/2*f*x+1/2*e)-1)^8-1/5*(236*A+168*B)/(tan(1/2*f*x+1/2*e)-1)^5-A/(tan(1/2*
f*x+1/2*e)-1)-1/9*(32*A+32*B)/(tan(1/2*f*x+1/2*e)-1)^9-1/2*(10*A+2*B)/(tan(
1/2*f*x+1/2*e)-1)^2-1/7*(248*A+232*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/6*(296*A+2
48*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/4*(128*A+72*B)/(tan(1/2*f*x+1/2*e)-1)^4)
```

maxima [B] time = 0.40, size = 1425, normalized size = 8.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm
="maxima")
```

```
[Out] -2/315*(A*a*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 33
60*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*sin(f*x
 + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*
c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9
```

```
*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e)
 + 1)^9) - 5*A*a*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 315*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 14
7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(cos(f*x + e) + 1
)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*
c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5
*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 5*B*a*(45*sin(f*x + e)/(cos(f*x + e
) + 1) - 117*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 - 315*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e
)^5/(cos(f*x + e) + 1)^5 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin
(f*x + e)^7/(cos(f*x + e) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x +
e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*
c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 14*B*a
*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 54*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 81*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 + 45*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 30*sin(f*x + e)^6/(cos(
f*x + e) + 1)^6 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^
5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^
5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e)
+ 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9))/f
```

mupad [B] time = 13.34, size = 310, normalized size = 1.76

$$\frac{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{1357\,A\,a}{4}-\frac{461\,B\,a}{16}-\frac{635\,A\,a\,\cos\left(e+f\,x\right)}{4}+\frac{5\,B\,a\,\cos\left(e+f\,x\right)}{2}-\frac{1575\,A\,a\,\sin\left(e+f\,x\right)}{4}+\frac{945\,B\,a\,\sin\left(e+f\,x\right)}{8}-\frac{625\,A\,a\,\mathrm{co}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^5,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((1357*A*a)/4 - (461*B*a)/16 - (635*A*a*cos(e + f*x))
/4 + (5*B*a*cos(e + f*x))/2 - (1575*A*a*sin(e + f*x))/4 + (945*B*a*sin(e +
f*x))/8 - (625*A*a*cos(2*e + 2*f*x))/4 + (121*A*a*cos(3*e + 3*f*x))/4 + (7*
A*a*cos(4*e + 4*f*x))/2 + (95*B*a*cos(2*e + 2*f*x))/4 - 8*B*a*cos(3*e + 3*f
```

```
*x) - (7*B*a*cos(4*e + 4*f*x))/16 + (399*A*a*sin(2*e + 2*f*x))/4 + (141*A*a
*sin(3*e + 3*f*x))/4 - (15*A*a*sin(4*e + 4*f*x))/4 - (231*B*a*sin(2*e + 2*f
*x))/8 - (39*B*a*sin(3*e + 3*f*x))/8 + (15*B*a*sin(4*e + 4*f*x))/16))/(315*
c^5*f*((63*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/8 - (21*2^(1/2)*cos((3*e)/2 -
 pi/4 + (3*f*x)/2))/4 - (9*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/4 + (9*
2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/16 + (2^(1/2)*cos((9*e)/2 + pi/4 +
 (9*f*x)/2))/16))
```

sympy [A] time = 52.38, size = 3232, normalized size = 18.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-630*A*a*tan(e/2 + f*x/2)**8/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2
835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c
**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f
*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(
e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 1890*A*a*tan
(e/2 + f*x/2)**7/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*
x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)*
*6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 +
26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*
c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 5250*A*a*tan(e/2 + f*x/2)**6/(315*c
**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*
tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e
/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 +
f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2)
 - 315*c**5*f) + 6930*A*a*tan(e/2 + f*x/2)**5/(315*c**5*f*tan(e/2 + f*x/2)*
*9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 2
6460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*
c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*
f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 7686*A
*a*tan(e/2 + f*x/2)**4/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/
2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f
*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)
**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 +
 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 4494*A*a*tan(e/2 + f*x/2)**3/
(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c
**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f
*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(
e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 +
f*x/2) - 315*c**5*f) - 2286*A*a*tan(e/2 + f*x/2)**2/(315*c**5*f*tan(e/2 + f
*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)*
```

*7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 -
39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340
*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) +
414*A*a*tan(e/2 + f*x/2)/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(
e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 +
f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/
2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2
+ 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 116*A*a/(315*c**5*f*tan(e/2
+ f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x
/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**
5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 1
1340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f
) - 630*B*a*tan(e/2 + f*x/2)**7/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5
*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*ta
n(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2
+ f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*
x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 630*B*a*tan(e/2 + f*
x/2)**6/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 +
11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 3969
0*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**
5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*ta
n(e/2 + f*x/2) - 315*c**5*f) - 1890*B*a*tan(e/2 + f*x/2)**5/(315*c**5*f*tan
(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 +
f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/
2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3
- 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c*
*5*f) + 882*B*a*tan(e/2 + f*x/2)**4/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*
c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*
f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan
(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2
+ f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 1218*B*a*tan(e/2
+ f*x/2)**3/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)
8 + 11340*c5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 +
39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 2646
0*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5
*f*tan(e/2 + f*x/2) - 315*c**5*f) + 162*B*a*tan(e/2 + f*x/2)**2/(315*c**5*f
*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e
/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 +
f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2
)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 31
5*c**5*f) - 198*B*a*tan(e/2 + f*x/2)/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835
*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5
*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*ta
n(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2
+ f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 22*B*a/(315*c**

```
5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*ta
n(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2
 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*
x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) -
 315*c**5*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)/(-c*sin(e) + c)**
5, True))
```

3.26 $\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c - c\sin(e + fx))^5\, dx$

Optimal. Leaf size=229

$$\frac{3a^2c^5(8A-3B)\cos^5(e+fx)}{80f}+\frac{3a^2(8A-3B)\cos^5(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{112f}+\frac{3a^2c^5(8A-3B)\sin(e+fx)\cos}{64f}$$

```
[Out] 9/128*a^2*(8*A-3*B)*c^5*x+3/80*a^2*(8*A-3*B)*c^5*cos(f*x+e)^5/f+9/128*a^2*(
8*A-3*B)*c^5*cos(f*x+e)*sin(f*x+e)/f+3/64*a^2*(8*A-3*B)*c^5*cos(f*x+e)^3*si
n(f*x+e)/f+1/56*a^2*(8*A-3*B)*c^3*cos(f*x+e)^5*(c-c*sin(f*x+e))^2/f-1/8*a^2
*B*c^2*cos(f*x+e)^5*(c-c*sin(f*x+e))^3/f+3/112*a^2*(8*A-3*B)*cos(f*x+e)^5*(
c^5-c^5*sin(f*x+e))/f
```

Rubi [A] time = 0.37, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2967, 2860, 2678, 2669, 2635, 8}

$$\frac{3a^2c^5(8A-3B)\cos^5(e+fx)}{80f}+\frac{3a^2c^5(8A-3B)\sin(e+fx)\cos^3(e+fx)}{64f}+\frac{a^2c^3(8A-3B)\cos^5(e+fx)(c-c\sin(}{56f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (9*a^2*(8*A - 3*B)*c^5*x)/128 + (3*a^2*(8*A - 3*B)*c^5*Cos[e + f*x]^5)/(80*
f) + (9*a^2*(8*A - 3*B)*c^5*Cos[e + f*x]*Sin[e + f*x])/(128*f) + (3*a^2*(8*
A - 3*B)*c^5*Cos[e + f*x]^3*Sin[e + f*x])/(64*f) + (a^2*(8*A - 3*B)*c^3*Cos
[e + f*x]^5*(c - c*Sin[e + f*x])^2)/(56*f) - (a^2*B*c^2*Cos[e + f*x]^5*(c -
 c*Sin[e + f*x])^3)/(8*f) + (3*a^2*(8*A - 3*B)*Cos[e + f*x]^5*(c^5 - c^5*Si
n[e + f*x]))/(112*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(A+B\sin(e+fx))(c-$$
$$= -\frac{a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^3}{8f}+\frac{1}{8}$$
$$= \frac{a^2(8A-3B)c^3\cos^5(e+fx)(c-c\sin(e+fx)}{56f}$$
$$= \frac{a^2(8A-3B)c^3\cos^5(e+fx)(c-c\sin(e+fx)}{56f}$$
$$= \frac{3a^2(8A-3B)c^5\cos^5(e+fx)}{80f}+\frac{a^2(8A-3B)}{}$$
$$= \frac{3a^2(8A-3B)c^5\cos^5(e+fx)}{80f}+\frac{3a^2(8A-3B}{}$$
$$= \frac{3a^2(8A-3B)c^5\cos^5(e+fx)}{80f}+\frac{9a^2(8A-3B}{}$$
$$= \frac{9}{128}a^2(8A-3B)c^5x+\frac{3a^2(8A-3B)c^5\cos^5(e}{80f}$$

Mathematica [A] time = 2.04, size = 219, normalized size = 0.96

$$\frac{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))^5(2520(8A-3B)(e+fx)+560(19A-3B)\sin(2(e+fx))-280(2A-7B)}{35}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
5,x]
```

```
[Out] ((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^5*(2520*(8*A - 3*B)*(e + f*x)
+ 560*(27*A - 17*B)*Cos[e + f*x] + 560*(13*A - 7*B)*Cos[3*(e + f*x)] + 112*
(11*A - B)*Cos[5*(e + f*x)] - 80*(A - 3*B)*Cos[7*(e + f*x)] + 560*(19*A - 3
*B)*Sin[2*(e + f*x)] - 280*(2*A - 7*B)*Sin[4*(e + f*x)] - 560*(A - B)*Sin[6
*(e + f*x)] - 35*B*Sin[8*(e + f*x)]))/(35840*f*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^10*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)
```

fricas [A] time = 0.46, size = 158, normalized size = 0.69

$$-\frac{640\,(A-3\,B)a^2c^5\cos\left(fx+e\right)^7-3584\,(A-B)a^2c^5\cos\left(fx+e\right)^5-315\,(8\,A-3\,B)a^2c^5fx+35\left(16\,Ba^2c^5\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorit
hm="fricas")
```

```
[Out] -1/4480*(640*(A - 3*B)*a^2*c^5*cos(f*x + e)^7 - 3584*(A - B)*a^2*c^5*cos(f*
x + e)^5 - 315*(8*A - 3*B)*a^2*c^5*f*x + 35*(16*B*a^2*c^5*cos(f*x + e)^7 +
8*(8*A - 11*B)*a^2*c^5*cos(f*x + e)^5 - 6*(8*A - 3*B)*a^2*c^5*cos(f*x + e)^
3 - 9*(8*A - 3*B)*a^2*c^5*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.23, size = 278, normalized size = 1.21

$$-\frac{Ba^2c^5\sin\left(8\,fx+8\,e\right)}{1024\,f}+\frac{9}{128}\left(8\,Aa^2c^5-3\,Ba^2c^5\right)x-\frac{\left(Aa^2c^5-3\,Ba^2c^5\right)\cos\left(7\,fx+7\,e\right)}{448\,f}+\frac{\left(11\,Aa^2c^5-Ba^2c^5\right)\cos\left(\right.}{320\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorit
hm="giac")
```

```
[Out] -1/1024*B*a^2*c^5*sin(8*f*x + 8*e)/f + 9/128*(8*A*a^2*c^5 - 3*B*a^2*c^5)*x
- 1/448*(A*a^2*c^5 - 3*B*a^2*c^5)*cos(7*f*x + 7*e)/f + 1/320*(11*A*a^2*c^5
- B*a^2*c^5)*cos(5*f*x + 5*e)/f + 1/64*(13*A*a^2*c^5 - 7*B*a^2*c^5)*cos(3*f
*x + 3*e)/f + 1/64*(27*A*a^2*c^5 - 17*B*a^2*c^5)*cos(f*x + e)/f - 1/64*(A*a
^2*c^5 - B*a^2*c^5)*sin(6*f*x + 6*e)/f - 1/128*(2*A*a^2*c^5 - 7*B*a^2*c^5)*
sin(4*f*x + 4*e)/f + 1/64*(19*A*a^2*c^5 - 3*B*a^2*c^5)*sin(2*f*x + 2*e)/f
```

maple [B] time = 0.67, size = 569, normalized size = 2.48

$$a^2A\,c^5\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{B\,a^2c^5\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}+3a^2A\,c^5\cos\left(fx+e\right)+a^2A\,c^5\left(fx+e\right)-B\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x)
```

```
[Out] 1/f*(a^2*A*c^5*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/3*B*a^2*c^5*(2+
sin(f*x+e)^2)*cos(f*x+e)+3*a^2*A*c^5*cos(f*x+e)+a^2*A*c^5*(f*x+e)-B*a^2*c^5
*cos(f*x+e)-3*B*a^2*c^5*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+1/7*a^2*
A*c^5*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)+3*a^
2*A*c^5*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/
16*f*x+5/16*e)+1/5*a^2*A*c^5*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)
-5*a^2*A*c^5*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-
```

```
5/3*a^2*A*c^5*(2+sin(f*x+e)^2)*cos(f*x+e)-B*a^2*c^5*(-1/8*(sin(f*x+e)^7+7/6
*sin(f*x+e)^5+35/24*sin(f*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)+35/128*f*x+35
/128*e)-3/7*B*a^2*c^5*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)
*cos(f*x+e)-B*a^2*c^5*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))
*cos(f*x+e)+5/16*f*x+5/16*e)+B*a^2*c^5*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*
cos(f*x+e)+5*B*a^2*c^5*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f
*x+3/8*e))
```

maxima [B] time = 0.54, size = 571, normalized size = 2.49

$$-\frac{3072\left(5\cos\left(fx+e\right)^{7}-21\cos\left(fx+e\right)^{5}+35\cos\left(fx+e\right)^{3}-35\cos\left(fx+e\right)\right)Aa^{2}c^{5}-7168\left(3\cos\left(fx+e\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorit
hm="maxima")
```

```
[Out] -1/107520*(3072*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 -
 35*cos(f*x + e))*A*a^2*c^5 - 7168*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 +
15*cos(f*x + e))*A*a^2*c^5 - 179200*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2
*c^5 - 1680*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48
*sin(2*f*x + 2*e))*A*a^2*c^5 + 16800*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*
sin(2*f*x + 2*e))*A*a^2*c^5 - 26880*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*
c^5 - 107520*(f*x + e)*A*a^2*c^5 - 9216*(5*cos(f*x + e)^7 - 21*cos(f*x + e)
^5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^2*c^5 - 35840*(3*cos(f*x + e)
^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*c^5 - 35840*(cos(f*x + e)^3
 - 3*cos(f*x + e))*B*a^2*c^5 + 35*(128*sin(2*f*x + 2*e)^3 + 840*f*x + 840*e
 + 3*sin(8*f*x + 8*e) + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2*e))*B*a^2*
c^5 + 560*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*s
in(2*f*x + 2*e))*B*a^2*c^5 - 16800*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*si
n(2*f*x + 2*e))*B*a^2*c^5 + 80640*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*c^
5 - 322560*A*a^2*c^5*cos(f*x + e) + 107520*B*a^2*c^5*cos(f*x + e))/f
```

mupad [B] time = 15.13, size = 661, normalized size = 2.89

$$\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{14}\left(6Aa^2c^5-2Ba^2c^5\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}\left(30Aa^2c^5-10Ba^2c^5\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}\left(22Aa^2c^5-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^5,x)
```

```
[Out] (tan(e/2 + (f*x)/2)^14*(6*A*a^2*c^5 - 2*B*a^2*c^5) + tan(e/2 + (f*x)/2)^10*
(30*A*a^2*c^5 - 10*B*a^2*c^5) + tan(e/2 + (f*x)/2)^12*(22*A*a^2*c^5 - 18*B*
a^2*c^5) + tan(e/2 + (f*x)/2)^8*(46*A*a^2*c^5 - 26*B*a^2*c^5) + tan(e/2 + (
f*x)/2)^4*((74*A*a^2*c^5)/5 - (14*B*a^2*c^5)/5) - tan(e/2 + (f*x)/2)^15*((7
*A*a^2*c^5)/8 + (27*B*a^2*c^5)/64) + tan(e/2 + (f*x)/2)^2*((158*A*a^2*c^5)/
35 - (138*B*a^2*c^5)/35) + tan(e/2 + (f*x)/2)^6*((218*A*a^2*c^5)/5 - (158*B
*a^2*c^5)/5) + tan(e/2 + (f*x)/2)^3*((75*A*a^2*c^5)/8 - (305*B*a^2*c^5)/64)
 - tan(e/2 + (f*x)/2)^13*((75*A*a^2*c^5)/8 - (305*B*a^2*c^5)/64) + tan(e/2
+ (f*x)/2)^5*((55*A*a^2*c^5)/8 - (437*B*a^2*c^5)/64) - tan(e/2 + (f*x)/2)^1
1*((55*A*a^2*c^5)/8 - (437*B*a^2*c^5)/64) - tan(e/2 + (f*x)/2)^7*((13*A*a^2
*c^5)/8 - (919*B*a^2*c^5)/64) + tan(e/2 + (f*x)/2)^9*((13*A*a^2*c^5)/8 - (9
19*B*a^2*c^5)/64) + tan(e/2 + (f*x)/2)*((7*A*a^2*c^5)/8 + (27*B*a^2*c^5)/64
) + (46*A*a^2*c^5)/35 - (26*B*a^2*c^5)/35)/(f*(8*tan(e/2 + (f*x)/2)^2 + 28*
tan(e/2 + (f*x)/2)^4 + 56*tan(e/2 + (f*x)/2)^6 + 70*tan(e/2 + (f*x)/2)^8 +
56*tan(e/2 + (f*x)/2)^10 + 28*tan(e/2 + (f*x)/2)^12 + 8*tan(e/2 + (f*x)/2)^
14 + tan(e/2 + (f*x)/2)^16 + 1)) + (9*a^2*c^5*atan((9*a^2*c^5*tan(e/2 + (f*
x)/2)*(8*A - 3*B))/(64*((9*A*a^2*c^5)/8 - (27*B*a^2*c^5)/64)))*(8*A - 3*B))
/(64*f)
```

sympy [A] time = 25.00, size = 1586, normalized size = 6.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((15*A*a**2*c**5*x*sin(e + f*x)**6/16 + 45*A*a**2*c**5*x*sin(e + f
*x)**4*cos(e + f*x)**2/16 - 15*A*a**2*c**5*x*sin(e + f*x)**4/8 + 45*A*a**2*
c**5*x*sin(e + f*x)**2*cos(e + f*x)**4/16 - 15*A*a**2*c**5*x*sin(e + f*x)**
2*cos(e + f*x)**2/4 + A*a**2*c**5*x*sin(e + f*x)**2/2 + 15*A*a**2*c**5*x*co
s(e + f*x)**6/16 - 15*A*a**2*c**5*x*cos(e + f*x)**4/8 + A*a**2*c**5*x*cos(e
 + f*x)**2/2 + A*a**2*c**5*x + A*a**2*c**5*sin(e + f*x)**6*cos(e + f*x)/f -
 33*A*a**2*c**5*sin(e + f*x)**5*cos(e + f*x)/(16*f) + 2*A*a**2*c**5*sin(e +
 f*x)**4*cos(e + f*x)**3/f + A*a**2*c**5*sin(e + f*x)**4*cos(e + f*x)/f - 5
*A*a**2*c**5*sin(e + f*x)**3*cos(e + f*x)**3/(2*f) + 25*A*a**2*c**5*sin(e +
 f*x)**3*cos(e + f*x)/(8*f) + 8*A*a**2*c**5*sin(e + f*x)**2*cos(e + f*x)**5
/(5*f) + 4*A*a**2*c**5*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 5*A*a**2*c**
5*sin(e + f*x)**2*cos(e + f*x)/f - 15*A*a**2*c**5*sin(e + f*x)*cos(e + f*x)
**5/(16*f) + 15*A*a**2*c**5*sin(e + f*x)*cos(e + f*x)**3/(8*f) - A*a**2*c**
5*sin(e + f*x)*cos(e + f*x)/(2*f) + 16*A*a**2*c**5*cos(e + f*x)**7/(35*f) +
 8*A*a**2*c**5*cos(e + f*x)**5/(15*f) - 10*A*a**2*c**5*cos(e + f*x)**3/(3*f
) + 3*A*a**2*c**5*cos(e + f*x)/f - 35*B*a**2*c**5*x*sin(e + f*x)**8/128 - 3
5*B*a**2*c**5*x*sin(e + f*x)**6*cos(e + f*x)**2/32 - 5*B*a**2*c**5*x*sin(e
+ f*x)**6/16 - 105*B*a**2*c**5*x*sin(e + f*x)**4*cos(e + f*x)**4/64 - 15*B*
a**2*c**5*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 15*B*a**2*c**5*x*sin(e + f
```

```
*x)**4/8 - 35*B*a**2*c**5*x*sin(e + f*x)**2*cos(e + f*x)**6/32 - 15*B*a**2*
c**5*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 15*B*a**2*c**5*x*sin(e + f*x)**
2*cos(e + f*x)**2/4 - 3*B*a**2*c**5*x*sin(e + f*x)**2/2 - 35*B*a**2*c**5*x*
cos(e + f*x)**8/128 - 5*B*a**2*c**5*x*cos(e + f*x)**6/16 + 15*B*a**2*c**5*x
*cos(e + f*x)**4/8 - 3*B*a**2*c**5*x*cos(e + f*x)**2/2 + 93*B*a**2*c**5*sin
(e + f*x)**7*cos(e + f*x)/(128*f) - 3*B*a**2*c**5*sin(e + f*x)**6*cos(e + f
*x)/f + 511*B*a**2*c**5*sin(e + f*x)**5*cos(e + f*x)**3/(384*f) + 11*B*a**2
*c**5*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 6*B*a**2*c**5*sin(e + f*x)**4*c
os(e + f*x)**3/f + 5*B*a**2*c**5*sin(e + f*x)**4*cos(e + f*x)/f + 385*B*a**
2*c**5*sin(e + f*x)**3*cos(e + f*x)**5/(384*f) + 5*B*a**2*c**5*sin(e + f*x)
**3*cos(e + f*x)**3/(6*f) - 25*B*a**2*c**5*sin(e + f*x)**3*cos(e + f*x)/(8*
f) - 24*B*a**2*c**5*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) + 20*B*a**2*c**5*
sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - B*a**2*c**5*sin(e + f*x)**2*cos(e +
 f*x)/f + 35*B*a**2*c**5*sin(e + f*x)*cos(e + f*x)**7/(128*f) + 5*B*a**2*c*
*5*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 15*B*a**2*c**5*sin(e + f*x)*cos(e
+ f*x)**3/(8*f) + 3*B*a**2*c**5*sin(e + f*x)*cos(e + f*x)/(2*f) - 48*B*a**2
*c**5*cos(e + f*x)**7/(35*f) + 8*B*a**2*c**5*cos(e + f*x)**5/(3*f) - 2*B*a*
*2*c**5*cos(e + f*x)**3/(3*f) - B*a**2*c**5*cos(e + f*x)/f, Ne(f, 0)), (x*(
A + B*sin(e))*(a*sin(e) + a)**2*(-c*sin(e) + c)**5, True))
```

3.27 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$

Optimal. Leaf size=189

$$\frac{a^2c^4(7A-2B)\cos^5(e+fx)}{30f}+\frac{a^2(7A-2B)\cos^5(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{42f}+\frac{a^2c^4(7A-2B)\sin(e+fx)\cos^3(e+$$

```
[Out] 1/16*a^2*(7*A-2*B)*c^4*x+1/30*a^2*(7*A-2*B)*c^4*cos(f*x+e)^5/f+1/16*a^2*(7*
A-2*B)*c^4*cos(f*x+e)*sin(f*x+e)/f+1/24*a^2*(7*A-2*B)*c^4*cos(f*x+e)^3*sin(
f*x+e)/f-1/7*a^2*B*cos(f*x+e)^5*(c^2-c^2*sin(f*x+e))^2/f+1/42*a^2*(7*A-2*B)
*cos(f*x+e)^5*(c^4-c^4*sin(f*x+e))/f
```

Rubi [A] time = 0.30, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2967, 2860, 2678, 2669, 2635, 8}

$$\frac{a^2c^4(7A-2B)\cos^5(e+fx)}{30f}+\frac{a^2(7A-2B)\cos^5(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{42f}+\frac{a^2c^4(7A-2B)\sin(e+fx)\cos^3(e+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a^2*(7*A - 2*B)*c^4*x)/16 + (a^2*(7*A - 2*B)*c^4*Cos[e + f*x]^5)/(30*f) +
(a^2*(7*A - 2*B)*c^4*Cos[e + f*x]*Sin[e + f*x])/(16*f) + (a^2*(7*A - 2*B)*c
^4*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - (a^2*B*Cos[e + f*x]^5*(c^2 - c^2*S
in[e + f*x])^2)/(7*f) + (a^2*(7*A - 2*B)*Cos[e + f*x]^5*(c^4 - c^4*Sin[e +
f*x]))/(42*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
```

```
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(A+B\sin(e+fx))(c-$$

$$= -\frac{a^2B\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{7f} + \frac{1}{7}($$

$$= -\frac{a^2B\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{7f} + \frac{a^2(}{}$$

$$= \frac{a^2(7A-2B)c^4\cos^5(e+fx)}{30f} - \frac{a^2B\cos^5(e+fx}{}$$

$$= \frac{a^2(7A-2B)c^4\cos^5(e+fx)}{30f} + \frac{a^2(7A-2B)c^4}{}$$

$$= \frac{a^2(7A-2B)c^4\cos^5(e+fx)}{30f} + \frac{a^2(7A-2B)c^4}{}$$

$$= \frac{1}{16}a^2(7A-2B)c^4x + \frac{a^2(7A-2B)c^4\cos^5(e+f}{30f}$$

Mathematica [A] time = 1.59, size = 163, normalized size = 0.86

$$\frac{a^2c^4(105(16A-11B)\cos(e+fx)+105(8A-5B)\cos(3(e+fx))+1785A\sin(2(e+fx))+105A\sin(4(e+fx))-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
4,x]
```

```
[Out] (a^2*c^4*(2940*A*e - 840*B*e + 2940*A*f*x - 840*B*f*x + 105*(16*A - 11*B)*C
os[e + f*x] + 105*(8*A - 5*B)*Cos[3*(e + f*x)] + 168*A*Cos[5*(e + f*x)] - 6
3*B*Cos[5*(e + f*x)] + 15*B*Cos[7*(e + f*x)] + 1785*A*Sin[2*(e + f*x)] - 21
0*B*Sin[2*(e + f*x)] + 105*A*Sin[4*(e + f*x)] + 210*B*Sin[4*(e + f*x)] - 35
*A*Sin[6*(e + f*x)] + 70*B*Sin[6*(e + f*x)]))/(6720*f)
```

fricas [A] time = 0.45, size = 135, normalized size = 0.71

$$\frac{240\,Ba^2c^4\cos\left(fx+e\right)^7+672\,(A-B)a^2c^4\cos\left(fx+e\right)^5+105\,(7\,A-2\,B)a^2c^4fx-35\left(8\,(A-2\,B)a^2c^4\cos\left(fx+\right.\right.}{1680\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorit
hm="fricas")
```

```
[Out] 1/1680*(240*B*a^2*c^4*cos(f*x + e)^7 + 672*(A - B)*a^2*c^4*cos(f*x + e)^5 +
 105*(7*A - 2*B)*a^2*c^4*f*x - 35*(8*(A - 2*B)*a^2*c^4*cos(f*x + e)^5 - 2*(
7*A - 2*B)*a^2*c^4*cos(f*x + e)^3 - 3*(7*A - 2*B)*a^2*c^4*cos(f*x + e))*sin
(f*x + e))/f
```

giac [A] time = 0.21, size = 244, normalized size = 1.29

$$\frac{Ba^2c^4\cos\left(7\,fx+7\,e\right)}{448\,f}+\frac{1}{16}\left(7\,Aa^2c^4-2\,Ba^2c^4\right)x+\frac{\left(8\,Aa^2c^4-3\,Ba^2c^4\right)\cos\left(5\,fx+5\,e\right)}{320\,f}+\frac{\left(8\,Aa^2c^4-5\,Ba^2c^4\right)\cos}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorit
hm="giac")
```

```
[Out] 1/448*B*a^2*c^4*cos(7*f*x + 7*e)/f + 1/16*(7*A*a^2*c^4 - 2*B*a^2*c^4)*x + 1
/320*(8*A*a^2*c^4 - 3*B*a^2*c^4)*cos(5*f*x + 5*e)/f + 1/64*(8*A*a^2*c^4 - 5
*B*a^2*c^4)*cos(3*f*x + 3*e)/f + 1/64*(16*A*a^2*c^4 - 11*B*a^2*c^4)*cos(f*x
 + e)/f - 1/192*(A*a^2*c^4 - 2*B*a^2*c^4)*sin(6*f*x + 6*e)/f + 1/64*(A*a^2*
c^4 + 2*B*a^2*c^4)*sin(4*f*x + 4*e)/f + 1/64*(17*A*a^2*c^4 - 2*B*a^2*c^4)*s
in(2*f*x + 2*e)/f
```

maple [B] time = 0.57, size = 463, normalized size = 2.45

$$a^2A\,c^4\left(fx+e\right)+4B\,a^2c^4\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+\frac{B\,a^2c^4\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}+2a^2A\,c^4\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x)
```

```
[Out] 1/f*(a^2*A*c^4*(f*x+e)+4*B*a^2*c^4*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(
f*x+e)+3/8*f*x+3/8*e)+1/3*B*a^2*c^4*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a^2*A*c^4
*cos(f*x+e)-2*B*a^2*c^4*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-4/3*a^2*
A*c^4*(2+sin(f*x+e)^2)*cos(f*x+e)-1/7*B*a^2*c^4*(16/5+sin(f*x+e)^6+6/5*sin(
f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)-2*B*a^2*c^4*(-1/6*(sin(f*x+e)^5+5/4*s
in(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+1/5*B*a^2*c^4*(8/3
+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-a^2*A*c^4*(-1/4*(sin(f*x+e)^3+3/
2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+a^2*A*c^4*(-1/6*(sin(f*x+e)^5+5/4*s
in(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+2/5*a^2*A*c^4*(8/3
```

```
+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-a^2*A*c^4*(-1/2*sin(f*x+e)*cos(f
*x+e)+1/2*f*x+1/2*e)-B*a^2*c^4*cos(f*x+e))
```

maxima [B] time = 0.34, size = 460, normalized size = 2.43

$$\frac{896\left(3\cos\left(fx+e\right)^{5}-10\cos\left(fx+e\right)^{3}+15\cos\left(fx+e\right)\right)Aa^{2}c^{4}+8960\left(\cos\left(fx+e\right)^{3}-3\cos\left(fx+e\right)\right)Aa^{2}c^{4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorit
hm="maxima")
```

```
[Out] 1/6720*(896*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^2*
c^4 + 8960*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c^4 + 35*(4*sin(2*f*x +
2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*A*a^2*c^
4 - 210*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*c^4 -
 1680*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^4 + 6720*(f*x + e)*A*a^2*c^4
 + 192*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 - 35*cos(f
*x + e))*B*a^2*c^4 + 448*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x
 + e))*B*a^2*c^4 - 2240*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^4 - 70*(4
*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2
*e))*B*a^2*c^4 + 840*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e)
)*B*a^2*c^4 - 3360*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*c^4 + 13440*A*a^2
*c^4*cos(f*x + e) - 6720*B*a^2*c^4*cos(f*x + e))/f
```

mupad [B] time = 14.89, size = 553, normalized size = 2.93

$$\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}\left(4\,A\,a^2\,c^4-2\,B\,a^2\,c^4\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{8}\left(12\,A\,a^2\,c^4-2\,B\,a^2\,c^4\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}\left(8\,A\,a^2\,c^4-8\,B\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^4,x)
```

```
[Out] (tan(e/2 + (f*x)/2)^12*(4*A*a^2*c^4 - 2*B*a^2*c^4) + tan(e/2 + (f*x)/2)^8*(
12*A*a^2*c^4 - 2*B*a^2*c^4) + tan(e/2 + (f*x)/2)^10*(8*A*a^2*c^4 - 8*B*a^2*
c^4) + tan(e/2 + (f*x)/2)^2*((8*A*a^2*c^4)/5 - (8*B*a^2*c^4)/5) - tan(e/2 +
 (f*x)/2)^13*((9*A*a^2*c^4)/8 + (B*a^2*c^4)/4) + tan(e/2 + (f*x)/2)^6*(16*A
*a^2*c^4 - 16*B*a^2*c^4) + tan(e/2 + (f*x)/2)^3*((29*A*a^2*c^4)/6 - (11*B*a
^2*c^4)/3) - tan(e/2 + (f*x)/2)^11*((29*A*a^2*c^4)/6 - (11*B*a^2*c^4)/3) +
tan(e/2 + (f*x)/2)^4*((44*A*a^2*c^4)/5 - (14*B*a^2*c^4)/5) + tan(e/2 + (f*x
```

```
)/2)^5*((23*A*a^2*c^4)/24 + (31*B*a^2*c^4)/12) - tan(e/2 + (f*x)/2)^9*((23*
A*a^2*c^4)/24 + (31*B*a^2*c^4)/12) + tan(e/2 + (f*x)/2)*((9*A*a^2*c^4)/8 +
(B*a^2*c^4)/4) + (4*A*a^2*c^4)/5 - (18*B*a^2*c^4)/35)/(f*(7*tan(e/2 + (f*x)
/2)^2 + 21*tan(e/2 + (f*x)/2)^4 + 35*tan(e/2 + (f*x)/2)^6 + 35*tan(e/2 + (f
*x)/2)^8 + 21*tan(e/2 + (f*x)/2)^10 + 7*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (
f*x)/2)^14 + 1)) + (a^2*c^4*atan((a^2*c^4*tan(e/2 + (f*x)/2)*(7*A - 2*B))/(
8*((7*A*a^2*c^4)/8 - (B*a^2*c^4)/4)))*(7*A - 2*B))/(8*f)
```

sympy [A] time = 15.02, size = 1210, normalized size = 6.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((5*A*a**2*c**4*x*sin(e + f*x)**6/16 + 15*A*a**2*c**4*x*sin(e + f*
x)**4*cos(e + f*x)**2/16 - 3*A*a**2*c**4*x*sin(e + f*x)**4/8 + 15*A*a**2*c*
*4*x*sin(e + f*x)**2*cos(e + f*x)**4/16 - 3*A*a**2*c**4*x*sin(e + f*x)**2*c
os(e + f*x)**2/4 - A*a**2*c**4*x*sin(e + f*x)**2/2 + 5*A*a**2*c**4*x*cos(e
+ f*x)**6/16 - 3*A*a**2*c**4*x*cos(e + f*x)**4/8 - A*a**2*c**4*x*cos(e + f*
x)**2/2 + A*a**2*c**4*x - 11*A*a**2*c**4*sin(e + f*x)**5*cos(e + f*x)/(16*f
) + 2*A*a**2*c**4*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**2*c**4*sin(e + f*
x)**3*cos(e + f*x)**3/(6*f) + 5*A*a**2*c**4*sin(e + f*x)**3*cos(e + f*x)/(8
*f) + 8*A*a**2*c**4*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 4*A*a**2*c**4*s
in(e + f*x)**2*cos(e + f*x)/f - 5*A*a**2*c**4*sin(e + f*x)*cos(e + f*x)**5/
(16*f) + 3*A*a**2*c**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) + A*a**2*c**4*sin
(e + f*x)*cos(e + f*x)/(2*f) + 16*A*a**2*c**4*cos(e + f*x)**5/(15*f) - 8*A*
a**2*c**4*cos(e + f*x)**3/(3*f) + 2*A*a**2*c**4*cos(e + f*x)/f - 5*B*a**2*c
**4*x*sin(e + f*x)**6/8 - 15*B*a**2*c**4*x*sin(e + f*x)**4*cos(e + f*x)**2/
8 + 3*B*a**2*c**4*x*sin(e + f*x)**4/2 - 15*B*a**2*c**4*x*sin(e + f*x)**2*co
s(e + f*x)**4/8 + 3*B*a**2*c**4*x*sin(e + f*x)**2*cos(e + f*x)**2 - B*a**2*
c**4*x*sin(e + f*x)**2 - 5*B*a**2*c**4*x*cos(e + f*x)**6/8 + 3*B*a**2*c**4*
x*cos(e + f*x)**4/2 - B*a**2*c**4*x*cos(e + f*x)**2 - B*a**2*c**4*sin(e + f
*x)**6*cos(e + f*x)/f + 11*B*a**2*c**4*sin(e + f*x)**5*cos(e + f*x)/(8*f) -
 2*B*a**2*c**4*sin(e + f*x)**4*cos(e + f*x)**3/f + B*a**2*c**4*sin(e + f*x)
**4*cos(e + f*x)/f + 5*B*a**2*c**4*sin(e + f*x)**3*cos(e + f*x)**3/(3*f) -
5*B*a**2*c**4*sin(e + f*x)**3*cos(e + f*x)/(2*f) - 8*B*a**2*c**4*sin(e + f*
x)**2*cos(e + f*x)**5/(5*f) + 4*B*a**2*c**4*sin(e + f*x)**2*cos(e + f*x)**3
/(3*f) + B*a**2*c**4*sin(e + f*x)**2*cos(e + f*x)/f + 5*B*a**2*c**4*sin(e +
 f*x)*cos(e + f*x)**5/(8*f) - 3*B*a**2*c**4*sin(e + f*x)*cos(e + f*x)**3/(2
*f) + B*a**2*c**4*sin(e + f*x)*cos(e + f*x)/f - 16*B*a**2*c**4*cos(e + f*x)
**7/(35*f) + 8*B*a**2*c**4*cos(e + f*x)**5/(15*f) + 2*B*a**2*c**4*cos(e + f
*x)**3/(3*f) - B*a**2*c**4*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*
sin(e) + a)**2*(-c*sin(e) + c)**4, True))
```

3.28 $\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c - c\sin(e + fx))^3\,dx$

Optimal. Leaf size=147

$$\frac{a^2c^3(6A-B)\cos^5(e+fx)}{30f}+\frac{a^2c^3(6A-B)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{a^2c^3(6A-B)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}a^2$$

```
[Out] 1/16*a^2*(6*A-B)*c^3*x+1/30*a^2*(6*A-B)*c^3*cos(f*x+e)^5/f+1/16*a^2*(6*A-B)
*c^3*cos(f*x+e)*sin(f*x+e)/f+1/24*a^2*(6*A-B)*c^3*cos(f*x+e)^3*sin(f*x+e)/f
-1/6*a^2*B*cos(f*x+e)^5*(c^3-c^3*sin(f*x+e))/f
```

Rubi [A] time = 0.22, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2860, 2669, 2635, 8}

$$\frac{a^2c^3(6A-B)\cos^5(e+fx)}{30f}+\frac{a^2c^3(6A-B)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{a^2c^3(6A-B)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}a^2$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a^2*(6*A - B)*c^3*x)/16 + (a^2*(6*A - B)*c^3*Cos[e + f*x]^5)/(30*f) + (a^2
*(6*A - B)*c^3*Cos[e + f*x]*Sin[e + f*x])/(16*f) + (a^2*(6*A - B)*c^3*Cos[e
 + f*x]^3*Sin[e + f*x])/(24*f) - (a^2*B*Cos[e + f*x]^5*(c^3 - c^3*Sin[e + f
*x]))/(6*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
```

```
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(A+B\sin(e+fx))(c-$$

$$= -\frac{a^2B\cos^5(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{6f}+\frac{1}{6}$$

$$= \frac{a^2(6A-B)c^3\cos^5(e+fx)}{30f}-\frac{a^2B\cos^5(e+f$$

$$= \frac{a^2(6A-B)c^3\cos^5(e+fx)}{30f}+\frac{a^2(6A-B)c^3\,\mathrm{c}$$

$$= \frac{a^2(6A-B)c^3\cos^5(e+fx)}{30f}+\frac{a^2(6A-B)c^3\,\mathrm{c}$$

$$= \frac{1}{16}a^2(6A-B)c^3x+\frac{a^2(6A-B)c^3\cos^5(e+f$$

Mathematica [A] time = 1.09, size = 137, normalized size = 0.93

$$\frac{a^2c^3(120(A-B)\cos(e+fx)+60(A-B)\cos(3(e+fx))+240A\sin(2(e+fx))+30A\sin(4(e+fx))+12A\,\mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
3,x]
```

```
[Out] (a^2*c^3*(360*A*e - 60*B*e + 360*A*f*x - 60*B*f*x + 120*(A - B)*Cos[e + f*x
] + 60*(A - B)*Cos[3*(e + f*x)] + 12*A*Cos[5*(e + f*x)] - 12*B*Cos[5*(e + f
*x)] + 240*A*Sin[2*(e + f*x)] - 15*B*Sin[2*(e + f*x)] + 30*A*Sin[4*(e + f*x
)] + 15*B*Sin[4*(e + f*x)] + 5*B*Sin[6*(e + f*x)]))/(960*f)
```

fricas [A] time = 0.44, size = 114, normalized size = 0.78

$$\frac{48\,(A-B)a^2c^3\cos\left(fx+e\right)^5+15\,(6\,A-B)a^2c^3fx+5\left(8\,Ba^2c^3\cos\left(fx+e\right)^5+2\,(6\,A-B)a^2c^3\cos\left(fx+e\right)^3+3\right.}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/240*(48*(A - B)*a^2*c^3*cos(f*x + e)^5 + 15*(6*A - B)*a^2*c^3*f*x + 5*(8*
B*a^2*c^3*cos(f*x + e)^5 + 2*(6*A - B)*a^2*c^3*cos(f*x + e)^3 + 3*(6*A - B)
*a^2*c^3*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.18, size = 208, normalized size = 1.41

$$\frac{Ba^2c^3\sin\left(6\,fx+6\,e\right)}{192\,f}+\frac{1}{16}\left(6\,Aa^2c^3-Ba^2c^3\right)x+\frac{\left(Aa^2c^3-Ba^2c^3\right)\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{\left(Aa^2c^3-Ba^2c^3\right)\cos\left(3\,fx+3\right.}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/192*B*a^2*c^3*sin(6*f*x + 6*e)/f + 1/16*(6*A*a^2*c^3 - B*a^2*c^3)*x + 1/8
0*(A*a^2*c^3 - B*a^2*c^3)*cos(5*f*x + 5*e)/f + 1/16*(A*a^2*c^3 - B*a^2*c^3)
*cos(3*f*x + 3*e)/f + 1/8*(A*a^2*c^3 - B*a^2*c^3)*cos(f*x + e)/f + 1/64*(2*
A*a^2*c^3 + B*a^2*c^3)*sin(4*f*x + 4*e)/f + 1/64*(16*A*a^2*c^3 - B*a^2*c^3)
*sin(2*f*x + 2*e)/f
```

maple [B] time = 0.57, size = 365, normalized size = 2.48

$$\frac{\frac{a^2Ac^3\left(\frac{8}{3}+\sin^4\left(fx+e\right)+\frac{4\left(\sin^2\left(fx+e\right)\right)}{3}\right)\cos\left(fx+e\right)}{5}+a^2Ac^3\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2a^2Ac^3\left(2+\sin^2\left(fx+e\right)\right)\cos}{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(1/5*a^2*A*c^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+a^2*A*c^3
*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/3*a^2*A*c^
3*(2+sin(f*x+e)^2)*cos(f*x+e)-2*a^2*A*c^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f
*x+1/2*e)-B*a^2*c^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*c
os(f*x+e)+5/16*f*x+5/16*e)-1/5*B*a^2*c^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2
)*cos(f*x+e)+2*B*a^2*c^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8
*f*x+3/8*e)+2/3*B*a^2*c^3*(2+sin(f*x+e)^2)*cos(f*x+e)+a^2*A*c^3*cos(f*x+e)-
B*a^2*c^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+a^2*A*c^3*(f*x+e)-B*a^
2*c^3*cos(f*x+e))
```

maxima [B] time = 0.47, size = 360, normalized size = 2.45

$$\frac{64\left(3\cos\left(fx+e\right)^5-10\cos\left(fx+e\right)^3+15\cos\left(fx+e\right)\right)Aa^2c^3+640\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)Aa^2c^3+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 1/960*(64*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^2*c^
3 + 640*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c^3 + 30*(12*f*x + 12*e + s
in(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*c^3 - 480*(2*f*x + 2*e - sin(2*
f*x + 2*e))*A*a^2*c^3 + 960*(f*x + e)*A*a^2*c^3 - 64*(3*cos(f*x + e)^5 - 10
*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*c^3 - 640*(cos(f*x + e)^3 - 3*cos(
f*x + e))*B*a^2*c^3 - 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x
 + 4*e) - 48*sin(2*f*x + 2*e))*B*a^2*c^3 + 60*(12*f*x + 12*e + sin(4*f*x +
4*e) - 8*sin(2*f*x + 2*e))*B*a^2*c^3 - 240*(2*f*x + 2*e - sin(2*f*x + 2*e))
*B*a^2*c^3 + 960*A*a^2*c^3*cos(f*x + e) - 960*B*a^2*c^3*cos(f*x + e))/f
```

mupad [B] time = 14.15, size = 542, normalized size = 3.69

$$\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(4\,A\,a^2\,c^3-4\,B\,a^2\,c^3\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8\left(2\,A\,a^2\,c^3-2\,B\,a^2\,c^3\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6\left(4\,A\,a^2\,c^3-4\,B\,a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^3,x)
```

```
[Out] (tan(e/2 + (f*x)/2)^4*(4*A*a^2*c^3 - 4*B*a^2*c^3) + tan(e/2 + (f*x)/2)^8*(2
*A*a^2*c^3 - 2*B*a^2*c^3) + tan(e/2 + (f*x)/2)^6*(4*A*a^2*c^3 - 4*B*a^2*c^3
) + tan(e/2 + (f*x)/2)^10*(2*A*a^2*c^3 - 2*B*a^2*c^3) + tan(e/2 + (f*x)/2)^
2*((2*A*a^2*c^3)/5 - (2*B*a^2*c^3)/5) + tan(e/2 + (f*x)/2)^5*((A*a^2*c^3)/2
 + (13*B*a^2*c^3)/4) - tan(e/2 + (f*x)/2)^7*((A*a^2*c^3)/2 + (13*B*a^2*c^3)
/4) - tan(e/2 + (f*x)/2)^11*((5*A*a^2*c^3)/4 + (B*a^2*c^3)/8) + tan(e/2 + (
f*x)/2)^3*((7*A*a^2*c^3)/4 - (47*B*a^2*c^3)/24) - tan(e/2 + (f*x)/2)^9*((7*
A*a^2*c^3)/4 - (47*B*a^2*c^3)/24) + tan(e/2 + (f*x)/2)*((5*A*a^2*c^3)/4 + (
B*a^2*c^3)/8) + (2*A*a^2*c^3)/5 - (2*B*a^2*c^3)/5)/(f*(6*tan(e/2 + (f*x)/2)
^2 + 15*tan(e/2 + (f*x)/2)^4 + 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)
/2)^8 + 6*tan(e/2 + (f*x)/2)^10 + tan(e/2 + (f*x)/2)^12 + 1)) + (a^2*c^3*at
an((a^2*c^3*tan(e/2 + (f*x)/2)*(6*A - B))/(8*((3*A*a^2*c^3)/4 - (B*a^2*c^3)
/8)))*(6*A - B))/(8*f) - (a^2*c^3*(6*A - B)*(atan(tan(e/2 + (f*x)/2)) - (f*
x)/2))/(8*f)
```

sympy [A] time = 9.14, size = 910, normalized size = 6.19

$$\begin{cases} \frac{3Aa^2c^3x\sin^4(e+fx)}{8} + \frac{3Aa^2c^3x\sin^2(e+fx)\cos^2(e+fx)}{4} - Aa^2c^3x\sin^2(e+fx) + \frac{3Aa^2c^3x\cos^4(e+fx)}{8} - Aa^2c^3x\cos^2(e+fx) \\ x(A+B\sin(e))(a\sin(e)+a)^2(-c\sin(e)+c)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((3*A*a**2*c**3*x*sin(e + f*x)**4/8 + 3*A*a**2*c**3*x*sin(e + f*x)
**2*cos(e + f*x)**2/4 - A*a**2*c**3*x*sin(e + f*x)**2 + 3*A*a**2*c**3*x*cos
(e + f*x)**4/8 - A*a**2*c**3*x*cos(e + f*x)**2 + A*a**2*c**3*x + A*a**2*c**
3*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**2*c**3*sin(e + f*x)**3*cos(e + f*
x)/(8*f) + 4*A*a**2*c**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 2*A*a**2*c
**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**2*c**3*sin(e + f*x)*cos(e + f*x
)**3/(8*f) + A*a**2*c**3*sin(e + f*x)*cos(e + f*x)/f + 8*A*a**2*c**3*cos(e
+ f*x)**5/(15*f) - 4*A*a**2*c**3*cos(e + f*x)**3/(3*f) + A*a**2*c**3*cos(e
+ f*x)/f - 5*B*a**2*c**3*x*sin(e + f*x)**6/16 - 15*B*a**2*c**3*x*sin(e + f*
x)**4*cos(e + f*x)**2/16 + 3*B*a**2*c**3*x*sin(e + f*x)**4/4 - 15*B*a**2*c*
*3*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 3*B*a**2*c**3*x*sin(e + f*x)**2*c
os(e + f*x)**2/2 - B*a**2*c**3*x*sin(e + f*x)**2/2 - 5*B*a**2*c**3*x*cos(e
+ f*x)**6/16 + 3*B*a**2*c**3*x*cos(e + f*x)**4/4 - B*a**2*c**3*x*cos(e + f*
x)**2/2 + 11*B*a**2*c**3*sin(e + f*x)**5*cos(e + f*x)/(16*f) - B*a**2*c**3*
sin(e + f*x)**4*cos(e + f*x)/f + 5*B*a**2*c**3*sin(e + f*x)**3*cos(e + f*x)
**3/(6*f) - 5*B*a**2*c**3*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 4*B*a**2*c**
3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 2*B*a**2*c**3*sin(e + f*x)**2*cos
(e + f*x)/f + 5*B*a**2*c**3*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 3*B*a**2*
c**3*sin(e + f*x)*cos(e + f*x)**3/(4*f) + B*a**2*c**3*sin(e + f*x)*cos(e +
```

```
f*x)/(2*f) - 8*B*a**2*c**3*cos(e + f*x)**5/(15*f) + 4*B*a**2*c**3*cos(e + f
*x)**3/(3*f) - B*a**2*c**3*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*
sin(e) + a)**2*(-c*sin(e) + c)**3, True))
```

3.29 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$

Optimal. Leaf size=89

$$\frac{a^2Ac^2\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3a^2Ac^2\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}a^2Ac^2x-\frac{a^2Bc^2\cos^5(e+fx)}{5f}$$

```
[Out] 3/8*a^2*A*c^2*x-1/5*a^2*B*c^2*cos(f*x+e)^5/f+3/8*a^2*A*c^2*cos(f*x+e)*sin(f
*x+e)/f+1/4*a^2*A*c^2*cos(f*x+e)^3*sin(f*x+e)/f
```

Rubi [A] time = 0.14, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2669, 2635, 8}

$$\frac{a^2Ac^2\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3a^2Ac^2\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}a^2Ac^2x-\frac{a^2Bc^2\cos^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (3*a^2*A*c^2*x)/8 - (a^2*B*c^2*Cos[e + f*x]^5)/(5*f) + (3*a^2*A*c^2*Cos[e +
 f*x]*Sin[e + f*x])/(8*f) + (a^2*A*c^2*Cos[e + f*x]^3*Sin[e + f*x])/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(A+B\sin(e+fx))\,dx$$
$$= -\frac{a^2Bc^2\cos^5(e+fx)}{5f} + \left(a^2Ac^2\right)\int \cos^4(e+$$
$$= -\frac{a^2Bc^2\cos^5(e+fx)}{5f} + \frac{a^2Ac^2\cos^3(e+fx)\,\mathrm{s}}{4f}$$
$$= -\frac{a^2Bc^2\cos^5(e+fx)}{5f} + \frac{3a^2Ac^2\cos(e+fx)\,\mathrm{s}}{8f}$$
$$= \frac{3}{8}a^2Ac^2x - \frac{a^2Bc^2\cos^5(e+fx)}{5f} + \frac{3a^2Ac^2\cos}{}$$

Mathematica [A] time = 0.15, size = 54, normalized size = 0.61

$$\frac{a^2c^2\left(5A(12(e+fx)+8\sin(2(e+fx))+\sin(4(e+fx)))-32B\cos^5(e+fx)\right)}{160f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
2,x]
```

```
[Out] (a^2*c^2*(-32*B*Cos[e + f*x]^5 + 5*A*(12*(e + f*x) + 8*Sin[2*(e + f*x)] + S
in[4*(e + f*x)])))/(160*f)
```

fricas [A] time = 0.42, size = 75, normalized size = 0.84

$$-\frac{8\,Ba^2c^2\cos\left(fx+e\right)^5 - 15\,Aa^2c^2fx - 5\left(2\,Aa^2c^2\cos\left(fx+e\right)^3 + 3\,Aa^2c^2\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{40\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/40*(8*B*a^2*c^2*cos(f*x + e)^5 - 15*A*a^2*c^2*f*x - 5*(2*A*a^2*c^2*cos(f
*x + e)^3 + 3*A*a^2*c^2*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.15, size = 118, normalized size = 1.33

$$\frac{3}{8}Aa^2c^2x - \frac{Ba^2c^2\cos\left(5fx+5e\right)}{80f} - \frac{Ba^2c^2\cos\left(3fx+3e\right)}{16f} - \frac{Ba^2c^2\cos\left(fx+e\right)}{8f} + \frac{Aa^2c^2\sin\left(4fx+4e\right)}{32f} + \frac{Aa^2c^2\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 3/8*A*a^2*c^2*x - 1/80*B*a^2*c^2*cos(5*f*x + 5*e)/f - 1/16*B*a^2*c^2*cos(3*
f*x + 3*e)/f - 1/8*B*a^2*c^2*cos(f*x + e)/f + 1/32*A*a^2*c^2*sin(4*f*x + 4*
e)/f + 1/4*A*a^2*c^2*sin(2*f*x + 2*e)/f
```

maple [B] time = 0.48, size = 166, normalized size = 1.87

$$\frac{-\frac{Ba^2c^2\left(\frac{8}{3}+\sin^4\left(fx+e\right)+\frac{4\left(\sin^2\left(fx+e\right)\right)}{3}\right)\cos\left(fx+e\right)}{5} + a^2Ac^2\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4} + \frac{3fx}{8} + \frac{3e}{8}\right) + \frac{2Ba^2c^2\left(2+\sin^2\left(fx+e\right)\right)\cos}{3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(-1/5*B*a^2*c^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+a^2*A*c^
2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+2/3*B*a^2*c
^2*(2+sin(f*x+e)^2)*cos(f*x+e)-2*a^2*A*c^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*
f*x+1/2*e)-B*a^2*c^2*cos(f*x+e)+a^2*A*c^2*(f*x+e))
```

maxima [B] time = 0.49, size = 164, normalized size = 1.84

$$\frac{15\left(12fx+12e+\sin\left(4fx+4e\right)-8\sin\left(2fx+2e\right)\right)Aa^2c^2 - 240\left(2fx+2e-\sin\left(2fx+2e\right)\right)Aa^2c^2 + 480\left(f\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/480*(15*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*c^2
 - 240*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^2 + 480*(f*x + e)*A*a^2*c^2
 - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*c^2 -
320*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^2 - 480*B*a^2*c^2*cos(f*x + e
))/f
```

mupad [B] time = 14.21, size = 238, normalized size = 2.67

$$\frac{3\,A\,a^2\,c^2\,x}{8}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8\,\left(\frac{a^2\,c^2\,\left(80\,B-75\,A\,\left(e+f\,x\right)\right)}{40}+\frac{15\,A\,a^2\,c^2\,\left(e+f\,x\right)}{8}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(\frac{a^2\,c^2\,\left(160\,B-150\,A\,\left(e+f\,x\right)\right)}{40}+\frac{15\,A}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^2,x)
```

```
[Out] (3*A*a^2*c^2*x)/8 - (tan(e/2 + (f*x)/2)^8*((a^2*c^2*(80*B - 75*A*(e + f*x))
)/40 + (15*A*a^2*c^2*(e + f*x))/8) + tan(e/2 + (f*x)/2)^4*((a^2*c^2*(160*B
- 150*A*(e + f*x)))/40 + (15*A*a^2*c^2*(e + f*x))/4) + (a^2*c^2*(16*B - 15*
A*(e + f*x)))/40 + (3*A*a^2*c^2*(e + f*x))/8 - (A*a^2*c^2*tan(e/2 + (f*x)/2
)^3)/2 + (A*a^2*c^2*tan(e/2 + (f*x)/2)^7)/2 + (5*A*a^2*c^2*tan(e/2 + (f*x)/
2)^9)/4 - (5*A*a^2*c^2*tan(e/2 + (f*x)/2))/4)/(f*(tan(e/2 + (f*x)/2)^2 + 1)
^5)
```

sympy [A] time = 3.70, size = 372, normalized size = 4.18

$$\begin{cases} \frac{3Aa^2c^2x\sin^4\left(e+fx\right)}{8} + \frac{3Aa^2c^2x\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4} - Aa^2c^2x\sin^2\left(e+fx\right) + \frac{3Aa^2c^2x\cos^4\left(e+fx\right)}{8} - Aa^2c^2x\cos^2\left(e+fx\right. \\ x\left(A+B\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right)^2\left(-c\sin\left(e\right)+c\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*A*a**2*c**2*x*sin(e + f*x)**4/8 + 3*A*a**2*c**2*x*sin(e + f*x)
**2*cos(e + f*x)**2/4 - A*a**2*c**2*x*sin(e + f*x)**2 + 3*A*a**2*c**2*x*cos
(e + f*x)**4/8 - A*a**2*c**2*x*cos(e + f*x)**2 + A*a**2*c**2*x - 5*A*a**2*c
**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*A*a**2*c**2*sin(e + f*x)*cos(e +
 f*x)**3/(8*f) + A*a**2*c**2*sin(e + f*x)*cos(e + f*x)/f - B*a**2*c**2*sin(
e + f*x)**4*cos(e + f*x)/f - 4*B*a**2*c**2*sin(e + f*x)**2*cos(e + f*x)**3/
(3*f) + 2*B*a**2*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 8*B*a**2*c**2*cos(e
+ f*x)**5/(15*f) + 4*B*a**2*c**2*cos(e + f*x)**3/(3*f) - B*a**2*c**2*cos(e
+ f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**2*(-c*sin(e) + c)**2
, True))
```

3.30 $\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c - c\sin(e + fx))\,dx$

Optimal. Leaf size=98

$$-\frac{a^2c(4A+B)\cos^3(e+fx)}{12f}+\frac{a^2c(4A+B)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}a^2cx(4A+B)-\frac{Bc\cos^3(e+fx)\left(a^2\sin(e+fx\right.}{4f}$$

[Out] 1/8*a^2*(4*A+B)*c*x-1/12*a^2*(4*A+B)*c*cos(f*x+e)^3/f+1/8*a^2*(4*A+B)*c*cos(f*x+e)*sin(f*x+e)/f-1/4*B*c*cos(f*x+e)^3*(a^2+a^2*sin(f*x+e))/f

Rubi [A] time = 0.15, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {2967, 2860, 2669, 2635, 8}

$$-\frac{a^2c(4A+B)\cos^3(e+fx)}{12f}+\frac{a^2c(4A+B)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}a^2cx(4A+B)-\frac{Bc\cos^3(e+fx)\left(a^2\sin(e+fx\right.}{4f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]

[Out] (a^2*(4*A + B)*c*x)/8 - (a^2*(4*A + B)*c*Cos[e + f*x]^3)/(12*f) + (a^2*(4*A + B)*c*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (B*c*Cos[e + f*x]^3*(a^2 + a^2*Sin[e + f*x]))/(4*f)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx = (ac)\int \cos^2(e+fx)(a+a\sin(e+fx))(A+B$$

$$= -\frac{Bc\cos^3(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{4f}+\frac{1}{4}(a$$

$$= -\frac{a^2(4A+B)c\cos^3(e+fx)}{12f}-\frac{Bc\cos^3(e+fx)}{}$$

$$= -\frac{a^2(4A+B)c\cos^3(e+fx)}{12f}+\frac{a^2(4A+B)c\cos}{}$$

$$= \frac{1}{8}a^2(4A+B)cx-\frac{a^2(4A+B)c\cos^3(e+fx)}{12f}+$$

Mathematica [A] time = 0.80, size = 67, normalized size = 0.68

$$-\frac{a^2c(24(A+B)\cos(e+fx)+8(A+B)\cos(3(e+fx))-12fx(4A+B)-24A\sin(2(e+fx))+3B\sin(4(e+fx}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),
x]
```

```
[Out] -1/96*(a^2*c*(-12*(4*A + B)*f*x + 24*(A + B)*Cos[e + f*x] + 8*(A + B)*Cos[3
*(e + f*x)] - 24*A*Sin[2*(e + f*x)] + 3*B*Sin[4*(e + f*x)]))/f
```

fricas [A] time = 0.43, size = 77, normalized size = 0.79

$$-\frac{8\,(A+B)a^2c\cos\left(fx+e\right)^3-3\,(4\,A+B)a^2cfx+3\left(2\,Ba^2c\cos\left(fx+e\right)^3-(4\,A+B)a^2c\cos\left(fx+e\right)\right)\sin\left(fx+\right.}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] -1/24*(8*(A + B)*a^2*c*cos(f*x + e)^3 - 3*(4*A + B)*a^2*c*f*x + 3*(2*B*a^2*
c*cos(f*x + e)^3 - (4*A + B)*a^2*c*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.15, size = 111, normalized size = 1.13

$$-\frac{Ba^2c\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{Aa^2c\sin\left(2\,fx+2\,e\right)}{4\,f}+\frac{1}{8}\left(4\,Aa^2c+Ba^2c\right)x-\frac{\left(Aa^2c+Ba^2c\right)\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{\left(Aa^2c+Ba\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/32*B*a^2*c*sin(4*f*x + 4*e)/f + 1/4*A*a^2*c*sin(2*f*x + 2*e)/f + 1/8*(4*
A*a^2*c + B*a^2*c)*x - 1/12*(A*a^2*c + B*a^2*c)*cos(3*f*x + 3*e)/f - 1/4*(A
*a^2*c + B*a^2*c)*cos(f*x + e)/f
```

maple [B] time = 0.39, size = 186, normalized size = 1.90

$$\frac{\frac{a^2Ac\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}-a^2Ac\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-B\,a^2c\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] 1/f*(1/3*a^2*A*c*(2+sin(f*x+e)^2)*cos(f*x+e)-a^2*A*c*(-1/2*sin(f*x+e)*cos(f
*x+e)+1/2*f*x+1/2*e)-B*a^2*c*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)
+3/8*f*x+3/8*e)+1/3*B*a^2*c*(2+sin(f*x+e)^2)*cos(f*x+e)-a^2*A*c*cos(f*x+e)+
B*a^2*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+a^2*A*c*(f*x+e)-B*a^2*c*
cos(f*x+e))
```

maxima [A] time = 0.37, size = 179, normalized size = 1.83

$$-\frac{32\left(\cos\left(fx+e\right)^{3}-3\cos\left(fx+e\right)\right)Aa^{2}c+24\left(2fx+2e-\sin\left(2fx+2e\right)\right)Aa^{2}c-96\left(fx+e\right)Aa^{2}c+32\left(\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -1/96*(32*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c + 24*(2*f*x + 2*e - sin
(2*f*x + 2*e))*A*a^2*c - 96*(f*x + e)*A*a^2*c + 32*(cos(f*x + e)^3 - 3*cos(
f*x + e))*B*a^2*c + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e
))*B*a^2*c - 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*c + 96*A*a^2*c*cos(f
*x + e) + 96*B*a^2*c*cos(f*x + e))/f
```

mupad [B] time = 13.70, size = 339, normalized size = 3.46

$$\frac{a^{2}\,c\,\mathrm{atan}\left(\frac{a^{2}\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(4\,A+B\right)}{4\,\left(A\,a^{2}\,c+\frac{B\,a^{2}\,c}{4}\right)}\right)\,\left(4\,A+B\right)}{4\,f}-\frac{a^{2}\,c\,\left(4\,A+B\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{f\,x}{2}\right)}{4\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}\,\left(2\,A\,a^{2}\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x)),x)
```

```
[Out] (a^2*c*atan((a^2*c*tan(e/2 + (f*x)/2)*(4*A + B))/(4*(A*a^2*c + (B*a^2*c)/4)
))*(4*A + B))/(4*f) - (a^2*c*(4*A + B)*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2)
)/(4*f) - (tan(e/2 + (f*x)/2)^4*(2*A*a^2*c + 2*B*a^2*c) - tan(e/2 + (f*x)/2
)*(A*a^2*c - (B*a^2*c)/4) + tan(e/2 + (f*x)/2)^6*(2*A*a^2*c + 2*B*a^2*c) +
tan(e/2 + (f*x)/2)^2*((2*A*a^2*c)/3 + (2*B*a^2*c)/3) + tan(e/2 + (f*x)/2)^7
*(A*a^2*c - (B*a^2*c)/4) - tan(e/2 + (f*x)/2)^3*(A*a^2*c + (7*B*a^2*c)/4) +
 tan(e/2 + (f*x)/2)^5*(A*a^2*c + (7*B*a^2*c)/4) + (2*A*a^2*c)/3 + (2*B*a^2*
c)/3)/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/2 + (f*
x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))
```

sympy [A] time = 2.43, size = 396, normalized size = 4.04

$$\begin{cases} -\frac{Aa^{2}cx\sin^{2}\left(e+fx\right)}{2}-\frac{Aa^{2}cx\cos^{2}\left(e+fx\right)}{2}+Aa^{2}cx+\frac{Aa^{2}c\sin^{2}\left(e+fx\right)\cos\left(e+fx\right)}{f}+\frac{Aa^{2}c\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}+\frac{2Aa^{2}c\cos^{3}\left(e+fx\right)}{3f}- \\ x\left(A+B\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right)^{2}\left(-c\sin\left(e\right)+c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-A*a**2*c*x*sin(e + f*x)**2/2 - A*a**2*c*x*cos(e + f*x)**2/2 + A
*a**2*c*x + A*a**2*c*sin(e + f*x)**2*cos(e + f*x)/f + A*a**2*c*sin(e + f*x)
*cos(e + f*x)/(2*f) + 2*A*a**2*c*cos(e + f*x)**3/(3*f) - A*a**2*c*cos(e + f
*x)/f - 3*B*a**2*c*x*sin(e + f*x)**4/8 - 3*B*a**2*c*x*sin(e + f*x)**2*cos(e
 + f*x)**2/4 + B*a**2*c*x*sin(e + f*x)**2/2 - 3*B*a**2*c*x*cos(e + f*x)**4/
8 + B*a**2*c*x*cos(e + f*x)**2/2 + 5*B*a**2*c*sin(e + f*x)**3*cos(e + f*x)/
(8*f) + B*a**2*c*sin(e + f*x)**2*cos(e + f*x)/f + 3*B*a**2*c*sin(e + f*x)*c
os(e + f*x)**3/(8*f) - B*a**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*B*a**2*
c*cos(e + f*x)**3/(3*f) - B*a**2*c*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin
(e))*(a*sin(e) + a)**2*(-c*sin(e) + c), True))
```

3.31 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=117

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}+\frac{3a^2(2A+3B)\cos(e+fx)}{2cf}+\frac{a^2(2A+3B)\cos^3(e+fx)}{2f(c-c\sin(e+fx))}-\frac{3a^2x(2A+3B)}{2c}$$

```
[Out] -3/2*a^2*(2*A+3*B)*x/c+3/2*a^2*(2*A+3*B)*cos(f*x+e)/c/f+a^2*(A+B)*c^2*cos(f
*x+e)^5/f/(c-c*sin(f*x+e))^3+1/2*a^2*(2*A+3*B)*cos(f*x+e)^3/f/(c-c*sin(f*x+
e))
```

Rubi [A] time = 0.29, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2859, 2679, 2682, 8}

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}+\frac{3a^2(2A+3B)\cos(e+fx)}{2cf}+\frac{a^2(2A+3B)\cos^3(e+fx)}{2f(c-c\sin(e+fx))}-\frac{3a^2x(2A+3B)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),x]
```

```
[Out] (-3*a^2*(2*A + 3*B)*x)/(2*c) + (3*a^2*(2*A + 3*B)*Cos[e + f*x])/(2*c*f) + (
a^2*(A + B)*c^2*Cos[e + f*x]^5)/(f*(c - c*Sin[e + f*x])^3) + (a^2*(2*A + 3*
B)*Cos[e + f*x]^3)/(2*f*(c - c*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
```

```
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3} - \left(a^2(2A+3B)c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx)}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3} + \frac{a^2(2A+3B)\cos^3(e+fx)}{2f(c-c\sin(e+fx))} - \frac{1}{2}\left(3a^2(\right.$$

$$= \frac{3a^2(2A+3B)\cos(e+fx)}{2cf} + \frac{a^2(A+B)c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3} + \frac{a^2(2A}{2f(c}$$

$$= -\frac{3a^2(2A+3B)x}{2c} + \frac{3a^2(2A+3B)\cos(e+fx)}{2cf} + \frac{a^2(A+B)c^2\cos}{f(c-c\sin(e}$$

Mathematica [A] time = 1.30, size = 191, normalized size = 1.63

$$\frac{a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)(6(2A+3B)(e+fx)-4(A+3B)\cos(e\right.}{4cf(\sin(e+fx)-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(Cos[(e + f
*x)/2]*(6*(2*A + 3*B)*(e + f*x) - 4*(A + 3*B)*Cos[e + f*x] - B*Sin[2*(e + f
*x)]) - Sin[(e + f*x)/2]*(4*A*(8 + 3*e + 3*f*x) + 2*B*(16 + 9*e + 9*f*x) -
4*(A + 3*B)*Cos[e + f*x] - B*Sin[2*(e + f*x)])))/(4*c*f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x]))
```

<u>fricas</u> [A] time = 0.43, size = 177, normalized size = 1.51

$$\frac{Ba^2\cos\left(fx+e\right)^3-3\left(2A+3B\right)a^2fx+2\left(A+3B\right)a^2\cos\left(fx+e\right)^2+8\left(A+B\right)a^2-\left(3\left(2A+3B\right)a^2fx-(10A\right.}{2\left(cf\cos\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/2*(B*a^2*cos(f*x + e)^3 - 3*(2*A + 3*B)*a^2*f*x + 2*(A + 3*B)*a^2*cos(f*x
 + e)^2 + 8*(A + B)*a^2 - (3*(2*A + 3*B)*a^2*f*x - (10*A + 13*B)*a^2)*cos(f
*x + e) + (3*(2*A + 3*B)*a^2*f*x + B*a^2*cos(f*x + e)^2 - (2*A + 5*B)*a^2*c
os(f*x + e) + 8*(A + B)*a^2)*sin(f*x + e))/(c*f*cos(f*x + e) - c*f*sin(f*x
+ e) + c*f)
```

<u>giac</u> [A] time = 0.18, size = 163, normalized size = 1.39

$$-\frac{\frac{3\left(2Aa^2+3Ba^2\right)\left(fx+e\right)}{c}+\frac{16\left(Aa^2+Ba^2\right)}{c\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}+\frac{2\left(Ba^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-6Ba^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-Ba^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2\right.}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)^2c}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/2*(3*(2*A*a^2 + 3*B*a^2)*(f*x + e)/c + 16*(A*a^2 + B*a^2)/(c*(tan(1/2*f*
x + 1/2*e) - 1)) + 2*(B*a^2*tan(1/2*f*x + 1/2*e)^3 - 2*A*a^2*tan(1/2*f*x +
1/2*e)^2 - 6*B*a^2*tan(1/2*f*x + 1/2*e)^2 - B*a^2*tan(1/2*f*x + 1/2*e) - 2*
A*a^2 - 6*B*a^2)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*c))/f
```

maple [B] time = 0.43, size = 299, normalized size = 2.56

$$-\frac{8a^2A}{fc\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{8a^2B}{fc\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{a^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)B}{fc\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{2a^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)A}{fc\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{6a^2\left(\tan^2\left(\cdots\right.\right.}{fc\left(1+\tan^2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] -8/f*a^2/c/(tan(1/2*f*x+1/2*e)-1)*A-8/f*a^2/c/(tan(1/2*f*x+1/2*e)-1)*B-1/f*
a^2/c/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^3*B+2/f*a^2/c/(1+tan(1/
2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*A+6/f*a^2/c/(1+tan(1/2*f*x+1/2*e)^2)
^2*tan(1/2*f*x+1/2*e)^2*B+1/f*a^2/c/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*
x+1/2*e)+2/f*a^2/c/(1+tan(1/2*f*x+1/2*e)^2)^2*A+6/f*a^2/c/(1+tan(1/2*f*x+1/
2*e)^2)^2*B-9/f*a^2/c*arctan(tan(1/2*f*x+1/2*e))*B-6/f*a^2/c*arctan(tan(1/2
*f*x+1/2*e))*A
```

maxima [B] time = 0.48, size = 624, normalized size = 5.33

$$-\frac{2\,Aa^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-2}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}\right)+4\,Ba^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-2}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\mathrm{arc}\cdots}{\cdots}\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -(2*A*a^2*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x +
 e)/(cos(f*x + e) + 1))/c) + 4*B*a^2*((sin(f*x + e)/(cos(f*x + e) + 1) - si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) +
1) + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + B*a^2*((sin(f*x + e
)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x +
e)^3/(cos(f*x + e) + 1)^3 - 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4)/(c -
 c*sin(f*x + e)/(cos(f*x + e) + 1) + 2*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 - 2*c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + c*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 - c*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(
cos(f*x + e) + 1))/c) + 4*A*a^2*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c
- 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) + 2*B*a^2*(arctan(sin(f*x + e)
```

```
/(cos(f*x + e) + 1))/c - 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) - 2*A*a
^2/(c - c*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

mupad [B] time = 14.70, size = 244, normalized size = 2.09

$$\frac{10\,A\,a^2-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,A\,a^2+5\,B\,a^2\right)+14\,B\,a^2-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(2\,A\,a^2+7\,B\,a^2\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(8\,A\,a^2+\right.}{f\,\left(-c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-2\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+2\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-c\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x)),x)
```

```
[Out] (10*A*a^2 - tan(e/2 + (f*x)/2)*(2*A*a^2 + 5*B*a^2) + 14*B*a^2 - tan(e/2 + (
f*x)/2)^3*(2*A*a^2 + 7*B*a^2) + tan(e/2 + (f*x)/2)^4*(8*A*a^2 + 9*B*a^2) +
tan(e/2 + (f*x)/2)^2*(18*A*a^2 + 21*B*a^2))/(f*(c - c*tan(e/2 + (f*x)/2) +
2*c*tan(e/2 + (f*x)/2)^2 - 2*c*tan(e/2 + (f*x)/2)^3 + c*tan(e/2 + (f*x)/2)^
4 - c*tan(e/2 + (f*x)/2)^5)) - (3*a^2*atan((3*a^2*tan(e/2 + (f*x)/2)*(2*A +
 3*B))/(6*A*a^2 + 9*B*a^2))*(2*A + 3*B))/(c*f)
```

sympy [A] time = 8.09, size = 2365, normalized size = 20.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-6*A*a**2*f*x*tan(e/2 + f*x/2)**5/(2*c*f*tan(e/2 + f*x/2)**5 - 2
*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/
2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 6*A*a**2*f*x*tan(e/2 + f*x/2)**4/
(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*
x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 12*
A*a**2*f*x*tan(e/2 + f*x/2)**3/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 +
 f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*
tan(e/2 + f*x/2) - 2*c*f) + 12*A*a**2*f*x*tan(e/2 + f*x/2)**2/(2*c*f*tan(e/
2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c
*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 6*A*a**2*f*x*tan
(e/2 + f*x/2)/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*
f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2)
- 2*c*f) + 6*A*a**2*f*x/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)
**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2
 + f*x/2) - 2*c*f) - 16*A*a**2*tan(e/2 + f*x/2)**4/(2*c*f*tan(e/2 + f*x/2)*
*5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2
```

```
+ f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 4*A*a**2*tan(e/2 + f*x/2)**
3/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 +
f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 3
6*A*a**2*tan(e/2 + f*x/2)**2/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f
*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*ta
n(e/2 + f*x/2) - 2*c*f) + 4*A*a**2*tan(e/2 + f*x/2)/(2*c*f*tan(e/2 + f*x/2)
**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2
 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 20*A*a**2/(2*c*f*tan(e/2 +
 f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*
tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 9*B*a**2*f*x*tan(e/
2 + f*x/2)**5/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*
f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2)
- 2*c*f) + 9*B*a**2*f*x*tan(e/2 + f*x/2)**4/(2*c*f*tan(e/2 + f*x/2)**5 - 2*
c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2
)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 18*B*a**2*f*x*tan(e/2 + f*x/2)**3/
(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*
x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 18*
B*a**2*f*x*tan(e/2 + f*x/2)**2/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 +
 f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*
tan(e/2 + f*x/2) - 2*c*f) - 9*B*a**2*f*x*tan(e/2 + f*x/2)/(2*c*f*tan(e/2 +
f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*t
an(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 9*B*a**2*f*x/(2*c*f*
tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3
 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 18*B*a**2*
tan(e/2 + f*x/2)**4/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4
+ 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f
*x/2) - 2*c*f) + 14*B*a**2*tan(e/2 + f*x/2)**3/(2*c*f*tan(e/2 + f*x/2)**5 -
 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*
x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 42*B*a**2*tan(e/2 + f*x/2)**2/(
2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x
/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 10*B
*a**2*tan(e/2 + f*x/2)/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)*
*4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2
+ f*x/2) - 2*c*f) - 28*B*a**2/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 +
f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*t
an(e/2 + f*x/2) - 2*c*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**2/(-
c*sin(e) + c), True))
```

3.32 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=109

$$-\frac{a^2(A+4B)\cos(e+fx)}{c^2f}+\frac{a^2c^2(A+B)\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}+\frac{a^2x(A+4B)}{c^2}-\frac{2a^2(A+4B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2}$$

```
[Out] a^2*(A+4*B)*x/c^2-a^2*(A+4*B)*cos(f*x+e)/c^2/f+1/3*a^2*(A+B)*c^2*cos(f*x+e)
^5/f/(c-c*sin(f*x+e))^4-2/3*a^2*(A+4*B)*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^2
```

Rubi [A] time = 0.28, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2859, 2680, 2682, 8}

$$-\frac{a^2(A+4B)\cos(e+fx)}{c^2f}+\frac{a^2c^2(A+B)\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}+\frac{a^2x(A+4B)}{c^2}-\frac{2a^2(A+4B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*(A + 4*B)*x)/c^2 - (a^2*(A + 4*B)*Cos[e + f*x])/(c^2*f) + (a^2*(A + B)
*c^2*Cos[e + f*x]^5)/(3*f*(c - c*Sin[e + f*x])^4) - (2*a^2*(A + 4*B)*Cos[e
+ f*x]^3)/(3*f*(c - c*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4} - \frac{1}{3}\left(a^2(A+4B)c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4} - \frac{2a^2(A+4B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2} + \frac{\left(a^2(A\right.}{}$$

$$= -\frac{a^2(A+4B)\cos(e+fx)}{c^2f} + \frac{a^2(A+B)c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4} - \frac{2a^2(A+}{3f(c-}$$

$$= \frac{a^2(A+4B)x}{c^2} - \frac{a^2(A+4B)\cos(e+fx)}{c^2f} + \frac{a^2(A+B)c^2\cos^5(e+}{3f(c-c\sin(e+fx))}$$

Mathematica [B] time = 0.63, size = 238, normalized size = 2.18

$$\frac{a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(8(A+B)\sin\left(\frac{1}{2}(e+fx)\right)+3(A+4B)(e+fx)\left(\cos\left(\frac{1}{2}(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^2,x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(4*(A + B)*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2]) + 3*(A + 4*B)*(e + f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^3 - 3*B*Cos[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 + 8*(A + B
)*Sin[(e + f*x)/2] - 8*(2*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*
Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^4*(c - c*Sin[e + f*x])^2)
```

fricas [B] time = 0.44, size = 237, normalized size = 2.17

$$-\frac{3\,Ba^2\cos\left(fx+e\right)^3+6\,(A+4\,B)a^2fx+4\,(A+B)a^2-\left(3\,(A+4\,B)a^2fx+(8\,A+23\,B)a^2\right)\cos\left(fx+e\right)^2+\left(3\right.}{3\left(c^2f\cos\left(fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/3*(3*B*a^2*cos(f*x + e)^3 + 6*(A + 4*B)*a^2*f*x + 4*(A + B)*a^2 - (3*(A
+ 4*B)*a^2*f*x + (8*A + 23*B)*a^2)*cos(f*x + e)^2 + (3*(A + 4*B)*a^2*f*x -
2*(2*A + 11*B)*a^2)*cos(f*x + e) - (6*(A + 4*B)*a^2*f*x - 3*B*a^2*cos(f*x +
 e)^2 - 4*(A + B)*a^2 + (3*(A + 4*B)*a^2*f*x - 2*(4*A + 13*B)*a^2)*cos(f*x
+ e))*sin(f*x + e))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f +
(c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

giac [A] time = 0.16, size = 135, normalized size = 1.24

$$\frac{\frac{3\left(Aa^2+4\,Ba^2\right)\left(fx+e\right)}{c^2}-\frac{6\,Ba^2}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)c^2}+\frac{8\left(3\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-9\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+Aa^2+4\,Ba^2\right)}{c^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/3*(3*(A*a^2 + 4*B*a^2)*(f*x + e)/c^2 - 6*B*a^2/((tan(1/2*f*x + 1/2*e)^2 +
 1)*c^2) + 8*(3*B*a^2*tan(1/2*f*x + 1/2*e)^2 - 3*A*a^2*tan(1/2*f*x + 1/2*e)
 - 9*B*a^2*tan(1/2*f*x + 1/2*e) + A*a^2 + 4*B*a^2)/(c^2*(tan(1/2*f*x + 1/2*
e) - 1)^3))/f
```

maple [A] time = 0.46, size = 198, normalized size = 1.82

$$-\frac{16a^2A}{3c^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{16a^2B}{3c^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{8a^2A}{c^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{8a^2B}{c^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}+\frac{}{c^2f\left(\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

```
[Out] -16/3*a^2/c^2/f/(tan(1/2*f*x+1/2*e)-1)^3*A-16/3*a^2/c^2/f/(tan(1/2*f*x+1/2*
e)-1)^3*B-8*a^2/c^2/f/(tan(1/2*f*x+1/2*e)-1)^2*A-8*a^2/c^2/f/(tan(1/2*f*x+1
/2*e)-1)^2*B+8*a^2/c^2/f*B/(tan(1/2*f*x+1/2*e)-1)-2*a^2/c^2/f*B/(1+tan(1/2*
f*x+1/2*e)^2)+2*a^2/c^2/f*arctan(tan(1/2*f*x+1/2*e))*A+8*a^2/c^2/f*arctan(t
an(1/2*f*x+1/2*e))*B
```

maxima [B] time = 0.44, size = 839, normalized size = 7.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 2/3*(2*B*a^2*((12*sin(f*x + e)/(cos(f*x + e) + 1) - 11*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 - 5)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*
c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^2*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 3*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^2*sin(f*x + e)^5/(
cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) + A*a
^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 - 4)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arcta
n(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) + 2*B*a^2*((9*sin(f*x + e)/(cos(f*x
 + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4)/(c^2 - 3*c^2*sin(f*
x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e)
 + 1))/c^2) - A*a^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 - 2)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c
^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3) + 2*A*a^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(c^2 - 3*c^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + B*a^2*(3*sin(f*x + e)/(cos(f*x + e
) + 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

mupad [B] time = 14.15, size = 246, normalized size = 2.26

$$\frac{2\,a^2\,\mathrm{atan}\left(\frac{2\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(A+4\,B)}{2\,A\,a^2+8\,B\,a^2}\right)\,(A+4\,B)}{c^2\,f}-\frac{\frac{8\,A\,a^2}{3}-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(8\,A\,a^2+30\,B\,a^2\right)+\frac{38\,B\,a^2}{3}-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(8\right.}{f\,\left(-c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-4\,c^2\,\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^2,x)
```

```
[Out] (2*a^2*atan((2*a^2*tan(e/2 + (f*x)/2)*(A + 4*B))/(2*A*a^2 + 8*B*a^2))*(A +
4*B))/(c^2*f) - ((8*A*a^2)/3 - tan(e/2 + (f*x)/2)*(8*A*a^2 + 30*B*a^2) + (3
8*B*a^2)/3 - tan(e/2 + (f*x)/2)^3*(8*A*a^2 + 26*B*a^2) + tan(e/2 + (f*x)/2)
^2*((8*A*a^2)/3 + (74*B*a^2)/3) + 8*B*a^2*tan(e/2 + (f*x)/2)^4)/(f*(4*c^2*t
an(e/2 + (f*x)/2)^2 - 4*c^2*tan(e/2 + (f*x)/2)^3 + 3*c^2*tan(e/2 + (f*x)/2)
^4 - c^2*tan(e/2 + (f*x)/2)^5 + c^2 - 3*c^2*tan(e/2 + (f*x)/2)))
```

sympy [A] time = 16.12, size = 2474, normalized size = 22.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*A*a**2*f*x*tan(e/2 + f*x/2)**5/(3*c**2*f*tan(e/2 + f*x/2)**5 -
 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*t
an(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 9*A*a**2*f*x*t
an(e/2 + f*x/2)**4/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2
)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c*
*2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 12*A*a**2*f*x*tan(e/2 + f*x/2)**3/(3*c*
*2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2
 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) -
3*c**2*f) - 12*A*a**2*f*x*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**5
 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f
*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 9*A*a**2*f*x
*tan(e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)
**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**
2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 3*A*a**2*f*x/(3*c**2*f*tan(e/2 + f*x/2)*
*5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2
*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 24*A*a**2*
tan(e/2 + f*x/2)**3/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/
2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c
**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 8*A*a**2*tan(e/2 + f*x/2)**2/(3*c**2*f
```

```
*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f
*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c*
*2*f) - 24*A*a**2*tan(e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f
*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 +
f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 8*A*a**2/(3*c**2*f*tan(
e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)
**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f)
 + 12*B*a**2*f*x*tan(e/2 + f*x/2)**5/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2
*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2
+ f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 36*B*a**2*f*x*tan(e/2
 + f*x/2)**4/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 +
 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*t
an(e/2 + f*x/2) - 3*c**2*f) + 48*B*a**2*f*x*tan(e/2 + f*x/2)**3/(3*c**2*f*t
an(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x
/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2
*f) - 48*B*a**2*f*x*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c
**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e
/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 36*B*a**2*f*x*tan(
e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 +
 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*t
an(e/2 + f*x/2) - 3*c**2*f) - 12*B*a**2*f*x/(3*c**2*f*tan(e/2 + f*x/2)**5 -
 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*t
an(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 24*B*a**2*tan(
e/2 + f*x/2)**4/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**
4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*
f*tan(e/2 + f*x/2) - 3*c**2*f) - 78*B*a**2*tan(e/2 + f*x/2)**3/(3*c**2*f*ta
n(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/
2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*
f) + 74*B*a**2*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f
*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 +
f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 90*B*a**2*tan(e/2 + f*x
/2)/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*
f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 +
f*x/2) - 3*c**2*f) + 38*B*a**2/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan
(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/
2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f), Ne(f, 0)), (x*(A + B*sin(e))
*(a*sin(e) + a)**2/(-c*sin(e) + c)**2, True))
```

3.33 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=112

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}+\frac{2a^2B\cos(e+fx)}{f\left(c^3-c^3\sin(e+fx)\right)}-\frac{a^2Bx}{c^3}-\frac{2a^2B\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}$$

```
[Out] -a^2*B*x/c^3+1/5*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^5-2/3*a^2*B*
cos(f*x+e)^3/f/(c-c*sin(f*x+e))^3+2*a^2*B*cos(f*x+e)/f/(c^3-c^3*sin(f*x+e))
```

Rubi [A] time = 0.28, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 2680, 8}

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}+\frac{2a^2B\cos(e+fx)}{f\left(c^3-c^3\sin(e+fx)\right)}-\frac{a^2Bx}{c^3}-\frac{2a^2B\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] -((a^2*B*x)/c^3) + (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x
])^5) - (2*a^2*B*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^3) + (2*a^2*B*Co
s[e + f*x])/(f*(c^3 - c^3*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
```

```
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} - \left(a^2Bc\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^4}\,dx$$
$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} - \frac{2a^2B\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3} + \frac{\left(a^2B\right)\int \frac{c}{(c-}}{}$$
$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} - \frac{2a^2B\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3} + \frac{2a^2B\cos}{f\left(c^3-c^3\sin\right.}$$
$$= -\frac{a^2Bx}{c^3} + \frac{a^2(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} - \frac{2a^2B\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3} + \frac{}{f}$$

Mathematica [B] time = 0.74, size = 278, normalized size = 2.48

$$\frac{a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(24(A+B)\sin\left(\frac{1}{2}(e+fx)\right)+2(3A+43B)\sin\left(\frac{1}{2}(e+fx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^3,x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(12*(A + B)*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]) - 4*(3*A + 8*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 -
 15*B*(e + f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 24*(A + B)*Sin[(e
 + f*x)/2] - 8*(3*A + 8*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e +
 f*x)/2] + 2*(3*A + 43*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e +
f*x)/2])*(1 + Sin[e + f*x])^2)/(15*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
4*(c - c*Sin[e + f*x])^3)
```

fricas [B] time = 0.44, size = 277, normalized size = 2.47

$$\frac{60\,Ba^{2}fx - \left(15\,Ba^{2}fx - (3\,A + 43\,B)a^{2}\right)\cos\left(fx + e\right)^{3} - 12\,(A + B)a^{2} - \left(45\,Ba^{2}fx - (9\,A - 11\,B)a^{2}\right)\cos\left(fx + \right.}{15\left(c^{3}f\cos\left(fx + e\right)^{3} + 3\,c^{3}f\cos\left(f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/15*(60*B*a^2*f*x - (15*B*a^2*f*x - (3*A + 43*B)*a^2)*cos(f*x + e)^3 - 12*
(A + B)*a^2 - (45*B*a^2*f*x - (9*A - 11*B)*a^2)*cos(f*x + e)^2 + 6*(5*B*a^2
*f*x - (A + 11*B)*a^2)*cos(f*x + e) - (60*B*a^2*f*x + 12*(A + B)*a^2 - (15*
B*a^2*f*x + (3*A + 43*B)*a^2)*cos(f*x + e)^2 + 6*(5*B*a^2*f*x + (A - 9*B)*a
^2)*cos(f*x + e))*sin(f*x + e))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e
)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(
f*x + e) - 4*c^3*f)*sin(f*x + e))
```

giac [A] time = 0.18, size = 159, normalized size = 1.42

$$-\frac{\frac{15\left(fx+e\right)Ba^{2}}{c^{3}} + \frac{2\left(15\,Aa^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+15\,Ba^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-60\,Ba^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+30\,Aa^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+170\,Ba^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right.}{c^{3}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{5}}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -1/15*(15*(f*x + e)*B*a^2/c^3 + 2*(15*A*a^2*tan(1/2*f*x + 1/2*e)^4 + 15*B*a
^2*tan(1/2*f*x + 1/2*e)^4 - 60*B*a^2*tan(1/2*f*x + 1/2*e)^3 + 30*A*a^2*tan(
1/2*f*x + 1/2*e)^2 + 170*B*a^2*tan(1/2*f*x + 1/2*e)^2 - 100*B*a^2*tan(1/2*f
*x + 1/2*e) + 3*A*a^2 + 23*B*a^2)/(c^3*(tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

maple [B] time = 0.50, size = 249, normalized size = 2.22

$$-\frac{2a^2A}{c^3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{2a^2B}{c^3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{32a^2A}{5c^3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{32a^2B}{5c^3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{1}{c^3f\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

```
[Out] -2*a^2/c^3/f/(tan(1/2*f*x+1/2*e)-1)*A-2*a^2/c^3/f/(tan(1/2*f*x+1/2*e)-1)*B-
32/5*a^2/c^3/f/(tan(1/2*f*x+1/2*e)-1)^5*A-32/5*a^2/c^3/f/(tan(1/2*f*x+1/2*e
)-1)^5*B-16*a^2/c^3/f/(tan(1/2*f*x+1/2*e)-1)^3*A-32/3*a^2/c^3/f/(tan(1/2*f*
x+1/2*e)-1)^3*B-16*a^2/c^3/f/(tan(1/2*f*x+1/2*e)-1)^4*A-16*a^2/c^3/f/(tan(1
/2*f*x+1/2*e)-1)^4*B-8*a^2/c^3/f*A/(tan(1/2*f*x+1/2*e)-1)^2-2*a^2/c^3/f*B*a
rctan(tan(1/2*f*x+1/2*e))
```

maxima [B] time = 0.63, size = 1139, normalized size = 10.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] -2/15*(B*a^2*((95*sin(f*x + e)/(cos(f*x + e) + 1) - 145*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 - 22)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1)
+ 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^3
) + A*a^2*(20*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c
^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(
cos(f*x + e) + 1)^5) - 6*A*a^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)
/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) -
3*B*a^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^3 - 5*c^3*sin(f*x +
 e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^
```

```
3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 2*A*a^2*(5*sin(f*x + e)
/(cos(f*x + e) + 1) - 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)/(c^3 - 5*
c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 4*B*a^2*(5
*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 -
 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
)/f
```

mupad [B] time = 14.68, size = 233, normalized size = 2.08

$$-\frac{B\,a^2\,x}{c^3}-\frac{\frac{a^2\left(6\,A+46\,B-15\,B\left(e+f\,x\right)\right)}{15}-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\left(\frac{a^2\left(120\,B-150\,B\left(e+f\,x\right)\right)}{15}+10\,B\,a^2\,\left(e+f\,x\right)\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\left(\frac{a^2\left(3\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^3,x)
```

```
[Out] - (B*a^2*x)/c^3 - ((a^2*(6*A + 46*B - 15*B*(e + f*x)))/15 - tan(e/2 + (f*x)
/2)^3*((a^2*(120*B - 150*B*(e + f*x)))/15 + 10*B*a^2*(e + f*x)) + tan(e/2 +
 (f*x)/2)^4*((a^2*(30*A + 30*B - 75*B*(e + f*x)))/15 + 5*B*a^2*(e + f*x)) +
 tan(e/2 + (f*x)/2)^2*((a^2*(60*A + 340*B - 150*B*(e + f*x)))/15 + 10*B*a^2
*(e + f*x)) - tan(e/2 + (f*x)/2)*((a^2*(200*B - 75*B*(e + f*x)))/15 + 5*B*a
^2*(e + f*x)) + B*a^2*(e + f*x))/(c^3*f*(tan(e/2 + (f*x)/2) - 1)^5)
```

sympy [A] time = 26.92, size = 1647, normalized size = 14.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*a**2*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**5 -
75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f
*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 60*A*a**2*
tan(e/2 + f*x/2)**2/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*
x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 +
 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 6*A*a**2/(15*c**3*f*tan(e/2 + f*
x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 -
150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) -
```

```
15*B*a**2*f*x*tan(e/2 + f*x/2)**5/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*
f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2
 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 75*B*a**2*f*x*tan(
e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)
**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*
c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 150*B*a**2*f*x*tan(e/2 + f*x/2)**3/(
15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*
tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 +
f*x/2) - 15*c**3*f) + 150*B*a**2*f*x*tan(e/2 + f*x/2)**2/(15*c**3*f*tan(e/2
 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)*
*3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*
f) - 75*B*a**2*f*x*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**
3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e
/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 15*B*a**2*f*x/(1
5*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*t
an(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f
*x/2) - 15*c**3*f) - 30*B*a**2*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x
/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 1
50*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 1
20*B*a**2*tan(e/2 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*ta
n(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f
*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 340*B*a**2*tan(e/2 + f
*x/2)**2/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 1
50*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*
tan(e/2 + f*x/2) - 15*c**3*f) + 200*B*a**2*tan(e/2 + f*x/2)/(15*c**3*f*tan(
e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/
2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c*
*3*f) - 46*B*a**2/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/
2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 7
5*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin
(e) + a)**2/(-c*sin(e) + c)**3, True))
```

3.34 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=75

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}+\frac{a^2c(A-6B)\cos^5(e+fx)}{35f(c-c\sin(e+fx))^5}$$

```
[Out] 1/7*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^6+1/35*a^2*(A-6*B)*c*cos(
f*x+e)^5/f/(c-c*sin(f*x+e))^5
```

Rubi [A] time = 0.23, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2967, 2859, 2671}

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}+\frac{a^2c(A-6B)\cos^5(e+fx)}{35f(c-c\sin(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(7*f*(c - c*Sin[e + f*x])^6) + (a^2*(A - 6
*B)*c*Cos[e + f*x]^5)/(35*f*(c - c*Sin[e + f*x])^5)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
```

```
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx\\ &= \frac{a^2(A+B)c^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}+\frac{1}{7}\left(a^2(A-6B)c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx}\\ &= \frac{a^2(A+B)c^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}+\frac{a^2(A-6B)c\cos^5(e+fx)}{35f(c-c\sin(e+fx))^5}\end{aligned}$$

Mathematica [B] time = 0.95, size = 191, normalized size = 2.55

$$-\frac{a^2\left(-35(A+4B)\cos\left(\frac{1}{2}(e+fx)\right)+7(2A+13B)\cos\left(\frac{3}{2}(e+fx)\right)-70A\sin\left(\frac{1}{2}(e+fx)\right)-35A\sin\left(\frac{3}{2}(e+fx)\right)+}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^4,x]
```

```
[Out] -1/140*(a^2*(-35*(A + 4*B)*Cos[(e + f*x)/2] + 7*(2*A + 13*B)*Cos[(3*(e + f*
x))/2] + 35*B*Cos[(5*(e + f*x))/2] + A*Cos[(7*(e + f*x))/2] - 6*B*Cos[(7*(e
 + f*x))/2] - 70*A*Sin[(e + f*x)/2] + 70*B*Sin[(e + f*x)/2] - 35*A*Sin[(3*(
e + f*x))/2] + 35*B*Sin[(3*(e + f*x))/2] + 7*A*Sin[(5*(e + f*x))/2] - 7*B*S
in[(5*(e + f*x))/2]))/(c^4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7)
```

fricas [B] time = 0.43, size = 263, normalized size = 3.51

$$-\frac{(A-6\,B)a^2\cos\left(fx+e\right)^4+(4\,A+11\,B)a^2\cos\left(fx+e\right)^3+(13\,A+27\,B)a^2\cos\left(fx+e\right)^2-10\,(A+B)a^2\cos\left(f\right.}{35\left(c^4f\cos\left(fx+e\right)^4-3\,c^4f\cos\left(fx+e\right)^3-8\,c^4f\cos\left(fx+e\right)^2+4\,c^4f\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorit
hm="fricas")
```

```
[Out] -1/35*((A - 6*B)*a^2*cos(f*x + e)^4 + (4*A + 11*B)*a^2*cos(f*x + e)^3 + (13
*A + 27*B)*a^2*cos(f*x + e)^2 - 10*(A + B)*a^2*cos(f*x + e) - 20*(A + B)*a^
2 - ((A - 6*B)*a^2*cos(f*x + e)^3 - (3*A + 17*B)*a^2*cos(f*x + e)^2 + 10*(A
 + B)*a^2*cos(f*x + e) + 20*(A + B)*a^2)*sin(f*x + e))/(c^4*f*cos(f*x + e)^
4 - 3*c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e)
+ 8*c^4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*
x + e) - 8*c^4*f)*sin(f*x + e))
```

giac [B] time = 0.19, size = 229, normalized size = 3.05

$$-\frac{2\left(35\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-35\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+35\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+140\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorit
hm="giac")
```

```
[Out] -2/35*(35*A*a^2*tan(1/2*f*x + 1/2*e)^6 - 35*A*a^2*tan(1/2*f*x + 1/2*e)^5 +
35*B*a^2*tan(1/2*f*x + 1/2*e)^5 + 140*A*a^2*tan(1/2*f*x + 1/2*e)^4 + 35*B*a
^2*tan(1/2*f*x + 1/2*e)^4 - 70*A*a^2*tan(1/2*f*x + 1/2*e)^3 + 70*B*a^2*tan(
1/2*f*x + 1/2*e)^3 + 91*A*a^2*tan(1/2*f*x + 1/2*e)^2 + 14*B*a^2*tan(1/2*f*x
 + 1/2*e)^2 - 7*A*a^2*tan(1/2*f*x + 1/2*e) + 7*B*a^2*tan(1/2*f*x + 1/2*e) +
 6*A*a^2 - B*a^2)/(c^4*f*(tan(1/2*f*x + 1/2*e) - 1)^7)
```

maple [B] time = 0.47, size = 161, normalized size = 2.15

$$\frac{2a^2\left(-\frac{128A+112B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{96A+64B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{96A+96B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{42A+18B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{32A+32B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{1(}{2(\tan(}\right.}{f\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x)
```

```
[Out] 2/f*a^2/c^4*(-1/5*(128*A+112*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/4*(96*A+64*B)/(t
an(1/2*f*x+1/2*e)-1)^4-1/6*(96*A+96*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/3*(42*A+1
8*B)/(tan(1/2*f*x+1/2*e)-1)^3-A/(tan(1/2*f*x+1/2*e)-1)-1/7*(32*A+32*B)/(tan
(1/2*f*x+1/2*e)-1)^7-1/2*(10*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^2)
```

maxima [B] time = 0.61, size = 1571, normalized size = 20.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorit
hm="maxima")
```

```
[Out] 2/105*(2*A*a^2*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 13)/
(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
+ 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x +
 e) + 1)^7) + B*a^2*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 -
 13)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) +
1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(
f*x + e) + 1)^7) - 3*A*a^2*(49*sin(f*x + e)/(cos(f*x + e) + 1) - 147*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 210*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 2
10*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) +
 1)^5 - 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 12)/(c^4 - 7*c^4*sin(f*x +
 e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^
4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) - 4*A*a^2*
(14*sin(f*x + e)/(cos(f*x + e) + 1) - 42*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 35*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 35*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 - 2)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e
)^7/(cos(f*x + e) + 1)^7) - 8*B*a^2*(14*sin(f*x + e)/(cos(f*x + e) + 1) - 4
2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2)/(c^4 - 7*c^4*sin(f*x + e)
/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 6*B*a^2*(7*
sin(f*x + e)/(cos(f*x + e) + 1) - 21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
35*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^4 - 7*c^4*sin(f*x + e)/(cos(
f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x
```

```
 + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7))/f
```

mupad [B] time = 13.35, size = 269, normalized size = 3.59

$$\frac{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{109\,A\,a^2}{4}+\frac{11\,B\,a^2}{4}-\frac{27\,A\,a^2\,\cos\left(2\,e+2\,f\,x\right)}{4}+\frac{5\,A\,a^2\,\cos\left(3\,e+3\,f\,x\right)}{8}-\frac{13\,B\,a^2\,\cos\left(2\,e+2\,f\,x\right)}{4}+\frac{5\,B\,a^2\,\cos\left(3\,e+3\,f\,x\right)}{8}+\right.}{35\,c^4\,f\left(\frac{35\,\sqrt{2}\,\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{f\,x}{2}\right)}{8}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^4,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((109*A*a^2)/4 + (11*B*a^2)/4 - (27*A*a^2*cos(2*e + 2
*f*x))/4 + (5*A*a^2*cos(3*e + 3*f*x))/8 - (13*B*a^2*cos(2*e + 2*f*x))/4 + (
5*B*a^2*cos(3*e + 3*f*x))/8 + (7*A*a^2*sin(2*e + 2*f*x))/2 + (7*A*a^2*sin(3
*e + 3*f*x))/8 - (7*B*a^2*sin(2*e + 2*f*x))/2 - (7*B*a^2*sin(3*e + 3*f*x))/
8 - (121*A*a^2*cos(e + f*x))/8 - (9*B*a^2*cos(e + f*x))/8 - (105*A*a^2*sin(
e + f*x))/8 + (105*B*a^2*sin(e + f*x))/8))/(35*c^4*f*((35*2^(1/2)*cos(e/2 +
 pi/4 + (f*x)/2))/8 - (21*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/8 - (7*2
^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/8 + (2^(1/2)*cos((7*e)/2 - pi/4 + (
7*f*x)/2))/8))
```

sympy [A] time = 45.07, size = 2008, normalized size = 26.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-70*A*a**2*tan(e/2 + f*x/2)**6/(35*c**4*f*tan(e/2 + f*x/2)**7 -
245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4
*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e
/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) + 70*A*a**2*tan(e
/2 + f*x/2)**5/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)
**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 12
25*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f
*tan(e/2 + f*x/2) - 35*c**4*f) - 280*A*a**2*tan(e/2 + f*x/2)**4/(35*c**4*f*
tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 +
 f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)
**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**
4*f) + 140*A*a**2*tan(e/2 + f*x/2)**3/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*
c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*t
an(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 +
```

```
f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) - 182*A*a**2*tan(e/2
+ f*x/2)**2/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6
 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*
c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*ta
n(e/2 + f*x/2) - 35*c**4*f) + 14*A*a**2*tan(e/2 + f*x/2)/(35*c**4*f*tan(e/2
 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)
**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 7
35*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) -
12*A*a**2/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 +
 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c*
*4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(
e/2 + f*x/2) - 35*c**4*f) - 70*B*a**2*tan(e/2 + f*x/2)**5/(35*c**4*f*tan(e/
2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2
)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 -
735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) -
 70*B*a**2*tan(e/2 + f*x/2)**4/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*
tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2
+ f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)
**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) - 140*B*a**2*tan(e/2 + f*x/2
)**3/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*
c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*
tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 +
 f*x/2) - 35*c**4*f) - 28*B*a**2*tan(e/2 + f*x/2)**2/(35*c**4*f*tan(e/2 + f
*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5
- 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c
**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) - 14*B
*a**2*tan(e/2 + f*x/2)/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2
+ f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)
**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 24
5*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) + 2*B*a**2/(35*c**4*f*tan(e/2 + f*x/
2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1
225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4
*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f), Ne(f, 0)
), (x*(A + B*sin(e))*(a*sin(e) + a)**2/(-c*sin(e) + c)**4, True))
```

3.35 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5} dx$

Optimal. Leaf size=115

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{a^2(2A-7B)\cos^5(e+fx)}{315f(c-c\sin(e+fx))^5}+\frac{a^2c(2A-7B)\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6}$$

```
[Out] 1/9*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^7+1/63*a^2*(2*A-7*B)*c*co
s(f*x+e)^5/f/(c-c*sin(f*x+e))^6+1/315*a^2*(2*A-7*B)*cos(f*x+e)^5/f/(c-c*sin
(f*x+e))^5
```

Rubi [A] time = 0.29, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 2672, 2671}

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{a^2(2A-7B)\cos^5(e+fx)}{315f(c-c\sin(e+fx))^5}+\frac{a^2c(2A-7B)\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(9*f*(c - c*Sin[e + f*x])^7) + (a^2*(2*A -
 7*B)*c*Cos[e + f*x]^5)/(63*f*(c - c*Sin[e + f*x])^6) + (a^2*(2*A - 7*B)*Co
s[e + f*x]^5)/(315*f*(c - c*Sin[e + f*x])^5)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx$$
$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7} + \frac{1}{9}\left(a^2(2A-7B)c\right)\int \frac{\cos^4(e+fx}{(c-c\sin(e+f}$$
$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7} + \frac{a^2(2A-7B)c\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6} + \frac{1}{63}\left(a^2\right.$$
$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7} + \frac{a^2(2A-7B)c\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6} + \frac{a^2(2A}{315f(}$$

Mathematica [B] time = 1.24, size = 261, normalized size = 2.27

$$-\frac{a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(315(2A+3B)\cos\left(\frac{1}{2}(e+fx)\right)-63(4A+11B)\cos\left(\frac{3}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^5,x]
```

```
[Out] -1/2520*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(31
5*(2*A + 3*B)*Cos[(e + f*x)/2] - 63*(4*A + 11*B)*Cos[(3*(e + f*x))/2] - 315
```

```
*B*Cos[(5*(e + f*x))/2] - 18*A*Cos[(7*(e + f*x))/2] + 63*B*Cos[(7*(e + f*x)
)/2] + 882*A*Sin[(e + f*x)/2] + 63*B*Sin[(e + f*x)/2] + 420*A*Sin[(3*(e + f
*x))/2] + 105*B*Sin[(3*(e + f*x))/2] - 72*A*Sin[(5*(e + f*x))/2] - 63*B*Sin
[(5*(e + f*x))/2] + 2*A*Sin[(9*(e + f*x))/2] - 7*B*Sin[(9*(e + f*x))/2]))/(
c^5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x])^5)
```

fricas [B] time = 0.41, size = 335, normalized size = 2.91

$$\frac{(2A-7B)a^2\cos\left(fx+e\right)^5-4(2A-7B)a^2\cos\left(fx+e\right)^4-5(5A+14B)a^2\cos\left(fx+e\right)^3-5(17A+35B)a^2}{315\left(c^5f\cos\left(fx+e\right)^5+5c^5f\cos\left(fx+e\right)^4-8c^5f\cos\left(fx+e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorit
hm="fricas")
```

```
[Out] 1/315*((2*A - 7*B)*a^2*cos(f*x + e)^5 - 4*(2*A - 7*B)*a^2*cos(f*x + e)^4 -
5*(5*A + 14*B)*a^2*cos(f*x + e)^3 - 5*(17*A + 35*B)*a^2*cos(f*x + e)^2 + 70
*(A + B)*a^2*cos(f*x + e) + 140*(A + B)*a^2 + ((2*A - 7*B)*a^2*cos(f*x + e)
^4 + 5*(2*A - 7*B)*a^2*cos(f*x + e)^3 - 15*(A + 7*B)*a^2*cos(f*x + e)^2 + 7
0*(A + B)*a^2*cos(f*x + e) + 140*(A + B)*a^2)*sin(f*x + e))/(c^5*f*cos(f*x
+ e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x
 + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f
*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f
)*sin(f*x + e))
```

giac [B] time = 0.23, size = 301, normalized size = 2.62

$$-\frac{2\left(315Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^8-630Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7+315Ba^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7+2310Aa^2\tan\left(\frac{1}{2}fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorit
hm="giac")
```

```
[Out] -2/315*(315*A*a^2*tan(1/2*f*x + 1/2*e)^8 - 630*A*a^2*tan(1/2*f*x + 1/2*e)^7
 + 315*B*a^2*tan(1/2*f*x + 1/2*e)^7 + 2310*A*a^2*tan(1/2*f*x + 1/2*e)^6 + 1
05*B*a^2*tan(1/2*f*x + 1/2*e)^6 - 2520*A*a^2*tan(1/2*f*x + 1/2*e)^5 + 945*B
*a^2*tan(1/2*f*x + 1/2*e)^5 + 3402*A*a^2*tan(1/2*f*x + 1/2*e)^4 + 63*B*a^2*
tan(1/2*f*x + 1/2*e)^4 - 1638*A*a^2*tan(1/2*f*x + 1/2*e)^3 + 693*B*a^2*tan(
1/2*f*x + 1/2*e)^3 + 1062*A*a^2*tan(1/2*f*x + 1/2*e)^2 + 63*B*a^2*tan(1/2*f
```

```
*x + 1/2*e)^2 - 108*A*a^2*tan(1/2*f*x + 1/2*e) + 63*B*a^2*tan(1/2*f*x + 1/2
*e) + 47*A*a^2 - 7*B*a^2)/(c^5*f*(tan(1/2*f*x + 1/2*e) - 1)^9)
```

maple [A] time = 0.53, size = 205, normalized size = 1.78

$$\frac{2a^2\left(-\frac{404A+276B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{64A+64B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{64A+22B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{480A+448B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{12A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{200A}{4\left(\tan\left(\frac{fx}{2}\right.\right.}\right.}{f\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x)
```

```
[Out] 2/f*a^2/c^5*(-1/5*(404*A+276*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/9*(64*A+64*B)/(t
an(1/2*f*x+1/2*e)-1)^9-1/3*(64*A+22*B)/(tan(1/2*f*x+1/2*e)-1)^3-A/(tan(1/2*
f*x+1/2*e)-1)-1/7*(480*A+448*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/2*(12*A+2*B)/(ta
n(1/2*f*x+1/2*e)-1)^2-1/4*(200*A+104*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/6*(544*A
+448*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/8*(256*A+256*B)/(tan(1/2*f*x+1/2*e)-1)^8
)
```

maxima [B] time = 0.42, size = 2087, normalized size = 18.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorit
hm="maxima")
```

```
[Out] -2/315*(A*a^2*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 -
3360*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*sin(f
*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 8
4*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 +
 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x +
e) + 1)^9) - 10*A*a^2*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 315*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
```

```
 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f
*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
- c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 5*B*a^2*(45*sin(f*x + e)/(cos(
f*x + e) + 1) - 117*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 - 315*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 +
 63*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos
(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f
*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) -
 10*B*a^2*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 84*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 63*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1)/(c^5 - 9*c^
5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) +
 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos
(f*x + e) + 1)^9) + 14*A*a^2*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 54*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 81
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 45*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5 - 30*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1)/(c^5 - 9*c^5*sin(f*x + e)/
(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/
(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*s
in(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^
9) + 28*B*a^2*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 54*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 81*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 45*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 30*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) +
 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) +
 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(
cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9))/f
```

mupad [B] time = 13.36, size = 331, normalized size = 2.88

$$-\frac{2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{265\,A\,a^2\,\cos\left(2\,e+2\,f\,x\right)}{2}-\frac{49\,B\,a^2}{8}-\frac{4967\,A\,a^2}{16}-\frac{89\,A\,a^2\,\cos\left(3\,e+3\,f\,x\right)}{4}-\frac{49\,A\,a^2\,\cos\left(4\,e+4\,f\,x\right)}{16}+\frac{35\,B\,a^2\,\cos\left(2\,e+2\,f\right.}{4}}{315\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^5,x)
```

```
[Out] -(2*cos(e/2 + (f*x)/2)*((265*A*a^2*cos(2*e + 2*f*x))/2 - (49*B*a^2)/8 - (49
67*A*a^2)/16 - (89*A*a^2*cos(3*e + 3*f*x))/4 - (49*A*a^2*cos(4*e + 4*f*x))/
16 + (35*B*a^2*cos(2*e + 2*f*x))/4 - (7*B*a^2*cos(3*e + 3*f*x))/8 + (7*B*a^
2*cos(4*e + 4*f*x))/8 - (567*A*a^2*sin(2*e + 2*f*x))/8 - (243*A*a^2*sin(3*e
 + 3*f*x))/8 + (45*A*a^2*sin(4*e + 4*f*x))/16 + (63*B*a^2*sin(2*e + 2*f*x))
/2 + (63*B*a^2*sin(3*e + 3*f*x))/8 + (625*A*a^2*cos(e + f*x))/4 + (35*B*a^2
*cos(e + f*x))/8 + (2205*A*a^2*sin(e + f*x))/8 - (945*B*a^2*sin(e + f*x))/8
))/(315*c^5*f*((63*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/8 - (21*2^(1/2)*cos((
3*e)/2 - pi/4 + (3*f*x)/2))/4 - (9*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))
/4 + (9*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/16 + (2^(1/2)*cos((9*e)/2
+ pi/4 + (9*f*x)/2))/16))
```

sympy [A] time = 78.09, size = 3262, normalized size = 28.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-630*A*a**2*tan(e/2 + f*x/2)**8/(315*c**5*f*tan(e/2 + f*x/2)**9
- 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 2646
0*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**
5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*t
an(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 1260*A*a*
*2*tan(e/2 + f*x/2)**7/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/
2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f
*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)
**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 +
 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 4620*A*a**2*tan(e/2 + f*x/2)*
*6/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 1134
0*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**
5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*t
an(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2
```

```
 + f*x/2) - 315*c**5*f) + 5040*A*a**2*tan(e/2 + f*x/2)**5/(315*c**5*f*tan(e
/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f
*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)
**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 -
 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5
*f) - 6804*A*a**2*tan(e/2 + f*x/2)**4/(315*c**5*f*tan(e/2 + f*x/2)**9 - 283
5*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**
5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*t
an(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/
2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 3276*A*a**2*ta
n(e/2 + f*x/2)**3/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f
*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)
**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 +
 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835
*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 2124*A*a**2*tan(e/2 + f*x/2)**2/(3
15*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**
5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*t
an(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/
2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*
x/2) - 315*c**5*f) + 216*A*a**2*tan(e/2 + f*x/2)/(315*c**5*f*tan(e/2 + f*x/
2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7
 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 396
90*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c*
*5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 94*
A*a**2/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 +
11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690
*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5
*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan
(e/2 + f*x/2) - 315*c**5*f) - 630*B*a**2*tan(e/2 + f*x/2)**7/(315*c**5*f*ta
n(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2
+ f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x
/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**
3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c
**5*f) - 210*B*a**2*tan(e/2 + f*x/2)**6/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2
835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c
**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f
*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(
e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 1890*B*a**2*
tan(e/2 + f*x/2)**5/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 +
 f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/
2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4
 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 28
35*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 126*B*a**2*tan(e/2 + f*x/2)**4/(
315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c*
*5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*
```

```
tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e
/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f
*x/2) - 315*c**5*f) - 1386*B*a**2*tan(e/2 + f*x/2)**3/(315*c**5*f*tan(e/2 +
 f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2
)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5
- 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 113
40*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f)
- 126*B*a**2*tan(e/2 + f*x/2)**2/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**
5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*t
an(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/
2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f
*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 126*B*a**2*tan(e/2
+ f*x/2)/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8
+ 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 396
90*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c*
*5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*t
an(e/2 + f*x/2) - 315*c**5*f) + 14*B*a**2/(315*c**5*f*tan(e/2 + f*x/2)**9 -
 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460
*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5
*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*ta
n(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f), Ne(f, 0)),
(x*(A + B*sin(e))*(a*sin(e) + a)**2/(-c*sin(e) + c)**5, True))
```

3.36 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$

Optimal. Leaf size=156

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}+\frac{2a^2(3A-8B)\cos^5(e+fx)}{3465cf(c-c\sin(e+fx))^5}+\frac{2a^2(3A-8B)\cos^5(e+fx)}{693f(c-c\sin(e+fx))^6}+\frac{a^2c(3A-8B)\cos^5(e+fx)}{99f(c-c\sin(e+fx))^7}$$

```
[Out] 1/11*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^8+1/99*a^2*(3*A-8*B)*c*c
os(f*x+e)^5/f/(c-c*sin(f*x+e))^7+2/693*a^2*(3*A-8*B)*cos(f*x+e)^5/f/(c-c*si
n(f*x+e))^6+2/3465*a^2*(3*A-8*B)*cos(f*x+e)^5/c/f/(c-c*sin(f*x+e))^5
```

Rubi [A] time = 0.37, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 2672, 2671}

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}+\frac{2a^2(3A-8B)\cos^5(e+fx)}{3465cf(c-c\sin(e+fx))^5}+\frac{2a^2(3A-8B)\cos^5(e+fx)}{693f(c-c\sin(e+fx))^6}+\frac{a^2c(3A-8B)\cos^5(e+fx)}{99f(c-c\sin(e+fx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^6,x]
```

```
[Out] (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(11*f*(c - c*Sin[e + f*x])^8) + (a^2*(3*A
- 8*B)*c*Cos[e + f*x]^5)/(99*f*(c - c*Sin[e + f*x])^7) + (2*a^2*(3*A - 8*B)
*Cos[e + f*x]^5)/(693*f*(c - c*Sin[e + f*x])^6) + (2*a^2*(3*A - 8*B)*Cos[e
+ f*x]^5)/(3465*c*f*(c - c*Sin[e + f*x])^5)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8} + \frac{1}{11}\left(a^2(3A-8B)c\right)\int \frac{\cos^4(e+f}{(c-c\sin(e+}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8} + \frac{a^2(3A-8B)c\cos^5(e+fx)}{99f(c-c\sin(e+fx))^7} + \frac{1}{99}\left(2a\right.$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8} + \frac{a^2(3A-8B)c\cos^5(e+fx)}{99f(c-c\sin(e+fx))^7} + \frac{2a^2(3A}{693f}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8} + \frac{a^2(3A-8B)c\cos^5(e+fx)}{99f(c-c\sin(e+fx))^7} + \frac{2a^2(3A}{693f}$$

Mathematica [A] time = 1.60, size = 285, normalized size = 1.83

$$\frac{a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(231(27A+28B)\cos\left(\frac{1}{2}(e+fx)\right)-2475(A+2B)\cos\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^6,x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(231*(27*A
+ 28*B)*Cos[(e + f*x)/2] - 2475*(A + 2*B)*Cos[(3*(e + f*x))/2] - 2310*B*Cos
[(5*(e + f*x))/2] - 165*A*Cos[(7*(e + f*x))/2] + 440*B*Cos[(7*(e + f*x))/2]
 + 3*A*Cos[(11*(e + f*x))/2] - 8*B*Cos[(11*(e + f*x))/2] + 7623*A*Sin[(e +
f*x)/2] + 2772*B*Sin[(e + f*x)/2] + 3465*A*Sin[(3*(e + f*x))/2] + 2310*B*Si
n[(3*(e + f*x))/2] - 495*A*Sin[(5*(e + f*x))/2] - 990*B*Sin[(5*(e + f*x))/2
] + 33*A*Sin[(9*(e + f*x))/2] - 88*B*Sin[(9*(e + f*x))/2]))/(27720*c^6*f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x])^6)
```

fricas [B] time = 0.42, size = 407, normalized size = 2.61

$$-\frac{2\,(3\,A-8\,B)a^2\cos\left(fx+e\right)^6+12\,(3\,A-8\,B)a^2\cos\left(fx+e\right)^5-25\,(3\,A-8\,B)a^2\cos\left(fx+e\right)^4-35\,(6\,A+17\,\ldots}{3465\left(c^6f\cos\left(fx+e\right)^6-5\,c^6f\cos\left(fx+e\right)^5-\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorit
hm="fricas")
```

```
[Out] -1/3465*(2*(3*A - 8*B)*a^2*cos(f*x + e)^6 + 12*(3*A - 8*B)*a^2*cos(f*x + e)
^5 - 25*(3*A - 8*B)*a^2*cos(f*x + e)^4 - 35*(6*A + 17*B)*a^2*cos(f*x + e)^3
 - 35*(21*A + 43*B)*a^2*cos(f*x + e)^2 + 630*(A + B)*a^2*cos(f*x + e) + 126
0*(A + B)*a^2 - (2*(3*A - 8*B)*a^2*cos(f*x + e)^5 - 10*(3*A - 8*B)*a^2*cos(
f*x + e)^4 - 35*(3*A - 8*B)*a^2*cos(f*x + e)^3 + 35*(3*A + 25*B)*a^2*cos(f*
x + e)^2 - 630*(A + B)*a^2*cos(f*x + e) - 1260*(A + B)*a^2)*sin(f*x + e))/(
c^6*f*cos(f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*c^6*f*cos(f*x + e)^4 + 2
0*c^6*f*cos(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 - 16*c^6*f*cos(f*x + e) -
32*c^6*f + (c^6*f*cos(f*x + e)^5 + 6*c^6*f*cos(f*x + e)^4 - 12*c^6*f*cos(f*
x + e)^3 - 32*c^6*f*cos(f*x + e)^2 + 16*c^6*f*cos(f*x + e) + 32*c^6*f)*sin(
f*x + e))
```

giac [B] time = 0.25, size = 373, normalized size = 2.39

$$-\frac{2\left(3465\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}-10395\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9+3465\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9+41580\,Aa^2\tan\left(\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorit
hm="giac")
```

```
[Out] -2/3465*(3465*A*a^2*tan(1/2*f*x + 1/2*e)^10 - 10395*A*a^2*tan(1/2*f*x + 1/2
*e)^9 + 3465*B*a^2*tan(1/2*f*x + 1/2*e)^9 + 41580*A*a^2*tan(1/2*f*x + 1/2*e
)^8 - 1155*B*a^2*tan(1/2*f*x + 1/2*e)^8 - 69300*A*a^2*tan(1/2*f*x + 1/2*e)^
7 + 16170*B*a^2*tan(1/2*f*x + 1/2*e)^7 + 112266*A*a^2*tan(1/2*f*x + 1/2*e)^
6 - 6006*B*a^2*tan(1/2*f*x + 1/2*e)^6 - 98406*A*a^2*tan(1/2*f*x + 1/2*e)^5
+ 22176*B*a^2*tan(1/2*f*x + 1/2*e)^5 + 81180*A*a^2*tan(1/2*f*x + 1/2*e)^4 -
 3960*B*a^2*tan(1/2*f*x + 1/2*e)^4 - 33660*A*a^2*tan(1/2*f*x + 1/2*e)^3 + 8
910*B*a^2*tan(1/2*f*x + 1/2*e)^3 + 14685*A*a^2*tan(1/2*f*x + 1/2*e)^2 + 110
*B*a^2*tan(1/2*f*x + 1/2*e)^2 - 1551*A*a^2*tan(1/2*f*x + 1/2*e) + 671*B*a^2
*tan(1/2*f*x + 1/2*e) + 456*A*a^2 - 61*B*a^2)/(c^6*f*(tan(1/2*f*x + 1/2*e)
- 1)^11)
```

maple [A] time = 0.51, size = 249, normalized size = 1.60

$$\frac{2a^2\left(-\frac{932A+528B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{128A+128B}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}}-\frac{1536A+1472B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{352A+152B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{640A+640B}{10\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}}-\frac{90A+26B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-}{f\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x)
```

```
[Out] 2/f*a^2/c^6*(-1/5*(932*A+528*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/11*(128*A+128*B)
/(tan(1/2*f*x+1/2*e)-1)^11-1/9*(1536*A+1472*B)/(tan(1/2*f*x+1/2*e)-1)^9-1/4
*(352*A+152*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/10*(640*A+640*B)/(tan(1/2*f*x+1/2
*e)-1)^10-1/3*(90*A+26*B)/(tan(1/2*f*x+1/2*e)-1)^3-A/(tan(1/2*f*x+1/2*e)-1)
-1/8*(2304*A+2048*B)/(tan(1/2*f*x+1/2*e)-1)^8-1/7*(2376*A+1896*B)/(tan(1/2*
f*x+1/2*e)-1)^7-1/6*(1752*A+1208*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/2*(14*A+2*B)
/(tan(1/2*f*x+1/2*e)-1)^2)
```

maxima [B] time = 0.45, size = 2604, normalized size = 16.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorit
hm="maxima")
```

```
[Out] -2/3465*(5*A*a^2*(913*sin(f*x + e)/(cos(f*x + e) + 1) - 4565*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 12540*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 25080*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33726*sin(f*x + e)^5/(cos(f*x + e) + 1
)^5 - 33726*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 23100*sin(f*x + e)^7/(cos
(f*x + e) + 1)^7 - 11550*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x
 + e)^9/(cos(f*x + e) + 1)^9 - 693*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 -
```

146)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^
6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5
5*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x
 + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 6*A*a^2*(671*s
in(f*x + e)/(cos(f*x + e) + 1) - 2200*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 6600*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 15246*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12936*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6 + 9240*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3465*s
in(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*sin(f*x + e)^9/(cos(f*x + e) + 1)
^9 - 61)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 33
0*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*
x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos
(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 3*B*a^2*(6
71*sin(f*x + e)/(cos(f*x + e) + 1) - 2200*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 6600*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 15246*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12936*sin(f*x
+ e)^6/(cos(f*x + e) + 1)^6 + 9240*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 34
65*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*sin(f*x + e)^9/(cos(f*x + e)
+ 1)^9 - 61)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
+ 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*si
n(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) +
1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/
(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 2*B*a^
2*(341*sin(f*x + e)/(cos(f*x + e) + 1) - 1705*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 5115*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6765*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 9471*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 4851*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 3465*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3
1)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*
sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*
c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x +
 e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 4*A*a^2*(253*sin
(f*x + e)/(cos(f*x + e) + 1) - 1265*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2
640*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5280*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + 5313*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 5313*sin(f*x + e)^6/(c

```
os(f*x + e) + 1)^6 + 2310*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 1155*sin(f*
x + e)^8/(cos(f*x + e) + 1)^8 - 23)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e
) + 1) + 55*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*
c^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 - 330*c^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x
+ e)^8/(cos(f*x + e) + 1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 +
11*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x
 + e) + 1)^11) + 8*B*a^2*(253*sin(f*x + e)/(cos(f*x + e) + 1) - 1265*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 2640*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 -
5280*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5313*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 - 5313*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 2310*sin(f*x + e)^7/(
cos(f*x + e) + 1)^7 - 1155*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 23)/(c^6 -
 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)^7/(cos(f
*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*c^6*sin(f
*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^
10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11))/f
```

mupad [B] time = 13.54, size = 423, normalized size = 2.71

$$2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{38163\,A\,a^2}{8}-\frac{1283\,B\,a^2}{8}-\frac{11931\,A\,a^2\,\cos\left(2\,e+2\,f\,x\right)}{4}+\frac{9609\,A\,a^2\,\cos\left(3\,e+3\,f\,x\right)}{16}+\frac{1383\,A\,a^2\,\cos\left(4\,e+4\,f\,x\right)}{8}-\frac{225\,A\,a^2\,c}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^6,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((38163*A*a^2)/8 - (1283*B*a^2)/8 - (11931*A*a^2*cos(
2*e + 2*f*x))/4 + (9609*A*a^2*cos(3*e + 3*f*x))/16 + (1383*A*a^2*cos(4*e +
4*f*x))/8 - (225*A*a^2*cos(5*e + 5*f*x))/16 + (631*B*a^2*cos(2*e + 2*f*x))/
4 - (1583*B*a^2*cos(3*e + 3*f*x))/32 - (223*B*a^2*cos(4*e + 4*f*x))/8 + (45
*B*a^2*cos(5*e + 5*f*x))/32 + 1386*A*a^2*sin(2*e + 2*f*x) + (14949*A*a^2*si
n(3*e + 3*f*x))/16 - (561*A*a^2*sin(4*e + 4*f*x))/4 - (231*A*a^2*sin(5*e +
5*f*x))/16 - (3003*B*a^2*sin(2*e + 2*f*x))/8 - (4653*B*a^2*sin(3*e + 3*f*x)
)/32 + (209*B*a^2*sin(4*e + 4*f*x))/16 + (77*B*a^2*sin(5*e + 5*f*x))/32 - 2
091*A*a^2*cos(e + f*x) + (281*B*a^2*cos(e + f*x))/16 - (22869*A*a^2*sin(e +
 f*x))/4 + (23331*B*a^2*sin(e + f*x))/16))/(3465*c^6*f*((231*2^(1/2)*cos(e/
2 + pi/4 + (f*x)/2))/16 - (165*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/16
- (165*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/32 + (55*2^(1/2)*cos((7*e)/
```

```
2 - pi/4 + (7*f*x)/2))/32 + (11*2^(1/2)*cos((9*e)/2 + pi/4 + (9*f*x)/2))/32
 - (2^(1/2)*cos((11*e)/2 - pi/4 + (11*f*x)/2))/32))
```

sympy [A] time = 125.92, size = 4816, normalized size = 30.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**6,x)

[Out] Piecewise((-6930*A*a**2*tan(e/2 + f*x/2)**10/(3465*c**6*f*tan(e/2 + f*x/2)*
*11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9
 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 -
 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 -
1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 19
0575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6
*f) + 20790*A*a**2*tan(e/2 + f*x/2)**9/(3465*c**6*f*tan(e/2 + f*x/2)**11 -
38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571
725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 16008
30*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 114345
0*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c
**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) -
83160*A*a**2*tan(e/2 + f*x/2)**8/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*
c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c*
*6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**
6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6
*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*
tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) + 138600
*A*a**2*tan(e/2 + f*x/2)**7/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*
f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*
tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*t
an(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*ta
n(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e
/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) - 224532*A*a*
*2*tan(e/2 + f*x/2)**6/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan
(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e
/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/
2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2
 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 +
f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) + 196812*A*a**2*ta
n(e/2 + f*x/2)**5/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2
+ f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 +
f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f
*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*
x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2
```

```
)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) - 162360*A*a**2*tan(e/2
 + f*x/2)**4/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x
/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2
)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)
**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)*
*4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2
+ 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) + 67320*A*a**2*tan(e/2 + f*x
/2)**3/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**1
0 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 +
 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 +
1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 5
71725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 3811
5*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) - 29370*A*a**2*tan(e/2 + f*x/2)**2
/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 19
0575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 11434
50*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 160083
0*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*
c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6
*f*tan(e/2 + f*x/2) - 3465*c**6*f) + 3102*A*a**2*tan(e/2 + f*x/2)/(3465*c**
6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*
f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*
tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*t
an(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan
(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2
 + f*x/2) - 3465*c**6*f) - 912*A*a**2/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 3
8115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 5717
25*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 160083
0*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450
*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c*
*6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) - 6
930*B*a**2*tan(e/2 + f*x/2)**9/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c*
*6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6
*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*
f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f
*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*ta
n(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) + 2310*B*a
**2*tan(e/2 + f*x/2)**8/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*ta
n(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(
e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e
/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/
2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 +
 f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) - 32340*B*a**2*ta
n(e/2 + f*x/2)**7/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2
+ f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 +
f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f
```

```
*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*
x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2
)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) + 12012*B*a**2*tan(e/2
+ f*x/2)**6/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/
2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)
**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)*
*6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**
4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2 +
 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) - 44352*B*a**2*tan(e/2 + f*x/
2)**5/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10
 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 +
1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1
600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 57
1725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115
*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) + 7920*B*a**2*tan(e/2 + f*x/2)**4/(
3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 1905
75*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450
*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*
c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c*
*6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f
*tan(e/2 + f*x/2) - 3465*c**6*f) - 17820*B*a**2*tan(e/2 + f*x/2)**3/(3465*c
**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**
6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*
f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f
*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*t
an(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e
/2 + f*x/2) - 3465*c**6*f) - 220*B*a**2*tan(e/2 + f*x/2)**2/(3465*c**6*f*ta
n(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(
e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/
2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2
 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 +
 f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x
/2) - 3465*c**6*f) - 1342*B*a**2*tan(e/2 + f*x/2)/(3465*c**6*f*tan(e/2 + f*
x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/
2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)
**7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)*
*5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3
 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465
*c**6*f) + 122*B*a**2/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(
e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/
2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2
 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2
+ f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f
*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f), Ne(f, 0)), (x*(A +
 B*sin(e))*(a*sin(e) + a)**2/(-c*sin(e) + c)**6, True))
```

3.37 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx$

Optimal. Leaf size=197

$$\frac{2a^2(4A-9B)\cos^5(e+fx)}{15015c^2f(c-c\sin(e+fx))^5}+\frac{a^2c^2(A+B)\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9}+\frac{2a^2(4A-9B)\cos^5(e+fx)}{3003cf(c-c\sin(e+fx))^6}+\frac{a^2(4A-9B)\cos^5(e+fx)}{429f(c-c\sin(e+fx))^7}$$

```
[Out] 1/13*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^9+1/143*a^2*(4*A-9*B)*c*
cos(f*x+e)^5/f/(c-c*sin(f*x+e))^8+1/429*a^2*(4*A-9*B)*cos(f*x+e)^5/f/(c-c*s
in(f*x+e))^7+2/3003*a^2*(4*A-9*B)*cos(f*x+e)^5/c/f/(c-c*sin(f*x+e))^6+2/150
15*a^2*(4*A-9*B)*cos(f*x+e)^5/c^2/f/(c-c*sin(f*x+e))^5
```

Rubi [A] time = 0.46, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 2672, 2671}

$$\frac{2a^2(4A-9B)\cos^5(e+fx)}{15015c^2f(c-c\sin(e+fx))^5}+\frac{a^2c^2(A+B)\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9}+\frac{2a^2(4A-9B)\cos^5(e+fx)}{3003cf(c-c\sin(e+fx))^6}+\frac{a^2(4A-9B)\cos^5(e+fx)}{429f(c-c\sin(e+fx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^7,x]
```

```
[Out] (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(13*f*(c - c*Sin[e + f*x])^9) + (a^2*(4*A
- 9*B)*c*Cos[e + f*x]^5)/(143*f*(c - c*Sin[e + f*x])^8) + (a^2*(4*A - 9*B)*
Cos[e + f*x]^5)/(429*f*(c - c*Sin[e + f*x])^7) + (2*a^2*(4*A - 9*B)*Cos[e +
 f*x]^5)/(3003*c*f*(c - c*Sin[e + f*x])^6) + (2*a^2*(4*A - 9*B)*Cos[e + f*x
]^5)/(15015*c^2*f*(c - c*Sin[e + f*x])^5)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^9}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9} + \frac{1}{13}\left(a^2(4A-9B)c\right)\int \frac{\cos^4(e+}{(c-c\sin(e-}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9} + \frac{a^2(4A-9B)c\cos^5(e+fx)}{143f(c-c\sin(e+fx))^8} + \frac{1}{143}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9} + \frac{a^2(4A-9B)c\cos^5(e+fx)}{143f(c-c\sin(e+fx))^8} + \frac{a^2(4}{429}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9} + \frac{a^2(4A-9B)c\cos^5(e+fx)}{143f(c-c\sin(e+fx))^8} + \frac{a^2(4}{429}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9} + \frac{a^2(4A-9B)c\cos^5(e+fx)}{143f(c-c\sin(e+fx))^8} + \frac{a^2(4}{429}$$

Mathematica [A] time = 3.65, size = 313, normalized size = 1.59

$$-\frac{a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6006(8A+7B)\cos\left(\frac{1}{2}(e+fx)\right)-1716(11A+19B)\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^7,x]
```

```
[Out] -1/240240*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(
6006*(8*A + 7*B)*Cos[(e + f*x)/2] - 1716*(11*A + 19*B)*Cos[(3*(e + f*x))/2]
 - 15015*B*Cos[(5*(e + f*x))/2] - 1144*A*Cos[(7*(e + f*x))/2] + 2574*B*Cos[
(7*(e + f*x))/2] + 52*A*Cos[(11*(e + f*x))/2] - 117*B*Cos[(11*(e + f*x))/2]
 + 54912*A*Sin[(e + f*x)/2] + 26598*B*Sin[(e + f*x)/2] + 24024*A*Sin[(3*(e
+ f*x))/2] + 21021*B*Sin[(3*(e + f*x))/2] - 2860*A*Sin[(5*(e + f*x))/2] - 8
580*B*Sin[(5*(e + f*x))/2] + 312*A*Sin[(9*(e + f*x))/2] - 702*B*Sin[(9*(e +
 f*x))/2] - 4*A*Sin[(13*(e + f*x))/2] + 9*B*Sin[(13*(e + f*x))/2]))/(c^7*f*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x])^7)
```

fricas [B] time = 0.44, size = 475, normalized size = 2.41

$$\frac{2\,(4\,A-9\,B)a^2\cos\left(fx+e\right)^7-12\,(4\,A-9\,B)a^2\cos\left(fx+e\right)^6-49\,(4\,A-9\,B)a^2\cos\left(fx+e\right)^5+70\,(4\,A-9\,B)a^2}{15015\left(c^7f\cos\left(fx+e\right)^7+7\,c^7f\cos\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorit
hm="fricas")
```

```
[Out] 1/15015*(2*(4*A - 9*B)*a^2*cos(f*x + e)^7 - 12*(4*A - 9*B)*a^2*cos(f*x + e)
^6 - 49*(4*A - 9*B)*a^2*cos(f*x + e)^5 + 70*(4*A - 9*B)*a^2*cos(f*x + e)^4
+ 105*(7*A + 20*B)*a^2*cos(f*x + e)^3 + 105*(25*A + 51*B)*a^2*cos(f*x + e)^
2 - 2310*(A + B)*a^2*cos(f*x + e) - 4620*(A + B)*a^2 + (2*(4*A - 9*B)*a^2*c
os(f*x + e)^6 + 14*(4*A - 9*B)*a^2*cos(f*x + e)^5 - 35*(4*A - 9*B)*a^2*cos(
f*x + e)^4 - 105*(4*A - 9*B)*a^2*cos(f*x + e)^3 + 105*(3*A + 29*B)*a^2*cos(
f*x + e)^2 - 2310*(A + B)*a^2*cos(f*x + e) - 4620*(A + B)*a^2)*sin(f*x + e)
)/(c^7*f*cos(f*x + e)^7 + 7*c^7*f*cos(f*x + e)^6 - 18*c^7*f*cos(f*x + e)^5
- 56*c^7*f*cos(f*x + e)^4 + 48*c^7*f*cos(f*x + e)^3 + 112*c^7*f*cos(f*x + e
)^2 - 32*c^7*f*cos(f*x + e) - 64*c^7*f - (c^7*f*cos(f*x + e)^6 - 6*c^7*f*co
s(f*x + e)^5 - 24*c^7*f*cos(f*x + e)^4 + 32*c^7*f*cos(f*x + e)^3 + 80*c^7*f
*cos(f*x + e)^2 - 32*c^7*f*cos(f*x + e) - 64*c^7*f)*sin(f*x + e))
```

giac [B] time = 0.33, size = 445, normalized size = 2.26

$$-\frac{2\left(15015\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{12}-60060\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}+15015\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}+270270\,Aa^2\,\mathrm{t}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorit
hm="giac")
```

```
[Out] -2/15015*(15015*A*a^2*tan(1/2*f*x + 1/2*e)^12 - 60060*A*a^2*tan(1/2*f*x + 1
/2*e)^11 + 15015*B*a^2*tan(1/2*f*x + 1/2*e)^11 + 270270*A*a^2*tan(1/2*f*x +
 1/2*e)^10 - 15015*B*a^2*tan(1/2*f*x + 1/2*e)^10 - 600600*A*a^2*tan(1/2*f*x
 + 1/2*e)^9 + 105105*B*a^2*tan(1/2*f*x + 1/2*e)^9 + 1174173*A*a^2*tan(1/2*f
*x + 1/2*e)^8 - 93093*B*a^2*tan(1/2*f*x + 1/2*e)^8 - 1465464*A*a^2*tan(1/2*
f*x + 1/2*e)^7 + 234234*B*a^2*tan(1/2*f*x + 1/2*e)^7 + 1559844*A*a^2*tan(1/
2*f*x + 1/2*e)^6 - 131274*B*a^2*tan(1/2*f*x + 1/2*e)^6 - 1094808*A*a^2*tan(
1/2*f*x + 1/2*e)^5 + 181038*B*a^2*tan(1/2*f*x + 1/2*e)^5 + 659945*A*a^2*tan
(1/2*f*x + 1/2*e)^4 - 47190*B*a^2*tan(1/2*f*x + 1/2*e)^4 - 233948*A*a^2*tan
(1/2*f*x + 1/2*e)^3 + 45903*B*a^2*tan(1/2*f*x + 1/2*e)^3 + 77454*A*a^2*tan(
1/2*f*x + 1/2*e)^2 - 1599*B*a^2*tan(1/2*f*x + 1/2*e)^2 - 7904*A*a^2*tan(1/2
*f*x + 1/2*e) + 2769*B*a^2*tan(1/2*f*x + 1/2*e) + 1763*A*a^2 - 213*B*a^2)/(
c^7*f*(tan(1/2*f*x + 1/2*e) - 1)^13)
```

maple [A] time = 0.57, size = 293, normalized size = 1.49

$$2a^2\left(-\frac{10896A+9360B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{1816A+884B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{4480A+4352B}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}}-\frac{1536A+1536B}{12\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{12}}-\frac{560A+208B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{16A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x)
```

```
[Out] 2/f*a^2/c^7*(-1/9*(10896*A+9360*B)/(tan(1/2*f*x+1/2*e)-1)^9-1/5*(1816*A+884
*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/11*(4480*A+4352*B)/(tan(1/2*f*x+1/2*e)-1)^11
-1/12*(1536*A+1536*B)/(tan(1/2*f*x+1/2*e)-1)^12-1/4*(560*A+208*B)/(tan(1/2*
f*x+1/2*e)-1)^4-1/2*(16*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^2-1/3*(120*A+30*B)/(t
an(1/2*f*x+1/2*e)-1)^3-A/(tan(1/2*f*x+1/2*e)-1)-1/7*(7744*A+5368*B)/(tan(1/
2*f*x+1/2*e)-1)^7-1/13*(256*A+256*B)/(tan(1/2*f*x+1/2*e)-1)^13-1/10*(8320*A
+7680*B)/(tan(1/2*f*x+1/2*e)-1)^10-1/6*(4320*A+2568*B)/(tan(1/2*f*x+1/2*e)-
1)^6-1/8*(10560*A+8256*B)/(tan(1/2*f*x+1/2*e)-1)^8)
```

maxima [B] time = 0.69, size = 3120, normalized size = 15.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorit
hm="maxima")
```

[Out] -2/45045*(2*A*a^2*(4771*sin(f*x + e)/(cos(f*x + e) + 1) - 28626*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 74932*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1873
30*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 265122*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 - 353496*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 276276*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7 - 207207*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 7507
5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 30030*sin(f*x + e)^10/(cos(f*x + e)
 + 1)^10 - 367)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*
c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x
+ e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x
 + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^
11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(co
s(f*x + e) + 1)^13) + 4*B*a^2*(4771*sin(f*x + e)/(cos(f*x + e) + 1) - 28626
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 74932*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 - 187330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 265122*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5 - 353496*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 276276*
sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 207207*sin(f*x + e)^8/(cos(f*x + e) +
 1)^8 + 75075*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 30030*sin(f*x + e)^10/(
cos(f*x + e) + 1)^10 - 367)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) +
 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) +
1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286
*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*
x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x
 + e)^13/(cos(f*x + e) + 1)^13) + 15*A*a^2*(3796*sin(f*x + e)/(cos(f*x + e)
 + 1) - 22776*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 77506*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 - 193765*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 339768*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5 - 453024*sin(f*x + e)^6/(cos(f*x + e) + 1
)^6 + 444444*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 333333*sin(f*x + e)^8/(c
os(f*x + e) + 1)^8 + 180180*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 72072*sin
(f*x + e)^10/(cos(f*x + e) + 1)^10 + 18018*sin(f*x + e)^11/(cos(f*x + e) +
1)^11 - 3003*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 523)/(c^7 - 13*c^7*sin
(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 -
 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e
)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 -
78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(
f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*x + e) + 1)^13) - 70*A*a^2*(6
11*sin(f*x + e)/(cos(f*x + e) + 1) - 2379*sin(f*x + e)^2/(cos(f*x + e) + 1)

```
^2 + 8723*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 18590*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 33462*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 40326*sin(f*x
+ e)^6/(cos(f*x + e) + 1)^6 + 40326*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 2
7027*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 15015*sin(f*x + e)^9/(cos(f*x +
e) + 1)^9 - 4719*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 1287*sin(f*x + e)^
11/(cos(f*x + e) + 1)^11 - 47)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1
) + 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e)
 + 1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x +
 e)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 +
286*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos
(f*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(
f*x + e)^13/(cos(f*x + e) + 1)^13) - 35*B*a^2*(611*sin(f*x + e)/(cos(f*x +
e) + 1) - 2379*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8723*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 - 18590*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33462*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 - 40326*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
 + 40326*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 27027*sin(f*x + e)^8/(cos(f*
x + e) + 1)^8 + 15015*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 4719*sin(f*x +
e)^10/(cos(f*x + e) + 1)^10 + 1287*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 -
47)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 715*c^7
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*sin(f*x
+ e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
- 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e)^10/(co
s(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 13*c^7*
sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*x + e) +
 1)^13) - 462*B*a^2*(13*sin(f*x + e)/(cos(f*x + e) + 1) - 78*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 286*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 520*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 936*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 -
858*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 858*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 - 351*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 195*sin(f*x + e)^9/(cos(
f*x + e) + 1)^9 - 1)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 -
1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos
(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*si
n(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e)
+ 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^1
3/(cos(f*x + e) + 1)^13))/f
```

mupad [B] time = 14.05, size = 500, normalized size = 2.54

$$-\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{994249\,A\,a^2}{32}-\frac{63639\,B\,a^2}{32}-\frac{1609013\,A\,a^2\,\cos\left(2\,e+2\,f\,x\right)}{64}+\frac{85687\,A\,a^2\,\cos\left(3\,e+3\,f\,x\right)}{16}+\frac{79591\,A\,a^2\,\cos\left(4\,e+4\,f\,x\right)}{32}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^7,x)
```

```
[Out] -(2*cos(e/2 + (f*x)/2)*((994249*A*a^2)/32 - (63639*B*a^2)/32 - (1609013*A*a
^2*cos(2*e + 2*f*x))/64 + (85687*A*a^2*cos(3*e + 3*f*x))/16 + (79591*A*a^2*
cos(4*e + 4*f*x))/32 - (5261*A*a^2*cos(5*e + 5*f*x))/16 - (1771*A*a^2*cos(6
*e + 6*f*x))/64 + (140553*B*a^2*cos(2*e + 2*f*x))/64 - (4431*B*a^2*cos(3*e
+ 3*f*x))/8 - (10161*B*a^2*cos(4*e + 4*f*x))/32 + 36*B*a^2*cos(5*e + 5*f*x)
 + (231*B*a^2*cos(6*e + 6*f*x))/64 + (636207*A*a^2*sin(2*e + 2*f*x))/64 + (
309309*A*a^2*sin(3*e + 3*f*x))/32 - (7007*A*a^2*sin(4*e + 4*f*x))/4 - (1238
9*A*a^2*sin(5*e + 5*f*x))/32 + (1755*A*a^2*sin(6*e + 6*f*x))/64 - (121407*B
*a^2*sin(2*e + 2*f*x))/64 - (39039*B*a^2*sin(3*e + 3*f*x))/32 + (3003*B*a^2
*sin(4*e + 4*f*x))/16 + (1599*B*a^2*sin(5*e + 5*f*x))/32 - (195*B*a^2*sin(6
*e + 6*f*x))/64 - (93221*A*a^2*cos(e + f*x))/8 + (3291*B*a^2*cos(e + f*x))/
8 - (704847*A*a^2*sin(e + f*x))/16 + (125697*B*a^2*sin(e + f*x))/16))/(1501
5*c^7*f*((1287*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/64 - (429*2^(1/2)*c
os(e/2 + pi/4 + (f*x)/2))/16 + (715*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2)
)/64 - (143*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/32 - (39*2^(1/2)*cos((
9*e)/2 + pi/4 + (9*f*x)/2))/32 + (13*2^(1/2)*cos((11*e)/2 - pi/4 + (11*f*x)
/2))/64 + (2^(1/2)*cos((13*e)/2 + pi/4 + (13*f*x)/2))/64))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**7,x)
```

```
[Out] Timed out
```

3.38 $\int (a + a\sin(e + fx))^3(A + B\sin(e + fx))(c - c\sin(e + fx))^6\,dx$

Optimal. Leaf size=265

$$\frac{11a^3c^6(10A-3B)\cos^7(e+fx)}{560f}+\frac{11a^3(10A-3B)\cos^7(e+fx)\left(c^6-c^6\sin(e+fx)\right)}{720f}+\frac{11a^3c^6(10A-3B)\sin(e+$$

```
[Out] 11/256*a^3*(10*A-3*B)*c^6*x+11/560*a^3*(10*A-3*B)*c^6*cos(f*x+e)^7/f+11/256
*a^3*(10*A-3*B)*c^6*cos(f*x+e)*sin(f*x+e)/f+11/384*a^3*(10*A-3*B)*c^6*cos(f
*x+e)^3*sin(f*x+e)/f+11/480*a^3*(10*A-3*B)*c^6*cos(f*x+e)^5*sin(f*x+e)/f-1/
10*a^3*B*cos(f*x+e)^7*(c^2-c^2*sin(f*x+e))^3/f+1/90*a^3*(10*A-3*B)*cos(f*x+
e)^7*(c^3-c^3*sin(f*x+e))^2/f+11/720*a^3*(10*A-3*B)*cos(f*x+e)^7*(c^6-c^6*s
in(f*x+e))/f
```

Rubi [A] time = 0.39, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2967, 2860, 2678, 2669, 2635, 8}

$$\frac{11a^3c^6(10A-3B)\cos^7(e+fx)}{560f}+\frac{a^3(10A-3B)\cos^7(e+fx)\left(c^3-c^3\sin(e+fx)\right)^2}{90f}+\frac{11a^3(10A-3B)\cos^7(e+fx}{720f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^6,x]
```

```
[Out] (11*a^3*(10*A - 3*B)*c^6*x)/256 + (11*a^3*(10*A - 3*B)*c^6*Cos[e + f*x]^7)/
(560*f) + (11*a^3*(10*A - 3*B)*c^6*Cos[e + f*x]*Sin[e + f*x])/(256*f) + (11
*a^3*(10*A - 3*B)*c^6*Cos[e + f*x]^3*Sin[e + f*x])/(384*f) + (11*a^3*(10*A
- 3*B)*c^6*Cos[e + f*x]^5*Sin[e + f*x])/(480*f) - (a^3*B*Cos[e + f*x]^7*(c^
2 - c^2*Sin[e + f*x])^3)/(10*f) + (a^3*(10*A - 3*B)*Cos[e + f*x]^7*(c^3 - c
^3*Sin[e + f*x])^2)/(90*f) + (11*a^3*(10*A - 3*B)*Cos[e + f*x]^7*(c^6 - c^6
*Sin[e + f*x]))/(720*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^6\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(A+B\sin(e+fx))(c-$$

$$= -\frac{a^3B\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)^3}{10f} + \frac{1}{1}$$

$$= -\frac{a^3B\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)^3}{10f} + \frac{a}{}$$

$$= -\frac{a^3B\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)^3}{10f} + \frac{a}{}$$

$$= \frac{11a^3(10A-3B)c^6\cos^7(e+fx)}{560f} - \frac{a^3B\cos^7(e}{}$$

$$= \frac{11a^3(10A-3B)c^6\cos^7(e+fx)}{560f} + \frac{11a^3(10A}{}$$

$$= \frac{11a^3(10A-3B)c^6\cos^7(e+fx)}{560f} + \frac{11a^3(10A}{}$$

$$= \frac{11a^3(10A-3B)c^6\cos^7(e+fx)}{560f} + \frac{11a^3(10A}{}$$

$$= \frac{11}{256}a^3(10A-3B)c^6x + \frac{11a^3(10A-3B)c^6\,\mathrm{co}}{560f}$$

Mathematica [A] time = 4.31, size = 255, normalized size = 0.96

$$\frac{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^6(27720(10A-3B)(e+fx)+1260(144A-25B)\sin(2(e+fx))+2520(6}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
6,x]
```

```
[Out] ((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^6*(27720*(10*A - 3*B)*(e + f*x
) + 5040*(33*A - 19*B)*Cos[e + f*x] + 3360*(29*A - 15*B)*Cos[3*(e + f*x)] +
 10080*(3*A - B)*Cos[5*(e + f*x)] + 360*(9*A + 5*B)*Cos[7*(e + f*x)] - 280*
(A - 3*B)*Cos[9*(e + f*x)] + 1260*(144*A - 25*B)*Sin[2*(e + f*x)] + 2520*(6
*A + 7*B)*Sin[4*(e + f*x)] - 210*(32*A - 51*B)*Sin[6*(e + f*x)] - 315*(6*A
```

```
- 5*B)*Sin[8*(e + f*x)] - 126*B*Sin[10*(e + f*x)]))/(645120*f*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2])^12*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

fricas [A] time = 0.50, size = 181, normalized size = 0.68

$$-\frac{8960\,(A-3\,B)a^3c^6\cos\left(fx+e\right)^9-46080\,(A-B)a^3c^6\cos\left(fx+e\right)^7-3465\,(10\,A-3\,B)a^3c^6fx+21\left(384\,Ba^3c^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x, algorit
hm="fricas")
```

```
[Out] -1/80640*(8960*(A - 3*B)*a^3*c^6*cos(f*x + e)^9 - 46080*(A - B)*a^3*c^6*cos
(f*x + e)^7 - 3465*(10*A - 3*B)*a^3*c^6*f*x + 21*(384*B*a^3*c^6*cos(f*x + e
)^9 + 48*(30*A - 41*B)*a^3*c^6*cos(f*x + e)^7 - 88*(10*A - 3*B)*a^3*c^6*cos
(f*x + e)^5 - 110*(10*A - 3*B)*a^3*c^6*cos(f*x + e)^3 - 165*(10*A - 3*B)*a^
3*c^6*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.27, size = 347, normalized size = 1.31

$$-\frac{Ba^3c^6\sin\left(10\,fx+10\,e\right)}{5120\,f}+\frac{11}{256}\left(10\,Aa^3c^6-3\,Ba^3c^6\right)x-\frac{\left(Aa^3c^6-3\,Ba^3c^6\right)\cos\left(9\,fx+9\,e\right)}{2304\,f}+\frac{\left(9\,Aa^3c^6+5\,Ba^3c^6\right)}{1792\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x, algorit
hm="giac")
```

```
[Out] -1/5120*B*a^3*c^6*sin(10*f*x + 10*e)/f + 11/256*(10*A*a^3*c^6 - 3*B*a^3*c^6
)*x - 1/2304*(A*a^3*c^6 - 3*B*a^3*c^6)*cos(9*f*x + 9*e)/f + 1/1792*(9*A*a^3
*c^6 + 5*B*a^3*c^6)*cos(7*f*x + 7*e)/f + 1/64*(3*A*a^3*c^6 - B*a^3*c^6)*cos
(5*f*x + 5*e)/f + 1/192*(29*A*a^3*c^6 - 15*B*a^3*c^6)*cos(3*f*x + 3*e)/f +
1/128*(33*A*a^3*c^6 - 19*B*a^3*c^6)*cos(f*x + e)/f - 1/2048*(6*A*a^3*c^6 -
5*B*a^3*c^6)*sin(8*f*x + 8*e)/f - 1/3072*(32*A*a^3*c^6 - 51*B*a^3*c^6)*sin(
6*f*x + 6*e)/f + 1/256*(6*A*a^3*c^6 + 7*B*a^3*c^6)*sin(4*f*x + 4*e)/f + 1/5
12*(144*A*a^3*c^6 - 25*B*a^3*c^6)*sin(2*f*x + 2*e)/f
```

maple [B] time = 0.85, size = 651, normalized size = 2.46

$$\frac{a^3A\,c^6\left(fx+e\right)-B\,a^3c^6\cos\left(fx+e\right)-\frac{8a^3A\,c^6\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}+8B\,a^3c^6\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3f}{8}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x)
```

```
[Out] 1/f*(a^3*A*c^6*(f*x+e)-B*a^3*c^6*cos(f*x+e)-8/3*a^3*A*c^6*(2+sin(f*x+e)^2)*
cos(f*x+e)+8*B*a^3*c^6*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f
*x+3/8*e)+3*a^3*A*c^6*cos(f*x+e)+6/5*B*a^3*c^6*(8/3+sin(f*x+e)^4+4/3*sin(f*
x+e)^2)*cos(f*x+e)+B*a^3*c^6*(-1/10*(sin(f*x+e)^9+9/8*sin(f*x+e)^7+21/16*si
n(f*x+e)^5+105/64*sin(f*x+e)^3+315/128*sin(f*x+e))*cos(f*x+e)+63/256*f*x+63
/256*e)+1/3*B*a^3*c^6*(128/35+sin(f*x+e)^8+8/7*sin(f*x+e)^6+48/35*sin(f*x+e
)^4+64/35*sin(f*x+e)^2)*cos(f*x+e)-8/7*B*a^3*c^6*(16/5+sin(f*x+e)^6+6/5*sin
(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)-6*B*a^3*c^6*(-1/6*(sin(f*x+e)^5+5/4*
sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)-1/9*a^3*A*c^6*(12
8/35+sin(f*x+e)^8+8/7*sin(f*x+e)^6+48/35*sin(f*x+e)^4+64/35*sin(f*x+e)^2)*c
os(f*x+e)-3*a^3*A*c^6*(-1/8*(sin(f*x+e)^7+7/6*sin(f*x+e)^5+35/24*sin(f*x+e)
^3+35/16*sin(f*x+e))*cos(f*x+e)+35/128*f*x+35/128*e)+8*a^3*A*c^6*(-1/6*(sin
(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+6/5
*a^3*A*c^6*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-6*a^3*A*c^6*(-1/4
*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3*B*a^3*c^6*(-1/2*
sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e))
```

maxima [B] time = 0.36, size = 661, normalized size = 2.49

$$-\frac{2048\left(35\cos\left(fx+e\right)^{9}-180\cos\left(fx+e\right)^{7}+378\cos\left(fx+e\right)^{5}-420\cos\left(fx+e\right)^{3}+315\cos\left(fx+e\right)\right)Aa^{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x, algorit
hm="maxima")
```

```
[Out] -1/645120*(2048*(35*cos(f*x + e)^9 - 180*cos(f*x + e)^7 + 378*cos(f*x + e)^
5 - 420*cos(f*x + e)^3 + 315*cos(f*x + e))*A*a^3*c^6 - 258048*(3*cos(f*x +
e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^3*c^6 - 1720320*(cos(f*x +
e)^3 - 3*cos(f*x + e))*A*a^3*c^6 + 630*(128*sin(2*f*x + 2*e)^3 + 840*f*x +
840*e + 3*sin(8*f*x + 8*e) + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2*e))*A
*a^3*c^6 - 26880*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e)
 - 48*sin(2*f*x + 2*e))*A*a^3*c^6 + 120960*(12*f*x + 12*e + sin(4*f*x + 4*e
) - 8*sin(2*f*x + 2*e))*A*a^3*c^6 - 645120*(f*x + e)*A*a^3*c^6 - 6144*(35*c
os(f*x + e)^9 - 180*cos(f*x + e)^7 + 378*cos(f*x + e)^5 - 420*cos(f*x + e)^
3 + 315*cos(f*x + e))*B*a^3*c^6 - 147456*(5*cos(f*x + e)^7 - 21*cos(f*x + e
)^5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*c^6 - 258048*(3*cos(f*x +
e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^6 + 63*(32*sin(2*f*x +
2*e)^5 - 640*sin(2*f*x + 2*e)^3 - 2520*f*x - 2520*e - 25*sin(8*f*x + 8*e) -
 600*sin(4*f*x + 4*e) + 2560*sin(2*f*x + 2*e))*B*a^3*c^6 + 20160*(4*sin(2*f
```

```
*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a
^3*c^6 - 161280*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a
^3*c^6 + 483840*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c^6 - 1935360*A*a^3*
c^6*cos(f*x + e) + 645120*B*a^3*c^6*cos(f*x + e))/f
```

mupad [B] time = 14.87, size = 812, normalized size = 3.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^6,x)
```

```
[Out] (tan(e/2 + (f*x)/2)^18*(6*A*a^3*c^6 - 2*B*a^3*c^6) + tan(e/2 + (f*x)/2)^16*
(22*A*a^3*c^6 - 18*B*a^3*c^6) + tan(e/2 + (f*x)/2)^8*(84*A*a^3*c^6 - 28*B*a
^3*c^6) + tan(e/2 + (f*x)/2)^14*((136*A*a^3*c^6)/3 - 8*B*a^3*c^6) + tan(e/2
 + (f*x)/2)^4*((136*A*a^3*c^6)/7 - (24*B*a^3*c^6)/7) + tan(e/2 + (f*x)/2)^1
0*(116*A*a^3*c^6 - 60*B*a^3*c^6) - tan(e/2 + (f*x)/2)^19*((73*A*a^3*c^6)/64
 + (33*B*a^3*c^6)/128) + tan(e/2 + (f*x)/2)^2*((202*A*a^3*c^6)/63 - (58*B*a
^3*c^6)/21) + tan(e/2 + (f*x)/2)^12*((328*A*a^3*c^6)/3 - 72*B*a^3*c^6) + ta
n(e/2 + (f*x)/2)^7*((341*A*a^3*c^6)/16 + (333*B*a^3*c^6)/32) - tan(e/2 + (f
*x)/2)^13*((341*A*a^3*c^6)/16 + (333*B*a^3*c^6)/32) + tan(e/2 + (f*x)/2)^6*
((456*A*a^3*c^6)/7 - (344*B*a^3*c^6)/7) + tan(e/2 + (f*x)/2)^5*((449*A*a^3*
c^6)/48 - (577*B*a^3*c^6)/160) - tan(e/2 + (f*x)/2)^15*((449*A*a^3*c^6)/48
- (577*B*a^3*c^6)/160) + tan(e/2 + (f*x)/2)^3*((2117*A*a^3*c^6)/192 - (705*
B*a^3*c^6)/128) - tan(e/2 + (f*x)/2)^17*((2117*A*a^3*c^6)/192 - (705*B*a^3*
c^6)/128) + tan(e/2 + (f*x)/2)^9*((699*A*a^3*c^6)/32 - (2749*B*a^3*c^6)/64)
 - tan(e/2 + (f*x)/2)^11*((699*A*a^3*c^6)/32 - (2749*B*a^3*c^6)/64) + tan(e
/2 + (f*x)/2)*((73*A*a^3*c^6)/64 + (33*B*a^3*c^6)/128) + (58*A*a^3*c^6)/63
- (10*B*a^3*c^6)/21)/(f*(10*tan(e/2 + (f*x)/2)^2 + 45*tan(e/2 + (f*x)/2)^4
+ 120*tan(e/2 + (f*x)/2)^6 + 210*tan(e/2 + (f*x)/2)^8 + 252*tan(e/2 + (f*x)
/2)^10 + 210*tan(e/2 + (f*x)/2)^12 + 120*tan(e/2 + (f*x)/2)^14 + 45*tan(e/2
 + (f*x)/2)^16 + 10*tan(e/2 + (f*x)/2)^18 + tan(e/2 + (f*x)/2)^20 + 1)) + (
11*a^3*c^6*atan((11*a^3*c^6*tan(e/2 + (f*x)/2)*(10*A - 3*B))/(128*((55*A*a^
3*c^6)/64 - (33*B*a^3*c^6)/128)))*(10*A - 3*B))/(128*f)
```

sympy [A] time = 52.76, size = 1948, normalized size = 7.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**6,x)
```

```
[Out] Piecewise((-105*A*a**3*c**6*x*sin(e + f*x)**8/128 - 105*A*a**3*c**6*x*sin(e
 + f*x)**6*cos(e + f*x)**2/32 + 5*A*a**3*c**6*x*sin(e + f*x)**6/2 - 315*A*a
```

```
**3*c**6*x*sin(e + f*x)**4*cos(e + f*x)**4/64 + 15*A*a**3*c**6*x*sin(e + f*
x)**4*cos(e + f*x)**2/2 - 9*A*a**3*c**6*x*sin(e + f*x)**4/4 - 105*A*a**3*c*
*6*x*sin(e + f*x)**2*cos(e + f*x)**6/32 + 15*A*a**3*c**6*x*sin(e + f*x)**2*
cos(e + f*x)**4/2 - 9*A*a**3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**2/2 - 105
*A*a**3*c**6*x*cos(e + f*x)**8/128 + 5*A*a**3*c**6*x*cos(e + f*x)**6/2 - 9*
A*a**3*c**6*x*cos(e + f*x)**4/4 + A*a**3*c**6*x - A*a**3*c**6*sin(e + f*x)*
*8*cos(e + f*x)/f + 279*A*a**3*c**6*sin(e + f*x)**7*cos(e + f*x)/(128*f) -
8*A*a**3*c**6*sin(e + f*x)**6*cos(e + f*x)**3/(3*f) + 511*A*a**3*c**6*sin(e
 + f*x)**5*cos(e + f*x)**3/(128*f) - 11*A*a**3*c**6*sin(e + f*x)**5*cos(e +
 f*x)/(2*f) - 16*A*a**3*c**6*sin(e + f*x)**4*cos(e + f*x)**5/(5*f) + 6*A*a*
*3*c**6*sin(e + f*x)**4*cos(e + f*x)/f + 385*A*a**3*c**6*sin(e + f*x)**3*co
s(e + f*x)**5/(128*f) - 20*A*a**3*c**6*sin(e + f*x)**3*cos(e + f*x)**3/(3*f
) + 15*A*a**3*c**6*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 64*A*a**3*c**6*sin(
e + f*x)**2*cos(e + f*x)**7/(35*f) + 8*A*a**3*c**6*sin(e + f*x)**2*cos(e +
f*x)**3/f - 8*A*a**3*c**6*sin(e + f*x)**2*cos(e + f*x)/f + 105*A*a**3*c**6*
sin(e + f*x)*cos(e + f*x)**7/(128*f) - 5*A*a**3*c**6*sin(e + f*x)*cos(e + f
*x)**5/(2*f) + 9*A*a**3*c**6*sin(e + f*x)*cos(e + f*x)**3/(4*f) - 128*A*a**
3*c**6*cos(e + f*x)**9/(315*f) + 16*A*a**3*c**6*cos(e + f*x)**5/(5*f) - 16*
A*a**3*c**6*cos(e + f*x)**3/(3*f) + 3*A*a**3*c**6*cos(e + f*x)/f + 63*B*a**
3*c**6*x*sin(e + f*x)**10/256 + 315*B*a**3*c**6*x*sin(e + f*x)**8*cos(e + f
*x)**2/256 + 315*B*a**3*c**6*x*sin(e + f*x)**6*cos(e + f*x)**4/128 - 15*B*a
**3*c**6*x*sin(e + f*x)**6/8 + 315*B*a**3*c**6*x*sin(e + f*x)**4*cos(e + f*
x)**6/128 - 45*B*a**3*c**6*x*sin(e + f*x)**4*cos(e + f*x)**2/8 + 3*B*a**3*c
**6*x*sin(e + f*x)**4 + 315*B*a**3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**8/2
56 - 45*B*a**3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**4/8 + 6*B*a**3*c**6*x*s
in(e + f*x)**2*cos(e + f*x)**2 - 3*B*a**3*c**6*x*sin(e + f*x)**2/2 + 63*B*a
**3*c**6*x*cos(e + f*x)**10/256 - 15*B*a**3*c**6*x*cos(e + f*x)**6/8 + 3*B*
a**3*c**6*x*cos(e + f*x)**4 - 3*B*a**3*c**6*x*cos(e + f*x)**2/2 - 193*B*a**
3*c**6*sin(e + f*x)**9*cos(e + f*x)/(256*f) + 3*B*a**3*c**6*sin(e + f*x)**8
*cos(e + f*x)/f - 237*B*a**3*c**6*sin(e + f*x)**7*cos(e + f*x)**3/(128*f) +
 8*B*a**3*c**6*sin(e + f*x)**6*cos(e + f*x)**3/f - 8*B*a**3*c**6*sin(e + f*
x)**6*cos(e + f*x)/f - 21*B*a**3*c**6*sin(e + f*x)**5*cos(e + f*x)**5/(10*f
) + 33*B*a**3*c**6*sin(e + f*x)**5*cos(e + f*x)/(8*f) + 48*B*a**3*c**6*sin(
e + f*x)**4*cos(e + f*x)**5/(5*f) - 16*B*a**3*c**6*sin(e + f*x)**4*cos(e +
f*x)**3/f + 6*B*a**3*c**6*sin(e + f*x)**4*cos(e + f*x)/f - 147*B*a**3*c**6*
sin(e + f*x)**3*cos(e + f*x)**7/(128*f) + 5*B*a**3*c**6*sin(e + f*x)**3*cos
(e + f*x)**3/f - 5*B*a**3*c**6*sin(e + f*x)**3*cos(e + f*x)/f + 192*B*a**3*
c**6*sin(e + f*x)**2*cos(e + f*x)**7/(35*f) - 64*B*a**3*c**6*sin(e + f*x)**
2*cos(e + f*x)**5/(5*f) + 8*B*a**3*c**6*sin(e + f*x)**2*cos(e + f*x)**3/f -
 63*B*a**3*c**6*sin(e + f*x)*cos(e + f*x)**9/(256*f) + 15*B*a**3*c**6*sin(e
 + f*x)*cos(e + f*x)**5/(8*f) - 3*B*a**3*c**6*sin(e + f*x)*cos(e + f*x)**3/
f + 3*B*a**3*c**6*sin(e + f*x)*cos(e + f*x)/(2*f) + 128*B*a**3*c**6*cos(e +
 f*x)**9/(105*f) - 128*B*a**3*c**6*cos(e + f*x)**7/(35*f) + 16*B*a**3*c**6*
cos(e + f*x)**5/(5*f) - B*a**3*c**6*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*si
n(e))*(a*sin(e) + a)**3*(-c*sin(e) + c)**6, True))
```

3.39 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx$

Optimal. Leaf size=222

$$\frac{a^3c^5(9A-2B)\cos^7(e+fx)}{56f}+\frac{a^3(9A-2B)\cos^7(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{72f}+\frac{a^3c^5(9A-2B)\sin(e+fx)\cos^5(e+\cdots}{48f}$$

```
[Out] 5/128*a^3*(9*A-2*B)*c^5*x+1/56*a^3*(9*A-2*B)*c^5*cos(f*x+e)^7/f+5/128*a^3*(
9*A-2*B)*c^5*cos(f*x+e)*sin(f*x+e)/f+5/192*a^3*(9*A-2*B)*c^5*cos(f*x+e)^3*s
in(f*x+e)/f+1/48*a^3*(9*A-2*B)*c^5*cos(f*x+e)^5*sin(f*x+e)/f-1/9*a^3*B*c^3*
cos(f*x+e)^7*(c-c*sin(f*x+e))^2/f+1/72*a^3*(9*A-2*B)*cos(f*x+e)^7*(c^5-c^5*
sin(f*x+e))/f
```

Rubi [A] time = 0.32, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2967, 2860, 2678, 2669, 2635, 8}

$$\frac{a^3c^5(9A-2B)\cos^7(e+fx)}{56f}+\frac{a^3(9A-2B)\cos^7(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{72f}+\frac{a^3c^5(9A-2B)\sin(e+fx)\cos^5(e+\cdots}{48f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (5*a^3*(9*A - 2*B)*c^5*x)/128 + (a^3*(9*A - 2*B)*c^5*Cos[e + f*x]^7)/(56*f)
 + (5*a^3*(9*A - 2*B)*c^5*Cos[e + f*x]*Sin[e + f*x])/(128*f) + (5*a^3*(9*A
- 2*B)*c^5*Cos[e + f*x]^3*Sin[e + f*x])/(192*f) + (a^3*(9*A - 2*B)*c^5*Cos[
e + f*x]^5*Sin[e + f*x])/(48*f) - (a^3*B*c^3*Cos[e + f*x]^7*(c - c*Sin[e +
f*x])^2)/(9*f) + (a^3*(9*A - 2*B)*Cos[e + f*x]^7*(c^5 - c^5*Sin[e + f*x]))/
(72*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(A+B\sin(e+fx))(c-$$
$$= -\frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^2}{9f} + \frac{1}{9}\left(a\right.$$
$$= -\frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^2}{9f} + \frac{a^3(9}{}$$
$$= \frac{a^3(9A-2B)c^5\cos^7(e+fx)}{56f} - \frac{a^3Bc^3\cos^7(e+}{}$$
$$= \frac{a^3(9A-2B)c^5\cos^7(e+fx)}{56f} + \frac{a^3(9A-2B)c^5}{}$$
$$= \frac{a^3(9A-2B)c^5\cos^7(e+fx)}{56f} + \frac{5a^3(9A-2B)c^5}{}$$
$$= \frac{a^3(9A-2B)c^5\cos^7(e+fx)}{56f} + \frac{5a^3(9A-2B)c^5}{}$$
$$= \frac{5}{128}a^3(9A-2B)c^5x + \frac{a^3(9A-2B)c^5\cos^7(e+}{56f}$$

Mathematica [**A**] time = 2.57, size = 232, normalized size = 1.05

$$\frac{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^5(2520(9A-2B)(e+fx)+2016(8A-B)\sin(2(e+fx))+504(5A+2B)\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
5,x]
```

```
[Out] ((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^5*(2520*(9*A - 2*B)*(e + f*x)
+ 504*(20*A - 13*B)*Cos[e + f*x] + 336*(18*A - 11*B)*Cos[3*(e + f*x)] + 100
8*(2*A - B)*Cos[5*(e + f*x)] + 36*(8*A - B)*Cos[7*(e + f*x)] + 28*B*Cos[9*(
e + f*x)] + 2016*(8*A - B)*Sin[2*(e + f*x)] + 504*(5*A + 2*B)*Sin[4*(e + f*
x)] + 672*B*Sin[6*(e + f*x)] - 63*(A - 2*B)*Sin[8*(e + f*x)]))/(64512*f*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^10*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^6)
```

fricas [**A**] time = 0.50, size = 158, normalized size = 0.71

$$\frac{896\,Ba^3c^5\cos\left(fx+e\right)^9+2304\,(A-B)a^3c^5\cos\left(fx+e\right)^7+315\,(9\,A-2\,B)a^3c^5fx-21\left(48\,(A-2\,B)a^3c^5\cos\left(f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorit
hm="fricas")
```

```
[Out] 1/8064*(896*B*a^3*c^5*cos(f*x + e)^9 + 2304*(A - B)*a^3*c^5*cos(f*x + e)^7
+ 315*(9*A - 2*B)*a^3*c^5*f*x - 21*(48*(A - 2*B)*a^3*c^5*cos(f*x + e)^7 - 8
*(9*A - 2*B)*a^3*c^5*cos(f*x + e)^5 - 10*(9*A - 2*B)*a^3*c^5*cos(f*x + e)^3
 - 15*(9*A - 2*B)*a^3*c^5*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.25, size = 301, normalized size = 1.36

$$\frac{Ba^3c^5\cos\left(9\,fx+9\,e\right)}{2304\,f}+\frac{Ba^3c^5\sin\left(6\,fx+6\,e\right)}{96\,f}+\frac{5}{128}\left(9\,Aa^3c^5-2\,Ba^3c^5\right)x+\frac{\left(8\,Aa^3c^5-Ba^3c^5\right)\cos\left(7\,fx+7\,e\right)}{1792\,f}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorit
hm="giac")
```

```
[Out] 1/2304*B*a^3*c^5*cos(9*f*x + 9*e)/f + 1/96*B*a^3*c^5*sin(6*f*x + 6*e)/f + 5
/128*(9*A*a^3*c^5 - 2*B*a^3*c^5)*x + 1/1792*(8*A*a^3*c^5 - B*a^3*c^5)*cos(7
*f*x + 7*e)/f + 1/64*(2*A*a^3*c^5 - B*a^3*c^5)*cos(5*f*x + 5*e)/f + 1/192*(
18*A*a^3*c^5 - 11*B*a^3*c^5)*cos(3*f*x + 3*e)/f + 1/128*(20*A*a^3*c^5 - 13*
B*a^3*c^5)*cos(f*x + e)/f - 1/1024*(A*a^3*c^5 - 2*B*a^3*c^5)*sin(8*f*x + 8*
e)/f + 1/128*(5*A*a^3*c^5 + 2*B*a^3*c^5)*sin(4*f*x + 4*e)/f + 1/32*(8*A*a^3
*c^5 - B*a^3*c^5)*sin(2*f*x + 2*e)/f
```

maple [B] time = 0.76, size = 611, normalized size = 2.75

$$a^3Ac^5\left(fx+e\right)-a^3Ac^5\left(-\frac{\left(\sin^7\left(fx+e\right)+\frac{7\left(\sin^5\left(fx+e\right)\right)}{6}+\frac{35\left(\sin^3\left(fx+e\right)\right)}{24}+\frac{35\sin\left(fx+e\right)}{16}\right)\cos\left(fx+e\right)}{8}+\frac{35fx}{128}+\frac{35e}{128}\right)-\frac{2a^3Ac^5\left(\frac{16}{5}+\sin^6\left(fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x)
```

```
[Out] 1/f*(a^3*A*c^5*(f*x+e)-a^3*A*c^5*(-1/8*(sin(f*x+e)^7+7/6*sin(f*x+e)^5+35/24
*sin(f*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)+35/128*f*x+35/128*e)-2/7*a^3*A*c
^5*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)+2*a^3*A
*c^5*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*
f*x+5/16*e)+6/5*a^3*A*c^5*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+1/
9*B*a^3*c^5*(128/35+sin(f*x+e)^8+8/7*sin(f*x+e)^6+48/35*sin(f*x+e)^4+64/35*
```

```
sin(f*x+e)^2)*cos(f*x+e)+2*B*a^3*c^5*(-1/8*(sin(f*x+e)^7+7/6*sin(f*x+e)^5+3
5/24*sin(f*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)+35/128*f*x+35/128*e)-2/7*B*a
^3*c^5*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)-6*B
*a^3*c^5*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5
/16*f*x+5/16*e)-B*a^3*c^5*cos(f*x+e)-2*a^3*A*c^5*(2+sin(f*x+e)^2)*cos(f*x+e
)+6*B*a^3*c^5*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)
-2*a^3*A*c^5*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2/3*B*a^3*c^5*(2+si
n(f*x+e)^2)*cos(f*x+e)+2*a^3*A*c^5*cos(f*x+e)-2*B*a^3*c^5*(-1/2*sin(f*x+e)*
cos(f*x+e)+1/2*f*x+1/2*e))
```

maxima [B] time = 0.35, size = 617, normalized size = 2.78

$$18432\left(5\cos\left(fx+e\right)^{7}-21\cos\left(fx+e\right)^{5}+35\cos\left(fx+e\right)^{3}-35\cos\left(fx+e\right)\right)Aa^{3}c^{5}+129024\left(3\cos\left(fx+e\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorit
hm="maxima")
```

```
[Out] 1/322560*(18432*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 -
 35*cos(f*x + e))*A*a^3*c^5 + 129024*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3
+ 15*cos(f*x + e))*A*a^3*c^5 + 645120*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a
^3*c^5 - 105*(128*sin(2*f*x + 2*e)^3 + 840*f*x + 840*e + 3*sin(8*f*x + 8*e)
 + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2*e))*A*a^3*c^5 + 3360*(4*sin(2*f
*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*A*a
^3*c^5 - 161280*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*c^5 + 322560*(f*x +
e)*A*a^3*c^5 + 1024*(35*cos(f*x + e)^9 - 180*cos(f*x + e)^7 + 378*cos(f*x +
 e)^5 - 420*cos(f*x + e)^3 + 315*cos(f*x + e))*B*a^3*c^5 + 18432*(5*cos(f*x
 + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*c^
5 - 215040*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*c^5 + 210*(128*sin(2*f*x
 + 2*e)^3 + 840*f*x + 840*e + 3*sin(8*f*x + 8*e) + 168*sin(4*f*x + 4*e) - 7
68*sin(2*f*x + 2*e))*B*a^3*c^5 - 10080*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*
e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^3*c^5 + 60480*(12*f*x + 1
2*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c^5 - 161280*(2*f*x + 2*
e - sin(2*f*x + 2*e))*B*a^3*c^5 + 645120*A*a^3*c^5*cos(f*x + e) - 322560*B*
a^3*c^5*cos(f*x + e))/f
```

mupad [B] time = 14.93, size = 705, normalized size = 3.18

$$\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{16}\left(4Aa^{3}c^{5}-2Ba^{3}c^{5}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{14}\left(8Aa^{3}c^{5}-8Ba^{3}c^{5}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}\left(\frac{8Aa^{3}c^{5}}{7}-\frac{8Ba^{3}c^{5}}{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^5,x)
```

```
[Out] (tan(e/2 + (f*x)/2)^16*(4*A*a^3*c^5 - 2*B*a^3*c^5) + tan(e/2 + (f*x)/2)^14*
(8*A*a^3*c^5 - 8*B*a^3*c^5) + tan(e/2 + (f*x)/2)^2*((8*A*a^3*c^5)/7 - (8*B*
a^3*c^5)/7) + tan(e/2 + (f*x)/2)^8*(32*A*a^3*c^5 - 4*B*a^3*c^5) + tan(e/2 +
 (f*x)/2)^6*(24*A*a^3*c^5 - 24*B*a^3*c^5) + tan(e/2 + (f*x)/2)^12*(24*A*a^3
*c^5 - (16*B*a^3*c^5)/3) + tan(e/2 + (f*x)/2)^10*(40*A*a^3*c^5 - 40*B*a^3*c
^5) + tan(e/2 + (f*x)/2)^4*((88*A*a^3*c^5)/7 - (32*B*a^3*c^5)/7) - tan(e/2
+ (f*x)/2)^17*((83*A*a^3*c^5)/64 + (5*B*a^3*c^5)/32) + tan(e/2 + (f*x)/2)^5
*((149*A*a^3*c^5)/32 + (83*B*a^3*c^5)/16) - tan(e/2 + (f*x)/2)^13*((149*A*a
^3*c^5)/32 + (83*B*a^3*c^5)/16) + tan(e/2 + (f*x)/2)^3*((189*A*a^3*c^5)/32
- (191*B*a^3*c^5)/48) - tan(e/2 + (f*x)/2)^15*((189*A*a^3*c^5)/32 - (191*B*
a^3*c^5)/48) + tan(e/2 + (f*x)/2)^7*((409*A*a^3*c^5)/32 - (145*B*a^3*c^5)/1
6) - tan(e/2 + (f*x)/2)^11*((409*A*a^3*c^5)/32 - (145*B*a^3*c^5)/16) + tan(
e/2 + (f*x)/2)*((83*A*a^3*c^5)/64 + (5*B*a^3*c^5)/32) + (4*A*a^3*c^5)/7 - (
22*B*a^3*c^5)/63)/(f*(9*tan(e/2 + (f*x)/2)^2 + 36*tan(e/2 + (f*x)/2)^4 + 84
*tan(e/2 + (f*x)/2)^6 + 126*tan(e/2 + (f*x)/2)^8 + 126*tan(e/2 + (f*x)/2)^1
0 + 84*tan(e/2 + (f*x)/2)^12 + 36*tan(e/2 + (f*x)/2)^14 + 9*tan(e/2 + (f*x)
/2)^16 + tan(e/2 + (f*x)/2)^18 + 1)) + (5*a^3*c^5*atan((5*a^3*c^5*tan(e/2 +
 (f*x)/2)*(9*A - 2*B))/(64*((45*A*a^3*c^5)/64 - (5*B*a^3*c^5)/32)))*(9*A -
2*B))/(64*f)
```

sympy [A] time = 37.38, size = 1753, normalized size = 7.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-35*A*a**3*c**5*x*sin(e + f*x)**8/128 - 35*A*a**3*c**5*x*sin(e +
 f*x)**6*cos(e + f*x)**2/32 + 5*A*a**3*c**5*x*sin(e + f*x)**6/8 - 105*A*a**
3*c**5*x*sin(e + f*x)**4*cos(e + f*x)**4/64 + 15*A*a**3*c**5*x*sin(e + f*x)
**4*cos(e + f*x)**2/8 - 35*A*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**6/32
 + 15*A*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**4/8 - A*a**3*c**5*x*sin(e
 + f*x)**2 - 35*A*a**3*c**5*x*cos(e + f*x)**8/128 + 5*A*a**3*c**5*x*cos(e +
 f*x)**6/8 - A*a**3*c**5*x*cos(e + f*x)**2 + A*a**3*c**5*x + 93*A*a**3*c**5
*sin(e + f*x)**7*cos(e + f*x)/(128*f) - 2*A*a**3*c**5*sin(e + f*x)**6*cos(e
 + f*x)/f + 511*A*a**3*c**5*sin(e + f*x)**5*cos(e + f*x)**3/(384*f) - 11*A*
a**3*c**5*sin(e + f*x)**5*cos(e + f*x)/(8*f) - 4*A*a**3*c**5*sin(e + f*x)**
4*cos(e + f*x)**3/f + 6*A*a**3*c**5*sin(e + f*x)**4*cos(e + f*x)/f + 385*A*
a**3*c**5*sin(e + f*x)**3*cos(e + f*x)**5/(384*f) - 5*A*a**3*c**5*sin(e + f
*x)**3*cos(e + f*x)**3/(3*f) - 16*A*a**3*c**5*sin(e + f*x)**2*cos(e + f*x)*
*5/(5*f) + 8*A*a**3*c**5*sin(e + f*x)**2*cos(e + f*x)**3/f - 6*A*a**3*c**5*
```

```
sin(e + f*x)**2*cos(e + f*x)/f + 35*A*a**3*c**5*sin(e + f*x)*cos(e + f*x)**
7/(128*f) - 5*A*a**3*c**5*sin(e + f*x)*cos(e + f*x)**5/(8*f) + A*a**3*c**5*
sin(e + f*x)*cos(e + f*x)/f - 32*A*a**3*c**5*cos(e + f*x)**7/(35*f) + 16*A*
a**3*c**5*cos(e + f*x)**5/(5*f) - 4*A*a**3*c**5*cos(e + f*x)**3/f + 2*A*a**
3*c**5*cos(e + f*x)/f + 35*B*a**3*c**5*x*sin(e + f*x)**8/64 + 35*B*a**3*c**
5*x*sin(e + f*x)**6*cos(e + f*x)**2/16 - 15*B*a**3*c**5*x*sin(e + f*x)**6/8
 + 105*B*a**3*c**5*x*sin(e + f*x)**4*cos(e + f*x)**4/32 - 45*B*a**3*c**5*x*
sin(e + f*x)**4*cos(e + f*x)**2/8 + 9*B*a**3*c**5*x*sin(e + f*x)**4/4 + 35*
B*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**6/16 - 45*B*a**3*c**5*x*sin(e +
 f*x)**2*cos(e + f*x)**4/8 + 9*B*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**
2/2 - B*a**3*c**5*x*sin(e + f*x)**2 + 35*B*a**3*c**5*x*cos(e + f*x)**8/64 -
 15*B*a**3*c**5*x*cos(e + f*x)**6/8 + 9*B*a**3*c**5*x*cos(e + f*x)**4/4 - B
*a**3*c**5*x*cos(e + f*x)**2 + B*a**3*c**5*sin(e + f*x)**8*cos(e + f*x)/f -
 93*B*a**3*c**5*sin(e + f*x)**7*cos(e + f*x)/(64*f) + 8*B*a**3*c**5*sin(e +
 f*x)**6*cos(e + f*x)**3/(3*f) - 2*B*a**3*c**5*sin(e + f*x)**6*cos(e + f*x)
/f - 511*B*a**3*c**5*sin(e + f*x)**5*cos(e + f*x)**3/(192*f) + 33*B*a**3*c*
*5*sin(e + f*x)**5*cos(e + f*x)/(8*f) + 16*B*a**3*c**5*sin(e + f*x)**4*cos(
e + f*x)**5/(5*f) - 4*B*a**3*c**5*sin(e + f*x)**4*cos(e + f*x)**3/f - 385*B
*a**3*c**5*sin(e + f*x)**3*cos(e + f*x)**5/(192*f) + 5*B*a**3*c**5*sin(e +
f*x)**3*cos(e + f*x)**3/f - 15*B*a**3*c**5*sin(e + f*x)**3*cos(e + f*x)/(4*
f) + 64*B*a**3*c**5*sin(e + f*x)**2*cos(e + f*x)**7/(35*f) - 16*B*a**3*c**5
*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) + 2*B*a**3*c**5*sin(e + f*x)**2*cos(
e + f*x)/f - 35*B*a**3*c**5*sin(e + f*x)*cos(e + f*x)**7/(64*f) + 15*B*a**3
*c**5*sin(e + f*x)*cos(e + f*x)**5/(8*f) - 9*B*a**3*c**5*sin(e + f*x)*cos(e
 + f*x)**3/(4*f) + B*a**3*c**5*sin(e + f*x)*cos(e + f*x)/f + 128*B*a**3*c**
5*cos(e + f*x)**9/(315*f) - 32*B*a**3*c**5*cos(e + f*x)**7/(35*f) + 4*B*a**
3*c**5*cos(e + f*x)**3/(3*f) - B*a**3*c**5*cos(e + f*x)/f, Ne(f, 0)), (x*(A
 + B*sin(e))*(a*sin(e) + a)**3*(-c*sin(e) + c)**5, True))
```

3.40 $\int (a + a\sin(e + fx))^3(A + B\sin(e + fx))(c - c\sin(e + fx))^4\,dx$

Optimal. Leaf size=181

$$\frac{a^3c^4(8A-B)\cos^7(e+fx)}{56f}+\frac{a^3c^4(8A-B)\sin(e+fx)\cos^5(e+fx)}{48f}+\frac{5a^3c^4(8A-B)\sin(e+fx)\cos^3(e+fx)}{192f}+\frac{5}{}$$

```
[Out] 5/128*a^3*(8*A-B)*c^4*x+1/56*a^3*(8*A-B)*c^4*cos(f*x+e)^7/f+5/128*a^3*(8*A-
B)*c^4*cos(f*x+e)*sin(f*x+e)/f+5/192*a^3*(8*A-B)*c^4*cos(f*x+e)^3*sin(f*x+e
)/f+1/48*a^3*(8*A-B)*c^4*cos(f*x+e)^5*sin(f*x+e)/f-1/8*a^3*B*cos(f*x+e)^7*(
c^4-c^4*sin(f*x+e))/f
```

Rubi [A] time = 0.23, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2860, 2669, 2635, 8}

$$\frac{a^3c^4(8A-B)\cos^7(e+fx)}{56f}+\frac{a^3c^4(8A-B)\sin(e+fx)\cos^5(e+fx)}{48f}+\frac{5a^3c^4(8A-B)\sin(e+fx)\cos^3(e+fx)}{192f}+\frac{5}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (5*a^3*(8*A - B)*c^4*x)/128 + (a^3*(8*A - B)*c^4*Cos[e + f*x]^7)/(56*f) + (
5*a^3*(8*A - B)*c^4*Cos[e + f*x]*Sin[e + f*x])/(128*f) + (5*a^3*(8*A - B)*c
^4*Cos[e + f*x]^3*Sin[e + f*x])/(192*f) + (a^3*(8*A - B)*c^4*Cos[e + f*x]^5
*Sin[e + f*x])/(48*f) - (a^3*B*Cos[e + f*x]^7*(c^4 - c^4*Sin[e + f*x]))/(8*
f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
```

```
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(A+B\sin(e+fx))(c-$$

$$= -\frac{a^3B\cos^7(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{8f}+\frac{1}{8}\big($$

$$= \frac{a^3(8A-B)c^4\cos^7(e+fx)}{56f}-\frac{a^3B\cos^7(e+fx)$$

$$= \frac{a^3(8A-B)c^4\cos^7(e+fx)}{56f}+\frac{a^3(8A-B)c^4\cos$$

$$= \frac{a^3(8A-B)c^4\cos^7(e+fx)}{56f}+\frac{5a^3(8A-B)c^4\mathrm{co}$$

$$= \frac{a^3(8A-B)c^4\cos^7(e+fx)}{56f}+\frac{5a^3(8A-B)c^4\mathrm{co}$$

$$= \frac{5}{128}a^3(8A-B)c^4x+\frac{a^3(8A-B)c^4\cos^7(e+f}{56f}$$

Mathematica [A] time = 1.95, size = 209, normalized size = 1.15

$$\frac{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^4(840(8A-B)(e+fx)+336(15A-B)\sin(2(e+fx))+168(6A+B)\sin(}{21504f\big(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
4,x]
```

```
[Out] ((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^4*(840*(8*A - B)*(e + f*x) + 1
680*(A - B)*Cos[e + f*x] + 1008*(A - B)*Cos[3*(e + f*x)] + 336*(A - B)*Cos[
5*(e + f*x)] + 48*(A - B)*Cos[7*(e + f*x)] + 336*(15*A - B)*Sin[2*(e + f*x)
] + 168*(6*A + B)*Sin[4*(e + f*x)] + 112*(A + B)*Sin[6*(e + f*x)] + 21*B*Si
n[8*(e + f*x)]))/(21504*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^6)
```

fricas [A] time = 0.47, size = 137, normalized size = 0.76

$$\frac{384\,(A-B)a^3c^4\cos\left(fx+e\right)^7+105\,(8\,A-B)a^3c^4fx+7\left(48\,Ba^3c^4\cos\left(fx+e\right)^7+8\,(8\,A-B)a^3c^4\cos\left(fx+e\right.\right.}{2688\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorit
hm="fricas")
```

```
[Out] 1/2688*(384*(A - B)*a^3*c^4*cos(f*x + e)^7 + 105*(8*A - B)*a^3*c^4*f*x + 7*
(48*B*a^3*c^4*cos(f*x + e)^7 + 8*(8*A - B)*a^3*c^4*cos(f*x + e)^5 + 10*(8*A
 - B)*a^3*c^4*cos(f*x + e)^3 + 15*(8*A - B)*a^3*c^4*cos(f*x + e))*sin(f*x +
 e))/f
```

giac [A] time = 0.23, size = 273, normalized size = 1.51

$$\frac{Ba^3c^4\sin\left(8\,fx+8\,e\right)}{1024\,f}+\frac{5}{128}\left(8\,Aa^3c^4-Ba^3c^4\right)x+\frac{\left(Aa^3c^4-Ba^3c^4\right)\cos\left(7\,fx+7\,e\right)}{448\,f}+\frac{\left(Aa^3c^4-Ba^3c^4\right)\cos\left(5\,fx\right.}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorit
hm="giac")
```

```
[Out] 1/1024*B*a^3*c^4*sin(8*f*x + 8*e)/f + 5/128*(8*A*a^3*c^4 - B*a^3*c^4)*x + 1
/448*(A*a^3*c^4 - B*a^3*c^4)*cos(7*f*x + 7*e)/f + 1/64*(A*a^3*c^4 - B*a^3*c
```

```
^4)*cos(5*f*x + 5*e)/f + 3/64*(A*a^3*c^4 - B*a^3*c^4)*cos(3*f*x + 3*e)/f +
5/64*(A*a^3*c^4 - B*a^3*c^4)*cos(f*x + e)/f + 1/192*(A*a^3*c^4 + B*a^3*c^4)
*sin(6*f*x + 6*e)/f + 1/128*(6*A*a^3*c^4 + B*a^3*c^4)*sin(4*f*x + 4*e)/f +
1/64*(15*A*a^3*c^4 - B*a^3*c^4)*sin(2*f*x + 2*e)/f
```

maple [B] time = 0.67, size = 568, normalized size = 3.14

$$\frac{a^3 A c^4 \left(fx+e\right) + B a^3 c^4 \left(-\frac{\left(\sin^7\left(fx+e\right)+\frac{7\left(\sin^5\left(fx+e\right)\right)}{6}+\frac{35\left(\sin^3\left(fx+e\right)\right)}{24}+\frac{35\sin\left(fx+e\right)}{16}\right)\cos\left(fx+e\right)}{8}+\frac{35fx}{128}+\frac{35e}{128}\right)+\frac{B a^3 c^4\left(\frac{16}{5}+\sin^6\left(fx+e\right)\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x)
```

```
[Out] 1/f*(a^3*A*c^4*(f*x+e)+B*a^3*c^4*(-1/8*(sin(f*x+e)^7+7/6*sin(f*x+e)^5+35/24
*sin(f*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)+35/128*f*x+35/128*e)+1/7*B*a^3*c
^4*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)-3*B*a^3
*c^4*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*
f*x+5/16*e)-3/5*B*a^3*c^4*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-a^
3*A*c^4*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/
16*f*x+5/16*e)+3/5*a^3*A*c^4*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)
+3*a^3*A*c^4*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-
1/7*a^3*A*c^4*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x
+e)+a^3*A*c^4*cos(f*x+e)-B*a^3*c^4*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*
e)-a^3*A*c^4*(2+sin(f*x+e)^2)*cos(f*x+e)+3*B*a^3*c^4*(-1/4*(sin(f*x+e)^3+3/
2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3*a^3*A*c^4*(-1/2*sin(f*x+e)*cos(f*
x+e)+1/2*f*x+1/2*e)+B*a^3*c^4*(2+sin(f*x+e)^2)*cos(f*x+e)-B*a^3*c^4*cos(f*x
+e))
```

maxima [B] time = 0.36, size = 571, normalized size = 3.15

$$\frac{3072\left(5\cos\left(fx+e\right)^7-21\cos\left(fx+e\right)^5+35\cos\left(fx+e\right)^3-35\cos\left(fx+e\right)\right)Aa^3c^4+21504\left(3\cos\left(fx+e\right)^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorit
hm="maxima")
```

```
[Out] 1/107520*(3072*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 -
35*cos(f*x + e))*A*a^3*c^4 + 21504*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 +
15*cos(f*x + e))*A*a^3*c^4 + 107520*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3
```

```
*c^4 - 560*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*
sin(2*f*x + 2*e))*A*a^3*c^4 + 10080*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*s
in(2*f*x + 2*e))*A*a^3*c^4 - 80640*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*c
^4 + 107520*(f*x + e)*A*a^3*c^4 - 3072*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^
5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*c^4 - 21504*(3*cos(f*x + e)^
5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^4 - 107520*(cos(f*x + e)^3
 - 3*cos(f*x + e))*B*a^3*c^4 + 35*(128*sin(2*f*x + 2*e)^3 + 840*f*x + 840*e
 + 3*sin(8*f*x + 8*e) + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2*e))*B*a^3*
c^4 - 1680*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*
sin(2*f*x + 2*e))*B*a^3*c^4 + 10080*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*s
in(2*f*x + 2*e))*B*a^3*c^4 - 26880*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c
^4 + 107520*A*a^3*c^4*cos(f*x + e) - 107520*B*a^3*c^4*cos(f*x + e))/f
```

mupad [B] time = 14.78, size = 661, normalized size = 3.65

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(6\,A\,a^3\,c^4-6\,B\,a^3\,c^4\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}\,\left(2\,A\,a^3\,c^4-2\,B\,a^3\,c^4\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,\left(6\,A\,a^3\,c^4-6\,B\,a^3 \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^4,x)
```

```
[Out] (tan(e/2 + (f*x)/2)^4*(6*A*a^3*c^4 - 6*B*a^3*c^4) + tan(e/2 + (f*x)/2)^12*(
2*A*a^3*c^4 - 2*B*a^3*c^4) + tan(e/2 + (f*x)/2)^6*(6*A*a^3*c^4 - 6*B*a^3*c^
4) + tan(e/2 + (f*x)/2)^14*(2*A*a^3*c^4 - 2*B*a^3*c^4) + tan(e/2 + (f*x)/2)
^2*((2*A*a^3*c^4)/7 - (2*B*a^3*c^4)/7) + tan(e/2 + (f*x)/2)^8*(10*A*a^3*c^4
 - 10*B*a^3*c^4) + tan(e/2 + (f*x)/2)^10*(10*A*a^3*c^4 - 10*B*a^3*c^4) - ta
n(e/2 + (f*x)/2)^15*((11*A*a^3*c^4)/8 + (5*B*a^3*c^4)/64) + tan(e/2 + (f*x)
/2)^3*((61*A*a^3*c^4)/24 - (397*B*a^3*c^4)/192) - tan(e/2 + (f*x)/2)^13*((6
1*A*a^3*c^4)/24 - (397*B*a^3*c^4)/192) + tan(e/2 + (f*x)/2)^5*((113*A*a^3*c
^4)/24 + (895*B*a^3*c^4)/192) - tan(e/2 + (f*x)/2)^11*((113*A*a^3*c^4)/24 +
 (895*B*a^3*c^4)/192) + tan(e/2 + (f*x)/2)^7*((85*A*a^3*c^4)/24 - (1765*B*a
^3*c^4)/192) - tan(e/2 + (f*x)/2)^9*((85*A*a^3*c^4)/24 - (1765*B*a^3*c^4)/1
92) + tan(e/2 + (f*x)/2)*((11*A*a^3*c^4)/8 + (5*B*a^3*c^4)/64) + (2*A*a^3*c
^4)/7 - (2*B*a^3*c^4)/7)/(f*(8*tan(e/2 + (f*x)/2)^2 + 28*tan(e/2 + (f*x)/2)
^4 + 56*tan(e/2 + (f*x)/2)^6 + 70*tan(e/2 + (f*x)/2)^8 + 56*tan(e/2 + (f*x)
/2)^10 + 28*tan(e/2 + (f*x)/2)^12 + 8*tan(e/2 + (f*x)/2)^14 + tan(e/2 + (f*
x)/2)^16 + 1)) + (5*a^3*c^4*atan((5*a^3*c^4*tan(e/2 + (f*x)/2)*(8*A - B))/(
64*((5*A*a^3*c^4)/8 - (5*B*a^3*c^4)/64)))*(8*A - B))/(64*f)
```

sympy [A] time = 23.43, size = 1579, normalized size = 8.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-5*A*a**3*c**4*x*sin(e + f*x)**6/16 - 15*A*a**3*c**4*x*sin(e + f
*x)**4*cos(e + f*x)**2/16 + 9*A*a**3*c**4*x*sin(e + f*x)**4/8 - 15*A*a**3*c
**4*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 9*A*a**3*c**4*x*sin(e + f*x)**2*
cos(e + f*x)**2/4 - 3*A*a**3*c**4*x*sin(e + f*x)**2/2 - 5*A*a**3*c**4*x*cos
(e + f*x)**6/16 + 9*A*a**3*c**4*x*cos(e + f*x)**4/8 - 3*A*a**3*c**4*x*cos(e
 + f*x)**2/2 + A*a**3*c**4*x - A*a**3*c**4*sin(e + f*x)**6*cos(e + f*x)/f +
 11*A*a**3*c**4*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 2*A*a**3*c**4*sin(e +
 f*x)**4*cos(e + f*x)**3/f + 3*A*a**3*c**4*sin(e + f*x)**4*cos(e + f*x)/f +
 5*A*a**3*c**4*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 15*A*a**3*c**4*sin(e
 + f*x)**3*cos(e + f*x)/(8*f) - 8*A*a**3*c**4*sin(e + f*x)**2*cos(e + f*x)*
*5/(5*f) + 4*A*a**3*c**4*sin(e + f*x)**2*cos(e + f*x)**3/f - 3*A*a**3*c**4*
sin(e + f*x)**2*cos(e + f*x)/f + 5*A*a**3*c**4*sin(e + f*x)*cos(e + f*x)**5
/(16*f) - 9*A*a**3*c**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 3*A*a**3*c**4*
sin(e + f*x)*cos(e + f*x)/(2*f) - 16*A*a**3*c**4*cos(e + f*x)**7/(35*f) + 8
*A*a**3*c**4*cos(e + f*x)**5/(5*f) - 2*A*a**3*c**4*cos(e + f*x)**3/f + A*a*
*3*c**4*cos(e + f*x)/f + 35*B*a**3*c**4*x*sin(e + f*x)**8/128 + 35*B*a**3*c
**4*x*sin(e + f*x)**6*cos(e + f*x)**2/32 - 15*B*a**3*c**4*x*sin(e + f*x)**6
/16 + 105*B*a**3*c**4*x*sin(e + f*x)**4*cos(e + f*x)**4/64 - 45*B*a**3*c**4
*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 9*B*a**3*c**4*x*sin(e + f*x)**4/8 +
 35*B*a**3*c**4*x*sin(e + f*x)**2*cos(e + f*x)**6/32 - 45*B*a**3*c**4*x*sin
(e + f*x)**2*cos(e + f*x)**4/16 + 9*B*a**3*c**4*x*sin(e + f*x)**2*cos(e + f
*x)**2/4 - B*a**3*c**4*x*sin(e + f*x)**2/2 + 35*B*a**3*c**4*x*cos(e + f*x)*
*8/128 - 15*B*a**3*c**4*x*cos(e + f*x)**6/16 + 9*B*a**3*c**4*x*cos(e + f*x)
**4/8 - B*a**3*c**4*x*cos(e + f*x)**2/2 - 93*B*a**3*c**4*sin(e + f*x)**7*co
s(e + f*x)/(128*f) + B*a**3*c**4*sin(e + f*x)**6*cos(e + f*x)/f - 511*B*a**
3*c**4*sin(e + f*x)**5*cos(e + f*x)**3/(384*f) + 33*B*a**3*c**4*sin(e + f*x
)**5*cos(e + f*x)/(16*f) + 2*B*a**3*c**4*sin(e + f*x)**4*cos(e + f*x)**3/f
- 3*B*a**3*c**4*sin(e + f*x)**4*cos(e + f*x)/f - 385*B*a**3*c**4*sin(e + f*
x)**3*cos(e + f*x)**5/(384*f) + 5*B*a**3*c**4*sin(e + f*x)**3*cos(e + f*x)*
*3/(2*f) - 15*B*a**3*c**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 8*B*a**3*c**
4*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) - 4*B*a**3*c**4*sin(e + f*x)**2*cos
(e + f*x)**3/f + 3*B*a**3*c**4*sin(e + f*x)**2*cos(e + f*x)/f - 35*B*a**3*c
**4*sin(e + f*x)*cos(e + f*x)**7/(128*f) + 15*B*a**3*c**4*sin(e + f*x)*cos(
e + f*x)**5/(16*f) - 9*B*a**3*c**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) + B*a
**3*c**4*sin(e + f*x)*cos(e + f*x)/(2*f) + 16*B*a**3*c**4*cos(e + f*x)**7/(
35*f) - 8*B*a**3*c**4*cos(e + f*x)**5/(5*f) + 2*B*a**3*c**4*cos(e + f*x)**3
/f - B*a**3*c**4*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a
)**3*(-c*sin(e) + c)**4, True))
```

3.41 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$

Optimal. Leaf size=117

$$\frac{a^3Ac^3\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{5a^3Ac^3\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{5a^3Ac^3\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}a^3Ac^3x$$

```
[Out] 5/16*a^3*A*c^3*x-1/7*a^3*B*c^3*cos(f*x+e)^7/f+5/16*a^3*A*c^3*cos(f*x+e)*sin
(f*x+e)/f+5/24*a^3*A*c^3*cos(f*x+e)^3*sin(f*x+e)/f+1/6*a^3*A*c^3*cos(f*x+e)
^5*sin(f*x+e)/f
```

Rubi [A] time = 0.15, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2669, 2635, 8}

$$\frac{a^3Ac^3\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{5a^3Ac^3\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{5a^3Ac^3\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}a^3Ac^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (5*a^3*A*c^3*x)/16 - (a^3*B*c^3*Cos[e + f*x]^7)/(7*f) + (5*a^3*A*c^3*Cos[e
+ f*x]*Sin[e + f*x])/(16*f) + (5*a^3*A*c^3*Cos[e + f*x]^3*Sin[e + f*x])/(24
*f) + (a^3*A*c^3*Cos[e + f*x]^5*Sin[e + f*x])/(6*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(A+B\sin(e+fx))\,dx$$
$$= -\frac{a^3Bc^3\cos^7(e+fx)}{7f} + \left(a^3Ac^3\right)\int \cos^6(e+f$$
$$= -\frac{a^3Bc^3\cos^7(e+fx)}{7f} + \frac{a^3Ac^3\cos^5(e+fx)\sin}{6f}$$
$$= -\frac{a^3Bc^3\cos^7(e+fx)}{7f} + \frac{5a^3Ac^3\cos^3(e+fx)\sin}{24f}$$
$$= -\frac{a^3Bc^3\cos^7(e+fx)}{7f} + \frac{5a^3Ac^3\cos(e+fx)\sin}{16f}$$
$$= \frac{5}{16}a^3Ac^3x - \frac{a^3Bc^3\cos^7(e+fx)}{7f} + \frac{5a^3Ac^3\cos}{}$$

Mathematica [A] time = 0.23, size = 64, normalized size = 0.55

$$\frac{a^3c^3\left(7A(45\sin(2(e+fx))+9\sin(4(e+fx))+\sin(6(e+fx))+60e+60fx)-192B\cos^7(e+fx)\right)}{1344f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
3,x]
```

```
[Out] (a^3*c^3*(-192*B*Cos[e + f*x]^7 + 7*A*(60*e + 60*f*x + 45*Sin[2*(e + f*x)]
+ 9*Sin[4*(e + f*x)] + Sin[6*(e + f*x)])))/(1344*f)
```

fricas [A] time = 0.45, size = 92, normalized size = 0.79

$$-\frac{48\,Ba^3c^3\cos\left(fx+e\right)^7-105\,Aa^3c^3fx-7\left(8\,Aa^3c^3\cos\left(fx+e\right)^5+10\,Aa^3c^3\cos\left(fx+e\right)^3+15\,Aa^3c^3\cos\left(fx\right.\right.}{336\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/336*(48*B*a^3*c^3*cos(f*x + e)^7 - 105*A*a^3*c^3*f*x - 7*(8*A*a^3*c^3*co
s(f*x + e)^5 + 10*A*a^3*c^3*cos(f*x + e)^3 + 15*A*a^3*c^3*cos(f*x + e))*sin
(f*x + e))/f
```

giac [A] time = 0.19, size = 162, normalized size = 1.38

$$\frac{5}{16}Aa^3c^3x - \frac{Ba^3c^3\cos\left(7fx+7e\right)}{448f} - \frac{Ba^3c^3\cos\left(5fx+5e\right)}{64f} - \frac{3Ba^3c^3\cos\left(3fx+3e\right)}{64f} - \frac{5Ba^3c^3\cos\left(fx+e\right)}{64f} + \frac{Aa^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 5/16*A*a^3*c^3*x - 1/448*B*a^3*c^3*cos(7*f*x + 7*e)/f - 1/64*B*a^3*c^3*cos(
5*f*x + 5*e)/f - 3/64*B*a^3*c^3*cos(3*f*x + 3*e)/f - 5/64*B*a^3*c^3*cos(f*x
 + e)/f + 1/192*A*a^3*c^3*sin(6*f*x + 6*e)/f + 3/64*A*a^3*c^3*sin(4*f*x + 4
*e)/f + 15/64*A*a^3*c^3*sin(2*f*x + 2*e)/f
```

maple [B] time = 0.67, size = 263, normalized size = 2.25

$$\frac{\frac{Ba^3c^3\left(\frac{16}{5}+\sin^6\left(fx+e\right)+\frac{6\left(\sin^4\left(fx+e\right)\right)}{5}+\frac{8\left(\sin^2\left(fx+e\right)\right)}{5}\right)\cos\left(fx+e\right)}{7} - a^3Ac^3\left(-\frac{\left(\sin^5\left(fx+e\right)+\frac{5\left(\sin^3\left(fx+e\right)\right)}{4}+\frac{15\sin\left(fx+e\right)}{8}\right)\cos\left(fx+e\right)}{6} + \frac{5fx}{16} + \frac{5e}{16}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(1/7*B*a^3*c^3*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*co
s(f*x+e)-a^3*A*c^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*co
s(f*x+e)+5/16*f*x+5/16*e)-3/5*B*a^3*c^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)
*cos(f*x+e)+3*a^3*A*c^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*
f*x+3/8*e)+B*a^3*c^3*(2+sin(f*x+e)^2)*cos(f*x+e)-3*a^3*A*c^3*(-1/2*sin(f*x+
e)*cos(f*x+e)+1/2*f*x+1/2*e)-B*a^3*c^3*cos(f*x+e)+a^3*A*c^3*(f*x+e))
```

maxima [B] time = 0.38, size = 264, normalized size = 2.26

$$-\frac{35\left(4\sin\left(2fx+2e\right)^3+60fx+60e+9\sin\left(4fx+4e\right)-48\sin\left(2fx+2e\right)\right)Aa^3c^3 - 630\left(12fx+12e+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] -1/6720*(35*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48
*sin(2*f*x + 2*e))*A*a^3*c^3 - 630*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*si
n(2*f*x + 2*e))*A*a^3*c^3 + 5040*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*c^3
 - 6720*(f*x + e)*A*a^3*c^3 + 192*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 3
5*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*c^3 + 1344*(3*cos(f*x + e)^5 - 10
*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^3 + 6720*(cos(f*x + e)^3 - 3*cos
(f*x + e))*B*a^3*c^3 + 6720*B*a^3*c^3*cos(f*x + e))/f
```

mupad [B] time = 14.29, size = 325, normalized size = 2.78

$$\frac{5\,A\,a^3\,c^3\,x}{16}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}\,\left(\frac{a^3\,c^3\,\left(672\,B-735\,A\,\left(e+f\,x\right)\right)}{336}+\frac{35\,A\,a^3\,c^3\,\left(e+f\,x\right)}{16}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(\frac{a^3\,c^3\,\left(2016\,B-2205\,A\,\left(e+f\,x\right)\right)}{336}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^3,x)
```

```
[Out] (5*A*a^3*c^3*x)/16 - (tan(e/2 + (f*x)/2)^12*((a^3*c^3*(672*B - 735*A*(e + f
*x)))/336 + (35*A*a^3*c^3*(e + f*x))/16) + tan(e/2 + (f*x)/2)^4*((a^3*c^3*(
2016*B - 2205*A*(e + f*x)))/336 + (105*A*a^3*c^3*(e + f*x))/16) + tan(e/2 +
 (f*x)/2)^8*((a^3*c^3*(3360*B - 3675*A*(e + f*x)))/336 + (175*A*a^3*c^3*(e
+ f*x))/16) + (a^3*c^3*(96*B - 105*A*(e + f*x)))/336 + (5*A*a^3*c^3*(e + f*
x))/16 - (7*A*a^3*c^3*tan(e/2 + (f*x)/2)^3)/6 - (85*A*a^3*c^3*tan(e/2 + (f*
x)/2)^5)/24 + (85*A*a^3*c^3*tan(e/2 + (f*x)/2)^9)/24 + (7*A*a^3*c^3*tan(e/2
 + (f*x)/2)^11)/6 + (11*A*a^3*c^3*tan(e/2 + (f*x)/2)^13)/8 - (11*A*a^3*c^3*
tan(e/2 + (f*x)/2))/8)/(f*(tan(e/2 + (f*x)/2)^2 + 1)^7)
```

sympy [A] time = 10.35, size = 682, normalized size = 5.83

$$\begin{cases} -\frac{5Aa^3c^3x\sin^6\left(e+fx\right)}{16}-\frac{15Aa^3c^3x\sin^4\left(e+fx\right)\cos^2\left(e+fx\right)}{16}+\frac{9Aa^3c^3x\sin^4\left(e+fx\right)}{8}-\frac{15Aa^3c^3x\sin^2\left(e+fx\right)\cos^4\left(e+fx\right)}{16}+\frac{9Aa^3c^3x\sin^2\left(e+f\right.}{4} \\ x\left(A+B\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right)^3\left(-c\sin\left(e\right)+c\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-5*A*a**3*c**3*x*sin(e + f*x)**6/16 - 15*A*a**3*c**3*x*sin(e + f
*x)**4*cos(e + f*x)**2/16 + 9*A*a**3*c**3*x*sin(e + f*x)**4/8 - 15*A*a**3*c
**3*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 9*A*a**3*c**3*x*sin(e + f*x)**2*
cos(e + f*x)**2/4 - 3*A*a**3*c**3*x*sin(e + f*x)**2/2 - 5*A*a**3*c**3*x*cos
(e + f*x)**6/16 + 9*A*a**3*c**3*x*cos(e + f*x)**4/8 - 3*A*a**3*c**3*x*cos(e
 + f*x)**2/2 + A*a**3*c**3*x + 11*A*a**3*c**3*sin(e + f*x)**5*cos(e + f*x)/
(16*f) + 5*A*a**3*c**3*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 15*A*a**3*c*
*3*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 5*A*a**3*c**3*sin(e + f*x)*cos(e +
f*x)**5/(16*f) - 9*A*a**3*c**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 3*A*a**
3*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) + B*a**3*c**3*sin(e + f*x)**6*cos(e
+ f*x)/f + 2*B*a**3*c**3*sin(e + f*x)**4*cos(e + f*x)**3/f - 3*B*a**3*c**3*
sin(e + f*x)**4*cos(e + f*x)/f + 8*B*a**3*c**3*sin(e + f*x)**2*cos(e + f*x)
**5/(5*f) - 4*B*a**3*c**3*sin(e + f*x)**2*cos(e + f*x)**3/f + 3*B*a**3*c**3
*sin(e + f*x)**2*cos(e + f*x)/f + 16*B*a**3*c**3*cos(e + f*x)**7/(35*f) - 8
*B*a**3*c**3*cos(e + f*x)**5/(5*f) + 2*B*a**3*c**3*cos(e + f*x)**3/f - B*a*
*3*c**3*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**3*(-c*
sin(e) + c)**3, True))
```

3.42 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$

Optimal. Leaf size=138

$$-\frac{a^3c^2(6A+B)\cos^5(e+fx)}{30f}+\frac{a^3c^2(6A+B)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{a^3c^2(6A+B)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}$$

```
[Out] 1/16*a^3*(6*A+B)*c^2*x-1/30*a^3*(6*A+B)*c^2*cos(f*x+e)^5/f+1/16*a^3*(6*A+B)
*c^2*cos(f*x+e)*sin(f*x+e)/f+1/24*a^3*(6*A+B)*c^2*cos(f*x+e)^3*sin(f*x+e)/f
-1/6*B*c^2*cos(f*x+e)^5*(a^3+a^3*sin(f*x+e))/f
```

Rubi [A] time = 0.20, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2860, 2669, 2635, 8}

$$-\frac{a^3c^2(6A+B)\cos^5(e+fx)}{30f}+\frac{a^3c^2(6A+B)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{a^3c^2(6A+B)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a^3*(6*A + B)*c^2*x)/16 - (a^3*(6*A + B)*c^2*Cos[e + f*x]^5)/(30*f) + (a^3
*(6*A + B)*c^2*Cos[e + f*x]*Sin[e + f*x])/(16*f) + (a^3*(6*A + B)*c^2*Cos[e
 + f*x]^3*Sin[e + f*x])/(24*f) - (B*c^2*Cos[e + f*x]^5*(a^3 + a^3*Sin[e + f
*x]))/(6*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
```

```
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(a+a\sin(e+fx))(A$$

$$= -\frac{Bc^2\cos^5(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{6f}+\frac{1}{6}$$

$$= -\frac{a^3(6A+B)c^2\cos^5(e+fx)}{30f}-\frac{Bc^2\cos^5(e+$$

$$= -\frac{a^3(6A+B)c^2\cos^5(e+fx)}{30f}+\frac{a^3(6A+B)c^2}{}$$

$$= -\frac{a^3(6A+B)c^2\cos^5(e+fx)}{30f}+\frac{a^3(6A+B)c^2}{}$$

$$= \frac{1}{16}a^3(6A+B)c^2x-\frac{a^3(6A+B)c^2\cos^5(e+f}{30f}$$

Mathematica [A] time = 1.09, size = 133, normalized size = 0.96

$$a^3c^2(-120(A+B)\cos(e+fx)-60(A+B)\cos(3(e+fx))+240A\sin(2(e+fx))+30A\sin(4(e+fx))-12A\,c$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
2,x]
```

```
[Out] (a^3*c^2*(360*A*e + 60*B*e + 360*A*f*x + 60*B*f*x - 120*(A + B)*Cos[e + f*x
] - 60*(A + B)*Cos[3*(e + f*x)] - 12*A*Cos[5*(e + f*x)] - 12*B*Cos[5*(e + f
*x)] + 240*A*Sin[2*(e + f*x)] + 15*B*Sin[2*(e + f*x)] + 30*A*Sin[4*(e + f*x
)] - 15*B*Sin[4*(e + f*x)] - 5*B*Sin[6*(e + f*x)]))/(960*f)
```

fricas [A] time = 0.44, size = 106, normalized size = 0.77

$$-\frac{48\,(A+B)a^3c^2\cos\left(fx+e\right)^5-15\,(6\,A+B)a^3c^2fx+5\left(8\,Ba^3c^2\cos\left(fx+e\right)^5-2\,(6\,A+B)a^3c^2\cos\left(fx+e\right)^3-\right.}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/240*(48*(A + B)*a^3*c^2*cos(f*x + e)^5 - 15*(6*A + B)*a^3*c^2*f*x + 5*(8
*B*a^3*c^2*cos(f*x + e)^5 - 2*(6*A + B)*a^3*c^2*cos(f*x + e)^3 - 3*(6*A + B
)*a^3*c^2*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.19, size = 204, normalized size = 1.48

$$-\frac{Ba^3c^2\sin\left(6\,fx+6\,e\right)}{192\,f}+\frac{1}{16}\left(6\,Aa^3c^2+Ba^3c^2\right)x-\frac{\left(Aa^3c^2+Ba^3c^2\right)\cos\left(5\,fx+5\,e\right)}{80\,f}-\frac{\left(Aa^3c^2+Ba^3c^2\right)\cos\left(3\,fx+\right.}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] -1/192*B*a^3*c^2*sin(6*f*x + 6*e)/f + 1/16*(6*A*a^3*c^2 + B*a^3*c^2)*x - 1/
80*(A*a^3*c^2 + B*a^3*c^2)*cos(5*f*x + 5*e)/f - 1/16*(A*a^3*c^2 + B*a^3*c^2
)*cos(3*f*x + 3*e)/f - 1/8*(A*a^3*c^2 + B*a^3*c^2)*cos(f*x + e)/f + 1/64*(2
*A*a^3*c^2 - B*a^3*c^2)*sin(4*f*x + 4*e)/f + 1/64*(16*A*a^3*c^2 + B*a^3*c^2
)*sin(2*f*x + 2*e)/f
```

maple [B] time = 0.58, size = 364, normalized size = 2.64

$$\frac{-\frac{a^3Ac^2\left(\frac{8}{3}+\sin^4\left(fx+e\right)+\frac{4\left(\sin^2\left(fx+e\right)\right)}{3}\right)\cos\left(fx+e\right)}{5}+a^3Ac^2\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+\frac{2a^3Ac^2\left(2+\sin^2\left(fx+e\right)\right)c}{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(-1/5*a^3*A*c^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+a^3*A*c^
2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+2/3*a^3*A*c
^2*(2+sin(f*x+e)^2)*cos(f*x+e)+B*a^3*c^2*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)
^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)-1/5*B*a^3*c^2*(8/3+sin(f*x+
e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-2*B*a^3*c^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f
*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2*a^3*A*c^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/
2*f*x+1/2*e)+2/3*B*a^3*c^2*(2+sin(f*x+e)^2)*cos(f*x+e)-a^3*A*c^2*cos(f*x+e)
+B*a^3*c^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+a^3*A*c^2*(f*x+e)-B*a
^3*c^2*cos(f*x+e))
```

maxima [B] time = 0.34, size = 360, normalized size = 2.61

$$-\frac{64\left(3\cos\left(fx+e\right)^{5}-10\cos\left(fx+e\right)^{3}+15\cos\left(fx+e\right)\right)Aa^{3}c^{2}+640\left(\cos\left(fx+e\right)^{3}-3\cos\left(fx+e\right)\right)Aa^{3}c^{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] -1/960*(64*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^3*c
^2 + 640*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c^2 - 30*(12*f*x + 12*e +
sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c^2 + 480*(2*f*x + 2*e - sin(2
*f*x + 2*e))*A*a^3*c^2 - 960*(f*x + e)*A*a^3*c^2 + 64*(3*cos(f*x + e)^5 - 1
0*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^2 + 640*(cos(f*x + e)^3 - 3*cos
(f*x + e))*B*a^3*c^2 - 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*
x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^3*c^2 + 60*(12*f*x + 12*e + sin(4*f*x +
 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c^2 - 240*(2*f*x + 2*e - sin(2*f*x + 2*e)
)*B*a^3*c^2 + 960*A*a^3*c^2*cos(f*x + e) + 960*B*a^3*c^2*cos(f*x + e))/f
```

mupad [B] time = 14.17, size = 536, normalized size = 3.88

$$\frac{a^{3}\,c^{2}\,\mathrm{atan}\left(\frac{a^{3}\,c^{2}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(6\,A+B\right)}{8\,\left(\frac{3\,A\,a^{3}\,c^{2}}{4}+\frac{B\,a^{3}\,c^{2}}{8}\right)}\right)\,\left(6\,A+B\right)}{8\,f}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{4}\,\left(4\,A\,a^{3}\,c^{2}+4\,B\,a^{3}\,c^{2}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{8}\,\left(2\,A\,a^{3}\,c^{2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^2,x)
```

```
[Out] (a^3*c^2*atan((a^3*c^2*tan(e/2 + (f*x)/2)*(6*A + B))/(8*((3*A*a^3*c^2)/4 +
(B*a^3*c^2)/8)))*(6*A + B))/(8*f) - (tan(e/2 + (f*x)/2)^4*(4*A*a^3*c^2 + 4*
B*a^3*c^2) + tan(e/2 + (f*x)/2)^8*(2*A*a^3*c^2 + 2*B*a^3*c^2) + tan(e/2 + (
f*x)/2)^6*(4*A*a^3*c^2 + 4*B*a^3*c^2) + tan(e/2 + (f*x)/2)^10*(2*A*a^3*c^2
+ 2*B*a^3*c^2) + tan(e/2 + (f*x)/2)^2*((2*A*a^3*c^2)/5 + (2*B*a^3*c^2)/5) -
 tan(e/2 + (f*x)/2)^5*((A*a^3*c^2)/2 - (13*B*a^3*c^2)/4) + tan(e/2 + (f*x)/
2)^7*((A*a^3*c^2)/2 - (13*B*a^3*c^2)/4) + tan(e/2 + (f*x)/2)^11*((5*A*a^3*c
^2)/4 - (B*a^3*c^2)/8) - tan(e/2 + (f*x)/2)^3*((7*A*a^3*c^2)/4 + (47*B*a^3*
c^2)/24) + tan(e/2 + (f*x)/2)^9*((7*A*a^3*c^2)/4 + (47*B*a^3*c^2)/24) - tan
(e/2 + (f*x)/2)*((5*A*a^3*c^2)/4 - (B*a^3*c^2)/8) + (2*A*a^3*c^2)/5 + (2*B*
a^3*c^2)/5)/(f*(6*tan(e/2 + (f*x)/2)^2 + 15*tan(e/2 + (f*x)/2)^4 + 20*tan(e
/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 + 6*tan(e/2 + (f*x)/2)^10 + tan(e
/2 + (f*x)/2)^12 + 1)) - (a^3*c^2*(6*A + B)*(atan(tan(e/2 + (f*x)/2)) - (f*
x)/2))/(8*f)
```

sympy [A] time = 8.10, size = 910, normalized size = 6.59

$$\begin{cases} \frac{3Aa^3c^2x\sin^4\left(e+fx\right)}{8} + \frac{3Aa^3c^2x\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4} - Aa^3c^2x\sin^2\left(e+fx\right) + \frac{3Aa^3c^2x\cos^4\left(e+fx\right)}{8} - Aa^3c^2x\cos^2\left(e+fx\right) \\ x\left(A+B\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right)^3\left(-c\sin\left(e\right)+c\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*A*a**3*c**2*x*sin(e + f*x)**4/8 + 3*A*a**3*c**2*x*sin(e + f*x)
**2*cos(e + f*x)**2/4 - A*a**3*c**2*x*sin(e + f*x)**2 + 3*A*a**3*c**2*x*cos
(e + f*x)**4/8 - A*a**3*c**2*x*cos(e + f*x)**2 + A*a**3*c**2*x - A*a**3*c**
2*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**3*c**2*sin(e + f*x)**3*cos(e + f*
x)/(8*f) - 4*A*a**3*c**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 2*A*a**3*c
**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**3*c**2*sin(e + f*x)*cos(e + f*x
)**3/(8*f) + A*a**3*c**2*sin(e + f*x)*cos(e + f*x)/f - 8*A*a**3*c**2*cos(e
+ f*x)**5/(15*f) + 4*A*a**3*c**2*cos(e + f*x)**3/(3*f) - A*a**3*c**2*cos(e
+ f*x)/f + 5*B*a**3*c**2*x*sin(e + f*x)**6/16 + 15*B*a**3*c**2*x*sin(e + f*
x)**4*cos(e + f*x)**2/16 - 3*B*a**3*c**2*x*sin(e + f*x)**4/4 + 15*B*a**3*c*
*2*x*sin(e + f*x)**2*cos(e + f*x)**4/16 - 3*B*a**3*c**2*x*sin(e + f*x)**2*c
os(e + f*x)**2/2 + B*a**3*c**2*x*sin(e + f*x)**2/2 + 5*B*a**3*c**2*x*cos(e
+ f*x)**6/16 - 3*B*a**3*c**2*x*cos(e + f*x)**4/4 + B*a**3*c**2*x*cos(e + f*
x)**2/2 - 11*B*a**3*c**2*sin(e + f*x)**5*cos(e + f*x)/(16*f) - B*a**3*c**2*
sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a**3*c**2*sin(e + f*x)**3*cos(e + f*x)
**3/(6*f) + 5*B*a**3*c**2*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 4*B*a**3*c**
2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 2*B*a**3*c**2*sin(e + f*x)**2*cos
(e + f*x)/f - 5*B*a**3*c**2*sin(e + f*x)*cos(e + f*x)**5/(16*f) + 3*B*a**3*
c**2*sin(e + f*x)*cos(e + f*x)**3/(4*f) - B*a**3*c**2*sin(e + f*x)*cos(e +
```

```
f*x)/(2*f) - 8*B*a**3*c**2*cos(e + f*x)**5/(15*f) + 4*B*a**3*c**2*cos(e + f
*x)**3/(3*f) - B*a**3*c**2*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*
sin(e) + a)**3*(-c*sin(e) + c)**2, True))
```

3.43 $\int (a + a\sin(e + fx))^3(A + B\sin(e + fx))(c - c\sin(e + fx))\,dx$

Optimal. Leaf size=140

$$-\frac{a^3c(5A+2B)\cos^3(e+fx)}{12f}-\frac{c(5A+2B)\cos^3(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{20f}+\frac{a^3c(5A+2B)\sin(e+fx)\cos(e+f}{8f}$$

```
[Out] 1/8*a^3*(5*A+2*B)*c*x-1/12*a^3*(5*A+2*B)*c*cos(f*x+e)^3/f+1/8*a^3*(5*A+2*B)
*c*cos(f*x+e)*sin(f*x+e)/f-1/5*a*B*c*cos(f*x+e)^3*(a+a*sin(f*x+e))^2/f-1/20
*(5*A+2*B)*c*cos(f*x+e)^3*(a^3+a^3*sin(f*x+e))/f
```

Rubi [A] time = 0.22, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2967, 2860, 2678, 2669, 2635, 8}

$$-\frac{a^3c(5A+2B)\cos^3(e+fx)}{12f}-\frac{c(5A+2B)\cos^3(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{20f}+\frac{a^3c(5A+2B)\sin(e+fx)\cos(e+f}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^3*(5*A + 2*B)*c*x)/8 - (a^3*(5*A + 2*B)*c*Cos[e + f*x]^3)/(12*f) + (a^3*
(5*A + 2*B)*c*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (a*B*c*Cos[e + f*x]^3*(a +
 a*Sin[e + f*x])^2)/(5*f) - ((5*A + 2*B)*c*Cos[e + f*x]^3*(a^3 + a^3*Sin[e
+ f*x]))/(20*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
```

```
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx = (ac)\int \cos^2(e+fx)(a+a\sin(e+fx))^2(A+B\sin$$
$$= -\frac{aBc\cos^3(e+fx)(a+a\sin(e+fx))^2}{5f} + \frac{1}{5}(a(5$$
$$= -\frac{aBc\cos^3(e+fx)(a+a\sin(e+fx))^2}{5f} - \frac{(5A+}{}$$
$$= -\frac{a^3(5A+2B)c\cos^3(e+fx)}{12f} - \frac{aBc\cos^3(e+fx}{}$$
$$= -\frac{a^3(5A+2B)c\cos^3(e+fx)}{12f} + \frac{a^3(5A+2B)c\cos}{}$$
$$= \frac{1}{8}a^3(5A+2B)cx - \frac{a^3(5A+2B)c\cos^3(e+fx)}{12f}$$

Mathematica [**A**] time = 0.85, size = 95, normalized size = 0.68

$$\frac{a^3c(15(-(A+2B)\sin(4(e+fx))+4fx(5A+2B)+8A\sin(2(e+fx)))-60(4A+3B)\cos(e+fx)-10(8A+5B}{480f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),
x]
```

```
[Out] (a^3*c*(-60*(4*A + 3*B)*Cos[e + f*x] - 10*(8*A + 5*B)*Cos[3*(e + f*x)] + 6*
B*Cos[5*(e + f*x)] + 15*(4*(5*A + 2*B)*f*x + 8*A*Sin[2*(e + f*x)] - (A + 2*
B)*Sin[4*(e + f*x)])))/(480*f)
```

fricas [**A**] time = 0.43, size = 100, normalized size = 0.71

$$\frac{24\,Ba^3c\cos\left(fx+e\right)^5 - 80\,(A+B)a^3c\cos\left(fx+e\right)^3 + 15\,(5\,A+2\,B)a^3cfx - 15\left(2\,(A+2\,B)a^3c\cos\left(fx+e\right)^3 - (\right.}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/120*(24*B*a^3*c*cos(f*x + e)^5 - 80*(A + B)*a^3*c*cos(f*x + e)^3 + 15*(5*
A + 2*B)*a^3*c*f*x - 15*(2*(A + 2*B)*a^3*c*cos(f*x + e)^3 - (5*A + 2*B)*a^3
*c*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.21, size = 145, normalized size = 1.04

$$\frac{Ba^3c\cos\left(5fx+5e\right)}{80f}+\frac{Aa^3c\sin\left(2fx+2e\right)}{4f}+\frac{1}{8}\left(5Aa^3c+2Ba^3c\right)x-\frac{\left(8Aa^3c+5Ba^3c\right)\cos\left(3fx+3e\right)}{48f}-\frac{\left(4Aa^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] 1/80*B*a^3*c*cos(5*f*x + 5*e)/f + 1/4*A*a^3*c*sin(2*f*x + 2*e)/f + 1/8*(5*A
*a^3*c + 2*B*a^3*c)*x - 1/48*(8*A*a^3*c + 5*B*a^3*c)*cos(3*f*x + 3*e)/f - 1
/8*(4*A*a^3*c + 3*B*a^3*c)*cos(f*x + e)/f - 1/32*(A*a^3*c + 2*B*a^3*c)*sin(
4*f*x + 4*e)/f
```

maple [A] time = 0.48, size = 208, normalized size = 1.49

$$\frac{-a^3Ac\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+\frac{2a^3Ac\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}+\frac{Ba^3c\left(\frac{8}{3}+\sin^4\left(fx+e\right)+\frac{4\left(\sin^2\left(fx+e\right)\right)}{3}\right)\cos}{5}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] 1/f*(-a^3*A*c*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)
+2/3*a^3*A*c*(2+sin(f*x+e)^2)*cos(f*x+e)+1/5*B*a^3*c*(8/3+sin(f*x+e)^4+4/3*
sin(f*x+e)^2)*cos(f*x+e)-2*B*a^3*c*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(
f*x+e)+3/8*f*x+3/8*e)-2*a^3*A*c*cos(f*x+e)+2*B*a^3*c*(-1/2*sin(f*x+e)*cos(f
*x+e)+1/2*f*x+1/2*e)+a^3*A*c*(f*x+e)-B*a^3*c*cos(f*x+e))
```

maxima [A] time = 0.52, size = 200, normalized size = 1.43

$$-\frac{320\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)Aa^3c+15\left(12fx+12e+\sin\left(4fx+4e\right)-8\sin\left(2fx+2e\right)\right)Aa^3c-48}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -1/480*(320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c + 15*(12*f*x + 12*e +
 sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c - 480*(f*x + e)*A*a^3*c - 3
```

```
2*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c + 30*(12
*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c - 240*(2*f*x +
 2*e - sin(2*f*x + 2*e))*B*a^3*c + 960*A*a^3*c*cos(f*x + e) + 480*B*a^3*c*c
os(f*x + e))/f
```

mupad [B] time = 13.66, size = 390, normalized size = 2.79

$$\frac{a^3\,c\,\mathrm{atan}\left(\frac{a^3\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(5\,A+2\,B)}{4\,\left(\frac{5\,A\,a^3\,c}{4}+\frac{B\,a^3\,c}{2}\right)}\right)\,(5\,A+2\,B)}{4\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8\,\left(4\,A\,a^3\,c+2\,B\,a^3\,c\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{3\,A\,a^3\,c}{4}-\frac{B\,a^3\,c}{2}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x)),x)
```

```
[Out] (a^3*c*atan((a^3*c*tan(e/2 + (f*x)/2)*(5*A + 2*B))/(4*((5*A*a^3*c)/4 + (B*a
^3*c)/2)))*(5*A + 2*B))/(4*f) - (tan(e/2 + (f*x)/2)^8*(4*A*a^3*c + 2*B*a^3*
c) - tan(e/2 + (f*x)/2)*((3*A*a^3*c)/4 - (B*a^3*c)/2) - tan(e/2 + (f*x)/2)^
3*((7*A*a^3*c)/2 + 3*B*a^3*c) + tan(e/2 + (f*x)/2)^7*((7*A*a^3*c)/2 + 3*B*a
^3*c) + tan(e/2 + (f*x)/2)^9*((3*A*a^3*c)/4 - (B*a^3*c)/2) + tan(e/2 + (f*x
)/2)^6*(8*A*a^3*c + 8*B*a^3*c) + tan(e/2 + (f*x)/2)^2*((8*A*a^3*c)/3 + (8*B
*a^3*c)/3) + tan(e/2 + (f*x)/2)^4*((16*A*a^3*c)/3 + (4*B*a^3*c)/3) + (4*A*a
^3*c)/3 + (14*B*a^3*c)/15)/(f*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/
2)^4 + 10*tan(e/2 + (f*x)/2)^6 + 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2
)^10 + 1)) - (a^3*c*(5*A + 2*B)*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2))/(4*f)
```

sympy [A] time = 4.21, size = 486, normalized size = 3.47

$$\begin{cases} -\frac{3Aa^3cx\sin^4\left(e+fx\right)}{8}-\frac{3Aa^3cx\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4}-\frac{3Aa^3cx\cos^4\left(e+fx\right)}{8}+Aa^3cx+\frac{5Aa^3c\sin^3\left(e+fx\right)\cos\left(e+fx\right)}{8f}+\frac{2Aa^3c\sin^2\left(e+\right.}{f} \\ x\left(A+B\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right)^3\left(-c\sin\left(e\right)+c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-3*A*a**3*c*x*sin(e + f*x)**4/8 - 3*A*a**3*c*x*sin(e + f*x)**2*c
os(e + f*x)**2/4 - 3*A*a**3*c*x*cos(e + f*x)**4/8 + A*a**3*c*x + 5*A*a**3*c
*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 2*A*a**3*c*sin(e + f*x)**2*cos(e + f*
x)/f + 3*A*a**3*c*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 4*A*a**3*c*cos(e + f
*x)**3/(3*f) - 2*A*a**3*c*cos(e + f*x)/f - 3*B*a**3*c*x*sin(e + f*x)**4/4 -
 3*B*a**3*c*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + B*a**3*c*x*sin(e + f*x)**
```

```
2 - 3*B*a**3*c*x*cos(e + f*x)**4/4 + B*a**3*c*x*cos(e + f*x)**2 + B*a**3*c*
sin(e + f*x)**4*cos(e + f*x)/f + 5*B*a**3*c*sin(e + f*x)**3*cos(e + f*x)/(4
*f) + 4*B*a**3*c*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 3*B*a**3*c*sin(e +
 f*x)*cos(e + f*x)**3/(4*f) - B*a**3*c*sin(e + f*x)*cos(e + f*x)/f + 8*B*a*
*3*c*cos(e + f*x)**5/(15*f) - B*a**3*c*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B
*sin(e))*(a*sin(e) + a)**3*(-c*sin(e) + c), True))
```

3.44 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=156

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{f(c-c\sin(e+fx))^4}+\frac{2a^3c^3(3A+4B)\cos^5(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{5a^3(3A+4B)\cos^3(e+fx)}{3cf}-\frac{5a^3(3A+4B)\sin(e+fx)}{2cf}$$

```
[Out] -5/2*a^3*(3*A+4*B)*x/c+5/3*a^3*(3*A+4*B)*cos(f*x+e)^3/c/f-5/2*a^3*(3*A+4*B)
*cos(f*x+e)*sin(f*x+e)/c/f+a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^4+
2*a^3*(3*A+4*B)*c^3*cos(f*x+e)^5/f/(c^2-c^2*sin(f*x+e))^2
```

Rubi [A] time = 0.31, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2967, 2859, 2680, 2682, 2635, 8}

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{f(c-c\sin(e+fx))^4}+\frac{2a^3c^3(3A+4B)\cos^5(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{5a^3(3A+4B)\cos^3(e+fx)}{3cf}-\frac{5a^3(3A+4B)\sin(e+fx)}{2cf}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),x]
```

```
[Out] (-5*a^3*(3*A + 4*B)*x)/(2*c) + (5*a^3*(3*A + 4*B)*Cos[e + f*x]^3)/(3*c*f) -
 (5*a^3*(3*A + 4*B)*Cos[e + f*x]*Sin[e + f*x])/(2*c*f) + (a^3*(A + B)*c^3*C
os[e + f*x]^7)/(f*(c - c*Sin[e + f*x])^4) + (2*a^3*(3*A + 4*B)*c^3*Cos[e +
f*x]^5)/(f*(c^2 - c^2*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
```

```
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{f(c-c\sin(e+fx))^4} - \left(a^3(3A+4B)c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{f(c-c\sin(e+fx))^4} + \frac{2a^3(3A+4B)c\cos^5(e+fx)}{f(c-c\sin(e+fx))^2} - \left(5a^3(\right.$$

$$= \frac{5a^3(3A+4B)\cos^3(e+fx)}{3cf} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{f(c-c\sin(e+fx))^4} + \frac{2a^3(3}{f(}$$

$$= \frac{5a^3(3A+4B)\cos^3(e+fx)}{3cf} - \frac{5a^3(3A+4B)\cos(e+fx)\sin(e+}{2cf}$$

$$= -\frac{5a^3(3A+4B)x}{2c} + \frac{5a^3(3A+4B)\cos^3(e+fx)}{3cf} - \frac{5a^3(3A+4B)}{}$$

Mathematica [A] time = 1.42, size = 223, normalized size = 1.43

$$\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)(30(3A+4B)(e+fx)-3(A+4B)\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(Cos[(e + f
*x)/2]*(30*(3*A + 4*B)*(e + f*x) - 3*(16*A + 31*B)*Cos[e + f*x] + B*Cos[3*(
e + f*x)] - 3*(A + 4*B)*Sin[2*(e + f*x)]) - Sin[(e + f*x)/2]*(24*B*(8 + 5*e
 + 5*f*x) + 6*A*(32 + 15*e + 15*f*x) - 3*(16*A + 31*B)*Cos[e + f*x] + B*Cos
[3*(e + f*x)] - 3*(A + 4*B)*Sin[2*(e + f*x)])))/(12*c*f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x]))
```

fricas [A] time = 0.44, size = 218, normalized size = 1.40

$$-\frac{2\,Ba^3\cos\left(fx+e\right)^4-(3\,A+10\,B)a^3\cos\left(fx+e\right)^3+15\,(3\,A+4\,B)a^3fx-24\,(A+2\,B)a^3\cos\left(fx+e\right)^2-48\,(A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] -1/6*(2*B*a^3*cos(f*x + e)^4 - (3*A + 10*B)*a^3*cos(f*x + e)^3 + 15*(3*A +
4*B)*a^3*f*x - 24*(A + 2*B)*a^3*cos(f*x + e)^2 - 48*(A + B)*a^3 + 3*(5*(3*A
 + 4*B)*a^3*f*x - (23*A + 28*B)*a^3)*cos(f*x + e) - (2*B*a^3*cos(f*x + e)^3
 + 15*(3*A + 4*B)*a^3*f*x + 3*(A + 4*B)*a^3*cos(f*x + e)^2 - 3*(7*A + 12*B)
*a^3*cos(f*x + e) + 48*(A + B)*a^3)*sin(f*x + e))/(c*f*cos(f*x + e) - c*f*s
in(f*x + e) + c*f)
```

giac [A] time = 0.22, size = 234, normalized size = 1.50

$$-\frac{\frac{15\left(3\,Aa^3+4\,Ba^3\right)\left(fx+e\right)}{c}+\frac{96\left(Aa^3+Ba^3\right)}{c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}+\frac{2\left(3\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+12\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-24\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-42\,Ba^3\tan\left(\frac{1}{2}\,fx\cdots\right.}{\cdots}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/6*(15*(3*A*a^3 + 4*B*a^3)*(f*x + e)/c + 96*(A*a^3 + B*a^3)/(c*(tan(1/2*f
*x + 1/2*e) - 1)) + 2*(3*A*a^3*tan(1/2*f*x + 1/2*e)^5 + 12*B*a^3*tan(1/2*f*
x + 1/2*e)^5 - 24*A*a^3*tan(1/2*f*x + 1/2*e)^4 - 42*B*a^3*tan(1/2*f*x + 1/2
*e)^4 - 48*A*a^3*tan(1/2*f*x + 1/2*e)^2 - 96*B*a^3*tan(1/2*f*x + 1/2*e)^2 -
 3*A*a^3*tan(1/2*f*x + 1/2*e) - 12*B*a^3*tan(1/2*f*x + 1/2*e) - 24*A*a^3 -
46*B*a^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^3*c))/f
```

maple [B] time = 0.44, size = 449, normalized size = 2.88

$$-\frac{16a^3A}{fc\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{16a^3B}{fc\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{a^3\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)A}{fc\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}-\frac{4a^3\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)B}{fc\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}+\frac{8a^3\left(\tan^4\left(\cdots\right.\right.}{fc\left(1+\tan\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] -16/f*a^3/c/(tan(1/2*f*x+1/2*e)-1)*A-16/f*a^3/c/(tan(1/2*f*x+1/2*e)-1)*B-1/
f*a^3/c/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*A-4/f*a^3/c/(1+tan(
1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*B+8/f*a^3/c/(1+tan(1/2*f*x+1/2*e)^
2)^3*tan(1/2*f*x+1/2*e)^4*A+14/f*a^3/c/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f
*x+1/2*e)^4*B+16/f*a^3/c/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2*A+
32/f*a^3/c/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2*B+1/f*a^3/c/(1+t
```

```
an(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*A+4/f*a^3/c/(1+tan(1/2*f*x+1/2*e)
^2)^3*tan(1/2*f*x+1/2*e)*B+8/f*a^3/c/(1+tan(1/2*f*x+1/2*e)^2)^3*A+46/3/f*a^
3/c/(1+tan(1/2*f*x+1/2*e)^2)^3*B-15/f*a^3/c*arctan(tan(1/2*f*x+1/2*e))*A-20
/f*a^3/c*arctan(tan(1/2*f*x+1/2*e))*B
```

maxima [B] time = 0.47, size = 1139, normalized size = 7.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -1/3*(B*a^3*((7*sin(f*x + e)/(cos(f*x + e) + 1) - 39*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 24*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 9*sin(f*x +
e)^6/(cos(f*x + e) + 1)^6 - 16)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + 3*
c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 3*c*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 3*c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3*c*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5 + c*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c*sin(f*x + e)^7/(c
os(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + 18*A*a
^3*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^2/(cos(f*x + e) + 1)^2
- 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(co
s(f*x + e) + 1))/c) + 18*B*a^3*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + 3*A*a^3*((sin(f*x + e)/(c
os(f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3 - 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4)/(c - c*s
in(f*x + e)/(cos(f*x + e) + 1) + 2*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 -
2*c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + c*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 - c*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(
f*x + e) + 1))/c) + 9*B*a^3*((sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 - 4)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1)
 + 2*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*c*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c*sin(f*x + e)^5/(cos(f
*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + 18*A*a^3*(
arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c - 1/(c - c*sin(f*x + e)/(cos(f*x
+ e) + 1))) + 6*B*a^3*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c - 1/(c - c
*sin(f*x + e)/(cos(f*x + e) + 1))) - 6*A*a^3/(c - c*sin(f*x + e)/(cos(f*x +
 e) + 1)))/f
```

mupad [B] time = 14.12, size = 323, normalized size = 2.07

$$\frac{24\,A\,a^3-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(7\,A\,a^3+\frac{34\,B\,a^3}{3}\right)+\frac{94\,B\,a^3}{3}-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(9\,A\,a^3+18\,B\,a^3\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,\left(17\,A\,a^3\right.}{f\,\left(-c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+3\,c\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x)),x)
```

```
[Out] (24*A*a^3 - tan(e/2 + (f*x)/2)*(7*A*a^3 + (34*B*a^3)/3) + (94*B*a^3)/3 - ta
n(e/2 + (f*x)/2)^5*(9*A*a^3 + 18*B*a^3) + tan(e/2 + (f*x)/2)^6*(17*A*a^3 +
20*B*a^3) - tan(e/2 + (f*x)/2)^3*(16*A*a^3 + 32*B*a^3) + tan(e/2 + (f*x)/2)
^4*(56*A*a^3 + 62*B*a^3) + tan(e/2 + (f*x)/2)^2*(63*A*a^3 + 76*B*a^3))/(f*(
c - c*tan(e/2 + (f*x)/2) + 3*c*tan(e/2 + (f*x)/2)^2 - 3*c*tan(e/2 + (f*x)/2
)^3 + 3*c*tan(e/2 + (f*x)/2)^4 - 3*c*tan(e/2 + (f*x)/2)^5 + c*tan(e/2 + (f*
x)/2)^6 - c*tan(e/2 + (f*x)/2)^7)) - (5*a^3*atan((5*a^3*tan(e/2 + (f*x)/2)*
(3*A + 4*B))/(15*A*a^3 + 20*B*a^3))*(3*A + 4*B))/(c*f)
```

sympy [A] time = 15.99, size = 4255, normalized size = 27.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-45*A*a**3*f*x*tan(e/2 + f*x/2)**7/(6*c*f*tan(e/2 + f*x/2)**7 -
6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f
*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*
tan(e/2 + f*x/2) - 6*c*f) + 45*A*a**3*f*x*tan(e/2 + f*x/2)**6/(6*c*f*tan(e/
2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18
*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*
x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 135*A*a**3*f*x*tan(e/2 + f*x/2)
**5/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2
 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18
*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 135*A*a**3*f*x
*tan(e/2 + f*x/2)**4/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6
 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2
 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f)
 - 135*A*a**3*f*x*tan(e/2 + f*x/2)**3/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*ta
n(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4
 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2
+ f*x/2) - 6*c*f) + 135*A*a**3*f*x*tan(e/2 + f*x/2)**2/(6*c*f*tan(e/2 + f*x
```

/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*ta
n(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2
 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 45*A*a**3*f*x*tan(e/2 + f*x/2)/(6*c*f*
tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**
5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/
2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 45*A*a**3*f*x/(6*c*f*tan(
e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 -
18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 +
f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 102*A*a**3*tan(e/2 + f*x/2)**
6/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 +
 f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c
*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 54*A*a**3*tan(e/
2 + f*x/2)**5/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c
*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/
2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 336*
A*a**3*tan(e/2 + f*x/2)**4/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x
/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*t
an(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) -
6*c*f) + 96*A*a**3*tan(e/2 + f*x/2)**3/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*t
an(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**
4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2
 + f*x/2) - 6*c*f) - 378*A*a**3*tan(e/2 + f*x/2)**2/(6*c*f*tan(e/2 + f*x/2)
7 - 6*c*f*tan(e/2 + f*x/2)6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e
/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 +
6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 42*A*a**3*tan(e/2 + f*x/2)/(6*c*f*tan(e/2
 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*
c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x
/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 144*A*a**3/(6*c*f*tan(e/2 + f*x/
2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan
(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2
+ 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 60*B*a**3*f*x*tan(e/2 + f*x/2)**7/(6*c*
f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)
5 - 18*c*f*tan(e/2 + f*x/2)4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(
e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 60*B*a**3*f*x*tan(e/2 +
 f*x/2)**6/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*
tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)*
*3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 180*B*a
3*f*x*tan(e/2 + f*x/2)5/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*
x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*
tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) -
 6*c*f) + 180*B*a**3*f*x*tan(e/2 + f*x/2)**4/(6*c*f*tan(e/2 + f*x/2)**7 - 6
*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*
x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*t
an(e/2 + f*x/2) - 6*c*f) - 180*B*a**3*f*x*tan(e/2 + f*x/2)**3/(6*c*f*tan(e/
2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18

```
*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*
x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 180*B*a**3*f*x*tan(e/2 + f*x/2)
**2/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2
 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18
*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 60*B*a**3*f*x*
tan(e/2 + f*x/2)/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 1
8*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f
*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 6
0*B*a**3*f*x/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*
f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2
)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 120*B
*a**3*tan(e/2 + f*x/2)**6/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/
2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*ta
n(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6
*c*f) + 108*B*a**3*tan(e/2 + f*x/2)**5/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*t
an(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**
4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2
 + f*x/2) - 6*c*f) - 372*B*a**3*tan(e/2 + f*x/2)**4/(6*c*f*tan(e/2 + f*x/2)
**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e
/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 +
6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 192*B*a**3*tan(e/2 + f*x/2)**3/(6*c*f*tan
(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 -
 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 +
 f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 456*B*a**3*tan(e/2 + f*x/2)*
*2/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2
+ f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*
c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 68*B*a**3*tan(e
/2 + f*x/2)/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f
*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)
**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 188*B*
a**3/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/
2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 1
8*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f), Ne(f, 0)), (x*
(A + B*sin(e))*(a*sin(e) + a)**3/(-c*sin(e) + c), True))
```

3.45 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=163

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}-\frac{5a^3(2A+5B)\cos(e+fx)}{2c^2f}-\frac{5a^3(2A+5B)\cos^3(e+fx)}{6f\left(c^2-c^2\sin(e+fx)\right)}+\frac{5a^3x(2A+5B)}{2c^2}-\frac{2a^3c(2A+5}{3f(c-c}$$

```
[Out] 5/2*a^3*(2*A+5*B)*x/c^2-5/2*a^3*(2*A+5*B)*cos(f*x+e)/c^2/f+1/3*a^3*(A+B)*c^
3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^5-2/3*a^3*(2*A+5*B)*c*cos(f*x+e)^5/f/(c-c
*sin(f*x+e))^3-5/6*a^3*(2*A+5*B)*cos(f*x+e)^3/f/(c^2-c^2*sin(f*x+e))
```

Rubi [A] time = 0.35, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2967, 2859, 2680, 2679, 2682, 8}

$$-\frac{5a^3(2A+5B)\cos(e+fx)}{2c^2f}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}-\frac{5a^3(2A+5B)\cos^3(e+fx)}{6f\left(c^2-c^2\sin(e+fx)\right)}+\frac{5a^3x(2A+5B)}{2c^2}-\frac{2a^3c(2A+}{3f(c-}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (5*a^3*(2*A + 5*B)*x)/(2*c^2) - (5*a^3*(2*A + 5*B)*Cos[e + f*x])/(2*c^2*f)
+ (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(3*f*(c - c*Sin[e + f*x])^5) - (2*a^3*(2
*A + 5*B)*c*Cos[e + f*x]^5)/(3*f*(c - c*Sin[e + f*x])^3) - (5*a^3*(2*A + 5*
B)*Cos[e + f*x]^3)/(6*f*(c^2 - c^2*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
```

```
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5} - \frac{1}{3}\left(a^3(2A+5B)c^2\right)\int \frac{\cos^6(e+f}{(c-c\sin(e+}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5} - \frac{2a^3(2A+5B)c\cos^5(e+fx)}{3f(c-c\sin(e+fx))^3} + \frac{1}{3}\left(5a\right.$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5} - \frac{2a^3(2A+5B)c\cos^5(e+fx)}{3f(c-c\sin(e+fx))^3} - \frac{5a^3(2}{6f}$$

$$= -\frac{5a^3(2A+5B)\cos(e+fx)}{2c^2f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5} - \frac{2a^3(2}{3f}$$

$$= \frac{5a^3(2A+5B)x}{2c^2} - \frac{5a^3(2A+5B)\cos(e+fx)}{2c^2f} + \frac{a^3(A+B)c^3\cos^7}{3f(c-c\sin(e+}$$

Mathematica [A] time = 0.88, size = 280, normalized size = 1.72

$$\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(64(A+B)\sin\left(\frac{1}{2}(e+fx)\right)+30(2A+5B)(e+fx)\left(\cos\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^2,x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(32*(A + B)
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + 30*(2*A + 5*B)*(e + f*x)*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^3 - 12*(A + 5*B)*Cos[e + f*x]*(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2])^3 + 64*(A + B)*Sin[(e + f*x)/2] - 32*(7*A + 13*B)*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] - 3*B*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)]))/(12*f*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^6*(c - c*Sin[e + f*x])^2)
```

fricas [A] time = 0.45, size = 286, normalized size = 1.75

$$\frac{3\,Ba^3\cos\left(fx+e\right)^4 - 6\,(A+4\,B)a^3\cos\left(fx+e\right)^3 - 30\,(2\,A+5\,B)a^3fx - 16\,(A+B)a^3 + \left(15\,(2\,A+5\,B)a^3fx + (\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] 1/6*(3*B*a^3*cos(f*x + e)^4 - 6*(A + 4*B)*a^3*cos(f*x + e)^3 - 30*(2*A + 5*
B)*a^3*f*x - 16*(A + B)*a^3 + (15*(2*A + 5*B)*a^3*f*x + (62*A + 131*B)*a^3)
*cos(f*x + e)^2 - (15*(2*A + 5*B)*a^3*f*x - 2*(26*A + 71*B)*a^3)*cos(f*x +
e) - (3*B*a^3*cos(f*x + e)^3 - 30*(2*A + 5*B)*a^3*f*x + 3*(2*A + 9*B)*a^3*c
os(f*x + e)^2 + 16*(A + B)*a^3 - (15*(2*A + 5*B)*a^3*f*x - 2*(34*A + 79*B)*
a^3)*cos(f*x + e))*sin(f*x + e))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e)
 - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

giac [A] time = 0.18, size = 233, normalized size = 1.43

$$\frac{\frac{15\left(2\,Aa^3+5\,Ba^3\right)\left(fx+e\right)}{c^2}+\frac{6\left(Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-10\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,Aa^3-10\,Ba^3\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2c^2}+\frac{16\left(3\,A\cdots\right.}{\cdots}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/6*(15*(2*A*a^3 + 5*B*a^3)*(f*x + e)/c^2 + 6*(B*a^3*tan(1/2*f*x + 1/2*e)^3
 - 2*A*a^3*tan(1/2*f*x + 1/2*e)^2 - 10*B*a^3*tan(1/2*f*x + 1/2*e)^2 - B*a^3
*tan(1/2*f*x + 1/2*e) - 2*A*a^3 - 10*B*a^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2
*c^2) + 16*(3*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 9*B*a^3*tan(1/2*f*x + 1/2*e)^2
 - 12*A*a^3*tan(1/2*f*x + 1/2*e) - 24*B*a^3*tan(1/2*f*x + 1/2*e) + 5*A*a^3
+ 11*B*a^3)/(c^2*(tan(1/2*f*x + 1/2*e) - 1)^3))/f
```

maple [B] time = 0.47, size = 399, normalized size = 2.45

$$\frac{8a^3A}{c^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{24a^3B}{c^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{32a^3A}{3c^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{32a^3B}{3c^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{1\cdots}{c^2f\left(\tan\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

```
[Out] 8*a^3/c^2/f/(tan(1/2*f*x+1/2*e)-1)*A+24*a^3/c^2/f/(tan(1/2*f*x+1/2*e)-1)*B-
32/3*a^3/c^2/f/(tan(1/2*f*x+1/2*e)-1)^3*A-32/3*a^3/c^2/f/(tan(1/2*f*x+1/2*e
)-1)^3*B-16*a^3/c^2/f/(tan(1/2*f*x+1/2*e)-1)^2*A-16*a^3/c^2/f/(tan(1/2*f*x+
```

```
1/2*e)-1)^2*B+a^3/c^2/f/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^3*B-2
*a^3/c^2/f/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*A-10*a^3/c^2/f/(
1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*B-a^3/c^2/f/(1+tan(1/2*f*x+1
/2*e)^2)^2*B*tan(1/2*f*x+1/2*e)-2*a^3/c^2/f/(1+tan(1/2*f*x+1/2*e)^2)^2*A-10
*a^3/c^2/f/(1+tan(1/2*f*x+1/2*e)^2)^2*B+25*a^3/c^2/f*arctan(tan(1/2*f*x+1/2
*e))*B+10*a^3/c^2/f*arctan(tan(1/2*f*x+1/2*e))*A
```

maxima [B] time = 0.61, size = 1386, normalized size = 8.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/3*(B*a^3*((75*sin(f*x + e)/(cos(f*x + e) + 1) - 97*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 98*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 21*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 - 32)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e)
+ 1) + 5*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 7*c^2*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 7*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 5*c^2*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + 3*c^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
- c^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f*
x + e) + 1))/c^2) + 4*A*a^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) - 11*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*
sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 5)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x
 + e) + 1) + 4*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^2*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3 + 3*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^2*s
in(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) +
 1))/c^2) + 12*B*a^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) - 11*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 - 5)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) +
 1) + 4*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^2*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 3*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^2*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^
2) + 6*A*a^3*((9*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 - 4)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)
 + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) + 6*B*a^3*((9*sin(f*x + e
)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4)/(c^2 - 3*
c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(co
s(f*x + e) + 1))/c^2) - 2*A*a^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e
) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(co
```

```
s(f*x + e) + 1)^3) + 6*A*a^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(c^2 -
 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 2*B*a^3*(3*sin(f*x + e
)/(cos(f*x + e) + 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*
c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3))/f
```

mupad [B] time = 14.00, size = 341, normalized size = 2.09

$$\frac{5\,a^3\,\mathrm{atan}\left(\frac{5\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(2\,A+5\,B)}{10\,A\,a^3+25\,B\,a^3}\right)\,(2\,A+5\,B)}{c^2\,f}-\frac{\frac{46\,A\,a^3}{3}-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(38\,A\,a^3+93\,B\,a^3\right)+\frac{118\,B\,a^3}{3}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right. \cdots}{f\,\left(-c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^2,x)
```

```
[Out] (5*a^3*atan((5*a^3*tan(e/2 + (f*x)/2)*(2*A + 5*B))/(10*A*a^3 + 25*B*a^3))*(
2*A + 5*B))/(c^2*f) - ((46*A*a^3)/3 - tan(e/2 + (f*x)/2)*(38*A*a^3 + 93*B*a
^3) + (118*B*a^3)/3 + tan(e/2 + (f*x)/2)^6*(8*A*a^3 + 25*B*a^3) - tan(e/2 +
 (f*x)/2)^5*(34*A*a^3 + 77*B*a^3) - tan(e/2 + (f*x)/2)^3*(72*A*a^3 + 166*B*
a^3) + tan(e/2 + (f*x)/2)^4*((106*A*a^3)/3 + (328*B*a^3)/3) + tan(e/2 + (f*
x)/2)^2*((128*A*a^3)/3 + (359*B*a^3)/3))/(f*(5*c^2*tan(e/2 + (f*x)/2)^2 - 7
*c^2*tan(e/2 + (f*x)/2)^3 + 7*c^2*tan(e/2 + (f*x)/2)^4 - 5*c^2*tan(e/2 + (f
*x)/2)^5 + 3*c^2*tan(e/2 + (f*x)/2)^6 - c^2*tan(e/2 + (f*x)/2)^7 + c^2 - 3*
c^2*tan(e/2 + (f*x)/2)))
```

sympy [A] time = 29.89, size = 4665, normalized size = 28.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((30*A*a**3*f*x*tan(e/2 + f*x/2)**7/(6*c**2*f*tan(e/2 + f*x/2)**7
- 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f
*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 +
f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 90*A*a**3*f*x*tan(e/2
+ f*x/2)**6/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 +
 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*
tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f
*x/2) - 6*c**2*f) + 150*A*a**3*f*x*tan(e/2 + f*x/2)**5/(6*c**2*f*tan(e/2 +
f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 -
```

```
 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*
tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 210*A*a**3*f
*x*tan(e/2 + f*x/2)**4/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 +
f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 +
 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*
tan(e/2 + f*x/2) - 6*c**2*f) + 210*A*a**3*f*x*tan(e/2 + f*x/2)**3/(6*c**2*f
*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 +
f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 -
 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 1
50*A*a**3*f*x*tan(e/2 + f*x/2)**2/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f
*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 +
f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 +
 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 90*A*a**3*f*x*tan(e/2 + f*x/2)/(6
*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan
(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/
2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2
*f) - 30*A*a**3*f*x/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x
/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42
*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan
(e/2 + f*x/2) - 6*c**2*f) + 48*A*a**3*tan(e/2 + f*x/2)**6/(6*c**2*f*tan(e/2
 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**
5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2
*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 204*A*a**
3*tan(e/2 + f*x/2)**5/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f
*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 +
42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*t
an(e/2 + f*x/2) - 6*c**2*f) + 212*A*a**3*tan(e/2 + f*x/2)**4/(6*c**2*f*tan(
e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2
)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c
**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 432*A*
a**3*tan(e/2 + f*x/2)**3/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2
+ f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4
 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*
f*tan(e/2 + f*x/2) - 6*c**2*f) + 256*A*a**3*tan(e/2 + f*x/2)**2/(6*c**2*f*t
an(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*
x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 3
0*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 228
*A*a**3*tan(e/2 + f*x/2)/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2
+ f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4
 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*
f*tan(e/2 + f*x/2) - 6*c**2*f) + 92*A*a**3/(6*c**2*f*tan(e/2 + f*x/2)**7 -
18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*t
an(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*
x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 75*B*a**3*f*x*tan(e/2 +
f*x/2)**7/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 3
```

```
0*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*ta
n(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x
/2) - 6*c**2*f) - 225*B*a**3*f*x*tan(e/2 + f*x/2)**6/(6*c**2*f*tan(e/2 + f*
x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 4
2*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*ta
n(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 375*B*a**3*f*x
*tan(e/2 + f*x/2)**5/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*
x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 4
2*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*ta
n(e/2 + f*x/2) - 6*c**2*f) - 525*B*a**3*f*x*tan(e/2 + f*x/2)**4/(6*c**2*f*t
an(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*
x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 3
0*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 525
*B*a**3*f*x*tan(e/2 + f*x/2)**3/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*t
an(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*
x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 1
8*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 375*B*a**3*f*x*tan(e/2 + f*x/2)**2/
(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*t
an(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*
x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c*
*2*f) + 225*B*a**3*f*x*tan(e/2 + f*x/2)/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*
c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(
e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2
)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 75*B*a**3*f*x/(6*c**2*f*tan
(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/
2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*
c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 150*B
*a**3*tan(e/2 + f*x/2)**6/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2
 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**
4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2
*f*tan(e/2 + f*x/2) - 6*c**2*f) - 462*B*a**3*tan(e/2 + f*x/2)**5/(6*c**2*f*
tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f
*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 -
30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 65
6*B*a**3*tan(e/2 + f*x/2)**4/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(
e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2
)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c
**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 996*B*a**3*tan(e/2 + f*x/2)**3/(6*c**2
*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2
+ f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3
 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) +
 718*B*a**3*tan(e/2 + f*x/2)**2/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*t
an(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*
x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 1
8*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 558*B*a**3*tan(e/2 + f*x/2)/(6*c**2
```

```
*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2
+ f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3
 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) +
 236*B*a**3/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 +
 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*
tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f
*x/2) - 6*c**2*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**3/(-c*sin(e
) + c)**2, True))
```

3.46 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=153

$$\frac{a^3(A+6B)\cos(e+fx)}{c^3f}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}+\frac{2a^3c^3(A+6B)\cos^3(e+fx)}{3f\left(c^3-c^3\sin(e+fx)\right)^2}-\frac{a^3x(A+6B)}{c^3}-\frac{2a^3c(A+6B)}{15f(c-c\sin}$$

```
[Out] -a^3*(A+6*B)*x/c^3+a^3*(A+6*B)*cos(f*x+e)/c^3/f+1/5*a^3*(A+B)*c^3*cos(f*x+e
)^7/f/(c-c*sin(f*x+e))^6-2/15*a^3*(A+6*B)*c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))
^4+2/3*a^3*(A+6*B)*c^3*cos(f*x+e)^3/f/(c^3-c^3*sin(f*x+e))^2
```

Rubi [A] time = 0.34, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2859, 2680, 2682, 8}

$$\frac{a^3(A+6B)\cos(e+fx)}{c^3f}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}+\frac{2a^3c^3(A+6B)\cos^3(e+fx)}{3f\left(c^3-c^3\sin(e+fx)\right)^2}-\frac{a^3x(A+6B)}{c^3}-\frac{2a^3c(A+6B)}{15f(c-c\sin}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] -((a^3*(A + 6*B)*x)/c^3) + (a^3*(A + 6*B)*Cos[e + f*x])/(c^3*f) + (a^3*(A +
 B)*c^3*Cos[e + f*x]^7)/(5*f*(c - c*Sin[e + f*x])^6) - (2*a^3*(A + 6*B)*c*C
os[e + f*x]^5)/(15*f*(c - c*Sin[e + f*x])^4) + (2*a^3*(A + 6*B)*c^3*Cos[e +
 f*x]^3)/(3*f*(c^3 - c^3*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
```

```
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6} - \frac{1}{5}\left(a^3(A+6B)c^2\right)\int \frac{\cos^6(e+fx}{(c-c\sin(e+f}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6} - \frac{2a^3(A+6B)c\cos^5(e+fx)}{15f(c-c\sin(e+fx))^4} + \frac{1}{3}\left(a^3(\right.$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6} - \frac{2a^3(A+6B)c\cos^5(e+fx)}{15f(c-c\sin(e+fx))^4} + \frac{2a^3(A}{3cf(}$$

$$= \frac{a^3(A+6B)\cos(e+fx)}{c^3f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6} - \frac{2a^3(A+}{15f(c-}$$

$$= -\frac{a^3(A+6B)x}{c^3} + \frac{a^3(A+6B)\cos(e+fx)}{c^3f} + \frac{a^3(A+B)c^3\cos^7(e-}{5f(c-c\sin(e+fx}$$

Mathematica [B] time = 1.10, size = 316, normalized size = 2.07

$$\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(48(A+B)\sin\left(\frac{1}{2}(e+fx)\right)-15(A+6B)(e+fx)\left(\cos\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^3,x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(24*(A + B)*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]) - 4*(11*A + 21*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3
 - 15*(A + 6*B)*(e + f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 15*B*Co
s[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 48*(A + B)*Sin[(e + f*
x)/2] - 8*(11*A + 21*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*
x)/2] + 4*(23*A + 93*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*
x)/2])*(1 + Sin[e + f*x])^3)/(15*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*
(c - c*Sin[e + f*x])^3)
```

fricas [B] time = 0.45, size = 337, normalized size = 2.20

$$\frac{15\,Ba^3\cos\left(fx+e\right)^4+60\,(A+6\,B)a^3fx-24\,(A+B)a^3-\left(15\,(A+6\,B)a^3fx-(46\,A+231\,B)a^3\right)\cos\left(fx+e\right)^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/15*(15*B*a^3*cos(f*x + e)^4 + 60*(A + 6*B)*a^3*f*x - 24*(A + B)*a^3 - (15
*(A + 6*B)*a^3*f*x - (46*A + 231*B)*a^3)*cos(f*x + e)^3 - (45*(A + 6*B)*a^3
*f*x + 2*(A + 66*B)*a^3)*cos(f*x + e)^2 + 6*(5*(A + 6*B)*a^3*f*x - 2*(6*A +
 31*B)*a^3)*cos(f*x + e) - (15*B*a^3*cos(f*x + e)^3 + 60*(A + 6*B)*a^3*f*x
+ 24*(A + B)*a^3 - (15*(A + 6*B)*a^3*f*x + 2*(23*A + 108*B)*a^3)*cos(f*x +
e)^2 + 6*(5*(A + 6*B)*a^3*f*x - 2*(4*A + 29*B)*a^3)*cos(f*x + e))*sin(f*x +
 e))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e)
- 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x
 + e))
```

giac [A] time = 0.20, size = 226, normalized size = 1.48

$$\frac{\frac{30\,Ba^{3}}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)c^{3}}-\frac{15\,\left(Aa^{3}+6\,Ba^{3}\right)\left(fx+e\right)}{c^{3}}-\frac{4\left(15\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+45\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-30\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-210\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\right.}{}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/15*(30*B*a^3/((tan(1/2*f*x + 1/2*e)^2 + 1)*c^3) - 15*(A*a^3 + 6*B*a^3)*(f
*x + e)/c^3 - 4*(15*A*a^3*tan(1/2*f*x + 1/2*e)^4 + 45*B*a^3*tan(1/2*f*x + 1
/2*e)^4 - 30*A*a^3*tan(1/2*f*x + 1/2*e)^3 - 210*B*a^3*tan(1/2*f*x + 1/2*e)^
3 + 100*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 420*B*a^3*tan(1/2*f*x + 1/2*e)^2 - 5
0*A*a^3*tan(1/2*f*x + 1/2*e) - 270*B*a^3*tan(1/2*f*x + 1/2*e) + 13*A*a^3 +
63*B*a^3)/(c^3*(tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

maple [B] time = 0.51, size = 323, normalized size = 2.11

$$-\frac{4a^{3}A}{c^{3}f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{12a^{3}B}{c^{3}f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{8a^{3}A}{c^{3}f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}+\frac{8a^{3}B}{c^{3}f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{64}{5c^{3}f\left(\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

```
[Out] -4*a^3/c^3/f/(tan(1/2*f*x+1/2*e)-1)*A-12*a^3/c^3/f/(tan(1/2*f*x+1/2*e)-1)*B
-8*a^3/c^3/f/(tan(1/2*f*x+1/2*e)-1)^2*A+8*a^3/c^3/f/(tan(1/2*f*x+1/2*e)-1)^
2*B-64/5*a^3/c^3/f/(tan(1/2*f*x+1/2*e)-1)^5*A-64/5*a^3/c^3/f/(tan(1/2*f*x+1
/2*e)-1)^5*B-80/3*a^3/c^3/f/(tan(1/2*f*x+1/2*e)-1)^3*A-16*a^3/c^3/f/(tan(1/
2*f*x+1/2*e)-1)^3*B-32*a^3/c^3/f/(tan(1/2*f*x+1/2*e)-1)^4*A-32*a^3/c^3/f/(t
an(1/2*f*x+1/2*e)-1)^4*B+2*a^3/c^3/f*B/(1+tan(1/2*f*x+1/2*e)^2)-2*a^3/c^3/f
*arctan(tan(1/2*f*x+1/2*e))*A-12*a^3/c^3/f*arctan(tan(1/2*f*x+1/2*e))*B
```

maxima [B] time = 0.70, size = 1685, normalized size = 11.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] -2/15*(3*B*a^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 189*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 160*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 15*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 24)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*
x + e) + 1) + 11*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 15*c^3*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + 15*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1
1*c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*c^3*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 - c^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x +
 e)/(cos(f*x + e) + 1))/c^3) + A*a^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) -
 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 22)/(c^3 - 5*c^3*sin(f*x
+ e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c
^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/c^3) + 3*B*a^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) -
145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 22)/(c^3 - 5*c^3*sin(f*x +
 e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^
3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/
(cos(f*x + e) + 1))/c^3) + A*a^3*(20*sin(f*x + e)/(cos(f*x + e) + 1) - 40*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*sin(f*x + e)/(c
os(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 9*A*a^3*(5*sin(f*x + e)/(cos(
f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 - 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c
^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(
cos(f*x + e) + 1)^5) - 3*B*a^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)
/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) +
6*A*a^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 - 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x +
e) + 1)^5) + 6*B*a^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 - 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 1
0*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^
5/(cos(f*x + e) + 1)^5))/f
```

mupad [B] time = 14.00, size = 336, normalized size = 2.20

$$\frac{\frac{52\,A\,a^3}{15}-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{40\,A\,a^3}{3}+82\,B\,a^3\right)+\frac{94\,B\,a^3}{5}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,\left(4\,A\,a^3+12\,B\,a^3\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(8\,A\,a^3+5\right.}{f\,\left(-c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+5\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-11\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+15\,c^3\,\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^3,x)
```

```
[Out] ((52*A*a^3)/15 - tan(e/2 + (f*x)/2)*((40*A*a^3)/3 + 82*B*a^3) + (94*B*a^3)/
5 + tan(e/2 + (f*x)/2)^6*(4*A*a^3 + 12*B*a^3) - tan(e/2 + (f*x)/2)^5*(8*A*a
^3 + 58*B*a^3) - tan(e/2 + (f*x)/2)^3*((64*A*a^3)/3 + 148*B*a^3) + tan(e/2
+ (f*x)/2)^4*((92*A*a^3)/3 + 134*B*a^3) + tan(e/2 + (f*x)/2)^2*((452*A*a^3)
/15 + (744*B*a^3)/5))/(f*(11*c^3*tan(e/2 + (f*x)/2)^2 - 15*c^3*tan(e/2 + (f
*x)/2)^3 + 15*c^3*tan(e/2 + (f*x)/2)^4 - 11*c^3*tan(e/2 + (f*x)/2)^5 + 5*c^
3*tan(e/2 + (f*x)/2)^6 - c^3*tan(e/2 + (f*x)/2)^7 + c^3 - 5*c^3*tan(e/2 + (
f*x)/2))) - (2*a^3*atan((2*a^3*tan(e/2 + (f*x)/2)*(A + 6*B))/(2*A*a^3 + 12*
B*a^3))*(A + 6*B))/(c^3*f)
```

sympy [A] time = 48.98, size = 4665, normalized size = 30.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-15*A*a**3*f*x*tan(e/2 + f*x/2)**7/(15*c**3*f*tan(e/2 + f*x/2)**
7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c*
*3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(
e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 75*A*a**3*f*x*t
an(e/2 + f*x/2)**6/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x
/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 +
225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f
*tan(e/2 + f*x/2) - 15*c**3*f) - 165*A*a**3*f*x*tan(e/2 + f*x/2)**5/(15*c**
3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/
2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2
)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**
3*f) + 225*A*a**3*f*x*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**7 -
75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f
*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2
+ f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 225*A*a**3*f*x*tan(
e/2 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)
```

```
**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225
*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*ta
n(e/2 + f*x/2) - 15*c**3*f) + 165*A*a**3*f*x*tan(e/2 + f*x/2)**2/(15*c**3*f
*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 +
 f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**
3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f
) - 75*A*a**3*f*x*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3
*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/
2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2
)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 15*A*a**3*f*x/(15*c**3*f*t
an(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f
*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3
- 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f)
- 60*A*a**3*tan(e/2 + f*x/2)**6/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*
tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 +
 f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**
2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 120*A*a**3*tan(e/2 + f*x/2)**
5/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3
*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/
2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2)
 - 15*c**3*f) - 460*A*a**3*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)*
*7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c
**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan
(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 320*A*a**3*tan
(e/2 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2
)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 22
5*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*t
an(e/2 + f*x/2) - 15*c**3*f) - 452*A*a**3*tan(e/2 + f*x/2)**2/(15*c**3*f*ta
n(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*
x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 -
 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) +
 200*A*a**3*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan
(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*
x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 +
 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 52*A*a**3/(15*c**3*f*tan(e/2 + f
*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 -
 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3
*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 90*B*a**
3*f*x*tan(e/2 + f*x/2)**7/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/
2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2
)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75
*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 450*B*a**3*f*x*tan(e/2 + f*x/2)**6/
(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f
*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2
+ f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) -
```

```
15*c**3*f) - 990*B*a**3*f*x*tan(e/2 + f*x/2)**5/(15*c**3*f*tan(e/2 + f*x/2
)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225
*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*t
an(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 1350*B*a**3*
f*x*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2
+ f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)*
*4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c
**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 1350*B*a**3*f*x*tan(e/2 + f*x/2)**3/(
15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*
tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 +
 f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) -
15*c**3*f) + 990*B*a**3*f*x*tan(e/2 + f*x/2)**2/(15*c**3*f*tan(e/2 + f*x/2)
**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*
c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*ta
n(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 450*B*a**3*f*
x*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x
/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 +
225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f
*tan(e/2 + f*x/2) - 15*c**3*f) + 90*B*a**3*f*x/(15*c**3*f*tan(e/2 + f*x/2)*
*7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c
**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan
(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 180*B*a**3*tan
(e/2 + f*x/2)**6/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2
)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 22
5*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*t
an(e/2 + f*x/2) - 15*c**3*f) + 870*B*a**3*tan(e/2 + f*x/2)**5/(15*c**3*f*ta
n(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*
x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 -
 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) -
 2010*B*a**3*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f
*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2
+ f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)*
*2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 2220*B*a**3*tan(e/2 + f*x/2)
**3/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c*
*3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(
e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/
2) - 15*c**3*f) - 2232*B*a**3*tan(e/2 + f*x/2)**2/(15*c**3*f*tan(e/2 + f*x/
2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 22
5*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*
tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 1230*B*a**3
*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/
2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 2
25*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*
tan(e/2 + f*x/2) - 15*c**3*f) - 282*B*a**3/(15*c**3*f*tan(e/2 + f*x/2)**7 -
 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*
```

```
f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2
 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f), Ne(f, 0)), (x*(A +
B*sin(e))*(a*sin(e) + a)**3/(-c*sin(e) + c)**3, True))
```

3.47 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=151

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}-\frac{2a^3B\cos(e+fx)}{f\left(c^4-c^4\sin(e+fx)\right)}+\frac{a^3Bx}{c^4}+\frac{2a^3Bc^2\cos^3(e+fx)}{3f\left(c^2-c^2\sin(e+fx)\right)^3}-\frac{2a^3Bc\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}$$

```
[Out] a^3*B*x/c^4+1/7*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^7-2/5*a^3*B*c
*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^5+2/3*a^3*B*c^2*cos(f*x+e)^3/f/(c^2-c^2*si
n(f*x+e))^3-2*a^3*B*cos(f*x+e)/f/(c^4-c^4*sin(f*x+e))
```

Rubi [A] time = 0.33, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 2680, 8}

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}+\frac{2a^3Bc^2\cos^3(e+fx)}{3f\left(c^2-c^2\sin(e+fx)\right)^3}-\frac{2a^3B\cos(e+fx)}{f\left(c^4-c^4\sin(e+fx)\right)}+\frac{a^3Bx}{c^4}-\frac{2a^3Bc\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a^3*B*x)/c^4 + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(7*f*(c - c*Sin[e + f*x])^
7) - (2*a^3*B*c*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x])^5) + (2*a^3*B*c^2
*Cos[e + f*x]^3)/(3*f*(c^2 - c^2*Sin[e + f*x])^3) - (2*a^3*B*Cos[e + f*x])/
(f*(c^4 - c^4*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
```

```
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7} - \left(a^3Bc^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^6}\,dx$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7} - \frac{2a^3Bc\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} + \left(a^3B\right)\int$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7} - \frac{2a^3Bc\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} + \frac{2a^3B\,\mathrm{co}}{3cf(c-c}$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7} - \frac{2a^3Bc\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} + \frac{2a^3B\,\mathrm{co}}{3cf(c-c}$$
$$= \frac{a^3Bx}{c^4} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7} - \frac{2a^3Bc\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} + \frac{}{3}$$

Mathematica [B] time = 1.21, size = 356, normalized size = 2.36

$$\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(240(A+B)\sin\left(\frac{1}{2}(e+fx)\right)-2(15A+337B)\sin\left(\frac{1}{2}(e\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^4,x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(120*(A + B)*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]) - 12*(15*A + 29*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^3 + 2*(45*A + 199*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 105*B*(e +
f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7 + 240*(A + B)*Sin[(e + f*x)/2]
 - 24*(15*A + 29*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2
] + 4*(45*A + 199*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/
2] - 2*(15*A + 337*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6*Sin[(e + f*x)
/2])*(1 + Sin[e + f*x])^3)/(105*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(
c - c*Sin[e + f*x])^4)
```

fricas [B] time = 0.44, size = 363, normalized size = 2.40

$$\frac{840\,Ba^3fx + \left(105\,Ba^3fx + (15\,A + 337\,B)a^3\right)\cos\left(fx + e\right)^4 + 120\,(A + B)a^3 - \left(315\,Ba^3fx + (45\,A - 613\,B)a^3\right)\mathrm{c}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorit
hm="fricas")
```

```
[Out] 1/105*(840*B*a^3*f*x + (105*B*a^3*f*x + (15*A + 337*B)*a^3)*cos(f*x + e)^4
+ 120*(A + B)*a^3 - (315*B*a^3*f*x + (45*A - 613*B)*a^3)*cos(f*x + e)^3 - 2
4*(35*B*a^3*f*x + (5*A + 26*B)*a^3)*cos(f*x + e)^2 + 60*(7*B*a^3*f*x + (A -
 13*B)*a^3)*cos(f*x + e) - (840*B*a^3*f*x - 120*(A + B)*a^3 - (105*B*a^3*f*
x - (15*A + 337*B)*a^3)*cos(f*x + e)^3 - 12*(35*B*a^3*f*x - (5*A - 23*B)*a^
3)*cos(f*x + e)^2 + 60*(7*B*a^3*f*x - (A + 15*B)*a^3)*cos(f*x + e))*sin(f*x
 + e))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e
)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(
f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x + e))
```

giac [A] time = 0.24, size = 213, normalized size = 1.41

$$\frac{\frac{105\left(fx+e\right)Ba^3}{c^4} - \frac{2\left(105\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6 - 105\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6 + 840\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 + 525\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 - 1925\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4}{c^4\left(\tan\left(\frac{1}{2}\,fx+\right.\right.}}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorit
hm="giac")
```

```
[Out] 1/105*(105*(f*x + e)*B*a^3/c^4 - 2*(105*A*a^3*tan(1/2*f*x + 1/2*e)^6 - 105*
B*a^3*tan(1/2*f*x + 1/2*e)^6 + 840*B*a^3*tan(1/2*f*x + 1/2*e)^5 + 525*A*a^3
*tan(1/2*f*x + 1/2*e)^4 - 1925*B*a^3*tan(1/2*f*x + 1/2*e)^4 + 3920*B*a^3*ta
n(1/2*f*x + 1/2*e)^3 + 315*A*a^3*tan(1/2*f*x + 1/2*e)^2 - 2667*B*a^3*tan(1/
2*f*x + 1/2*e)^2 + 1064*B*a^3*tan(1/2*f*x + 1/2*e) + 15*A*a^3 - 167*B*a^3)/
(c^4*(tan(1/2*f*x + 1/2*e) - 1)^7))/f
```

maple [B] time = 0.48, size = 374, normalized size = 2.48

$$-\frac{2a^3A}{c^4f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{2a^3B}{c^4f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{12a^3A}{c^4f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{4a^3B}{c^4f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{4\cdots}{c^4f\left(\tan\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x)
```

```
[Out] -2*a^3/c^4/f/(tan(1/2*f*x+1/2*e)-1)*A+2*a^3/c^4/f/(tan(1/2*f*x+1/2*e)-1)*B-
12*a^3/c^4/f/(tan(1/2*f*x+1/2*e)-1)^2*A-4*a^3/c^4/f/(tan(1/2*f*x+1/2*e)-1)^
2*B-40*a^3/c^4/f/(tan(1/2*f*x+1/2*e)-1)^3*A-40/3*a^3/c^4/f/(tan(1/2*f*x+1/2
*e)-1)^3*B-128/7*a^3/c^4/f/(tan(1/2*f*x+1/2*e)-1)^7*A-128/7*a^3/c^4/f/(tan(
1/2*f*x+1/2*e)-1)^7*B-80*a^3/c^4/f/(tan(1/2*f*x+1/2*e)-1)^4*A-48*a^3/c^4/f/
(tan(1/2*f*x+1/2*e)-1)^4*B-64*a^3/c^4/f/(tan(1/2*f*x+1/2*e)-1)^6*A-64*a^3/c
^4/f/(tan(1/2*f*x+1/2*e)-1)^6*B-96*a^3/c^4/f/(tan(1/2*f*x+1/2*e)-1)^5*A-416
/5*a^3/c^4/f/(tan(1/2*f*x+1/2*e)-1)^5*B+2*a^3/c^4/f*B*arctan(tan(1/2*f*x+1/
2*e))
```

maxima [B] time = 0.58, size = 2118, normalized size = 14.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorit
hm="maxima")
```

```
[Out] 2/105*(5*B*a^3*((203*sin(f*x + e)/(cos(f*x + e) + 1) - 525*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 686*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 434*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + 147*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 21
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 32)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f
*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x
+ e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x
+ e)/(cos(f*x + e) + 1))/c^4) + 3*A*a^3*(91*sin(f*x + e)/(cos(f*x + e) + 1)
 - 168*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x +
```

```
e) + 1)^3 - 175*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5 - 13)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*
c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e
)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*
sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + B*a^3*(91*sin(f*x + e)/(cos(f*x + e)
 + 1) - 168*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 - 175*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)
^5/(cos(f*x + e) + 1)^5 - 13)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1)
+ 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 -
 c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) - 3*A*a^3*(49*sin(f*x + e)/(cos(f
*x + e) + 1) - 147*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 210*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3 - 210*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 - 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1
2)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*
x + e) + 1)^7) - 12*A*a^3*(14*sin(f*x + e)/(cos(f*x + e) + 1) - 42*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 35*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x
+ e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*
c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) - 12*B*a^3*(14*sin(f*x
+ e)/(cos(f*x + e) + 1) - 42*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2
)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x
 + e) + 1)^7) + 6*A*a^3*(7*sin(f*x + e)/(cos(f*x + e) + 1) - 21*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^4
 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*
c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7) + 18*B*a^3*(7*sin(f*x + e)/(cos(f*x + e) + 1) - 21*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^4 - 7*
c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^
```

7))/f

mupad [B] time = 15.98, size = 316, normalized size = 2.09

$$\frac{B\,a^3\,x}{c^4}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(\frac{a^3\,\left(1680\,B-2205\,B\,\left(e+f\,x\right)\right)}{105}+21\,B\,a^3\,\left(e+f\,x\right)\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(\frac{a^3\,\left(7840\,B-3675\,B\,\left(e+f\,x\right)\right)}{105}+35\,B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^4,x)
```

```
[Out] (B*a^3*x)/c^4 - (tan(e/2 + (f*x)/2)^5*((a^3*(1680*B - 2205*B*(e + f*x)))/10
5 + 21*B*a^3*(e + f*x)) + tan(e/2 + (f*x)/2)^3*((a^3*(7840*B - 3675*B*(e +
f*x)))/105 + 35*B*a^3*(e + f*x)) + (a^3*(30*A - 334*B + 105*B*(e + f*x)))/1
05 + tan(e/2 + (f*x)/2)^6*((a^3*(210*A - 210*B + 735*B*(e + f*x)))/105 - 7*
B*a^3*(e + f*x)) + tan(e/2 + (f*x)/2)^2*((a^3*(630*A - 5334*B + 2205*B*(e +
 f*x)))/105 - 21*B*a^3*(e + f*x)) + tan(e/2 + (f*x)/2)^4*((a^3*(1050*A - 38
50*B + 3675*B*(e + f*x)))/105 - 35*B*a^3*(e + f*x)) + tan(e/2 + (f*x)/2)*((
a^3*(2128*B - 735*B*(e + f*x)))/105 + 7*B*a^3*(e + f*x)) - B*a^3*(e + f*x))
/(c^4*f*(tan(e/2 + (f*x)/2) - 1)^7)
```

sympy [A] time = 76.73, size = 2951, normalized size = 19.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-210*A*a**3*tan(e/2 + f*x/2)**6/(105*c**4*f*tan(e/2 + f*x/2)**7
- 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c
**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*t
an(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 1050*A*a**
3*tan(e/2 + f*x/2)**4/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2
+ f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2
)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 +
735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 630*A*a**3*tan(e/2 + f*x/2)**2/
(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**
4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan
(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f
*x/2) - 105*c**4*f) - 30*A*a**3/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*
f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e
/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*
x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 105*B*a**3*f*x*tan(e/
```

```
2 + f*x/2)**7/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)
**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3
675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4
*f*tan(e/2 + f*x/2) - 105*c**4*f) - 735*B*a**3*f*x*tan(e/2 + f*x/2)**6/(105
*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*
tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2
 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2
) - 105*c**4*f) + 2205*B*a**3*f*x*tan(e/2 + f*x/2)**5/(105*c**4*f*tan(e/2 +
 f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)*
*5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 22
05*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) -
 3675*B*a**3*f*x*tan(e/2 + f*x/2)**4/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*
c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*
tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2
 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 3675*B*a**3*f*x*
tan(e/2 + f*x/2)**3/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 +
f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)*
*4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 73
5*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 2205*B*a**3*f*x*tan(e/2 + f*x/2)*
*2/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*
c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*
tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2
+ f*x/2) - 105*c**4*f) + 735*B*a**3*f*x*tan(e/2 + f*x/2)/(105*c**4*f*tan(e/
2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/
2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 -
 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f
) - 105*B*a**3*f*x/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f
*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**
4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735
*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 210*B*a**3*tan(e/2 + f*x/2)**6/(10
5*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f
*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/
2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/
2) - 105*c**4*f) - 1680*B*a**3*tan(e/2 + f*x/2)**5/(105*c**4*f*tan(e/2 + f*
x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5
- 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*
c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 38
50*B*a**3*tan(e/2 + f*x/2)**4/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*
tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2
 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/
2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 7840*B*a**3*tan(e/2 + f
*x/2)**3/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 +
 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c
**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*ta
n(e/2 + f*x/2) - 105*c**4*f) + 5334*B*a**3*tan(e/2 + f*x/2)**2/(105*c**4*f*
```

```
tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2
+ f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2
)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*
c**4*f) - 2128*B*a**3*tan(e/2 + f*x/2)/(105*c**4*f*tan(e/2 + f*x/2)**7 - 73
5*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*
f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e
/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 334*B*a**3/(10
5*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f
*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/
2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/
2) - 105*c**4*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**3/(-c*sin(e)
 + c)**4, True))
```

3.48 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$

Optimal. Leaf size=77

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}+\frac{a^3c^2(A-8B)\cos^7(e+fx)}{63f(c-c\sin(e+fx))^7}$$

```
[Out] 1/9*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^8+1/63*a^3*(A-8*B)*c^2*co
s(f*x+e)^7/f/(c-c*sin(f*x+e))^7
```

Rubi [A] time = 0.24, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2967, 2859, 2671}

$$\frac{a^3c^2(A-8B)\cos^7(e+fx)}{63f(c-c\sin(e+fx))^7}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(9*f*(c - c*Sin[e + f*x])^8) + (a^3*(A - 8
*B)*c^2*Cos[e + f*x]^7)/(63*f*(c - c*Sin[e + f*x])^7)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
```

```
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8} + \frac{1}{9}\left(a^3(A-8B)c^2\right)\int \frac{\cos^6(e+f}{(c-c\sin(e+}$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8} + \frac{a^3(A-8B)c^2\cos^7(e+fx)}{63f(c-c\sin(e+fx))^7}$$

Mathematica [B] time = 2.52, size = 283, normalized size = 3.68

$$-\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(315(A-B)\cos\left(\frac{1}{2}(e+fx)\right)-189(A-B)\cos\left(\frac{3}{2}(e+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^5,x]
```

```
[Out] -1/504*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(315
*(A - B)*Cos[(e + f*x)/2] - 189*(A - B)*Cos[(3*(e + f*x))/2] - 63*A*Cos[(5*
(e + f*x))/2] + 63*B*Cos[(5*(e + f*x))/2] + 9*A*Cos[(7*(e + f*x))/2] - 9*B*
Cos[(7*(e + f*x))/2] + 189*A*Sin[(e + f*x)/2] + 693*B*Sin[(e + f*x)/2] + 10
5*A*Sin[(3*(e + f*x))/2] + 483*B*Sin[(3*(e + f*x))/2] - 27*A*Sin[(5*(e + f*
x))/2] - 225*B*Sin[(5*(e + f*x))/2] - 63*B*Sin[(7*(e + f*x))/2] - A*Sin[(9*
(e + f*x))/2] + 8*B*Sin[(9*(e + f*x))/2]))/(c^5*f*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^6*(-1 + Sin[e + f*x])^5)
```

fricas [B] time = 0.43, size = 331, normalized size = 4.30

$$-\frac{(A-8B)a^3\cos\left(fx+e\right)^5-(4A+31B)a^3\cos\left(fx+e\right)^4+(19A+37B)a^3\cos\left(fx+e\right)^3+4(13A+22B)a^3}{63\left(c^5f\cos\left(fx+e\right)^5+5c^5f\cos\left(fx+e\right)^4-8c^5f\cos\left(fx+e\right)^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorit
hm="fricas")
```

```
[Out] -1/63*((A - 8*B)*a^3*cos(f*x + e)^5 - (4*A + 31*B)*a^3*cos(f*x + e)^4 + (19
*A + 37*B)*a^3*cos(f*x + e)^3 + 4*(13*A + 22*B)*a^3*cos(f*x + e)^2 - 28*(A
+ B)*a^3*cos(f*x + e) - 56*(A + B)*a^3 + ((A - 8*B)*a^3*cos(f*x + e)^4 + (5
*A + 23*B)*a^3*cos(f*x + e)^3 + 12*(2*A + 5*B)*a^3*cos(f*x + e)^2 - 28*(A +
 B)*a^3*cos(f*x + e) - 56*(A + B)*a^3)*sin(f*x + e))/(c^5*f*cos(f*x + e)^5
+ 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x + e)^2
 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*
x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f)*sin(f
*x + e))
```

giac [B] time = 0.27, size = 301, normalized size = 3.91

$$-\frac{2\left(63\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-63\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+63\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+483\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorit
hm="giac")
```

```
[Out] -2/63*(63*A*a^3*tan(1/2*f*x + 1/2*e)^8 - 63*A*a^3*tan(1/2*f*x + 1/2*e)^7 +
63*B*a^3*tan(1/2*f*x + 1/2*e)^7 + 483*A*a^3*tan(1/2*f*x + 1/2*e)^6 + 105*B*
a^3*tan(1/2*f*x + 1/2*e)^6 - 315*A*a^3*tan(1/2*f*x + 1/2*e)^5 + 315*B*a^3*t
an(1/2*f*x + 1/2*e)^5 + 693*A*a^3*tan(1/2*f*x + 1/2*e)^4 + 189*B*a^3*tan(1/
2*f*x + 1/2*e)^4 - 189*A*a^3*tan(1/2*f*x + 1/2*e)^3 + 189*B*a^3*tan(1/2*f*x
 + 1/2*e)^3 + 225*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 27*B*a^3*tan(1/2*f*x + 1/2
*e)^2 - 9*A*a^3*tan(1/2*f*x + 1/2*e) + 9*B*a^3*tan(1/2*f*x + 1/2*e) + 8*A*a
^3 - B*a^3)/(c^5*f*(tan(1/2*f*x + 1/2*e) - 1)^9)
```

maple [B] time = 0.53, size = 205, normalized size = 2.66

$$\frac{2a^3\left(-\frac{992A+800B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{304A+144B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{928A+864B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{86A+26B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{512A+512B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{14A\cdots}{2\left(\tan\left(\frac{fx}{2}+\cdots\right.\right.}\cdots\right.}{f\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x)
```

```
[Out] 2/f*a^3/c^5*(-1/6*(992*A+800*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/4*(304*A+144*B)/
(tan(1/2*f*x+1/2*e)-1)^4-A/(tan(1/2*f*x+1/2*e)-1)-1/7*(928*A+864*B)/(tan(1/
```

```
2*f*x+1/2*e)-1)^7-1/3*(86*A+26*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/8*(512*A+512*B
)/(tan(1/2*f*x+1/2*e)-1)^8-1/2*(14*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^2-1/5*(680
*A+440*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/9*(128*A+128*B)/(tan(1/2*f*x+1/2*e)-1)
^9)
```

maxima [B] time = 0.54, size = 2701, normalized size = 35.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorit
hm="maxima")
```

```
[Out] -2/315*(A*a^3*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 -
3360*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*sin(f
*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 8
4*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 +
 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x +
e) + 1)^9) - 15*A*a^3*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 315*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f
*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
- c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 5*B*a^3*(45*sin(f*x + e)/(cos(
f*x + e) + 1) - 117*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 - 315*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 +
 63*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos
(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f
*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) -
 10*A*a^3*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 84*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 63*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1)/(c^5 - 9*c^
```

```
5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) +
 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos
(f*x + e) + 1)^9) - 30*B*a^3*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 84*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 63
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c
^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x +
e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e
)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*
sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 8*B*a^3*(9*sin(f*x + e)/(cos(f*x + e
) + 1) - 36*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 84*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 - 126*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1)/(c^5 - 9*c^5*s
in(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin
(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)
^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*
x + e) + 1)^9) + 42*A*a^3*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 54*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 81*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 + 45*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
- 30*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(co
s(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(co
s(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(
f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9)
+ 42*B*a^3*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 54*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 81*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 45*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 30*sin(f*x +
e)^6/(cos(f*x + e) + 1)^6 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1)
 + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)
^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos
(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9))/f
```

mupad [B] time = 13.39, size = 346, normalized size = 4.49

$$2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{1013\,A\,a^3}{16}+\frac{149\,B\,a^3}{16}-\frac{113\,A\,a^3\,\cos\left(2\,e+2\,f\,x\right)}{4}+\frac{37\,A\,a^3\,\cos\left(3\,e+3\,f\,x\right)}{8}+\frac{7\,A\,a^3\,\cos\left(4\,e+4\,f\,x\right)}{16}-\frac{41\,B\,a^3\,\cos\left(2\,e+2\,f\right.}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^5,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((1013*A*a^3)/16 + (149*B*a^3)/16 - (113*A*a^3*cos(2*
e + 2*f*x))/4 + (37*A*a^3*cos(3*e + 3*f*x))/8 + (7*A*a^3*cos(4*e + 4*f*x))/
16 - (41*B*a^3*cos(2*e + 2*f*x))/4 + (19*B*a^3*cos(3*e + 3*f*x))/8 + (7*B*a
^3*cos(4*e + 4*f*x))/16 + (63*A*a^3*sin(2*e + 2*f*x))/8 + (9*A*a^3*sin(3*e
+ 3*f*x))/2 - (9*A*a^3*sin(4*e + 4*f*x))/16 - (63*B*a^3*sin(2*e + 2*f*x))/8
 - (9*B*a^3*sin(3*e + 3*f*x))/2 + (9*B*a^3*sin(4*e + 4*f*x))/16 - (257*A*a^
3*cos(e + f*x))/8 - (23*B*a^3*cos(e + f*x))/8 - (63*A*a^3*sin(e + f*x))/2 +
 (63*B*a^3*sin(e + f*x))/2))/(63*c^5*f*((63*2^(1/2)*cos(e/2 + pi/4 + (f*x)/
2))/8 - (21*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/4 - (9*2^(1/2)*cos((5*
e)/2 + pi/4 + (5*f*x)/2))/4 + (9*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/1
6 + (2^(1/2)*cos((9*e)/2 + pi/4 + (9*f*x)/2))/16))
```

sympy [A] time = 116.40, size = 3262, normalized size = 42.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-126*A*a**3*tan(e/2 + f*x/2)**8/(63*c**5*f*tan(e/2 + f*x/2)**9 -
 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c*
*5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*ta
n(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 +
 f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) + 126*A*a**3*tan(e/2
+ f*x/2)**7/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8
 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938
*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f
*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2
 + f*x/2) - 63*c**5*f) - 966*A*a**3*tan(e/2 + f*x/2)**6/(63*c**5*f*tan(e/2
+ f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)
**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7
938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**
5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) + 630*A*
```

a**3*tan(e/2 + f*x/2)**5/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/
2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x
/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4
+ 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c
5*f*tan(e/2 + f*x/2) - 63*c5*f) - 1386*A*a**3*tan(e/2 + f*x/2)**4/(63*c
5*f*tan(e/2 + f*x/2)9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*ta
n(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 +
f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)
3 - 2268*c5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c*
*5*f) + 378*A*a**3*tan(e/2 + f*x/2)**3/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567
*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f
*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/
2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x
/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) - 450*A*a**3*tan(e/2 + f*
x/2)**2/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2
268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**
5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan
(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f
*x/2) - 63*c**5*f) + 18*A*a**3*tan(e/2 + f*x/2)/(63*c**5*f*tan(e/2 + f*x/2)
9 - 567*c5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 52
92*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5
*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(
e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) - 16*A*a**3/(63*
c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*t
an(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2
+ f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2
)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c
5*f) - 126*B*a3*tan(e/2 + f*x/2)**7/(63*c**5*f*tan(e/2 + f*x/2)**9 - 56
7*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*
f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e
/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*
x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) - 210*B*a**3*tan(e/2 + f
*x/2)**6/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 +
2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c*
*5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*ta
n(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 +
f*x/2) - 63*c**5*f) - 630*B*a**3*tan(e/2 + f*x/2)**5/(63*c**5*f*tan(e/2 + f
*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7
- 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938
*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f
*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) - 378*B*a**
3*tan(e/2 + f*x/2)**4/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 +
f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)
6 + 7938*c5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5
292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5

```
*f*tan(e/2 + f*x/2) - 63*c**5*f) - 378*B*a**3*tan(e/2 + f*x/2)**3/(63*c**5*
f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/
2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x
/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3
- 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f
) - 54*B*a**3*tan(e/2 + f*x/2)**2/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5
*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(
e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f
*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**
2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) - 18*B*a**3*tan(e/2 + f*x/2)/(
63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*
f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e
/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*
x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 6
3*c**5*f) + 2*B*a**3/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 +
f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)*
*6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 52
92*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*
f*tan(e/2 + f*x/2) - 63*c**5*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a
)**3/(-c*sin(e) + c)**5, True))
```

3.49 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$

Optimal. Leaf size=118

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}+\frac{a^3c^2(2A-9B)\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8}+\frac{a^3c(2A-9B)\cos^7(e+fx)}{693f(c-c\sin(e+fx))^7}$$

```
[Out] 1/11*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^9+1/99*a^3*(2*A-9*B)*c^2
*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^8+1/693*a^3*(2*A-9*B)*c*cos(f*x+e)^7/f/(c-
c*sin(f*x+e))^7
```

Rubi [A] time = 0.29, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 2672, 2671}

$$\frac{a^3c^2(2A-9B)\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}+\frac{a^3c(2A-9B)\cos^7(e+fx)}{693f(c-c\sin(e+fx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^6,x]
```

```
[Out] (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(11*f*(c - c*Sin[e + f*x])^9) + (a^3*(2*A
- 9*B)*c^2*Cos[e + f*x]^7)/(99*f*(c - c*Sin[e + f*x])^8) + (a^3*(2*A - 9*B)
*c*Cos[e + f*x]^7)/(693*f*(c - c*Sin[e + f*x])^7)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^9}\,dx$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9} + \frac{1}{11}\left(a^3(2A-9B)c^2\right)\int \frac{\cos^6(e+}{(c-c\sin(e}$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9} + \frac{a^3(2A-9B)c^2\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8} + \frac{1}{99}($$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9} + \frac{a^3(2A-9B)c^2\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8} + \frac{a^3(2}{69}$$

Mathematica [B] time = 2.94, size = 313, normalized size = 2.65

$$\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(462(11A+3B)\cos\left(\frac{1}{2}(e+fx)\right)-594(5A+2B)\cos\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^6,x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(462*(11*A
+ 3*B)*Cos[(e + f*x)/2] - 594*(5*A + 2*B)*Cos[(3*(e + f*x))/2] - 924*A*Cos[
```

```
(5*(e + f*x))/2] - 693*B*Cos[(5*(e + f*x))/2] + 110*A*Cos[(7*(e + f*x))/2]
+ 198*B*Cos[(7*(e + f*x))/2] - 2*A*Cos[(11*(e + f*x))/2] + 9*B*Cos[(11*(e +
 f*x))/2] + 4158*A*Sin[(e + f*x)/2] + 5544*B*Sin[(e + f*x)/2] + 2310*A*Sin[
(3*(e + f*x))/2] + 4158*B*Sin[(3*(e + f*x))/2] - 594*A*Sin[(5*(e + f*x))/2]
 - 2178*B*Sin[(5*(e + f*x))/2] - 693*B*Sin[(7*(e + f*x))/2] - 22*A*Sin[(9*(
e + f*x))/2] + 99*B*Sin[(9*(e + f*x))/2]))/(11088*c^6*f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x])^6)
```

fricas [B] time = 0.45, size = 405, normalized size = 3.43

$$\frac{(2A-9B)a^3\cos\left(fx+e\right)^6+6(2A-9B)a^3\cos\left(fx+e\right)^5-(25A+234B)a^3\cos\left(fx+e\right)^4+7(23A+45B)a^3}{693\left(c^6f\cos\left(fx+e\right)^6-5c^6f\cos\left(fx+e\right)^5-18c^6f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorit
hm="fricas")
```

```
[Out] 1/693*((2*A - 9*B)*a^3*cos(f*x + e)^6 + 6*(2*A - 9*B)*a^3*cos(f*x + e)^5 -
(25*A + 234*B)*a^3*cos(f*x + e)^4 + 7*(23*A + 45*B)*a^3*cos(f*x + e)^3 + 28
*(16*A + 27*B)*a^3*cos(f*x + e)^2 - 252*(A + B)*a^3*cos(f*x + e) - 504*(A +
 B)*a^3 - ((2*A - 9*B)*a^3*cos(f*x + e)^5 - 5*(2*A - 9*B)*a^3*cos(f*x + e)^
4 - 7*(5*A + 27*B)*a^3*cos(f*x + e)^3 - 28*(7*A + 18*B)*a^3*cos(f*x + e)^2
+ 252*(A + B)*a^3*cos(f*x + e) + 504*(A + B)*a^3)*sin(f*x + e))/(c^6*f*cos(
f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*c^6*f*cos(f*x + e)^4 + 20*c^6*f*co
s(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 - 16*c^6*f*cos(f*x + e) - 32*c^6*f +
 (c^6*f*cos(f*x + e)^5 + 6*c^6*f*cos(f*x + e)^4 - 12*c^6*f*cos(f*x + e)^3 -
 32*c^6*f*cos(f*x + e)^2 + 16*c^6*f*cos(f*x + e) + 32*c^6*f)*sin(f*x + e))
```

giac [B] time = 0.31, size = 373, normalized size = 3.16

$$-\frac{2\left(693\,Aa^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{10}-1386\,Aa^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9+693\,Ba^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9+8085\,Aa^3\tan\left(\frac{1}{2}fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorit
hm="giac")
```

```
[Out] -2/693*(693*A*a^3*tan(1/2*f*x + 1/2*e)^10 - 1386*A*a^3*tan(1/2*f*x + 1/2*e)
^9 + 693*B*a^3*tan(1/2*f*x + 1/2*e)^9 + 8085*A*a^3*tan(1/2*f*x + 1/2*e)^8 +
 693*B*a^3*tan(1/2*f*x + 1/2*e)^8 - 10626*A*a^3*tan(1/2*f*x + 1/2*e)^7 + 41
58*B*a^3*tan(1/2*f*x + 1/2*e)^7 + 21252*A*a^3*tan(1/2*f*x + 1/2*e)^6 + 1386
```

```
*B*a^3*tan(1/2*f*x + 1/2*e)^6 - 15246*A*a^3*tan(1/2*f*x + 1/2*e)^5 + 5544*B
*a^3*tan(1/2*f*x + 1/2*e)^5 + 15444*A*a^3*tan(1/2*f*x + 1/2*e)^4 + 1188*B*a
^3*tan(1/2*f*x + 1/2*e)^4 - 4950*A*a^3*tan(1/2*f*x + 1/2*e)^3 + 2178*B*a^3*
tan(1/2*f*x + 1/2*e)^3 + 2959*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 198*B*a^3*tan(
1/2*f*x + 1/2*e)^2 - 176*A*a^3*tan(1/2*f*x + 1/2*e) + 99*B*a^3*tan(1/2*f*x
+ 1/2*e) + 79*A*a^3 - 9*B*a^3)/(c^6*f*(tan(1/2*f*x + 1/2*e) - 1)^11)
```

maple [B] time = 0.52, size = 249, normalized size = 2.11

$$\frac{2a^3\left(-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{504A+200B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{4352A+3840B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{4272A+3344B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{116A+30B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{1280A+1280B}{10\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}}-\frac{29}{6(\mathrm{ta}}\right.}{f\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x)
```

```
[Out] 2/f*a^3/c^6*(-A/(tan(1/2*f*x+1/2*e)-1)-1/4*(504*A+200*B)/(tan(1/2*f*x+1/2*e
)-1)^4-1/8*(4352*A+3840*B)/(tan(1/2*f*x+1/2*e)-1)^8-1/7*(4272*A+3344*B)/(ta
n(1/2*f*x+1/2*e)-1)^7-1/3*(116*A+30*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/10*(1280*
A+1280*B)/(tan(1/2*f*x+1/2*e)-1)^10-1/6*(2960*A+1968*B)/(tan(1/2*f*x+1/2*e)
-1)^6-1/5*(1460*A+780*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/2*(16*A+2*B)/(tan(1/2*f
*x+1/2*e)-1)^2-1/11*(256*A+256*B)/(tan(1/2*f*x+1/2*e)-1)^11-1/9*(3008*A+288
0*B)/(tan(1/2*f*x+1/2*e)-1)^9)
```

maxima [B] time = 0.53, size = 3390, normalized size = 28.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorit
hm="maxima")
```

```
[Out] -2/3465*(5*A*a^3*(913*sin(f*x + e)/(cos(f*x + e) + 1) - 4565*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 12540*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 25080*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33726*sin(f*x + e)^5/(cos(f*x + e) + 1
)^5 - 33726*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 23100*sin(f*x + e)^7/(cos
(f*x + e) + 1)^7 - 11550*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x
 + e)^9/(cos(f*x + e) + 1)^9 - 693*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 -
146)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^
6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5
```

5*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x
 + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 9*A*a^3*(671*s
in(f*x + e)/(cos(f*x + e) + 1) - 2200*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 6600*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 15246*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12936*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6 + 9240*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3465*s
in(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*sin(f*x + e)^9/(cos(f*x + e) + 1)
^9 - 61)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 33
0*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*
x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos
(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 3*B*a^3*(6
71*sin(f*x + e)/(cos(f*x + e) + 1) - 2200*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 6600*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 15246*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12936*sin(f*x
+ e)^6/(cos(f*x + e) + 1)^6 + 9240*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 34
65*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*sin(f*x + e)^9/(cos(f*x + e)
+ 1)^9 - 61)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
+ 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*si
n(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) +
1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/
(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 2*A*a^
3*(341*sin(f*x + e)/(cos(f*x + e) + 1) - 1705*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 5115*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6765*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 9471*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 4851*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 3465*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3
1)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*
sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*
c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x +
 e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 6*B*a^3*(341*sin
(f*x + e)/(cos(f*x + e) + 1) - 1705*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5
115*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6765*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + 9471*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 4851*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 3465*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 31)/(c^6 -
11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 4
62*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)^7/(cos(f*

```
x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*c^6*sin(f*
x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^1
0 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 12*A*a^3*(253*sin(f*x + e)
/(cos(f*x + e) + 1) - 1265*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2640*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 - 5280*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
 5313*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 5313*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 + 2310*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 1155*sin(f*x + e)^8/
(cos(f*x + e) + 1)^8 - 23)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) +
55*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^
6 - 330*c^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(c
os(f*x + e) + 1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*si
n(f*x + e)^10/(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1
)^11) + 12*B*a^3*(253*sin(f*x + e)/(cos(f*x + e) + 1) - 1265*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 2640*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5280*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5313*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
 - 5313*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 2310*sin(f*x + e)^7/(cos(f*x
+ e) + 1)^7 - 1155*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 23)/(c^6 - 11*c^6*
sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 462*c^6*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*c^6*sin(f*x + e)^
9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - c^6
*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 48*B*a^3*(11*sin(f*x + e)/(cos(f*
x + e) + 1) - 55*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 165*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3 - 330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 231*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5 - 231*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1)
/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e) +
1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)^7
/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*c^
6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x + e
) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11))/f
```

mupad [B] time = 13.54, size = 408, normalized size = 3.46

$$-\frac{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(565\,A\,a^3\,\cos\left(2\,e+2\,f\,x\right)-\frac{837\,B\,a^3}{16}-922\,A\,a^3-\frac{3527\,A\,a^3\,\cos\left(3\,e+3\,f\,x\right)}{32}-29\,A\,a^3\,\cos\left(4\,e+4\,f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^6,x)
```

```
[Out] -(2*cos(e/2 + (f*x)/2)*(565*A*a^3*cos(2*e + 2*f*x) - (837*B*a^3)/16 - 922*A
*a^3 - (3527*A*a^3*cos(3*e + 3*f*x))/32 - 29*A*a^3*cos(4*e + 4*f*x) + (81*A
*a^3*cos(5*e + 5*f*x))/32 + (225*B*a^3*cos(2*e + 2*f*x))/4 - (207*B*a^3*cos
(3*e + 3*f*x))/16 + (9*B*a^3*cos(4*e + 4*f*x))/16 - (9*B*a^3*cos(5*e + 5*f*
x))/16 - (1617*A*a^3*sin(2*e + 2*f*x))/8 - (5049*A*a^3*sin(3*e + 3*f*x))/32
 + (407*A*a^3*sin(4*e + 4*f*x))/16 + (77*A*a^3*sin(5*e + 5*f*x))/32 + (693*
B*a^3*sin(2*e + 2*f*x))/8 + (99*B*a^3*sin(3*e + 3*f*x))/2 - (99*B*a^3*sin(4
*e + 4*f*x))/16 + (6635*A*a^3*cos(e + f*x))/16 + 18*B*a^3*cos(e + f*x) + (1
3629*A*a^3*sin(e + f*x))/16 - (693*B*a^3*sin(e + f*x))/2))/(693*c^6*f*((231
*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/16 - (165*2^(1/2)*cos((3*e)/2 - pi/4 +
(3*f*x)/2))/16 - (165*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/32 + (55*2^(
1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/32 + (11*2^(1/2)*cos((9*e)/2 + pi/4 +
 (9*f*x)/2))/32 - (2^(1/2)*cos((11*e)/2 - pi/4 + (11*f*x)/2))/32))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**6,x)
```

```
[Out] Timed out
```

3.50 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx$

Optimal. Leaf size=156

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{a^3c^2(3A-10B)\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9}+\frac{2a^3(3A-10B)\cos^7(e+fx)}{9009f(c-c\sin(e+fx))^7}+\frac{2a^3c(3A-10B)\cos^7(e}{1287f(c-c\sin(e+f}$$

```
[Out] 1/13*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^10+1/143*a^3*(3*A-10*B)*
c^2*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^9+2/1287*a^3*(3*A-10*B)*c*cos(f*x+e)^7/
f/(c-c*sin(f*x+e))^8+2/9009*a^3*(3*A-10*B)*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^
7
```

Rubi [A] time = 0.38, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 2672, 2671}

$$\frac{a^3c^2(3A-10B)\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{2a^3(3A-10B)\cos^7(e+fx)}{9009f(c-c\sin(e+fx))^7}+\frac{2a^3c(3A-10B)\cos^7(e}{1287f(c-c\sin(e+f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^7,x]
```

```
[Out] (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(13*f*(c - c*Sin[e + f*x])^10) + (a^3*(3*A
 - 10*B)*c^2*Cos[e + f*x]^7)/(143*f*(c - c*Sin[e + f*x])^9) + (2*a^3*(3*A -
 10*B)*c*Cos[e + f*x]^7)/(1287*f*(c - c*Sin[e + f*x])^8) + (2*a^3*(3*A - 10
*B)*Cos[e + f*x]^7)/(9009*f*(c - c*Sin[e + f*x])^7)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{10}}\,dx$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}} + \frac{1}{13}\left(a^3(3A-10B)c^2\right)\int \frac{\cos^6(e+}{(c-c\sin(e}$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}} + \frac{a^3(3A-10B)c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9} + \frac{1}{143}$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}} + \frac{a^3(3A-10B)c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9} + \frac{2a^3(}{12}$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}} + \frac{a^3(3A-10B)c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9} + \frac{2a^3(}{12}$$

Mathematica [B] time = 5.29, size = 339, normalized size = 2.17

$$-\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6006(9A+5B)\cos\left(\frac{1}{2}(e+fx)\right)-7722(4A+3B)\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^7,x]
```

```
[Out] -1/144144*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(
6006*(9*A + 5*B)*Cos[(e + f*x)/2] - 7722*(4*A + 3*B)*Cos[(3*(e + f*x))/2] -
 9009*A*Cos[(5*(e + f*x))/2] - 12012*B*Cos[(5*(e + f*x))/2] + 858*A*Cos[(7*
(e + f*x))/2] + 3146*B*Cos[(7*(e + f*x))/2] - 39*A*Cos[(11*(e + f*x))/2] +
130*B*Cos[(11*(e + f*x))/2] + 48906*A*Sin[(e + f*x)/2] + 47190*B*Sin[(e + f
*x)/2] + 27027*A*Sin[(3*(e + f*x))/2] + 36036*B*Sin[(3*(e + f*x))/2] - 6864
*A*Sin[(5*(e + f*x))/2] - 19162*B*Sin[(5*(e + f*x))/2] - 6006*B*Sin[(7*(e +
 f*x))/2] - 234*A*Sin[(9*(e + f*x))/2] + 780*B*Sin[(9*(e + f*x))/2] + 3*A*S
in[(13*(e + f*x))/2] - 10*B*Sin[(13*(e + f*x))/2]))/(c^7*f*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x])^7)
```

fricas [B] time = 0.45, size = 475, normalized size = 3.04

$$-\frac{2\,(3\,A-10\,B)a^3\cos\left(fx+e\right)^7-12\,(3\,A-10\,B)a^3\cos\left(fx+e\right)^6-49\,(3\,A-10\,B)a^3\cos\left(fx+e\right)^5+7\,(30\,A+}{9009\left(c^7f\cos\left(fx+e\right)^7+7\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorit
hm="fricas")
```

```
[Out] -1/9009*(2*(3*A - 10*B)*a^3*cos(f*x + e)^7 - 12*(3*A - 10*B)*a^3*cos(f*x +
e)^6 - 49*(3*A - 10*B)*a^3*cos(f*x + e)^5 + 7*(30*A + 329*B)*a^3*cos(f*x +
e)^4 - 63*(27*A + 53*B)*a^3*cos(f*x + e)^3 - 252*(19*A + 32*B)*a^3*cos(f*x
+ e)^2 + 2772*(A + B)*a^3*cos(f*x + e) + 5544*(A + B)*a^3 + (2*(3*A - 10*B)
*a^3*cos(f*x + e)^6 + 14*(3*A - 10*B)*a^3*cos(f*x + e)^5 - 35*(3*A - 10*B)*
a^3*cos(f*x + e)^4 - 63*(5*A + 31*B)*a^3*cos(f*x + e)^3 - 252*(8*A + 21*B)*
a^3*cos(f*x + e)^2 + 2772*(A + B)*a^3*cos(f*x + e) + 5544*(A + B)*a^3)*sin(
f*x + e))/(c^7*f*cos(f*x + e)^7 + 7*c^7*f*cos(f*x + e)^6 - 18*c^7*f*cos(f*x
 + e)^5 - 56*c^7*f*cos(f*x + e)^4 + 48*c^7*f*cos(f*x + e)^3 + 112*c^7*f*cos
(f*x + e)^2 - 32*c^7*f*cos(f*x + e) - 64*c^7*f - (c^7*f*cos(f*x + e)^6 - 6*
c^7*f*cos(f*x + e)^5 - 24*c^7*f*cos(f*x + e)^4 + 32*c^7*f*cos(f*x + e)^3 +
80*c^7*f*cos(f*x + e)^2 - 32*c^7*f*cos(f*x + e) - 64*c^7*f)*sin(f*x + e))
```

giac [B] time = 0.36, size = 445, normalized size = 2.85

$$-\frac{2\left(9009\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{12}-27027\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}+9009\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}+153153\,Aa^3\,\mathrm{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorit
hm="giac")
```

```
[Out] -2/9009*(9009*A*a^3*tan(1/2*f*x + 1/2*e)^12 - 27027*A*a^3*tan(1/2*f*x + 1/2
*e)^11 + 9009*B*a^3*tan(1/2*f*x + 1/2*e)^11 + 153153*A*a^3*tan(1/2*f*x + 1/
2*e)^10 + 3003*B*a^3*tan(1/2*f*x + 1/2*e)^10 - 297297*A*a^3*tan(1/2*f*x + 1
/2*e)^9 + 69069*B*a^3*tan(1/2*f*x + 1/2*e)^9 + 648648*A*a^3*tan(1/2*f*x + 1
/2*e)^8 - 9009*B*a^3*tan(1/2*f*x + 1/2*e)^8 - 738738*A*a^3*tan(1/2*f*x + 1/
2*e)^7 + 150150*B*a^3*tan(1/2*f*x + 1/2*e)^7 + 857142*A*a^3*tan(1/2*f*x + 1
/2*e)^6 - 16302*B*a^3*tan(1/2*f*x + 1/2*e)^6 - 548262*A*a^3*tan(1/2*f*x + 1
/2*e)^5 + 115830*B*a^3*tan(1/2*f*x + 1/2*e)^5 + 367653*A*a^3*tan(1/2*f*x +
1/2*e)^4 - 286*B*a^3*tan(1/2*f*x + 1/2*e)^4 - 112827*A*a^3*tan(1/2*f*x + 1/
2*e)^3 + 30745*B*a^3*tan(1/2*f*x + 1/2*e)^3 + 45513*A*a^3*tan(1/2*f*x + 1/2
*e)^2 + 1443*B*a^3*tan(1/2*f*x + 1/2*e)^2 - 3081*A*a^3*tan(1/2*f*x + 1/2*e)
 + 1261*B*a^3*tan(1/2*f*x + 1/2*e) + 930*A*a^3 - 97*B*a^3)/(c^7*f*(tan(1/2*
f*x + 1/2*e) - 1)^13)
```

maple [A] time = 0.57, size = 293, normalized size = 1.88

$$2a^3\left(-\frac{16000A+14720B}{10\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}}-\frac{512A+512B}{13\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{13}}-\frac{6888A+3928B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{150A+34B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{13112A+8840B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{3}{12(t}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x)
```

```
[Out] 2/f*a^3/c^7*(-1/10*(16000*A+14720*B)/(tan(1/2*f*x+1/2*e)-1)^10-1/13*(512*A+
512*B)/(tan(1/2*f*x+1/2*e)-1)^13-1/6*(6888*A+3928*B)/(tan(1/2*f*x+1/2*e)-1)
^6-A/(tan(1/2*f*x+1/2*e)-1)-1/3*(150*A+34*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/7*(
13112*A+8840*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/12*(3072*A+3072*B)/(tan(1/2*f*x+
1/2*e)-1)^12-1/5*(2700*A+1240*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/8*(18816*A+1446
4*B)/(tan(1/2*f*x+1/2*e)-1)^8-1/4*(768*A+264*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/
11*(8832*A+8576*B)/(tan(1/2*f*x+1/2*e)-1)^11-1/2*(18*A+2*B)/(tan(1/2*f*x+1/
2*e)-1)^2-1/9*(20256*A+17248*B)/(tan(1/2*f*x+1/2*e)-1)^9)
```

maxima [B] time = 0.57, size = 4078, normalized size = 26.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorit
hm="maxima")
```

```
[Out] -2/45045*(6*A*a^3*(4771*sin(f*x + e)/(cos(f*x + e) + 1) - 28626*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 74932*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1873
```

30*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 265122*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 - 353496*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 276276*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7 - 207207*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 7507
5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 30030*sin(f*x + e)^10/(cos(f*x + e)
 + 1)^10 - 367)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*
c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x
+ e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x
 + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^
11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(co
s(f*x + e) + 1)^13) + 6*B*a^3*(4771*sin(f*x + e)/(cos(f*x + e) + 1) - 28626
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 74932*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 - 187330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 265122*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5 - 353496*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 276276*
sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 207207*sin(f*x + e)^8/(cos(f*x + e) +
 1)^8 + 75075*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 30030*sin(f*x + e)^10/(
cos(f*x + e) + 1)^10 - 367)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) +
 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) +
1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286
*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*
x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x
 + e)^13/(cos(f*x + e) + 1)^13) + 15*A*a^3*(3796*sin(f*x + e)/(cos(f*x + e)
 + 1) - 22776*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 77506*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 - 193765*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 339768*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5 - 453024*sin(f*x + e)^6/(cos(f*x + e) + 1
)^6 + 444444*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 333333*sin(f*x + e)^8/(c
os(f*x + e) + 1)^8 + 180180*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 72072*sin
(f*x + e)^10/(cos(f*x + e) + 1)^10 + 18018*sin(f*x + e)^11/(cos(f*x + e) +
1)^11 - 3003*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 523)/(c^7 - 13*c^7*sin
(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 -
 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e
)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 -
78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(
f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*x + e) + 1)^13) - 105*A*a^3*(
611*sin(f*x + e)/(cos(f*x + e) + 1) - 2379*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 8723*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 18590*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 + 33462*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 40326*sin(f*x

+ e)^6/(cos(f*x + e) + 1)^6 + 40326*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 27027*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 15015*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 4719*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 1287*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 47)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*x + e) + 1)^13) - 35*B*a^3*(611*sin(f*x + e)/(cos(f*x + e) + 1) - 2379*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8723*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 18590*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33462*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 40326*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 40326*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 27027*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 15015*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 4719*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 1287*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 47)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*x + e) + 1)^13) + 8*B*a^3*(559*sin(f*x + e)/(cos(f*x + e) + 1) - 3354*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 12298*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 30745*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 37323*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 49764*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24024*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 18018*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 43)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*x + e) + 1)^13) - 462*A*a^3*(13*sin(f*x + e)/(cos(f*x + e) + 1) - 78*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 286*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 520*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 936*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 858*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 858*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 351*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 195*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 1)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x +

```
 e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*s
in(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e)
+ 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e)
^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 +
13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*x
 + e) + 1)^13) - 1386*B*a^3*(13*sin(f*x + e)/(cos(f*x + e) + 1) - 78*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 286*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5
20*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 936*sin(f*x + e)^5/(cos(f*x + e) +
 1)^5 - 858*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 858*sin(f*x + e)^7/(cos(f
*x + e) + 1)^7 - 351*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 195*sin(f*x + e)
^9/(cos(f*x + e) + 1)^9 - 1)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1)
+ 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) +
 1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e
)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 28
6*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f
*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*
x + e)^13/(cos(f*x + e) + 1)^13))/f
```

mupad [B] time = 13.79, size = 500, normalized size = 3.21

$$2\cos\left(\frac{e}{2}+\frac{f x}{2}\right)\left(\frac{2363 B a^3}{32}-\frac{279183 A a^3}{16}+\frac{220269 A a^3 \cos\left(2 e+2 f x\right)}{16}-\frac{46095 A a^3 \cos\left(3 e+3 f x\right)}{16}-\frac{20829 A a^3 \cos\left(4 e+4 f x\right)}{16}+\frac{281}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^7,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((2363*B*a^3)/32 - (279183*A*a^3)/16 + (220269*A*a^3*
cos(2*e + 2*f*x))/16 - (46095*A*a^3*cos(3*e + 3*f*x))/16 - (20829*A*a^3*cos
(4*e + 4*f*x))/16 + (2811*A*a^3*cos(5*e + 5*f*x))/16 + (231*A*a^3*cos(6*e +
 6*f*x))/16 - (8995*B*a^3*cos(2*e + 2*f*x))/64 + (497*B*a^3*cos(3*e + 3*f*x
))/16 + (3725*B*a^3*cos(4*e + 4*f*x))/32 - (361*B*a^3*cos(5*e + 5*f*x))/16
- (77*B*a^3*cos(6*e + 6*f*x))/64 - (19305*A*a^3*sin(2*e + 2*f*x))/4 - (8108
1*A*a^3*sin(3*e + 3*f*x))/16 + (15015*A*a^3*sin(4*e + 4*f*x))/16 + (3237*A*
a^3*sin(5*e + 5*f*x))/16 - (117*A*a^3*sin(6*e + 6*f*x))/8 + (77649*B*a^3*si
n(2*e + 2*f*x))/64 + (27027*B*a^3*sin(3*e + 3*f*x))/32 - (1001*B*a^3*sin(4*
e + 4*f*x))/8 - (559*B*a^3*sin(5*e + 5*f*x))/32 + (117*B*a^3*sin(6*e + 6*f*
x))/64 + (26979*A*a^3*cos(e + f*x))/4 + 40*B*a^3*cos(e + f*x) + (173745*A*a
^3*sin(e + f*x))/8 - (80223*B*a^3*sin(e + f*x))/16))/(9009*c^7*f*((1287*2^(
```

```
1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/64 - (429*2^(1/2)*cos(e/2 + pi/4 + (f
*x)/2))/16 + (715*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/64 - (143*2^(1/2
)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/32 - (39*2^(1/2)*cos((9*e)/2 + pi/4 + (9
*f*x)/2))/32 + (13*2^(1/2)*cos((11*e)/2 - pi/4 + (11*f*x)/2))/64 + (2^(1/2)
*cos((13*e)/2 + pi/4 + (13*f*x)/2))/64))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**7,x)
```

```
[Out] Timed out
```

3.51 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8} dx$

Optimal. Leaf size=197

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}+\frac{a^3c^2(4A-11B)\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}}+\frac{2a^3(4A-11B)\cos^7(e+fx)}{45045cf(c-c\sin(e+fx))^7}+\frac{2a^3(4A-11B)\cos^7(e+}{6435f(c-c\sin(e+fx}$$

```
[Out] 1/15*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^11+1/195*a^3*(4*A-11*B)*
c^2*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^10+1/715*a^3*(4*A-11*B)*c*cos(f*x+e)^7/
f/(c-c*sin(f*x+e))^9+2/6435*a^3*(4*A-11*B)*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^
8+2/45045*a^3*(4*A-11*B)*cos(f*x+e)^7/c/f/(c-c*sin(f*x+e))^7
```

Rubi [A] time = 0.44, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 2672, 2671}

$$\frac{a^3c^2(4A-11B)\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}+\frac{2a^3(4A-11B)\cos^7(e+fx)}{45045cf(c-c\sin(e+fx))^7}+\frac{2a^3(4A-11B)\cos^7(e+}{6435f(c-c\sin(e+fx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^8,x]
```

```
[Out] (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(15*f*(c - c*Sin[e + f*x])^11) + (a^3*(4*A
 - 11*B)*c^2*Cos[e + f*x]^7)/(195*f*(c - c*Sin[e + f*x])^10) + (a^3*(4*A -
11*B)*c*Cos[e + f*x]^7)/(715*f*(c - c*Sin[e + f*x])^9) + (2*a^3*(4*A - 11*B
)*Cos[e + f*x]^7)/(6435*f*(c - c*Sin[e + f*x])^8) + (2*a^3*(4*A - 11*B)*Cos
[e + f*x]^7)/(45045*c*f*(c - c*Sin[e + f*x])^7)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11}}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{1}{15}\left(a^3(4A-11B)c^2\right)\int \frac{\cos^6(e+}{(c-c\sin(e+}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{a^3(4A-11B)c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}} + \frac{1}{65}\Big($$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{a^3(4A-11B)c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}} + \frac{a^3(4}{71}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{a^3(4A-11B)c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}} + \frac{a^3(4}{71}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{a^3(4A-11B)c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}} + \frac{a^3(4}{71}$$

Mathematica [A] time = 6.69, size = 378, normalized size = 1.92

$$\frac{(a\sin(e+fx)+a)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(437580A\sin\left(\frac{1}{2}(e+fx)\right)+240240A\sin\left(\frac{3}{2}(e+fx)\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^8,x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a + a*Sin[e + f*x])^3*(463320*A*Cos
[(e + f*x)/2] + 302445*B*Cos[(e + f*x)/2] - 260260*A*Cos[(3*(e + f*x))/2] -
 230230*B*Cos[(3*(e + f*x))/2] - 72072*A*Cos[(5*(e + f*x))/2] - 117117*B*Co
s[(5*(e + f*x))/2] + 5460*A*Cos[(7*(e + f*x))/2] + 30030*B*Cos[(7*(e + f*x)
)/2] - 420*A*Cos[(11*(e + f*x))/2] + 1155*B*Cos[(11*(e + f*x))/2] + 4*A*Cos
[(15*(e + f*x))/2] - 11*B*Cos[(15*(e + f*x))/2] + 437580*A*Sin[(e + f*x)/2]
 + 373230*B*Sin[(e + f*x)/2] + 240240*A*Sin[(3*(e + f*x))/2] + 285285*B*Sin
[(3*(e + f*x))/2] - 60060*A*Sin[(5*(e + f*x))/2] - 150150*B*Sin[(5*(e + f*x
))/2] - 45045*B*Sin[(7*(e + f*x))/2] - 1820*A*Sin[(9*(e + f*x))/2] + 5005*B
*Sin[(9*(e + f*x))/2] + 60*A*Sin[(13*(e + f*x))/2] - 165*B*Sin[(13*(e + f*x
))/2]))/(1441440*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f
*x])^8)
```

fricas [B] time = 0.46, size = 541, normalized size = 2.75

$$\frac{2\,(4\,A-11\,B)a^3\cos\left(fx+e\right)^8+16\,(4\,A-11\,B)a^3\cos\left(fx+e\right)^7-49\,(4\,A-11\,B)a^3\cos\left(fx+e\right)^6-168\,(4\,A-1}{45045\left(c^8f\cos\left(fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x, algorit
hm="fricas")
```

```
[Out] 1/45045*(2*(4*A - 11*B)*a^3*cos(f*x + e)^8 + 16*(4*A - 11*B)*a^3*cos(f*x +
e)^7 - 49*(4*A - 11*B)*a^3*cos(f*x + e)^6 - 168*(4*A - 11*B)*a^3*cos(f*x +
e)^5 + 105*(7*A + 88*B)*a^3*cos(f*x + e)^4 - 231*(31*A + 61*B)*a^3*cos(f*x
+ e)^3 - 924*(22*A + 37*B)*a^3*cos(f*x + e)^2 + 12012*(A + B)*a^3*cos(f*x +
 e) + 24024*(A + B)*a^3 - (2*(4*A - 11*B)*a^3*cos(f*x + e)^7 - 14*(4*A - 11
*B)*a^3*cos(f*x + e)^6 - 63*(4*A - 11*B)*a^3*cos(f*x + e)^5 + 105*(4*A - 11
*B)*a^3*cos(f*x + e)^4 + 1155*(A + 7*B)*a^3*cos(f*x + e)^3 + 2772*(3*A + 8*
B)*a^3*cos(f*x + e)^2 - 12012*(A + B)*a^3*cos(f*x + e) - 24024*(A + B)*a^3)
*sin(f*x + e))/(c^8*f*cos(f*x + e)^8 - 7*c^8*f*cos(f*x + e)^7 - 32*c^8*f*co
s(f*x + e)^6 + 56*c^8*f*cos(f*x + e)^5 + 160*c^8*f*cos(f*x + e)^4 - 112*c^8
*f*cos(f*x + e)^3 - 256*c^8*f*cos(f*x + e)^2 + 64*c^8*f*cos(f*x + e) + 128*
c^8*f + (c^8*f*cos(f*x + e)^7 + 8*c^8*f*cos(f*x + e)^6 - 24*c^8*f*cos(f*x +
 e)^5 - 80*c^8*f*cos(f*x + e)^4 + 80*c^8*f*cos(f*x + e)^3 + 192*c^8*f*cos(f
*x + e)^2 - 64*c^8*f*cos(f*x + e) - 128*c^8*f)*sin(f*x + e))
```

giac [B] time = 0.39, size = 517, normalized size = 2.62

$$-\frac{2\left(45045\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{14}-180180\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{13}+45045\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{13}+1066065\,Aa\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x, algorit
hm="giac")
```

```
[Out] -2/45045*(45045*A*a^3*tan(1/2*f*x + 1/2*e)^14 - 180180*A*a^3*tan(1/2*f*x +
1/2*e)^13 + 45045*B*a^3*tan(1/2*f*x + 1/2*e)^13 + 1066065*A*a^3*tan(1/2*f*x
 + 1/2*e)^12 - 15015*B*a^3*tan(1/2*f*x + 1/2*e)^12 - 2702700*A*a^3*tan(1/2*
f*x + 1/2*e)^11 + 450450*B*a^3*tan(1/2*f*x + 1/2*e)^11 + 6675669*A*a^3*tan(
1/2*f*x + 1/2*e)^10 - 306306*B*a^3*tan(1/2*f*x + 1/2*e)^10 - 10210200*A*a^3
*tan(1/2*f*x + 1/2*e)^9 + 1456455*B*a^3*tan(1/2*f*x + 1/2*e)^9 + 14124825*A
*a^3*tan(1/2*f*x + 1/2*e)^8 - 791505*B*a^3*tan(1/2*f*x + 1/2*e)^8 - 1317888
0*A*a^3*tan(1/2*f*x + 1/2*e)^7 + 1827540*B*a^3*tan(1/2*f*x + 1/2*e)^7 + 110
26015*A*a^3*tan(1/2*f*x + 1/2*e)^6 - 580580*B*a^3*tan(1/2*f*x + 1/2*e)^6 -
6066060*A*a^3*tan(1/2*f*x + 1/2*e)^5 + 915915*B*a^3*tan(1/2*f*x + 1/2*e)^5
+ 3088995*A*a^3*tan(1/2*f*x + 1/2*e)^4 - 105105*B*a^3*tan(1/2*f*x + 1/2*e)^
4 - 864500*A*a^3*tan(1/2*f*x + 1/2*e)^3 + 170170*B*a^3*tan(1/2*f*x + 1/2*e)
^3 + 265335*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 2310*B*a^3*tan(1/2*f*x + 1/2*e)^
2 - 18600*A*a^3*tan(1/2*f*x + 1/2*e) + 6105*B*a^3*tan(1/2*f*x + 1/2*e) + 42
43*A*a^3 - 407*B*a^3)/(c^8*f*(tan(1/2*f*x + 1/2*e) - 1)^15)
```

maple [A] time = 0.58, size = 337, normalized size = 1.71

$$\frac{2a^{3}\left(-\frac{4536A+1836B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{20A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{81344A+72512B}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}}-\frac{24320A+23808B}{13\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{13}}-\frac{1104A+336B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{322}{7(\mathrm{ta}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x)
```

```
[Out] 2/f*a^3/c^8*(-1/5*(4536*A+1836*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/2*(20*A+2*B)/(
tan(1/2*f*x+1/2*e)-1)^2-1/11*(81344*A+72512*B)/(tan(1/2*f*x+1/2*e)-1)^11-1/
13*(24320*A+23808*B)/(tan(1/2*f*x+1/2*e)-1)^13-1/4*(1104*A+336*B)/(tan(1/2*
f*x+1/2*e)-1)^4-A/(tan(1/2*f*x+1/2*e)-1)-1/7*(32288*A+19176*B)/(tan(1/2*f*x
+1/2*e)-1)^7-1/15*(1024*A+1024*B)/(tan(1/2*f*x+1/2*e)-1)^15-1/6*(13824*A+69
36*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/14*(7168*A+7168*B)/(tan(1/2*f*x+1/2*e)-1)^
```

```
14-1/8*(58816*A+40000*B)/(tan(1/2*f*x+1/2*e)-1)^8-1/12*(52736*A+49664*B)/(t
an(1/2*f*x+1/2*e)-1)^12-1/3*(188*A+38*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/9*(8411
2*A+63856*B)/(tan(1/2*f*x+1/2*e)-1)^9-1/10*(94144*A+78144*B)/(tan(1/2*f*x+1
/2*e)-1)^10)
```

maxima [B] time = 0.68, size = 4765, normalized size = 24.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x, algorit
hm="maxima")
```

```
[Out] 2/45045*(3*A*a^3*(17715*sin(f*x + e)/(cos(f*x + e) + 1) - 78960*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 342160*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 891
345*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1960959*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5 - 3043040*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 3912480*sin(f*x
+ e)^7/(cos(f*x + e) + 1)^7 - 3687255*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 +
 2867865*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 1585584*sin(f*x + e)^10/(cos
(f*x + e) + 1)^10 + 720720*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 195195*s
in(f*x + e)^12/(cos(f*x + e) + 1)^12 + 45045*sin(f*x + e)^13/(cos(f*x + e)
+ 1)^13 - 1181)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e) + 1) + 105*c^8*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8*sin(f*x + e)^
5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 643
5*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*sin(f*x + e)^8/(cos(f*
x + e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3003*c^8*sin
(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x + e)^11/(cos(f*x + e)
 + 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 105*c^8*sin(f*x
+ e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/(cos(f*x + e) + 1)^1
4 - c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15) + B*a^3*(17715*sin(f*x + e)/
(cos(f*x + e) + 1) - 78960*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 342160*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 - 891345*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + 1960959*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 3043040*sin(f*x + e)^6/(
cos(f*x + e) + 1)^6 + 3912480*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3687255
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 2867865*sin(f*x + e)^9/(cos(f*x + e)
 + 1)^9 - 1585584*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 720720*sin(f*x +
e)^11/(cos(f*x + e) + 1)^11 - 195195*sin(f*x + e)^12/(cos(f*x + e) + 1)^12
+ 45045*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - 1181)/(c^8 - 15*c^8*sin(f*x
 + e)/(cos(f*x + e) + 1) + 105*c^8*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 45
5*c^8*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 - 3003*c^8*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin
(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6435*c^8*sin(f*x + e)^7/(cos(f*x + e) +
1)^7 + 6435*c^8*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5005*c^8*sin(f*x + e)
^9/(cos(f*x + e) + 1)^9 + 3003*c^8*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 -
```

1365*c^8*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 455*c^8*sin(f*x + e)^12/(c
os(f*x + e) + 1)^12 - 105*c^8*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 15*c^
8*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 - c^8*sin(f*x + e)^15/(cos(f*x + e)
 + 1)^15) - 7*A*a^3*(7845*sin(f*x + e)/(cos(f*x + e) + 1) - 54915*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 222950*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6
68850*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1444443*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5 - 2407405*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 3063060*sin(f*
x + e)^7/(cos(f*x + e) + 1)^7 - 3063060*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
 + 2357355*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 1414413*sin(f*x + e)^10/(c
os(f*x + e) + 1)^10 + 630630*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 210210
*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 45045*sin(f*x + e)^13/(cos(f*x + e
) + 1)^13 - 6435*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 - 952)/(c^8 - 15*c^8
*sin(f*x + e)/(cos(f*x + e) + 1) + 105*c^8*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1365*c^8*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 - 3003*c^8*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5005
*c^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6435*c^8*sin(f*x + e)^7/(cos(f*x
 + e) + 1)^7 + 6435*c^8*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5005*c^8*sin(
f*x + e)^9/(cos(f*x + e) + 1)^9 + 3003*c^8*sin(f*x + e)^10/(cos(f*x + e) +
1)^10 - 1365*c^8*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 455*c^8*sin(f*x +
e)^12/(cos(f*x + e) + 1)^12 - 105*c^8*sin(f*x + e)^13/(cos(f*x + e) + 1)^13
 + 15*c^8*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 - c^8*sin(f*x + e)^15/(cos(
f*x + e) + 1)^15) - 12*A*a^3*(1740*sin(f*x + e)/(cos(f*x + e) + 1) - 12180*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 37765*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 - 113295*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 204204*sin(f*x + e)^5/(
cos(f*x + e) + 1)^5 - 340340*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 373230*s
in(f*x + e)^7/(cos(f*x + e) + 1)^7 - 373230*sin(f*x + e)^8/(cos(f*x + e) +
1)^8 + 240240*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 144144*sin(f*x + e)^10/
(cos(f*x + e) + 1)^10 + 45045*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 15015
*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 116)/(c^8 - 15*c^8*sin(f*x + e)/(c
os(f*x + e) + 1) + 105*c^8*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) +
 1)^4 - 3003*c^8*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e
)^6/(cos(f*x + e) + 1)^6 - 6435*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6
435*c^8*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(
f*x + e) + 1)^9 + 3003*c^8*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8
*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x +
 e) + 1)^12 - 105*c^8*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*
x + e)^14/(cos(f*x + e) + 1)^14 - c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15
) - 12*B*a^3*(1740*sin(f*x + e)/(cos(f*x + e) + 1) - 12180*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 37765*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 113295*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 + 204204*sin(f*x + e)^5/(cos(f*x + e) + 1
)^5 - 340340*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 373230*sin(f*x + e)^7/(c
os(f*x + e) + 1)^7 - 373230*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 240240*si
n(f*x + e)^9/(cos(f*x + e) + 1)^9 - 144144*sin(f*x + e)^10/(cos(f*x + e) +
1)^10 + 45045*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 15015*sin(f*x + e)^12

/(cos(f*x + e) + 1)^12 - 116)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e) + 1)
 + 105*c^8*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 - 6435*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*sin(f*x
+ e)^8/(cos(f*x + e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9
+ 3003*c^8*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x + e)^11
/(cos(f*x + e) + 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 10
5*c^8*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/(cos(f
*x + e) + 1)^14 - c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15) + 6*A*a^3*(675
*sin(f*x + e)/(cos(f*x + e) + 1) - 4725*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 20475*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 46410*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 + 102102*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 130130*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 167310*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 -
 122265*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 95095*sin(f*x + e)^9/(cos(f*x
 + e) + 1)^9 - 33033*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 15015*sin(f*x
+ e)^11/(cos(f*x + e) + 1)^11 - 45)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e
) + 1) + 105*c^8*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 30
03*c^8*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f
*x + e) + 1)^6 - 6435*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*si
n(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) +
 1)^9 + 3003*c^8*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x +
 e)^11/(cos(f*x + e) + 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^1
2 - 105*c^8*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/
(cos(f*x + e) + 1)^14 - c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15) + 18*B*a
^3*(675*sin(f*x + e)/(cos(f*x + e) + 1) - 4725*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 20475*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 46410*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 102102*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 130130*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 167310*sin(f*x + e)^7/(cos(f*x + e) +
 1)^7 - 122265*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 95095*sin(f*x + e)^9/(
cos(f*x + e) + 1)^9 - 33033*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 15015*s
in(f*x + e)^11/(cos(f*x + e) + 1)^11 - 45)/(c^8 - 15*c^8*sin(f*x + e)/(cos(
f*x + e) + 1) + 105*c^8*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 - 3003*c^8*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 - 6435*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435
*c^8*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x
 + e) + 1)^9 + 3003*c^8*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*si
n(f*x + e)^11/(cos(f*x + e) + 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x + e)
 + 1)^12 - 105*c^8*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*x +
 e)^14/(cos(f*x + e) + 1)^14 - c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15) -
 48*B*a^3*(60*sin(f*x + e)/(cos(f*x + e) + 1) - 420*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 1820*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5460*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 9009*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 15015*

```
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 12870*sin(f*x + e)^7/(cos(f*x + e) +
1)^7 - 12870*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 5005*sin(f*x + e)^9/(cos
(f*x + e) + 1)^9 - 3003*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 4)/(c^8 - 1
5*c^8*sin(f*x + e)/(cos(f*x + e) + 1) + 105*c^8*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1365*c^8*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
 5005*c^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6435*c^8*sin(f*x + e)^7/(co
s(f*x + e) + 1)^7 + 6435*c^8*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5005*c^8
*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3003*c^8*sin(f*x + e)^10/(cos(f*x +
e) + 1)^10 - 1365*c^8*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 455*c^8*sin(f
*x + e)^12/(cos(f*x + e) + 1)^12 - 105*c^8*sin(f*x + e)^13/(cos(f*x + e) +
1)^13 + 15*c^8*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 - c^8*sin(f*x + e)^15/
(cos(f*x + e) + 1)^15))/f
```

mupad [B] time = 13.96, size = 577, normalized size = 2.93

$$2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{544369\,A\,a^3}{4}-\frac{21791\,B\,a^3}{4}-\frac{257861\,A\,a^3\,\cos\left(2\,e+2\,f\,x\right)}{2}+\frac{3497111\,A\,a^3\,\cos\left(3\,e+3\,f\,x\right)}{128}+\frac{72047\,A\,a^3\,\cos\left(4\,e+4\,f\,x\right)}{4}-\frac{37\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^8,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((544369*A*a^3)/4 - (21791*B*a^3)/4 - (257861*A*a^3*c
os(2*e + 2*f*x))/2 + (3497111*A*a^3*cos(3*e + 3*f*x))/128 + (72047*A*a^3*co
s(4*e + 4*f*x))/4 - (378579*A*a^3*cos(5*e + 5*f*x))/128 - (1059*A*a^3*cos(6
*e + 6*f*x))/2 + (4251*A*a^3*cos(7*e + 7*f*x))/128 + (219769*B*a^3*cos(2*e
+ 2*f*x))/32 - (191389*B*a^3*cos(3*e + 3*f*x))/128 - 1672*B*a^3*cos(4*e + 4
*f*x) + (38841*B*a^3*cos(5*e + 5*f*x))/128 + (1551*B*a^3*cos(6*e + 6*f*x))/
32 - (429*B*a^3*cos(7*e + 7*f*x))/128 + (2633345*A*a^3*sin(2*e + 2*f*x))/64
 + (7210775*A*a^3*sin(3*e + 3*f*x))/128 - (89375*A*a^3*sin(4*e + 4*f*x))/8
- (504205*A*a^3*sin(5*e + 5*f*x))/128 + (29765*A*a^3*sin(6*e + 6*f*x))/64 +
 (4235*A*a^3*sin(7*e + 7*f*x))/128 - (451165*B*a^3*sin(2*e + 2*f*x))/64 - (
854425*B*a^3*sin(3*e + 3*f*x))/128 + (9295*B*a^3*sin(4*e + 4*f*x))/8 + (464
75*B*a^3*sin(5*e + 5*f*x))/128 - (3025*B*a^3*sin(6*e + 6*f*x))/64 - (385*B*
a^3*sin(7*e + 7*f*x))/128 - (5734111*A*a^3*cos(e + f*x))/128 + (126929*B*a^
3*cos(e + f*x))/128 - (25501905*A*a^3*sin(e + f*x))/128 + (3970395*B*a^3*si
n(e + f*x))/128))/(45045*c^8*f*((6435*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/12
8 - (5005*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/128 - (3003*2^(1/2)*cos(
(5*e)/2 + pi/4 + (5*f*x)/2))/128 + (1365*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*
x)/2))/128 + (455*2^(1/2)*cos((9*e)/2 + pi/4 + (9*f*x)/2))/128 - (105*2^(1/
2)*cos((11*e)/2 - pi/4 + (11*f*x)/2))/128 - (15*2^(1/2)*cos((13*e)/2 + pi/4
 + (13*f*x)/2))/128 + (2^(1/2)*cos((15*e)/2 - pi/4 + (15*f*x)/2))/128))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**8,x)
[Out] Timed out
```

3.52 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=190

$$-\frac{a^4c^4(A-B)\cos^9(e+fx)}{f(a\sin(e+fx)+a)^5}-\frac{2a^2c^4(4A-5B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^3}-\frac{35c^4(4A-5B)\cos^3(e+fx)}{12af}-\frac{7c^4(4A-5B)\cos^5(e+}{4f(a\sin(e+fx)+a)}$$

```
[Out] -35/8*(4*A-5*B)*c^4*x/a-35/12*(4*A-5*B)*c^4*cos(f*x+e)^3/a/f-35/8*(4*A-5*B)
*c^4*cos(f*x+e)*sin(f*x+e)/a/f-a^4*(A-B)*c^4*cos(f*x+e)^9/f/(a+a*sin(f*x+e)
)^5-2*a^2*(4*A-5*B)*c^4*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^3-7/4*(4*A-5*B)*c^4
*cos(f*x+e)^5/f/(a+a*sin(f*x+e))
```

Rubi [A] time = 0.36, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2967, 2859, 2680, 2679, 2682, 2635, 8}

$$-\frac{a^4c^4(A-B)\cos^9(e+fx)}{f(a\sin(e+fx)+a)^5}-\frac{2a^2c^4(4A-5B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^3}-\frac{35c^4(4A-5B)\cos^3(e+fx)}{12af}-\frac{7c^4(4A-5B)\cos^5(e+}{4f(a\sin(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-35*(4*A - 5*B)*c^4*x)/(8*a) - (35*(4*A - 5*B)*c^4*Cos[e + f*x]^3)/(12*a*f
) - (35*(4*A - 5*B)*c^4*Cos[e + f*x]*Sin[e + f*x])/(8*a*f) - (a^4*(A - B)*c
^4*Cos[e + f*x]^9)/(f*(a + a*Sin[e + f*x])^5) - (2*a^2*(4*A - 5*B)*c^4*Cos[
e + f*x]^7)/(f*(a + a*Sin[e + f*x])^3) - (7*(4*A - 5*B)*c^4*Cos[e + f*x]^5)
/(4*f*(a + a*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
```

```
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx = \left(a^4c^4\right)\int \frac{\cos^8(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^5}\,dx$$
$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{f(a+a\sin(e+fx))^5} - \left(a^3(4A-5B)c^4\right)\int \frac{\cos^8(e+fx}{(a+a\sin(e+f}$$
$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{f(a+a\sin(e+fx))^5} - \frac{2a^2(4A-5B)c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^3} - \left(7a\right.$$
$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{f(a+a\sin(e+fx))^5} - \frac{2a^2(4A-5B)c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^3} - \frac{7(4}{4}$$
$$= -\frac{35(4A-5B)c^4\cos^3(e+fx)}{12af} - \frac{a^4(A-B)c^4\cos^9(e+fx)}{f(a+a\sin(e+fx))^5} - \frac{2a^2(}{}$$
$$= -\frac{35(4A-5B)c^4\cos^3(e+fx)}{12af} - \frac{35(4A-5B)c^4\cos(e+fx)\sin(e}{8af}$$
$$= -\frac{35(4A-5B)c^4x}{8a} - \frac{35(4A-5B)c^4\cos^3(e+fx)}{12af} - \frac{35(4A-5B)c}{}$$

Mathematica [A] time = 2.36, size = 274, normalized size = 1.44

$$\frac{(c-c\sin(e+fx))^4\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(3072(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-420(4A-5B)(e+fx)\left(\text{si}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x]
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^4*(3072*(A - B)
*Sin[(e + f*x)/2] - 420*(4*A - 5*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2]) - 24*(47*A - 75*B)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2]) + 8*(A - 5*B)*Cos[3*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) +
24*(5*A - 12*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sin[2*(e + f*x)] + 3*
B*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sin[4*(e + f*x)]))/(96*a*f*(Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])^8*(1 + Sin[e + f*x]))
```

fricas [A] time = 0.46, size = 261, normalized size = 1.37

$$-\frac{6\,Bc^4\cos\left(fx+e\right)^5 - 8\,(A-5\,B)c^4\cos\left(fx+e\right)^4 + (52\,A-113\,B)c^4\cos\left(fx+e\right)^3 + 105\,(4\,A-5\,B)c^4fx + 96\,(3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] -1/24*(6*B*c^4*cos(f*x + e)^5 - 8*(A - 5*B)*c^4*cos(f*x + e)^4 + (52*A - 11
3*B)*c^4*cos(f*x + e)^3 + 105*(4*A - 5*B)*c^4*f*x + 96*(3*A - 5*B)*c^4*cos(
f*x + e)^2 + 384*(A - B)*c^4 + 3*(35*(4*A - 5*B)*c^4*f*x + (204*A - 239*B)*
c^4)*cos(f*x + e) - (6*B*c^4*cos(f*x + e)^4 + 2*(4*A - 17*B)*c^4*cos(f*x +
e)^3 - 105*(4*A - 5*B)*c^4*f*x + 3*(20*A - 49*B)*c^4*cos(f*x + e)^2 - 3*(76
*A - 111*B)*c^4*cos(f*x + e) + 384*(A - B)*c^4)*sin(f*x + e))/(a*f*cos(f*x
+ e) + a*f*sin(f*x + e) + a*f)
```

giac [A] time = 0.20, size = 343, normalized size = 1.81

$$-\frac{\frac{105\left(4\,Ac^4-5\,Bc^4\right)\left(fx+e\right)}{a}+\frac{768\left(Ac^4-Bc^4\right)}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{2\left(60\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-141\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+264\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-360\,Bc^4\tan(\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/24*(105*(4*A*c^4 - 5*B*c^4)*(f*x + e)/a + 768*(A*c^4 - B*c^4)/(a*(tan(1/
2*f*x + 1/2*e) + 1)) + 2*(60*A*c^4*tan(1/2*f*x + 1/2*e)^7 - 141*B*c^4*tan(1
/2*f*x + 1/2*e)^7 + 264*A*c^4*tan(1/2*f*x + 1/2*e)^6 - 360*B*c^4*tan(1/2*f*
x + 1/2*e)^6 + 60*A*c^4*tan(1/2*f*x + 1/2*e)^5 - 165*B*c^4*tan(1/2*f*x + 1/
2*e)^5 + 840*A*c^4*tan(1/2*f*x + 1/2*e)^4 - 1320*B*c^4*tan(1/2*f*x + 1/2*e)
^4 - 60*A*c^4*tan(1/2*f*x + 1/2*e)^3 + 165*B*c^4*tan(1/2*f*x + 1/2*e)^3 + 8
56*A*c^4*tan(1/2*f*x + 1/2*e)^2 - 1400*B*c^4*tan(1/2*f*x + 1/2*e)^2 - 60*A*
c^4*tan(1/2*f*x + 1/2*e) + 141*B*c^4*tan(1/2*f*x + 1/2*e) + 280*A*c^4 - 440
*B*c^4)/((tan(1/2*f*x + 1/2*e)^2 + 1)^4*a))/f
```

maple [B] time = 0.43, size = 678, normalized size = 3.57

$$-\frac{5c^4\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)A}{fa\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^4}+\frac{47c^4\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)B}{4fa\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^4}-\frac{22c^4\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)A}{fa\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^4}+\frac{30c^4\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)B}{fa\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^4}-\frac{5c^4\cdots}{fa\left(1\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x)
```

```
[Out] -5/f*c^4/a/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^7*A+47/4/f*c^4/a/(
1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^7*B-22/f*c^4/a/(1+tan(1/2*f*x+
1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^6*A+30/f*c^4/a/(1+tan(1/2*f*x+1/2*e)^2)^4*ta
n(1/2*f*x+1/2*e)^6*B-5/f*c^4/a/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e
)^5*A+55/4/f*c^4/a/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^5*B-70/f*c
^4/a/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^4*A+110/f*c^4/a/(1+tan(1
/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^4*B+5/f*c^4/a/(1+tan(1/2*f*x+1/2*e)^2
)^4*tan(1/2*f*x+1/2*e)^3*A-55/4/f*c^4/a/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*
f*x+1/2*e)^3*B-214/3/f*c^4/a/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^
2*A+350/3/f*c^4/a/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^2*B+5/f*c^4
/a/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)*A-47/4/f*c^4/a/(1+tan(1/2*
f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)*B-70/3/f*c^4/a/(1+tan(1/2*f*x+1/2*e)^2)^
4*A+110/3/f*c^4/a/(1+tan(1/2*f*x+1/2*e)^2)^4*B-35/f*c^4/a*arctan(tan(1/2*f*
x+1/2*e))*A+175/4/f*c^4/a*arctan(tan(1/2*f*x+1/2*e))*B-32/f*c^4/a/(tan(1/2*
f*x+1/2*e)+1)*A+32/f*c^4/a/(tan(1/2*f*x+1/2*e)+1)*B
```

maxima [B] time = 0.64, size = 1796, normalized size = 9.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] 1/12*(B*c^4*((19*sin(f*x + e)/(cos(f*x + e) + 1) + 211*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 91*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 219*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 165*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 165*sin
(f*x + e)^6/(cos(f*x + e) + 1)^6 + 45*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 +
 45*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 64)/(a + a*sin(f*x + e)/(cos(f*x
+ e) + 1) + 4*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 6*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 6*a*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5 + 4*a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 4*a
*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + a*sin(f*x + e)^8/(cos(f*x + e) + 1)^
8 + a*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 45*arctan(sin(f*x + e)/(cos(f*
x + e) + 1))/a) - 4*A*c^4*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 39*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 24*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
+ 9*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x + e)/(cos(f*x
+ e) + 1) + 3*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a*sin
(f*x + e)^7/(cos(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1
))/a) + 16*B*c^4*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 39*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 24*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9*sin(f
```

```
*x + e)^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)
 + 3*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 4
8*A*c^4*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*
arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 72*B*c^4*((sin(f*x + e)/(cos(f
*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4)/(a + a*sin(f
*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2*a*
sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x
+ e) + 1))/a) - 144*A*c^4*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) +
arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 96*B*c^4*((sin(f*x + e)/(cos(f
*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)
/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 96
*A*c^4*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(
cos(f*x + e) + 1))) + 24*B*c^4*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a +
 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) - 24*A*c^4/(a + a*sin(f*x + e)/
(cos(f*x + e) + 1)))/f
```

mupad [B] time = 14.78, size = 397, normalized size = 2.09

$$-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{55\,A\,c^4}{3}-\frac{299\,B\,c^4}{12}\right)+\frac{166\,A\,c^4}{3}-\frac{206\,B\,c^4}{3}+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7\,\left(27\,A\,c^4-\frac{167\,B\,c^4}{4}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8\,\left(37\,A\right.}{f\,\left(a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^9+a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8+4\,a\,\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^4)/(a + a*sin(e + f*x)),x)
```

```
[Out] - (tan(e/2 + (f*x)/2)*((55*A*c^4)/3 - (299*B*c^4)/12) + (166*A*c^4)/3 - (20
6*B*c^4)/3 + tan(e/2 + (f*x)/2)^7*(27*A*c^4 - (167*B*c^4)/4) + tan(e/2 + (f
*x)/2)^8*(37*A*c^4 - (175*B*c^4)/4) + tan(e/2 + (f*x)/2)^5*(75*A*c^4 - (495
*B*c^4)/4) + tan(e/2 + (f*x)/2)^6*(155*A*c^4 - (687*B*c^4)/4) + tan(e/2 + (
f*x)/2)^4*(257*A*c^4 - (1153*B*c^4)/4) + tan(e/2 + (f*x)/2)^3*((199*A*c^4)/
3 - (1235*B*c^4)/12) + tan(e/2 + (f*x)/2)^2*((583*A*c^4)/3 - (2795*B*c^4)/1
```

```
2))/(f*(a + a*tan(e/2 + (f*x)/2) + 4*a*tan(e/2 + (f*x)/2)^2 + 4*a*tan(e/2 +
 (f*x)/2)^3 + 6*a*tan(e/2 + (f*x)/2)^4 + 6*a*tan(e/2 + (f*x)/2)^5 + 4*a*tan
(e/2 + (f*x)/2)^6 + 4*a*tan(e/2 + (f*x)/2)^7 + a*tan(e/2 + (f*x)/2)^8 + a*t
an(e/2 + (f*x)/2)^9)) - (35*c^4*atan((35*c^4*tan(e/2 + (f*x)/2)*(4*A - 5*B)
)/(140*A*c^4 - 175*B*c^4))*(4*A - 5*B))/(4*a*f)
```

sympy [A] time = 26.87, size = 6690, normalized size = 35.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-420*A*c**4*f*x*tan(e/2 + f*x/2)**9/(24*a*f*tan(e/2 + f*x/2)**9
+ 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2
+ f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 9
6*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f
*x/2) + 24*a*f) - 420*A*c**4*f*x*tan(e/2 + f*x/2)**8/(24*a*f*tan(e/2 + f*x/
2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*ta
n(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)*
*4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e
/2 + f*x/2) + 24*a*f) - 1680*A*c**4*f*x*tan(e/2 + f*x/2)**7/(24*a*f*tan(e/2
 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96
*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 +
f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*
f*tan(e/2 + f*x/2) + 24*a*f) - 1680*A*c**4*f*x*tan(e/2 + f*x/2)**6/(24*a*f*
tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)*
*7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan
(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2
+ 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 2520*A*c**4*f*x*tan(e/2 + f*x/2)**5/(
24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 +
f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*
a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x
/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 2520*A*c**4*f*x*tan(e/2 + f*x/
2)**4/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan
(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5
 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/
2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 1680*A*c**4*f*x*tan(e/2
 + f*x/2)**3/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*
a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*
x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f
*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 1680*A*c**4*f*x*
tan(e/2 + f*x/2)**2/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**
8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e
/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 +
```

96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 420*A*c**4*f*x*tan(e/2 + f*x/2)/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 420*A*c**4*f*x/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 888*A*c**4*tan(e/2 + f*x/2)**8/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 648*A*c**4*tan(e/2 + f*x/2)**7/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 3720*A*c**4*tan(e/2 + f*x/2)**6/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 1800*A*c**4*tan(e/2 + f*x/2)**5/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 6168*A*c**4*tan(e/2 + f*x/2)**4/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 1592*A*c**4*tan(e/2 + f*x/2)**3/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 4664*A*c**4*tan(e/2 + f*x/2)**2/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 440*A*c**4*tan(e/2 + f*x/2)/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 1328*A*c**4/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) + 525*B*c**4*f*x*tan(

e/2 + f*x/2)**9/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 +
96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 +
 f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*
a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) + 525*B*c**4*f*
x*tan(e/2 + f*x/2)**8/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)
8 + 96*a*f*tan(e/2 + f*x/2)7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan
(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3
 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) + 2100*B*
c**4*f*x*tan(e/2 + f*x/2)**7/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 +
 f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*
a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*
x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) +
2100*B*c**4*f*x*tan(e/2 + f*x/2)**6/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*ta
n(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6
 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e
/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*
a*f) + 3150*B*c**4*f*x*tan(e/2 + f*x/2)**5/(24*a*f*tan(e/2 + f*x/2)**9 + 24
*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*
x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*
f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2
) + 24*a*f) + 3150*B*c**4*f*x*tan(e/2 + f*x/2)**4/(24*a*f*tan(e/2 + f*x/2)*
*9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e
/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4
+ 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2
+ f*x/2) + 24*a*f) + 2100*B*c**4*f*x*tan(e/2 + f*x/2)**3/(24*a*f*tan(e/2 +
f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*
f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x
/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*t
an(e/2 + f*x/2) + 24*a*f) + 2100*B*c**4*f*x*tan(e/2 + f*x/2)**2/(24*a*f*tan
(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7
+ 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/
2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 2
4*a*f*tan(e/2 + f*x/2) + 24*a*f) + 525*B*c**4*f*x*tan(e/2 + f*x/2)/(24*a*f*
tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)*
*7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan
(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2
+ 24*a*f*tan(e/2 + f*x/2) + 24*a*f) + 525*B*c**4*f*x/(24*a*f*tan(e/2 + f*x/
2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*ta
n(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)*
*4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e
/2 + f*x/2) + 24*a*f) + 1050*B*c**4*tan(e/2 + f*x/2)**8/(24*a*f*tan(e/2 + f
*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f
*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/
2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*ta
n(e/2 + f*x/2) + 24*a*f) + 1002*B*c**4*tan(e/2 + f*x/2)**7/(24*a*f*tan(e/2

```
+ f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*
a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f
*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f
*tan(e/2 + f*x/2) + 24*a*f) + 4122*B*c**4*tan(e/2 + f*x/2)**6/(24*a*f*tan(e
/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 +
96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2
+ f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*
a*f*tan(e/2 + f*x/2) + 24*a*f) + 2970*B*c**4*tan(e/2 + f*x/2)**5/(24*a*f*ta
n(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7
 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e
/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 +
24*a*f*tan(e/2 + f*x/2) + 24*a*f) + 6918*B*c**4*tan(e/2 + f*x/2)**4/(24*a*f
*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)
**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*ta
n(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2
 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) + 2470*B*c**4*tan(e/2 + f*x/2)**3/(24*
a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x
/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f
*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)
**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) + 5590*B*c**4*tan(e/2 + f*x/2)**2/(
24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 +
f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*
a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x
/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) + 598*B*c**4*tan(e/2 + f*x/2)/(2
4*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f
*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a
*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/
2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) + 1648*B*c**4/(24*a*f*tan(e/2 + f
*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f
*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/
2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*ta
n(e/2 + f*x/2) + 24*a*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c)**4/(
a*sin(e) + a), True))
```

3.53 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=157

$$-\frac{a^3c^3(A-B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}-\frac{2a^3c^3(3A-4B)\cos^5(e+fx)}{f\left(a^2\sin(e+fx)+a^2\right)^2}-\frac{5c^3(3A-4B)\cos^3(e+fx)}{3af}-\frac{5c^3(3A-4B)\sin(e+fx)}{2af}$$

```
[Out] -5/2*(3*A-4*B)*c^3*x/a-5/3*(3*A-4*B)*c^3*cos(f*x+e)^3/a/f-5/2*(3*A-4*B)*c^3
*cos(f*x+e)*sin(f*x+e)/a/f-a^3*(A-B)*c^3*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^4-
2*a^3*(3*A-4*B)*c^3*cos(f*x+e)^5/f/(a^2+a^2*sin(f*x+e))^2
```

Rubi [A] time = 0.32, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2967, 2859, 2680, 2682, 2635, 8}

$$-\frac{a^3c^3(A-B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}-\frac{2a^3c^3(3A-4B)\cos^5(e+fx)}{f\left(a^2\sin(e+fx)+a^2\right)^2}-\frac{5c^3(3A-4B)\cos^3(e+fx)}{3af}-\frac{5c^3(3A-4B)\sin(e+fx)}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-5*(3*A - 4*B)*c^3*x)/(2*a) - (5*(3*A - 4*B)*c^3*Cos[e + f*x]^3)/(3*a*f) -
 (5*(3*A - 4*B)*c^3*Cos[e + f*x]*Sin[e + f*x])/(2*a*f) - (a^3*(A - B)*c^3*C
os[e + f*x]^7)/(f*(a + a*Sin[e + f*x])^4) - (2*a^3*(3*A - 4*B)*c^3*Cos[e +
f*x]^5)/(f*(a^2 + a^2*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
```

```
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^4}\,dx$$
$$= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} - \left(a^2(3A-4B)c^3\right)\int \frac{\cos^6(e+fx}{(a+a\sin(e+f}$$
$$= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} - \frac{2a(3A-4B)c^3\cos^5(e+fx)}{f(a+a\sin(e+fx))^2} - \left(5(3\right.$$
$$= -\frac{5(3A-4B)c^3\cos^3(e+fx)}{3af} - \frac{a^3(A-B)c^3\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} - \frac{2a(3A}{f(}$$
$$= -\frac{5(3A-4B)c^3\cos^3(e+fx)}{3af} - \frac{5(3A-4B)c^3\cos(e+fx)\sin(e+}{2af}$$
$$= -\frac{5(3A-4B)c^3x}{2a} - \frac{5(3A-4B)c^3\cos^3(e+fx)}{3af} - \frac{5(3A-4B)c^3\,\mathrm{co}}{}$$

Mathematica [A] time = 1.40, size = 220, normalized size = 1.40

$$\frac{c^3(\sin(e+fx)-1)^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)(30(3A-4B)(e+fx)-3(A-4B)\sin(2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
),x]
```

```
[Out] (c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^3*(Cos[(e +
f*x)/2]*(30*(3*A - 4*B)*(e + f*x) + (48*A - 93*B)*Cos[e + f*x] + B*Cos[3*(e
 + f*x)] - 3*(A - 4*B)*Sin[2*(e + f*x)]) + Sin[(e + f*x)/2]*(-24*B*(-8 + 5*
e + 5*f*x) + 6*A*(-32 + 15*e + 15*f*x) + (48*A - 93*B)*Cos[e + f*x] + B*Cos
[3*(e + f*x)] - 3*(A - 4*B)*Sin[2*(e + f*x)])))/(12*a*f*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^6*(1 + Sin[e + f*x]))
```

fricas [A] time = 0.46, size = 218, normalized size = 1.39

$$-\frac{2\,Bc^3\cos\left(fx+e\right)^4+(3\,A-10\,B)c^3\cos\left(fx+e\right)^3+15\,(3\,A-4\,B)c^3fx+24\,(A-2\,B)c^3\cos\left(fx+e\right)^2+48\,(A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] -1/6*(2*B*c^3*cos(f*x + e)^4 + (3*A - 10*B)*c^3*cos(f*x + e)^3 + 15*(3*A -
4*B)*c^3*f*x + 24*(A - 2*B)*c^3*cos(f*x + e)^2 + 48*(A - B)*c^3 + 3*(5*(3*A
 - 4*B)*c^3*f*x + (23*A - 28*B)*c^3)*cos(f*x + e) + (2*B*c^3*cos(f*x + e)^3
 + 15*(3*A - 4*B)*c^3*f*x - 3*(A - 4*B)*c^3*cos(f*x + e)^2 + 3*(7*A - 12*B)
*c^3*cos(f*x + e) - 48*(A - B)*c^3)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*s
in(f*x + e) + a*f)
```

giac [A] time = 0.16, size = 235, normalized size = 1.50

$$-\frac{\frac{15\left(3\,Ac^3-4\,Bc^3\right)(fx+e)}{a}+\frac{96\left(Ac^3-Bc^3\right)}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{2\left(3\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-12\,Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+24\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-42\,Bc^3\tan\left(\frac{1}{2}\,fx+\right.}{}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/6*(15*(3*A*c^3 - 4*B*c^3)*(f*x + e)/a + 96*(A*c^3 - B*c^3)/(a*(tan(1/2*f
*x + 1/2*e) + 1)) + 2*(3*A*c^3*tan(1/2*f*x + 1/2*e)^5 - 12*B*c^3*tan(1/2*f*
x + 1/2*e)^5 + 24*A*c^3*tan(1/2*f*x + 1/2*e)^4 - 42*B*c^3*tan(1/2*f*x + 1/2
*e)^4 + 48*A*c^3*tan(1/2*f*x + 1/2*e)^2 - 96*B*c^3*tan(1/2*f*x + 1/2*e)^2 -
 3*A*c^3*tan(1/2*f*x + 1/2*e) + 12*B*c^3*tan(1/2*f*x + 1/2*e) + 24*A*c^3 -
46*B*c^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^3*a))/f
```

maple [B] time = 0.41, size = 449, normalized size = 2.86

$$-\frac{c^3\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)A}{fa\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}+\frac{4c^3\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)B}{fa\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}-\frac{8c^3\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)A}{fa\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}+\frac{14c^3\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)B}{fa\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}-\frac{16c^3\left(\right.}{fa\left(1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x)
```

```
[Out] -1/f*c^3/a/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*A+4/f*c^3/a/(1+t
an(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*B-8/f*c^3/a/(1+tan(1/2*f*x+1/2*
e)^2)^3*tan(1/2*f*x+1/2*e)^4*A+14/f*c^3/a/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/
2*f*x+1/2*e)^4*B-16/f*c^3/a/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2
*A+32/f*c^3/a/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2*B+1/f*c^3/a/(
1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*A-4/f*c^3/a/(1+tan(1/2*f*x+1/2
```

```
*e)^2)^3*tan(1/2*f*x+1/2*e)*B-8/f*c^3/a/(1+tan(1/2*f*x+1/2*e)^2)^3*A+46/3/f
*c^3/a/(1+tan(1/2*f*x+1/2*e)^2)^3*B-15/f*c^3/a*arctan(tan(1/2*f*x+1/2*e))*A
+20/f*c^3/a*arctan(tan(1/2*f*x+1/2*e))*B-16/f*c^3/a/(tan(1/2*f*x+1/2*e)+1)*
A+16/f*c^3/a/(tan(1/2*f*x+1/2*e)+1)*B
```

maxima [B] time = 0.46, size = 1120, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] 1/3*(B*c^3*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 39*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 24*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9*sin(f*x + e
)^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a*sin(f*x + e)^7/(co
s(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 3*A*c^3
*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
+ 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(
sin(f*x + e)/(cos(f*x + e) + 1))/a) + 9*B*c^3*((sin(f*x + e)/(cos(f*x + e)
+ 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4)/(a + a*sin(f*x + e)/
(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2*a*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1
))/a) - 18*A*c^3*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(si
n(f*x + e)/(cos(f*x + e) + 1))/a) + 18*B*c^3*((sin(f*x + e)/(cos(f*x + e) +
 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x
 + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 18*A*c^3*(a
rctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x +
 e) + 1))) + 6*B*c^3*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*
sin(f*x + e)/(cos(f*x + e) + 1))) - 6*A*c^3/(a + a*sin(f*x + e)/(cos(f*x +
e) + 1)))/f
```

mupad [B] time = 14.01, size = 319, normalized size = 2.03

$$-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(7\,A\,c^3-\frac{34\,B\,c^3}{3}\right)+24\,A\,c^3-\frac{94\,B\,c^3}{3}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(9\,A\,c^3-18\,B\,c^3\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,\left(17\,A\,c^3\right.}{f\,\left(a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+3\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+3\,a\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^3)/(a + a*sin(e + f*x)),x)
```

```
[Out] - (tan(e/2 + (f*x)/2)*(7*A*c^3 - (34*B*c^3)/3) + 24*A*c^3 - (94*B*c^3)/3 +
tan(e/2 + (f*x)/2)^5*(9*A*c^3 - 18*B*c^3) + tan(e/2 + (f*x)/2)^6*(17*A*c^3
- 20*B*c^3) + tan(e/2 + (f*x)/2)^3*(16*A*c^3 - 32*B*c^3) + tan(e/2 + (f*x)/
2)^4*(56*A*c^3 - 62*B*c^3) + tan(e/2 + (f*x)/2)^2*(63*A*c^3 - 76*B*c^3))/(f
*(a + a*tan(e/2 + (f*x)/2) + 3*a*tan(e/2 + (f*x)/2)^2 + 3*a*tan(e/2 + (f*x)
/2)^3 + 3*a*tan(e/2 + (f*x)/2)^4 + 3*a*tan(e/2 + (f*x)/2)^5 + a*tan(e/2 + (
f*x)/2)^6 + a*tan(e/2 + (f*x)/2)^7)) - (5*c^3*atan((5*c^3*tan(e/2 + (f*x)/2
)*(3*A - 4*B))/(15*A*c^3 - 20*B*c^3))*(3*A - 4*B))/(a*f)
```

sympy [A] time = 16.31, size = 4255, normalized size = 27.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-45*A*c**3*f*x*tan(e/2 + f*x/2)**7/(6*a*f*tan(e/2 + f*x/2)**7 +
6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f
*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*
tan(e/2 + f*x/2) + 6*a*f) - 45*A*c**3*f*x*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/
2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18
*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*
x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 135*A*c**3*f*x*tan(e/2 + f*x/2)
**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2
 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18
*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 135*A*c**3*f*x
*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6
 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2
 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f)
 - 135*A*c**3*f*x*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*ta
n(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4
 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
+ f*x/2) + 6*a*f) - 135*A*c**3*f*x*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x
```

/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*ta
n(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2
 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 45*A*c**3*f*x*tan(e/2 + f*x/2)/(6*a*f*
tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**
5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/
2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 45*A*c**3*f*x/(6*a*f*tan(
e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 +
18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 +
f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 102*A*c**3*tan(e/2 + f*x/2)**
6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 +
 f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a
*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 54*A*c**3*tan(e/
2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a
*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/
2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 336*
A*c**3*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x
/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*t
an(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) +
6*a*f) - 96*A*c**3*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*t
an(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**
4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
 + f*x/2) + 6*a*f) - 378*A*c**3*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/2)
7 + 6*a*f*tan(e/2 + f*x/2)6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e
/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 +
6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 42*A*c**3*tan(e/2 + f*x/2)/(6*a*f*tan(e/2
 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*
a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x
/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 144*A*c**3/(6*a*f*tan(e/2 + f*x/
2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan
(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2
+ 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 60*B*c**3*f*x*tan(e/2 + f*x/2)**7/(6*a*
f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)
5 + 18*a*f*tan(e/2 + f*x/2)4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(
e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 60*B*c**3*f*x*tan(e/2 +
 f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*
tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)*
*3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 180*B*c
3*f*x*tan(e/2 + f*x/2)5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*
x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*
tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) +
 6*a*f) + 180*B*c**3*f*x*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6
*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*
x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*t
an(e/2 + f*x/2) + 6*a*f) + 180*B*c**3*f*x*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/
2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18

```
*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*
x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 180*B*c**3*f*x*tan(e/2 + f*x/2)
**2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2
 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18
*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 60*B*c**3*f*x*
tan(e/2 + f*x/2)/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 1
8*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f
*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 6
0*B*c**3*f*x/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*
f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2
)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 120*B
*c**3*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/
2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*ta
n(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6
*a*f) + 108*B*c**3*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*t
an(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**
4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
 + f*x/2) + 6*a*f) + 372*B*c**3*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)
**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e
/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 +
6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 192*B*c**3*tan(e/2 + f*x/2)**3/(6*a*f*tan
(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 +
 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 +
 f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 456*B*c**3*tan(e/2 + f*x/2)*
*2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2
+ f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*
a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 68*B*c**3*tan(e
/2 + f*x/2)/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f
*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)
**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 188*B*
c**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/
2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 1
8*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f), Ne(f, 0)), (x*
(A + B*sin(e))*(-c*sin(e) + c)**3/(a*sin(e) + a), True))
```

3.54 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=118

$$-\frac{a^2c^2(A-B)\cos^5(e+fx)}{f(a\sin(e+fx)+a)^3}-\frac{3c^2(2A-3B)\cos(e+fx)}{2af}-\frac{c^2(2A-3B)\cos^3(e+fx)}{2f(a\sin(e+fx)+a)}-\frac{3c^2x(2A-3B)}{2a}$$

```
[Out] -3/2*(2*A-3*B)*c^2*x/a-3/2*(2*A-3*B)*c^2*cos(f*x+e)/a/f-a^2*(A-B)*c^2*cos(f
*x+e)^5/f/(a+a*sin(f*x+e))^3-1/2*(2*A-3*B)*c^2*cos(f*x+e)^3/f/(a+a*sin(f*x+
e))
```

Rubi [A] time = 0.28, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2859, 2679, 2682, 8}

$$-\frac{a^2c^2(A-B)\cos^5(e+fx)}{f(a\sin(e+fx)+a)^3}-\frac{3c^2(2A-3B)\cos(e+fx)}{2af}-\frac{c^2(2A-3B)\cos^3(e+fx)}{2f(a\sin(e+fx)+a)}-\frac{3c^2x(2A-3B)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-3*(2*A - 3*B)*c^2*x)/(2*a) - (3*(2*A - 3*B)*c^2*Cos[e + f*x])/(2*a*f) - (
a^2*(A - B)*c^2*Cos[e + f*x]^5)/(f*(a + a*Sin[e + f*x])^3) - ((2*A - 3*B)*c
^2*Cos[e + f*x]^3)/(2*f*(a + a*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
```

```
&& EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$$

$$= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{f(a+a\sin(e+fx))^3} - \left(a(2A-3B)c^2\right)\int \frac{\cos^4(e+f}{(a+a\sin(e+}$$

$$= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{f(a+a\sin(e+fx))^3} - \frac{(2A-3B)c^2\cos^3(e+fx)}{2f(a+a\sin(e+fx))} - \frac{1}{2}\left(3(\right.$$

$$= -\frac{3(2A-3B)c^2\cos(e+fx)}{2af} - \frac{a^2(A-B)c^2\cos^5(e+fx)}{f(a+a\sin(e+fx))^3} - \frac{(2A-}{2f}$$

$$= -\frac{3(2A-3B)c^2x}{2a} - \frac{3(2A-3B)c^2\cos(e+fx)}{2af} - \frac{a^2(A-B)c^2\cos}{f(a+a\sin(e}$$

Mathematica [A] time = 1.33, size = 188, normalized size = 1.59

$$-\frac{c^2(\sin(e+fx)-1)^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)(6(2A-3B)(e+fx)+4(A-3B)\cos\right.}{4af(\sin(e+fx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
),x]
```

```
[Out] -1/4*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*(Cos[
(e + f*x)/2]*(6*(2*A - 3*B)*(e + f*x) + 4*(A - 3*B)*Cos[e + f*x] + B*Sin[2*
(e + f*x)]) + Sin[(e + f*x)/2]*(4*A*(-8 + 3*e + 3*f*x) - 2*B*(-16 + 9*e + 9
*f*x) + 4*(A - 3*B)*Cos[e + f*x] + B*Sin[2*(e + f*x)])))/(a*f*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2])^4*(1 + Sin[e + f*x]))
```

fricas [A] time = 0.44, size = 179, normalized size = 1.52

$$\frac{Bc^2\cos\left(fx+e\right)^3-3\,(2\,A-3\,B)c^2fx-2\,(A-3\,B)c^2\cos\left(fx+e\right)^2-8\,(A-B)c^2-\left(3\,(2\,A-3\,B)c^2fx+(10\,A-\right.}{2\left(af\cos\left(fx+e\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/2*(B*c^2*cos(f*x + e)^3 - 3*(2*A - 3*B)*c^2*f*x - 2*(A - 3*B)*c^2*cos(f*x
 + e)^2 - 8*(A - B)*c^2 - (3*(2*A - 3*B)*c^2*f*x + (10*A - 13*B)*c^2)*cos(f
*x + e) - (3*(2*A - 3*B)*c^2*f*x + B*c^2*cos(f*x + e)^2 + (2*A - 5*B)*c^2*c
os(f*x + e) - 8*(A - B)*c^2)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x
+ e) + a*f)
```

giac [A] time = 0.16, size = 164, normalized size = 1.39

$$-\frac{\frac{3\left(2\,Ac^2-3\,Bc^2\right)\left(fx+e\right)}{a}+\frac{16\left(Ac^2-Bc^2\right)}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}-\frac{2\left(Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+6\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,Ac^2\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/2*(3*(2*A*c^2 - 3*B*c^2)*(f*x + e)/a + 16*(A*c^2 - B*c^2)/(a*(tan(1/2*f*
x + 1/2*e) + 1)) - 2*(B*c^2*tan(1/2*f*x + 1/2*e)^3 - 2*A*c^2*tan(1/2*f*x +
1/2*e)^2 + 6*B*c^2*tan(1/2*f*x + 1/2*e)^2 - B*c^2*tan(1/2*f*x + 1/2*e) - 2*
A*c^2 + 6*B*c^2)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*a))/f
```

maple [B] time = 0.40, size = 299, normalized size = 2.53

$$\frac{c^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)B}{fa\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{2c^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)A}{fa\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{6c^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)B}{fa\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{c^2B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{fa\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{}{fa\left(1+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x)
```

```
[Out] 1/f*c^2/a/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^3*B-2/f*c^2/a/(1+ta
n(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*A+6/f*c^2/a/(1+tan(1/2*f*x+1/2*e
)^2)^2*tan(1/2*f*x+1/2*e)^2*B-1/f*c^2/a/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/
2*f*x+1/2*e)-2/f*c^2/a/(1+tan(1/2*f*x+1/2*e)^2)^2*A+6/f*c^2/a/(1+tan(1/2*f*
x+1/2*e)^2)^2*B+9/f*c^2/a*arctan(tan(1/2*f*x+1/2*e))*B-6/f*c^2/a*arctan(tan
(1/2*f*x+1/2*e))*A-8/f*c^2/a/(tan(1/2*f*x+1/2*e)+1)*A+8/f*c^2/a/(tan(1/2*f*
x+1/2*e)+1)*B
```

maxima [B] time = 0.43, size = 608, normalized size = 5.15

$$Bc^2\left(\frac{\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{5\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{3\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{3\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+4}{a+\frac{a\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{2a\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{2a\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{a\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{a\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}}+\frac{3\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{a}\right)-2Ac^2\left(\frac{\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\cdots}{a+\frac{a\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{a\sin\left(\cdots\right.}{\left(\cos\left(f\cdots\right.\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] (B*c^2*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*a
rctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 2*A*c^2*((sin(f*x + e)/(cos(f*x
 + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(
cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 4*B*
c^2*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(c
os(f*x + e) + 1))/a) - 4*A*c^2*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a +
 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + 2*B*c^2*(arctan(sin(f*x + e)/
```

```
(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) - 2*A*c^
2/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

mupad [B] time = 14.58, size = 241, normalized size = 2.04

$$-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(2\,A\,c^2-5\,B\,c^2\right)+10\,A\,c^2-14\,B\,c^2+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(2\,A\,c^2-7\,B\,c^2\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(8\,A\,c^2-9\right.}{f\left(a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5+a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+2\,a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3+2\,a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+a\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^2)/(a + a*sin(e + f*x)),x)
```

```
[Out] - (tan(e/2 + (f*x)/2)*(2*A*c^2 - 5*B*c^2) + 10*A*c^2 - 14*B*c^2 + tan(e/2 +
 (f*x)/2)^3*(2*A*c^2 - 7*B*c^2) + tan(e/2 + (f*x)/2)^4*(8*A*c^2 - 9*B*c^2)
+ tan(e/2 + (f*x)/2)^2*(18*A*c^2 - 21*B*c^2))/(f*(a + a*tan(e/2 + (f*x)/2)
+ 2*a*tan(e/2 + (f*x)/2)^2 + 2*a*tan(e/2 + (f*x)/2)^3 + a*tan(e/2 + (f*x)/2
)^4 + a*tan(e/2 + (f*x)/2)^5)) - (3*c^2*atan((3*c^2*tan(e/2 + (f*x)/2)*(2*A
 - 3*B))/(6*A*c^2 - 9*B*c^2))*(2*A - 3*B))/(a*f)
```

sympy [A] time = 8.10, size = 2365, normalized size = 20.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-6*A*c**2*f*x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2
*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/
2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 6*A*c**2*f*x*tan(e/2 + f*x/2)**4/
(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*
x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 12*
A*c**2*f*x*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 +
 f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*
tan(e/2 + f*x/2) + 2*a*f) - 12*A*c**2*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/
2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a
*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 6*A*c**2*f*x*tan
(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*
f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2)
+ 2*a*f) - 6*A*c**2*f*x/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)
**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2
 + f*x/2) + 2*a*f) - 16*A*c**2*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)*
*5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2
```

```
+ f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*A*c**2*tan(e/2 + f*x/2)**
3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 +
f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 3
6*A*c**2*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f
*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*ta
n(e/2 + f*x/2) + 2*a*f) - 4*A*c**2*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)
**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2
 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 20*A*c**2/(2*a*f*tan(e/2 +
 f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*
tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 9*B*c**2*f*x*tan(e/
2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*
f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2)
+ 2*a*f) + 9*B*c**2*f*x*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*
a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2
)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 18*B*c**2*f*x*tan(e/2 + f*x/2)**3/
(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*
x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 18*
B*c**2*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 +
 f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*
tan(e/2 + f*x/2) + 2*a*f) + 9*B*c**2*f*x*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 +
f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*t
an(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 9*B*c**2*f*x/(2*a*f*
tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3
 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 18*B*c**2*
tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4
+ 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f
*x/2) + 2*a*f) + 14*B*c**2*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 +
 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*
x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 42*B*c**2*tan(e/2 + f*x/2)**2/(
2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x
/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 10*B
*c**2*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)*
*4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2
+ f*x/2) + 2*a*f) + 28*B*c**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 +
f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*t
an(e/2 + f*x/2) + 2*a*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c)**2/(
a*sin(e) + a), True))
```

3.55 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=57

$$-\frac{2c(A-B)\cos(e+fx)}{f(a\sin(e+fx)+a)}-\frac{cx(A-2B)}{a}+\frac{Bc\cos(e+fx)}{af}$$

```
[Out] -(A-2*B)*c*x/a+B*c*cos(f*x+e)/a/f-2*(A-B)*c*cos(f*x+e)/f/(a+a*sin(f*x+e))
```

Rubi [A] time = 0.16, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2967, 2857, 2638}

$$-\frac{2c(A-B)\cos(e+fx)}{f(a\sin(e+fx)+a)}-\frac{cx(A-2B)}{a}+\frac{Bc\cos(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x]),x]
```

```
[Out] -(((A - 2*B)*c*x)/a) + (B*c*Cos[e + f*x])/(a*f) - (2*(A - B)*c*Cos[e + f*x]
)/(f*(a + a*Sin[e + f*x]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{a+a\sin(e+fx)}\,dx &= (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx\\ &= -\frac{2(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))}-\frac{c\int (aA-2aB+aB\sin(e+fx))\,dx}{a^2}\\ &= -\frac{(A-2B)cx}{a}-\frac{2(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))}-\frac{(Bc)\int \sin(e+fx)\,dx}{a}\\ &= -\frac{(A-2B)cx}{a}+\frac{Bc\cos(e+fx)}{af}-\frac{2(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))}\end{aligned}$$

Mathematica [B] time = 0.59, size = 127, normalized size = 2.23

$$\frac{(c-c\sin(e+fx))\left(\frac{4(A-B)\sin\left(\frac{fx}{2}\right)}{f\left(\sin\left(\frac{e}{2}\right)+\cos\left(\frac{e}{2}\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}-(x(A-2B))-\frac{B\sin(e)\sin(fx)}{f}+\frac{B\cos(e)\cos(fx)}{f}\right)}{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x]), 
x]
```

```
[Out] ((-((A - 2*B)*x) + (B*Cos[e]*Cos[f*x])/f - (B*Sin[e]*Sin[f*x])/f + (4*(A -
B)*Sin[(f*x)/2])/(f*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])))*(c - c*Sin[e + f*x]))/(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2)
```

fricas [B] time = 0.43, size = 117, normalized size = 2.05

$$-\frac{(A-2\,B)cfx-Bc\cos\left(fx+e\right)^2+2\,(A-B)c+\left((A-2\,B)cfx+(2\,A-3\,B)c\right)\cos\left(fx+e\right)+\left((A-2\,B)cfx-\right.}{af\cos\left(fx+e\right)+af\sin\left(fx+e\right)+af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -((A - 2*B)*c*f*x - B*c*cos(f*x + e)^2 + 2*(A - B)*c + ((A - 2*B)*c*f*x + (
2*A - 3*B)*c)*cos(f*x + e) + ((A - 2*B)*c*f*x - B*c*cos(f*x + e) - 2*(A - B
)*c)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

giac [B] time = 0.14, size = 122, normalized size = 2.14

$$-\frac{\frac{(Ac-2\,Bc)(fx+e)}{a}+\frac{2\left(2\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-2\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,Ac-3\,Bc\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] -((A*c - 2*B*c)*(f*x + e)/a + 2*(2*A*c*tan(1/2*f*x + 1/2*e)^2 - 2*B*c*tan(1
/2*f*x + 1/2*e)^2 - B*c*tan(1/2*f*x + 1/2*e) + 2*A*c - 3*B*c)/((tan(1/2*f*x
 + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e) + 1)*a))/f
```

maple [A] time = 0.38, size = 113, normalized size = 1.98

$$\frac{2cB}{fa\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}-\frac{2c\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)A}{fa}+\frac{4c\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)B}{fa}-\frac{4cA}{fa\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{}{fa\left(\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] 2/f*c/a*B/(1+tan(1/2*f*x+1/2*e)^2)-2/f*c/a*arctan(tan(1/2*f*x+1/2*e))*A+4/f
*c/a*arctan(tan(1/2*f*x+1/2*e))*B-4/f*c/a/(tan(1/2*f*x+1/2*e)+1)*A+4/f*c/a/
(tan(1/2*f*x+1/2*e)+1)*B
```

maxima [B] time = 0.55, size = 256, normalized size = 4.49

$$\frac{2\left(Bc\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+2}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{a\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{a\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)-Ac\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)+Bc\left(\frac{\arctan\left(\frac{}{\mathrm{co}}\right.}{}\right.\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] 2*(B*c*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x
```

```
 + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/a) - A*c*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a +
1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + B*c*(arctan(sin(f*x + e)/(cos(
f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) - A*c/(a + a*
sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

mupad [B] time = 12.94, size = 110, normalized size = 1.93

$$-\frac{(4\,A\,c-4\,B\,c)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-2\,B\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+4\,A\,c-6\,B\,c}{f\,\left(a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\right)}-\frac{A\,c\,f\,x-2\,B\,c\,f\,x}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x)))/(a + a*sin(e + f*x)),x)
```

```
[Out] - (4*A*c - 6*B*c + tan(e/2 + (f*x)/2)^2*(4*A*c - 4*B*c) - 2*B*c*tan(e/2 + (
f*x)/2))/(f*(a + a*tan(e/2 + (f*x)/2) + a*tan(e/2 + (f*x)/2)^2 + a*tan(e/2
+ (f*x)/2)^3)) - (A*c*f*x - 2*B*c*f*x)/(a*f)
```

sympy [A] time = 3.94, size = 828, normalized size = 14.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-A*c*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(
e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - A*c*f*x*tan(e/2 + f*x/2)**2
/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2)
+ a*f) - A*c*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 +
f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - A*c*f*x/(a*f*tan(e/2 + f*x/2)**3
+ a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 4*A*c*tan(e/2 + f
*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 +
 f*x/2) + a*f) - 4*A*c/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 +
 a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 +
 f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*
f*x*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2
+ a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f
*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*f*
x/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2)
 + a*f) + 4*B*c*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2
+ f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*tan(e/2 + f*x/2)/(a*f*tan
(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) +
```

```
6*B*c/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*
x/2) + a*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c)/(a*sin(e) + a), T
rue))
```

3.56 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx$

Optimal. Leaf size=35

$$\frac{A\tan(e+fx)}{acf}+\frac{B\sec(e+fx)}{acf}$$

```
[Out] B*sec(f*x+e)/a/c/f+A*tan(f*x+e)/a/c/f
```

Rubi [A] time = 0.14, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2669, 3767, 8}

$$\frac{A\tan(e+fx)}{acf}+\frac{B\sec(e+fx)}{acf}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])),x]
```

```
[Out] (B*Sec[e + f*x])/(a*c*f) + (A*Tan[e + f*x])/(a*c*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx &= \frac{\int \sec^2(e+fx)(A+B\sin(e+fx))\,dx}{ac}\\ &= \frac{B\sec(e+fx)}{acf}+\frac{A\int \sec^2(e+fx)\,dx}{ac}\\ &= \frac{B\sec(e+fx)}{acf}-\frac{A\,\mathrm{Subst}\left(\int 1\,dx,x,-\tan(e+fx)\right)}{acf}\\ &= \frac{B\sec(e+fx)}{acf}+\frac{A\tan(e+fx)}{acf}\end{aligned}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 1.00

$$\frac{A\tan(e+fx)}{acf}+\frac{B\sec(e+fx)}{acf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])),
x]
[Out] (B*Sec[e + f*x])/(a*c*f) + (A*Tan[e + f*x])/(a*c*f)
```

fricas [A] time = 0.41, size = 28, normalized size = 0.80

$$\frac{A\sin\left(fx+e\right)+B}{acf\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="
fricas")
[Out] (A*sin(f*x + e) + B)/(a*c*f*cos(f*x + e))
```

giac [A] time = 0.18, size = 41, normalized size = 1.17

$$-\frac{2\left(A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+B\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)acf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] -2*(A*tan(1/2*f*x + 1/2*e) + B)/((tan(1/2*f*x + 1/2*e)^2 - 1)*a*c*f)
```

maple [A] time = 0.37, size = 57, normalized size = 1.63

$$\frac{-\frac{2\left(\frac{A}{2}+\frac{B}{2}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{2\left(\frac{A}{2}-\frac{B}{2}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{acf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] 2/f/c/a*(-(1/2*A+1/2*B)/(tan(1/2*f*x+1/2*e)-1)-(1/2*A-1/2*B)/(tan(1/2*f*x+1
/2*e)+1))
```

maxima [A] time = 0.32, size = 35, normalized size = 1.00

$$\frac{\frac{A\tan\left(fx+e\right)}{ac}+\frac{B}{ac\cos\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] (A*tan(f*x + e)/(a*c) + B/(a*c*cos(f*x + e)))/f
```

mupad [B] time = 12.55, size = 39, normalized size = 1.11

$$-\frac{2\left(B+A\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{acf\left({\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))),x)
```

```
[Out] -(2*(B + A*tan(e/2 + (f*x)/2)))/(a*c*f*(tan(e/2 + (f*x)/2)^2 - 1))
```

sympy [A] time = 2.53, size = 83, normalized size = 2.37

$$\begin{cases} -\frac{2A\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{acf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-acf} - \frac{2B}{acf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-acf} & \text{for } f \neq 0 \\ \frac{x(A+B\sin(e))}{(a\sin(e)+a)(-c\sin(e)+c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-2*A*tan(e/2 + f*x/2)/(a*c*f*tan(e/2 + f*x/2)**2 - a*c*f) - 2*B/
(a*c*f*tan(e/2 + f*x/2)**2 - a*c*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e
) + a)*(-c*sin(e) + c)), True))
```

3.57 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=63

$$\frac{(2A-B)\tan(e+fx)}{3ac^2f}+\frac{(A+B)\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)}$$

```
[Out] 1/3*(A+B)*sec(f*x+e)/a/f/(c^2-c^2*sin(f*x+e))+1/3*(2*A-B)*tan(f*x+e)/a/c^2/
f
```

Rubi [A] time = 0.20, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 3767, 8}

$$\frac{(2A-B)\tan(e+fx)}{3ac^2f}+\frac{(A+B)\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^2),x]
```

```
[Out] ((A + B)*Sec[e + f*x])/(3*a*f*(c^2 - c^2*Sin[e + f*x])) + ((2*A - B)*Tan[e
+ f*x])/(3*a*c^2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
```

```
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx = \frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx}{ac}$$
$$= \frac{(A+B)\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)} + \frac{(2A-B)\int \sec^2(e+fx)\,dx}{3ac^2}$$
$$= \frac{(A+B)\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)} - \frac{(2A-B)\operatorname{Subst}(\int 1\,dx, x, -\tan(e+fx$$
$$3ac^2f$$
$$= \frac{(A+B)\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)} + \frac{(2A-B)\tan(e+fx)}{3ac^2f}$$

Mathematica [A] time = 0.59, size = 108, normalized size = 1.71

$$\cos(e+fx)(-2(A+B)\cos(e+fx)+(4A-2B)\cos(2(e+fx))+8A\sin(e+fx)+A\sin(2(e+fx))-4B\sin(e+$$
$$12ac^2f(\sin(e+fx)-1)^2(\sin(e+fx)+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^2
),x]
```

```
[Out] (Cos[e + f*x]*(6*B - 2*(A + B)*Cos[e + f*x] + (4*A - 2*B)*Cos[2*(e + f*x)]
+ 8*A*Sin[e + f*x] - 4*B*Sin[e + f*x] + A*Sin[2*(e + f*x)] + B*Sin[2*(e + f
*x)]))/(12*a*c^2*f*(-1 + Sin[e + f*x])^2*(1 + Sin[e + f*x]))
```

fricas [A] time = 0.42, size = 73, normalized size = 1.16

$$-\frac{(2\,A-B)\cos\left(fx+e\right)^2+(2\,A-B)\sin\left(fx+e\right)-A+2\,B}{3\left(ac^2f\cos\left(fx+e\right)\sin\left(fx+e\right)-ac^2f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] -1/3*((2*A - B)*cos(f*x + e)^2 + (2*A - B)*sin(f*x + e) - A + 2*B)/(a*c^2*f
*cos(f*x + e)*sin(f*x + e) - a*c^2*f*cos(f*x + e))
```

giac [A] time = 0.29, size = 102, normalized size = 1.62

$$-\frac{\frac{3\,(A-B)}{ac^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{9\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-12\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+7\,A+B}{ac^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] -1/6*(3*(A - B)/(a*c^2*(tan(1/2*f*x + 1/2*e) + 1)) + (9*A*tan(1/2*f*x + 1/2
*e)^2 + 3*B*tan(1/2*f*x + 1/2*e)^2 - 12*A*tan(1/2*f*x + 1/2*e) + 7*A + B)/(
a*c^2*(tan(1/2*f*x + 1/2*e) - 1)^3))/f
```

maple [A] time = 0.45, size = 93, normalized size = 1.48

$$\frac{-\frac{2(A+B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{A+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{2\left(\frac{3A}{4}+\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{2\left(\frac{A}{4}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{f\,a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

```
[Out] 2/f/a/c^2*(-1/3*(A+B)/(tan(1/2*f*x+1/2*e)-1)^3-1/2*(A+B)/(tan(1/2*f*x+1/2*e
)-1)^2-(3/4*A+1/4*B)/(tan(1/2*f*x+1/2*e)-1)-(1/4*A-1/4*B)/(tan(1/2*f*x+1/2*
e)+1))
```

maxima [B] time = 0.63, size = 266, normalized size = 4.22

$$-\frac{2\left(\frac{B\left(\frac{2\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-1\right)}{ac^2-\frac{2\,ac^2\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{2\,ac^2\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}-\frac{ac^2\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}}-\frac{A\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{3\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+1\right)}{ac^2-\frac{2\,ac^2\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{2\,ac^2\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}-\frac{ac^2\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}}\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] -2/3*(B*(2*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 - 1)/(a*c^2 - 2*a*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*c^2*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 - a*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4) - A*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a*c^2 - 2*a*c^2*sin(f*x
+ e)/(cos(f*x + e) + 1) + 2*a*c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - a*c
^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4))/f
```

mupad [B] time = 12.32, size = 118, normalized size = 1.87

$$\frac{2\left(\frac{3B}{2}+A\cos\left(e+fx\right)+B\cos\left(e+fx\right)+2A\sin\left(e+fx\right)-B\sin\left(e+fx\right)+A\cos\left(2e+2fx\right)-\frac{B\cos(2e+2}{2}\right.}{3ac^2f\left(2\cos\left(e+fx\right)-\sin\left(2e+2fx\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^2),x)
```

```
[Out] (2*((3*B)/2 + A*cos(e + f*x) + B*cos(e + f*x) + 2*A*sin(e + f*x) - B*sin(e
+ f*x) + A*cos(2*e + 2*f*x) - (B*cos(2*e + 2*f*x))/2 - (A*sin(2*e + 2*f*x))
/2 - (B*sin(2*e + 2*f*x))/2))/(3*a*c^2*f*(2*cos(e + f*x) - sin(2*e + 2*f*x)
))
```

sympy [A] time = 7.81, size = 578, normalized size = 9.17

$$\begin{cases} -\frac{6A\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3ac^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-6ac^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+6ac^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3ac^2f}+\frac{6A\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3ac^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)-6ac^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+6ac^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3ac^2f}-\frac{}{3ac} \\ \frac{x(A+B\sin(e))}{(a\sin(e)+a)(-c\sin(e)+c)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*A*tan(e/2 + f*x/2)**3/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c
**2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) + 6*A
*tan(e/2 + f*x/2)**2/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 +
 f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) - 2*A*tan(e/2 + f*x/
2)/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 + f*x/2)**3 + 6*a*c
**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) - 2*A/(3*a*c**2*f*tan(e/2 + f*x/2)**4
- 6*a*c**2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f
```

```
) - 6*B*tan(e/2 + f*x/2)**2/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*ta
n(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) + 4*B*tan(e/2
 + f*x/2)/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 + f*x/2)**3
+ 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) - 2*B/(3*a*c**2*f*tan(e/2 + f*x
/2)**4 - 6*a*c**2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a
*c**2*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)*(-c*sin(e) + c)**2),
 True))
```

3.58 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=102

$$\frac{2(3A-2B)\tan(e+fx)}{15ac^3f}+\frac{(3A-2B)\sec(e+fx)}{15af\left(c^3-c^3\sin(e+fx)\right)}+\frac{(A+B)\sec(e+fx)}{5acf(c-c\sin(e+fx))^2}$$

```
[Out] 1/5*(A+B)*sec(f*x+e)/a/c/f/(c-c*sin(f*x+e))^2+1/15*(3*A-2*B)*sec(f*x+e)/a/f
/(c^3-c^3*sin(f*x+e))+2/15*(3*A-2*B)*tan(f*x+e)/a/c^3/f
```

Rubi [A] time = 0.26, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2859, 2672, 3767, 8}

$$\frac{2(3A-2B)\tan(e+fx)}{15ac^3f}+\frac{(3A-2B)\sec(e+fx)}{15af\left(c^3-c^3\sin(e+fx)\right)}+\frac{(A+B)\sec(e+fx)}{5acf(c-c\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^3),x]
```

```
[Out] ((A + B)*Sec[e + f*x])/(5*a*c*f*(c - c*Sin[e + f*x])^2) + ((3*A - 2*B)*Sec[
e + f*x])/(15*a*f*(c^3 - c^3*Sin[e + f*x])) + (2*(3*A - 2*B)*Tan[e + f*x])/
(15*a*c^3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
```

```
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx = \frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx}{ac}$$

$$= \frac{(A+B)\sec(e+fx)}{5acf(c-c\sin(e+fx))^2} + \frac{(3A-2B)\int \frac{\sec^2(e+fx)}{c-c\sin(e+fx)}\,dx}{5ac^2}$$

$$= \frac{(A+B)\sec(e+fx)}{5acf(c-c\sin(e+fx))^2} + \frac{(3A-2B)\sec(e+fx)}{15af\left(c^3-c^3\sin(e+fx)\right)} + \frac{(2(3A-$$

$$= \frac{(A+B)\sec(e+fx)}{5acf(c-c\sin(e+fx))^2} + \frac{(3A-2B)\sec(e+fx)}{15af\left(c^3-c^3\sin(e+fx)\right)} - \frac{(2(3A-$$

$$= \frac{(A+B)\sec(e+fx)}{5acf(c-c\sin(e+fx))^2} + \frac{(3A-2B)\sec(e+fx)}{15af\left(c^3-c^3\sin(e+fx)\right)} + \frac{2(3A-$$

Mathematica [A] time = 0.89, size = 157, normalized size = 1.54

$$-\frac{\cos(e+fx)(5(B-9A)\cos(e+fx)+32(3A-2B)\cos(2(e+fx))+120A\sin(e+fx)+36A\sin(2(e+fx))-}{240ac^3f(\mathrm{si}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^3
),x]
```

```
[Out] -1/240*(Cos[e + f*x]*(80*B + 5*(-9*A + B)*Cos[e + f*x] + 32*(3*A - 2*B)*Cos
[2*(e + f*x)] + 9*A*Cos[3*(e + f*x)] - B*Cos[3*(e + f*x)] + 120*A*Sin[e + f
*x] - 80*B*Sin[e + f*x] + 36*A*Sin[2*(e + f*x)] - 4*B*Sin[2*(e + f*x)] - 24
*A*Sin[3*(e + f*x)] + 16*B*Sin[3*(e + f*x)]))/(a*c^3*f*(-1 + Sin[e + f*x])^
3*(1 + Sin[e + f*x]))
```

fricas [A] time = 0.42, size = 107, normalized size = 1.05

$$-\frac{4\,(3\,A-2\,B)\cos\left(fx+e\right)^{2}-\left(2\,(3\,A-2\,B)\cos\left(fx+e\right)^{2}-9\,A+6\,B\right)\sin\left(fx+e\right)-6\,A+9\,B}{15\left(ac^{3}f\cos\left(fx+e\right)^{3}+2\,ac^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,ac^{3}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -1/15*(4*(3*A - 2*B)*cos(f*x + e)^2 - (2*(3*A - 2*B)*cos(f*x + e)^2 - 9*A +
 6*B)*sin(f*x + e) - 6*A + 9*B)/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(f*x
 + e)*sin(f*x + e) - 2*a*c^3*f*cos(f*x + e))
```

giac [A] time = 0.26, size = 177, normalized size = 1.74

$$-\frac{\frac{15\,(A-B)}{ac^{3}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{105\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+15\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-270\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+30\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+360\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-40\,B\cdots}{ac^{3}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{5}}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -1/60*(15*(A - B)/(a*c^3*(tan(1/2*f*x + 1/2*e) + 1)) + (105*A*tan(1/2*f*x +
 1/2*e)^4 + 15*B*tan(1/2*f*x + 1/2*e)^4 - 270*A*tan(1/2*f*x + 1/2*e)^3 + 30
*B*tan(1/2*f*x + 1/2*e)^3 + 360*A*tan(1/2*f*x + 1/2*e)^2 - 40*B*tan(1/2*f*x
 + 1/2*e)^2 - 210*A*tan(1/2*f*x + 1/2*e) + 50*B*tan(1/2*f*x + 1/2*e) + 63*A
 - 7*B)/(a*c^3*(tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

maple [A] time = 0.55, size = 145, normalized size = 1.42

$$\frac{-\frac{2(2A+2B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{\frac{5A}{2}+\frac{3B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{2\left(\frac{7A}{8}+\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{2\left(\frac{9A}{2}+\frac{7B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{2\left(\frac{A}{8}-\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{f\,a\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

```
[Out] 2/f/a/c^3*(-1/5*(2*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/4*(4*A+4*B)/(tan(1/2*f
*x+1/2*e)-1)^4-1/2*(5/2*A+3/2*B)/(tan(1/2*f*x+1/2*e)-1)^2-(7/8*A+1/8*B)/(ta
n(1/2*f*x+1/2*e)-1)-1/3*(9/2*A+7/2*B)/(tan(1/2*f*x+1/2*e)-1)^3-(1/8*A-1/8*B
)/(tan(1/2*f*x+1/2*e)+1))
```

maxima [B] time = 0.64, size = 423, normalized size = 4.15

$$-\frac{2\left(\frac{B\left(\frac{4\sin(fx+e)}{\cos(fx+e)+1}-\frac{20\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-1\right)}{ac^3-\frac{4ac^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{5ac^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{5ac^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{4ac^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{ac^3\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}+\frac{3A\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{10\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right.}{ac^3-\frac{4ac^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{5ac^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{5ac^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{4a}{(c}}\right.}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] -2/15*(B*(4*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 - 1)/(a*c^3 - 4*a*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 5*
a*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*a*c^3*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 + 4*a*c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - a*c^3*sin(f*x
+ e)^6/(cos(f*x + e) + 1)^6) + 3*A*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 10*
sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 10*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 2)/(a*c^3 - 4*a*c^3*sin(f*x + e
)/(cos(f*x + e) + 1) + 5*a*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*a*c^
3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4*a*c^3*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 - a*c^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6))/f
```

mupad [B] time = 12.47, size = 178, normalized size = 1.75

$$\frac{2\left(\frac{5B\sin(e+fx)}{2}-\frac{15A\cos(e+fx)}{4}-\frac{5B\cos(e+fx)}{8}-\frac{15A\sin(e+fx)}{4}-\frac{5B}{2}-3A\cos\left(2e+2fx\right)+\frac{3A\cos(3e+3fx)}{4}+2B\right.}{15ac^3f\left(\frac{\cos(3e+3fx)}{4}-\frac{5\cos(e+}{4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^3),x)
```

```
[Out] (2*((5*B*sin(e + f*x))/2 - (15*A*cos(e + f*x))/4 - (5*B*cos(e + f*x))/8 - (
15*A*sin(e + f*x))/4 - (5*B)/2 - 3*A*cos(2*e + 2*f*x) + (3*A*cos(3*e + 3*f*
```

```
x))/4 + 2*B*cos(2*e + 2*f*x) + (B*cos(3*e + 3*f*x))/8 + 3*A*sin(2*e + 2*f*x
) + (3*A*sin(3*e + 3*f*x))/4 + (B*sin(2*e + 2*f*x))/2 - (B*sin(3*e + 3*f*x)
)/2))/(15*a*c^3*f*(cos(3*e + 3*f*x)/4 - (5*cos(e + f*x))/4 + sin(2*e + 2*f*
x)))
```

sympy [A] time = 15.15, size = 1236, normalized size = 12.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*tan(e/2 + f*x/2)**5/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*
a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*
f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) + 60*A*
tan(e/2 + f*x/2)**4/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2
+ f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2
)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) - 60*A*tan(e/2 + f*x/2)*
*3/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*
a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*
f*tan(e/2 + f*x/2) - 15*a*c**3*f) + 18*A*tan(e/2 + f*x/2)/(15*a*c**3*f*tan(
e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f
*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) -
 15*a*c**3*f) - 12*A/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2
 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/
2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) - 30*B*tan(e/2 + f*x/2)
**4/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75
*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3
*f*tan(e/2 + f*x/2) - 15*a*c**3*f) + 40*B*tan(e/2 + f*x/2)**3/(15*a*c**3*f*
tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2
 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/
2) - 15*a*c**3*f) - 40*B*tan(e/2 + f*x/2)**2/(15*a*c**3*f*tan(e/2 + f*x/2)*
*6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75
*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f)
 + 8*B*tan(e/2 + f*x/2)/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(
e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f
*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) - 2*B/(15*a*c**3*f*t
an(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2
+ f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2
) - 15*a*c**3*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)*(-c*sin(e) +
 c)**3), True))
```

3.59 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=142

$$\frac{2(4A-3B)\tan(e+fx)}{35ac^4f}+\frac{(4A-3B)\sec(e+fx)}{35af\left(c^4-c^4\sin(e+fx)\right)}+\frac{(4A-3B)\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3}$$

```
[Out] 1/7*(A+B)*sec(f*x+e)/a/c/f/(c-c*sin(f*x+e))^3+1/35*(4*A-3*B)*sec(f*x+e)/a/f
/(c^2-c^2*sin(f*x+e))^2+1/35*(4*A-3*B)*sec(f*x+e)/a/f/(c^4-c^4*sin(f*x+e))+
2/35*(4*A-3*B)*tan(f*x+e)/a/c^4/f
```

Rubi [A] time = 0.31, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2859, 2672, 3767, 8}

$$\frac{2(4A-3B)\tan(e+fx)}{35ac^4f}+\frac{(4A-3B)\sec(e+fx)}{35af\left(c^4-c^4\sin(e+fx)\right)}+\frac{(4A-3B)\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] ((A + B)*Sec[e + f*x])/(7*a*c*f*(c - c*Sin[e + f*x])^3) + ((4*A - 3*B)*Sec[
e + f*x])/(35*a*f*(c^2 - c^2*Sin[e + f*x])^2) + ((4*A - 3*B)*Sec[e + f*x])/
(35*a*f*(c^4 - c^4*Sin[e + f*x])) + (2*(4*A - 3*B)*Tan[e + f*x])/(35*a*c^4*
f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
```

```
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx = \frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx}{ac}$$

$$= \frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{(4A-3B)\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{7ac^2}$$

$$= \frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{(4A-3B)\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(3(4A-}{}$$

$$= \frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{(4A-3B)\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(4A-}{35af\left(c^4\right.}$$

$$= \frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{(4A-3B)\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(4A-}{35af\left(c^4\right.}$$

$$= \frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{(4A-3B)\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(4A-}{35af\left(c^4\right.}$$

Mathematica [A] time = 1.14, size = 240, normalized size = 1.69

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)((182B-406A)\cos(e+fx)+224(4A}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^4
),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(560*B + (-406*A + 182*B)*Cos[e + f*x] + 224*(4*A - 3*B)*Cos[2*(e + f*x)]
 + 174*A*Cos[3*(e + f*x)] - 78*B*Cos[3*(e + f*x)] - 64*A*Cos[4*(e + f*x)] +
 48*B*Cos[4*(e + f*x)] + 896*A*Sin[e + f*x] - 672*B*Sin[e + f*x] + 406*A*Si
n[2*(e + f*x)] - 182*B*Sin[2*(e + f*x)] - 384*A*Sin[3*(e + f*x)] + 288*B*Si
n[3*(e + f*x)] - 29*A*Sin[4*(e + f*x)] + 13*B*Sin[4*(e + f*x)]))/(2240*a*c^
4*f*(-1 + Sin[e + f*x])^4*(1 + Sin[e + f*x]))
```

fricas [A] time = 0.43, size = 141, normalized size = 0.99

$$\frac{2\,(4\,A-3\,B)\cos\left(fx+e\right)^{4}-9\,(4\,A-3\,B)\cos\left(fx+e\right)^{2}+\left(6\,(4\,A-3\,B)\cos\left(fx+e\right)^{2}-20\,A+15\,B\right)\sin\left(fx+\right.}{35\left(3\,ac^{4}f\cos\left(fx+e\right)^{3}-4\,ac^{4}f\cos\left(fx+e\right)-\left(ac^{4}f\cos\left(fx+e\right)^{3}-4\,ac^{4}f\cos\left(fx+e\right)\right)\sin\left(fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm
="fricas")
```

```
[Out] 1/35*(2*(4*A - 3*B)*cos(f*x + e)^4 - 9*(4*A - 3*B)*cos(f*x + e)^2 + (6*(4*A
 - 3*B)*cos(f*x + e)^2 - 20*A + 15*B)*sin(f*x + e) + 15*A - 20*B)/(3*a*c^4*
f*cos(f*x + e)^3 - 4*a*c^4*f*cos(f*x + e) - (a*c^4*f*cos(f*x + e)^3 - 4*a*c
^4*f*cos(f*x + e))*sin(f*x + e))
```

giac [A] time = 0.20, size = 237, normalized size = 1.67

$$-\frac{\frac{35\,(A-B)}{ac^{4}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{525\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}+35\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-1960\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+280\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+4025\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm
="giac")
```

```
[Out] -1/280*(35*(A - B)/(a*c^4*(tan(1/2*f*x + 1/2*e) + 1)) + (525*A*tan(1/2*f*x
+ 1/2*e)^6 + 35*B*tan(1/2*f*x + 1/2*e)^6 - 1960*A*tan(1/2*f*x + 1/2*e)^5 +
280*B*tan(1/2*f*x + 1/2*e)^5 + 4025*A*tan(1/2*f*x + 1/2*e)^4 - 665*B*tan(1/
2*f*x + 1/2*e)^4 - 4480*A*tan(1/2*f*x + 1/2*e)^3 + 1120*B*tan(1/2*f*x + 1/2
*e)^3 + 3143*A*tan(1/2*f*x + 1/2*e)^2 - 791*B*tan(1/2*f*x + 1/2*e)^2 - 1176
*A*tan(1/2*f*x + 1/2*e) + 392*B*tan(1/2*f*x + 1/2*e) + 243*A - 51*B)/(a*c^4
*(tan(1/2*f*x + 1/2*e) - 1)^7))/f
```

maple [A] time = 0.49, size = 189, normalized size = 1.33

$$\frac{-\frac{2(4A+4B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{12A+12B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{18A+14B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{2(19A+17B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{2\left(\frac{15A}{16}+\frac{B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{\frac{17A}{4}+\frac{7B}{4}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{2\left(\frac{45A}{4}+\frac{27B}{4}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right.}}{f\,a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x)
```

```
[Out] 2/f/a/c^4*(-1/7*(4*A+4*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/6*(12*A+12*B)/(tan(1/2
*f*x+1/2*e)-1)^6-1/4*(18*A+14*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/5*(19*A+17*B)/(
tan(1/2*f*x+1/2*e)-1)^5-(15/16*A+1/16*B)/(tan(1/2*f*x+1/2*e)-1)-1/2*(17/4*A
+7/4*B)/(tan(1/2*f*x+1/2*e)-1)^2-1/3*(45/4*A+27/4*B)/(tan(1/2*f*x+1/2*e)-1)
^3-(1/16*A-1/16*B)/(tan(1/2*f*x+1/2*e)+1))
```

maxima [B] time = 0.35, size = 619, normalized size = 4.36

$$-\frac{2\left(\frac{A\left(\frac{43\sin(fx+e)}{\cos(fx+e)+1}-\frac{77\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{7\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{105\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{175\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{105\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{35\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-13\right)}{ac^4-\frac{6ac^4\sin(fx+e)}{\cos(fx+e)+1}+\frac{14ac^4\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{14ac^4\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{14ac^4\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{14ac^4\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{6ac^4\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-\frac{ac^4\sin(fx+e)^8}{(\cos(fx+e)+1)^8}}-\frac{B\left(\frac{6\sin(f\right.}{\cos(fx+\right.}}{ac^4-\frac{6ac^4\sin(fx+e)}{\cos(fx+e)+1}+\cdots}\right)}{35f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm
="maxima")
```

```
[Out] -2/35*(A*(43*sin(f*x + e)/(cos(f*x + e) + 1) - 77*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 105*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 - 175*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 105*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 - 35*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 13)/(
a*c^4 - 6*a*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 14*a*c^4*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 - 14*a*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 14*a*c
^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 14*a*c^4*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 + 6*a*c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a*c^4*sin(f*x +
```

```
e)^8/(cos(f*x + e) + 1)^8) - B*(6*sin(f*x + e)/(cos(f*x + e) + 1) + 21*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 56*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
105*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 70*sin(f*x + e)^5/(cos(f*x + e) +
 1)^5 + 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1)/(a*c^4 - 6*a*c^4*sin(f*
x + e)/(cos(f*x + e) + 1) + 14*a*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 -
14*a*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 14*a*c^4*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 - 14*a*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 6*a*c^4*s
in(f*x + e)^7/(cos(f*x + e) + 1)^7 - a*c^4*sin(f*x + e)^8/(cos(f*x + e) + 1
)^8))/f
```

mupad [B] time = 13.23, size = 239, normalized size = 1.68

$$2\left(\frac{35\,B}{4}+\frac{91\,A\,\cos\left(e+f\,x\right)}{4}-\frac{7\,B\,\cos\left(e+f\,x\right)}{4}+14\,A\,\sin\left(e+f\,x\right)-\frac{21\,B\,\sin\left(e+f\,x\right)}{2}+14\,A\,\cos\left(2\,e+2\,f\,x\right)-\frac{39\,A\,\cos\left(3\,e+3\right.}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^4),x)
```

```
[Out] (2*((35*B)/4 + (91*A*cos(e + f*x))/4 - (7*B*cos(e + f*x))/4 + 14*A*sin(e +
f*x) - (21*B*sin(e + f*x))/2 + 14*A*cos(2*e + 2*f*x) - (39*A*cos(3*e + 3*f*
x))/4 - A*cos(4*e + 4*f*x) - (21*B*cos(2*e + 2*f*x))/2 + (3*B*cos(3*e + 3*f
*x))/4 + (3*B*cos(4*e + 4*f*x))/4 - (91*A*sin(2*e + 2*f*x))/4 - 6*A*sin(3*e
 + 3*f*x) + (13*A*sin(4*e + 4*f*x))/8 + (7*B*sin(2*e + 2*f*x))/4 + (9*B*sin
(3*e + 3*f*x))/2 - (B*sin(4*e + 4*f*x))/8))/(35*a*c^4*f*((7*cos(e + f*x))/2
 - (3*cos(3*e + 3*f*x))/2 - (7*sin(2*e + 2*f*x))/2 + sin(4*e + 4*f*x)/4))
```

sympy [A] time = 28.58, size = 2468, normalized size = 17.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-70*A*tan(e/2 + f*x/2)**7/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210
*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c*
*4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*
tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) + 210*A*
tan(e/2 + f*x/2)**6/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2
 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*
x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)*
*2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) - 350*A*tan(e/2 + f*x/2)*
*5/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 49
```

0*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c
4*f*tan(e/2 + f*x/2)3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f
*tan(e/2 + f*x/2) - 35*a*c**4*f) + 210*A*tan(e/2 + f*x/2)**4/(35*a*c**4*f*t
an(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/
2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f
*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2)
 - 35*a*c**4*f) + 14*A*tan(e/2 + f*x/2)**3/(35*a*c**4*f*tan(e/2 + f*x/2)**8
 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 49
0*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c
4*f*tan(e/2 + f*x/2)2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) -
154*A*tan(e/2 + f*x/2)**2/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*t
an(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/
2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f
*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) + 86*A*tan(e/2 + f*
x/2)/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 +
490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a
*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4
*f*tan(e/2 + f*x/2) - 35*a*c**4*f) - 26*A/(35*a*c**4*f*tan(e/2 + f*x/2)**8
- 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490
*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c*
*4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) - 7
0*B*tan(e/2 + f*x/2)**6/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan
(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2
+ f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x
/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) + 140*B*tan(e/2 + f*x
/2)**5/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7
+ 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490
*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c*
*4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) - 210*B*tan(e/2 + f*x/2)**4/(35*a*c**4
*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*ta
n(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2
 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*
x/2) - 35*a*c**4*f) + 112*B*tan(e/2 + f*x/2)**3/(35*a*c**4*f*tan(e/2 + f*x/
2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6
 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 49
0*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*
f) - 42*B*tan(e/2 + f*x/2)**2/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4
*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*ta
n(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2
 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) - 12*B*tan(e/2
+ f*x/2)/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**
7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 4
90*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*
c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) + 2*B/(35*a*c**4*f*tan(e/2 + f*x/2)*
*8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 -

```
490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a
*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f),
 Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)*(-c*sin(e) + c)**4), True))
```

3.60 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=240

$$-\frac{a^5c^5(A-B)\cos^{11}(e+fx)}{3f(a\sin(e+fx)+a)^7}+\frac{2a^3c^5(4A-7B)\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^5}+\frac{35c^5(4A-7B)\cos^3(e+fx)}{4a^2f}+\frac{21c^5(4A-7B)\cos^5(e}{4f\left(a^2\sin(e+fx)+\right.}$$

```
[Out] 105/8*(4*A-7*B)*c^5*x/a^2+35/4*(4*A-7*B)*c^5*cos(f*x+e)^3/a^2/f+105/8*(4*A-
7*B)*c^5*cos(f*x+e)*sin(f*x+e)/a^2/f-1/3*a^5*(A-B)*c^5*cos(f*x+e)^11/f/(a+a
*sin(f*x+e))^7+2/3*a^3*(4*A-7*B)*c^5*cos(f*x+e)^9/f/(a+a*sin(f*x+e))^5+6*a^
4*(4*A-7*B)*c^5*cos(f*x+e)^7/f/(a^2+a^2*sin(f*x+e))^3+21/4*(4*A-7*B)*c^5*co
s(f*x+e)^5/f/(a^2+a^2*sin(f*x+e))
```

Rubi [A] time = 0.41, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2967, 2859, 2680, 2679, 2682, 2635, 8}

$$\frac{35c^5(4A-7B)\cos^3(e+fx)}{4a^2f}-\frac{a^5c^5(A-B)\cos^{11}(e+fx)}{3f(a\sin(e+fx)+a)^7}+\frac{2a^3c^5(4A-7B)\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^5}+\frac{6a^4c^5(4A-7B)\cos^7(e+}{f\left(a^2\sin(e+fx)+a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (105*(4*A - 7*B)*c^5*x)/(8*a^2) + (35*(4*A - 7*B)*c^5*Cos[e + f*x]^3)/(4*a^
2*f) + (105*(4*A - 7*B)*c^5*Cos[e + f*x]*Sin[e + f*x])/(8*a^2*f) - (a^5*(A
- B)*c^5*Cos[e + f*x]^11)/(3*f*(a + a*Sin[e + f*x])^7) + (2*a^3*(4*A - 7*B)
*c^5*Cos[e + f*x]^9)/(3*f*(a + a*Sin[e + f*x])^5) + (6*a^4*(4*A - 7*B)*c^5*
Cos[e + f*x]^7)/(f*(a^2 + a^2*Sin[e + f*x])^3) + (21*(4*A - 7*B)*c^5*Cos[e
+ f*x]^5)/(4*f*(a^2 + a^2*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx = \left(a^5c^5\right)\int \frac{\cos^{10}(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^7}\,dx$$

$$= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{3f(a+a\sin(e+fx))^7} - \frac{1}{3}\left(a^4(4A-7B)c^5\right)\int \frac{\cos^{10}(e+}{(a+a\sin(e}$$

$$= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{3f(a+a\sin(e+fx))^7} + \frac{2a^3(4A-7B)c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^5} + \left(3\right.$$

$$= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{3f(a+a\sin(e+fx))^7} + \frac{2a^3(4A-7B)c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{6a}{}$$

$$= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{3f(a+a\sin(e+fx))^7} + \frac{2a^3(4A-7B)c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{6a}{}$$

$$= \frac{35(4A-7B)c^5\cos^3(e+fx)}{4a^2f} - \frac{a^5(A-B)c^5\cos^{11}(e+fx)}{3f(a+a\sin(e+fx))^7} + \frac{2a^3(}{3}$$

$$= \frac{35(4A-7B)c^5\cos^3(e+fx)}{4a^2f} + \frac{105(4A-7B)c^5\cos(e+fx)\sin(e}{8a^2f}$$

$$= \frac{105(4A-7B)c^5x}{8a^2} + \frac{35(4A-7B)c^5\cos^3(e+fx)}{4a^2f} + \frac{105(4A-7B)}{}$$

Mathematica [A] time = 2.07, size = 354, normalized size = 1.48

$$\frac{(c-c\sin(e+fx))^5\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2048(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+1260(4A-7B)(e+fx)\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^5*(2048*(A - B)
*Sin[(e + f*x)/2] - 1024*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 10
24*(13*A - 19*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 +
 1260*(4*A - 7*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 24*(9
5*A - 217*B)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 8*(A -
7*B)*Cos[3*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 24*(7*A - 2
4*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)] - 3*B*(Cos[(e
```

```
 + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[4*(e + f*x)]))/(96*a^2*f*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2])^10*(1 + Sin[e + f*x])^2)
```

fricas [A] time = 0.46, size = 370, normalized size = 1.54

$$-\frac{6\,Bc^5\cos\left(fx+e\right)^6+4\,(2\,A-11\,B)c^5\cos\left(fx+e\right)^5+(76\,A-241\,B)c^5\cos\left(fx+e\right)^4-2\,(212\,A-431\,B)c^5\cos\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/24*(6*B*c^5*cos(f*x + e)^6 + 4*(2*A - 11*B)*c^5*cos(f*x + e)^5 + (76*A -
 241*B)*c^5*cos(f*x + e)^4 - 2*(212*A - 431*B)*c^5*cos(f*x + e)^3 + 630*(4*
A - 7*B)*c^5*f*x - 256*(A - B)*c^5 - (315*(4*A - 7*B)*c^5*f*x - (2156*A - 3
485*B)*c^5)*cos(f*x + e)^2 + (315*(4*A - 7*B)*c^5*f*x + 2*(1196*A - 2141*B)
*c^5)*cos(f*x + e) + (6*B*c^5*cos(f*x + e)^5 - 2*(4*A - 25*B)*c^5*cos(f*x +
 e)^4 + (68*A - 191*B)*c^5*cos(f*x + e)^3 + 630*(4*A - 7*B)*c^5*f*x + 3*(16
4*A - 351*B)*c^5*cos(f*x + e)^2 + 256*(A - B)*c^5 + (315*(4*A - 7*B)*c^5*f*
x + 2*(1324*A - 2269*B)*c^5)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e
)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x
 + e))
```

giac [A] time = 0.26, size = 412, normalized size = 1.72

$$\frac{\frac{315\left(4\,Ac^5-7\,Bc^5\right)\left(fx+e\right)}{a^2}+\frac{256\left(9\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-15\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+24\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-36\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+11\,Ac^5-17\,Bc^5\right)}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}+\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/24*(315*(4*A*c^5 - 7*B*c^5)*(f*x + e)/a^2 + 256*(9*A*c^5*tan(1/2*f*x + 1/
2*e)^2 - 15*B*c^5*tan(1/2*f*x + 1/2*e)^2 + 24*A*c^5*tan(1/2*f*x + 1/2*e) -
36*B*c^5*tan(1/2*f*x + 1/2*e) + 11*A*c^5 - 17*B*c^5)/(a^2*(tan(1/2*f*x + 1/
2*e) + 1)^3) + 2*(84*A*c^5*tan(1/2*f*x + 1/2*e)^7 - 285*B*c^5*tan(1/2*f*x +
 1/2*e)^7 + 552*A*c^5*tan(1/2*f*x + 1/2*e)^6 - 1176*B*c^5*tan(1/2*f*x + 1/2
*e)^6 + 84*A*c^5*tan(1/2*f*x + 1/2*e)^5 - 309*B*c^5*tan(1/2*f*x + 1/2*e)^5
+ 1704*A*c^5*tan(1/2*f*x + 1/2*e)^4 - 3864*B*c^5*tan(1/2*f*x + 1/2*e)^4 - 8
4*A*c^5*tan(1/2*f*x + 1/2*e)^3 + 309*B*c^5*tan(1/2*f*x + 1/2*e)^3 + 1720*A*
```

```
c^5*tan(1/2*f*x + 1/2*e)^2 - 3976*B*c^5*tan(1/2*f*x + 1/2*e)^2 - 84*A*c^5*t
an(1/2*f*x + 1/2*e) + 285*B*c^5*tan(1/2*f*x + 1/2*e) + 568*A*c^5 - 1288*B*c
^5)/((tan(1/2*f*x + 1/2*e)^2 + 1)^4*a^2))/f
```

maple [B] time = 0.52, size = 778, normalized size = 3.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 7*c^5/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^7*A-95/4*c^5/a^2/
f/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^7*B+46*c^5/a^2/f/(1+tan(1/2
*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^6*A-98*c^5/a^2/f/(1+tan(1/2*f*x+1/2*e)^
2)^4*tan(1/2*f*x+1/2*e)^6*B+7*c^5/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*
f*x+1/2*e)^5*A-103/4*c^5/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e
)^5*B+142*c^5/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^4*A-322*c
^5/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^4*B-7*c^5/a^2/f/(1+t
an(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^3*A+103/4*c^5/a^2/f/(1+tan(1/2*f*
x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^3*B+430/3*c^5/a^2/f/(1+tan(1/2*f*x+1/2*e)^
2)^4*tan(1/2*f*x+1/2*e)^2*A-994/3*c^5/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(
1/2*f*x+1/2*e)^2*B-7*c^5/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e
)*A+95/4*c^5/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)*B+142/3*c^
5/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^4*A-322/3*c^5/a^2/f/(1+tan(1/2*f*x+1/2*e)^
2)^4*B+105*c^5/a^2/f*arctan(tan(1/2*f*x+1/2*e))*A-735/4*c^5/a^2/f*arctan(ta
n(1/2*f*x+1/2*e))*B+64*c^5/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*A-64*c^5/a^2/f/(t
an(1/2*f*x+1/2*e)+1)^2*B+96*c^5/a^2/f/(tan(1/2*f*x+1/2*e)+1)*A-160*c^5/a^2/
f/(tan(1/2*f*x+1/2*e)+1)*B-128/3*c^5/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*A+128/3
*c^5/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*B
```

maxima [B] time = 0.76, size = 2982, normalized size = 12.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] -1/12*(B*c^5*((603*sin(f*x + e)/(cos(f*x + e) + 1) + 1297*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 2228*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 2628*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 3014*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
2618*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1980*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 + 1100*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 495*sin(f*x + e)^9/(c
os(f*x + e) + 1)^9 + 165*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 256)/(a^2
+ 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 7*a^2*sin(f*x + e)^2/(cos(f*x + e
```

```
) + 1)^2 + 13*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 18*a^2*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 22*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 22*a
^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 18*a^2*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 + 13*a^2*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 7*a^2*sin(f*x + e)^
9/(cos(f*x + e) + 1)^9 + 3*a^2*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + a^2*
sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 165*arctan(sin(f*x + e)/(cos(f*x +
 e) + 1))/a^2) - 20*A*c^5*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5 + 21*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)
/(cos(f*x + e) + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 + 5*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f
*x + e) + 1)^6 + a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f
*x + e)/(cos(f*x + e) + 1))/a^2) + 40*B*c^5*((75*sin(f*x + e)/(cos(f*x + e)
 + 1) + 97*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 98*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5 + 21*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 +
3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 7*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 5*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin
(f*x + e)^6/(cos(f*x + e) + 1)^6 + a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)
 + 21*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 8*A*c^5*((57*sin(f*x +
 e)/(cos(f*x + e) + 1) + 99*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 155*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + 153*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
135*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 85*sin(f*x + e)^6/(cos(f*x + e) +
 1)^6 + 45*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 15*sin(f*x + e)^8/(cos(f*x
 + e) + 1)^8 + 24)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 6*a^2*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 12*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 12*a^2*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5 + 10*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 6*a^2*sin(
f*x + e)^7/(cos(f*x + e) + 1)^7 + 3*a^2*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
 + a^2*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 15*arctan(sin(f*x + e)/(cos(f
*x + e) + 1))/a^2) + 40*B*c^5*((57*sin(f*x + e)/(cos(f*x + e) + 1) + 99*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 155*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
+ 153*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 135*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 + 85*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 45*sin(f*x + e)^7/(cos(
f*x + e) + 1)^7 + 15*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 24)/(a^2 + 3*a^2
*sin(f*x + e)/(cos(f*x + e) + 1) + 6*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 10*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 12*a^2*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + 12*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 10*a^2*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6 + 6*a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7
 + 3*a^2*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^2*sin(f*x + e)^9/(cos(f*x
+ e) + 1)^9) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 160*A*c^5*
((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos(f*x + e) + 1)
```

```
^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*a^2*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*
a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 160*B*c^5*((12*s
in(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*a^2*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a^2*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 80*A*c^5*((9*sin(f*x +
 e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 +
3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(
cos(f*x + e) + 1))/a^2) + 40*B*c^5*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x
 + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 8
*A*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - 40*A
*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos
(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x +
e)^3/(cos(f*x + e) + 1)^3) + 8*B*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1
)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

mupad [B] time = 14.83, size = 500, normalized size = 2.08

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(391\,A\,c^5-\frac{2729\,B\,c^5}{4}\right)+\frac{494\,A\,c^5}{3}-\frac{866\,B\,c^5}{3}+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}\,\left(103\,A\,c^5-\frac{735\,B\,c^5}{4}\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{9}\,\left(323\right.}{f\,\left(a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{11}+3\,a^2\,\tan\left(\frac{e}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^5)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(391*A*c^5 - (2729*B*c^5)/4) + (494*A*c^5)/3 - (866*B*c
^5)/3 + tan(e/2 + (f*x)/2)^10*(103*A*c^5 - (735*B*c^5)/4) + tan(e/2 + (f*x)
/2)^9*(323*A*c^5 - (2213*B*c^5)/4) + tan(e/2 + (f*x)/2)^7*(1332*A*c^5 - 225
3*B*c^5) + tan(e/2 + (f*x)/2)^4*(1632*A*c^5 - 2943*B*c^5) + tan(e/2 + (f*x)
/2)^8*((2002*A*c^5)/3 - (3637*B*c^5)/3) + tan(e/2 + (f*x)/2)^3*((4420*A*c^5
)/3 - (7621*B*c^5)/3) + tan(e/2 + (f*x)/2)^2*((2489*A*c^5)/3 - (17609*B*c^5
)/12) + tan(e/2 + (f*x)/2)^6*((4594*A*c^5)/3 - (16805*B*c^5)/6) + tan(e/2 +
```

```
 (f*x)/2)^5*((6274*A*c^5)/3 - (21299*B*c^5)/6))/(f*(7*a^2*tan(e/2 + (f*x)/2
)^2 + 13*a^2*tan(e/2 + (f*x)/2)^3 + 18*a^2*tan(e/2 + (f*x)/2)^4 + 22*a^2*ta
n(e/2 + (f*x)/2)^5 + 22*a^2*tan(e/2 + (f*x)/2)^6 + 18*a^2*tan(e/2 + (f*x)/2
)^7 + 13*a^2*tan(e/2 + (f*x)/2)^8 + 7*a^2*tan(e/2 + (f*x)/2)^9 + 3*a^2*tan(
e/2 + (f*x)/2)^10 + a^2*tan(e/2 + (f*x)/2)^11 + a^2 + 3*a^2*tan(e/2 + (f*x)
/2))) + (105*c^5*atan((105*c^5*tan(e/2 + (f*x)/2)*(4*A - 7*B))/(420*A*c^5 -
 735*B*c^5))*(4*A - 7*B))/(4*a^2*f)
```

sympy [A] time = 68.94, size = 10608, normalized size = 44.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**5/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((1260*A*c**5*f*x*tan(e/2 + f*x/2)**11/(24*a**2*f*tan(e/2 + f*x/2)
**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 31
2*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*
tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 +
 f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**
2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) + 3780*A*c**5*f*x*tan(e/2 + f*x
/2)**10/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 +
168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*
f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2
 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)
**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2
*f) + 8820*A*c**5*f*x*tan(e/2 + f*x/2)**9/(24*a**2*f*tan(e/2 + f*x/2)**11 +
 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2
*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/
2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2
)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72
*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) + 16380*A*c**5*f*x*tan(e/2 + f*x/2)**
8/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a*
*2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(
e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x
/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 +
168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) +
22680*A*c**5*f*x*tan(e/2 + f*x/2)**7/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a
**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*ta
n(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f
*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4
+ 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2
*f*tan(e/2 + f*x/2) + 24*a**2*f) + 27720*A*c**5*f*x*tan(e/2 + f*x/2)**6/(24
*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*
tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 +
```

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 f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**
5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a
**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) + 27720
*A*c**5*f*x*tan(e/2 + f*x/2)**5/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f
*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2
 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)
**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312
*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*ta
n(e/2 + f*x/2) + 24*a**2*f) + 22680*A*c**5*f*x*tan(e/2 + f*x/2)**4/(24*a**2
*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e
/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/
2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 4
32*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f
*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) + 16380*A*c*
*5*f*x*tan(e/2 + f*x/2)**3/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(
e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*
x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 +
 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2
*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2
 + f*x/2) + 24*a**2*f) + 8820*A*c**5*f*x*tan(e/2 + f*x/2)**2/(24*a**2*f*tan
(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f
*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7
+ 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**
2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e
/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) + 3780*A*c**5*f*x*
tan(e/2 + f*x/2)/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/
2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 +
432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*
f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2
 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2)
+ 24*a**2*f) + 1260*A*c**5*f*x/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*
tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2
+ f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)*
*6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*
a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan
(e/2 + f*x/2) + 24*a**2*f) + 2472*A*c**5*tan(e/2 + f*x/2)**10/(24*a**2*f*ta
n(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 +
f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7
 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a*
*2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(
e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) + 7752*A*c**5*tan
(e/2 + f*x/2)**9/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/
2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 +
432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*
f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2
```

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 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2)
+ 24*a**2*f) + 16016*A*c**5*tan(e/2 + f*x/2)**8/(24*a**2*f*tan(e/2 + f*x/2)
**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 31
2*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*
tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 +
 f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**
2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) + 31968*A*c**5*tan(e/2 + f*x/2)
**7/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*
a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*ta
n(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f
*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3
+ 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f)
+ 36752*A*c**5*tan(e/2 + f*x/2)**6/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**
2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(
e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x
/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 +
312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f
*tan(e/2 + f*x/2) + 24*a**2*f) + 50192*A*c**5*tan(e/2 + f*x/2)**5/(24*a**2*
f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/
2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2
)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 43
2*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*
tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) + 39168*A*c**
5*tan(e/2 + f*x/2)**4/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 +
 f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)*
*8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*
a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*ta
n(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*
x/2) + 24*a**2*f) + 35360*A*c**5*tan(e/2 + f*x/2)**3/(24*a**2*f*tan(e/2 + f
*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9
 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a*
*2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(
e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x
/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) + 19912*A*c**5*tan(e/2 + f
*x/2)**2/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 +
 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2
*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/
2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2
)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**
2*f) + 9384*A*c**5*tan(e/2 + f*x/2)/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a*
*2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan
(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*
x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 +
 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*
f*tan(e/2 + f*x/2) + 24*a**2*f) + 3952*A*c**5/(24*a**2*f*tan(e/2 + f*x/2)**
```

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11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*
a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*ta
n(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f
*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2
+ 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) - 2205*B*c**5*f*x*tan(e/2 + f*x/2
)**11/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 16
8*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*
tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 +
 f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**
3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f
) - 6615*B*c**5*f*x*tan(e/2 + f*x/2)**10/(24*a**2*f*tan(e/2 + f*x/2)**11 +
72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*
f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2
 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)
**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*
a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) - 15435*B*c**5*f*x*tan(e/2 + f*x/2)**9
/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**
2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e
/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/
2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 1
68*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) - 2
8665*B*c**5*f*x*tan(e/2 + f*x/2)**8/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a*
*2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan
(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*
x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 +
 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*
f*tan(e/2 + f*x/2) + 24*a**2*f) - 39690*B*c**5*f*x*tan(e/2 + f*x/2)**7/(24*
a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*t
an(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 +
f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5
 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a*
*2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) - 48510*
B*c**5*f*x*tan(e/2 + f*x/2)**6/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*
tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2
+ f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)*
*6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*
a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan
(e/2 + f*x/2) + 24*a**2*f) - 48510*B*c**5*f*x*tan(e/2 + f*x/2)**5/(24*a**2*
f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/
2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2
)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 43
2*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*
tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) - 39690*B*c**
5*f*x*tan(e/2 + f*x/2)**4/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e
/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x
```

/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 +
528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*
f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2
+ f*x/2) + 24*a**2*f) - 28665*B*c**5*f*x*tan(e/2 + f*x/2)**3/(24*a**2*f*tan
(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f
*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7
+ 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**
2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e
/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) - 15435*B*c**5*f*x
*tan(e/2 + f*x/2)**2/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 +
f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**
8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a
2*f*tan(e/2 + f*x/2)5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan
(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x
/2) + 24*a**2*f) - 6615*B*c**5*f*x*tan(e/2 + f*x/2)/(24*a**2*f*tan(e/2 + f*
x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9
+ 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**
2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e
/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/
2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) - 2205*B*c**5*f*x/(24*a**2*
f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/
2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2
)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 43
2*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*
tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) - 4410*B*c**5
*tan(e/2 + f*x/2)**10/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 +
f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)*
*8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*
a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*ta
n(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*
x/2) + 24*a**2*f) - 13278*B*c**5*tan(e/2 + f*x/2)**9/(24*a**2*f*tan(e/2 + f
*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9
+ 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a*
*2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(
e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x
/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) - 29096*B*c**5*tan(e/2 + f
*x/2)**8/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 +
168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2
*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/
2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2
)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**
2*f) - 54072*B*c**5*tan(e/2 + f*x/2)**7/(24*a**2*f*tan(e/2 + f*x/2)**11 + 7
2*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f
*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2
+ f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)*

```
*4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a
**2*f*tan(e/2 + f*x/2) + 24*a**2*f) - 67220*B*c**5*tan(e/2 + f*x/2)**6/(24*
a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*t
an(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 +
f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5
 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a*
*2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) - 85196*
B*c**5*tan(e/2 + f*x/2)**5/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(
e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*
x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 +
 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2
*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2
 + f*x/2) + 24*a**2*f) - 70632*B*c**5*tan(e/2 + f*x/2)**4/(24*a**2*f*tan(e/
2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/
2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 5
28*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f
*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2
+ f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) - 60968*B*c**5*tan(e/
2 + f*x/2)**3/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)*
*10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432
*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*t
an(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 +
f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 2
4*a**2*f) - 35218*B*c**5*tan(e/2 + f*x/2)**2/(24*a**2*f*tan(e/2 + f*x/2)**1
1 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a
**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan
(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*
x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 +
 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) - 16374*B*c**5*tan(e/2 + f*x/2)/(2
4*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f
*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2
+ f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)*
*5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*
a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) - 6928
*B*c**5/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 +
168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*
f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2
 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)
**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2
*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c)**5/(a*sin(e) + a)**2, Tru
e))
```

3.61 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=180

$$-\frac{a^4c^4(A-B)\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^6}+\frac{35c^4(A-2B)\cos^3(e+fx)}{3a^2f}+\frac{2a^2c^4(A-2B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}+\frac{35c^4(A-2B)\sin(e+fx)}{2a^2f}$$

```
[Out] 35/2*(A-2*B)*c^4*x/a^2+35/3*(A-2*B)*c^4*cos(f*x+e)^3/a^2/f+35/2*(A-2*B)*c^4
*cos(f*x+e)*sin(f*x+e)/a^2/f-1/3*a^4*(A-B)*c^4*cos(f*x+e)^9/f/(a+a*sin(f*x+
e))^6+2*a^2*(A-2*B)*c^4*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^4+14*(A-2*B)*c^4*co
s(f*x+e)^5/f/(a+a*sin(f*x+e))^2
```

Rubi [A] time = 0.36, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2967, 2859, 2680, 2682, 2635, 8}

$$\frac{35c^4(A-2B)\cos^3(e+fx)}{3a^2f}-\frac{a^4c^4(A-B)\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^6}+\frac{2a^2c^4(A-2B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}+\frac{35c^4(A-2B)\sin(e+fx)}{2a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (35*(A - 2*B)*c^4*x)/(2*a^2) + (35*(A - 2*B)*c^4*Cos[e + f*x]^3)/(3*a^2*f)
+ (35*(A - 2*B)*c^4*Cos[e + f*x]*Sin[e + f*x])/(2*a^2*f) - (a^4*(A - B)*c^4
*Cos[e + f*x]^9)/(3*f*(a + a*Sin[e + f*x])^6) + (2*a^2*(A - 2*B)*c^4*Cos[e
+ f*x]^7)/(f*(a + a*Sin[e + f*x])^4) + (14*(A - 2*B)*c^4*Cos[e + f*x]^5)/(f
*(a + a*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
```

```
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx = \left(a^4c^4\right)\int \frac{\cos^8(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^6}\,dx$$
$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} - \left(a^3(A-2B)c^4\right)\int \frac{\cos^8(e+fx}{(a+a\sin(e+f}$$
$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} + \frac{2a^2(A-2B)c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} + \left(7a\right.$$
$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} + \frac{2a^2(A-2B)c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} + \frac{14}{}$$
$$= \frac{35(A-2B)c^4\cos^3(e+fx)}{3a^2f} - \frac{a^4(A-B)c^4\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} + \frac{2a^2(}{f(}$$
$$= \frac{35(A-2B)c^4\cos^3(e+fx)}{3a^2f} + \frac{35(A-2B)c^4\cos(e+fx)\sin(e+}{2a^2f}$$
$$= \frac{35(A-2B)c^4x}{2a^2} + \frac{35(A-2B)c^4\cos^3(e+fx)}{3a^2f} + \frac{35(A-2B)c^4\cos}{}$$

Mathematica [A] time = 1.32, size = 311, normalized size = 1.73

$$\frac{(c-c\sin(e+fx))^4\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(128(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+210(A-2B)(e+fx)\left(\sin\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^4*(128*(A - B)*
Sin[(e + f*x)/2] - 64*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 128*(
5*A - 8*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 210*(
A - 2*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 3*(24*A - 71*B
)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + B*Cos[3*(e + f*x)]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 3*(A - 6*B)*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)]))/(12*a^2*f*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^8*(1 + Sin[e + f*x])^2)
```

fricas [A] time = 0.44, size = 322, normalized size = 1.79

$$\frac{2\,Bc^{4}\cos\left(fx+e\right)^{5}-(3\,A-16\,B)c^{4}\cos\left(fx+e\right)^{4}+2\,(15\,A-38\,B)c^{4}\cos\left(fx+e\right)^{3}-210\,(A-2\,B)c^{4}fx+32\,(A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] 1/6*(2*B*c^4*cos(f*x + e)^5 - (3*A - 16*B)*c^4*cos(f*x + e)^4 + 2*(15*A - 3
8*B)*c^4*cos(f*x + e)^3 - 210*(A - 2*B)*c^4*f*x + 32*(A - B)*c^4 + (105*(A
- 2*B)*c^4*f*x - (193*A - 346*B)*c^4)*cos(f*x + e)^2 - (105*(A - 2*B)*c^4*f
*x + 2*(97*A - 202*B)*c^4)*cos(f*x + e) - (2*B*c^4*cos(f*x + e)^4 + (3*A -
14*B)*c^4*cos(f*x + e)^3 + 210*(A - 2*B)*c^4*f*x + 3*(11*A - 30*B)*c^4*cos(
f*x + e)^2 + 32*(A - B)*c^4 + (105*(A - 2*B)*c^4*f*x + 2*(113*A - 218*B)*c^
4)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) -
 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

giac [B] time = 0.22, size = 369, normalized size = 2.05

$$\frac{\frac{105\left(Ac^{4}-2\,Bc^{4}\right)\left(fx+e\right)}{a^{2}}+\frac{2\left(99\,Ac^{4}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-210\,Bc^{4}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}+333\,Ac^{4}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}-636\,Bc^{4}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+533\,Ac^{4}\tan\left(\frac{1}{2}\,fx\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/6*(105*(A*c^4 - 2*B*c^4)*(f*x + e)/a^2 + 2*(99*A*c^4*tan(1/2*f*x + 1/2*e)
^8 - 210*B*c^4*tan(1/2*f*x + 1/2*e)^8 + 333*A*c^4*tan(1/2*f*x + 1/2*e)^7 -
636*B*c^4*tan(1/2*f*x + 1/2*e)^7 + 533*A*c^4*tan(1/2*f*x + 1/2*e)^6 - 1160*
B*c^4*tan(1/2*f*x + 1/2*e)^6 + 1047*A*c^4*tan(1/2*f*x + 1/2*e)^5 - 1980*B*c
^4*tan(1/2*f*x + 1/2*e)^5 + 921*A*c^4*tan(1/2*f*x + 1/2*e)^4 - 1980*B*c^4*t
an(1/2*f*x + 1/2*e)^4 + 1107*A*c^4*tan(1/2*f*x + 1/2*e)^3 - 2140*B*c^4*tan(
1/2*f*x + 1/2*e)^3 + 651*A*c^4*tan(1/2*f*x + 1/2*e)^2 - 1344*B*c^4*tan(1/2*
f*x + 1/2*e)^2 + 393*A*c^4*tan(1/2*f*x + 1/2*e) - 780*B*c^4*tan(1/2*f*x + 1
/2*e) + 164*A*c^4 - 330*B*c^4)/((tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2
*e)^2 + tan(1/2*f*x + 1/2*e) + 1)^3*a^2))/f
```

maple [B] time = 0.46, size = 549, normalized size = 3.05

$$\frac{c^4\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)A}{a^2 f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}-\frac{6c^4\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)B}{a^2 f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}+\frac{12c^4\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)A}{a^2 f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}-\frac{34c^4\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)B}{a^2 f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3}+\frac{24c}{a^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x)
```

```
[Out] c^4/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*A-6*c^4/a^2/f/(1+
tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*B+12*c^4/a^2/f/(1+tan(1/2*f*x+
1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^4*A-34*c^4/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^3*
tan(1/2*f*x+1/2*e)^4*B+24*c^4/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+
1/2*e)^2*A-72*c^4/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2*B-c
^4/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*A+6*c^4/a^2/f/(1+tan
(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*B+12*c^4/a^2/f/(1+tan(1/2*f*x+1/2*e
)^2)^3*A-106/3*c^4/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^3*B+35*c^4/a^2/f*arctan(t
an(1/2*f*x+1/2*e))*A-70*c^4/a^2/f*arctan(tan(1/2*f*x+1/2*e))*B+32*c^4/a^2/f
/(tan(1/2*f*x+1/2*e)+1)^2*A-32*c^4/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*B+32*c^4/
a^2/f/(tan(1/2*f*x+1/2*e)+1)*A-64*c^4/a^2/f/(tan(1/2*f*x+1/2*e)+1)*B-64/3*c
^4/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*A+64/3*c^4/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3
*B
```

maxima [B] time = 0.52, size = 2094, normalized size = 11.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/3*(A*c^4*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e)
+ 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
+ a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f*
x + e) + 1))/a^2) - 4*B*c^4*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
98*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) +
1)^5 + 21*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x +
```

```
e)/(cos(f*x + e) + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1
)^4 + 5*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 + a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin
(f*x + e)/(cos(f*x + e) + 1))/a^2) - 2*B*c^4*((57*sin(f*x + e)/(cos(f*x + e
) + 1) + 99*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 155*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 153*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 135*sin(f*x + e)
^5/(cos(f*x + e) + 1)^5 + 85*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 45*sin(f
*x + e)^7/(cos(f*x + e) + 1)^7 + 15*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 2
4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 6*a^2*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 10*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 12*a^2*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + 12*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5 + 10*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 6*a^2*sin(f*x + e)^7/(cos(
f*x + e) + 1)^7 + 3*a^2*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^2*sin(f*x +
 e)^9/(cos(f*x + e) + 1)^9) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^
2) + 16*A*c^4*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4
*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 24
*B*c^4*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*a^2*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 12*A*c^4*
((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(s
in(f*x + e)/(cos(f*x + e) + 1))/a^2) - 8*B*c^4*((9*sin(f*x + e)/(cos(f*x +
e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x +
 e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e) +
1))/a^2) - 2*A*c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^
2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) +
1)^3) + 8*A*c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*
x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - 2*B*c^4*(3*sin(f*x + e)/(cos(f*x +
e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

mupad [B] time = 14.95, size = 414, normalized size = 2.30

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(131\,A\,c^4-260\,B\,c^4\right)+\frac{164\,A\,c^4}{3}-110\,B\,c^4+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8\,\left(33\,A\,c^4-70\,B\,c^4\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7\,(11}{f\,\left(a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9+3\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+6\,a^2\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^4)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(131*A*c^4 - 260*B*c^4) + (164*A*c^4)/3 - 110*B*c^4 + t
an(e/2 + (f*x)/2)^8*(33*A*c^4 - 70*B*c^4) + tan(e/2 + (f*x)/2)^7*(111*A*c^4
 - 212*B*c^4) + tan(e/2 + (f*x)/2)^2*(217*A*c^4 - 448*B*c^4) + tan(e/2 + (f
*x)/2)^4*(307*A*c^4 - 660*B*c^4) + tan(e/2 + (f*x)/2)^5*(349*A*c^4 - 660*B*
c^4) + tan(e/2 + (f*x)/2)^6*((533*A*c^4)/3 - (1160*B*c^4)/3) + tan(e/2 + (f
*x)/2)^3*(369*A*c^4 - (2140*B*c^4)/3))/(f*(6*a^2*tan(e/2 + (f*x)/2)^2 + 10*
a^2*tan(e/2 + (f*x)/2)^3 + 12*a^2*tan(e/2 + (f*x)/2)^4 + 12*a^2*tan(e/2 + (
f*x)/2)^5 + 10*a^2*tan(e/2 + (f*x)/2)^6 + 6*a^2*tan(e/2 + (f*x)/2)^7 + 3*a^
2*tan(e/2 + (f*x)/2)^8 + a^2*tan(e/2 + (f*x)/2)^9 + a^2 + 3*a^2*tan(e/2 + (
f*x)/2))) + (35*c^4*atan((35*c^4*tan(e/2 + (f*x)/2)*(A - 2*B))/(35*A*c^4 -
70*B*c^4))*(A - 2*B))/(a^2*f)
```

sympy [A] time = 44.15, size = 7337, normalized size = 40.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((105*A*c**4*f*x*tan(e/2 + f*x/2)**9/(6*a**2*f*tan(e/2 + f*x/2)**9
 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*
f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 +
 f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2
+ 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 315*A*c**4*f*x*tan(e/2 + f*x/2)*
*8/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*
f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 +
 f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3
+ 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) +
630*A*c**4*f*x*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*
f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 +
 f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4
+ 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f
*tan(e/2 + f*x/2) + 6*a**2*f) + 1050*A*c**4*f*x*tan(e/2 + f*x/2)**6/(6*a**2
```

*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2
+ f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5
+ 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*
f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 1260*A*c**
4*f*x*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2
+ f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**
6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2
*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2
+ f*x/2) + 6*a**2*f) + 1260*A*c**4*f*x*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/
2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)*
*7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**
2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2
+ f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 1050*A*c**4*f*x*tan
(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)
8 + 36*a2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a*
*2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/
2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2)
+ 6*a**2*f) + 630*A*c**4*f*x*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)
9 + 18*a2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a*
*2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/
2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)*
*2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 315*A*c**4*f*x*tan(e/2 + f*x/
2)/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*
f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 +
f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3
+ 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) +
105*A*c**4*f*x/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**
8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2
*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2
+ f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) +
6*a**2*f) + 198*A*c**4*tan(e/2 + f*x/2)**8/(6*a**2*f*tan(e/2 + f*x/2)**9 +
18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*t
an(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*
x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 1
8*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 666*A*c**4*tan(e/2 + f*x/2)**7/(6*a
2*f*tan(e/2 + f*x/2)9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e
/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)
5 + 72*a2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a*
*2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 1066*A*
c**4*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2
+ f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6
+ 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*
f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 +
f*x/2) + 6*a**2*f) + 2094*A*c**4*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f
*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 +

60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*t
an(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*
x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 1842*A*c**4*tan(e/2 + f*
x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*
a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(
e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2
)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*
f) + 2214*A*c**4*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**
2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2
 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**
4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2
*f*tan(e/2 + f*x/2) + 6*a**2*f) + 1302*A*c**4*tan(e/2 + f*x/2)**2/(6*a**2*f
*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 +
f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 +
 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*
tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 786*A*c**4*t
an(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)*
*8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**
2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2
 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) +
 6*a**2*f) + 328*A*c**4/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 +
 f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6
+ 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f
*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 +
f*x/2) + 6*a**2*f) - 210*B*c**4*f*x*tan(e/2 + f*x/2)**9/(6*a**2*f*tan(e/2 +
 f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7
+ 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f
*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 +
f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 630*B*c**4*f*x*tan(e/2
 + f*x/2)**8/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8
+ 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f
*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 +
f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*
a**2*f) - 1260*B*c**4*f*x*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**9
 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*
f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 +
 f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2
+ 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 2100*B*c**4*f*x*tan(e/2 + f*x/2)
6/(6*a2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2
*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2
+ f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3
 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) -
 2520*B*c**4*f*x*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**
2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2
 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**

```
4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2
*f*tan(e/2 + f*x/2) + 6*a**2*f) - 2520*B*c**4*f*x*tan(e/2 + f*x/2)**4/(6*a*
*2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/
2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)*
*5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**
2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 2100*B*c
**4*f*x*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e
/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)
**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a*
*2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/
2 + f*x/2) + 6*a**2*f) - 1260*B*c**4*f*x*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(
e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2
)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a
**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e
/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 630*B*c**4*f*x*ta
n(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**
8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2
*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2
 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) +
6*a**2*f) - 210*B*c**4*f*x/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/
2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)*
*6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**
2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2
 + f*x/2) + 6*a**2*f) - 420*B*c**4*tan(e/2 + f*x/2)**8/(6*a**2*f*tan(e/2 +
f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 +
 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*
tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f
*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 1272*B*c**4*tan(e/2 + f
*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36
*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan
(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/
2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2
*f) - 2320*B*c**4*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a*
*2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/
2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)*
*4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**
2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 3960*B*c**4*tan(e/2 + f*x/2)**5/(6*a**2*
f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 +
 f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5
+ 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f
*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 3960*B*c**4
*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*
x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 7
2*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*ta
n(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x
```

```
/2) + 6*a**2*f) - 4280*B*c**4*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2
)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a
**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e
/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)
**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 2688*B*c**4*tan(e/2 + f*x/2)
**2/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2
*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2
+ f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3
 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) -
 1560*B*c**4*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan
(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/
2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*
a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(
e/2 + f*x/2) + 6*a**2*f) - 660*B*c**4/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a*
*2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/
2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)*
*4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**
2*f*tan(e/2 + f*x/2) + 6*a**2*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) +
 c)**4/(a*sin(e) + a)**2, True))
```

3.62 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=162

$$-\frac{a^3c^3(A-B)\cos^7(e+fx)}{3f(a\sin(e+fx)+a)^5}+\frac{5c^3(2A-5B)\cos(e+fx)}{2a^2f}+\frac{5c^3(2A-5B)\cos^3(e+fx)}{6f\left(a^2\sin(e+fx)+a^2\right)}+\frac{5c^3x(2A-5B)}{2a^2}+\frac{2ac^3(2A-}{3f(a\sin}$$

```
[Out] 5/2*(2*A-5*B)*c^3*x/a^2+5/2*(2*A-5*B)*c^3*cos(f*x+e)/a^2/f-1/3*a^3*(A-B)*c^
3*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^5+2/3*a*(2*A-5*B)*c^3*cos(f*x+e)^5/f/(a+a
*sin(f*x+e))^3+5/6*(2*A-5*B)*c^3*cos(f*x+e)^3/f/(a^2+a^2*sin(f*x+e))
```

Rubi [A] time = 0.33, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2967, 2859, 2680, 2679, 2682, 8}

$$\frac{5c^3(2A-5B)\cos(e+fx)}{2a^2f}-\frac{a^3c^3(A-B)\cos^7(e+fx)}{3f(a\sin(e+fx)+a)^5}+\frac{5c^3(2A-5B)\cos^3(e+fx)}{6f\left(a^2\sin(e+fx)+a^2\right)}+\frac{5c^3x(2A-5B)}{2a^2}+\frac{2ac^3(2A-5}{3f(a\sin(e}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (5*(2*A - 5*B)*c^3*x)/(2*a^2) + (5*(2*A - 5*B)*c^3*Cos[e + f*x])/(2*a^2*f)
- (a^3*(A - B)*c^3*Cos[e + f*x]^7)/(3*f*(a + a*Sin[e + f*x])^5) + (2*a*(2*A
 - 5*B)*c^3*Cos[e + f*x]^5)/(3*f*(a + a*Sin[e + f*x])^3) + (5*(2*A - 5*B)*c
^3*Cos[e + f*x]^3)/(6*f*(a^2 + a^2*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
```

```
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^5}\,dx$$

$$= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} - \frac{1}{3}\left(a^2(2A-5B)c^3\right)\int \frac{\cos^6(e+}{(a+a\sin(e+}$$

$$= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{2a(2A-5B)c^3\cos^5(e+fx)}{3f(a+a\sin(e+fx))^3} + \frac{1}{3}(5$$

$$= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{2a(2A-5B)c^3\cos^5(e+fx)}{3f(a+a\sin(e+fx))^3} + \frac{5(2}{6f}$$

$$= \frac{5(2A-5B)c^3\cos(e+fx)}{2a^2f} - \frac{a^3(A-B)c^3\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{2a(2A-}{3f(a}$$

$$= \frac{5(2A-5B)c^3x}{2a^2} + \frac{5(2A-5B)c^3\cos(e+fx)}{2a^2f} - \frac{a^3(A-B)c^3\cos^7(}{3f(a+a\sin(e+}$$

Mathematica [A] time = 0.88, size = 274, normalized size = 1.69

$$\frac{(c-c\sin(e+fx))^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(64(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+30(2A-5B)(e+fx)\left(\sin\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^3*(64*(A - B)*S
in[(e + f*x)/2] - 32*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 32*(7*
A - 13*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 30*(2*
A - 5*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 12*(A - 5*B)*C
os[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 3*B*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)]))/(12*a^2*f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^6*(1 + Sin[e + f*x])^2)
```

fricas [A] time = 0.45, size = 291, normalized size = 1.80

$$\frac{3\,Bc^3\cos\left(fx+e\right)^4+6\,(A-4\,B)c^3\cos\left(fx+e\right)^3-30\,(2\,A-5\,B)c^3fx+16\,(A-B)c^3+\left(15\,(2\,A-5\,B)c^3fx-(62\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] 1/6*(3*B*c^3*cos(f*x + e)^4 + 6*(A - 4*B)*c^3*cos(f*x + e)^3 - 30*(2*A - 5*
B)*c^3*f*x + 16*(A - B)*c^3 + (15*(2*A - 5*B)*c^3*f*x - (62*A - 131*B)*c^3)
*cos(f*x + e)^2 - (15*(2*A - 5*B)*c^3*f*x + 2*(26*A - 71*B)*c^3)*cos(f*x +
e) + (3*B*c^3*cos(f*x + e)^3 - 30*(2*A - 5*B)*c^3*f*x - 3*(2*A - 9*B)*c^3*c
os(f*x + e)^2 - 16*(A - B)*c^3 - (15*(2*A - 5*B)*c^3*f*x + 2*(34*A - 79*B)*
c^3)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e)
 - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

giac [A] time = 0.20, size = 233, normalized size = 1.44

$$\frac{\frac{15\left(2\,Ac^3-5\,Bc^3\right)\left(fx+e\right)}{a^2}-\frac{6\left(Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+10\,Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,Ac^3+10\,Bc^3\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a^2}+\frac{16\left(3\,Ac^3 \cdots\right.}{\cdots}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/6*(15*(2*A*c^3 - 5*B*c^3)*(f*x + e)/a^2 - 6*(B*c^3*tan(1/2*f*x + 1/2*e)^3
 - 2*A*c^3*tan(1/2*f*x + 1/2*e)^2 + 10*B*c^3*tan(1/2*f*x + 1/2*e)^2 - B*c^3
*tan(1/2*f*x + 1/2*e) - 2*A*c^3 + 10*B*c^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2
*a^2) + 16*(3*A*c^3*tan(1/2*f*x + 1/2*e)^2 - 9*B*c^3*tan(1/2*f*x + 1/2*e)^2
 + 12*A*c^3*tan(1/2*f*x + 1/2*e) - 24*B*c^3*tan(1/2*f*x + 1/2*e) + 5*A*c^3
- 11*B*c^3)/(a^2*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

maple [B] time = 0.45, size = 399, normalized size = 2.46

$$-\frac{c^3\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)B}{a^2f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{2c^3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)A}{a^2f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{10c^3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)B}{a^2f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{c^3B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a^2f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x)
```

```
[Out] -c^3/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^3*B+2*c^3/a^2/f/(1
+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*A-10*c^3/a^2/f/(1+tan(1/2*f*x
+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*B+c^3/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^2*B*
```

```
tan(1/2*f*x+1/2*e)+2*c^3/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^2*A-10*c^3/a^2/f/(1
+tan(1/2*f*x+1/2*e)^2)^2*B-25*c^3/a^2/f*arctan(tan(1/2*f*x+1/2*e))*B+10*c^3
/a^2/f*arctan(tan(1/2*f*x+1/2*e))*A+16*c^3/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*A
-16*c^3/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*B+8*c^3/a^2/f/(tan(1/2*f*x+1/2*e)+1)
*A-24*c^3/a^2/f/(tan(1/2*f*x+1/2*e)+1)*B-32/3*c^3/a^2/f/(tan(1/2*f*x+1/2*e)
+1)^3*A+32/3*c^3/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*B
```

maxima [B] time = 0.48, size = 1378, normalized size = 8.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] -1/3*(B*c^3*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(f*
x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e)
 + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
 + a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f
*x + e) + 1))/a^2) - 4*A*c^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*
x + e) + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e)
+ 1))/a^2) + 12*B*c^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e)
+ 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a
^2) - 6*A*c^3*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 6*B*c^3*((9*sin(f*x +
e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3
*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(c
os(f*x + e) + 1))/a^2) + 2*A*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(c
```

```
os(f*x + e) + 1)^3) - 6*A*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2
+ 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 2*B*c^3*(3*sin(f*x +
e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3
*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3))/f
```

mupad [B] time = 14.34, size = 336, normalized size = 2.07

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(38\,A\,c^3-93\,B\,c^3\right)+\frac{46\,A\,c^3}{3}-\frac{118\,B\,c^3}{3}+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(8\,A\,c^3-25\,B\,c^3\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,\left(34\,A\,c^3\right.}{f\,\left(a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7+3\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+5\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+7\,a^2\,\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^3)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(38*A*c^3 - 93*B*c^3) + (46*A*c^3)/3 - (118*B*c^3)/3 +
tan(e/2 + (f*x)/2)^6*(8*A*c^3 - 25*B*c^3) + tan(e/2 + (f*x)/2)^5*(34*A*c^3
- 77*B*c^3) + tan(e/2 + (f*x)/2)^3*(72*A*c^3 - 166*B*c^3) + tan(e/2 + (f*x)
/2)^4*((106*A*c^3)/3 - (328*B*c^3)/3) + tan(e/2 + (f*x)/2)^2*((128*A*c^3)/3
 - (359*B*c^3)/3))/(f*(5*a^2*tan(e/2 + (f*x)/2)^2 + 7*a^2*tan(e/2 + (f*x)/2
)^3 + 7*a^2*tan(e/2 + (f*x)/2)^4 + 5*a^2*tan(e/2 + (f*x)/2)^5 + 3*a^2*tan(e
/2 + (f*x)/2)^6 + a^2*tan(e/2 + (f*x)/2)^7 + a^2 + 3*a^2*tan(e/2 + (f*x)/2)
)) + (5*c^3*atan((5*c^3*tan(e/2 + (f*x)/2)*(2*A - 5*B))/(10*A*c^3 - 25*B*c^
3))*(2*A - 5*B))/(a^2*f)
```

sympy [A] time = 25.78, size = 4665, normalized size = 28.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((30*A*c**3*f*x*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**7
+ 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f
*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 +
f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 90*A*c**3*f*x*tan(e/2
+ f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 +
 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*
tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f
*x/2) + 6*a**2*f) + 150*A*c**3*f*x*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 +
f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 +
```

```
 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*
tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 210*A*c**3*f
*x*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 +
f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 +
 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*
tan(e/2 + f*x/2) + 6*a**2*f) + 210*A*c**3*f*x*tan(e/2 + f*x/2)**3/(6*a**2*f
*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 +
f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 +
 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 1
50*A*c**3*f*x*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f
*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 +
f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 +
 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 90*A*c**3*f*x*tan(e/2 + f*x/2)/(6
*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan
(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/
2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2
*f) + 30*A*c**3*f*x/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x
/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42
*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan
(e/2 + f*x/2) + 6*a**2*f) + 48*A*c**3*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2
 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**
5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2
*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 204*A*c**
3*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f
*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 +
42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*t
an(e/2 + f*x/2) + 6*a**2*f) + 212*A*c**3*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(
e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2
)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a
**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 432*A*
c**3*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2
+ f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4
 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*
f*tan(e/2 + f*x/2) + 6*a**2*f) + 256*A*c**3*tan(e/2 + f*x/2)**2/(6*a**2*f*t
an(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*
x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 3
0*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 228
*A*c**3*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2
+ f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4
 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*
f*tan(e/2 + f*x/2) + 6*a**2*f) + 92*A*c**3/(6*a**2*f*tan(e/2 + f*x/2)**7 +
18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*t
an(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*
x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 75*B*c**3*f*x*tan(e/2 +
f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 3
```

0*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*ta
n(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x
/2) + 6*a**2*f) - 225*B*c**3*f*x*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*
x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 4
2*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*ta
n(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 375*B*c**3*f*x
*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*
x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 4
2*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*ta
n(e/2 + f*x/2) + 6*a**2*f) - 525*B*c**3*f*x*tan(e/2 + f*x/2)**4/(6*a**2*f*t
an(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*
x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 3
0*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 525
*B*c**3*f*x*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*t
an(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*
x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 1
8*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 375*B*c**3*f*x*tan(e/2 + f*x/2)**2/
(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*t
an(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*
x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a*
*2*f) - 225*B*c**3*f*x*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*
a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(
e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2
)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 75*B*c**3*f*x/(6*a**2*f*tan
(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/
2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*
a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 150*B
*c**3*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2
 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**
4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2
*f*tan(e/2 + f*x/2) + 6*a**2*f) - 462*B*c**3*tan(e/2 + f*x/2)**5/(6*a**2*f*
tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f
*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 +
30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 65
6*B*c**3*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(
e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2
)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a
2*f*tan(e/2 + f*x/2) + 6*a2*f) - 996*B*c**3*tan(e/2 + f*x/2)**3/(6*a**2
*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2
+ f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3
 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) -
 718*B*c**3*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*t
an(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*
x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 1
8*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 558*B*c**3*tan(e/2 + f*x/2)/(6*a**2

```
*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2
+ f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3
 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) -
 236*B*c**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 +
 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*
tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f
*x/2) + 6*a**2*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c)**3/(a*sin(e
) + a)**2, True))
```

3.63 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=108

$$\frac{c^2(A-4B)\cos(e+fx)}{a^2f}-\frac{a^2c^2(A-B)\cos^5(e+fx)}{3f(a\sin(e+fx)+a)^4}+\frac{c^2x(A-4B)}{a^2}+\frac{2c^2(A-4B)\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

```
[Out] (A-4*B)*c^2*x/a^2+(A-4*B)*c^2*cos(f*x+e)/a^2/f-1/3*a^2*(A-B)*c^2*cos(f*x+e)
^5/f/(a+a*sin(f*x+e))^4+2/3*(A-4*B)*c^2*cos(f*x+e)^3/f/(a+a*sin(f*x+e))^2
```

Rubi [A] time = 0.28, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2859, 2680, 2682, 8}

$$\frac{c^2(A-4B)\cos(e+fx)}{a^2f}-\frac{a^2c^2(A-B)\cos^5(e+fx)}{3f(a\sin(e+fx)+a)^4}+\frac{c^2x(A-4B)}{a^2}+\frac{2c^2(A-4B)\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((A - 4*B)*c^2*x)/a^2 + ((A - 4*B)*c^2*Cos[e + f*x])/(a^2*f) - (a^2*(A - B)
*c^2*Cos[e + f*x]^5)/(3*f*(a + a*Sin[e + f*x])^4) + (2*(A - 4*B)*c^2*Cos[e
+ f*x]^3)/(3*f*(a + a*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^4}\,dx$$

$$= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4} - \frac{1}{3}\left(a(A-4B)c^2\right)\int \frac{\cos^4(e+fx}{(a+a\sin(e+f}$$

$$= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4} + \frac{2(A-4B)c^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^2} + \frac{\left((A-\right.}{}$$

$$= \frac{(A-4B)c^2\cos(e+fx)}{a^2f} - \frac{a^2(A-B)c^2\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4} + \frac{2(A-4B)}{3f(a+a}$$

$$= \frac{(A-4B)c^2x}{a^2} + \frac{(A-4B)c^2\cos(e+fx)}{a^2f} - \frac{a^2(A-B)c^2\cos^5(e+f}{3f(a+a\sin(e+fx))^}$$

Mathematica [B] time = 0.60, size = 234, normalized size = 2.17

$$\frac{(c-c\sin(e+fx))^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(8(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+3(A-4B)(e+fx)\left(\sin\left(\frac{1}{2}(e\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*Sin[(e + f*x)/2] - 4*(A -
 B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 8*(2*A - 5*B)*Sin[(e + f*x)/2]*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 3*(A - 4*B)*(e + f*x)*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])^3 - 3*B*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^3)*(c - c*Sin[e + f*x])^2)/(3*a^2*f*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^4*(1 + Sin[e + f*x])^2)
```

fricas [B] time = 0.45, size = 242, normalized size = 2.24

$$-\frac{3\,Bc^2\cos\left(fx+e\right)^3+6\,(A-4\,B)c^2fx-4\,(A-B)c^2-\left(3\,(A-4\,B)c^2fx-(8\,A-23\,B)c^2\right)\cos\left(fx+e\right)^2+\left(3\,(\ldots\right.}{3\left(a^2f\cos\left(fx+\ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/3*(3*B*c^2*cos(f*x + e)^3 + 6*(A - 4*B)*c^2*f*x - 4*(A - B)*c^2 - (3*(A
- 4*B)*c^2*f*x - (8*A - 23*B)*c^2)*cos(f*x + e)^2 + (3*(A - 4*B)*c^2*f*x +
2*(2*A - 11*B)*c^2)*cos(f*x + e) + (6*(A - 4*B)*c^2*f*x - 3*B*c^2*cos(f*x +
 e)^2 + 4*(A - B)*c^2 + (3*(A - 4*B)*c^2*f*x + 2*(4*A - 13*B)*c^2)*cos(f*x
+ e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f -
(a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

giac [A] time = 0.19, size = 136, normalized size = 1.26

$$\frac{\frac{3\left(Ac^2-4\,Bc^2\right)\left(fx+e\right)}{a^2}-\frac{6\,Bc^2}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^2}-\frac{8\left(3\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+9\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-Ac^2+4\,Bc^2\right)}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/3*(3*(A*c^2 - 4*B*c^2)*(f*x + e)/a^2 - 6*B*c^2/((tan(1/2*f*x + 1/2*e)^2 +
 1)*a^2) - 8*(3*B*c^2*tan(1/2*f*x + 1/2*e)^2 - 3*A*c^2*tan(1/2*f*x + 1/2*e)
 + 9*B*c^2*tan(1/2*f*x + 1/2*e) - A*c^2 + 4*B*c^2)/(a^2*(tan(1/2*f*x + 1/2*
e) + 1)^3))/f
```

maple [A] time = 0.43, size = 198, normalized size = 1.83

$$-\frac{2c^2B}{a^2f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}+\frac{2c^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)A}{a^2f}-\frac{8c^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)B}{a^2f}+\frac{8c^2A}{a^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x)
```

```
[Out] -2*c^2/a^2/f*B/(1+tan(1/2*f*x+1/2*e)^2)+2*c^2/a^2/f*arctan(tan(1/2*f*x+1/2*
e))*A-8*c^2/a^2/f*arctan(tan(1/2*f*x+1/2*e))*B+8*c^2/a^2/f/(tan(1/2*f*x+1/2
*e)+1)^2*A-8*c^2/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*B-16/3*c^2/a^2/f/(tan(1/2*f
*x+1/2*e)+1)^3*A+16/3*c^2/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*B-8*c^2/a^2/f*B/(t
an(1/2*f*x+1/2*e)+1)
```

maxima [B] time = 0.46, size = 833, normalized size = 7.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] -2/3*(2*B*c^2*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4
*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - A*
c^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arct
an(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 2*B*c^2*((9*sin(f*x + e)/(cos(f*
x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e
) + 1))/a^2) + A*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*
a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3) - 2*A*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(
f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a
^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + B*c^2*(3*sin(f*x + e)/(cos(f*x +
e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

mupad [B] time = 14.53, size = 242, normalized size = 2.24

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(8\,A\,c^2-30\,B\,c^2\right)+\frac{8\,A\,c^2}{3}-\frac{38\,B\,c^2}{3}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(8\,A\,c^2-26\,B\,c^2\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{8\,A\,c^2}{3}-\frac{74}{3}\right.}{f\,\left(a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+3\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+4\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+4\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+3\,a^2\,\tan\left(\frac{e}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^2)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(8*A*c^2 - 30*B*c^2) + (8*A*c^2)/3 - (38*B*c^2)/3 + tan
(e/2 + (f*x)/2)^3*(8*A*c^2 - 26*B*c^2) + tan(e/2 + (f*x)/2)^2*((8*A*c^2)/3
- (74*B*c^2)/3) - 8*B*c^2*tan(e/2 + (f*x)/2)^4)/(f*(4*a^2*tan(e/2 + (f*x)/2
)^2 + 4*a^2*tan(e/2 + (f*x)/2)^3 + 3*a^2*tan(e/2 + (f*x)/2)^4 + a^2*tan(e/2
 + (f*x)/2)^5 + a^2 + 3*a^2*tan(e/2 + (f*x)/2))) + (2*c^2*atan((2*c^2*tan(e
/2 + (f*x)/2)*(A - 4*B))/(2*A*c^2 - 8*B*c^2))*(A - 4*B))/(a^2*f)
```

sympy [A] time = 15.06, size = 2474, normalized size = 22.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*A*c**2*f*x*tan(e/2 + f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 +
 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*t
an(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*A*c**2*f*x*t
an(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2
)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a*
*2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 12*A*c**2*f*x*tan(e/2 + f*x/2)**3/(3*a*
*2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2
 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) +
3*a**2*f) + 12*A*c**2*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5
 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f
*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*A*c**2*f*x
*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)
**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**
2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 3*A*c**2*f*x/(3*a**2*f*tan(e/2 + f*x/2)*
*5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2
*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 24*A*c**2*
tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/
2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a
**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 8*A*c**2*tan(e/2 + f*x/2)**2/(3*a**2*f
```

```
*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f
*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a*
*2*f) + 24*A*c**2*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f
*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 +
f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 8*A*c**2/(3*a**2*f*tan(
e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)
**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f)
 - 12*B*c**2*f*x*tan(e/2 + f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2
*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 36*B*c**2*f*x*tan(e/2
 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 +
 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*t
an(e/2 + f*x/2) + 3*a**2*f) - 48*B*c**2*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*t
an(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x
/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2
*f) - 48*B*c**2*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a
**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e
/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 36*B*c**2*f*x*tan(
e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 +
 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*t
an(e/2 + f*x/2) + 3*a**2*f) - 12*B*c**2*f*x/(3*a**2*f*tan(e/2 + f*x/2)**5 +
 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*t
an(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 24*B*c**2*tan(
e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**
4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*
f*tan(e/2 + f*x/2) + 3*a**2*f) - 78*B*c**2*tan(e/2 + f*x/2)**3/(3*a**2*f*ta
n(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/
2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*
f) - 74*B*c**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f
*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 +
f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 90*B*c**2*tan(e/2 + f*x
/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*
f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 +
f*x/2) + 3*a**2*f) - 38*B*c**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan
(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/
2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f), Ne(f, 0)), (x*(A + B*sin(e))
*(-c*sin(e) + c)**2/(a*sin(e) + a)**2, True))
```

3.64 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=72

$$\frac{c(A-7B)\cos(e+fx)}{3a^2f(\sin(e+fx)+1)}-\frac{Bcx}{a^2}-\frac{2c(A-B)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

```
[Out] -B*c*x/a^2+1/3*(A-7*B)*c*cos(f*x+e)/a^2/f/(1+sin(f*x+e))-2/3*(A-B)*c*cos(f*
x+e)/f/(a+a*sin(f*x+e))^2
```

Rubi [A] time = 0.21, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2967, 2857, 2735, 2648}

$$\frac{c(A-7B)\cos(e+fx)}{3a^2f(\sin(e+fx)+1)}-\frac{Bcx}{a^2}-\frac{2c(A-B)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -((B*c*x)/a^2) + ((A - 7*B)*c*Cos[e + f*x])/(3*a^2*f*(1 + Sin[e + f*x])) -
(2*(A - B)*c*Cos[e + f*x])/(3*f*(a + a*Sin[e + f*x])^2)
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx &= (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx\\ &= -\frac{2(A-B)c\cos(e+fx)}{3f(a+a\sin(e+fx))^2}-\frac{c\int \frac{aA-4aB+3aB\sin(e+fx)}{a+a\sin(e+fx)}\,dx}{3a^2}\\ &= -\frac{Bcx}{a^2}-\frac{2(A-B)c\cos(e+fx)}{3f(a+a\sin(e+fx))^2}-\frac{((A-7B)c)\int \frac{1}{a+a\sin(e+fx)}\,dx}{3a}\\ &= -\frac{Bcx}{a^2}-\frac{2(A-B)c\cos(e+fx)}{3f(a+a\sin(e+fx))^2}+\frac{(A-7B)c\cos(e+fx)}{3f\left(a^2+a^2\sin(e+fx)\right)}\end{aligned}$$

Mathematica [B] time = 0.58, size = 156, normalized size = 2.17

$$\frac{c\left(-6(A-3B)\cos\left(e+\frac{fx}{2}\right)+2A\cos\left(e+\frac{3fx}{2}\right)-9Bfx\sin\left(e+\frac{fx}{2}\right)-3Bfx\sin\left(e+\frac{3fx}{2}\right)-14B\cos\left(e+\frac{3fx}{2}\right)+3\right.}{6a^2f\left(\sin\left(\frac{e}{2}\right)+\cos\left(\frac{e}{2}\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])^
2,x]
```

```
[Out] (c*(-9*B*f*x*Cos[(f*x)/2] - 6*(A - 3*B)*Cos[e + (f*x)/2] + 2*A*Cos[e + (3*f
*x)/2] - 14*B*Cos[e + (3*f*x)/2] + 3*B*f*x*Cos[2*e + (3*f*x)/2] + 24*B*Sin[
(f*x)/2] - 9*B*f*x*Sin[e + (f*x)/2] - 3*B*f*x*Sin[e + (3*f*x)/2]))/(6*a^2*f
*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

fricas [B] time = 0.44, size = 166, normalized size = 2.31

$$\frac{6\,Bcfx-\left(3\,Bcfx+(A-7\,B)c\right)\cos\left(fx+e\right)^2+2\,(A-B)c+\left(3\,Bcfx+(A+5\,B)c\right)\cos\left(fx+e\right)+\left(6\,Bcfx-2\,(A\right.}{3\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-2\,a^2f-\left(a^2f\cos\left(fx+e\right)+2\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] 1/3*(6*B*c*f*x - (3*B*c*f*x + (A - 7*B)*c)*cos(f*x + e)^2 + 2*(A - B)*c + (
3*B*c*f*x + (A + 5*B)*c)*cos(f*x + e) + (6*B*c*f*x - 2*(A - B)*c + (3*B*c*f
*x - (A - 7*B)*c)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f
*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

giac [A] time = 0.16, size = 92, normalized size = 1.28

$$-\frac{\frac{3\left(fx+e\right)Bc}{a^2}+\frac{2\left(3\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+12\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+Ac+5\,Bc\right)}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] -1/3*(3*(f*x + e)*B*c/a^2 + 2*(3*A*c*tan(1/2*f*x + 1/2*e)^2 + 3*B*c*tan(1/2
*f*x + 1/2*e)^2 + 12*B*c*tan(1/2*f*x + 1/2*e) + A*c + 5*B*c)/(a^2*(tan(1/2*
f*x + 1/2*e) + 1)^3))/f
```

maple [B] time = 0.42, size = 160, normalized size = 2.22

$$-\frac{2cB\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^2f}+\frac{4cA}{a^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{4cB}{a^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2cA}{a^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{}{a^2f\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)
```

```
[Out] -2*c/a^2/f*B*arctan(tan(1/2*f*x+1/2*e))+4*c/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*
A-4*c/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*B-2*c/a^2/f/(tan(1/2*f*x+1/2*e)+1)*A-2
*c/a^2/f/(tan(1/2*f*x+1/2*e)+1)*B-8/3*c/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*A+8/
3*c/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*B
```

maxima [B] time = 0.46, size = 452, normalized size = 6.28

$$-\frac{2\left(Bc\left(\frac{\frac{9\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+4}{a^2+\frac{3a^2\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3a^2\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{a^2\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}}+\frac{3\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{a^2}\right)+\frac{Ac\left(\frac{3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+2\right)}{a^2+\frac{3a^2\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3a^2\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{a^2\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}}-\frac{}{a^2+}\right.}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] -2/3*(B*c*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) +
3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + A*c*(3*sin(f*x + e)/(cos(f
*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(
f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a
^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - A*c*(3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + B*c*(
3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
 e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3))/f
```

mupad [B] time = 12.83, size = 133, normalized size = 1.85

$$-\frac{B\,c\,x}{a^2}-\frac{\left(\frac{c\,\left(6\,A+6\,B+9\,B\,\left(e+f\,x\right)\right)}{3}-3\,B\,c\,\left(e+f\,x\right)\right)\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+\left(\frac{c\,\left(24\,B+9\,B\,\left(e+f\,x\right)\right)}{3}-3\,B\,c\,\left(e+f\,x\right)\right)\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^2\,f\,{\left(\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x)))/(a + a*sin(e + f*x))^2,x)
```

```
[Out] - (B*c*x)/a^2 - (tan(e/2 + (f*x)/2)^2*((c*(6*A + 6*B + 9*B*(e + f*x)))/3 -
3*B*c*(e + f*x)) + tan(e/2 + (f*x)/2)*((c*(24*B + 9*B*(e + f*x)))/3 - 3*B*c
*(e + f*x)) + (c*(2*A + 10*B + 3*B*(e + f*x)))/3 - B*c*(e + f*x))/(a^2*f*(t
an(e/2 + (f*x)/2) + 1)^3)
```

sympy [A] time = 7.39, size = 702, normalized size = 9.75

$$\begin{cases} -\frac{6Ac\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f}-\frac{2Ac}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f}-\frac{}{3a^2f\tan^3\left(\right.} \\ \frac{x(A+B\sin(e))(-c\sin(e)+c)}{(a\sin(e)+a)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*A*c*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**
2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*A*c/(3*
a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/
2 + f*x/2) + 3*a**2*f) - 3*B*c*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 +
f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a*
*2*f) - 9*B*c*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**
2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 9*B*c*f*x
*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)
**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 3*B*c*f*x/(3*a**2*f*tan(e/2 +
 f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a
**2*f) - 6*B*c*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f
*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 24*B*c*tan(e
/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 +
9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 10*B*c/(3*a**2*f*tan(e/2 + f*x/2)**
3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f), N
e(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c)/(a*sin(e) + a)**2, True))
```

3.65 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx$

Optimal. Leaf size=62

$$\frac{(2A+B)\tan(e+fx)}{3a^2cf}-\frac{(A-B)\sec(e+fx)}{3cf\left(a^2\sin(e+fx)+a^2\right)}$$

```
[Out] -1/3*(A-B)*sec(f*x+e)/c/f/(a^2+a^2*sin(f*x+e))+1/3*(2*A+B)*tan(f*x+e)/a^2/c
/f
```

Rubi [A] time = 0.20, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 3767, 8}

$$\frac{(2A+B)\tan(e+fx)}{3a^2cf}-\frac{(A-B)\sec(e+fx)}{3cf\left(a^2\sin(e+fx)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])),x]
```

```
[Out] -((A - B)*Sec[e + f*x])/(3*c*f*(a^2 + a^2*Sin[e + f*x])) + ((2*A + B)*Tan[e
 + f*x])/(3*a^2*c*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
```

```
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx = \frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx}{ac}$$

$$= -\frac{(A-B)\sec(e+fx)}{3cf\left(a^2+a^2\sin(e+fx)\right)} + \frac{(2A+B)\int \sec^2(e+fx)\,dx}{3a^2c}$$

$$= -\frac{(A-B)\sec(e+fx)}{3cf\left(a^2+a^2\sin(e+fx)\right)} - \frac{(2A+B)\,\text{Subst}(\int 1\,dx, x, -\tan(e+$$

$$= -\frac{(A-B)\sec(e+fx)}{3cf\left(a^2+a^2\sin(e+fx)\right)} + \frac{(2A+B)\tan(e+fx)}{3a^2cf}$$

Mathematica [A] time = 0.51, size = 110, normalized size = 1.77

$$\frac{\cos(e+fx)(-2(A-B)\cos(e+fx)+2(2A+B)\cos(2(e+fx))-8A\sin(e+fx)-A\sin(2(e+fx))-4B\sin(e}{12a^2cf(\sin(e+fx)-1)(\sin(e+fx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
),x]
```

```
[Out] (Cos[e + f*x]*(-6*B - 2*(A - B)*Cos[e + f*x] + 2*(2*A + B)*Cos[2*(e + f*x)]
 - 8*A*Sin[e + f*x] - 4*B*Sin[e + f*x] - A*Sin[2*(e + f*x)] + B*Sin[2*(e +
f*x)]))/(12*a^2*c*f*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])^2)
```

fricas [A] time = 0.41, size = 69, normalized size = 1.11

$$-\frac{(2\,A+B)\cos\left(fx+e\right)^2-(2\,A+B)\sin\left(fx+e\right)-A-2\,B}{3\left(a^2cf\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2cf\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] -1/3*((2*A + B)*cos(f*x + e)^2 - (2*A + B)*sin(f*x + e) - A - 2*B)/(a^2*c*f
*cos(f*x + e)*sin(f*x + e) + a^2*c*f*cos(f*x + e))
```

giac [A] time = 0.18, size = 102, normalized size = 1.65

$$-\frac{\frac{3\,(A+B)}{a^2c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}+\frac{9\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+12\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+7\,A-B}{a^2c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/6*(3*(A + B)/(a^2*c*(tan(1/2*f*x + 1/2*e) - 1)) + (9*A*tan(1/2*f*x + 1/2
*e)^2 - 3*B*tan(1/2*f*x + 1/2*e)^2 + 12*A*tan(1/2*f*x + 1/2*e) + 7*A - B)/(
a^2*c*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

maple [A] time = 0.44, size = 97, normalized size = 1.56

$$\frac{-\frac{2\left(\frac{A}{4}+\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{-A+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(A-B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(\frac{3A}{4}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{a^2cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x)
```

```
[Out] 2/f/a^2/c*(-(1/4*A+1/4*B)/(tan(1/2*f*x+1/2*e)-1)-1/2*(-A+B)/(tan(1/2*f*x+1/
2*e)+1)^2-1/3*(A-B)/(tan(1/2*f*x+1/2*e)+1)^3-(3/4*A-1/4*B)/(tan(1/2*f*x+1/2
*e)+1))
```

maxima [B] time = 0.35, size = 265, normalized size = 4.27

$$\frac{2\left(\frac{B\left(\frac{2\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+1\right)}{a^2c+\frac{2\,a^2c\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{2\,a^2c\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}-\frac{a^2c\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}}+\frac{A\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{3\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}-1\right)}{a^2c+\frac{2\,a^2c\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{2\,a^2c\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}-\frac{a^2c\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}}\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] 2/3*(B*(2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 1)/(a^2*c + 2*a^2*c*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^2*c*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3 - a^2*c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
) + A*(sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(a^2*c + 2*a^2*c*sin(f*x +
 e)/(cos(f*x + e) + 1) - 2*a^2*c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - a^2*
c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4))/f
```

mupad [B] time = 12.29, size = 117, normalized size = 1.89

$$\frac{2\left(\frac{3B}{2} - A\cos\left(e+fx\right) + B\cos\left(e+fx\right) + 2A\sin\left(e+fx\right) + B\sin\left(e+fx\right) - A\cos\left(2e+2fx\right) - \frac{B\cos\left(2e+\right.}{2}\right.}{3a^2cf\left(2\cos\left(e+fx\right)+\sin\left(2e+2fx\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))),x)
```

```
[Out] (2*((3*B)/2 - A*cos(e + f*x) + B*cos(e + f*x) + 2*A*sin(e + f*x) + B*sin(e
+ f*x) - A*cos(2*e + 2*f*x) - (B*cos(2*e + 2*f*x))/2 - (A*sin(2*e + 2*f*x))
/2 + (B*sin(2*e + 2*f*x))/2))/(3*a^2*c*f*(2*cos(e + f*x) + sin(2*e + 2*f*x)
))
```

sympy [A] time = 7.23, size = 578, normalized size = 9.32

$$\begin{cases} -\frac{6A\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2cf\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+6a^2cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-6a^2cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2cf} - \frac{6A\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2cf\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+6a^2cf\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)-6a^2cf\tan\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2cf} - \frac{}{3a} \\ \frac{x(A+B\sin(e))}{(a\sin(e)+a)^2(-c\sin(e)+c)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-6*A*tan(e/2 + f*x/2)**3/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**
2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 6*A
*tan(e/2 + f*x/2)**2/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 +
 f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 2*A*tan(e/2 + f*x/
2)/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 + f*x/2)**3 - 6*a**
2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) + 2*A/(3*a**2*c*f*tan(e/2 + f*x/2)**4
+ 6*a**2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f
```

```
) - 6*B*tan(e/2 + f*x/2)**2/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*ta
n(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 4*B*tan(e/2
 + f*x/2)/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 + f*x/2)**3
- 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 2*B/(3*a**2*c*f*tan(e/2 + f*x
/2)**4 + 6*a**2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a
**2*c*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)**2*(-c*sin(e) + c)),
 True))
```

3.66 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=62

$$\frac{A\tan^3(e+fx)}{3a^2c^2f}+\frac{A\tan(e+fx)}{a^2c^2f}+\frac{B\sec^3(e+fx)}{3a^2c^2f}$$

```
[Out] 1/3*B*sec(f*x+e)^3/a^2/c^2/f+A*tan(f*x+e)/a^2/c^2/f+1/3*A*tan(f*x+e)^3/a^2/
c^2/f
```

Rubi [A] time = 0.14, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2967, 2669, 3767}

$$\frac{A\tan^3(e+fx)}{3a^2c^2f}+\frac{A\tan(e+fx)}{a^2c^2f}+\frac{B\sec^3(e+fx)}{3a^2c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^2),x]
```

```
[Out] (B*Sec[e + f*x]^3)/(3*a^2*c^2*f) + (A*Tan[e + f*x])/(a^2*c^2*f) + (A*Tan[e
+ f*x]^3)/(3*a^2*c^2*f)
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))\,dx}{a^2c^2}$$
$$= \frac{B\sec^3(e+fx)}{3a^2c^2f} + \frac{A\int \sec^4(e+fx)\,dx}{a^2c^2}$$
$$= \frac{B\sec^3(e+fx)}{3a^2c^2f} - \frac{A\,\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -\tan(e+fx)\right)}{a^2c^2f}$$
$$= \frac{B\sec^3(e+fx)}{3a^2c^2f} + \frac{A\tan(e+fx)}{a^2c^2f} + \frac{A\tan^3(e+fx)}{3a^2c^2f}$$

Mathematica [A] time = 0.12, size = 53, normalized size = 0.85

$$\frac{A\left(\frac{1}{3}\tan^3(e+fx)+\tan(e+fx)\right)}{a^2c^2f} + \frac{B\sec^3(e+fx)}{3a^2c^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
^2),x]
```

```
[Out] (B*Sec[e + f*x]^3)/(3*a^2*c^2*f) + (A*(Tan[e + f*x] + Tan[e + f*x]^3/3))/(a
^2*c^2*f)
```

fricas [A] time = 0.42, size = 41, normalized size = 0.66

$$\frac{\left(2\,A\cos\left(fx+e\right)^2+A\right)\sin\left(fx+e\right)+B}{3\,a^2c^2f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] 1/3*((2*A*cos(f*x + e)^2 + A)*sin(f*x + e) + B)/(a^2*c^2*f*cos(f*x + e)^3)
```

giac [A] time = 0.20, size = 87, normalized size = 1.40

$$-\frac{2\left(3\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+3\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-2\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+3\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+B\right)}{3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)^{3}a^{2}c^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] -2/3*(3*A*tan(1/2*f*x + 1/2*e)^5 + 3*B*tan(1/2*f*x + 1/2*e)^4 - 2*A*tan(1/2
*f*x + 1/2*e)^3 + 3*A*tan(1/2*f*x + 1/2*e) + B)/((tan(1/2*f*x + 1/2*e)^2 -
1)^3*a^2*c^2*f)
```

maple [B] time = 0.35, size = 145, normalized size = 2.34

$$\frac{-\frac{2\left(\frac{A}{2}+\frac{B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{\frac{A}{2}+\frac{B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{2\left(\frac{A}{2}+\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{-\frac{A}{2}+\frac{B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{A}{2}-\frac{B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2\left(\frac{A}{2}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{f\,a^{2}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x)
```

```
[Out] 2/f/a^2/c^2*(-1/3*(1/2*A+1/2*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/2*(1/2*A+1/2*B)/
(tan(1/2*f*x+1/2*e)-1)^2-(1/2*A+1/4*B)/(tan(1/2*f*x+1/2*e)-1)-1/2*(-1/2*A+1
/2*B)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(1/2*A-1/2*B)/(tan(1/2*f*x+1/2*e)+1)^3-(
1/2*A-1/4*B)/(tan(1/2*f*x+1/2*e)+1))
```

maxima [A] time = 0.39, size = 47, normalized size = 0.76

$$\frac{\frac{\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)A}{a^{2}c^{2}}+\frac{B}{a^{2}c^{2}\cos\left(fx+e\right)^{3}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/3*((tan(f*x + e)^3 + 3*tan(f*x + e))*A/(a^2*c^2) + B/(a^2*c^2*cos(f*x + e
)^3))/f
```

mupad [B] time = 12.38, size = 82, normalized size = 1.32

$$-\frac{2\,\left(3\,A\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+3\,B\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-2\,A\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+3\,A\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+B\right)}{3\,a^2\,c^2\,f\,{\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^2),x)
```

```
[Out] -(2*(B + 3*A*tan(e/2 + (f*x)/2) - 2*A*tan(e/2 + (f*x)/2)^3 + 3*A*tan(e/2 +
(f*x)/2)^5 + 3*B*tan(e/2 + (f*x)/2)^4))/(3*a^2*c^2*f*(tan(e/2 + (f*x)/2)^2
- 1)^3)
```

sympy [A] time = 7.63, size = 469, normalized size = 7.56

$$\begin{cases} -\frac{6A\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2c^2f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)-9a^2c^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2c^2f}+\frac{4A\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2c^2f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)-9a^2c^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2 \\ \frac{x(A+B\sin(e))}{(a\sin(e)+a)^2(-c\sin(e)+c)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*A*tan(e/2 + f*x/2)**5/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*
a**2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**
2*c**2*f) + 4*A*tan(e/2 + f*x/2)**3/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*
a**2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**
2*c**2*f) - 6*A*tan(e/2 + f*x/2)/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a**
2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*c
**2*f) - 6*B*tan(e/2 + f*x/2)**4/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a**
2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*c
**2*f) - 2*B/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a**2*c**2*f*tan(e/2 + f
*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*c**2*f), Ne(f, 0)), (
x*(A + B*sin(e))/((a*sin(e) + a)**2*(-c*sin(e) + c)**2), True))
```

3.67 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=93

$$\frac{(4A-B)\tan^3(e+fx)}{15a^2c^3f}+\frac{(4A-B)\tan(e+fx)}{5a^2c^3f}+\frac{(A+B)\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)}$$

```
[Out] 1/5*(A+B)*sec(f*x+e)^3/a^2/f/(c^3-c^3*sin(f*x+e))+1/5*(4*A-B)*tan(f*x+e)/a^
2/c^3/f+1/15*(4*A-B)*tan(f*x+e)^3/a^2/c^3/f
```

Rubi [A] time = 0.22, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2967, 2859, 3767}

$$\frac{(4A-B)\tan^3(e+fx)}{15a^2c^3f}+\frac{(4A-B)\tan(e+fx)}{5a^2c^3f}+\frac{(A+B)\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^3),x]
```

```
[Out] ((A + B)*Sec[e + f*x]^3)/(5*a^2*f*(c^3 - c^3*Sin[e + f*x])) + ((4*A - B)*Ta
n[e + f*x])/(5*a^2*c^3*f) + ((4*A - B)*Tan[e + f*x]^3)/(15*a^2*c^3*f)
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx = \frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx}{a^2c^2}$$

$$= \frac{(A+B)\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)} + \frac{(4A-B)\int \sec^4(e+fx)\,dx}{5a^2c^3}$$

$$= \frac{(A+B)\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)} - \frac{(4A-B)\,\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -$$

$$= \frac{(A+B)\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)} + \frac{(4A-B)\tan(e+fx)}{5a^2c^3f} + \frac{(4A-B)\,\text{ta}}{15a^2}$$

Mathematica [B] time = 1.02, size = 237, normalized size = 2.55

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(54(A+B)\cos(e+fx)-32(4A-B)\cos}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
^3),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-240*B + 54*(A + B)*Cos[e + f*x] - 32*(4*A - B)*Cos[2*(e + f*x)] + 18*A*
Cos[3*(e + f*x)] + 18*B*Cos[3*(e + f*x)] - 64*A*Cos[4*(e + f*x)] + 16*B*Cos
[4*(e + f*x)] - 384*A*Sin[e + f*x] + 96*B*Sin[e + f*x] - 18*A*Sin[2*(e + f*
x)] - 18*B*Sin[2*(e + f*x)] - 128*A*Sin[3*(e + f*x)] + 32*B*Sin[3*(e + f*x)
] - 9*A*Sin[4*(e + f*x)] - 9*B*Sin[4*(e + f*x)]))/(960*a^2*c^3*f*(-1 + Sin[
e + f*x])^3*(1 + Sin[e + f*x])^2)
```

fricas [A] time = 0.42, size = 116, normalized size = 1.25

$$-\frac{2\,(4\,A-B)\cos\left(fx+e\right)^4-(4\,A-B)\cos\left(fx+e\right)^2+\left(2\,(4\,A-B)\cos\left(fx+e\right)^2+4\,A-B\right)\sin\left(fx+e\right)-A+4}{15\left(a^2c^3f\cos\left(fx+e\right)^3\sin\left(fx+e\right)-a^2c^3f\cos\left(fx+e\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/15*(2*(4*A - B)*cos(f*x + e)^4 - (4*A - B)*cos(f*x + e)^2 + (2*(4*A - B)
*cos(f*x + e)^2 + 4*A - B)*sin(f*x + e) - A + 4*B)/(a^2*c^3*f*cos(f*x + e)^
3*sin(f*x + e) - a^2*c^3*f*cos(f*x + e)^3)
```

giac [B] time = 0.20, size = 235, normalized size = 2.53

$$-\frac{\frac{5\left(15\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-9\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+24\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-12\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+13\,A-7\,B\right)}{a^{2}c^{3}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{3}}+\frac{165\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+45\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right.}{}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -1/120*(5*(15*A*tan(1/2*f*x + 1/2*e)^2 - 9*B*tan(1/2*f*x + 1/2*e)^2 + 24*A*
tan(1/2*f*x + 1/2*e) - 12*B*tan(1/2*f*x + 1/2*e) + 13*A - 7*B)/(a^2*c^3*(ta
n(1/2*f*x + 1/2*e) + 1)^3) + (165*A*tan(1/2*f*x + 1/2*e)^4 + 45*B*tan(1/2*f
*x + 1/2*e)^4 - 480*A*tan(1/2*f*x + 1/2*e)^3 - 60*B*tan(1/2*f*x + 1/2*e)^3
+ 650*A*tan(1/2*f*x + 1/2*e)^2 + 70*B*tan(1/2*f*x + 1/2*e)^2 - 400*A*tan(1/
2*f*x + 1/2*e) - 20*B*tan(1/2*f*x + 1/2*e) + 113*A + 13*B)/(a^2*c^3*(tan(1/
2*f*x + 1/2*e) - 1)^5))/f
```

maple [B] time = 0.50, size = 183, normalized size = 1.97

$$\frac{-\frac{2(A+B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{2A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{\frac{3A}{2}+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{2\left(\frac{5A}{2}+2B\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{2\left(\frac{11A}{16}+\frac{3B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{-\frac{A}{4}+\frac{B}{4}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{A}{4}-\frac{B}{4}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\right.}}{f\,a^{2}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x)
```

```
[Out] 2/f/a^2/c^3*(-1/5*(A+B)/(tan(1/2*f*x+1/2*e)-1)^5-1/4*(2*A+2*B)/(tan(1/2*f*x
+1/2*e)-1)^4-1/2*(3/2*A+B)/(tan(1/2*f*x+1/2*e)-1)^2-1/3*(5/2*A+2*B)/(tan(1/
2*f*x+1/2*e)-1)^3-(11/16*A+3/16*B)/(tan(1/2*f*x+1/2*e)-1)-1/2*(-1/4*A+1/4*B
)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(1/4*A-1/4*B)/(tan(1/2*f*x+1/2*e)+1)^3-(5/16
*A-3/16*B)/(tan(1/2*f*x+1/2*e)+1))
```

maxima [B] time = 0.47, size = 651, normalized size = 7.00

$$\frac{2\left(\frac{A\left(\frac{9\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{21\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{13\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{25\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-\frac{5\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}-\frac{15\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}+\frac{15\sin\left(fx+e\right)^{7}}{\left(\cos\left(fx+e\right)+1\right)^{7}}+3\right)}{a^{2}c^{3}-\frac{2a^{2}c^{3}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{2a^{2}c^{3}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{6a^{2}c^{3}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}-\frac{6a^{2}c^{3}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+\frac{2a^{2}c^{3}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}+\frac{2a^{2}c^{3}\sin\left(fx+e\right)^{7}}{\left(\cos\left(fx+e\right)+1\right)^{7}}-\frac{a^{2}c^{3}\sin\left(fx+e\right)^{8}}{\left(\cos\left(fx+e\right)+1\right)^{8}}}-\frac{B\left(\frac{6\,\mathrm{s}}{\cos}\right.}{a^{2}c^{3}-\frac{2a^{2}c^{3}\sin\left(fx+e\right.}{\cos\left(fx+e\right)+1}}\right.}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 2/15*(A*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 21*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 13*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 25*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 15*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 + 15*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 3)/(a^2*c^
3 - 2*a^2*c^3*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^2*c^3*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 6*a^2*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6*a^2*
c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2*a^2*c^3*sin(f*x + e)^6/(cos(f*x
 + e) + 1)^6 + 2*a^2*c^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^2*c^3*sin(
f*x + e)^8/(cos(f*x + e) + 1)^8) - B*(6*sin(f*x + e)/(cos(f*x + e) + 1) - 9
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 8*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3 - 5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 10*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5 - 15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3)/(a^2*c^3 - 2*a^2*c^3*
sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^2*c^3*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 6*a^2*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6*a^2*c^3*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5 + 2*a^2*c^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 +
 2*a^2*c^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^2*c^3*sin(f*x + e)^8/(co
s(f*x + e) + 1)^8))/f
```

mupad [B] time = 12.45, size = 183, normalized size = 1.97

$$\frac{\left(\frac{8\,A}{15}-\frac{2\,B}{15}-\frac{16\,A\,\sin\left(e+f\,x\right)}{15}+\frac{4\,B\,\sin\left(e+f\,x\right)}{15}\right)\,\cos\left(e+f\,x\right)^{2}+\frac{2\,A}{15}-\frac{8\,B}{15}-\frac{8\,A\,\sin\left(e+f\,x\right)}{15}+\frac{2\,B\,\sin\left(e+f\,x\right)}{15}}{a^{2}\,c^{3}\,f\,\left(2\cos\left(e+f\,x\right)^{3}\,\sin\left(e+f\,x\right)-2\cos\left(e+f\,x\right)^{3}\right)}-\frac{\frac{2\,A}{5}+\frac{2\,B}{5}-\frac{2\,A\,\mathrm{si}}{}}{a^{2}\,c^{3}\,f\,\left(2\,\mathrm{s}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^3),x)
```

```
[Out] ((2*A)/15 - (8*B)/15 - (8*A*sin(e + f*x))/15 + (2*B*sin(e + f*x))/15 + cos(
e + f*x)^2*((8*A)/15 - (2*B)/15 - (16*A*sin(e + f*x))/15 + (4*B*sin(e + f*x
))/15))/(a^2*c^3*f*(2*cos(e + f*x)^3*sin(e + f*x) - 2*cos(e + f*x)^3)) - ((
```

```
2*A)/5 + (2*B)/5 - (2*A*sin(e + f*x))/5 - (2*B*sin(e + f*x))/5)/(a^2*c^3*f*
(2*sin(e + f*x) - 2)) - (cos(e + f*x)*((16*A)/15 - (4*B)/15))/(a^2*c^3*f*(2
*sin(e + f*x) - 2))
```

sympy [A] time = 27.73, size = 2674, normalized size = 28.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*tan(e/2 + f*x/2)**7/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 -
30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 9
0*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30
*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**
2*c**3*f) + 30*A*tan(e/2 + f*x/2)**6/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 -
30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 9
0*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30
*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**
2*c**3*f) + 10*A*tan(e/2 + f*x/2)**5/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 -
30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 9
0*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30
*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**
2*c**3*f) - 50*A*tan(e/2 + f*x/2)**4/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 -
30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 9
0*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30
*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**
2*c**3*f) - 26*A*tan(e/2 + f*x/2)**3/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 -
30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 9
0*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30
*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**
2*c**3*f) + 42*A*tan(e/2 + f*x/2)**2/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 -
30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 9
0*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30
*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**
2*c**3*f) - 18*A*tan(e/2 + f*x/2)/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*
a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a
**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a*
*2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c
**3*f) - 6*A/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 +
 f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 +
f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f
*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) - 30*B*tan(e/2
 + f*x/2)**6/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 +
 f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 +
```

```
f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f
*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) + 20*B*tan(e/2
 + f*x/2)**5/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 +
 f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 +
f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f
*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) - 10*B*tan(e/2
 + f*x/2)**4/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 +
 f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 +
f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f
*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) - 16*B*tan(e/2
 + f*x/2)**3/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 +
 f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 +
f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f
*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) - 18*B*tan(e/2
 + f*x/2)**2/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 +
 f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 +
f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f
*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) + 12*B*tan(e/2
 + f*x/2)/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 + f*
x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 + f*x
/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f*x/
2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) - 6*B/(15*a**2*c*
*3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**
3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3
*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*
f*tan(e/2 + f*x/2) - 15*a**2*c**3*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(
e) + a)**2*(-c*sin(e) + c)**3), True))
```

3.68 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=135

$$\frac{4(5A-2B)\tan^3(e+fx)}{105a^2c^4f}+\frac{4(5A-2B)\tan(e+fx)}{35a^2c^4f}+\frac{(5A-2B)\sec^3(e+fx)}{35a^2f\left(c^4-c^4\sin(e+fx)\right)}+\frac{(A+B)\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2}$$

```
[Out] 1/7*(A+B)*sec(f*x+e)^3/a^2/f/(c^2-c^2*sin(f*x+e))^2+1/35*(5*A-2*B)*sec(f*x+
e)^3/a^2/f/(c^4-c^4*sin(f*x+e))+4/35*(5*A-2*B)*tan(f*x+e)/a^2/c^4/f+4/105*(
5*A-2*B)*tan(f*x+e)^3/a^2/c^4/f
```

Rubi [A] time = 0.27, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 2672, 3767}

$$\frac{4(5A-2B)\tan^3(e+fx)}{105a^2c^4f}+\frac{4(5A-2B)\tan(e+fx)}{35a^2c^4f}+\frac{(5A-2B)\sec^3(e+fx)}{35a^2f\left(c^4-c^4\sin(e+fx)\right)}+\frac{(A+B)\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] ((A + B)*Sec[e + f*x]^3)/(7*a^2*f*(c^2 - c^2*Sin[e + f*x])^2) + ((5*A - 2*B
)*Sec[e + f*x]^3)/(35*a^2*f*(c^4 - c^4*Sin[e + f*x])) + (4*(5*A - 2*B)*Tan[
e + f*x])/(35*a^2*c^4*f) + (4*(5*A - 2*B)*Tan[e + f*x]^3)/(105*a^2*c^4*f)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
```

```
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx = \frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx}{a^2c^2}$$

$$= \frac{(A+B)\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(5A-2B)\int \frac{\sec^4(e+fx)}{c-c\sin(e+fx)}\,dx}{7a^2c^3}$$

$$= \frac{(A+B)\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(5A-2B)\sec^3(e+fx)}{35a^2f\left(c^4-c^4\sin(e+fx)\right)} + \frac{(4(5A}{}$$

$$= \frac{(A+B)\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(5A-2B)\sec^3(e+fx)}{35a^2f\left(c^4-c^4\sin(e+fx)\right)} - \frac{(4(5A}{}$$

$$= \frac{(A+B)\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(5A-2B)\sec^3(e+fx)}{35a^2f\left(c^4-c^4\sin(e+fx)\right)} + \frac{4(5A}{}$$

Mathematica [B] time = 0.96, size = 285, normalized size = 2.11

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(42(25A+4B)\cos(e+fx)-512(5A-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
^4),x]
```

```
[Out] -1/13440*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])*(-2688*B + 42*(25*A + 4*B)*Cos[e + f*x] - 512*(5*A - 2*B)*Cos[2*
(e + f*x)] + 225*A*Cos[3*(e + f*x)] + 36*B*Cos[3*(e + f*x)] - 1280*A*Cos[4*
(e + f*x)] + 512*B*Cos[4*(e + f*x)] - 75*A*Cos[5*(e + f*x)] - 12*B*Cos[5*(e
 + f*x)] - 4480*A*Sin[e + f*x] + 1792*B*Sin[e + f*x] - 600*A*Sin[2*(e + f*x
)] - 96*B*Sin[2*(e + f*x)] - 960*A*Sin[3*(e + f*x)] + 384*B*Sin[3*(e + f*x)
] - 300*A*Sin[4*(e + f*x)] - 48*B*Sin[4*(e + f*x)] + 320*A*Sin[5*(e + f*x)]
 - 128*B*Sin[5*(e + f*x)]))/(a^2*c^4*f*(-1 + Sin[e + f*x])^4*(1 + Sin[e + f
*x])^2)
```

fricas [A] time = 0.43, size = 151, normalized size = 1.12

$$-\frac{16\,(5\,A-2\,B)\cos\left(fx+e\right)^{4}-8\,(5\,A-2\,B)\cos\left(fx+e\right)^{2}-\left(8\,(5\,A-2\,B)\cos\left(fx+e\right)^{4}-12\,(5\,A-2\,B)\cos\left(f\right.\right.}{105\left(a^{2}c^{4}f\cos\left(fx+e\right)^{5}+2\,a^{2}c^{4}f\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)-2\,a^{2}c^{4}f\,\mathrm{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorit
hm="fricas")
```

```
[Out] -1/105*(16*(5*A - 2*B)*cos(f*x + e)^4 - 8*(5*A - 2*B)*cos(f*x + e)^2 - (8*(
5*A - 2*B)*cos(f*x + e)^4 - 12*(5*A - 2*B)*cos(f*x + e)^2 - 25*A + 10*B)*si
n(f*x + e) - 10*A + 25*B)/(a^2*c^4*f*cos(f*x + e)^5 + 2*a^2*c^4*f*cos(f*x +
 e)^3*sin(f*x + e) - 2*a^2*c^4*f*cos(f*x + e)^3)
```

giac [B] time = 0.23, size = 295, normalized size = 2.19

$$-\frac{\frac{35\left(9\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-6\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+15\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-9\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+8\,A-5\,B\right)}{a^{2}c^{4}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{3}}+\frac{1365\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}+210\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorit
hm="giac")
```

```
[Out] -1/840*(35*(9*A*tan(1/2*f*x + 1/2*e)^2 - 6*B*tan(1/2*f*x + 1/2*e)^2 + 15*A*
tan(1/2*f*x + 1/2*e) - 9*B*tan(1/2*f*x + 1/2*e) + 8*A - 5*B)/(a^2*c^4*(tan(
1/2*f*x + 1/2*e) + 1)^3) + (1365*A*tan(1/2*f*x + 1/2*e)^6 + 210*B*tan(1/2*f
*x + 1/2*e)^6 - 5775*A*tan(1/2*f*x + 1/2*e)^5 - 105*B*tan(1/2*f*x + 1/2*e)^
5 + 12250*A*tan(1/2*f*x + 1/2*e)^4 - 175*B*tan(1/2*f*x + 1/2*e)^4 - 14350*A
```

```
*tan(1/2*f*x + 1/2*e)^3 + 910*B*tan(1/2*f*x + 1/2*e)^3 + 10185*A*tan(1/2*f*
x + 1/2*e)^2 - 756*B*tan(1/2*f*x + 1/2*e)^2 - 3955*A*tan(1/2*f*x + 1/2*e) +
 427*B*tan(1/2*f*x + 1/2*e) + 760*A - 31*B)/(a^2*c^4*(tan(1/2*f*x + 1/2*e)
- 1)^7))/f
```

maple [A] time = 0.50, size = 233, normalized size = 1.73

$$\frac{-\frac{2(2A+2B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{6A+6B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{10A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{2(10A+9B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{2\left(\frac{13A}{16}+\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{\frac{23A}{8}+\frac{11B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{2\left(\frac{55A}{8}+\frac{35B}{8}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right.}}{f\,a^{2}c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x)
```

```
[Out] 2/f/a^2/c^4*(-1/7*(2*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/6*(6*A+6*B)/(tan(1/2
*f*x+1/2*e)-1)^6-1/4*(10*A+8*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/5*(10*A+9*B)/(ta
n(1/2*f*x+1/2*e)-1)^5-(13/16*A+1/8*B)/(tan(1/2*f*x+1/2*e)-1)-1/2*(23/8*A+11
/8*B)/(tan(1/2*f*x+1/2*e)-1)^2-1/3*(55/8*A+35/8*B)/(tan(1/2*f*x+1/2*e)-1)^3
-1/2*(-1/8*A+1/8*B)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(1/8*A-1/8*B)/(tan(1/2*f*x
+1/2*e)+1)^3-(3/16*A-1/8*B)/(tan(1/2*f*x+1/2*e)+1))
```

maxima [B] time = 0.37, size = 835, normalized size = 6.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorit
hm="maxima")
```

```
[Out] -2/105*(B*(36*sin(f*x + e)/(cos(f*x + e) + 1) - 132*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 68*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 14*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 84*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 140*sin(f*x
+ e)^6/(cos(f*x + e) + 1)^6 + 140*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 105
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 9)/(a^2*c^4 - 4*a^2*c^4*sin(f*x + e)
/(cos(f*x + e) + 1) + 3*a^2*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8*a^2
*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 14*a^2*c^4*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 14*a^2*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 8*a^2*c^4
*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3*a^2*c^4*sin(f*x + e)^8/(cos(f*x +
e) + 1)^8 + 4*a^2*c^4*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - a^2*c^4*sin(f*x
 + e)^10/(cos(f*x + e) + 1)^10) + 5*A*(3*sin(f*x + e)/(cos(f*x + e) + 1) +
24*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 76*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 28*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 42*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 - 56*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 28*sin(f*x + e)^7/(c
```

```
os(f*x + e) + 1)^7 + 42*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 21*sin(f*x +
e)^9/(cos(f*x + e) + 1)^9 - 6)/(a^2*c^4 - 4*a^2*c^4*sin(f*x + e)/(cos(f*x +
 e) + 1) + 3*a^2*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8*a^2*c^4*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 - 14*a^2*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1
)^4 + 14*a^2*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 8*a^2*c^4*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7 - 3*a^2*c^4*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 +
 4*a^2*c^4*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - a^2*c^4*sin(f*x + e)^10/(c
os(f*x + e) + 1)^10))/f
```

mupad [B] time = 12.75, size = 197, normalized size = 1.46

$$-\frac{\left(\frac{32\,A}{21}-\frac{64\,B}{105}-\frac{16\,A\,\sin\left(e+f\,x\right)}{21}+\frac{32\,B\,\sin\left(e+f\,x\right)}{105}\right)\cos\left(e+f\,x\right)^4+\left(\frac{8\,A}{7}+\frac{12\,B}{35}-\frac{8\,A\,\sin\left(e+f\,x\right)}{7}-\frac{12\,B\,\sin\left(e+f\,x\right)}{35}+\frac{\left(4\,\sin\left(e\cdots\right.\right.}{}\right.}{a^2\,c^4\,f\,\left(4\cos\left(e+f\,x\right)^3\,\mathrm{si}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^4),x)
```

```
[Out] -((10*B)/21 - (4*A)/21 + (10*A*sin(e + f*x))/21 - (4*B*sin(e + f*x))/21 + c
os(e + f*x)^3*((8*A)/7 + (12*B)/35 - (8*A*sin(e + f*x))/7 - (12*B*sin(e + f
*x))/35 + ((4*sin(e + f*x) - 4)*((4*A)/7 + (6*B)/35))/2) - cos(e + f*x)^2*(
(16*A)/21 - (32*B)/105 - (8*A*sin(e + f*x))/7 + (16*B*sin(e + f*x))/35) + c
os(e + f*x)^4*((32*A)/21 - (64*B)/105 - (16*A*sin(e + f*x))/21 + (32*B*sin(
e + f*x))/105))/(a^2*c^4*f*(4*cos(e + f*x)^3*sin(e + f*x) - 4*cos(e + f*x)^
3 + 2*cos(e + f*x)^5))
```

sympy [A] time = 49.83, size = 4228, normalized size = 31.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-210*A*tan(e/2 + f*x/2)**9/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10
 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**
8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)
**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/
2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x
/2) - 105*a**2*c**4*f) + 420*A*tan(e/2 + f*x/2)**8/(105*a**2*c**4*f*tan(e/2
 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e
/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan
(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*t
an(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*
```

tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 280*A*tan(e/2 + f*x/2)**7/(105*a**2*c **4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2 *c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a* *2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840* a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420 *a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 560*A*tan(e/2 + f*x/2)** 6/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)* *9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2) **7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x /2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f* x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) + 420*A*tan(e /2 + f*x/2)**5/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan (e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f* tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f *tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) + 280*A*tan(e/2 + f*x/2)**4/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a** 2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a* *2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470 *a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 31 5*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105* a**2*c**4*f) - 760*A*tan(e/2 + f*x/2)**3/(105*a**2*c**4*f*tan(e/2 + f*x/2)* *10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x /2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f *x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) + 240*A*tan(e/2 + f*x/2)**2/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*ta n(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f* tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4* f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4 *f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) + 30*A*tan(e/2 + f*x/2)/(105*a**2*c* *4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2* c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a** 2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a **2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420* a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 60*A/(105*a**2*c**4*f*tan (e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*t an(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f *tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4 *f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c** 4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 210*B*tan(e/2 + f*x/2)**8/(105*a* *2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315* a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 147 0*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 -

```
840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 +
 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) + 280*B*tan(e/2 + f*x/
2)**7/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x
/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*
x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 +
 f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2
+ f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 280*B*t
an(e/2 + f*x/2)**6/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*
tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f
*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**
4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c*
*4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4
*f) - 168*B*tan(e/2 + f*x/2)**5/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420
*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 84
0*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 +
1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3
- 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) -
105*a**2*c**4*f) - 28*B*tan(e/2 + f*x/2)**4/(105*a**2*c**4*f*tan(e/2 + f*x/
2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*
x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 +
f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2
+ f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2
 + f*x/2) - 105*a**2*c**4*f) + 136*B*tan(e/2 + f*x/2)**3/(105*a**2*c**4*f*t
an(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f
*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4
*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c*
*4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c
**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 264*B*tan(e/2 + f*x/2)**2/(105*
a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 31
5*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1
470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4
- 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2
 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) + 72*B*tan(e/2 + f*x
/2)/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2
)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/
2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f
*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 +
f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 18*B/(105
*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 3
15*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 -
1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4
 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**
2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f), Ne(f, 0)), (x*(A +
 B*sin(e))/((a*sin(e) + a)**2*(-c*sin(e) + c)**4), True))
```

3.69 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx$

Optimal. Leaf size=175

$$\frac{4(2A-B)\tan^3(e+fx)}{63a^2c^5f}+\frac{4(2A-B)\tan(e+fx)}{21a^2c^5f}+\frac{(2A-B)\sec^3(e+fx)}{21a^2f\left(c^5-c^5\sin(e+fx)\right)}+\frac{(2A-B)\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2}+\frac{(A}{9a^2c}$$

```
[Out] 1/9*(A+B)*sec(f*x+e)^3/a^2/c^2/f/(c-c*sin(f*x+e))^3+1/21*(2*A-B)*sec(f*x+e)
^3/a^2/c^3/f/(c-c*sin(f*x+e))^2+1/21*(2*A-B)*sec(f*x+e)^3/a^2/f/(c^5-c^5*si
n(f*x+e))+4/21*(2*A-B)*tan(f*x+e)/a^2/c^5/f+4/63*(2*A-B)*tan(f*x+e)^3/a^2/c
^5/f
```

Rubi [A] time = 0.32, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 2672, 3767}

$$\frac{4(2A-B)\tan^3(e+fx)}{63a^2c^5f}+\frac{4(2A-B)\tan(e+fx)}{21a^2c^5f}+\frac{(2A-B)\sec^3(e+fx)}{21a^2f\left(c^5-c^5\sin(e+fx)\right)}+\frac{(2A-B)\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2}+\frac{(A}{9a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^5),x]
```

```
[Out] ((A + B)*Sec[e + f*x]^3)/(9*a^2*c^2*f*(c - c*Sin[e + f*x])^3) + ((2*A - B)*
Sec[e + f*x]^3)/(21*a^2*c^3*f*(c - c*Sin[e + f*x])^2) + ((2*A - B)*Sec[e +
f*x]^3)/(21*a^2*f*(c^5 - c^5*Sin[e + f*x])) + (4*(2*A - B)*Tan[e + f*x])/(2
1*a^2*c^5*f) + (4*(2*A - B)*Tan[e + f*x]^3)/(63*a^2*c^5*f)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
```

```
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx = \frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx}{a^2c^2}$$

$$= \frac{(A+B)\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{(2A-B)\int \frac{\sec^4(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{3a^2c^3}$$

$$= \frac{(A+B)\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{(2A-B)\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2} + \frac{(5(2}{\ }$$

$$= \frac{(A+B)\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{(2A-B)\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2} + \frac{(}{21a}$$

$$= \frac{(A+B)\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{(2A-B)\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2} + \frac{(}{21a}$$

$$= \frac{(A+B)\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{(2A-B)\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2} + \frac{(}{21a}$$

Mathematica [A] time = 1.16, size = 329, normalized size = 1.88

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(180(31A-5B)\cos(e+fx)-6912(2A}{\ }$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
^5),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-10752*B + 180*(31*A - 5*B)*Cos[e + f*x] - 6912*(2*A - B)*Cos[2*(e + f*x
)] + 310*A*Cos[3*(e + f*x)] - 50*B*Cos[3*(e + f*x)] - 6144*A*Cos[4*(e + f*x
)] + 3072*B*Cos[4*(e + f*x)] - 930*A*Cos[5*(e + f*x)] + 150*B*Cos[5*(e + f*
x)] + 512*A*Cos[6*(e + f*x)] - 256*B*Cos[6*(e + f*x)] - 18432*A*Sin[e + f*x
] + 9216*B*Sin[e + f*x] - 4185*A*Sin[2*(e + f*x)] + 675*B*Sin[2*(e + f*x)]
- 1024*A*Sin[3*(e + f*x)] + 512*B*Sin[3*(e + f*x)] - 1860*A*Sin[4*(e + f*x)
] + 300*B*Sin[4*(e + f*x)] + 3072*A*Sin[5*(e + f*x)] - 1536*B*Sin[5*(e + f*
x)] + 155*A*Sin[6*(e + f*x)] - 25*B*Sin[6*(e + f*x)]))/(64512*a^2*c^5*f*(-1
 + Sin[e + f*x])^5*(1 + Sin[e + f*x])^2)
```

fricas [A] time = 0.43, size = 187, normalized size = 1.07

$$\frac{8\,(2\,A-B)\cos\left(fx+e\right)^{6}-36\,(2\,A-B)\cos\left(fx+e\right)^{4}+15\,(2\,A-B)\cos\left(fx+e\right)^{2}+\left(24\,(2\,A-B)\cos\left(fx+e\right)^{4}\right.}{63\left(3\,a^{2}c^{5}f\cos\left(fx+e\right)^{5}-4\,a^{2}c^{5}f\cos\left(fx+e\right)^{3}-\left(a^{2}c^{5}f\cos\left(fx+e\right)^{5}-4\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorit
hm="fricas")
```

```
[Out] 1/63*(8*(2*A - B)*cos(f*x + e)^6 - 36*(2*A - B)*cos(f*x + e)^4 + 15*(2*A -
B)*cos(f*x + e)^2 + (24*(2*A - B)*cos(f*x + e)^4 - 20*(2*A - B)*cos(f*x + e
)^2 - 14*A + 7*B)*sin(f*x + e) + 7*A - 14*B)/(3*a^2*c^5*f*cos(f*x + e)^5 -
4*a^2*c^5*f*cos(f*x + e)^3 - (a^2*c^5*f*cos(f*x + e)^5 - 4*a^2*c^5*f*cos(f*
x + e)^3)*sin(f*x + e))
```

giac [B] time = 0.25, size = 355, normalized size = 2.03

$$-\frac{\frac{21\left(21\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-15\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+36\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-24\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+19\,A-13\,B\right)}{a^{2}c^{5}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{3}}+\frac{3591\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}+315\,B\tan\left(\frac{1}{2}\,fx\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorit
hm="giac")
```

```
[Out] -1/2016*(21*(21*A*tan(1/2*f*x + 1/2*e)^2 - 15*B*tan(1/2*f*x + 1/2*e)^2 + 36
*A*tan(1/2*f*x + 1/2*e) - 24*B*tan(1/2*f*x + 1/2*e) + 19*A - 13*B)/(a^2*c^5
```

```
*(tan(1/2*f*x + 1/2*e) + 1)^3) + (3591*A*tan(1/2*f*x + 1/2*e)^8 + 315*B*tan
(1/2*f*x + 1/2*e)^8 - 19656*A*tan(1/2*f*x + 1/2*e)^7 + 756*B*tan(1/2*f*x +
1/2*e)^7 + 56196*A*tan(1/2*f*x + 1/2*e)^6 - 4200*B*tan(1/2*f*x + 1/2*e)^6 -
 95760*A*tan(1/2*f*x + 1/2*e)^5 + 11340*B*tan(1/2*f*x + 1/2*e)^5 + 107730*A
*tan(1/2*f*x + 1/2*e)^4 - 14994*B*tan(1/2*f*x + 1/2*e)^4 - 79464*A*tan(1/2*
f*x + 1/2*e)^3 + 13356*B*tan(1/2*f*x + 1/2*e)^3 + 38484*A*tan(1/2*f*x + 1/2
*e)^2 - 6768*B*tan(1/2*f*x + 1/2*e)^2 - 10944*A*tan(1/2*f*x + 1/2*e) + 2196
*B*tan(1/2*f*x + 1/2*e) + 1615*A - 209*B)/(a^2*c^5*(tan(1/2*f*x + 1/2*e) -
1)^9))/f
```

maple [A] time = 0.53, size = 277, normalized size = 1.58

$$\frac{-\frac{2(4A+4B)}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}}-\frac{16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\frac{2(34A+32B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{46A+40B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{\frac{9A}{2}+\frac{13B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{2\left(\frac{57A}{64}+\frac{5B}{64}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{\frac{59A}{2}+\frac{39B}{2}}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\cdots\right.}}{f\,a^{2}c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x)
```

```
[Out] 2/f/a^2/c^5*(-1/9*(4*A+4*B)/(tan(1/2*f*x+1/2*e)-1)^9-1/8*(16*A+16*B)/(tan(1
/2*f*x+1/2*e)-1)^8-1/7*(34*A+32*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/6*(46*A+40*B)
/(tan(1/2*f*x+1/2*e)-1)^6-1/2*(9/2*A+13/8*B)/(tan(1/2*f*x+1/2*e)-1)^2-(57/6
4*A+5/64*B)/(tan(1/2*f*x+1/2*e)-1)-1/4*(59/2*A+39/2*B)/(tan(1/2*f*x+1/2*e)-
1)^4-1/3*(57/4*A+59/8*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/5*(175/4*A+135/4*B)/(ta
n(1/2*f*x+1/2*e)-1)^5-1/2*(-1/16*A+1/16*B)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(1/
16*A-1/16*B)/(tan(1/2*f*x+1/2*e)+1)^3-(7/64*A-5/64*B)/(tan(1/2*f*x+1/2*e)+1
))
```

maxima [B] time = 0.57, size = 998, normalized size = 5.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorit
hm="maxima")
```

```
[Out] -2/63*(A*(51*sin(f*x + e)/(cos(f*x + e) + 1) - 39*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 235*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 450*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 306*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 294*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 378*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 63
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 273*sin(f*x + e)^9/(cos(f*x + e) + 1
)^9 + 189*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 63*sin(f*x + e)^11/(cos(f
*x + e) + 1)^11 - 19)/(a^2*c^5 - 6*a^2*c^5*sin(f*x + e)/(cos(f*x + e) + 1)
```

```
+ 12*a^2*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*a^2*c^5*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3 - 27*a^2*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 36
*a^2*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 36*a^2*c^5*sin(f*x + e)^7/(c
os(f*x + e) + 1)^7 + 27*a^2*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 2*a^2
*c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 12*a^2*c^5*sin(f*x + e)^10/(cos(
f*x + e) + 1)^10 + 6*a^2*c^5*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - a^2*c^
5*sin(f*x + e)^12/(cos(f*x + e) + 1)^12) + B*(6*sin(f*x + e)/(cos(f*x + e)
+ 1) - 75*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 128*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 - 162*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 36*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5 + 42*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 189*sin(f*x
 + e)^8/(cos(f*x + e) + 1)^8 + 126*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 63
*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1)/(a^2*c^5 - 6*a^2*c^5*sin(f*x +
e)/(cos(f*x + e) + 1) + 12*a^2*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*
a^2*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 27*a^2*c^5*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 36*a^2*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 36*a^2
*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 27*a^2*c^5*sin(f*x + e)^8/(cos(f
*x + e) + 1)^8 + 2*a^2*c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 12*a^2*c^5
*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 6*a^2*c^5*sin(f*x + e)^11/(cos(f*x
 + e) + 1)^11 - a^2*c^5*sin(f*x + e)^12/(cos(f*x + e) + 1)^12))/f
```

mupad [B]   time = 12.88, size = 337, normalized size = 1.93

$$\frac{2\left(7\,A-14\,B-14\,A\,\sin\left(e+f\,x\right)+7\,B\,\sin\left(e+f\,x\right)+30\,A\cos\left(e+f\,x\right)^2-76\,A\cos\left(e+f\,x\right)^3-72\,A\cos\left(e+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^5),x)
```

```
[Out] (2*(7*A - 14*B - 14*A*sin(e + f*x) + 7*B*sin(e + f*x) + 30*A*cos(e + f*x)^2
 - 76*A*cos(e + f*x)^3 - 72*A*cos(e + f*x)^4 + 57*A*cos(e + f*x)^5 + 16*A*c
os(e + f*x)^6 - 15*B*cos(e + f*x)^2 - 4*B*cos(e + f*x)^3 + 36*B*cos(e + f*x
)^4 + 3*B*cos(e + f*x)^5 - 8*B*cos(e + f*x)^6 - 40*A*cos(e + f*x)^2*sin(e +
 f*x) + 76*A*cos(e + f*x)^3*sin(e + f*x) + 48*A*cos(e + f*x)^4*sin(e + f*x)
 - 19*A*cos(e + f*x)^5*sin(e + f*x) + 20*B*cos(e + f*x)^2*sin(e + f*x) + 4*
B*cos(e + f*x)^3*sin(e + f*x) - 24*B*cos(e + f*x)^4*sin(e + f*x) - B*cos(e
+ f*x)^5*sin(e + f*x)))/(63*a^2*c^5*f*(8*cos(e + f*x)^3*sin(e + f*x) - 2*co
s(e + f*x)^5*sin(e + f*x) - 8*cos(e + f*x)^3 + 6*cos(e + f*x)^5))
```

sympy [A]   time = 90.73, size = 5868, normalized size = 33.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-126*A*tan(e/2 + f*x/2)**11/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12
 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)*
*10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/
2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f
*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 +
 f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2
+ f*x/2) - 63*a**2*c**5*f) + 378*A*tan(e/2 + f*x/2)**10/(63*a**2*c**5*f*tan
(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*
tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5
*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c
**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2
*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**
2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) - 546*A*tan(e/2 + f*x/2)**9/(63
*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 +
756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9
- 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)*
*7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/
2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x
/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) - 126*A*tan(e/2
 + f*x/2)**8/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2
 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(
e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*t
an(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*
f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5
*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f)
 + 756*A*tan(e/2 + f*x/2)**7/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**
2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*
a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 22
68*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 +
 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3
 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) -
 63*a**2*c**5*f) - 588*A*tan(e/2 + f*x/2)**6/(63*a**2*c**5*f*tan(e/2 + f*x/
2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f
*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2
+ f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e
/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan
(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*ta
n(e/2 + f*x/2) - 63*a**2*c**5*f) - 612*A*tan(e/2 + f*x/2)**5/(63*a**2*c**5*
f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c*
*5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2
*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a
**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126
*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 37
```

8*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) + 900*A*tan(e/2 + f*x/2)**
4/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**
11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2
)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*
x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 +
f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2
+ f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) - 470*A*ta
n(e/2 + f*x/2)**3/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*ta
n(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f
*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**
5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*
c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2
*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c*
*5*f) - 78*A*tan(e/2 + f*x/2)**2/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378
*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 -
126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8
+ 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)*
*5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2
)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/
2) - 63*a**2*c**5*f) + 102*A*tan(e/2 + f*x/2)/(63*a**2*c**5*f*tan(e/2 + f*x
/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 +
f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2
+ f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(
e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*ta
n(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*t
an(e/2 + f*x/2) - 63*a**2*c**5*f) - 38*A/(63*a**2*c**5*f*tan(e/2 + f*x/2)**
12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2
)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*
x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 +
f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2
+ f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/
2 + f*x/2) - 63*a**2*c**5*f) - 126*B*tan(e/2 + f*x/2)**10/(63*a**2*c**5*f*t
an(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*
f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c*
*5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2
*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a*
*2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a
2*c5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) + 252*B*tan(e/2 + f*x/2)**9/(
63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11
+ 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**
9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2
)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*
x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f
*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) - 378*B*tan(e
/2 + f*x/2)**8/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e

/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*ta
n(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f
*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**
5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c*
*5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*
f) + 84*B*tan(e/2 + f*x/2)**6/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a*
*2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126
*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2
268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5
+ 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**
3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2)
- 63*a**2*c**5*f) - 72*B*tan(e/2 + f*x/2)**5/(63*a**2*c**5*f*tan(e/2 + f*x/
2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f
*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2
+ f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e
/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan
(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*ta
n(e/2 + f*x/2) - 63*a**2*c**5*f) - 324*B*tan(e/2 + f*x/2)**4/(63*a**2*c**5*
f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c*
*5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2
*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a
2*c5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126
*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 37
8*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) + 256*B*tan(e/2 + f*x/2)**
3/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**
11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2
)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*
x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 +
 f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2
+ f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) - 150*B*ta
n(e/2 + f*x/2)**2/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*ta
n(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f
*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**
5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*
c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2
*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c*
*5*f) + 12*B*tan(e/2 + f*x/2)/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a*
*2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126
*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2
268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5
+ 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**
3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2)
- 63*a**2*c**5*f) - 2*B/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**
5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*
c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a*

```
*2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701
*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 75
6*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a
**2*c**5*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)**2*(-c*sin(e) + c
)**5), True))
```

3.70 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=243

$$-\frac{a^5c^5(A-B)\cos^{11}(e+fx)}{5f(a\sin(e+fx)+a)^8}-\frac{7c^5(3A-8B)\cos^3(e+fx)}{a^3f}+\frac{2a^3c^5(3A-8B)\cos^9(e+fx)}{15f(a\sin(e+fx)+a)^6}-\frac{21c^5(3A-8B)\sin(e+}{2a^3f}$$

```
[Out] -21/2*(3*A-8*B)*c^5*x/a^3-7*(3*A-8*B)*c^5*cos(f*x+e)^3/a^3/f-21/2*(3*A-8*B)
*c^5*cos(f*x+e)*sin(f*x+e)/a^3/f-1/5*a^5*(A-B)*c^5*cos(f*x+e)^11/f/(a+a*sin
(f*x+e))^8+2/15*a^3*(3*A-8*B)*c^5*cos(f*x+e)^9/f/(a+a*sin(f*x+e))^6-6/5*a^5
*(3*A-8*B)*c^5*cos(f*x+e)^7/f/(a^2+a^2*sin(f*x+e))^4-42/5*a^5*(3*A-8*B)*c^5
*cos(f*x+e)^5/f/(a^4+a^4*sin(f*x+e))^2
```

Rubi [A] time = 0.41, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2967, 2859, 2680, 2682, 2635, 8}

$$-\frac{7c^5(3A-8B)\cos^3(e+fx)}{a^3f}-\frac{a^5c^5(A-B)\cos^{11}(e+fx)}{5f(a\sin(e+fx)+a)^8}+\frac{2a^3c^5(3A-8B)\cos^9(e+fx)}{15f(a\sin(e+fx)+a)^6}-\frac{6a^5c^5(3A-8B)\cos^7(e}{5f\left(a^2\sin(e+fx)+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-21*(3*A - 8*B)*c^5*x)/(2*a^3) - (7*(3*A - 8*B)*c^5*Cos[e + f*x]^3)/(a^3*f
) - (21*(3*A - 8*B)*c^5*Cos[e + f*x]*Sin[e + f*x])/(2*a^3*f) - (a^5*(A - B)
*c^5*Cos[e + f*x]^11)/(5*f*(a + a*Sin[e + f*x])^8) + (2*a^3*(3*A - 8*B)*c^5
*Cos[e + f*x]^9)/(15*f*(a + a*Sin[e + f*x])^6) - (6*a^5*(3*A - 8*B)*c^5*Cos
[e + f*x]^7)/(5*f*(a^2 + a^2*Sin[e + f*x])^4) - (42*a^5*(3*A - 8*B)*c^5*Cos
[e + f*x]^5)/(5*f*(a^4 + a^4*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx = \left(a^5c^5\right)\int \frac{\cos^{10}(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^8}\,dx$$
$$= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{5f(a+a\sin(e+fx))^8} - \frac{1}{5}\left(a^4(3A-8B)c^5\right)\int \frac{\cos^{10}(e}{(a+a\sin(}$$
$$= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{5f(a+a\sin(e+fx))^8} + \frac{2a^3(3A-8B)c^5\cos^9(e+fx)}{15f(a+a\sin(e+fx))^6} +$$
$$= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{5f(a+a\sin(e+fx))^8} + \frac{2a^3(3A-8B)c^5\cos^9(e+fx)}{15f(a+a\sin(e+fx))^6} -$$
$$= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{5f(a+a\sin(e+fx))^8} + \frac{2a^3(3A-8B)c^5\cos^9(e+fx)}{15f(a+a\sin(e+fx))^6} -$$
$$= -\frac{7(3A-8B)c^5\cos^3(e+fx)}{a^3f} - \frac{a^5(A-B)c^5\cos^{11}(e+fx)}{5f(a+a\sin(e+fx))^8} + \frac{2a^3}{}$$
$$= -\frac{7(3A-8B)c^5\cos^3(e+fx)}{a^3f} - \frac{21(3A-8B)c^5\cos(e+fx)\sin(e}{2a^3f}$$
$$= -\frac{21(3A-8B)c^5x}{2a^3} - \frac{7(3A-8B)c^5\cos^3(e+fx)}{a^3f} - \frac{21(3A-8B)c}{}$$

Mathematica [A] time = 2.50, size = 388, normalized size = 1.60

$$\frac{(c-c\sin(e+fx))^5\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(768(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-630(3A-8B)(e+fx)\left(\mathrm{s}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x]
)^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^5*(768*(A - B)*
Sin[(e + f*x)/2] - 384*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 128*
(21*A - 31*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 64
*(21*A - 31*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 128*(54*A - 119*B)
*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 630*(3*A - 8*B)
*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 15*(32*A - 127*B)*Cos[
e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 5*B*Cos[3*(e + f*x)]*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + 15*(A - 8*B)*(Cos[(e + f*x)/2] + Sin
```

```
[(e + f*x)/2])^5*Sin[2*(e + f*x)]))/(60*a^3*f*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])^10*(1 + Sin[e + f*x])^3)
```

fricas [A] time = 0.46, size = 429, normalized size = 1.77

$$-\frac{10\,Bc^5\cos\left(fx+e\right)^6+15\,(A-6\,B)c^5\cos\left(fx+e\right)^5+10\,(21\,A-74\,B)c^5\cos\left(fx+e\right)^4-1260\,(3\,A-8\,B)c^5fx-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/30*(10*B*c^5*cos(f*x + e)^6 + 15*(A - 6*B)*c^5*cos(f*x + e)^5 + 10*(21*A
 - 74*B)*c^5*cos(f*x + e)^4 - 1260*(3*A - 8*B)*c^5*f*x - 192*(A - B)*c^5 +
(315*(3*A - 8*B)*c^5*f*x + (2373*A - 6128*B)*c^5)*cos(f*x + e)^3 + (945*(3*
A - 8*B)*c^5*f*x - 2*(753*A - 2248*B)*c^5)*cos(f*x + e)^2 - 6*(105*(3*A - 8
*B)*c^5*f*x + 2*(323*A - 848*B)*c^5)*cos(f*x + e) + (10*B*c^5*cos(f*x + e)^
5 - 5*(3*A - 20*B)*c^5*cos(f*x + e)^4 + 5*(39*A - 128*B)*c^5*cos(f*x + e)^3
 - 1260*(3*A - 8*B)*c^5*f*x + 192*(A - B)*c^5 + (315*(3*A - 8*B)*c^5*f*x -
2*(1089*A - 2744*B)*c^5)*cos(f*x + e)^2 - 6*(105*(3*A - 8*B)*c^5*f*x + 2*(3
07*A - 832*B)*c^5)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^
3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2
 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

giac [A] time = 0.28, size = 376, normalized size = 1.55

$$-\frac{\frac{315\left(3\,Ac^5-8\,Bc^5\right)\left(fx+e\right)}{a^3}+\frac{10\left(3\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-24\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+48\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-186\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+96\,Ac^5\tan\left(\frac{1}{2}\,fx\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^3a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -1/30*(315*(3*A*c^5 - 8*B*c^5)*(f*x + e)/a^3 + 10*(3*A*c^5*tan(1/2*f*x + 1/
2*e)^5 - 24*B*c^5*tan(1/2*f*x + 1/2*e)^5 + 48*A*c^5*tan(1/2*f*x + 1/2*e)^4
- 186*B*c^5*tan(1/2*f*x + 1/2*e)^4 + 96*A*c^5*tan(1/2*f*x + 1/2*e)^2 - 384*
B*c^5*tan(1/2*f*x + 1/2*e)^2 - 3*A*c^5*tan(1/2*f*x + 1/2*e) + 24*B*c^5*tan(
1/2*f*x + 1/2*e) + 48*A*c^5 - 190*B*c^5)/((tan(1/2*f*x + 1/2*e)^2 + 1)^3*a^
3) + 64*(30*A*c^5*tan(1/2*f*x + 1/2*e)^4 - 75*B*c^5*tan(1/2*f*x + 1/2*e)^4
+ 135*A*c^5*tan(1/2*f*x + 1/2*e)^3 - 345*B*c^5*tan(1/2*f*x + 1/2*e)^3 + 255
```

```
*A*c^5*tan(1/2*f*x + 1/2*e)^2 - 595*B*c^5*tan(1/2*f*x + 1/2*e)^2 + 165*A*c^
5*tan(1/2*f*x + 1/2*e) - 395*B*c^5*tan(1/2*f*x + 1/2*e) + 39*A*c^5 - 94*B*c
^5)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

maple [B] time = 0.52, size = 649, normalized size = 2.67

$$-\frac{c^{5}\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)A}{a^{3}f\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{3}}+\frac{8c^{5}\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)B}{a^{3}f\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{3}}-\frac{16c^{5}\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)A}{a^{3}f\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{3}}+\frac{62c^{5}\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)B}{a^{3}f\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{3}}-\frac{3}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x)
```

```
[Out] -c^5/a^3/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*A+8*c^5/a^3/f/(1
+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*B-16*c^5/a^3/f/(1+tan(1/2*f*x
+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^4*A+62*c^5/a^3/f/(1+tan(1/2*f*x+1/2*e)^2)^3
*tan(1/2*f*x+1/2*e)^4*B-32*c^5/a^3/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x
+1/2*e)^2*A+128*c^5/a^3/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2*B
+c^5/a^3/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*A-8*c^5/a^3/f/(1+t
an(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*B-16*c^5/a^3/f/(1+tan(1/2*f*x+1/2
*e)^2)^3*A+190/3*c^5/a^3/f/(1+tan(1/2*f*x+1/2*e)^2)^3*B-63*c^5/a^3/f*arctan
(tan(1/2*f*x+1/2*e))*A+168*c^5/a^3/f*arctan(tan(1/2*f*x+1/2*e))*B+128*c^5/a
^3/f/(tan(1/2*f*x+1/2*e)+1)^4*A-128*c^5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*B-32
*c^5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^2*A+96*c^5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^2
*B-64*c^5/a^3/f/(tan(1/2*f*x+1/2*e)+1)*A+160*c^5/a^3/f/(tan(1/2*f*x+1/2*e)+
1)*B-64*c^5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3*A+64/3*c^5/a^3/f/(tan(1/2*f*x+1/
2*e)+1)^3*B-256/5*c^5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5*A+256/5*c^5/a^3/f/(tan
(1/2*f*x+1/2*e)+1)^5*B
```

maxima [B] time = 0.57, size = 3282, normalized size = 13.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 1/15*(B*c^5*((2375*sin(f*x + e)/(cos(f*x + e) + 1) + 5347*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 9230*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 12622*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 13340*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
+ 11684*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 8050*sin(f*x + e)^7/(cos(f*x
+ e) + 1)^7 + 4370*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1725*sin(f*x + e)^
9/(cos(f*x + e) + 1)^9 + 345*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 544)/(
a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 13*a^3*sin(f*x + e)^2/(cos(f*
```

x + e) + 1)^2 + 25*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 38*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 46*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 46*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 38*a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 25*a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 13*a^3*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 5*a^3*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + a^3*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 345*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) - A*c^5*((1325*sin(f*x + e)/(cos(f*x + e) + 1) + 2673*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3805*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 4329*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3575*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2275*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 975*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 195*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 304)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 12*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 20*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 26*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 26*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 12*a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 5*a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^3*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 195*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + 5*B*c^5*((1325*sin(f*x + e)/(cos(f*x + e) + 1) + 2673*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3805*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 4329*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3575*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2275*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 975*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 195*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 304)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 12*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 20*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 26*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 26*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 12*a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 5*a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^3*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 195*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) - 30*A*c^5*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + 60*B*c^5*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) - 20*A*c^5*((95*sin(f*x + e)/(cos(f*x +

```
e) + 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3
*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin
(f*x + e)/(cos(f*x + e) + 1))/a^3) + 20*B*c^5*((95*sin(f*x + e)/(cos(f*x +
e) + 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3
*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin
(f*x + e)/(cos(f*x + e) + 1))/a^3) - 2*A*c^5*(20*sin(f*x + e)/(cos(f*x + e)
 + 1) + 40*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin
(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 40*A*c^5*(5*sin(f
*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a
^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 20*
B*c^5*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5
*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5) + 30*A*c^5*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a
^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 6*B*c^5*(5*
sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*
x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^
3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

mupad [B] time = 14.75, size = 501, normalized size = 2.06

$$-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(431\,A\,c^5-\frac{3454\,B\,c^5}{3}\right)+\frac{496\,A\,c^5}{5}-\frac{3958\,B\,c^5}{15}+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}\,\left(65\,A\,c^5-168\,B\,c^5\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^9}{f\,\left(a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{11}+5\,a^3\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^5)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] - (tan(e/2 + (f*x)/2)*(431*A*c^5 - (3454*B*c^5)/3) + (496*A*c^5)/5 - (3958*
B*c^5)/15 + tan(e/2 + (f*x)/2)^10*(65*A*c^5 - 168*B*c^5) + tan(e/2 + (f*x)/
2)^9*(309*A*c^5 - 838*B*c^5) + tan(e/2 + (f*x)/2)^8*(826*A*c^5 - (6418*B*c^
5)/3) + tan(e/2 + (f*x)/2)^7*(1418*A*c^5 - (11636*B*c^5)/3) + tan(e/2 + (f*
x)/2)^3*(1654*A*c^5 - (13372*B*c^5)/3) + tan(e/2 + (f*x)/2)^5*(2332*A*c^5 -
 (19072*B*c^5)/3) + tan(e/2 + (f*x)/2)^2*((4903*A*c^5)/5 - (38884*B*c^5)/15
) + tan(e/2 + (f*x)/2)^6*((11156*A*c^5)/5 - (86708*B*c^5)/15) + tan(e/2 + (
f*x)/2)^4*((11758*A*c^5)/5 - (92224*B*c^5)/15))/(f*(13*a^3*tan(e/2 + (f*x)/
2)^2 + 25*a^3*tan(e/2 + (f*x)/2)^3 + 38*a^3*tan(e/2 + (f*x)/2)^4 + 46*a^3*t
an(e/2 + (f*x)/2)^5 + 46*a^3*tan(e/2 + (f*x)/2)^6 + 38*a^3*tan(e/2 + (f*x)/
2)^7 + 25*a^3*tan(e/2 + (f*x)/2)^8 + 13*a^3*tan(e/2 + (f*x)/2)^9 + 5*a^3*ta
n(e/2 + (f*x)/2)^10 + a^3*tan(e/2 + (f*x)/2)^11 + a^3 + 5*a^3*tan(e/2 + (f*
x)/2))) - (21*c^5*atan((21*c^5*tan(e/2 + (f*x)/2)*(3*A - 8*B))/(63*A*c^5 -
168*B*c^5))*(3*A - 8*B))/(a^3*f)
```

sympy [A] time = 112.09, size = 10608, normalized size = 43.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**5/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-945*A*c**5*f*x*tan(e/2 + f*x/2)**11/(30*a**3*f*tan(e/2 + f*x/2)
**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 7
50*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3
*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(
e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x
/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 4725*A*c**5*f*x*tan(e/2
 + f*x/2)**10/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)
**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 11
40*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3
*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e
/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/
2) + 30*a**3*f) - 12285*A*c**5*f*x*tan(e/2 + f*x/2)**9/(30*a**3*f*tan(e/2 +
 f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)
**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 13
80*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3
*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/
2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 23625*A*c**5*f*x
*tan(e/2 + f*x/2)**8/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 +
 f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)*
*8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 13
80*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*
```

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f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2
 + f*x/2) + 30*a**3*f) - 35910*A*c**5*f*x*tan(e/2 + f*x/2)**7/(30*a**3*f*ta
n(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 +
 f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)*
*7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 11
40*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f
*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 43470*A*c
**5*f*x*tan(e/2 + f*x/2)**6/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*ta
n(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 +
f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)*
*6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 75
0*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*
tan(e/2 + f*x/2) + 30*a**3*f) - 43470*A*c**5*f*x*tan(e/2 + f*x/2)**5/(30*a*
*3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*ta
n(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 +
f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)*
*5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390
*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 35
910*A*c**5*f*x*tan(e/2 + f*x/2)**4/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a*
*3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan
(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 +
f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)*
*4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*
a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 23625*A*c**5*f*x*tan(e/2 + f*x/2)**3
/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a*
*3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan
(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 +
f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**
3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*
f) - 12285*A*c**5*f*x*tan(e/2 + f*x/2)**2/(30*a**3*f*tan(e/2 + f*x/2)**11 +
 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**
3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan
(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 +
f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2
 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 4725*A*c**5*f*x*tan(e/2 + f*x
/2)/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390
*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*
tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2
 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2
)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a*
*3*f) - 945*A*c**5*f*x/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2
 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2
)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 +
1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**
3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e
```

```
/2 + f*x/2) + 30*a**3*f) - 1950*A*c**5*tan(e/2 + f*x/2)**10/(30*a**3*f*tan(
e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f
*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7
 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140
*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*t
an(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 9270*A*c**5
*tan(e/2 + f*x/2)**9/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 +
 f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)*
*8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 13
80*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*
f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2
 + f*x/2) + 30*a**3*f) - 24780*A*c**5*tan(e/2 + f*x/2)**8/(30*a**3*f*tan(e/
2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x
/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 +
 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a
**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan
(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 42540*A*c**5*
tan(e/2 + f*x/2)**7/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 +
f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**
8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 138
0*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f
*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2
+ f*x/2) + 30*a**3*f) - 66936*A*c**5*tan(e/2 + f*x/2)**6/(30*a**3*f*tan(e/2
 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/
2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 +
1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a*
*3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(
e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 69960*A*c**5*t
an(e/2 + f*x/2)**5/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f
*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8
 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380
*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*
tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 +
 f*x/2) + 30*a**3*f) - 70548*A*c**5*tan(e/2 + f*x/2)**4/(30*a**3*f*tan(e/2
+ f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2
)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1
380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**
3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e
/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 49620*A*c**5*ta
n(e/2 + f*x/2)**3/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*
x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8
+ 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*
a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*t
an(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 +
f*x/2) + 30*a**3*f) - 29418*A*c**5*tan(e/2 + f*x/2)**2/(30*a**3*f*tan(e/2 +
```

```
 f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)
**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 13
80*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3
*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/
2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 12930*A*c**5*tan
(e/2 + f*x/2)/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)
**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 11
40*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3
*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e
/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/
2) + 30*a**3*f) - 2976*A*c**5/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*
tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2
+ f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2
)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 +
750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*
f*tan(e/2 + f*x/2) + 30*a**3*f) + 2520*B*c**5*f*x*tan(e/2 + f*x/2)**11/(30*
a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*
tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2
+ f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2
)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 3
90*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) +
12600*B*c**5*f*x*tan(e/2 + f*x/2)**10/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150
*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*
tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2
 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/
2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 1
50*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 32760*B*c**5*f*x*tan(e/2 + f*x/2)
**9/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390
*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*
tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2
 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2
)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a*
*3*f) + 63000*B*c**5*f*x*tan(e/2 + f*x/2)**8/(30*a**3*f*tan(e/2 + f*x/2)**1
1 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*
a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*
tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2
 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)
**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 95760*B*c**5*f*x*tan(e/2 +
 f*x/2)**7/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**1
0 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*
a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*
tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2
+ f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2)
+ 30*a**3*f) + 115920*B*c**5*f*x*tan(e/2 + f*x/2)**6/(30*a**3*f*tan(e/2 + f
*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**
```

```
9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380
*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f
*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2
+ f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 115920*B*c**5*f*x*
tan(e/2 + f*x/2)**5/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 +
f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**
8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 138
0*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f
*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2
+ f*x/2) + 30*a**3*f) + 95760*B*c**5*f*x*tan(e/2 + f*x/2)**4/(30*a**3*f*tan
(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 +
f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**
7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 114
0*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*
tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 63000*B*c*
*5*f*x*tan(e/2 + f*x/2)**3/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan
(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f
*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**
6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750
*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*t
an(e/2 + f*x/2) + 30*a**3*f) + 32760*B*c**5*f*x*tan(e/2 + f*x/2)**2/(30*a**
3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan
(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f
*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**
5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*
a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 126
00*B*c**5*f*x*tan(e/2 + f*x/2)/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f
*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2
 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/
2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 +
 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3
*f*tan(e/2 + f*x/2) + 30*a**3*f) + 2520*B*c**5*f*x/(30*a**3*f*tan(e/2 + f*x
/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9
+ 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a
**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*t
an(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 +
f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 5040*B*c**5*tan(e/2
+ f*x/2)**10/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)*
*10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 114
0*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*
f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/
2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2
) + 30*a**3*f) + 25140*B*c**5*tan(e/2 + f*x/2)**9/(30*a**3*f*tan(e/2 + f*x/
2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 +
 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a*
```

*3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*ta
n(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f
*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 64180*B*c**5*tan(e/2
+ f*x/2)**8/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**
10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140
*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f
*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2
 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2)
 + 30*a**3*f) + 116360*B*c**5*tan(e/2 + f*x/2)**7/(30*a**3*f*tan(e/2 + f*x/
2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 +
 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a*
*3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*ta
n(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f
*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 173416*B*c**5*tan(e/2
 + f*x/2)**6/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)*
*10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 114
0*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*
f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/
2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2
) + 30*a**3*f) + 190720*B*c**5*tan(e/2 + f*x/2)**5/(30*a**3*f*tan(e/2 + f*x
/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9
+ 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a
3*f*tan(e/2 + f*x/2)6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*t
an(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 +
f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 184448*B*c**5*tan(e/
2 + f*x/2)**4/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)
10 + 390*a3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 11
40*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3
*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e
/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/
2) + 30*a**3*f) + 133720*B*c**5*tan(e/2 + f*x/2)**3/(30*a**3*f*tan(e/2 + f*
x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9
 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*
a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*
tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 +
 f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 77768*B*c**5*tan(e/
2 + f*x/2)**2/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)
10 + 390*a3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 11
40*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3
*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e
/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/
2) + 30*a**3*f) + 34540*B*c**5*tan(e/2 + f*x/2)/(30*a**3*f*tan(e/2 + f*x/2)
11 + 150*a3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 7
50*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3
*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(

```
e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x
/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 7916*B*c**5/(30*a**3*f*
tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2
 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2
)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 +
1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3
*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f), Ne(f, 0)
), (x*(A + B*sin(e))*(-c*sin(e) + c)**5/(a*sin(e) + a)**3, True))
```

3.71 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=201

$$-\frac{a^4c^4(A-B)\cos^9(e+fx)}{5f(a\sin(e+fx)+a)^7}-\frac{7c^4(2A-7B)\cos(e+fx)}{2a^3f}-\frac{7c^4(2A-7B)\cos^3(e+fx)}{6f\left(a^3\sin(e+fx)+a^3\right)}-\frac{7c^4x(2A-7B)}{2a^3}+\frac{2a^2c^4(2A}{15f(a\,s}$$

```
[Out] -7/2*(2*A-7*B)*c^4*x/a^3-7/2*(2*A-7*B)*c^4*cos(f*x+e)/a^3/f-1/5*a^4*(A-B)*c
^4*cos(f*x+e)^9/f/(a+a*sin(f*x+e))^7+2/15*a^2*(2*A-7*B)*c^4*cos(f*x+e)^7/f/
(a+a*sin(f*x+e))^5-14/15*(2*A-7*B)*c^4*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^3-7/
6*(2*A-7*B)*c^4*cos(f*x+e)^3/f/(a^3+a^3*sin(f*x+e))
```

Rubi [A] time = 0.39, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2967, 2859, 2680, 2679, 2682, 8}

$$-\frac{7c^4(2A-7B)\cos(e+fx)}{2a^3f}-\frac{a^4c^4(A-B)\cos^9(e+fx)}{5f(a\sin(e+fx)+a)^7}+\frac{2a^2c^4(2A-7B)\cos^7(e+fx)}{15f(a\sin(e+fx)+a)^5}-\frac{7c^4(2A-7B)\cos^3(e+f}{6f\left(a^3\sin(e+fx)+a^3\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-7*(2*A - 7*B)*c^4*x)/(2*a^3) - (7*(2*A - 7*B)*c^4*Cos[e + f*x])/(2*a^3*f)
 - (a^4*(A - B)*c^4*Cos[e + f*x]^9)/(5*f*(a + a*Sin[e + f*x])^7) + (2*a^2*(
2*A - 7*B)*c^4*Cos[e + f*x]^7)/(15*f*(a + a*Sin[e + f*x])^5) - (14*(2*A - 7
*B)*c^4*Cos[e + f*x]^5)/(15*f*(a + a*Sin[e + f*x])^3) - (7*(2*A - 7*B)*c^4*
Cos[e + f*x]^3)/(6*f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx = \left(a^4c^4\right)\int \frac{\cos^8(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^7}\,dx$$

$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} - \frac{1}{5}\left(a^3(2A-7B)c^4\right)\int \frac{\cos^8(e+}{(a+a\sin(e}$$

$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} + \frac{2a^2(2A-7B)c^4\cos^7(e+fx)}{15f(a+a\sin(e+fx))^5} + \frac{1}{15}$$

$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} + \frac{2a^2(2A-7B)c^4\cos^7(e+fx)}{15f(a+a\sin(e+fx))^5} - \frac{14}{}$$

$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} + \frac{2a^2(2A-7B)c^4\cos^7(e+fx)}{15f(a+a\sin(e+fx))^5} - \frac{14}{}$$

$$= -\frac{7(2A-7B)c^4\cos(e+fx)}{2a^3f} - \frac{a^4(A-B)c^4\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} + \frac{2a^2(2}{15}$$

$$= -\frac{7(2A-7B)c^4x}{2a^3} - \frac{7(2A-7B)c^4\cos(e+fx)}{2a^3f} - \frac{a^4(A-B)c^4\cos}{5f(a+a\sin(e}$$

Mathematica [A] time = 1.61, size = 348, normalized size = 1.73

$$\frac{(c-c\sin(e+fx))^4\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(384(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-210(2A-7B)(e+fx)\left(\mathrm{s}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x]
)^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^4*(384*(A - B)*
Sin[(e + f*x)/2] - 192*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 128*
(8*A - 13*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 64*
(8*A - 13*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 64*(29*A - 79*B)*Sin
[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 210*(2*A - 7*B)*(e
+ f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 60*(A - 7*B)*Cos[e + f*x]*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 15*B*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^5*Sin[2*(e + f*x)]))/(60*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^8*(1 + Sin[e + f*x])^3)
```

fricas [B] time = 0.46, size = 392, normalized size = 1.95

$$\frac{15\,Bc^4\cos\left(fx+e\right)^5-30\,(A-6\,B)c^4\cos\left(fx+e\right)^4+420\,(2\,A-7\,B)c^4fx+96\,(A-B)c^4-\left(105\,(2\,A-7\,B)c^4fx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/30*(15*B*c^4*cos(f*x + e)^5 - 30*(A - 6*B)*c^4*cos(f*x + e)^4 + 420*(2*A
- 7*B)*c^4*f*x + 96*(A - B)*c^4 - (105*(2*A - 7*B)*c^4*f*x + (554*A - 1819*
B)*c^4)*cos(f*x + e)^3 - (315*(2*A - 7*B)*c^4*f*x - 2*(134*A - 619*B)*c^4)*
cos(f*x + e)^2 + 6*(35*(2*A - 7*B)*c^4*f*x + 2*(74*A - 249*B)*c^4)*cos(f*x
+ e) - (15*B*c^4*cos(f*x + e)^4 + 15*(2*A - 11*B)*c^4*cos(f*x + e)^3 - 420*
(2*A - 7*B)*c^4*f*x + 96*(A - B)*c^4 + (105*(2*A - 7*B)*c^4*f*x - 2*(262*A
- 827*B)*c^4)*cos(f*x + e)^2 - 6*(35*(2*A - 7*B)*c^4*f*x + 2*(66*A - 241*B)
*c^4)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x +
 e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*co
s(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

giac [A] time = 0.26, size = 305, normalized size = 1.52

$$-\frac{\frac{105\left(2\,Ac^4-7\,Bc^4\right)\left(fx+e\right)}{a^3}-\frac{30\left(Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+14\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,Ac^4+14\,Bc^4\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a^3}+\frac{32\left(15\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -1/30*(105*(2*A*c^4 - 7*B*c^4)*(f*x + e)/a^3 - 30*(B*c^4*tan(1/2*f*x + 1/2*
e)^3 - 2*A*c^4*tan(1/2*f*x + 1/2*e)^2 + 14*B*c^4*tan(1/2*f*x + 1/2*e)^2 - B
*c^4*tan(1/2*f*x + 1/2*e) - 2*A*c^4 + 14*B*c^4)/((tan(1/2*f*x + 1/2*e)^2 +
1)^2*a^3) + 32*(15*A*c^4*tan(1/2*f*x + 1/2*e)^4 - 45*B*c^4*tan(1/2*f*x + 1/
2*e)^4 + 60*A*c^4*tan(1/2*f*x + 1/2*e)^3 - 210*B*c^4*tan(1/2*f*x + 1/2*e)^3
 + 130*A*c^4*tan(1/2*f*x + 1/2*e)^2 - 380*B*c^4*tan(1/2*f*x + 1/2*e)^2 + 80
*A*c^4*tan(1/2*f*x + 1/2*e) - 250*B*c^4*tan(1/2*f*x + 1/2*e) + 19*A*c^4 - 5
9*B*c^4)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

maple [B] time = 0.50, size = 474, normalized size = 2.36

$$\frac{c^4\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)B}{a^3 f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{2c^4\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)A}{a^3 f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{14c^4\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)B}{a^3 f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{c^4 B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a^3 f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{}{a^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x)
```

```
[Out] c^4/a^3/f/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^3*B-2*c^4/a^3/f/(1+
tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*A+14*c^4/a^3/f/(1+tan(1/2*f*x+
1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*B-c^4/a^3/f/(1+tan(1/2*f*x+1/2*e)^2)^2*B*t
an(1/2*f*x+1/2*e)-2*c^4/a^3/f/(1+tan(1/2*f*x+1/2*e)^2)^2*A+14*c^4/a^3/f/(1+
tan(1/2*f*x+1/2*e)^2)^2*B+49*c^4/a^3/f*arctan(tan(1/2*f*x+1/2*e))*B-14*c^4/
a^3/f*arctan(tan(1/2*f*x+1/2*e))*A+64*c^4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*A-
64*c^4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*B-16*c^4/a^3/f/(tan(1/2*f*x+1/2*e)+1)
*A+48*c^4/a^3/f/(tan(1/2*f*x+1/2*e)+1)*B-128/5*c^4/a^3/f/(tan(1/2*f*x+1/2*e
)+1)^5*A+128/5*c^4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5*B-128/3*c^4/a^3/f/(tan(1/
2*f*x+1/2*e)+1)^3*A+64/3*c^4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3*B+32*c^4/a^3/f*
B/(tan(1/2*f*x+1/2*e)+1)^2
```

maxima [B] time = 0.52, size = 2394, normalized size = 11.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 1/15*(B*c^4*((1325*sin(f*x + e)/(cos(f*x + e) + 1) + 2673*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 3805*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 4329*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 3575*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
2275*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 975*sin(f*x + e)^7/(cos(f*x + e)
 + 1)^7 + 195*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 304)/(a^3 + 5*a^3*sin(f
*x + e)/(cos(f*x + e) + 1) + 12*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2
0*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 26*a^3*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4 + 26*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20*a^3*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 + 12*a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 5
*a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^3*sin(f*x + e)^9/(cos(f*x + e)
 + 1)^9) + 195*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) - 6*A*c^4*((105
*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
+ 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*x + e)^6/(cos(
```

f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 11*a^3
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11*a^3*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a^3*sin
(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x + e)/(cos(f*x + e) +
1))/a^3) + 24*B*c^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
+ 15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(c
os(f*x + e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 + 11*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(
f*x + e) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(
f*x + e)/(cos(f*x + e) + 1))/a^3) - 8*A*c^4*((95*sin(f*x + e)/(cos(f*x + e)
+ 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*s
in(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
+ 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f
*x + e)/(cos(f*x + e) + 1))/a^3) + 12*B*c^4*((95*sin(f*x + e)/(cos(f*x + e)
+ 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*s
in(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
+ 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f
*x + e)/(cos(f*x + e) + 1))/a^3) - 2*A*c^4*(20*sin(f*x + e)/(cos(f*x + e) +
1) + 40*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f
*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1
0*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 24*A*c^4*(5*sin(f*x
+ e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3
+ 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 16*B*
c^4*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a
^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) +
1)^5) + 24*A*c^4*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3
*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 6*B*c^4*(5*si
n(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*s

```
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x
+ e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

mupad [B] time = 14.71, size = 419, normalized size = 2.08

$$-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{286\,A\,c^4}{3}-\frac{1007\,B\,c^4}{3}\right)+\frac{334\,A\,c^4}{15}-\frac{1154\,B\,c^4}{15}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8\left(16\,A\,c^4-49\,B\,c^4\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7\left(66\right.}{f\left(a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9+5\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+12\,a^3\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^4)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] - (tan(e/2 + (f*x)/2)*((286*A*c^4)/3 - (1007*B*c^4)/3) + (334*A*c^4)/15 - (
1154*B*c^4)/15 + tan(e/2 + (f*x)/2)^8*(16*A*c^4 - 49*B*c^4) + tan(e/2 + (f*
x)/2)^7*(66*A*c^4 - 243*B*c^4) + tan(e/2 + (f*x)/2)^6*((542*A*c^4)/3 - (174
1*B*c^4)/3) + tan(e/2 + (f*x)/2)^5*((706*A*c^4)/3 - (2621*B*c^4)/3) + tan(e
/2 + (f*x)/2)^3*((794*A*c^4)/3 - (2875*B*c^4)/3) + tan(e/2 + (f*x)/2)^2*((1
006*A*c^4)/5 - (3401*B*c^4)/5) + tan(e/2 + (f*x)/2)^4*((1718*A*c^4)/5 - (56
33*B*c^4)/5))/(f*(12*a^3*tan(e/2 + (f*x)/2)^2 + 20*a^3*tan(e/2 + (f*x)/2)^3
 + 26*a^3*tan(e/2 + (f*x)/2)^4 + 26*a^3*tan(e/2 + (f*x)/2)^5 + 20*a^3*tan(e
/2 + (f*x)/2)^6 + 12*a^3*tan(e/2 + (f*x)/2)^7 + 5*a^3*tan(e/2 + (f*x)/2)^8
+ a^3*tan(e/2 + (f*x)/2)^9 + a^3 + 5*a^3*tan(e/2 + (f*x)/2))) - (7*c^4*atan
((7*c^4*tan(e/2 + (f*x)/2)*(2*A - 7*B))/(14*A*c^4 - 49*B*c^4))*(2*A - 7*B))
/(a^3*f)
```

sympy [A] time = 74.02, size = 7337, normalized size = 36.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-210*A*c**4*f*x*tan(e/2 + f*x/2)**9/(30*a**3*f*tan(e/2 + f*x/2)*
*9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*
a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*ta
n(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f
*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 1050*A*c**4*f*x*tan(e
/2 + f*x/2)**8/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)
**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780
*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*t
```

an(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 2520*A*c**4*f*x*tan(e/2 + f*x/2)**7/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 4200*A*c**4*f*x*tan(e/2 + f*x/2)**6/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 5460*A*c**4*f*x*tan(e/2 + f*x/2)**5/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 5460*A*c**4*f*x*tan(e/2 + f*x/2)**4/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 4200*A*c**4*f*x*tan(e/2 + f*x/2)**3/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 2520*A*c**4*f*x*tan(e/2 + f*x/2)**2/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 1050*A*c**4*f*x*tan(e/2 + f*x/2)/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 210*A*c**4*f*x/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 480*A*c**4*tan(e/2 + f*x/2)**8/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 1980*A*c**4*tan(e/2 + f*x/2)**7/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*

x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 +
150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 5420*A*c**4*tan(e/2 + f*x/2)**6
/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3
*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/
2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2
)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a*
*3*f) - 7060*A*c**4*tan(e/2 + f*x/2)**5/(30*a**3*f*tan(e/2 + f*x/2)**9 + 15
0*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*
tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 +
f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**
2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 10308*A*c**4*tan(e/2 + f*x/2
)**4/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*
a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*ta
n(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f
*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 3
0*a**3*f) - 7940*A*c**4*tan(e/2 + f*x/2)**3/(30*a**3*f*tan(e/2 + f*x/2)**9
+ 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**
3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e
/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/
2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 6036*A*c**4*tan(e/2 + f*
x/2)**2/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 3
60*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f
*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2
+ f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2)
+ 30*a**3*f) - 2860*A*c**4*tan(e/2 + f*x/2)/(30*a**3*f*tan(e/2 + f*x/2)**9
+ 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**
3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e
/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/
2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 668*A*c**4/(30*a**3*f*ta
n(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f
*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5
+ 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**
3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 735*B*
c**4*f*x*tan(e/2 + f*x/2)**9/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*ta
n(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f
*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4
+ 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**
3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 3675*B*c**4*f*x*tan(e/2 + f*x/2)**8/(30
*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*t
an(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 +
f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3
+ 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f
) + 8820*B*c**4*f*x*tan(e/2 + f*x/2)**7/(30*a**3*f*tan(e/2 + f*x/2)**9 + 15
0*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*
tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 +

```
 f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**
2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 14700*B*c**4*f*x*tan(e/2 + f
*x/2)**6/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 +
360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*
f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2
 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2)
 + 30*a**3*f) + 19110*B*c**4*f*x*tan(e/2 + f*x/2)**5/(30*a**3*f*tan(e/2 + f
*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7
+ 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**
3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e
/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 19110*B*c**4*f*
x*tan(e/2 + f*x/2)**4/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 +
 f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**
6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a
**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan
(e/2 + f*x/2) + 30*a**3*f) + 14700*B*c**4*f*x*tan(e/2 + f*x/2)**3/(30*a**3*
f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2
 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)
**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360
*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 88
20*B*c**4*f*x*tan(e/2 + f*x/2)**2/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3
*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/
2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2
)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 15
0*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 3675*B*c**4*f*x*tan(e/2 + f*x/2)/(
30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f
*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2
+ f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)*
*3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3
*f) + 735*B*c**4*f*x/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 +
f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6
 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a*
*3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(
e/2 + f*x/2) + 30*a**3*f) + 1470*B*c**4*tan(e/2 + f*x/2)**8/(30*a**3*f*tan(
e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x
/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 +
780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*
f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 7290*B*c
**4*tan(e/2 + f*x/2)**7/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2
 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)
**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600
*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*t
an(e/2 + f*x/2) + 30*a**3*f) + 17410*B*c**4*tan(e/2 + f*x/2)**6/(30*a**3*f*
tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 +
 f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**
```

```
5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a
**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 2621
0*B*c**4*tan(e/2 + f*x/2)**5/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*ta
n(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f
*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4
+ 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**
3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 33798*B*c**4*tan(e/2 + f*x/2)**4/(30*a*
*3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(
e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x
/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 +
360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) +
 28750*B*c**4*tan(e/2 + f*x/2)**3/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3
*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/
2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2
)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 15
0*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 20406*B*c**4*tan(e/2 + f*x/2)**2/(
30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f
*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2
+ f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)*
*3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3
*f) + 10070*B*c**4*tan(e/2 + f*x/2)/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a*
*3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(
e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x
/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 +
150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 2308*B*c**4/(30*a**3*f*tan(e/2 +
 f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**
7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a
**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan
(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f), Ne(f, 0)), (x*
(A + B*sin(e))*(-c*sin(e) + c)**4/(a*sin(e) + a)**3, True))
```

3.72 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=153

$$-\frac{c^3(A-6B)\cos(e+fx)}{a^3f}-\frac{a^3c^3(A-B)\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^6}-\frac{2a^3c^3(A-6B)\cos^3(e+fx)}{3f\left(a^3\sin(e+fx)+a^3\right)^2}-\frac{c^3x(A-6B)}{a^3}+\frac{2ac^3(A-6B)\cos^5(e+fx)}{15f(a\sin(e+fx)+a)^4}$$

```
[Out] -(A-6*B)*c^3*x/a^3-(A-6*B)*c^3*cos(f*x+e)/a^3/f-1/5*a^3*(A-B)*c^3*cos(f*x+e
)^7/f/(a+a*sin(f*x+e))^6+2/15*a*(A-6*B)*c^3*cos(f*x+e)^5/f/(a+a*sin(f*x+e))
^4-2/3*a^3*(A-6*B)*c^3*cos(f*x+e)^3/f/(a^3+a^3*sin(f*x+e))^2
```

Rubi [A] time = 0.33, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2859, 2680, 2682, 8}

$$-\frac{c^3(A-6B)\cos(e+fx)}{a^3f}-\frac{a^3c^3(A-B)\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^6}-\frac{2a^3c^3(A-6B)\cos^3(e+fx)}{3f\left(a^3\sin(e+fx)+a^3\right)^2}-\frac{c^3x(A-6B)}{a^3}+\frac{2ac^3(A-6B)\cos^5(e+fx)}{15f(a\sin(e+fx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -(((A - 6*B)*c^3*x)/a^3) - ((A - 6*B)*c^3*Cos[e + f*x])/(a^3*f) - (a^3*(A -
 B)*c^3*Cos[e + f*x]^7)/(5*f*(a + a*Sin[e + f*x])^6) + (2*a*(A - 6*B)*c^3*C
os[e + f*x]^5)/(15*f*(a + a*Sin[e + f*x])^4) - (2*a^3*(A - 6*B)*c^3*Cos[e +
 f*x]^3)/(3*f*(a^3 + a^3*Sin[e + f*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
```

```
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^6}\,dx$$

$$= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} - \frac{1}{5}\left(a^2(A-6B)c^3\right)\int \frac{\cos^6(e+}{(a+a\sin(e+}$$

$$= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} + \frac{2a(A-6B)c^3\cos^5(e+fx)}{15f(a+a\sin(e+fx))^4} + \frac{1}{3}\Big((\;$$

$$= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} + \frac{2a(A-6B)c^3\cos^5(e+fx)}{15f(a+a\sin(e+fx))^4} - \frac{2(A}{3a}$$

$$= -\frac{(A-6B)c^3\cos(e+fx)}{a^3f} - \frac{a^3(A-B)c^3\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} + \frac{2a(A-}{15f(a}$$

$$= -\frac{(A-6B)c^3x}{a^3} - \frac{(A-6B)c^3\cos(e+fx)}{a^3f} - \frac{a^3(A-B)c^3\cos^7(e+}{5f(a+a\sin(e+f}$$

Mathematica [B] time = 1.09, size = 308, normalized size = 2.01

$$(c - c\sin(e+fx))^3 \left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right)\right)\left(48(A-B)\sin\left(\frac{1}{2}(e+fx)\right) - 15(A-6B)(e+fx)\left(\sin\left(\frac{1}{2}(\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
)^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(48*(A - B)*Sin[(e + f*x)/2] - 24*(A
 - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 8*(11*A - 21*B)*Sin[(e + f*x)
/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 4*(11*A - 21*B)*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^3 + 4*(23*A - 93*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^4 - 15*(A - 6*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^5 + 15*B*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
5)*(c - c*Sin[e + f*x])^3)/(15*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
6*(1 + Sin[e + f*x])^3)
```

fricas [B] time = 0.44, size = 338, normalized size = 2.21

$$15\,Bc^3\cos\left(fx+e\right)^4 + 60\,(A-6\,B)c^3fx + 24\,(A-B)c^3 - \left(15\,(A-6\,B)c^3fx + (46\,A-231\,B)c^3\right)\cos\left(fx+e\right)^3 -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/15*(15*B*c^3*cos(f*x + e)^4 + 60*(A - 6*B)*c^3*f*x + 24*(A - B)*c^3 - (15
*(A - 6*B)*c^3*f*x + (46*A - 231*B)*c^3)*cos(f*x + e)^3 - (45*(A - 6*B)*c^3
*f*x - 2*(A - 66*B)*c^3)*cos(f*x + e)^2 + 6*(5*(A - 6*B)*c^3*f*x + 2*(6*A -
 31*B)*c^3)*cos(f*x + e) + (15*B*c^3*cos(f*x + e)^3 + 60*(A - 6*B)*c^3*f*x
- 24*(A - B)*c^3 - (15*(A - 6*B)*c^3*f*x - 2*(23*A - 108*B)*c^3)*cos(f*x +
e)^2 + 6*(5*(A - 6*B)*c^3*f*x + 2*(4*A - 29*B)*c^3)*cos(f*x + e))*sin(f*x +
 e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e)
- 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x
 + e))
```

giac [A] time = 0.21, size = 226, normalized size = 1.48

$$\frac{\frac{30\,Bc^3}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^3} - \frac{15\left(Ac^3-6\,Bc^3\right)\left(fx+e\right)}{a^3} - \frac{4\left(15\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-45\,Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+30\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-210\,Bc^3\tan\left(\frac{1}{2}\,fx\right.}{}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/15*(30*B*c^3/((tan(1/2*f*x + 1/2*e)^2 + 1)*a^3) - 15*(A*c^3 - 6*B*c^3)*(f
*x + e)/a^3 - 4*(15*A*c^3*tan(1/2*f*x + 1/2*e)^4 - 45*B*c^3*tan(1/2*f*x + 1
/2*e)^4 + 30*A*c^3*tan(1/2*f*x + 1/2*e)^3 - 210*B*c^3*tan(1/2*f*x + 1/2*e)^
3 + 100*A*c^3*tan(1/2*f*x + 1/2*e)^2 - 420*B*c^3*tan(1/2*f*x + 1/2*e)^2 + 5
0*A*c^3*tan(1/2*f*x + 1/2*e) - 270*B*c^3*tan(1/2*f*x + 1/2*e) + 13*A*c^3 -
63*B*c^3)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

maple [B] time = 0.48, size = 323, normalized size = 2.11

$$\frac{2c^3B}{a^3f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)} - \frac{2c^3\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)A}{a^3f} + \frac{12c^3\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)B}{a^3f} + \frac{32c^3A}{a^3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2*c^3/a^3/f*B/(1+tan(1/2*f*x+1/2*e)^2)-2*c^3/a^3/f*arctan(tan(1/2*f*x+1/2*e
))*A+12*c^3/a^3/f*arctan(tan(1/2*f*x+1/2*e))*B+32*c^3/a^3/f/(tan(1/2*f*x+1/
2*e)+1)^4*A-32*c^3/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*B+8*c^3/a^3/f/(tan(1/2*f*
x+1/2*e)+1)^2*A+8*c^3/a^3/f/(tan(1/2*f*x+1/2*e)+1)^2*B-4*c^3/a^3/f/(tan(1/2
*f*x+1/2*e)+1)*A+12*c^3/a^3/f/(tan(1/2*f*x+1/2*e)+1)*B-64/5*c^3/a^3/f/(tan(
1/2*f*x+1/2*e)+1)^5*A+64/5*c^3/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5*B-80/3*c^3/a^
3/f/(tan(1/2*f*x+1/2*e)+1)^3*A+16*c^3/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3*B
```

maxima [B] time = 0.52, size = 1679, normalized size = 10.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 2/15*(3*B*c^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x
 + e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x +
e)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11
*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x +
e)/(cos(f*x + e) + 1))/a^3) - A*c^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) +
145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x +
 e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^
3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/
(cos(f*x + e) + 1))/a^3) + 3*B*c^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 1
45*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x +
e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) +
 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(
cos(f*x + e) + 1))/a^3) - A*c^3*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
+ 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(co
s(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
+ a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 6*A*c^3*(5*sin(f*x + e)/(cos(f
*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin
(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 6*B*c^3*(5*sin(f*
x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^
3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 9*A*
c^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/
(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 3*B*c^3*(5*sin(f*x + e)/(co
s(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10
*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5))/f
```

mupad [B] time = 14.36, size = 333, normalized size = 2.18

$$-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{40\,A\,c^3}{3}-82\,B\,c^3\right)+\frac{52\,A\,c^3}{15}-\frac{94\,B\,c^3}{5}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\left(4\,A\,c^3-12\,B\,c^3\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\left(8\,A\,c^3-\right.}{f\left(a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+5\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+11\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+15\,a^3\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^3)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] - (tan(e/2 + (f*x)/2)*((40*A*c^3)/3 - 82*B*c^3) + (52*A*c^3)/15 - (94*B*c^3
)/5 + tan(e/2 + (f*x)/2)^6*(4*A*c^3 - 12*B*c^3) + tan(e/2 + (f*x)/2)^5*(8*A
*c^3 - 58*B*c^3) + tan(e/2 + (f*x)/2)^3*((64*A*c^3)/3 - 148*B*c^3) + tan(e/
2 + (f*x)/2)^4*((92*A*c^3)/3 - 134*B*c^3) + tan(e/2 + (f*x)/2)^2*((452*A*c^
3)/15 - (744*B*c^3)/5))/(f*(11*a^3*tan(e/2 + (f*x)/2)^2 + 15*a^3*tan(e/2 +
(f*x)/2)^3 + 15*a^3*tan(e/2 + (f*x)/2)^4 + 11*a^3*tan(e/2 + (f*x)/2)^5 + 5*
a^3*tan(e/2 + (f*x)/2)^6 + a^3*tan(e/2 + (f*x)/2)^7 + a^3 + 5*a^3*tan(e/2 +
 (f*x)/2))) - (2*c^3*atan((2*c^3*tan(e/2 + (f*x)/2)*(A - 6*B))/(2*A*c^3 - 1
2*B*c^3))*(A - 6*B))/(a^3*f)
```

sympy [A] time = 46.58, size = 4665, normalized size = 30.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-15*A*c**3*f*x*tan(e/2 + f*x/2)**7/(15*a**3*f*tan(e/2 + f*x/2)**
7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a*
*3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(
e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 75*A*c**3*f*x*t
an(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x
/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 +
225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f
*tan(e/2 + f*x/2) + 15*a**3*f) - 165*A*c**3*f*x*tan(e/2 + f*x/2)**5/(15*a**
3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/
2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2
)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**
3*f) - 225*A*c**3*f*x*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 +
75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f
*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2
+ f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 225*A*c**3*f*x*tan(
e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)
```

```
**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225
*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*ta
n(e/2 + f*x/2) + 15*a**3*f) - 165*A*c**3*f*x*tan(e/2 + f*x/2)**2/(15*a**3*f
*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 +
 f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**
3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f
) - 75*A*c**3*f*x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3
*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/
2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2
)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 15*A*c**3*f*x/(15*a**3*f*t
an(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f
*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3
+ 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f)
- 60*A*c**3*tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*
tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 +
 f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**
2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 120*A*c**3*tan(e/2 + f*x/2)**
5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3
*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/
2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)
 + 15*a**3*f) - 460*A*c**3*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)*
*7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a
**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan
(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 320*A*c**3*tan
(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2
)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 22
5*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*t
an(e/2 + f*x/2) + 15*a**3*f) - 452*A*c**3*tan(e/2 + f*x/2)**2/(15*a**3*f*ta
n(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*
x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 +
 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) -
 200*A*c**3*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan
(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*
x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 +
 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 52*A*c**3/(15*a**3*f*tan(e/2 + f
*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 +
 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3
*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 90*B*c**
3*f*x*tan(e/2 + f*x/2)**7/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/
2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2
)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75
*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 450*B*c**3*f*x*tan(e/2 + f*x/2)**6/
(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f
*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2
+ f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
```

```
 15*a**3*f) + 990*B*c**3*f*x*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2
)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225
*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*t
an(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 1350*B*c**3*
f*x*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2
+ f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)*
*4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a
**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 1350*B*c**3*f*x*tan(e/2 + f*x/2)**3/(
15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*
tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 +
 f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
15*a**3*f) + 990*B*c**3*f*x*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)
**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*
a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*ta
n(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 450*B*c**3*f*
x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x
/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 +
225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f
*tan(e/2 + f*x/2) + 15*a**3*f) + 90*B*c**3*f*x/(15*a**3*f*tan(e/2 + f*x/2)*
*7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a
**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan
(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 180*B*c**3*tan
(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2
)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 22
5*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*t
an(e/2 + f*x/2) + 15*a**3*f) + 870*B*c**3*tan(e/2 + f*x/2)**5/(15*a**3*f*ta
n(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*
x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 +
 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) +
 2010*B*c**3*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f
*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2
+ f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)*
*2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 2220*B*c**3*tan(e/2 + f*x/2)
**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a*
*3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(
e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/
2) + 15*a**3*f) + 2232*B*c**3*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/
2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 22
5*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*
tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 1230*B*c**3
*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/
2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 2
25*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*
tan(e/2 + f*x/2) + 15*a**3*f) + 282*B*c**3/(15*a**3*f*tan(e/2 + f*x/2)**7 +
 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*
```

```
f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2
 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(A +
B*sin(e))*(-c*sin(e) + c)**3/(a*sin(e) + a)**3, True))
```

3.73 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=110

$$\frac{2Bc^2\cos(e+fx)}{f\left(a^3\sin(e+fx)+a^3\right)}+\frac{Bc^2x}{a^3}-\frac{a^2c^2(A-B)\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}-\frac{2Bc^2\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^3}$$

```
[Out] B*c^2*x/a^3-1/5*a^2*(A-B)*c^2*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^5-2/3*B*c^2*c
os(f*x+e)^3/f/(a+a*sin(f*x+e))^3+2*B*c^2*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e))
```

Rubi [A] time = 0.26, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 2680, 8}

$$-\frac{a^2c^2(A-B)\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}+\frac{2Bc^2\cos(e+fx)}{f\left(a^3\sin(e+fx)+a^3\right)}+\frac{Bc^2x}{a^3}-\frac{2Bc^2\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (B*c^2*x)/a^3 - (a^2*(A - B)*c^2*Cos[e + f*x]^5)/(5*f*(a + a*Sin[e + f*x])^
5) - (2*B*c^2*Cos[e + f*x]^3)/(3*f*(a + a*Sin[e + f*x])^3) + (2*B*c^2*Cos[e
 + f*x])/(f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
```

```
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^5}\,dx$$

$$= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5} + \left(aBc^2\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^4}\,dx$$

$$= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5} - \frac{2Bc^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3} - \frac{\left(Bc^2\right)\int \ldots}{\ldots}$$

$$= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5} - \frac{2Bc^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3} + \frac{2Bc^2\cos\ldots}{f\left(a^3+a^3\ldots\right.}$$

$$= \frac{Bc^2x}{a^3} - \frac{a^2(A-B)c^2\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5} - \frac{2Bc^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3} + \frac{\ldots}{f\left(\ldots\right.}$$

Mathematica [B] time = 0.70, size = 272, normalized size = 2.47

$$\frac{(c-c\sin(e+fx))^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(24(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+2(3A-43B)\sin\left(\frac{1}{2}(e+fx)\right)\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
)^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(24*(A - B)*Sin[(e + f*x)/2] - 12*(A
 - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 8*(3*A - 8*B)*Sin[(e + f*x)/2
]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 4*(3*A - 8*B)*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^3 + 2*(3*A - 43*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^4 + 15*B*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^5)*(c - c*Sin[e + f*x])^2)/(15*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^4*(1 + Sin[e + f*x])^3)
```

fricas [B] time = 0.42, size = 279, normalized size = 2.54

$$-\frac{60\,Bc^2fx - \left(15\,Bc^2fx - (3\,A - 43\,B)c^2\right)\cos\left(fx+e\right)^3 - 12\,(A-B)c^2 - \left(45\,Bc^2fx - (9\,A + 11\,B)c^2\right)\cos\left(fx + \cdots\right.}{15\left(a^3f\cos\left(fx+e\right)^3 + 3\,a^3f\cos\left(f\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/15*(60*B*c^2*f*x - (15*B*c^2*f*x - (3*A - 43*B)*c^2)*cos(f*x + e)^3 - 12
*(A - B)*c^2 - (45*B*c^2*f*x - (9*A + 11*B)*c^2)*cos(f*x + e)^2 + 6*(5*B*c^
2*f*x - (A - 11*B)*c^2)*cos(f*x + e) + (60*B*c^2*f*x + 12*(A - B)*c^2 - (15
*B*c^2*f*x + (3*A - 43*B)*c^2)*cos(f*x + e)^2 + 6*(5*B*c^2*f*x + (A + 9*B)*
c^2)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x +
e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos
(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

giac [A] time = 0.21, size = 159, normalized size = 1.45

$$\frac{\frac{15\left(fx+e\right)Bc^2}{a^3} - \frac{2\left(15\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 - 15\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 - 60\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 30\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 - 170\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 - 100\cdots\right.}{a^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/15*(15*(f*x + e)*B*c^2/a^3 - 2*(15*A*c^2*tan(1/2*f*x + 1/2*e)^4 - 15*B*c^
2*tan(1/2*f*x + 1/2*e)^4 - 60*B*c^2*tan(1/2*f*x + 1/2*e)^3 + 30*A*c^2*tan(1
/2*f*x + 1/2*e)^2 - 170*B*c^2*tan(1/2*f*x + 1/2*e)^2 - 100*B*c^2*tan(1/2*f*
x + 1/2*e) + 3*A*c^2 - 23*B*c^2)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

maple [B] time = 0.47, size = 249, normalized size = 2.26

$$\frac{2c^2B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^3f}+\frac{16c^2A}{a^3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{16c^2B}{a^3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2c^2A}{a^3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{}{a^3f\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2*c^2/a^3/f*B*arctan(tan(1/2*f*x+1/2*e))+16*c^2/a^3/f/(tan(1/2*f*x+1/2*e)+1
)^4*A-16*c^2/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*B-2*c^2/a^3/f/(tan(1/2*f*x+1/2*
e)+1)*A+2*c^2/a^3/f/(tan(1/2*f*x+1/2*e)+1)*B-32/5*c^2/a^3/f/(tan(1/2*f*x+1/
2*e)+1)^5*A+32/5*c^2/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5*B-16*c^2/a^3/f/(tan(1/2
*f*x+1/2*e)+1)^3*A+32/3*c^2/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3*B+8*c^2/a^3/f*A/
(tan(1/2*f*x+1/2*e)+1)^2
```

maxima [B] time = 0.47, size = 1134, normalized size = 10.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 2/15*(B*c^2*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) +
 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e
)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3)
 - A*c^2*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5) - 2*A*c^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5) + 4*B*c^2*(5*sin(f*x + e)/(cos(f*x + e) +
1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/
(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)
```

```
^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 6*A*c^2*(5*sin(f*x + e)/(co
s(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10
*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5) - 3*B*c^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))
/f
```

mupad [B] time = 15.08, size = 230, normalized size = 2.09

$$\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(\frac{c^2\,\left(120\,B+150\,B\,\left(e+f\,x\right)\right)}{15}-10\,B\,c^2\,\left(e+f\,x\right)\right)+\frac{c^2\,\left(46\,B-6\,A+15\,B\,\left(e+f\,x\right)\right)}{15}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(\frac{c^2\,\left(30\,B-30\,A+75\right.}{15}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^2)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (tan(e/2 + (f*x)/2)^3*((c^2*(120*B + 150*B*(e + f*x)))/15 - 10*B*c^2*(e + f
*x)) + (c^2*(46*B - 6*A + 15*B*(e + f*x)))/15 + tan(e/2 + (f*x)/2)^4*((c^2*
(30*B - 30*A + 75*B*(e + f*x)))/15 - 5*B*c^2*(e + f*x)) + tan(e/2 + (f*x)/2
)^2*((c^2*(340*B - 60*A + 150*B*(e + f*x)))/15 - 10*B*c^2*(e + f*x)) + tan(
e/2 + (f*x)/2)*((c^2*(200*B + 75*B*(e + f*x)))/15 - 5*B*c^2*(e + f*x)) - B*
c^2*(e + f*x))/(a^3*f*(tan(e/2 + (f*x)/2) + 1)^5) + (B*c^2*x)/a^3
```

sympy [A] time = 26.03, size = 1647, normalized size = 14.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*c**2*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 +
75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f
*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c**2*
tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*
x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 +
 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 6*A*c**2/(15*a**3*f*tan(e/2 + f*
x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 +
150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) +
```

```
15*B*c**2*f*x*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*
f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2
 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 75*B*c**2*f*x*tan(
e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)
**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*
a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 150*B*c**2*f*x*tan(e/2 + f*x/2)**3/(
15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*
tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 +
f*x/2) + 15*a**3*f) + 150*B*c**2*f*x*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2
 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)*
*3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*
f) + 75*B*c**2*f*x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**
3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e
/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 15*B*c**2*f*x/(1
5*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*t
an(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f
*x/2) + 15*a**3*f) + 30*B*c**2*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x
/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 1
50*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 1
20*B*c**2*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*ta
n(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f
*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 340*B*c**2*tan(e/2 + f
*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 1
50*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*
tan(e/2 + f*x/2) + 15*a**3*f) + 200*B*c**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(
e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/
2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a*
*3*f) + 46*B*c**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/
2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 7
5*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*si
n(e) + c)**2/(a*sin(e) + a)**3, True))
```

3.74 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=103

$$\frac{c(A+4B)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}+\frac{ac(A-11B)\cos(e+fx)}{15f\left(a^2\sin(e+fx)+a^2\right)^2}-\frac{2c(A-B)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

```
[Out] -2/5*(A-B)*c*cos(f*x+e)/f/(a+a*sin(f*x+e))^3+1/15*a*(A-11*B)*c*cos(f*x+e)/f
/(a^2+a^2*sin(f*x+e))^2+1/15*(A+4*B)*c*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e))
```

Rubi [A] time = 0.23, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2967, 2857, 2750, 2648}

$$\frac{c(A+4B)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}+\frac{ac(A-11B)\cos(e+fx)}{15f\left(a^2\sin(e+fx)+a^2\right)^2}-\frac{2c(A-B)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-2*(A - B)*c*Cos[e + f*x])/(5*f*(a + a*Sin[e + f*x])^3) + (a*(A - 11*B)*c*
Cos[e + f*x])/(15*f*(a^2 + a^2*Sin[e + f*x])^2) + ((A + 4*B)*c*Cos[e + f*x]
)/(15*f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
```

```
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^4}\,dx$$

$$= -\frac{2(A-B)c\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{c\int \frac{aA-6aB+5aB\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx}{5a^2}$$

$$= -\frac{2(A-B)c\cos(e+fx)}{5f(a+a\sin(e+fx))^3} + \frac{(A-11B)c\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{((A+4B)c}{}$$

$$= -\frac{2(A-B)c\cos(e+fx)}{5f(a+a\sin(e+fx))^3} + \frac{(A-11B)c\cos(e+fx)}{15af(a+a\sin(e+fx))^2} + \frac{(A+4B)}{15f\left(a^3+a\right.}$$

Mathematica [A] time = 0.82, size = 139, normalized size = 1.35

$$\frac{c\left(-15(A+B)\cos\left(e+\frac{fx}{2}\right)+5(A+B)\cos\left(e+\frac{3fx}{2}\right)+A\sin\left(2e+\frac{5fx}{2}\right)+5A\sin\left(\frac{fx}{2}\right)-15B\sin\left(2e+\frac{3fx}{2}\right)+4B\right.}{30a^3f\left(\sin\left(\frac{e}{2}\right)+\cos\left(\frac{e}{2}\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])^
3,x]
```

```
[Out] (c*(-15*(A + B)*Cos[e + (f*x)/2] + 5*(A + B)*Cos[e + (3*f*x)/2] + 5*A*Sin[(
f*x)/2] - 25*B*Sin[(f*x)/2] - 15*B*Sin[2*e + (3*f*x)/2] + A*Sin[2*e + (5*f*
x)/2] + 4*B*Sin[2*e + (5*f*x)/2]))/(30*a^3*f*(Cos[e/2] + Sin[e/2])*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^5)
```

fricas [A] time = 0.42, size = 191, normalized size = 1.85

$$\frac{(A+4B)c\cos\left(fx+e\right)^{3}-(2A-7B)c\cos\left(fx+e\right)^{2}+3(A-B)c\cos\left(fx+e\right)+6(A-B)c-\Big((A+4B)c\cos\big(}{15\left(a^{3}f\cos\left(fx+e\right)^{3}+3a^{3}f\cos\left(fx+e\right)^{2}-2a^{3}f\cos\left(fx+e\right)-4a^{3}f+\left(a^{3}f\cos\left(fx+e\right)^{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] 1/15*((A + 4*B)*c*cos(f*x + e)^3 - (2*A - 7*B)*c*cos(f*x + e)^2 + 3*(A - B)
*c*cos(f*x + e) + 6*(A - B)*c - ((A + 4*B)*c*cos(f*x + e)^2 + 3*(A - B)*c*c
os(f*x + e) + 6*(A - B)*c)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*co
s(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a
^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

giac [A] time = 0.17, size = 138, normalized size = 1.34

$$-\frac{2\left(15Ac\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+15Ac\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+15Bc\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+25Ac\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-5B\right.}{15a^{3}f\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -2/15*(15*A*c*tan(1/2*f*x + 1/2*e)^4 + 15*A*c*tan(1/2*f*x + 1/2*e)^3 + 15*B
*c*tan(1/2*f*x + 1/2*e)^3 + 25*A*c*tan(1/2*f*x + 1/2*e)^2 - 5*B*c*tan(1/2*f
*x + 1/2*e)^2 + 5*A*c*tan(1/2*f*x + 1/2*e) + 5*B*c*tan(1/2*f*x + 1/2*e) + 4
*A*c + B*c)/(a^3*f*(tan(1/2*f*x + 1/2*e) + 1)^5)
```

maple [A] time = 0.45, size = 115, normalized size = 1.12

$$\frac{2c\left(-\frac{-16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{8A-8B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{14A-10B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{-6A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)}{f\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/f*c/a^3*(-1/4*(-16*A+16*B)/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(8*A-8*B)/(tan(1/
2*f*x+1/2*e)+1)^5-1/3*(14*A-10*B)/(tan(1/2*f*x+1/2*e)+1)^3-1/2*(-6*A+2*B)/(
tan(1/2*f*x+1/2*e)+1)^2-A/(tan(1/2*f*x+1/2*e)+1))
```

maxima [B] time = 0.35, size = 733, normalized size = 7.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] -2/15*(A*c*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*
a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) - 2*B*c*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5) - 3*A*c*(5*sin(f*x + e)/(cos(f*x + e) + 1)
 + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*
a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5) + 3*B*c*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*s
in(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

mupad [B] time = 13.03, size = 172, normalized size = 1.67

$$\frac{2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{41\,A\,c}{4}-\frac{B\,c}{4}-\frac{11\,A\,c\,\cos\left(e+f\,x\right)}{2}+\frac{B\,c\,\cos\left(e+f\,x\right)}{2}+5\,A\,c\,\sin\left(e+f\,x\right)+5\,B\,c\,\sin\left(e+f\,x\right)-\frac{3\,A\,c\,\cos\left(2\,e+\right.}{4}}{15\,a^3\,f\,\left(\frac{5\,\sqrt{2}\,\cos\left(\frac{3\,e}{2}+\frac{\pi}{4}+\frac{3\,f\,x}{2}\right)}{4}-\frac{5\,\sqrt{2}\,\cos\left(\frac{e}{2}-\frac{\pi}{4}+\frac{f\,x}{2}\right)}{2}+\frac{\sqrt{2}\,\cos\left(\frac{5\,e}{2}-\frac{\pi}{4}\right.}{4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x)))/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((41*A*c)/4 - (B*c)/4 - (11*A*c*cos(e + f*x))/2 + (B*
c*cos(e + f*x))/2 + 5*A*c*sin(e + f*x) + 5*B*c*sin(e + f*x) - (3*A*c*cos(2*
```

```
e + 2*f*x))/4 + (3*B*c*cos(2*e + 2*f*x))/4 - (5*A*c*sin(2*e + 2*f*x))/4 - (
5*B*c*sin(2*e + 2*f*x))/4))/(15*a^3*f*((5*2^(1/2)*cos((3*e)/2 + pi/4 + (3*f
*x)/2))/4 - (5*2^(1/2)*cos(e/2 - pi/4 + (f*x)/2))/2 + (2^(1/2)*cos((5*e)/2
- pi/4 + (5*f*x)/2))/4))
```

sympy [A] time = 13.79, size = 1035, normalized size = 10.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*c*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*
a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*ta
n(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*A*c*tan(e/
2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**
4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a*
*3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 50*A*c*tan(e/2 + f*x/2)**2/(15*a**3*f*
tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 +
f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 1
5*a**3*f) - 10*A*c*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**
3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e
/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 8*A*c/(15*a**3*f
*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 +
 f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
15*a**3*f) - 30*B*c*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75
*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*t
an(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 10*B*c*tan(e
/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)*
*4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a
**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 10*B*c*tan(e/2 + f*x/2)/(15*a**3*f*ta
n(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*
x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*
a**3*f) - 2*B*c/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)
**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*
a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(
e) + c)/(a*sin(e) + a)**3, True))
```

3.75 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx$

Optimal. Leaf size=102

$$\frac{2(3A+2B)\tan(e+fx)}{15a^3cf}-\frac{(3A+2B)\sec(e+fx)}{15cf\left(a^3\sin(e+fx)+a^3\right)}-\frac{(A-B)\sec(e+fx)}{5acf(a\sin(e+fx)+a)^2}$$

```
[Out] -1/5*(A-B)*sec(f*x+e)/a/c/f/(a+a*sin(f*x+e))^2-1/15*(3*A+2*B)*sec(f*x+e)/c/
f/(a^3+a^3*sin(f*x+e))+2/15*(3*A+2*B)*tan(f*x+e)/a^3/c/f
```

Rubi [A] time = 0.25, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2859, 2672, 3767, 8}

$$\frac{2(3A+2B)\tan(e+fx)}{15a^3cf}-\frac{(3A+2B)\sec(e+fx)}{15cf\left(a^3\sin(e+fx)+a^3\right)}-\frac{(A-B)\sec(e+fx)}{5acf(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])),x]
```

```
[Out] -((A - B)*Sec[e + f*x])/(5*a*c*f*(a + a*Sin[e + f*x])^2) - ((3*A + 2*B)*Sec
[e + f*x])/(15*c*f*(a^3 + a^3*Sin[e + f*x])) + (2*(3*A + 2*B)*Tan[e + f*x])
/(15*a^3*c*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
```

```
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx = \frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx}{ac}$$

$$= -\frac{(A-B)\sec(e+fx)}{5acf(a+a\sin(e+fx))^2} + \frac{(3A+2B)\int \frac{\sec^2(e+fx)}{a+a\sin(e+fx)}\,dx}{5a^2c}$$

$$= -\frac{(A-B)\sec(e+fx)}{5acf(a+a\sin(e+fx))^2} - \frac{(3A+2B)\sec(e+fx)}{15cf\left(a^3+a^3\sin(e+fx)\right)} + \frac{(2(3A \cdots}{\cdots}$$

$$= -\frac{(A-B)\sec(e+fx)}{5acf(a+a\sin(e+fx))^2} - \frac{(3A+2B)\sec(e+fx)}{15cf\left(a^3+a^3\sin(e+fx)\right)} - \frac{(2(3A \cdots}{\cdots}$$

$$= -\frac{(A-B)\sec(e+fx)}{5acf(a+a\sin(e+fx))^2} - \frac{(3A+2B)\sec(e+fx)}{15cf\left(a^3+a^3\sin(e+fx)\right)} + \frac{2(3A \cdots}{\cdots}$$

Mathematica [A] time = 0.84, size = 156, normalized size = 1.53

$$\frac{\cos(e+fx)(-5(9A+B)\cos(e+fx)+32(3A+2B)\cos(2(e+fx))-120A\sin(e+fx)-36A\sin(2(e+fx))+ \cdots}{240a^3cf(\sin(e \cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
),x]
```

```
[Out] (Cos[e + f*x]*(-80*B - 5*(9*A + B)*Cos[e + f*x] + 32*(3*A + 2*B)*Cos[2*(e +
 f*x)] + 9*A*Cos[3*(e + f*x)] + B*Cos[3*(e + f*x)] - 120*A*Sin[e + f*x] - 8
0*B*Sin[e + f*x] - 36*A*Sin[2*(e + f*x)] - 4*B*Sin[2*(e + f*x)] + 24*A*Sin[
3*(e + f*x)] + 16*B*Sin[3*(e + f*x)]))/(240*a^3*c*f*(-1 + Sin[e + f*x])*(1
+ Sin[e + f*x])^3)
```

fricas [A] time = 0.41, size = 106, normalized size = 1.04

$$\frac{4\,(3\,A+2\,B)\cos\left(fx+e\right)^{2}+\left(2\,(3\,A+2\,B)\cos\left(fx+e\right)^{2}-9\,A-6\,B\right)\sin\left(fx+e\right)-6\,A-9\,B}{15\left(a^{3}cf\cos\left(fx+e\right)^{3}-2\,a^{3}cf\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^{3}cf\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/15*(4*(3*A + 2*B)*cos(f*x + e)^2 + (2*(3*A + 2*B)*cos(f*x + e)^2 - 9*A -
6*B)*sin(f*x + e) - 6*A - 9*B)/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f*x
+ e)*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e))
```

giac [A] time = 0.18, size = 175, normalized size = 1.72

$$-\frac{\frac{15\,(A+B)}{a^{3}c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}+\frac{105\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-15\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+270\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+30\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+360\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+40\,B}{a^{3}c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{5}}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/60*(15*(A + B)/(a^3*c*(tan(1/2*f*x + 1/2*e) - 1)) + (105*A*tan(1/2*f*x +
 1/2*e)^4 - 15*B*tan(1/2*f*x + 1/2*e)^4 + 270*A*tan(1/2*f*x + 1/2*e)^3 + 30
*B*tan(1/2*f*x + 1/2*e)^3 + 360*A*tan(1/2*f*x + 1/2*e)^2 + 40*B*tan(1/2*f*x
 + 1/2*e)^2 + 210*A*tan(1/2*f*x + 1/2*e) + 50*B*tan(1/2*f*x + 1/2*e) + 63*A
 + 7*B)/(a^3*c*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

maple [A] time = 0.52, size = 145, normalized size = 1.42

$$\frac{-\frac{2\left(\frac{A}{8}+\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{-4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{2(2A-2B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{-\frac{5A}{2}+\frac{3B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{7A}{8}-\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{9A}{2}-\frac{7B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}}{a^{3}cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x)
```

```
[Out] 2/f/a^3/c*(-(1/8*A+1/8*B)/(tan(1/2*f*x+1/2*e)-1)-1/4*(-4*A+4*B)/(tan(1/2*f*
x+1/2*e)+1)^4-1/5*(2*A-2*B)/(tan(1/2*f*x+1/2*e)+1)^5-1/2*(-5/2*A+3/2*B)/(ta
n(1/2*f*x+1/2*e)+1)^2-(7/8*A-1/8*B)/(tan(1/2*f*x+1/2*e)+1)-1/3*(9/2*A-7/2*B
)/(tan(1/2*f*x+1/2*e)+1)^3)
```

maxima [B] time = 0.42, size = 423, normalized size = 4.15

$$\frac{2\left(\frac{B\left(\frac{4\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{20\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{20\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{15\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+1\right)}{a^3c+\frac{4a^3c\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{5a^3c\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{5a^3c\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-\frac{4a^3c\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}-\frac{a^3c\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}}-\frac{3A\left(\frac{3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{10\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}-\frac{10\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-\cdots\right.}{a^3c+\frac{4a^3c\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{5a^3c\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{5a^3c\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-\cdots}\cdots\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] 2/15*(B*(4*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 + 1)/(a^3*c + 4*a^3*c*sin(f*x + e)/(cos(f*x + e) + 1) + 5*a
^3*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*a^3*c*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4 - 4*a^3*c*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - a^3*c*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6) - 3*A*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 10*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2)/(a^3*c + 4*a^3*c*sin(f*x + e)
/(cos(f*x + e) + 1) + 5*a^3*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*a^3*c
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4*a^3*c*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5 - a^3*c*sin(f*x + e)^6/(cos(f*x + e) + 1)^6))/f
```

mupad [B] time = 12.43, size = 178, normalized size = 1.75

$$-\frac{2\left(\frac{15A\cos\left(e+fx\right)}{4}-\frac{5B}{2}-\frac{5B\cos\left(e+fx\right)}{8}-\frac{15A\sin\left(e+fx\right)}{4}-\frac{5B\sin\left(e+fx\right)}{2}+3A\cos\left(2e+2fx\right)-\frac{3A\cos\left(3e+3fx\right)}{4}+2\cdots\right.}{15a^3cf\left(\frac{5\cos\left(e+fx\right)}{4}-\frac{\cos\left(3e\cdots\right)}{4}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))),x)
```

```
[Out] -(2*((15*A*cos(e + f*x))/4 - (5*B)/2 - (5*B*cos(e + f*x))/8 - (15*A*sin(e +
 f*x))/4 - (5*B*sin(e + f*x))/2 + 3*A*cos(2*e + 2*f*x) - (3*A*cos(3*e + 3*f
```

```
*x))/4 + 2*B*cos(2*e + 2*f*x) + (B*cos(3*e + 3*f*x))/8 + 3*A*sin(2*e + 2*f*
x) + (3*A*sin(3*e + 3*f*x))/4 - (B*sin(2*e + 2*f*x))/2 + (B*sin(3*e + 3*f*x
))/2))/(15*a^3*c*f*((5*cos(e + f*x))/4 - cos(3*e + 3*f*x)/4 + sin(2*e + 2*f
*x)))
```

sympy [A] time = 14.44, size = 1236, normalized size = 12.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-30*A*tan(e/2 + f*x/2)**5/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*
a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*
f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 60*A*
tan(e/2 + f*x/2)**4/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2
+ f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2
)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 60*A*tan(e/2 + f*x/2)*
*3/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*
a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*
f*tan(e/2 + f*x/2) - 15*a**3*c*f) + 18*A*tan(e/2 + f*x/2)/(15*a**3*c*f*tan(
e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f
*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) -
 15*a**3*c*f) + 12*A/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2
 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/
2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 30*B*tan(e/2 + f*x/2)
**4/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75
*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c
*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 40*B*tan(e/2 + f*x/2)**3/(15*a**3*c*f*
tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2
 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/
2) - 15*a**3*c*f) - 40*B*tan(e/2 + f*x/2)**2/(15*a**3*c*f*tan(e/2 + f*x/2)*
*6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75
*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f)
 - 8*B*tan(e/2 + f*x/2)/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(
e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f
*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 2*B/(15*a**3*c*f*t
an(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2
+ f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2
) - 15*a**3*c*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)**3*(-c*sin(e
) + c)), True))
```

3.76 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=90

$$\frac{(4A+B)\tan^3(e+fx)}{15a^3c^2f}+\frac{(4A+B)\tan(e+fx)}{5a^3c^2f}-\frac{(A-B)\sec^3(e+fx)}{5c^2f\left(a^3\sin(e+fx)+a^3\right)}$$

```
[Out] -1/5*(A-B)*sec(f*x+e)^3/c^2/f/(a^3+a^3*sin(f*x+e))+1/5*(4*A+B)*tan(f*x+e)/a
^3/c^2/f+1/15*(4*A+B)*tan(f*x+e)^3/a^3/c^2/f
```

Rubi [A] time = 0.20, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2967, 2859, 3767}

$$\frac{(4A+B)\tan^3(e+fx)}{15a^3c^2f}+\frac{(4A+B)\tan(e+fx)}{5a^3c^2f}-\frac{(A-B)\sec^3(e+fx)}{5c^2f\left(a^3\sin(e+fx)+a^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^2),x]
```

```
[Out] -((A - B)*Sec[e + f*x]^3)/(5*c^2*f*(a^3 + a^3*Sin[e + f*x])) + ((4*A + B)*T
an[e + f*x])/(5*a^3*c^2*f) + ((4*A + B)*Tan[e + f*x]^3)/(15*a^3*c^2*f)
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx = \frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx}{a^2c^2}$$
$$= -\frac{(A-B)\sec^3(e+fx)}{5c^2f\left(a^3+a^3\sin(e+fx)\right)} + \frac{(4A+B)\int \sec^4(e+fx)\,dx}{5a^3c^2}$$
$$= -\frac{(A-B)\sec^3(e+fx)}{5c^2f\left(a^3+a^3\sin(e+fx)\right)} - \frac{(4A+B)\,\mathrm{Subst}\left(\int \left(1+x^2\right)\,dx, x,$$
$$= -\frac{(A-B)\sec^3(e+fx)}{5c^2f\left(a^3+a^3\sin(e+fx)\right)} + \frac{(4A+B)\tan(e+fx)}{5a^3c^2f} + \frac{(4A+B)}{15$$

Mathematica [B] time = 1.05, size = 237, normalized size = 2.63

$$\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(54(A-B)\cos(e+fx)-32(4A+B)\cos$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(240*B + 54*(A - B)*Cos[e + f*x] - 32*(4*A + B)*Cos[2*(e + f*x)] + 18*A*C
os[3*(e + f*x)] - 18*B*Cos[3*(e + f*x)] - 64*A*Cos[4*(e + f*x)] - 16*B*Cos[
4*(e + f*x)] + 384*A*Sin[e + f*x] + 96*B*Sin[e + f*x] + 18*A*Sin[2*(e + f*x
)] - 18*B*Sin[2*(e + f*x)] + 128*A*Sin[3*(e + f*x)] + 32*B*Sin[3*(e + f*x)]
 + 9*A*Sin[4*(e + f*x)] - 9*B*Sin[4*(e + f*x)]))/(960*a^3*c^2*f*(-1 + Sin[e
 + f*x])^2*(1 + Sin[e + f*x])^3)
```

fricas [A] time = 0.42, size = 108, normalized size = 1.20

$$-\frac{2\,(4\,A+B)\cos\left(fx+e\right)^4-(4\,A+B)\cos\left(fx+e\right)^2-\left(2\,(4\,A+B)\cos\left(fx+e\right)^2+4\,A+B\right)\sin\left(fx+e\right)-A-}{15\left(a^3c^2f\cos\left(fx+e\right)^3\sin\left(fx+e\right)+a^3c^2f\cos\left(fx+e\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/15*(2*(4*A + B)*cos(f*x + e)^4 - (4*A + B)*cos(f*x + e)^2 - (2*(4*A + B)
*cos(f*x + e)^2 + 4*A + B)*sin(f*x + e) - A - 4*B)/(a^3*c^2*f*cos(f*x + e)^
3*sin(f*x + e) + a^3*c^2*f*cos(f*x + e)^3)
```

giac [B] time = 0.20, size = 235, normalized size = 2.61

$$-\frac{\frac{5\left(15\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+9\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-24\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-12\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+13\,A+7\,B\right)}{a^{3}c^{2}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{3}}+\frac{165\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-45\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e}{}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] -1/120*(5*(15*A*tan(1/2*f*x + 1/2*e)^2 + 9*B*tan(1/2*f*x + 1/2*e)^2 - 24*A*
tan(1/2*f*x + 1/2*e) - 12*B*tan(1/2*f*x + 1/2*e) + 13*A + 7*B)/(a^3*c^2*(ta
n(1/2*f*x + 1/2*e) - 1)^3) + (165*A*tan(1/2*f*x + 1/2*e)^4 - 45*B*tan(1/2*f
*x + 1/2*e)^4 + 480*A*tan(1/2*f*x + 1/2*e)^3 - 60*B*tan(1/2*f*x + 1/2*e)^3
+ 650*A*tan(1/2*f*x + 1/2*e)^2 - 70*B*tan(1/2*f*x + 1/2*e)^2 + 400*A*tan(1/
2*f*x + 1/2*e) - 20*B*tan(1/2*f*x + 1/2*e) + 113*A - 13*B)/(a^3*c^2*(tan(1/
2*f*x + 1/2*e) + 1)^5))/f
```

maple [B] time = 0.46, size = 185, normalized size = 2.06

$$\frac{-\frac{2\left(\frac{A}{4}+\frac{B}{4}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{\frac{A}{4}+\frac{B}{4}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{2\left(\frac{5A}{16}+\frac{3B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{-2A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{2(A-B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{-\frac{3A}{2}+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{5A}{2}-2B\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\right.}}{f\,a^{3}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x)
```

```
[Out] 2/f/a^3/c^2*(-1/3*(1/4*A+1/4*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/2*(1/4*A+1/4*B)/
(tan(1/2*f*x+1/2*e)-1)^2-(5/16*A+3/16*B)/(tan(1/2*f*x+1/2*e)-1)-1/4*(-2*A+2
*B)/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(A-B)/(tan(1/2*f*x+1/2*e)+1)^5-1/2*(-3/2*A
+B)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(5/2*A-2*B)/(tan(1/2*f*x+1/2*e)+1)^3-(11/1
6*A-3/16*B)/(tan(1/2*f*x+1/2*e)+1))
```

maxima [B] time = 0.37, size = 650, normalized size = 7.22

$$2\left(\frac{A\left(\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{13\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{25\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{5\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{15\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{15\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-3\right)}{a^3c^2+\frac{2a^3c^2\sin(fx+e)}{\cos(fx+e)+1}-\frac{2a^3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{6a^3c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{6a^3c^2\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{2a^3c^2\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{2a^3c^2\sin(fx+e)^7}{(\cos(fx+e)+1)^7}-\frac{a^3c^2\sin(fx+e)^8}{(\cos(fx+e)+1)^8}}+\frac{B\left(\frac{6\sin}{\cos}\right.}{a^3c^2+\frac{2a^3c^2\sin(fx+}{\cos(fx+e)+1}}\right.$$
$$15f$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 2/15*(A*(9*sin(f*x + e)/(cos(f*x + e) + 1) + 21*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 13*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 25*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 + 15*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3)/(a^3*c^
2 + 2*a^3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^3*c^2*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 6*a^3*c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 6*a^3*
c^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2*a^3*c^2*sin(f*x + e)^6/(cos(f*x
 + e) + 1)^6 - 2*a^3*c^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^3*c^2*sin(
f*x + e)^8/(cos(f*x + e) + 1)^8) + B*(6*sin(f*x + e)/(cos(f*x + e) + 1) + 9
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 8*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3 + 5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 10*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5 + 15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 3)/(a^3*c^2 + 2*a^3*c^2*
sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^3*c^2*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 - 6*a^3*c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 6*a^3*c^2*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5 + 2*a^3*c^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 -
 2*a^3*c^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^3*c^2*sin(f*x + e)^8/(co
s(f*x + e) + 1)^8))/f
```

mupad [B] time = 12.47, size = 183, normalized size = 2.03

$$\frac{\left(\frac{8A}{15}+\frac{2B}{15}+\frac{16A\sin(e+fx)}{15}+\frac{4B\sin(e+fx)}{15}\right)\cos\left(e+fx\right)^2+\frac{2A}{15}+\frac{8B}{15}+\frac{8A\sin(e+fx)}{15}+\frac{2B\sin(e+fx)}{15}}{a^3c^2f\left(2\cos\left(e+fx\right)^3\sin\left(e+fx\right)+2\cos\left(e+fx\right)^3\right)}-\frac{\frac{2A}{5}-\frac{2B}{5}+\frac{2A\,s}{}}{a^3c^2f\left(2\,s\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^2),x)
```

```
[Out] ((2*A)/15 + (8*B)/15 + (8*A*sin(e + f*x))/15 + (2*B*sin(e + f*x))/15 + cos(
e + f*x)^2*((8*A)/15 + (2*B)/15 + (16*A*sin(e + f*x))/15 + (4*B*sin(e + f*x
))/15))/(a^3*c^2*f*(2*cos(e + f*x)^3*sin(e + f*x) + 2*cos(e + f*x)^3)) - ((
```

```
2*A)/5 - (2*B)/5 + (2*A*sin(e + f*x))/5 - (2*B*sin(e + f*x))/5)/(a^3*c^2*f*
(2*sin(e + f*x) + 2)) - (cos(e + f*x)*((16*A)/15 + (4*B)/15))/(a^3*c^2*f*(2
*sin(e + f*x) + 2))
```

sympy [A] time = 27.39, size = 2674, normalized size = 29.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-30*A*tan(e/2 + f*x/2)**7/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 +
30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 9
0*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30
*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**
3*c**2*f) - 30*A*tan(e/2 + f*x/2)**6/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 +
30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 9
0*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30
*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**
3*c**2*f) + 10*A*tan(e/2 + f*x/2)**5/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 +
30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 9
0*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30
*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**
3*c**2*f) + 50*A*tan(e/2 + f*x/2)**4/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 +
30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 9
0*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30
*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**
3*c**2*f) - 26*A*tan(e/2 + f*x/2)**3/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 +
30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 9
0*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30
*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**
3*c**2*f) - 42*A*tan(e/2 + f*x/2)**2/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 +
30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 9
0*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30
*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**
3*c**2*f) - 18*A*tan(e/2 + f*x/2)/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*
a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a
**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a*
*3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c
**2*f) + 6*A/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 +
 f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 +
f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f
*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) - 30*B*tan(e/2
 + f*x/2)**6/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 +
 f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 +
```

```
f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f
*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) - 20*B*tan(e/2
 + f*x/2)**5/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 +
 f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 +
f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f
*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) - 10*B*tan(e/2
 + f*x/2)**4/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 +
 f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 +
f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f
*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) + 16*B*tan(e/2
 + f*x/2)**3/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 +
 f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 +
f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f
*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) - 18*B*tan(e/2
 + f*x/2)**2/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 +
 f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 +
f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f
*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) - 12*B*tan(e/2
 + f*x/2)/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 + f*
x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 + f*x
/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f*x/
2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) - 6*B/(15*a**3*c*
*2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**
2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2
*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*
f*tan(e/2 + f*x/2) - 15*a**3*c**2*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(
e) + a)**3*(-c*sin(e) + c)**2), True))
```

3.77 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=84

$$\frac{A\tan^5(e+fx)}{5a^3c^3f}+\frac{2A\tan^3(e+fx)}{3a^3c^3f}+\frac{A\tan(e+fx)}{a^3c^3f}+\frac{B\sec^5(e+fx)}{5a^3c^3f}$$

```
[Out] 1/5*B*sec(f*x+e)^5/a^3/c^3/f+A*tan(f*x+e)/a^3/c^3/f+2/3*A*tan(f*x+e)^3/a^3/
c^3/f+1/5*A*tan(f*x+e)^5/a^3/c^3/f
```

Rubi [A] time = 0.15, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2967, 2669, 3767}

$$\frac{A\tan^5(e+fx)}{5a^3c^3f}+\frac{2A\tan^3(e+fx)}{3a^3c^3f}+\frac{A\tan(e+fx)}{a^3c^3f}+\frac{B\sec^5(e+fx)}{5a^3c^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^3),x]
```

```
[Out] (B*Sec[e + f*x]^5)/(5*a^3*c^3*f) + (A*Tan[e + f*x])/(a^3*c^3*f) + (2*A*Tan[
e + f*x]^3)/(3*a^3*c^3*f) + (A*Tan[e + f*x]^5)/(5*a^3*c^3*f)
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))\,dx}{a^3c^3}$$

$$= \frac{B\sec^5(e+fx)}{5a^3c^3f} + \frac{A\int \sec^6(e+fx)\,dx}{a^3c^3}$$

$$= \frac{B\sec^5(e+fx)}{5a^3c^3f} - \frac{A\,\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -\tan(e+fx)\right)}{a^3c^3f}$$

$$= \frac{B\sec^5(e+fx)}{5a^3c^3f} + \frac{A\tan(e+fx)}{a^3c^3f} + \frac{2A\tan^3(e+fx)}{3a^3c^3f} + \frac{A\tan^5(e+}{5a^3c^3f}$$

Mathematica [A] time = 0.21, size = 65, normalized size = 0.77

$$\frac{A\left(\frac{1}{5}\tan^5(e+fx)+\frac{2}{3}\tan^3(e+fx)+\tan(e+fx)\right)}{a^3c^3f}+\frac{B\sec^5(e+fx)}{5a^3c^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^3),x]
```

```
[Out] (B*Sec[e + f*x]^5)/(5*a^3*c^3*f) + (A*(Tan[e + f*x] + (2*Tan[e + f*x]^3)/3
+ Tan[e + f*x]^5/5))/(a^3*c^3*f)
```

fricas [A] time = 0.41, size = 56, normalized size = 0.67

$$\frac{\left(8\,A\cos\left(fx+e\right)^4+4\,A\cos\left(fx+e\right)^2+3\,A\right)\sin\left(fx+e\right)+3\,B}{15\,a^3c^3f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/15*((8*A*cos(f*x + e)^4 + 4*A*cos(f*x + e)^2 + 3*A)*sin(f*x + e) + 3*B)/(
a^3*c^3*f*cos(f*x + e)^5)
```

giac [A] time = 0.35, size = 134, normalized size = 1.60

$$-\frac{2\left(15\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}+15\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-20\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+58\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+30\,B\,\mathrm{ta}\right.}{15\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)^{5}a^{3}c^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -2/15*(15*A*tan(1/2*f*x + 1/2*e)^9 + 15*B*tan(1/2*f*x + 1/2*e)^8 - 20*A*tan
(1/2*f*x + 1/2*e)^7 + 58*A*tan(1/2*f*x + 1/2*e)^5 + 30*B*tan(1/2*f*x + 1/2*
e)^4 - 20*A*tan(1/2*f*x + 1/2*e)^3 + 15*A*tan(1/2*f*x + 1/2*e) + 3*B)/((tan
(1/2*f*x + 1/2*e)^2 - 1)^5*a^3*c^3*f)
```

maple [B] time = 0.42, size = 227, normalized size = 2.70

$$\frac{-\frac{A+B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{2\left(\frac{A}{2}+\frac{B}{2}\right)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{\frac{7A}{8}+\frac{5B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{2\left(\frac{A}{2}+\frac{3B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{2\left(\frac{11A}{8}+\frac{9B}{8}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{-A+B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{-\frac{7A}{8}+\frac{5B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\right.}}{f\,a^{3}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x)
```

```
[Out] 2/f/a^3/c^3*(-1/4*(A+B)/(tan(1/2*f*x+1/2*e)-1)^4-1/5*(1/2*A+1/2*B)/(tan(1/2
*f*x+1/2*e)-1)^5-1/2*(7/8*A+5/8*B)/(tan(1/2*f*x+1/2*e)-1)^2-(1/2*A+3/16*B)/
(tan(1/2*f*x+1/2*e)-1)-1/3*(11/8*A+9/8*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/4*(-A+
B)/(tan(1/2*f*x+1/2*e)+1)^4-1/2*(-7/8*A+5/8*B)/(tan(1/2*f*x+1/2*e)+1)^2-1/5
*(1/2*A-1/2*B)/(tan(1/2*f*x+1/2*e)+1)^5-(1/2*A-3/16*B)/(tan(1/2*f*x+1/2*e)+
1)-1/3*(11/8*A-9/8*B)/(tan(1/2*f*x+1/2*e)+1)^3)
```

maxima [A] time = 0.36, size = 60, normalized size = 0.71

$$\frac{\frac{\left(3\tan\left(fx+e\right)^{5}+10\tan\left(fx+e\right)^{3}+15\tan\left(fx+e\right)\right)A}{a^{3}c^{3}}+\frac{3\,B}{a^{3}c^{3}\cos\left(fx+e\right)^{5}}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 1/15*((3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*A/(a^3*c^3)
+ 3*B/(a^3*c^3*cos(f*x + e)^5))/f
```

mupad [B] time = 14.42, size = 126, normalized size = 1.50

$$-\frac{2\left(15\,A\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9+15\,B\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-20\,A\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+58\,A\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+30\,B\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-\right.}{15\,a^3\,c^3\,f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^3),x)
```

```
[Out] -(2*(3*B + 15*A*tan(e/2 + (f*x)/2) - 20*A*tan(e/2 + (f*x)/2)^3 + 58*A*tan(e
/2 + (f*x)/2)^5 - 20*A*tan(e/2 + (f*x)/2)^7 + 15*A*tan(e/2 + (f*x)/2)^9 + 3
0*B*tan(e/2 + (f*x)/2)^4 + 15*B*tan(e/2 + (f*x)/2)^8))/(15*a^3*c^3*f*(tan(e
/2 + (f*x)/2)^2 - 1)^5)
```

sympy [A] time = 20.93, size = 1098, normalized size = 13.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*tan(e/2 + f*x/2)**9/(15*a**3*c**3*f*tan(e/2 + f*x/2)**10 -
 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e/2 + f*x/2)**6 -
 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e/2 + f*x/2)**2 -
 15*a**3*c**3*f) + 40*A*tan(e/2 + f*x/2)**7/(15*a**3*c**3*f*tan(e/2 + f*x/2
)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e/2 + f*x/
2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e/2 + f*x/
2)**2 - 15*a**3*c**3*f) - 116*A*tan(e/2 + f*x/2)**5/(15*a**3*c**3*f*tan(e/2
 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e/
2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e/
2 + f*x/2)**2 - 15*a**3*c**3*f) + 40*A*tan(e/2 + f*x/2)**3/(15*a**3*c**3*f*
tan(e/2 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f
*tan(e/2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f
*tan(e/2 + f*x/2)**2 - 15*a**3*c**3*f) - 30*A*tan(e/2 + f*x/2)/(15*a**3*c**
3*f*tan(e/2 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c*
*3*f*tan(e/2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c*
*3*f*tan(e/2 + f*x/2)**2 - 15*a**3*c**3*f) - 30*B*tan(e/2 + f*x/2)**8/(15*a
**3*c**3*f*tan(e/2 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*
a**3*c**3*f*tan(e/2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*
a**3*c**3*f*tan(e/2 + f*x/2)**2 - 15*a**3*c**3*f) - 60*B*tan(e/2 + f*x/2)**
```

```
4/(15*a**3*c**3*f*tan(e/2 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8
 + 150*a**3*c**3*f*tan(e/2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**
4 + 75*a**3*c**3*f*tan(e/2 + f*x/2)**2 - 15*a**3*c**3*f) - 6*B/(15*a**3*c**
3*f*tan(e/2 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c*
*3*f*tan(e/2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c*
*3*f*tan(e/2 + f*x/2)**2 - 15*a**3*c**3*f), Ne(f, 0)), (x*(A + B*sin(e))/((
a*sin(e) + a)**3*(-c*sin(e) + c)**3), True))
```

3.78 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=121

$$\frac{(6A-B)\tan^5(e+fx)}{35a^3c^4f}+\frac{2(6A-B)\tan^3(e+fx)}{21a^3c^4f}+\frac{(6A-B)\tan(e+fx)}{7a^3c^4f}+\frac{(A+B)\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)}$$

```
[Out] 1/7*(A+B)*sec(f*x+e)^5/a^3/f/(c^4-c^4*sin(f*x+e))+1/7*(6*A-B)*tan(f*x+e)/a^
3/c^4/f+2/21*(6*A-B)*tan(f*x+e)^3/a^3/c^4/f+1/35*(6*A-B)*tan(f*x+e)^5/a^3/c
^4/f
```

Rubi [A] time = 0.22, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2967, 2859, 3767}

$$\frac{(6A-B)\tan^5(e+fx)}{35a^3c^4f}+\frac{2(6A-B)\tan^3(e+fx)}{21a^3c^4f}+\frac{(6A-B)\tan(e+fx)}{7a^3c^4f}+\frac{(A+B)\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] ((A + B)*Sec[e + f*x]^5)/(7*a^3*f*(c^4 - c^4*Sin[e + f*x])) + ((6*A - B)*Ta
n[e + f*x])/(7*a^3*c^4*f) + (2*(6*A - B)*Tan[e + f*x]^3)/(21*a^3*c^4*f) + (
(6*A - B)*Tan[e + f*x]^5)/(35*a^3*c^4*f)
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx = \frac{\int \frac{\sec^6(e+fx)(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx}{a^3c^3}$$

$$= \frac{(A+B)\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)} + \frac{(6A-B)\int \sec^6(e+fx)\,dx}{7a^3c^4}$$

$$= \frac{(A+B)\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)} - \frac{(6A-B)\,\text{Subst}\left(\int\left(1+2x^2+x^4\right)\right.}{7a^3c^4f}$$

$$= \frac{(A+B)\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)} + \frac{(6A-B)\tan(e+fx)}{7a^3c^4f} + \frac{2(6A-B)}{21}$$

Mathematica [B] time = 1.14, size = 325, normalized size = 2.69

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(1500(A+B)\cos(e+fx)-640(6A-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^4),x]
```

```
[Out] -1/53760*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])*(-8960*B + 1500*(A + B)*Cos[e + f*x] - 640*(6*A - B)*Cos[2*(e +
f*x)] + 750*A*Cos[3*(e + f*x)] + 750*B*Cos[3*(e + f*x)] - 3072*A*Cos[4*(e +
 f*x)] + 512*B*Cos[4*(e + f*x)] + 150*A*Cos[5*(e + f*x)] + 150*B*Cos[5*(e +
 f*x)] - 768*A*Cos[6*(e + f*x)] + 128*B*Cos[6*(e + f*x)] - 15360*A*Sin[e +
f*x] + 2560*B*Sin[e + f*x] - 375*A*Sin[2*(e + f*x)] - 375*B*Sin[2*(e + f*x)
] - 7680*A*Sin[3*(e + f*x)] + 1280*B*Sin[3*(e + f*x)] - 300*A*Sin[4*(e + f*
x)] - 300*B*Sin[4*(e + f*x)] - 1536*A*Sin[5*(e + f*x)] + 256*B*Sin[5*(e + f
*x)] - 75*A*Sin[6*(e + f*x)] - 75*B*Sin[6*(e + f*x)]))/(a^3*c^4*f*(-1 + Sin
[e + f*x])^4*(1 + Sin[e + f*x])^3)
```

fricas [A] time = 0.44, size = 150, normalized size = 1.24

$$-\frac{8\,(6\,A-B)\cos\left(fx+e\right)^{6}-4\,(6\,A-B)\cos\left(fx+e\right)^{4}-(6\,A-B)\cos\left(fx+e\right)^{2}+\left(8\,(6\,A-B)\cos\left(fx+e\right)^{4}+4\right.}{105\left(a^{3}c^{4}f\cos\left(fx+e\right)^{5}\sin\left(fx+e\right)-a^{3}c^{4}f\cos\left(fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorit
hm="fricas")
```

```
[Out] -1/105*(8*(6*A - B)*cos(f*x + e)^6 - 4*(6*A - B)*cos(f*x + e)^4 - (6*A - B)
*cos(f*x + e)^2 + (8*(6*A - B)*cos(f*x + e)^4 + 4*(6*A - B)*cos(f*x + e)^2
+ 18*A - 3*B)*sin(f*x + e) - 3*A + 18*B)/(a^3*c^4*f*cos(f*x + e)^5*sin(f*x
+ e) - a^3*c^4*f*cos(f*x + e)^5)
```

giac [B] time = 0.27, size = 355, normalized size = 2.93

$$-\frac{\frac{7\left(165\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-75\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+540\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-210\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+750\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-280\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+480\,A\right.}{a^{3}c^{4}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{5}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorit
hm="giac")
```

```
[Out] -1/1680*(7*(165*A*tan(1/2*f*x + 1/2*e)^4 - 75*B*tan(1/2*f*x + 1/2*e)^4 + 54
0*A*tan(1/2*f*x + 1/2*e)^3 - 210*B*tan(1/2*f*x + 1/2*e)^3 + 750*A*tan(1/2*f
*x + 1/2*e)^2 - 280*B*tan(1/2*f*x + 1/2*e)^2 + 480*A*tan(1/2*f*x + 1/2*e) -
 170*B*tan(1/2*f*x + 1/2*e) + 129*A - 49*B)/(a^3*c^4*(tan(1/2*f*x + 1/2*e)
+ 1)^5) + (2205*A*tan(1/2*f*x + 1/2*e)^6 + 525*B*tan(1/2*f*x + 1/2*e)^6 - 1
0080*A*tan(1/2*f*x + 1/2*e)^5 - 1470*B*tan(1/2*f*x + 1/2*e)^5 + 21945*A*tan
(1/2*f*x + 1/2*e)^4 + 2555*B*tan(1/2*f*x + 1/2*e)^4 - 26460*A*tan(1/2*f*x +
 1/2*e)^3 - 2240*B*tan(1/2*f*x + 1/2*e)^3 + 18963*A*tan(1/2*f*x + 1/2*e)^2
+ 1407*B*tan(1/2*f*x + 1/2*e)^2 - 7476*A*tan(1/2*f*x + 1/2*e) - 434*B*tan(1
/2*f*x + 1/2*e) + 1383*A + 137*B)/(a^3*c^4*(tan(1/2*f*x + 1/2*e) - 1)^7))/f
```

maple [B] time = 0.51, size = 271, normalized size = 2.24

$$\frac{-\frac{2(A+B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{3A+3B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{\frac{11A}{2}+\frac{9B}{2}}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{\frac{15A}{8}+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{2\left(\frac{21A}{32}+\frac{5B}{32}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{2\left(\frac{21A}{4}+\frac{19B}{4}\right)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{2\left(\frac{33A}{8}+\frac{11B}{4}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right.}}{f\,a^{3}c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x)
```

```
[Out] 2/f/a^3/c^4*(-1/7*(A+B)/(tan(1/2*f*x+1/2*e)-1)^7-1/6*(3*A+3*B)/(tan(1/2*f*x
+1/2*e)-1)^6-1/4*(11/2*A+9/2*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/2*(15/8*A+B)/(ta
n(1/2*f*x+1/2*e)-1)^2-(21/32*A+5/32*B)/(tan(1/2*f*x+1/2*e)-1)-1/5*(21/4*A+1
9/4*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/3*(33/8*A+11/4*B)/(tan(1/2*f*x+1/2*e)-1)^
3-1/2*(-1/2*A+3/8*B)/(tan(1/2*f*x+1/2*e)+1)^2-1/4*(-1/2*A+1/2*B)/(tan(1/2*f
*x+1/2*e)+1)^4-1/5*(1/4*A-1/4*B)/(tan(1/2*f*x+1/2*e)+1)^5-1/3*(3/4*A-5/8*B)
/(tan(1/2*f*x+1/2*e)+1)^3-(11/32*A-5/32*B)/(tan(1/2*f*x+1/2*e)+1))
```

maxima [B] time = 0.41, size = 1019, normalized size = 8.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorit
hm="maxima")
```

```
[Out] -2/105*(B*(30*sin(f*x + e)/(cos(f*x + e) + 1) - 45*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 - 80*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 110*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 188*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 266*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 - 112*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 35
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 70*sin(f*x + e)^9/(cos(f*x + e) + 1)
^9 - 105*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 15)/(a^3*c^4 - 2*a^3*c^4*s
in(f*x + e)/(cos(f*x + e) + 1) - 4*a^3*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 10*a^3*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*c^4*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 - 20*a^3*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
+ 20*a^3*c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 5*a^3*c^4*sin(f*x + e)^8
/(cos(f*x + e) + 1)^8 - 10*a^3*c^4*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 4*
a^3*c^4*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 2*a^3*c^4*sin(f*x + e)^11/(
cos(f*x + e) + 1)^11 - a^3*c^4*sin(f*x + e)^12/(cos(f*x + e) + 1)^12) - 3*A
*(25*sin(f*x + e)/(cos(f*x + e) + 1) - 55*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 15*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 130*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 26*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 182*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 126*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 105*sin(f*x
+ e)^8/(cos(f*x + e) + 1)^8 - 35*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 35*s
in(f*x + e)^10/(cos(f*x + e) + 1)^10 + 35*sin(f*x + e)^11/(cos(f*x + e) + 1
)^11 + 5)/(a^3*c^4 - 2*a^3*c^4*sin(f*x + e)/(cos(f*x + e) + 1) - 4*a^3*c^4*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*c^4*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 5*a^3*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 20*a^3*c^4*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 + 20*a^3*c^4*sin(f*x + e)^7/(cos(f*x + e) +
 1)^7 - 5*a^3*c^4*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 10*a^3*c^4*sin(f*x
+ e)^9/(cos(f*x + e) + 1)^9 + 4*a^3*c^4*sin(f*x + e)^10/(cos(f*x + e) + 1)^
```

```
10 + 2*a^3*c^4*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - a^3*c^4*sin(f*x + e)
^12/(cos(f*x + e) + 1)^12))/f
```

mupad [B] time = 13.16, size = 217, normalized size = 1.79

$$\frac{\left(\frac{16\,A}{35}-\frac{8\,B}{105}-\frac{32\,A\,\sin\left(e+f\,x\right)}{35}+\frac{16\,B\,\sin\left(e+f\,x\right)}{105}\right)\cos\left(e+f\,x\right)^4+\left(\frac{4\,A}{35}-\frac{2\,B}{105}-\frac{16\,A\,\sin\left(e+f\,x\right)}{35}+\frac{8\,B\,\sin\left(e+f\,x\right)}{105}\right)\cos\left(e+f\,x\right.}{a^3\,c^4\,f\,\left(2\cos\left(e+f\,x\right)^5\,\sin\left(e+f\,x\right)-2\cos\left(e+f\,x\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^4),x)
```

```
[Out] ((2*A)/35 - (12*B)/35 - (12*A*sin(e + f*x))/35 + (2*B*sin(e + f*x))/35 + co
s(e + f*x)^2*((4*A)/35 - (2*B)/105 - (16*A*sin(e + f*x))/35 + (8*B*sin(e +
f*x))/105) + cos(e + f*x)^4*((16*A)/35 - (8*B)/105 - (32*A*sin(e + f*x))/35
 + (16*B*sin(e + f*x))/105))/(a^3*c^4*f*(2*cos(e + f*x)^5*sin(e + f*x) - 2*
cos(e + f*x)^5)) - ((2*A)/7 + (2*B)/7 - (2*A*sin(e + f*x))/7 - (2*B*sin(e +
 f*x))/7)/(a^3*c^4*f*(2*sin(e + f*x) - 2)) - (cos(e + f*x)*((32*A)/35 - (16
*B)/105))/(a^3*c^4*f*(2*sin(e + f*x) - 2))
```

sympy [A] time = 92.70, size = 6135, normalized size = 50.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-210*A*tan(e/2 + f*x/2)**11/(105*a**3*c**4*f*tan(e/2 + f*x/2)**1
2 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)
**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x
/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 +
f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2
+ f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2
 + f*x/2) - 105*a**3*c**4*f) + 210*A*tan(e/2 + f*x/2)**10/(105*a**3*c**4*f*
tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4
*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c
**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**
3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a
**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*
a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) + 210*A*tan(e/2 + f*x/2)**9
/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**
11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/
2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*
```

```
x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 +
f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2
+ f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) - 630*A*t
an(e/2 + f*x/2)**8/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*
tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**
4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c
**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3
*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a*
*3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3
*c**4*f) - 756*A*tan(e/2 + f*x/2)**7/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12
- 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**
10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2
)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*
x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 +
f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 +
 f*x/2) - 105*a**3*c**4*f) + 1092*A*tan(e/2 + f*x/2)**6/(105*a**3*c**4*f*ta
n(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f
*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**
4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*
c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**
3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*
*3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) - 156*A*tan(e/2 + f*x/2)**5/(
105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11
 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)
**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/
2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*
x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 +
f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) - 780*A*tan
(e/2 + f*x/2)**4/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*ta
n(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*
f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**
4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c
**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3
*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c
**4*f) - 90*A*tan(e/2 + f*x/2)**3/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 2
10*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10
+ 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**
8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2
)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x
/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*
x/2) - 105*a**3*c**4*f) + 330*A*tan(e/2 + f*x/2)**2/(105*a**3*c**4*f*tan(e/
2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan
(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*
tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4
*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c*
```

```
*4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c
**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) - 150*A*tan(e/2 + f*x/2)/(105*a**
3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*
a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 5
25*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 +
 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4
 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)*
*2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) - 30*A/(105*a**3*c
**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**
3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*
a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 21
00*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 -
1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2
+ 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) - 210*B*tan(e/2 + f*x
/2)**10/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f
*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2
 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e
/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan
(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*t
an(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) +
140*B*tan(e/2 + f*x/2)**9/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*
c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a
**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100
*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 5
25*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 +
 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 1
05*a**3*c**4*f) - 70*B*tan(e/2 + f*x/2)**8/(105*a**3*c**4*f*tan(e/2 + f*x/2
)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*
x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 +
 f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/
2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(
e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan
(e/2 + f*x/2) - 105*a**3*c**4*f) - 224*B*tan(e/2 + f*x/2)**7/(105*a**3*c**4
*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c
**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**
3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*
a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 105
0*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 2
10*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) - 532*B*tan(e/2 + f*x/2)
**6/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2
)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f
*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 +
 f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2
 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e
/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) + 376*
```

```
B*tan(e/2 + f*x/2)**5/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4
*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*
c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**
3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a
**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420
*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a
**3*c**4*f) - 220*B*tan(e/2 + f*x/2)**4/(105*a**3*c**4*f*tan(e/2 + f*x/2)**
12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2
)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*
x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 +
 f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2
 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/
2 + f*x/2) - 105*a**3*c**4*f) - 160*B*tan(e/2 + f*x/2)**3/(105*a**3*c**4*f*
tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4
*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c
**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**
3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a
**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*
a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) - 90*B*tan(e/2 + f*x/2)**2/
(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**1
1 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2
)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x
/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f
*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 +
 f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) + 60*B*tan
(e/2 + f*x/2)/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e
/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*t
an(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f
*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4
*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c*
*4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4
*f) - 30*B/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2
+ f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(
e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*ta
n(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*
tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*
f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f)
, Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)**3*(-c*sin(e) + c)**4), True
))
```

3.79 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx$

Optimal. Leaf size=162

$$\frac{2(7A-2B)\tan^5(e+fx)}{105a^3c^5f}+\frac{4(7A-2B)\tan^3(e+fx)}{63a^3c^5f}+\frac{2(7A-2B)\tan(e+fx)}{21a^3c^5f}+\frac{(7A-2B)\sec^5(e+fx)}{63a^3f\left(c^5-c^5\sin(e+fx)\right)}+\frac{(A}{9a^3c}$$

```
[Out] 1/9*(A+B)*sec(f*x+e)^5/a^3/c^3/f/(c-c*sin(f*x+e))^2+1/63*(7*A-2*B)*sec(f*x+
e)^5/a^3/f/(c^5-c^5*sin(f*x+e))+2/21*(7*A-2*B)*tan(f*x+e)/a^3/c^5/f+4/63*(7
*A-2*B)*tan(f*x+e)^3/a^3/c^5/f+2/105*(7*A-2*B)*tan(f*x+e)^5/a^3/c^5/f
```

Rubi [A] time = 0.29, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 2672, 3767}

$$\frac{2(7A-2B)\tan^5(e+fx)}{105a^3c^5f}+\frac{4(7A-2B)\tan^3(e+fx)}{63a^3c^5f}+\frac{2(7A-2B)\tan(e+fx)}{21a^3c^5f}+\frac{(7A-2B)\sec^5(e+fx)}{63a^3f\left(c^5-c^5\sin(e+fx)\right)}+\frac{(A}{9a^3c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^5),x]
```

```
[Out] ((A + B)*Sec[e + f*x]^5)/(9*a^3*c^3*f*(c - c*Sin[e + f*x])^2) + ((7*A - 2*B
)*Sec[e + f*x]^5)/(63*a^3*f*(c^5 - c^5*Sin[e + f*x])) + (2*(7*A - 2*B)*Tan[
e + f*x])/(21*a^3*c^5*f) + (4*(7*A - 2*B)*Tan[e + f*x]^3)/(63*a^3*c^5*f) +
(2*(7*A - 2*B)*Tan[e + f*x]^5)/(105*a^3*c^5*f)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
```

```
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx = \frac{\int \frac{\sec^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx}{a^3c^3}$$

$$= \frac{(A+B)\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{(7A-2B)\int \frac{\sec^6(e+fx)}{c-c\sin(e+fx)}\,dx}{9a^3c^4}$$

$$= \frac{(A+B)\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{(7A-2B)\sec^5(e+fx)}{63a^3f\left(c^5-c^5\sin(e+fx)\right)} + \frac{(2(7}{}$$

$$= \frac{(A+B)\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{(7A-2B)\sec^5(e+fx)}{63a^3f\left(c^5-c^5\sin(e+fx)\right)} - \frac{(2(7}{}$$

$$= \frac{(A+B)\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{(7A-2B)\sec^5(e+fx)}{63a^3f\left(c^5-c^5\sin(e+fx)\right)} + \frac{2(7}{}$$

Mathematica [B] time = 1.41, size = 373, normalized size = 2.30

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(1125(49A+13B)\cos(e+fx)-20480}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^5),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-184320*B + 1125*(49*A + 13*B)*Cos[e + f*x] - 20480*(7*A - 2*B)*Cos[2*(e
 + f*x)] + 23275*A*Cos[3*(e + f*x)] + 6175*B*Cos[3*(e + f*x)] - 114688*A*Co
s[4*(e + f*x)] + 32768*B*Cos[4*(e + f*x)] + 1225*A*Cos[5*(e + f*x)] + 325*B
*Cos[5*(e + f*x)] - 28672*A*Cos[6*(e + f*x)] + 8192*B*Cos[6*(e + f*x)] - 12
25*A*Cos[7*(e + f*x)] - 325*B*Cos[7*(e + f*x)] - 322560*A*Sin[e + f*x] + 92
160*B*Sin[e + f*x] - 24500*A*Sin[2*(e + f*x)] - 6500*B*Sin[2*(e + f*x)] - 1
36192*A*Sin[3*(e + f*x)] + 38912*B*Sin[3*(e + f*x)] - 19600*A*Sin[4*(e + f*
x)] - 5200*B*Sin[4*(e + f*x)] - 7168*A*Sin[5*(e + f*x)] + 2048*B*Sin[5*(e +
 f*x)] - 4900*A*Sin[6*(e + f*x)] - 1300*B*Sin[6*(e + f*x)] + 7168*A*Sin[7*(
e + f*x)] - 2048*B*Sin[7*(e + f*x)]))/(1290240*a^3*c^5*f*(-1 + Sin[e + f*x]
)^5*(1 + Sin[e + f*x])^3)
```

fricas [A] time = 0.44, size = 185, normalized size = 1.14

$$-\frac{32\,(7\,A-2\,B)\cos\left(fx+e\right)^{6}-16\,(7\,A-2\,B)\cos\left(fx+e\right)^{4}-4\,(7\,A-2\,B)\cos\left(fx+e\right)^{2}-\left(16\,(7\,A-2\,B)\cos\left(f\right.\right.}{315\left(a^{3}c^{5}f\cos\left(fx+e\right)^{7}+2\,a^{3}c^{5}f\cos\left(fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorit
hm="fricas")
```

```
[Out] -1/315*(32*(7*A - 2*B)*cos(f*x + e)^6 - 16*(7*A - 2*B)*cos(f*x + e)^4 - 4*(
7*A - 2*B)*cos(f*x + e)^2 - (16*(7*A - 2*B)*cos(f*x + e)^6 - 24*(7*A - 2*B)
*cos(f*x + e)^4 - 10*(7*A - 2*B)*cos(f*x + e)^2 - 49*A + 14*B)*sin(f*x + e)
 - 14*A + 49*B)/(a^3*c^5*f*cos(f*x + e)^7 + 2*a^3*c^5*f*cos(f*x + e)^5*sin(
f*x + e) - 2*a^3*c^5*f*cos(f*x + e)^5)
```

giac [B] time = 0.27, size = 415, normalized size = 2.56

$$-\frac{\frac{21\left(435\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-225\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+1470\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-690\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+2060\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-940\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+\right.}{a^{3}c^{5}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{5}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorit
hm="giac")
```

```
[Out] -1/20160*(21*(435*A*tan(1/2*f*x + 1/2*e)^4 - 225*B*tan(1/2*f*x + 1/2*e)^4 +
 1470*A*tan(1/2*f*x + 1/2*e)^3 - 690*B*tan(1/2*f*x + 1/2*e)^3 + 2060*A*tan(
```

```
1/2*f*x + 1/2*e)^2 - 940*B*tan(1/2*f*x + 1/2*e)^2 + 1330*A*tan(1/2*f*x + 1/
2*e) - 590*B*tan(1/2*f*x + 1/2*e) + 353*A - 163*B)/(a^3*c^5*(tan(1/2*f*x +
1/2*e) + 1)^5) + (31185*A*tan(1/2*f*x + 1/2*e)^8 + 4725*B*tan(1/2*f*x + 1/2
*e)^8 - 185220*A*tan(1/2*f*x + 1/2*e)^7 - 11340*B*tan(1/2*f*x + 1/2*e)^7 +
546840*A*tan(1/2*f*x + 1/2*e)^6 + 15120*B*tan(1/2*f*x + 1/2*e)^6 - 961380*A
*tan(1/2*f*x + 1/2*e)^5 + 3780*B*tan(1/2*f*x + 1/2*e)^5 + 1101618*A*tan(1/2
*f*x + 1/2*e)^4 - 24318*B*tan(1/2*f*x + 1/2*e)^4 - 828492*A*tan(1/2*f*x + 1
/2*e)^3 + 33852*B*tan(1/2*f*x + 1/2*e)^3 + 404208*A*tan(1/2*f*x + 1/2*e)^2
- 19368*B*tan(1/2*f*x + 1/2*e)^2 - 116172*A*tan(1/2*f*x + 1/2*e) + 6732*B*t
an(1/2*f*x + 1/2*e) + 16373*A - 223*B)/(a^3*c^5*(tan(1/2*f*x + 1/2*e) - 1)^
9))/f
```

maple [B] time = 0.56, size = 321, normalized size = 1.98

$$-\frac{2(2A+2B)}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}}-\frac{8A+8B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\frac{\frac{35A}{2}+12B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{2\left(\frac{35A}{2}+\frac{33B}{2}\right)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{\frac{49A}{2}+\frac{43B}{2}}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{\frac{51A}{16}+\frac{21B}{16}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{2\left(\frac{49A}{2}\right.}{5\left(\tan\left(\frac{fx}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x)
```

```
[Out] 2/f/a^3/c^5*(-1/9*(2*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^9-1/8*(8*A+8*B)/(tan(1/2
*f*x+1/2*e)-1)^8-1/4*(35/2*A+12*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/7*(35/2*A+33/
2*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/6*(49/2*A+43/2*B)/(tan(1/2*f*x+1/2*e)-1)^6-
1/2*(51/16*A+21/16*B)/(tan(1/2*f*x+1/2*e)-1)^2-1/5*(49/2*A+77/4*B)/(tan(1/2
*f*x+1/2*e)-1)^5-(99/128*A+15/128*B)/(tan(1/2*f*x+1/2*e)-1)-1/3*(147/16*A+8
1/16*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/2*(-9/32*A+7/32*B)/(tan(1/2*f*x+1/2*e)+1
)^2-1/4*(-1/4*A+1/4*B)/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(1/8*A-1/8*B)/(tan(1/2*
f*x+1/2*e)+1)^5-1/3*(13/32*A-11/32*B)/(tan(1/2*f*x+1/2*e)+1)^3-(29/128*A-15
/128*B)/(tan(1/2*f*x+1/2*e)+1))
```

maxima [B] time = 0.57, size = 1201, normalized size = 7.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorit
hm="maxima")
```

```
[Out] -2/315*(B*(100*sin(f*x + e)/(cos(f*x + e) + 1) - 340*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 55*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 - 88*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1608*sin(f*
x + e)^6/(cos(f*x + e) + 1)^6 + 1032*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 -
```

```
483*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 588*sin(f*x + e)^9/(cos(f*x + e)
+ 1)^9 - 420*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 420*sin(f*x + e)^11/(c
os(f*x + e) + 1)^11 - 315*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 25)/(a^3*
c^5 - 4*a^3*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + a^3*c^5*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 16*a^3*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 19*a^
3*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 20*a^3*c^5*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5 + 45*a^3*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 45*a^3*c
^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 20*a^3*c^5*sin(f*x + e)^9/(cos(f*x
 + e) + 1)^9 + 19*a^3*c^5*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 16*a^3*c^
5*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - a^3*c^5*sin(f*x + e)^12/(cos(f*x
+ e) + 1)^12 + 4*a^3*c^5*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - a^3*c^5*si
n(f*x + e)^14/(cos(f*x + e) + 1)^14) - 7*A*(5*sin(f*x + e)/(cos(f*x + e) +
1) - 80*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 190*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 50*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 269*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5 + 96*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 516*sin(f*x +
 e)^7/(cos(f*x + e) + 1)^7 - 354*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 69*s
in(f*x + e)^9/(cos(f*x + e) + 1)^9 + 240*sin(f*x + e)^10/(cos(f*x + e) + 1)
^10 + 30*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 90*sin(f*x + e)^12/(cos(f*
x + e) + 1)^12 + 45*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 10)/(a^3*c^5 -
4*a^3*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + a^3*c^5*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 16*a^3*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 19*a^3*c^5*
sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 20*a^3*c^5*sin(f*x + e)^5/(cos(f*x +
e) + 1)^5 + 45*a^3*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 45*a^3*c^5*sin
(f*x + e)^8/(cos(f*x + e) + 1)^8 + 20*a^3*c^5*sin(f*x + e)^9/(cos(f*x + e)
+ 1)^9 + 19*a^3*c^5*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 16*a^3*c^5*sin(
f*x + e)^11/(cos(f*x + e) + 1)^11 - a^3*c^5*sin(f*x + e)^12/(cos(f*x + e) +
 1)^12 + 4*a^3*c^5*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - a^3*c^5*sin(f*x
+ e)^14/(cos(f*x + e) + 1)^14))/f
```

mupad [B] time = 13.14, size = 231, normalized size = 1.43

$$\frac{\left(\frac{128\,B}{315}-\frac{64\,A}{45}+\frac{32\,A\,\sin\left(e+f\,x\right)}{45}-\frac{64\,B\,\sin\left(e+f\,x\right)}{315}\right)\,\cos\left(e+f\,x\right)^6+\left(\frac{8\,A\,\sin\left(e+f\,x\right)}{9}-\frac{20\,B}{63}-\frac{8\,A}{9}+\frac{20\,B\,\sin\left(e+f\,x\right)}{63}-\frac{\left(4\,\sin\left(e+f\right.\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^5),x)
```

```
[Out] ((4*A)/45 - (14*B)/45 - (14*A*sin(e + f*x))/45 + (4*B*sin(e + f*x))/45 - co
s(e + f*x)^5*((8*A)/9 + (20*B)/63 - (8*A*sin(e + f*x))/9 - (20*B*sin(e + f*
x))/63 + ((4*sin(e + f*x) - 4)*((4*A)/9 + (10*B)/63))/2) + cos(e + f*x)^2*(
(8*A)/45 - (16*B)/315 - (4*A*sin(e + f*x))/9 + (8*B*sin(e + f*x))/63) + cos
(e + f*x)^4*((32*A)/45 - (64*B)/315 - (16*A*sin(e + f*x))/15 + (32*B*sin(e
```

```
+ f*x))/105) - cos(e + f*x)^6*((64*A)/45 - (128*B)/315 - (32*A*sin(e + f*x)
)/45 + (64*B*sin(e + f*x))/315))/(a^3*c^5*f*(4*cos(e + f*x)^5*sin(e + f*x)
- 4*cos(e + f*x)^5 + 2*cos(e + f*x)^7))
```

sympy [A] time = 155.47, size = 8396, normalized size = 51.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-630*A*tan(e/2 + f*x/2)**13/(315*a**3*c**5*f*tan(e/2 + f*x/2)**1
4 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2
)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 +
f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e
/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*t
an(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*
f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**
5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) + 1260*A*tan(e/2 + f*x/2)**12/(315*
a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 +
315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**1
1 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/
2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 +
 f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/
2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(
e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) - 42
0*A*tan(e/2 + f*x/2)**11/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*
c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a
**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6
300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8
 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2
)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*
x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 +
f*x/2) - 315*a**3*c**5*f) - 3360*A*tan(e/2 + f*x/2)**10/(315*a**3*c**5*f*ta
n(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*
f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*
c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*
a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6
300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4
- 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**
2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) + 966*A*tan(e/2 +
f*x/2)**9/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2
+ f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(
e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f
*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c
```

5*f*tan(e/2 + f*x/2)6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**
3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a
3*c5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a*
*3*c**5*f) + 4956*A*tan(e/2 + f*x/2)**8/(315*a**3*c**5*f*tan(e/2 + f*x/2)**
14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/
2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 +
f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(
e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*
tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5
*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c*
*5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) - 7224*A*tan(e/2 + f*x/2)**7/(315*
a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 +
315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**1
1 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/
2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 +
f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/
2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(
e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) - 13
44*A*tan(e/2 + f*x/2)**6/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*
c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a
3*c5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6
300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8
- 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2
)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*
x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 +
f*x/2) - 315*a**3*c**5*f) + 3766*A*tan(e/2 + f*x/2)**5/(315*a**3*c**5*f*tan
(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f
*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c
5*f*tan(e/2 + f*x/2)10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a
3*c5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 63
00*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 -
5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2
+ 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) - 700*A*tan(e/2 + f
*x/2)**4/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 +
f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e
/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*
tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c*
*5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3
*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a*
*3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**
3*c**5*f) - 2660*A*tan(e/2 + f*x/2)**3/(315*a**3*c**5*f*tan(e/2 + f*x/2)**1
4 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2
)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 +
f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e
/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*t

```
an(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*
f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**
5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) + 1120*A*tan(e/2 + f*x/2)**2/(315*a
**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 3
15*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11
 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2
)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 +
f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2
 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e
/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) - 70*
A*tan(e/2 + f*x/2)/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f
*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c*
*5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a*
*3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 141
75*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 +
 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**
3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2)
 - 315*a**3*c**5*f) - 140*A/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a*
*3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 504
0*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10
- 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)
**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*
x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 +
 f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2
 + f*x/2) - 315*a**3*c**5*f) - 630*B*tan(e/2 + f*x/2)**12/(315*a**3*c**5*f*
tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**
5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**
3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 1417
5*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 +
 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**
4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)
**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) + 840*B*tan(e/2
+ f*x/2)**11/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e
/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*t
an(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**
5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**
3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*
a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 31
5*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315
*a**3*c**5*f) - 840*B*tan(e/2 + f*x/2)**10/(315*a**3*c**5*f*tan(e/2 + f*x/2
)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f
*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/
2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*t
an(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5
*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c
```

```
**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3
*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) - 1176*B*tan(e/2 + f*x/2)**9/(3
15*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13
 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)
**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f
*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/
2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan
(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*t
an(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) -
 966*B*tan(e/2 + f*x/2)**8/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**
3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040
*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 -
 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)*
*8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x
/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 +
f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2
+ f*x/2) - 315*a**3*c**5*f) + 2064*B*tan(e/2 + f*x/2)**7/(315*a**3*c**5*f*t
an(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5
*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3
*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175
*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 +
6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4
 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)*
*2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) - 3216*B*tan(e/2
+ f*x/2)**6/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/
2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*ta
n(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5
*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3
*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a
**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315
*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*
a**3*c**5*f) - 176*B*tan(e/2 + f*x/2)**5/(315*a**3*c**5*f*tan(e/2 + f*x/2)*
*14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x
/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2
+ f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan
(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f
*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**
5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c
**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) + 110*B*tan(e/2 + f*x/2)**4/(315*
a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 +
315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**1
1 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/
2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 +
 f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/
2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(
```

```
e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) + 40
*B*tan(e/2 + f*x/2)**3/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c*
*5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**
3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 630
0*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 -
 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)*
*5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/
2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*
x/2) - 315*a**3*c**5*f) - 680*B*tan(e/2 + f*x/2)**2/(315*a**3*c**5*f*tan(e/
2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*ta
n(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5
*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3
*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*
a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 50
40*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 +
1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) + 200*B*tan(e/2 + f*x/
2)/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2
)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f
*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/
2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*t
an(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*
f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**
5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5
*f) - 50*B/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2
 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan
(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*
f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*
c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a*
*3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*
a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a
**3*c**5*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)**3*(-c*sin(e) + c
)**5), True))
```

3.80 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx$

Optimal. Leaf size=205

$$\frac{2(8A-3B)\tan^5(e+fx)}{165a^3c^6f}+\frac{4(8A-3B)\tan^3(e+fx)}{99a^3c^6f}+\frac{2(8A-3B)\tan(e+fx)}{33a^3c^6f}+\frac{(8A-3B)\sec^5(e+fx)}{99a^3f\left(c^6-c^6\sin(e+fx)\right)}+\frac{(8.}{99a^3}$$

```
[Out] 1/11*(A+B)*sec(f*x+e)^5/a^3/f/(c^2-c^2*sin(f*x+e))^3+1/99*(8*A-3*B)*sec(f*x
+e)^5/a^3/f/(c^3-c^3*sin(f*x+e))^2+1/99*(8*A-3*B)*sec(f*x+e)^5/a^3/f/(c^6-c
^6*sin(f*x+e))+2/33*(8*A-3*B)*tan(f*x+e)/a^3/c^6/f+4/99*(8*A-3*B)*tan(f*x+e
)^3/a^3/c^6/f+2/165*(8*A-3*B)*tan(f*x+e)^5/a^3/c^6/f
```

Rubi [A] time = 0.35, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 2672, 3767}

$$\frac{2(8A-3B)\tan^5(e+fx)}{165a^3c^6f}+\frac{4(8A-3B)\tan^3(e+fx)}{99a^3c^6f}+\frac{2(8A-3B)\tan(e+fx)}{33a^3c^6f}+\frac{(8A-3B)\sec^5(e+fx)}{99a^3f\left(c^6-c^6\sin(e+fx)\right)}+\frac{(8.}{99a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^6),x]
```

```
[Out] ((A + B)*Sec[e + f*x]^5)/(11*a^3*f*(c^2 - c^2*Sin[e + f*x])^3) + ((8*A - 3*
B)*Sec[e + f*x]^5)/(99*a^3*f*(c^3 - c^3*Sin[e + f*x])^2) + ((8*A - 3*B)*Sec
[e + f*x]^5)/(99*a^3*f*(c^6 - c^6*Sin[e + f*x])) + (2*(8*A - 3*B)*Tan[e + f
*x])/(33*a^3*c^6*f) + (4*(8*A - 3*B)*Tan[e + f*x]^3)/(99*a^3*c^6*f) + (2*(8
*A - 3*B)*Tan[e + f*x]^5)/(165*a^3*c^6*f)
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
```

```
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx = \frac{\int \frac{\sec^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx}{a^3c^3}$$

$$= \frac{(A+B)\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3} + \frac{(8A-3B)\int \frac{\sec^6(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{11a^3c^4}$$

$$= \frac{(A+B)\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3} + \frac{(8A-3B)\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2} + \ldots$$

$$= \frac{(A+B)\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3} + \frac{(8A-3B)\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2} + \ldots$$

$$= \frac{(A+B)\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3} + \frac{(8A-3B)\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2} + \ldots$$

$$= \frac{(A+B)\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3} + \frac{(8A-3B)\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2} + \ldots$$

Mathematica [A] time = 3.55, size = 401, normalized size = 1.96

$$\frac{-3850(107A-3B)\cos(e+fx)+135168(8A-3B)\cos(2(e+fx))+1802240A\sin(e+fx)+247170A\sin(2(e+\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^6),x]
```

```
[Out] (1013760*B - 3850*(107*A - 3*B)*Cos[e + f*x] + 135168*(8*A - 3*B)*Cos[2*(e
+ f*x)] - 127330*A*Cos[3*(e + f*x)] + 3570*B*Cos[3*(e + f*x)] + 819200*A*Co
s[4*(e + f*x)] - 307200*B*Cos[4*(e + f*x)] + 37450*A*Cos[5*(e + f*x)] - 105
0*B*Cos[5*(e + f*x)] + 163840*A*Cos[6*(e + f*x)] - 61440*B*Cos[6*(e + f*x)]
 + 22470*A*Cos[7*(e + f*x)] - 630*B*Cos[7*(e + f*x)] - 16384*A*Cos[8*(e + f
*x)] + 6144*B*Cos[8*(e + f*x)] + 1802240*A*Sin[e + f*x] - 675840*B*Sin[e +
f*x] + 247170*A*Sin[2*(e + f*x)] - 6930*B*Sin[2*(e + f*x)] + 557056*A*Sin[3
*(e + f*x)] - 208896*B*Sin[3*(e + f*x)] + 187250*A*Sin[4*(e + f*x)] - 5250*
B*Sin[4*(e + f*x)] - 163840*A*Sin[5*(e + f*x)] + 61440*B*Sin[5*(e + f*x)] +
 37450*A*Sin[6*(e + f*x)] - 1050*B*Sin[6*(e + f*x)] - 98304*A*Sin[7*(e + f*
x)] + 36864*B*Sin[7*(e + f*x)] - 3745*A*Sin[8*(e + f*x)] + 105*B*Sin[8*(e +
 f*x)])/(8110080*a^3*c^6*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^11*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^5)
```

fricas [A] time = 0.46, size = 221, normalized size = 1.08

$$\frac{16\,(8\,A-3\,B)\cos\left(fx+e\right)^{8}-72\,(8\,A-3\,B)\cos\left(fx+e\right)^{6}+30\,(8\,A-3\,B)\cos\left(fx+e\right)^{4}+7\,(8\,A-3\,B)\cos\left(fx\cdots\right.}{495\left(3\,a^{3}c^{6}f\cos\left(fx+e\right)^{7}-4\,a^{3}c^{6}f\cos\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorit
hm="fricas")
```

```
[Out] 1/495*(16*(8*A - 3*B)*cos(f*x + e)^8 - 72*(8*A - 3*B)*cos(f*x + e)^6 + 30*(
8*A - 3*B)*cos(f*x + e)^4 + 7*(8*A - 3*B)*cos(f*x + e)^2 + (48*(8*A - 3*B)*
cos(f*x + e)^6 - 40*(8*A - 3*B)*cos(f*x + e)^4 - 14*(8*A - 3*B)*cos(f*x + e
)^2 - 72*A + 27*B)*sin(f*x + e) + 27*A - 72*B)/(3*a^3*c^6*f*cos(f*x + e)^7
- 4*a^3*c^6*f*cos(f*x + e)^5 - (a^3*c^6*f*cos(f*x + e)^7 - 4*a^3*c^6*f*cos(
f*x + e)^5)*sin(f*x + e))
```

giac [B] time = 0.30, size = 475, normalized size = 2.32

$$-\frac{\frac{33\left(555\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-315\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+1920\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-1020\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+2710\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1410\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right.}{a^{3}c^{6}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{5}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorit
hm="giac")
```

```
[Out] -1/63360*(33*(555*A*tan(1/2*f*x + 1/2*e)^4 - 315*B*tan(1/2*f*x + 1/2*e)^4 +
 1920*A*tan(1/2*f*x + 1/2*e)^3 - 1020*B*tan(1/2*f*x + 1/2*e)^3 + 2710*A*tan
(1/2*f*x + 1/2*e)^2 - 1410*B*tan(1/2*f*x + 1/2*e)^2 + 1760*A*tan(1/2*f*x +
1/2*e) - 900*B*tan(1/2*f*x + 1/2*e) + 463*A - 243*B)/(a^3*c^6*(tan(1/2*f*x
+ 1/2*e) + 1)^5) + (108405*A*tan(1/2*f*x + 1/2*e)^10 + 10395*B*tan(1/2*f*x
+ 1/2*e)^10 - 784080*A*tan(1/2*f*x + 1/2*e)^9 - 5940*B*tan(1/2*f*x + 1/2*e)
^9 + 2901195*A*tan(1/2*f*x + 1/2*e)^8 - 79695*B*tan(1/2*f*x + 1/2*e)^8 - 66
52800*A*tan(1/2*f*x + 1/2*e)^7 + 388080*B*tan(1/2*f*x + 1/2*e)^7 + 10407474
*A*tan(1/2*f*x + 1/2*e)^6 - 816354*B*tan(1/2*f*x + 1/2*e)^6 - 11435424*A*ta
n(1/2*f*x + 1/2*e)^5 + 1114344*B*tan(1/2*f*x + 1/2*e)^5 + 8949270*A*tan(1/2
*f*x + 1/2*e)^4 - 990990*B*tan(1/2*f*x + 1/2*e)^4 - 4899840*A*tan(1/2*f*x +
 1/2*e)^3 + 609840*B*tan(1/2*f*x + 1/2*e)^3 + 1816265*A*tan(1/2*f*x + 1/2*e
)^2 - 235785*B*tan(1/2*f*x + 1/2*e)^2 - 411664*A*tan(1/2*f*x + 1/2*e) + 563
64*B*tan(1/2*f*x + 1/2*e) + 47279*A - 4179*B)/(a^3*c^6*(tan(1/2*f*x + 1/2*e
) - 1)^11))/f
```

maple [A] time = 0.51, size = 365, normalized size = 1.78

$$\frac{-\frac{2(4A+4B)}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}}-\frac{20A+20B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}}-\frac{2(53A+51B)}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}}-\frac{92A+84B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\frac{\frac{169A}{4}+\frac{99B}{4}}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{\frac{217A}{2}+84B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{2\left(\frac{219}{25}\right.}{\tan\left(\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x)
```

```
[Out] 2/f/a^3/c^6*(-1/11*(4*A+4*B)/(tan(1/2*f*x+1/2*e)-1)^11-1/10*(20*A+20*B)/(ta
n(1/2*f*x+1/2*e)-1)^10-1/9*(53*A+51*B)/(tan(1/2*f*x+1/2*e)-1)^9-1/8*(92*A+8
4*B)/(tan(1/2*f*x+1/2*e)-1)^8-1/4*(169/4*A+99/4*B)/(tan(1/2*f*x+1/2*e)-1)^4
-1/6*(217/2*A+84*B)/(tan(1/2*f*x+1/2*e)-1)^6-(219/256*A+21/256*B)/(tan(1/2*
f*x+1/2*e)-1)-1/7*(231/2*A+98*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/2*(303/64*A+99/
64*B)/(tan(1/2*f*x+1/2*e)-1)^2-1/5*(623/8*A+427/8*B)/(tan(1/2*f*x+1/2*e)-1)
```

```
^5-1/3*(1095/64*A+507/64*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/2*(-5/32*A+1/8*B)/(t
an(1/2*f*x+1/2*e)+1)^2-1/4*(-1/8*A+1/8*B)/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(1/1
6*A-1/16*B)/(tan(1/2*f*x+1/2*e)+1)^5-1/3*(7/32*A-3/16*B)/(tan(1/2*f*x+1/2*e
)+1)^3-(37/256*A-21/256*B)/(tan(1/2*f*x+1/2*e)+1))
```

maxima [B] time = 0.58, size = 1387, normalized size = 6.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorit
hm="maxima")
```

```
[Out] -2/495*(A*(255*sin(f*x + e)/(cos(f*x + e) + 1) + 235*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 - 3065*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3775*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + 667*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 8217*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 + 2035*sin(f*x + e)^7/(cos(f*x + e) + 1)
^7 + 8745*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 11715*sin(f*x + e)^9/(cos(f
*x + e) + 1)^9 + 33*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 4917*sin(f*x +
e)^11/(cos(f*x + e) + 1)^11 - 2475*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 -
1815*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 1485*sin(f*x + e)^14/(cos(f*x
+ e) + 1)^14 - 495*sin(f*x + e)^15/(cos(f*x + e) + 1)^15 - 125)/(a^3*c^6 -
6*a^3*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*c^6*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 10*a^3*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 50*a^3*c
^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 34*a^3*c^6*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5 + 66*a^3*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 110*a^3*c^6
*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 110*a^3*c^6*sin(f*x + e)^9/(cos(f*x
+ e) + 1)^9 - 66*a^3*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 34*a^3*c^6
*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 50*a^3*c^6*sin(f*x + e)^12/(cos(f*
x + e) + 1)^12 - 10*a^3*c^6*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - 10*a^3*
c^6*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 + 6*a^3*c^6*sin(f*x + e)^15/(cos(
f*x + e) + 1)^15 - a^3*c^6*sin(f*x + e)^16/(cos(f*x + e) + 1)^16) + 3*B*(30
*sin(f*x + e)/(cos(f*x + e) + 1) - 215*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
+ 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 245*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 - 434*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 231*sin(f*x + e)^6/(co
s(f*x + e) + 1)^6 + 880*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 1815*sin(f*x
+ e)^8/(cos(f*x + e) + 1)^8 + 330*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 99*
sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 264*sin(f*x + e)^11/(cos(f*x + e) +
 1)^11 - 495*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 330*sin(f*x + e)^13/(c
os(f*x + e) + 1)^13 - 165*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 - 5)/(a^3*c
^6 - 6*a^3*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*c^6*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 10*a^3*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 50*
a^3*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 34*a^3*c^6*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5 + 66*a^3*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 110*a^
3*c^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 110*a^3*c^6*sin(f*x + e)^9/(cos
```

```
(f*x + e) + 1)^9 - 66*a^3*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 34*a^
3*c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 50*a^3*c^6*sin(f*x + e)^12/(c
os(f*x + e) + 1)^12 - 10*a^3*c^6*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - 10
*a^3*c^6*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 + 6*a^3*c^6*sin(f*x + e)^15/
(cos(f*x + e) + 1)^15 - a^3*c^6*sin(f*x + e)^16/(cos(f*x + e) + 1)^16))/f
```

mupad [B] time = 14.52, size = 474, normalized size = 2.31

$$2\left(\frac{165\,B\,\sin\left(e+f\,x\right)}{4}-\frac{6875\,A\,\cos\left(e+f\,x\right)}{64}-\frac{825\,B\,\cos\left(e+f\,x\right)}{64}-110\,A\,\sin\left(e+f\,x\right)-\frac{495\,B}{8}-66\,A\,\cos\left(2\,e+2\,f\,x\right)-\frac{2125\,A}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^6),x)
```

```
[Out] (2*((165*B*sin(e + f*x))/4 - (6875*A*cos(e + f*x))/64 - (825*B*cos(e + f*x)
)/64 - 110*A*sin(e + f*x) - (495*B)/8 - 66*A*cos(2*e + 2*f*x) - (2125*A*cos
(3*e + 3*f*x))/64 - 50*A*cos(4*e + 4*f*x) + (625*A*cos(5*e + 5*f*x))/64 - 1
0*A*cos(6*e + 6*f*x) + (375*A*cos(7*e + 7*f*x))/64 + A*cos(8*e + 8*f*x) + (
99*B*cos(2*e + 2*f*x))/4 - (255*B*cos(3*e + 3*f*x))/64 + (75*B*cos(4*e + 4*
f*x))/4 + (75*B*cos(5*e + 5*f*x))/64 + (15*B*cos(6*e + 6*f*x))/4 + (45*B*co
s(7*e + 7*f*x))/64 - (3*B*cos(8*e + 8*f*x))/8 + (4125*A*sin(2*e + 2*f*x))/6
4 - 34*A*sin(3*e + 3*f*x) + (3125*A*sin(4*e + 4*f*x))/64 + 10*A*sin(5*e + 5
*f*x) + (625*A*sin(6*e + 6*f*x))/64 + 6*A*sin(7*e + 7*f*x) - (125*A*sin(8*e
 + 8*f*x))/128 + (495*B*sin(2*e + 2*f*x))/64 + (51*B*sin(3*e + 3*f*x))/4 +
(375*B*sin(4*e + 4*f*x))/64 - (15*B*sin(5*e + 5*f*x))/4 + (75*B*sin(6*e + 6
*f*x))/64 - (9*B*sin(7*e + 7*f*x))/4 - (15*B*sin(8*e + 8*f*x))/128))/(495*a
^3*c^6*f*((5*cos(5*e + 5*f*x))/32 - (17*cos(3*e + 3*f*x))/32 - (55*cos(e +
f*x))/32 + (3*cos(7*e + 7*f*x))/32 + (33*sin(2*e + 2*f*x))/32 + (25*sin(4*e
 + 4*f*x))/32 + (5*sin(6*e + 6*f*x))/32 - sin(8*e + 8*f*x)/64))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**6,x)
```

```
[Out] Timed out
```

3.81 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=198

$$\frac{256ac^5(11A-5B)\cos^3(e+fx)}{3465f(c-c\sin(e+fx))^{3/2}}+\frac{64ac^4(11A-5B)\cos^3(e+fx)}{1155f\sqrt{c-c\sin(e+fx)}}+\frac{8ac^3(11A-5B)\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{231f}$$

```
[Out] 256/3465*a*(11*A-5*B)*c^5*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)+2/99*a*(11*
A-5*B)*c^2*cos(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/f-2/11*a*B*c*cos(f*x+e)^3*(c
-c*sin(f*x+e))^(5/2)/f+64/1155*a*(11*A-5*B)*c^4*cos(f*x+e)^3/f/(c-c*sin(f*x
+e))^(1/2)+8/231*a*(11*A-5*B)*c^3*cos(f*x+e)^3*(c-c*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.49, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2856, 2674, 2673}

$$\frac{256ac^5(11A-5B)\cos^3(e+fx)}{3465f(c-c\sin(e+fx))^{3/2}}+\frac{64ac^4(11A-5B)\cos^3(e+fx)}{1155f\sqrt{c-c\sin(e+fx)}}+\frac{8ac^3(11A-5B)\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{231f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (256*a*(11*A - 5*B)*c^5*Cos[e + f*x]^3)/(3465*f*(c - c*Sin[e + f*x])^(3/2))
 + (64*a*(11*A - 5*B)*c^4*Cos[e + f*x]^3)/(1155*f*Sqrt[c - c*Sin[e + f*x]])
 + (8*a*(11*A - 5*B)*c^3*Cos[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(231*f) +
 (2*a*(11*A - 5*B)*c^2*Cos[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(99*f) -
(2*a*B*c*Cos[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(11*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
```

```
[m + p, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = (ac)\int \cos^2(e+fx)(A+B\sin(e+fx))(c-c$$

$$= -\frac{2aBc\cos^3(e+fx)(c-c\sin(e+fx))^{5/2}}{11f} + \frac{}{1}$$

$$= \frac{2a(11A-5B)c^2\cos^3(e+fx)(c-c\sin(e+f}{99f}$$

$$= \frac{8a(11A-5B)c^3\cos^3(e+fx)\sqrt{c-c\sin(e+}}{231f}$$

$$= \frac{64a(11A-5B)c^4\cos^3(e+fx)}{1155f\sqrt{c-c\sin(e+fx)}} + \frac{8a(11A-5}{}$$

$$= \frac{256a(11A-5B)c^5\cos^3(e+fx)}{3465f(c-c\sin(e+fx))^{3/2}} + \frac{64a(11A-}{1155f\sqrt{}}$$

Mathematica [A] time = 2.99, size = 149, normalized size = 0.75

$$-\frac{ac^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3(60(121A-202B)\cos(2(e+fx))+30558A\sin(e+}{13860f\left(\cos\left(\frac{1}{2}(e\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] -1/13860*(a*c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e +
f*x]]*(-35332*A + 27085*B + 60*(121*A - 202*B)*Cos[2*(e + f*x)] + 315*B*Cos
[4*(e + f*x)] + 30558*A*Sin[e + f*x] - 31530*B*Sin[e + f*x] - 770*A*Sin[3*(
e + f*x)] + 2870*B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2]))
```

fricas [A] time = 0.44, size = 287, normalized size = 1.45

$$\frac{2\left(315\,Bac^3\cos\left(fx+e\right)^6-35\,(11\,A-32\,B)ac^3\cos\left(fx+e\right)^5+5\,(209\,A-221\,B)ac^3\cos\left(fx+e\right)^4+2\,(1243\,A-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 2/3465*(315*B*a*c^3*cos(f*x + e)^6 - 35*(11*A - 32*B)*a*c^3*cos(f*x + e)^5
+ 5*(209*A - 221*B)*a*c^3*cos(f*x + e)^4 + 2*(1243*A - 1195*B)*a*c^3*cos(f*
x + e)^3 - 32*(11*A - 5*B)*a*c^3*cos(f*x + e)^2 + 128*(11*A - 5*B)*a*c^3*co
s(f*x + e) + 256*(11*A - 5*B)*a*c^3 - (315*B*a*c^3*cos(f*x + e)^5 + 35*(11*
A - 23*B)*a*c^3*cos(f*x + e)^4 + 10*(143*A - 191*B)*a*c^3*cos(f*x + e)^3 -
96*(11*A - 5*B)*a*c^3*cos(f*x + e)^2 - 128*(11*A - 5*B)*a*c^3*cos(f*x + e)
- 256*(11*A - 5*B)*a*c^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*
x + e) - f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*c)*(-112*f*(2*A*a*c^3*sign(sin(
1/2*(f*x+exp(1))-1/4*pi))-4*B*a*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*cos
(1/4*(14*f*x+14*exp(1)+pi))/(112*f)^2+48*f*(8*A*a*c^3*sign(sin(1/2*(f*x+exp
(1))-1/4*pi))-6*B*a*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(6*f*x+
6*exp(1)-pi))/(48*f)^2-8*f*(10*A*a*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))-4
*B*a*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(2*f*x-pi)+1/2*exp(1))
/(8*f)^2+224*f*(16*A*a*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))-10*B*a*c^3*si
gn(sin(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(14*f*x+14*exp(1)-pi))/(224*f)^2-
144*f*(-2*A*a*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))+4*B*a*c^3*sign(sin(1/2
*(f*x+exp(1))-1/4*pi)))*cos(1/4*(18*f*x+18*exp(1)-pi))/(144*f)^2+16*f*(-8*A
*a*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))+6*B*a*c^3*sign(sin(1/2*(f*x+exp(1
))-1/4*pi)))*sin(1/4*(2*f*x+2*exp(1)+pi))/(16*f)^2+160*f*(-16*A*a*c^3*sign(
sin(1/2*(f*x+exp(1))-1/4*pi))+10*B*a*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))
)*sin(1/4*(10*f*x+10*exp(1)+pi))/(160*f)^2+24*A*a*c^3*f*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*cos(1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2-40*A*a*c^3*f*sign(sin
(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(10*f*x+10*exp(1)-pi))/(20*f)^2-576*B*a*
c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(18*f*x+18*exp(1)+pi))/(28
8*f)^2+704*B*a*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(22*f*x+22*
exp(1)-pi))/(352*f)^2)
```

maple [A] time = 1.30, size = 119, normalized size = 0.60

$$\frac{2\left(\sin\left(fx+e\right)-1\right)c^{4}\left(1+\sin\left(fx+e\right)\right)^{2}a\left(\left(-385A+1435B\right)\sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)+\left(3916A-4300B\right)\right.}{3465\cos\left(fx+e\right)\sqrt{c-c\sin\left(\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 2/3465*(sin(f*x+e)-1)*c^4*(1+sin(f*x+e))^2*a*((-385*A+1435*B)*sin(f*x+e)*co
s(f*x+e)^2+(3916*A-4300*B)*sin(f*x+e)+315*B*cos(f*x+e)^4+(1815*A-3345*B)*co
s(f*x+e)^2-5324*A+4940*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(
7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.82 $\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^{5/2}\,dx$

Optimal. Leaf size=157

$$\frac{64ac^4(3A-B)\cos^3(e+fx)}{315f(c-c\sin(e+fx))^{3/2}}+\frac{16ac^3(3A-B)\cos^3(e+fx)}{105f\sqrt{c-c\sin(e+fx)}}+\frac{2ac^2(3A-B)\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{21f}-\frac{2aBc}{}$$

```
[Out] 64/315*a*(3*A-B)*c^4*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)-2/9*a*B*c*cos(f*
x+e)^3*(c-c*sin(f*x+e))^(3/2)/f+16/105*a*(3*A-B)*c^3*cos(f*x+e)^3/f/(c-c*si
n(f*x+e))^(1/2)+2/21*a*(3*A-B)*c^2*cos(f*x+e)^3*(c-c*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.41, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2856, 2674, 2673}

$$\frac{64ac^4(3A-B)\cos^3(e+fx)}{315f(c-c\sin(e+fx))^{3/2}}+\frac{16ac^3(3A-B)\cos^3(e+fx)}{105f\sqrt{c-c\sin(e+fx)}}+\frac{2ac^2(3A-B)\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{21f}-\frac{2aBc}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (64*a*(3*A - B)*c^4*Cos[e + f*x]^3)/(315*f*(c - c*Sin[e + f*x])^(3/2)) + (1
6*a*(3*A - B)*c^3*Cos[e + f*x]^3)/(105*f*Sqrt[c - c*Sin[e + f*x]]) + (2*a*(
3*A - B)*c^2*Cos[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(21*f) - (2*a*B*c*Cos
[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(9*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = (ac)\int \cos^2(e+fx)(A+B\sin(e+fx))(c-c\,\mathrm{s}$$

$$= -\frac{2aBc\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{9f}+\frac{1}{3}($$

$$= \frac{2a(3A-B)c^2\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{21f}$$

$$= \frac{16a(3A-B)c^3\cos^3(e+fx)}{105f\sqrt{c-c\sin(e+fx)}}+\frac{2a(3A-B)c^2\,\mathrm{c}}{}$$

$$= \frac{64a(3A-B)c^4\cos^3(e+fx)}{315f(c-c\sin(e+fx))^{3/2}}+\frac{16a(3A-B)c^3}{105f\sqrt{c-c\,\mathrm{s}}}$$

Mathematica [A] time = 1.52, size = 123, normalized size = 0.78

$$-\frac{ac^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3((648A-741B)\sin(e+fx)+30(3A-8B)\cos(2(e}{630f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] -1/630*(a*c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*
x]]*(-942*A + 664*B + 30*(3*A - 8*B)*Cos[2*(e + f*x)] + (648*A - 741*B)*Sin
[e + f*x] + 35*B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
))
```

fricas [A] time = 0.43, size = 243, normalized size = 1.55

$$\frac{2\left(35\,Bac^2\cos\left(fx+e\right)^5+5\,(9\,A-10\,B)ac^2\cos\left(fx+e\right)^4+(117\,A-109\,B)ac^2\cos\left(fx+e\right)^3-8\,(3\,A-B)ac^2\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 2/315*(35*B*a*c^2*cos(f*x + e)^5 + 5*(9*A - 10*B)*a*c^2*cos(f*x + e)^4 + (1
17*A - 109*B)*a*c^2*cos(f*x + e)^3 - 8*(3*A - B)*a*c^2*cos(f*x + e)^2 + 32*
(3*A - B)*a*c^2*cos(f*x + e) + 64*(3*A - B)*a*c^2 + (35*B*a*c^2*cos(f*x + e
)^4 - 5*(9*A - 17*B)*a*c^2*cos(f*x + e)^3 + 24*(3*A - B)*a*c^2*cos(f*x + e)
^2 + 32*(3*A - B)*a*c^2*cos(f*x + e) + 64*(3*A - B)*a*c^2)*sin(f*x + e))*sq
rt(-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)sqrt(2*c)*(24*f*(2*A*a*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))-2*
B*a*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(6*f*x+6*exp(1)-pi))/(2
4*f)^2+56*f*(2*A*a*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))-2*B*a*c^2*sign(si
```

```
n(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(14*f*x+14*exp(1)-pi))/(56*f)^2-8*f*(8
*A*a*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))-2*B*a*c^2*sign(sin(1/2*(f*x+exp
(1))-1/4*pi)))*cos(1/4*(2*f*x-pi)+1/2*exp(1))/(8*f)^2+8*f*(-2*A*a*c^2*sign(
sin(1/2*(f*x+exp(1))-1/4*pi))+2*B*a*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))
*sin(1/4*(2*f*x+2*exp(1)+pi))/(8*f)^2+40*f*(-2*A*a*c^2*sign(sin(1/2*(f*x+ex
p(1))-1/4*pi))+2*B*a*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(10*f*
x+10*exp(1)+pi))/(40*f)^2+24*A*a*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*c
os(1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2-40*A*a*c^2*f*sign(sin(1/2*(f*x+exp(1))
-1/4*pi))*cos(1/4*(10*f*x+10*exp(1)-pi))/(20*f)^2+224*B*a*c^2*f*sign(sin(1/
2*(f*x+exp(1))-1/4*pi))*cos(1/4*(14*f*x+14*exp(1)+pi))/(112*f)^2-288*B*a*c^
2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(18*f*x+18*exp(1)-pi))/(144*
f)^2)
```

maple [A] time = 1.23, size = 103, normalized size = 0.66

$$-\frac{2\left(\sin\left(fx+e\right)-1\right)c^{3}\left(1+\sin\left(fx+e\right)\right)^{2}a\left(-35B\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)+\left(-162A+194B\right)\sin\left(fx+e\right)+\cdots\right.}{315\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/315*(sin(f*x+e)-1)*c^3*(1+sin(f*x+e))^2*a*(-35*B*cos(f*x+e)^2*sin(f*x+e)
+(-162*A+194*B)*sin(f*x+e)+(-45*A+120*B)*cos(f*x+e)^2+258*A-226*B)/cos(f*x+
e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(
5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.83 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=116

$$\frac{8ac^3(7A-B)\cos^3(e+fx)}{105f(c-c\sin(e+fx))^{3/2}}+\frac{2ac^2(7A-B)\cos^3(e+fx)}{35f\sqrt{c-c\sin(e+fx)}}-\frac{2aBc\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}$$

```
[Out] 8/105*a*(7*A-B)*c^3*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)+2/35*a*(7*A-B)*c^
2*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(1/2)-2/7*a*B*c*cos(f*x+e)^3*(c-c*sin(f*x
+e))^(1/2)/f
```

Rubi [A] time = 0.32, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2856, 2674, 2673}

$$\frac{2ac^2(7A-B)\cos^3(e+fx)}{35f\sqrt{c-c\sin(e+fx)}}+\frac{8ac^3(7A-B)\cos^3(e+fx)}{105f(c-c\sin(e+fx))^{3/2}}-\frac{2aBc\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (8*a*(7*A - B)*c^3*Cos[e + f*x]^3)/(105*f*(c - c*Sin[e + f*x])^(3/2)) + (2*
a*(7*A - B)*c^2*Cos[e + f*x]^3)/(35*f*Sqrt[c - c*Sin[e + f*x]]) - (2*a*B*c*
Cos[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(7*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=(ac)\int \cos^2(e+fx)(A+B\sin(e+fx))\sqrt{c-}$$

$$=-\frac{2aBc\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}+\frac{1}{7}($$

$$=\frac{2a(7A-B)c^2\cos^3(e+fx)}{35f\sqrt{c-c\sin(e+fx)}}-\frac{2aBc\cos^3(e+}{}$$

$$=\frac{8a(7A-B)c^3\cos^3(e+fx)}{105f(c-c\sin(e+fx))^{3/2}}+\frac{2a(7A-B)c^2}{35f\sqrt{c-c\,\mathrm{si}}}$$

Mathematica [A] time = 0.96, size = 104, normalized size = 0.90

$$\frac{ac\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3((66B-42A)\sin(e+fx)+98A+15B\cos(2(e+fx}{105f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] (a*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(98*A - 59*B + 15*B*Cos[2*(e +
 f*x)] + (-42*A + 66*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(105*f*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

fricas [A] time = 0.42, size = 184, normalized size = 1.59

$$-\frac{2\left(15\,Bac\cos\left(fx+e\right)^{4}-3\,(7\,A-6\,B)ac\cos\left(fx+e\right)^{3}+(7\,A-B)ac\cos\left(fx+e\right)^{2}-4\,(7\,A-B)ac\cos\left(fx+e\right)\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -2/105*(15*B*a*c*cos(f*x + e)^4 - 3*(7*A - 6*B)*a*c*cos(f*x + e)^3 + (7*A -
 B)*a*c*cos(f*x + e)^2 - 4*(7*A - B)*a*c*cos(f*x + e) - 8*(7*A - B)*a*c - (
15*B*a*c*cos(f*x + e)^3 + 3*(7*A - B)*a*c*cos(f*x + e)^2 + 4*(7*A - B)*a*c*
cos(f*x + e) + 8*(7*A - B)*a*c)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*
cos(f*x + e) - f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)sqrt(2*c)*(24*A*a*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(6*f*x+
6*exp(1)+pi))/(12*f)^2-40*A*a*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/
4*(10*f*x+10*exp(1)-pi))/(20*f)^2-4*A*a*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*p
i))*cos(1/4*(2*f*x-pi)+1/2*exp(1))/(2*f)^2+16*B*a*c*f*sign(sin(1/2*(f*x+exp
(1))-1/4*pi))*sin(1/4*(2*f*x+2*exp(1)+pi))/(8*f)^2-48*B*a*c*f*sign(sin(1/2*
(f*x+exp(1))-1/4*pi))*sin(1/4*(6*f*x+6*exp(1)-pi))/(24*f)^2+80*B*a*c*f*sign
(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(10*f*x+10*exp(1)+pi))/(40*f)^2-112*
B*a*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(14*f*x+14*exp(1)-pi))/(
56*f)^2)
```

maple [A] time = 1.11, size = 81, normalized size = 0.70

$$\frac{2\left(\sin\left(fx+e\right)-1\right)c^{2}\left(1+\sin\left(fx+e\right)\right)^{2}a\left(\sin\left(fx+e\right)(21A-33B)-15B\left(\cos^{2}\left(fx+e\right)\right)-49A+37B\right)}{105\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/105*(sin(f*x+e)-1)*c^2*(1+sin(f*x+e))^2*a*(sin(f*x+e)*(21*A-33*B)-15*B*co
s(f*x+e)^2-49*A+37*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(
3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int Ac\sqrt{-c\sin\left(e+fx\right)+c}\,dx+\int\left(-Ac\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^{2}\left(e+fx\right)\right)dx+\int Bc\sqrt{-c\sin\left(e+fx\right)+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] a*(Integral(A*c*sqrt(-c*sin(e + f*x) + c), x) + Integral(-A*c*sqrt(-c*sin(e
 + f*x) + c)*sin(e + f*x)**2, x) + Integral(B*c*sqrt(-c*sin(e + f*x) + c)*s
in(e + f*x), x) + Integral(-B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**3,
x))
```

3.84 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\, d$

Optimal. Leaf size=73

$$\frac{2ac^2(5A+B)\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}}-\frac{2aBc\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 2/15*a*(5*A+B)*c^2*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)-2/5*a*B*c*cos(f*x+
e)^3/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2967, 2856, 2673}

$$\frac{2ac^2(5A+B)\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}}-\frac{2aBc\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a*(5*A + B)*c^2*Cos[e + f*x]^3)/(15*f*(c - c*Sin[e + f*x])^(3/2)) - (2*a
*B*c*Cos[e + f*x]^3)/(5*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
```

```
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= -\frac{2aBc\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}} + \frac{1}{5}(a(5A+B)c)\int -$$
$$= \frac{2a(5A+B)c^2\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}} - \frac{2aBc\cos^3(e+}{5f\sqrt{c-c\sin(e+}}$$

Mathematica [A] time = 0.42, size = 87, normalized size = 1.19

$$\frac{2a\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3(5A+3B\sin(e+fx)-2B)}{15f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] (2*a*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(5*A - 2*B + 3*B*Sin[e + f*x])
*Sqrt[c - c*Sin[e + f*x]])/(15*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

fricas [A] time = 0.42, size = 130, normalized size = 1.78

$$-\frac{2\left(3\,Ba\cos\left(fx+e\right)^3+(5\,A+4\,B)a\cos\left(fx+e\right)^2-(5\,A+B)a\cos\left(fx+e\right)-2\,(5\,A+B)a+\left(3\,Ba\cos\left(fx+\right.\right.\right.}{15\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] -2/15*(3*B*a*cos(f*x + e)^3 + (5*A + 4*B)*a*cos(f*x + e)^2 - (5*A + B)*a*co
s(f*x + e) - 2*(5*A + B)*a + (3*B*a*cos(f*x + e)^2 - (5*A + B)*a*cos(f*x +
```

```
e) - 2*(5*A + B)*a)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e)
 - f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*c)*(2*f*(2*A*a*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))+2*B*a*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*sin(
1/4*(2*f*x+2*exp(1)+pi))/(2*f)^2-2*f*(4*A*a*sign(sin(1/2*(f*x+exp(1))-1/4*p
i))+2*B*a*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(2*f*x-pi)+1/2*exp(1)
)/(2*f)^2+6*f*(-2*A*a*sign(sin(1/2*(f*x+exp(1))-1/4*pi))-2*B*a*sign(sin(1/2
*(f*x+exp(1))-1/4*pi)))*sin(1/4*(6*f*x+6*exp(1)-pi))/(6*f)^2-24*B*a*f*sign(
sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2+40*B*a*
f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(10*f*x+10*exp(1)-pi))/(20*f)^
2)
```

maple [A] time = 1.31, size = 63, normalized size = 0.86

$$-\frac{2\left(\sin\left(fx+e\right)-1\right)c\left(1+\sin\left(fx+e\right)\right)^{2}a\left(3B\sin\left(fx+e\right)+5A-2B\right)}{15\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/15*(sin(f*x+e)-1)*c*(1+sin(f*x+e))^2*a*(3*B*sin(f*x+e)+5*A-2*B)/cos(f*x+
e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sqrt{-c\sin\left(e+fx\right)+c}\,dx+\int A\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)dx+\int B\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a*(Integral(A*sqrt(-c*sin(e + f*x) + c), x) + Integral(A*sqrt(-c*sin(e + f*
x) + c)*sin(e + f*x), x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x
), x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x))
```

3.85 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=122

$$-\frac{2a(3A+5B)\cos(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}+\frac{2\sqrt{2}\,a(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f}+\frac{2aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3cf}$$

```
[Out] 2*a*(A+B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))*2^
(1/2)/f/c^(1/2)-2/3*a*(3*A+5*B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(1/2)+2/3*a*B
*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/c/f
```

Rubi [A] time = 0.34, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2858, 2751, 2649, 206}

$$-\frac{2a(3A+5B)\cos(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}+\frac{2\sqrt{2}\,a(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f}+\frac{2aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3cf}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*Sqrt[2]*a*(A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin
[e + f*x]])])/(Sqrt[c]*f) - (2*a*(3*A + 5*B)*Cos[e + f*x])/(3*f*Sqrt[c - c*
Sin[e + f*x]]) + (2*a*B*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*c*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2858

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(d*Cos[e + f*x]*(
a + b*Sin[e + f*x])^(m + 2))/(b^2*f*(m + 3)), x] - Dist[1/(b^2*(m + 3)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(b*d*(m + 2) - a*c*(m + 3) + (b*c*(m + 3) -
a*d*(m + 4))*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a
^2 - b^2, 0] && GeQ[m, -3/2] && LtQ[m, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$$

$$= \frac{2aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3cf} - \frac{(2a)\int \frac{-\frac{3Ac}{2}-\frac{Bc}{2}+\left(-\frac{3Ac}{2}-\frac{5Bc}{2}\right)s}{\sqrt{c-c\sin(e+fx)}}}{3c}$$

$$= -\frac{2a(3A+5B)\cos(e+fx)}{3f\sqrt{c-c\sin(e+fx)}} + \frac{2aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3cf}$$

$$= -\frac{2a(3A+5B)\cos(e+fx)}{3f\sqrt{c-c\sin(e+fx)}} + \frac{2aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3cf}$$

$$= \frac{2\sqrt{2}\,a(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f} - \frac{2a(3A+5B)\cos(e+}{3f\sqrt{c-c\sin(e+f}}$$

Mathematica [A] time = 1.26, size = 166, normalized size = 1.36

$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6\sqrt{2}\,(A+B)\sqrt{-c(\sin(e+fx)+1)}\,\tan^{-1}\left(\frac{\sqrt{-c(\sin(e+fx)+1)}}{\sqrt{2}\sqrt{c}}\right)+\sqrt{c}\,(2(3A+\right.}{3\sqrt{c}\,f\sqrt{c-c\sin(e+fx)}\,\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] -1/3*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(6*Sqrt[2]*(A + B)*ArcTan[Sqr
t[-(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*Sqrt[-(c*(1 + Sin[e + f*x]))]
 + Sqrt[c]*(6*A + 9*B - B*Cos[2*(e + f*x)] + 2*(3*A + 5*B)*Sin[e + f*x])))/
(Sqrt[c]*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]])
```

fricas [B] time = 0.44, size = 254, normalized size = 2.08

$$\frac{\frac{3\sqrt{2}\left((A+B)ac\cos(fx+e)-(A+B)ac\sin(fx+e)+(A+B)ac\right)\log\left(-\frac{\cos(fx+e)^2+(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\,(\cos(fx+e)+\sin(fx+e)+1)}{\sqrt{c}}+3\cos(fx\right.}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}}{\sqrt{c}}}{3\,(cf\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(3*sqrt(2)*((A + B)*a*c*cos(f*x + e) - (A + B)*a*c*sin(f*x + e) + (A +
B)*a*c)*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*
sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(c) + 3*cos
(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x
+ e) - 2))/sqrt(c) + 2*(B*a*cos(f*x + e)^2 - (3*A + 4*B)*a*cos(f*x + e) - (
3*A + 5*B)*a - (B*a*cos(f*x + e) + (3*A + 5*B)*a)*sin(f*x + e))*sqrt(-c*sin
(f*x + e) + c))/(c*f*cos(f*x + e) - c*f*sin(f*x + e) + c*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2/sqrt(c*t
an((f*x+exp(1))/2)^2+c)/(c*tan((f*x+exp(1))/2)^2+c)*(tan((f*x+exp(1))/2)*(t
an((f*x+exp(1))/2)*(-1/144*tan((f*x+exp(1))/2)*(-72*A*a*c*sign(tan((f*x+exp
(1))/2)-1)-96*B*a*c*sign(tan((f*x+exp(1))/2)-1))-1/144*(-72*A*a*c*sign(tan(
(f*x+exp(1))/2)-1)-144*B*a*c*sign(tan((f*x+exp(1))/2)-1)))-1/144*(-72*A*a*c
*sign(tan((f*x+exp(1))/2)-1)-144*B*a*c*sign(tan((f*x+exp(1))/2)-1)))-1/144*
(-72*A*a*c*sign(tan((f*x+exp(1))/2)-1)-96*B*a*c*sign(tan((f*x+exp(1))/2)-1)
))+sqrt(2)*(2*A*a+2*B*a)*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*
tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(-c)/sign(tan((f*x+exp(1))/
2)-1)+(-6*A*a*c*sqrt(2)*atan(sqrt(c)/sqrt(-c))-3*A*a*sqrt(-c)*sqrt(2)*sqrt(
c)-6*B*a*c*sqrt(2)*atan(sqrt(c)/sqrt(-c))-5*B*a*sqrt(-c)*sqrt(2)*sqrt(c))/3
/c/sqrt(-c)*sign(tan((f*x+exp(1))/2)-1))
```

maple [A] time = 1.49, size = 159, normalized size = 1.30

$$-\frac{2\left(\sin\left(fx+e\right)-1\right)\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,a\left(3c^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{2}}{2\sqrt{c}}\right)A+3c^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+}}{}\right.\right.}{3c^{2}\cos\left(fx+e\right)\sqrt{c-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/3*(sin(f*x+e)-1)*(c*(1+sin(f*x+e)))^(1/2)*a*(3*c^(3/2)*2^(1/2)*arctanh(1
/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*A+3*c^(3/2)*2^(1/2)*arctanh(1/
2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*B-B*(c*(1+sin(f*x+e)))^(3/2)-3*
A*c*(c*(1+sin(f*x+e)))^(1/2)-3*B*c*(c*(1+sin(f*x+e)))^(1/2))/c^2/cos(f*x+e)
/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/sqrt(-c*sin(f*x + e) +
c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(1/2),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(1/2),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{A}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int \frac{A\sin\left(e+fx\right)}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int \frac{B\sin\left(e+fx\right)}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int \frac{B\sin^2\left(e+f\right.}{\sqrt{-c\sin\left(e+fx\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a*(Integral(A/sqrt(-c*sin(e + f*x) + c), x) + Integral(A*sin(e + f*x)/sqrt(
-c*sin(e + f*x) + c), x) + Integral(B*sin(e + f*x)/sqrt(-c*sin(e + f*x) + c
), x) + Integral(B*sin(e + f*x)**2/sqrt(-c*sin(e + f*x) + c), x))
```

3.86 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{a(A+5B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}\,c^{3/2}f}+\frac{a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}}+\frac{2aB\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(3/2)-1/2*a*(A+5*B)*arctanh(1/2*cos(f
*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(3/2)/f*2^(1/2)+2*a*B*cos(f
*x+e)/c/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2857, 2751, 2649, 206}

$$-\frac{a(A+5B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}\,c^{3/2}f}+\frac{a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}}+\frac{2aB\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2),
x]
```

```
[Out] -((a*(A + 5*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f
*x]])])/(Sqrt[2]*c^(3/2)*f)) + (a*(A + B)*Cos[e + f*x])/(f*(c - c*Sin[e + f
*x])^(3/2)) + (2*a*B*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$$

$$= \frac{a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}} + \frac{a\int \frac{-Ac-3Bc-2Bc\sin(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx}{2c^2}$$

$$= \frac{a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}} + \frac{2aB\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}} - \frac{(a(A+5B))\int -}{2c}$$

$$= \frac{a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}} + \frac{2aB\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}} + \frac{(a(A+5B))\,\mathrm{Sul}}{}$$

$$= -\frac{a(A+5B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}\,c^{3/2}f} + \frac{a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}} +$$

Mathematica [A] time = 1.61, size = 157, normalized size = 1.37

$$\frac{a\sec(e+fx)\left(2\sqrt{c}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2(A-2B\sin(e+fx)+3B)+\sqrt{2}(A+5B)\sqrt{-c(\sin(e+}}{2c^{3/2}f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (a*Sec[e + f*x]*(Sqrt[2]*(A + 5*B)*ArcTan[Sqrt[-(c*(1 + Sin[e + f*x]))]/(Sq
rt[2]*Sqrt[c])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sqrt[-(c*(1 + Sin[e
 + f*x]))] + 2*Sqrt[c]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*(A + 3*B - 2
*B*Sin[e + f*x])))/(2*c^(3/2)*f*Sqrt[c - c*Sin[e + f*x]])
```

fricas [B] time = 0.45, size = 318, normalized size = 2.77

$$\frac{\frac{\sqrt{2}\left((A+5B)ac\cos\left(fx+e\right)^2-(A+5B)ac\cos\left(fx+e\right)-2(A+5B)ac+\left((A+5B)ac\cos\left(fx+e\right)+2(A+5B)ac\right)\sin\left(fx+e\right)\right)\log\left(-\frac{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)-2\right)\sin\left(f\right.}{\cos\left(f\right.}\right.}{\sqrt{c}}}{4\left(c^2f\cos\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*((A + 5*B)*a*c*cos(f*x + e)^2 - (A + 5*B)*a*c*cos(f*x + e) - 2
*(A + 5*B)*a*c + ((A + 5*B)*a*c*cos(f*x + e) + 2*(A + 5*B)*a*c)*sin(f*x + e
))*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(
-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(c) + 3*cos(f*x
+ e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e)
- 2))/sqrt(c) - 4*(2*B*a*cos(f*x + e)^2 + (A + 3*B)*a*cos(f*x + e) + (A + B
)*a - (2*B*a*cos(f*x + e) - (A + B)*a)*sin(f*x + e))*sqrt(-c*sin(f*x + e) +
 c))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f + (c^2*f*cos(f*x
+ e) + 2*c^2*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)2/f*(2*sqrt(c*tan((f*x+exp(1))/2)^2+c)*(-1/2*B*a/c/sig
n(tan((f*x+exp(1))/2)-1)-1/2*B*a*tan((f*x+exp(1))/2)/c/sign(tan((f*x+exp(1)
)/2)-1))/(c*tan((f*x+exp(1))/2)^2+c)+2*(1/2*(-3*A*a*(-sqrt(c)*tan((f*x+exp(
1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-3*B*a*(-sqrt(c)*tan((f*x+exp(1))/
2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3+A*a*c*(-sqrt(c)*tan((f*x+exp(1))/2)+s
qrt(c*tan((f*x+exp(1))/2)^2+c))-A*a*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+s
qrt(c*tan((f*x+exp(1))/2)^2+c))^2+B*a*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(
c*tan((f*x+exp(1))/2)^2+c))-B*a*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(
c*tan((f*x+exp(1))/2)^2+c))^2-A*a*sqrt(c)*c-B*a*sqrt(c)*c)/(-(-sqrt(c)*tan(
(f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan(
(f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^2/c/sign(tan((f*x+exp(1
))/2)-1)+1/2*(-A*a-5*B*a)*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c
*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/sqrt(-c)/c/sign(tan((f
*x+exp(1))/2)-1)))
```

maple [B] time = 1.24, size = 227, normalized size = 1.97

$$\frac{a\left(A\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\sin\left(fx+e\right)c+5B\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\sin\left(fx+e\right)c-A\sqrt{2}\ a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/2*a/c^(5/2)*(A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/
2))*sin(f*x+e)*c+5*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c
^(1/2))*sin(f*x+e)*c-A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)
/c^(1/2))*c-4*(c*(1+sin(f*x+e)))^(1/2)*c^(1/2)*B*sin(f*x+e)-5*B*2^(1/2)*arc
tanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c+2*(c*(1+sin(f*x+e)))^(
1/2)*c^(1/2)*A+6*(c*(1+sin(f*x+e)))^(1/2)*c^(1/2)*B)*(c*(1+sin(f*x+e)))^(1/
2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(
3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(3/2),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(3/2),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int \frac{A}{- c \sqrt{- c \sin{\left(e + f x \right)} + c} \sin{\left(e + f x \right)} + c \sqrt{- c \sin{\left(e + f x \right)} + c}}\, dx + \int \frac{A \sin{\left(e + f x \right)}}{- c \sqrt{- c \sin{\left(e + f x \right)} + c} \sin{\left(e + f x \right)} + c \sqrt{- c \sin{\left(e + f x \right)} + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] a*(Integral(A/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e
+ f*x) + c)), x) + Integral(A*sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x) + c)*si
n(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)), x) + Integral(B*sin(e + f*x)/(-c
*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)), x)
+ Integral(B*sin(e + f*x)**2/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c
*sqrt(-c*sin(e + f*x) + c)), x))
```

3.87 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{a(A-7B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f}-\frac{a(A+9B)\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}}+\frac{a(A+B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 1/2*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(5/2)-1/8*a*(A+9*B)*cos(f*x+e)/c/
f/(c-c*sin(f*x+e))^(3/2)-1/16*a*(A-7*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1
/2)/(c-c*sin(f*x+e))^(1/2))/c^(5/2)/f*2^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2857, 2750, 2649, 206}

$$-\frac{a(A-7B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f}-\frac{a(A+9B)\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}}+\frac{a(A+B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/2),
x]
```

```
[Out] -(a*(A - 7*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*
x]])])/(8*Sqrt[2]*c^(5/2)*f) + (a*(A + B)*Cos[e + f*x])/(2*f*(c - c*Sin[e +
 f*x])^(5/2)) - (a*(A + 9*B)*Cos[e + f*x])/(8*c*f*(c - c*Sin[e + f*x])^(3/2
))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx &= (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx \\
&= \frac{a(A+B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}} + \frac{a\int \frac{-Ac-5Bc-4Bc\sin(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx}{4c^2} \\
&= \frac{a(A+B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+9B)\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}} - \frac{(a(A-7B}{} \\
&= \frac{a(A+B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+9B)\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}} + \frac{(a(A-7B}{} \\
&= -\frac{a(A-7B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f} + \frac{a(A+B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}}
\end{aligned}$$

Mathematica [A] time = 2.24, size = 199, normalized size = 1.58

$$-\frac{a(\sin(e+fx)-1)(\sin(e+fx)+1)\left(\frac{2\sqrt{c}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)((A+9B)\sin(e+fx)+3A-5B)}{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}+\sqrt{2}(A-7B)\sec(e+fx\cdots}{16c^{5/2}f\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] -1/16*(a*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])*(Sqrt[2]*(A - 7*B)*ArcTan[S
qrt[-(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*Sec[e + f*x]*Sqrt[-(c*(1 +
Sin[e + f*x]))] + (2*Sqrt[c]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*A - 5
*B + (A + 9*B)*Sin[e + f*x]))/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5))/(c^
(5/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sqrt[c - c*Sin[e + f*x]])
```

fricas [B] time = 0.46, size = 394, normalized size = 3.13

$$-\frac{\sqrt{2}\left((A-7B)a\cos\left(fx+e\right)^3+3(A-7B)a\cos\left(fx+e\right)^2-2(A-7B)a\cos\left(fx+e\right)-4(A-7B)a-\left((A-7\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/32*(sqrt(2)*((A - 7*B)*a*cos(f*x + e)^3 + 3*(A - 7*B)*a*cos(f*x + e)^2 -
 2*(A - 7*B)*a*cos(f*x + e) - 4*(A - 7*B)*a - ((A - 7*B)*a*cos(f*x + e)^2 -
 2*(A - 7*B)*a*cos(f*x + e) - 4*(A - 7*B)*a)*sin(f*x + e))*sqrt(c)*log(-(c*
cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e)
+ sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e
) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) -
 2)) - 4*((A + 9*B)*a*cos(f*x + e)^2 - (3*A - 5*B)*a*cos(f*x + e) - 4*(A +
B)*a - ((A + 9*B)*a*cos(f*x + e) + 4*(A + B)*a)*sin(f*x + e))*sqrt(-c*sin(f
*x + e) + c))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(
f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f
)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*2*(1/16*(-17*A*a*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+
exp(1))/2)^2+c))^7+7*B*a*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(
1))/2)^2+c))^7-23*A*a*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x
+exp(1))/2)^2+c))^6+81*B*a*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan
((f*x+exp(1))/2)^2+c))^6-19*A*a*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan(
(f*x+exp(1))/2)^2+c))^5+53*B*a*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((
f*x+exp(1))/2)^2+c))^5+39*A*a*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(
c*tan((f*x+exp(1))/2)^2+c))^4-65*B*a*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2
)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4+5*A*a*c^2*(-sqrt(c)*tan((f*x+exp(1))/2
)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3+13*B*a*c^2*(-sqrt(c)*tan((f*x+exp(1))/
2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3+7*A*a*c^3*(-sqrt(c)*tan((f*x+exp(1))/
2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-37*A*a*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+e
xp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-33*B*a*c^3*(-sqrt(c)*tan((f*x+
exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+19*B*a*sqrt(c)*c^2*(-sqrt(c)*ta
n((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-3*A*a*sqrt(c)*c^3+5*B*
a*sqrt(c)*c^3)/c^2/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/
2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/
2)^2+c))+c)^4/sign(tan((f*x+exp(1))/2)-1)+1/16*(-A*a+7*B*a)*atan((-sqrt(c)*
tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-
c))/sqrt(2)/c^2/sqrt(-c)/sign(tan((f*x+exp(1))/2)-1))
```

maple [B] time = 1.61, size = 268, normalized size = 2.13

$$a\left(-2\sin\left(fx+e\right)\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c+c\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{c}}\right)c^2\left(A-7B\right)-\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c+c\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{c}}\right)c^2\left(A-7B\right)\left(\mathrm{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/16*a*(-2*sin(f*x+e)*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^
(1/2))*c^2*(A-7*B)-2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/
2))*c^2*(A-7*B)*cos(f*x+e)^2+2*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)
*2^(1/2)/c^(1/2))*c^2-2*A*(c+c*sin(f*x+e))^(3/2)*c^(1/2)-4*A*(c+c*sin(f*x+e
))^(1/2)*c^(3/2)-14*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^
(1/2))*c^2-18*B*(c+c*sin(f*x+e))^(3/2)*c^(1/2)+28*B*(c+c*sin(f*x+e))^(1/2)*
```

```
c^(3/2))*(c*(1+sin(f*x+e)))^(1/2)/c^(9/2)/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*si
n(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(
5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(5/2),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(5/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.88 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=163

$$-\frac{a(A-3B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}\,c^{7/2}f}-\frac{a(A-3B)\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{3/2}}-\frac{a(A+13B)\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}}+\frac{a(A+B)\cos(e}{3f(c-c\sin(e+}$$

```
[Out] 1/3*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(7/2)-1/24*a*(A+13*B)*cos(f*x+e)/
c/f/(c-c*sin(f*x+e))^(5/2)-1/32*a*(A-3*B)*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))
^(3/2)-1/64*a*(A-3*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e
))^(1/2))/c^(7/2)/f*2^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2967, 2857, 2750, 2650, 2649, 206}

$$-\frac{a(A-3B)\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{3/2}}-\frac{a(A-3B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}\,c^{7/2}f}-\frac{a(A+13B)\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}}+\frac{a(A+B)\cos(e}{3f(c-c\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/2),
x]
```

```
[Out] -(a*(A - 3*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*
x]])])/(32*Sqrt[2]*c^(7/2)*f) + (a*(A + B)*Cos[e + f*x])/(3*f*(c - c*Sin[e
+ f*x])^(7/2)) - (a*(A + 13*B)*Cos[e + f*x])/(24*c*f*(c - c*Sin[e + f*x])^(
5/2)) - (a*(A - 3*B)*Cos[e + f*x])/(32*c^2*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$$

$$= \frac{a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}} + \frac{a\int \frac{-Ac-7Bc-6Bc\sin(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx}{6c^2}$$

$$= \frac{a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+13B)\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}} - \frac{(a(A-3$$

$$= \frac{a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+13B)\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}} - \frac{a(A-}{32c^2f(c-}$$

$$= \frac{a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+13B)\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}} - \frac{a(A-}{32c^2f(c-}$$

$$= -\frac{a(A-3B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}\,c^{7/2}f} + \frac{a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}}$$

Mathematica [A] time = 3.45, size = 217, normalized size = 1.33

$$-\frac{a(\sin(e+fx)-1)(\sin(e+fx)+1)\left(\frac{\sqrt{c}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(4(5A+17B)\sin(e+fx)+3(A-3B)\cos(2(e+fx))+47A-13B)}{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}+\right.}{192c^{7/2}f\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] -1/192*(a*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])*(3*Sqrt[2]*(A - 3*B)*ArcTa
n[Sqrt[-(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*Sec[e + f*x]*Sqrt[-(c*(1
 + Sin[e + f*x]))] + (Sqrt[c]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(47*A -
 13*B + 3*(A - 3*B)*Cos[2*(e + f*x)] + 4*(5*A + 17*B)*Sin[e + f*x]))/(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^7))/(c^(7/2)*f*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^2*Sqrt[c - c*Sin[e + f*x]])
```

fricas [B] time = 0.45, size = 490, normalized size = 3.01

$$-\frac{3\sqrt{2}\left((A-3B)a\cos\left(fx+e\right)^{4}-3(A-3B)a\cos\left(fx+e\right)^{3}-8(A-3B)a\cos\left(fx+e\right)^{2}+4(A-3B)a\cos\left(fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algor
ithm="fricas")
```

```
[Out] -1/384*(3*sqrt(2)*((A - 3*B)*a*cos(f*x + e)^4 - 3*(A - 3*B)*a*cos(f*x + e)^
3 - 8*(A - 3*B)*a*cos(f*x + e)^2 + 4*(A - 3*B)*a*cos(f*x + e) + 8*(A - 3*B)
*a + ((A - 3*B)*a*cos(f*x + e)^3 + 4*(A - 3*B)*a*cos(f*x + e)^2 - 4*(A - 3*
B)*a*cos(f*x + e) - 8*(A - 3*B)*a)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x +
e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x
+ e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(
cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(
3*(A - 3*B)*a*cos(f*x + e)^3 - (7*A + 43*B)*a*cos(f*x + e)^2 + 2*(11*A - B)
*a*cos(f*x + e) + 32*(A + B)*a + (3*(A - 3*B)*a*cos(f*x + e)^2 + 2*(5*A + 1
7*B)*a*cos(f*x + e) + 32*(A + B)*a)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)
)/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 +
 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x +
 e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*2*(1/192*(-195*A*a*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*
x+exp(1))/2)^2+c))^11+9*B*a*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+e
xp(1))/2)^2+c))^11-609*A*a*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan
((f*x+exp(1))/2)^2+c))^10+483*B*a*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqr
t(c*tan((f*x+exp(1))/2)^2+c))^10-1347*A*a*c*(-sqrt(c)*tan((f*x+exp(1))/2)+s
qrt(c*tan((f*x+exp(1))/2)^2+c))^9+393*B*a*c*(-sqrt(c)*tan((f*x+exp(1))/2)+s
qrt(c*tan((f*x+exp(1))/2)^2+c))^9+3*A*a*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1)
)/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^8+183*B*a*sqrt(c)*c*(-sqrt(c)*tan((f*
x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^8+1842*A*a*c^2*(-sqrt(c)*tan(
```

```
(f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7-534*B*a*c^2*(-sqrt(c)*ta
n((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7+950*A*a*sqrt(c)*c^2*(-
sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6-418*B*a*sqrt
(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6-23
58*A*a*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5
+1314*B*a*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c)
)^5-378*A*a*sqrt(c)*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1
))/2)^2+c))^4-18*B*a*sqrt(c)*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((
f*x+exp(1))/2)^2+c))^4+1153*A*a*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*ta
n((f*x+exp(1))/2)^2+c))^3-707*B*a*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*
tan((f*x+exp(1))/2)^2+c))^3+105*A*a*c^5*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(
c*tan((f*x+exp(1))/2)^2+c))-741*A*a*sqrt(c)*c^4*(-sqrt(c)*tan((f*x+exp(1))/
2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-123*B*a*c^5*(-sqrt(c)*tan((f*x+exp(1)
)/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+111*B*a*sqrt(c)*c^4*(-sqrt(c)*tan((f*
x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-25*A*a*sqrt(c)*c^5+11*B*a*s
qrt(c)*c^5)/c^3/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^
2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^
2+c))+c)^6/sign(tan((f*x+exp(1))/2)-1)+1/64*(-A*a+3*B*a)*atan((-sqrt(c)*tan
((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c))
/sqrt(2)/c^3/sqrt(-c)/sign(tan((f*x+exp(1))/2)-1))
```

maple [B] time = 1.73, size = 352, normalized size = 2.16

$$\frac{a\left(-3\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c+c\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{c}}\right)c^{4}\left(A-3B\right)\sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)+12\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c+c\sin\left(fx+e\right)}\ \sqrt{}}{2\sqrt{c}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/192*a*(-3*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4
*(A-3*B)*sin(f*x+e)*cos(f*x+e)^2+12*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1
/2)*2^(1/2)/c^(1/2))*c^4*(A-3*B)*sin(f*x+e)+9*2^(1/2)*arctanh(1/2*(c+c*sin(
f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4*(A-3*B)*cos(f*x+e)^2+24*A*(c+c*sin(f*x+e
))^(1/2)*c^(7/2)+32*A*(c+c*sin(f*x+e))^(3/2)*c^(5/2)-6*A*(c+c*sin(f*x+e))^(
5/2)*c^(3/2)-72*B*(c+c*sin(f*x+e))^(1/2)*c^(7/2)+32*B*(c+c*sin(f*x+e))^(3/2
)*c^(5/2)+18*B*(c+c*sin(f*x+e))^(5/2)*c^(3/2)-12*A*2^(1/2)*arctanh(1/2*(c+c
*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4+36*B*2^(1/2)*arctanh(1/2*(c+c*sin(f
*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4)*(c*(1+sin(f*x+e)))^(1/2)/c^(15/2)/(sin(f
*x+e)-1)^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(
7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(7/2),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(7/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.89 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=210

$$\frac{256a^2c^6(13A-3B)\cos^5(e+fx)}{15015f(c-c\sin(e+fx))^{5/2}}+\frac{64a^2c^5(13A-3B)\cos^5(e+fx)}{3003f(c-c\sin(e+fx))^{3/2}}+\frac{8a^2c^4(13A-3B)\cos^5(e+fx)}{429f\sqrt{c-c\sin(e+fx)}}+2a^2c^3(13A$$

```
[Out] 256/15015*a^2*(13*A-3*B)*c^6*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+64/3003*
a^2*(13*A-3*B)*c^5*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(3/2)-2/13*a^2*B*c^2*cos
(f*x+e)^5*(c-c*sin(f*x+e))^(3/2)/f+8/429*a^2*(13*A-3*B)*c^4*cos(f*x+e)^5/f/
(c-c*sin(f*x+e))^(1/2)+2/143*a^2*(13*A-3*B)*c^3*cos(f*x+e)^5*(c-c*sin(f*x+e
))^(1/2)/f
```

Rubi [A] time = 0.55, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2856, 2674, 2673}

$$\frac{256a^2c^6(13A-3B)\cos^5(e+fx)}{15015f(c-c\sin(e+fx))^{5/2}}+\frac{64a^2c^5(13A-3B)\cos^5(e+fx)}{3003f(c-c\sin(e+fx))^{3/2}}+\frac{8a^2c^4(13A-3B)\cos^5(e+fx)}{429f\sqrt{c-c\sin(e+fx)}}+2a^2c^3(13A$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2),
x]
```

```
[Out] (256*a^2*(13*A - 3*B)*c^6*Cos[e + f*x]^5)/(15015*f*(c - c*Sin[e + f*x])^(5/
2)) + (64*a^2*(13*A - 3*B)*c^5*Cos[e + f*x]^5)/(3003*f*(c - c*Sin[e + f*x])
^(3/2)) + (8*a^2*(13*A - 3*B)*c^4*Cos[e + f*x]^5)/(429*f*Sqrt[c - c*Sin[e +
 f*x]]) + (2*a^2*(13*A - 3*B)*c^3*Cos[e + f*x]^5*Sqrt[c - c*Sin[e + f*x]])/
(143*f) - (2*a^2*B*c^2*Cos[e + f*x]^5*(c - c*Sin[e + f*x])^(3/2))/(13*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
```

```
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(A+B\sin(e+fx))(c-$$

$$= -\frac{2a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^{3/2}}{13f} +$$

$$= \frac{2a^2(13A-3B)c^3\cos^5(e+fx)\sqrt{c-c\sin(e+}}{143f}$$

$$= \frac{8a^2(13A-3B)c^4\cos^5(e+fx)}{429f\sqrt{c-c\sin(e+fx)}} + \frac{2a^2(13A-}{}$$

$$= \frac{64a^2(13A-3B)c^5\cos^5(e+fx)}{3003f(c-c\sin(e+fx))^{3/2}} + \frac{8a^2(13A-}{429f\sqrt{}}$$

$$= \frac{256a^2(13A-3B)c^6\cos^5(e+fx)}{15015f(c-c\sin(e+fx))^{5/2}} + \frac{64a^2(13A}{3003f(}$$

Mathematica [B] time = 6.64, size = 1355, normalized size = 6.45

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] ((7*A - 2*B)*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(
7/2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^4) - ((4*A + B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(
c - c*Sin[e + f*x])^(7/2))/(32*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((22*A - 7*B)*Cos[(5*(e + f*x))/2]
*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(160*f*(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((A - 4*
B)*Cos[(7*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/
(112*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^4) + (A*Cos[(9*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e
+ f*x])^(7/2))/(48*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^4) + ((2*A - 3*B)*Cos[(11*(e + f*x))/2]*(a + a*Sin[
e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(352*f*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + (B*Cos[(13*(e + f*x)
)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(416*f*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((7*
A - 2*B)*Sin[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2)
)/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^4) + ((4*A + B)*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2)
*Sin[(3*(e + f*x))/2])/(32*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((22*A - 7*B)*(a + a*Sin[e + f*x])^2*(
c - c*Sin[e + f*x])^(7/2)*Sin[(5*(e + f*x))/2])/(160*f*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) - ((A - 4*B)*(
a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2)*Sin[(7*(e + f*x))/2])/(112
*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^4) + (A*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2)*Sin[(9*(e +
 f*x))/2])/(48*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^4) - ((2*A - 3*B)*(a + a*Sin[e + f*x])^2*(c - c*Sin[e +
 f*x])^(7/2)*Sin[(11*(e + f*x))/2])/(352*f*(Cos[(e + f*x)/2] - Sin[(e + f*x
)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + (B*(a + a*Sin[e + f*x])^
2*(c - c*Sin[e + f*x])^(7/2)*Sin[(13*(e + f*x))/2])/(416*f*(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)
```

fricas [A] time = 0.45, size = 358, normalized size = 1.70

$$\frac{2\left(1155\,Ba^2c^3\cos\left(fx+e\right)^7+105\,(13\,A-14\,B)a^2c^3\cos\left(fx+e\right)^6+35\,(91\,A-87\,B)a^2c^3\cos\left(fx+e\right)^5-20\,(13\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] 2/15015*(1155*B*a^2*c^3*cos(f*x + e)^7 + 105*(13*A - 14*B)*a^2*c^3*cos(f*x
+ e)^6 + 35*(91*A - 87*B)*a^2*c^3*cos(f*x + e)^5 - 20*(13*A - 3*B)*a^2*c^3*
cos(f*x + e)^4 + 32*(13*A - 3*B)*a^2*c^3*cos(f*x + e)^3 - 64*(13*A - 3*B)*a
^2*c^3*cos(f*x + e)^2 + 256*(13*A - 3*B)*a^2*c^3*cos(f*x + e) + 512*(13*A -
 3*B)*a^2*c^3 + (1155*B*a^2*c^3*cos(f*x + e)^6 - 105*(13*A - 25*B)*a^2*c^3*
cos(f*x + e)^5 + 140*(13*A - 3*B)*a^2*c^3*cos(f*x + e)^4 + 160*(13*A - 3*B)
*a^2*c^3*cos(f*x + e)^3 + 192*(13*A - 3*B)*a^2*c^3*cos(f*x + e)^2 + 256*(13
*A - 3*B)*a^2*c^3*cos(f*x + e) + 512*(13*A - 3*B)*a^2*c^3)*sin(f*x + e))*sq
rt(-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)sqrt(2*c)*(48*f*(2*A*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))-2*B*a^2*c^
3*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(6*f*x+6*exp(1)-pi))/(48*f)^2
+352*f*(2*A*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))-2*B*a^2*c^3*sign(sin
(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(22*f*x+22*exp(1)-pi))/(352*f)^2-288*f*
(4*A*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))+2*B*a^2*c^3*sign(sin(1/2*(f
*x+exp(1))-1/4*pi)))*cos(1/4*(18*f*x+18*exp(1)-pi))/(288*f)^2+224*f*(6*A*a^
2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))-6*B*a^2*c^3*sign(sin(1/2*(f*x+exp(
1))-1/4*pi)))*sin(1/4*(14*f*x+14*exp(1)-pi))/(224*f)^2-16*f*(12*A*a^2*c^3*s
ign(sin(1/2*(f*x+exp(1))-1/4*pi))-2*B*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4
```

```
*pi)))*cos(1/4*(2*f*x-pi)+1/2*exp(1))/(16*f)^2-320*f*(32*A*a^2*c^3*sign(sin
(1/2*(f*x+exp(1))-1/4*pi))-2*B*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*
cos(1/4*(10*f*x+10*exp(1)-pi))/(320*f)^2+16*f*(-2*A*a^2*c^3*sign(sin(1/2*(f
*x+exp(1))-1/4*pi))+2*B*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4
*(2*f*x+2*exp(1)+pi))/(16*f)^2+288*f*(-2*A*a^2*c^3*sign(sin(1/2*(f*x+exp(1)
)-1/4*pi))+2*B*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(18*f*x+
18*exp(1)+pi))/(288*f)^2-224*f*(-4*A*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*
pi))-2*B*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(14*f*x+14*exp
(1)+pi))/(224*f)^2+160*f*(-6*A*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))+6
*B*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(10*f*x+10*exp(1)+pi
))/(160*f)^2-192*f*(-32*A*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))+2*B*a^
2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(6*f*x+6*exp(1)+pi))/(192
*f)^2+1408*B*a^2*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(22*f*x+2
2*exp(1)+pi))/(704*f)^2-1664*B*a^2*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))
*cos(1/4*(26*f*x+26*exp(1)-pi))/(832*f)^2)
```

maple [A] time = 1.16, size = 121, normalized size = 0.58

$$\frac{2\left(\sin\left(fx+e\right)-1\right)c^4\left(1+\sin\left(fx+e\right)\right)^3a^2\left(\left(-1365A+4935B\right)\sin\left(fx+e\right)\left(\cos^2\left(fx+e\right)\right)+\left(11180A-1182\right.\right.}{15015\cos\left(fx+e\right)\sqrt{c-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 2/15015*(sin(f*x+e)-1)*c^4*(1+sin(f*x+e))^3*a^2*((-1365*A+4935*B)*sin(f*x+e
)*cos(f*x+e)^2+(11180*A-11820*B)*sin(f*x+e)+1155*B*cos(f*x+e)^4+(5915*A-106
05*B)*cos(f*x+e)^2-12844*A+12204*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^2\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)
^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^2\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(7/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(7/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.90 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=167

$$\frac{64a^2c^5(11A-B)\cos^5(e+fx)}{3465f(c-c\sin(e+fx))^{5/2}}+\frac{16a^2c^4(11A-B)\cos^5(e+fx)}{693f(c-c\sin(e+fx))^{3/2}}+\frac{2a^2c^3(11A-B)\cos^5(e+fx)}{99f\sqrt{c-c\sin(e+fx)}}-\frac{2a^2Bc^2\cos^5(e+\ldots}{}$$

```
[Out] 64/3465*a^2*(11*A-B)*c^5*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+16/693*a^2*(
11*A-B)*c^4*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(3/2)+2/99*a^2*(11*A-B)*c^3*cos
(f*x+e)^5/f/(c-c*sin(f*x+e))^(1/2)-2/11*a^2*B*c^2*cos(f*x+e)^5*(c-c*sin(f*x
+e))^(1/2)/f
```

Rubi [A] time = 0.45, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2856, 2674, 2673}

$$\frac{2a^2c^3(11A-B)\cos^5(e+fx)}{99f\sqrt{c-c\sin(e+fx)}}+\frac{16a^2c^4(11A-B)\cos^5(e+fx)}{693f(c-c\sin(e+fx))^{3/2}}+\frac{64a^2c^5(11A-B)\cos^5(e+fx)}{3465f(c-c\sin(e+fx))^{5/2}}-\frac{2a^2Bc^2\cos^5(e+\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (64*a^2*(11*A - B)*c^5*Cos[e + f*x]^5)/(3465*f*(c - c*Sin[e + f*x])^(5/2))
+ (16*a^2*(11*A - B)*c^4*Cos[e + f*x]^5)/(693*f*(c - c*Sin[e + f*x])^(3/2))
 + (2*a^2*(11*A - B)*c^3*Cos[e + f*x]^5)/(99*f*Sqrt[c - c*Sin[e + f*x]]) -
(2*a^2*B*c^2*Cos[e + f*x]^5*Sqrt[c - c*Sin[e + f*x]])/(11*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
```

```
[m + p, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(A+B\sin(e+fx))\sqrt{c$$

$$= -\frac{2a^2Bc^2\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f} + \frac{1}{1}$$

$$= \frac{2a^2(11A-B)c^3\cos^5(e+fx)}{99f\sqrt{c-c\sin(e+fx)}} - \frac{2a^2Bc^2\cos^5(}{}$$

$$= \frac{16a^2(11A-B)c^4\cos^5(e+fx)}{693f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2(11A-}{99f\sqrt{c-}}$$

$$= \frac{64a^2(11A-B)c^5\cos^5(e+fx)}{3465f(c-c\sin(e+fx))^{5/2}} + \frac{16a^2(11A-}{693f(c-}$$

Mathematica [B] time = 6.51, size = 1173, normalized size = 7.02

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] ((6*A - B)*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/
2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])^4) - ((4*A + B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c
- c*Sin[e + f*x])^(5/2))/(24*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((8*A - 3*B)*Cos[(5*(e + f*x))/2]*(a
 + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2))/(80*f*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) - ((2*A + 3*B)
*Cos[(7*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2))/(1
12*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^4) + ((2*A - B)*Cos[(9*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*
Sin[e + f*x])^(5/2))/(144*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^4) - (B*Cos[(11*(e + f*x))/2]*(a + a*Sin[e +
 f*x])^2*(c - c*Sin[e + f*x])^(5/2))/(176*f*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((6*A - B)*Sin[(e + f*x
)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2))/(8*f*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((4*A
+ B)*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)*Sin[(3*(e + f*x))/2]
)/(24*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^4) + ((8*A - 3*B)*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5
/2)*Sin[(5*(e + f*x))/2])/(80*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((2*A + 3*B)*(a + a*Sin[e + f*x])^2
*(c - c*Sin[e + f*x])^(5/2)*Sin[(7*(e + f*x))/2])/(112*f*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((2*A - B)
*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)*Sin[(9*(e + f*x))/2])/(1
44*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^4) + (B*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)*Sin[(11*(
e + f*x))/2])/(176*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^4)
```

fricas [B] time = 0.44, size = 313, normalized size = 1.87

$$-\frac{2\left(315\,Ba^2c^2\cos\left(fx+e\right)^6-35\,(11\,A-10\,B)a^2c^2\cos\left(fx+e\right)^5+5\,(11\,A-B)a^2c^2\cos\left(fx+e\right)^4-8\,(11\,A-B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] -2/3465*(315*B*a^2*c^2*cos(f*x + e)^6 - 35*(11*A - 10*B)*a^2*c^2*cos(f*x +
e)^5 + 5*(11*A - B)*a^2*c^2*cos(f*x + e)^4 - 8*(11*A - B)*a^2*c^2*cos(f*x +
 e)^3 + 16*(11*A - B)*a^2*c^2*cos(f*x + e)^2 - 64*(11*A - B)*a^2*c^2*cos(f*
```

```
x + e) - 128*(11*A - B)*a^2*c^2 - (315*B*a^2*c^2*cos(f*x + e)^5 + 35*(11*A
- B)*a^2*c^2*cos(f*x + e)^4 + 40*(11*A - B)*a^2*c^2*cos(f*x + e)^3 + 48*(11
*A - B)*a^2*c^2*cos(f*x + e)^2 + 64*(11*A - B)*a^2*c^2*cos(f*x + e) + 128*(
11*A - B)*a^2*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e)
- f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*c)*(24*A*a^2*c^2*f*sign(sin(1/2*(f*x+ex
p(1))-1/4*pi))*cos(1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2-40*A*a^2*c^2*f*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(10*f*x+10*exp(1)-pi))/(20*f)^2+224*A*a
^2*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(14*f*x+14*exp(1)+pi))/
(112*f)^2-288*A*a^2*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(18*f*
x+18*exp(1)-pi))/(144*f)^2-48*A*a^2*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi)
)*cos(1/4*(2*f*x-pi)+1/2*exp(1))/(8*f)^2+32*B*a^2*c^2*f*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*sin(1/4*(2*f*x+2*exp(1)+pi))/(16*f)^2-96*B*a^2*c^2*f*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(6*f*x+6*exp(1)-pi))/(48*f)^2+960*B*a^
2*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(10*f*x+10*exp(1)+pi))/(
160*f)^2-1344*B*a^2*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(14*f*
x+14*exp(1)-pi))/(224*f)^2+576*B*a^2*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi
))*sin(1/4*(18*f*x+18*exp(1)+pi))/(288*f)^2-704*B*a^2*c^2*f*sign(sin(1/2*(f
*x+exp(1))-1/4*pi))*sin(1/4*(22*f*x+22*exp(1)-pi))/(352*f)^2)
```

maple [A] time = 1.26, size = 105, normalized size = 0.63

$$-\frac{2\left(\sin\left(fx+e\right)-1\right)c^{3}\left(1+\sin\left(fx+e\right)\right)^{3}a^{2}\left(-315B\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)+\left(-1210A+1370B\right)\sin\left(fx+\right.\right.}{3465\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/3465*(sin(f*x+e)-1)*c^3*(1+sin(f*x+e))^3*a^2*(-315*B*cos(f*x+e)^2*sin(f*
x+e)+(-1210*A+1370*B)*sin(f*x+e)+(-385*A+980*B)*cos(f*x+e)^2+1562*A-1402*B)
/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{2}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)
^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^2\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.91 $\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2}\,dx$

Optimal. Leaf size=120

$$\frac{8a^2c^4(9A+B)\cos^5(e+fx)}{315f(c-c\sin(e+fx))^{5/2}}+\frac{2a^2c^3(9A+B)\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}}-\frac{2a^2Bc^2\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 8/315*a^2*(9*A+B)*c^4*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+2/63*a^2*(9*A+B
)*c^3*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(3/2)-2/9*a^2*B*c^2*cos(f*x+e)^5/f/(c
-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2856, 2674, 2673}

$$\frac{2a^2c^3(9A+B)\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}}+\frac{8a^2c^4(9A+B)\cos^5(e+fx)}{315f(c-c\sin(e+fx))^{5/2}}-\frac{2a^2Bc^2\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (8*a^2*(9*A + B)*c^4*Cos[e + f*x]^5)/(315*f*(c - c*Sin[e + f*x])^(5/2)) + (
2*a^2*(9*A + B)*c^3*Cos[e + f*x]^5)/(63*f*(c - c*Sin[e + f*x])^(3/2)) - (2*
a^2*B*c^2*Cos[e + f*x]^5)/(9*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,d$$
$$= -\frac{2a^2Bc^2\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}+\frac{1}{9}\left(a^2(9A+B)c^2\right)$$
$$= \frac{2a^2(9A+B)c^3\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}}-\frac{2a^2Bc^2\cos^5}{9f\sqrt{c-c\sin}}$$
$$= \frac{8a^2(9A+B)c^4\cos^5(e+fx)}{315f(c-c\sin(e+fx))^{5/2}}+\frac{2a^2(9A+B)}{63f(c-c\,\mathrm{s}}$$

Mathematica [A] time = 4.59, size = 106, normalized size = 0.88

$$\frac{a^2c\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5((130B-90A)\sin(e+fx)+162A+35B\cos(2(e+}{315f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (a^2*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(162*A - 87*B + 35*B*Cos[2*(
e + f*x)] + (-90*A + 130*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(315*f*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

fricas [B] time = 0.43, size = 228, normalized size = 1.90

$$-\frac{2\left(35\,Ba^2c\cos\left(fx+e\right)^5+5\,(9\,A+8\,B)a^2c\cos\left(fx+e\right)^4-(9\,A+B)a^2c\cos\left(fx+e\right)^3+2\,(9\,A+B)a^2c\cos\left(fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] -2/315*(35*B*a^2*c*cos(f*x + e)^5 + 5*(9*A + 8*B)*a^2*c*cos(f*x + e)^4 - (9
*A + B)*a^2*c*cos(f*x + e)^3 + 2*(9*A + B)*a^2*c*cos(f*x + e)^2 - 8*(9*A +
B)*a^2*c*cos(f*x + e) - 16*(9*A + B)*a^2*c + (35*B*a^2*c*cos(f*x + e)^4 - 5
*(9*A + B)*a^2*c*cos(f*x + e)^3 - 6*(9*A + B)*a^2*c*cos(f*x + e)^2 - 8*(9*A
 + B)*a^2*c*cos(f*x + e) - 16*(9*A + B)*a^2*c)*sin(f*x + e))*sqrt(-c*sin(f*
x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)sqrt(2*c)*(8*f*(2*A*a^2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))+2*B
*a^2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(2*f*x+2*exp(1)+pi))/(8*
f)^2+40*f*(2*A*a^2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))+2*B*a^2*c*sign(sin(
1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(10*f*x+10*exp(1)+pi))/(40*f)^2-8*f*(8*A
*a^2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))+2*B*a^2*c*sign(sin(1/2*(f*x+exp(1
))-1/4*pi)))*cos(1/4*(2*f*x-pi)+1/2*exp(1))/(8*f)^2+24*f*(-2*A*a^2*c*sign(s
```

```
in(1/2*(f*x+exp(1))-1/4*pi))-2*B*a^2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*
sin(1/4*(6*f*x+6*exp(1)-pi))/(24*f)^2+56*f*(-2*A*a^2*c*sign(sin(1/2*(f*x+ex
p(1))-1/4*pi))-2*B*a^2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(14*f*
x+14*exp(1)-pi))/(56*f)^2+24*A*a^2*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*c
os(1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2-40*A*a^2*c*f*sign(sin(1/2*(f*x+exp(1))
-1/4*pi))*cos(1/4*(10*f*x+10*exp(1)-pi))/(20*f)^2-224*B*a^2*c*f*sign(sin(1/
2*(f*x+exp(1))-1/4*pi))*cos(1/4*(14*f*x+14*exp(1)+pi))/(112*f)^2+288*B*a^2*
c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(18*f*x+18*exp(1)-pi))/(144*
f)^2)
```

maple [A] time = 1.27, size = 83, normalized size = 0.69

$$\frac{2\left(\sin\left(fx+e\right)-1\right)c^{2}\left(1+\sin\left(fx+e\right)\right)^{3}a^{2}\left(\sin\left(fx+e\right)\left(45A-65B\right)-35B\left(\cos^{2}\left(fx+e\right)\right)-81A+61B\right)}{315\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/315*(sin(f*x+e)-1)*c^2*(1+sin(f*x+e))^3*a^2*(sin(f*x+e)*(45*A-65*B)-35*B*
cos(f*x+e)^2-81*A+61*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{2}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)
^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^2\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int Ac\sqrt{-c\sin\left(e+fx\right)+c}\,dx+\int Ac\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)dx+\int\left(-Ac\sqrt{-c\sin\left(e+fx\right)+c}\,\mathrm{s}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] a**2*(Integral(A*c*sqrt(-c*sin(e + f*x) + c), x) + Integral(A*c*sqrt(-c*sin
(e + f*x) + c)*sin(e + f*x), x) + Integral(-A*c*sqrt(-c*sin(e + f*x) + c)*s
in(e + f*x)**2, x) + Integral(-A*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**
3, x) + Integral(B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(
B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(-B*c*sqrt(-c*s
in(e + f*x) + c)*sin(e + f*x)**3, x) + Integral(-B*c*sqrt(-c*sin(e + f*x) +
 c)*sin(e + f*x)**4, x))
```

3.92 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}$

Optimal. Leaf size=81

$$\frac{2a^2c^3(7A+3B)\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}}-\frac{2a^2Bc^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 2/35*a^2*(7*A+3*B)*c^3*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)-2/7*a^2*B*c^2*
cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(3/2)
```

Rubi [A] time = 0.33, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2967, 2856, 2673}

$$\frac{2a^2c^3(7A+3B)\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}}-\frac{2a^2Bc^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a^2*(7*A + 3*B)*c^3*Cos[e + f*x]^5)/(35*f*(c - c*Sin[e + f*x])^(5/2)) -
(2*a^2*B*c^2*Cos[e + f*x]^5)/(7*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
```

```
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$$
$$= -\frac{2a^2Bc^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}} + \frac{1}{7}\left(a^2(7A+3B)c^2\right)$$
$$= \frac{2a^2(7A+3B)c^3\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}} - \frac{2a^2Bc^2\cos^5(}{7f(c-c\sin(e}$$

Mathematica [A] time = 0.58, size = 89, normalized size = 1.10

$$\frac{2a^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5(7A+5B\sin(e+fx)-2B)}{35f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (2*a^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(7*A - 2*B + 5*B*Sin[e + f*x
])*Sqrt[c - c*Sin[e + f*x]])/(35*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

fricas [B] time = 0.44, size = 193, normalized size = 2.38

$$\frac{2\left(5\,Ba^2\cos\left(fx+e\right)^4-(7\,A+8\,B)a^2\cos\left(fx+e\right)^3-(21\,A+19\,B)a^2\cos\left(fx+e\right)^2+2\,(7\,A+3\,B)a^2\cos\left(fx+e\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/35*(5*B*a^2*cos(f*x + e)^4 - (7*A + 8*B)*a^2*cos(f*x + e)^3 - (21*A + 19*
B)*a^2*cos(f*x + e)^2 + 2*(7*A + 3*B)*a^2*cos(f*x + e) + 4*(7*A + 3*B)*a^2
- (5*B*a^2*cos(f*x + e)^3 + (7*A + 13*B)*a^2*cos(f*x + e)^2 - 2*(7*A + 3*B)
*a^2*cos(f*x + e) - 4*(7*A + 3*B)*a^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) +
 c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sq
rt(2*c)*(-12*f*(2*A*a^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))+4*B*a^2*sign(sin
(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2-2*f*(6*A*
a^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))+4*B*a^2*sign(sin(1/2*(f*x+exp(1))-1/
4*pi)))*cos(1/4*(2*f*x-pi)+1/2*exp(1))/(2*f)^2+8*f*(16*A*a^2*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))+14*B*a^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(
2*f*x+2*exp(1)+pi))/(8*f)^2-20*f*(-2*A*a^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi
))-4*B*a^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(10*f*x+10*exp(1)-pi
))/(20*f)^2+24*f*(-16*A*a^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))-14*B*a^2*sig
n(sin(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(6*f*x+6*exp(1)-pi))/(24*f)^2-80*B
*a^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(10*f*x+10*exp(1)+pi))/(4
0*f)^2+112*B*a^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(14*f*x+14*ex
p(1)-pi))/(56*f)^2)
```

maple [A] time = 1.30, size = 65, normalized size = 0.80

$$-\frac{2\left(\sin\left(fx+e\right)-1\right)c\left(1+\sin\left(fx+e\right)\right)^{3}a^{2}\left(5B\sin\left(fx+e\right)+7A-2B\right)}{35\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/35*(sin(f*x+e)-1)*c*(1+sin(f*x+e))^3*a^2*(5*B*sin(f*x+e)+7*A-2*B)/cos(f*
x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{2}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*sqrt(-c*sin(f*x + e)
+ c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int A\sqrt{-c\sin\left(e+fx\right)+c}\,dx+\int 2A\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)dx+\int A\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^2\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a**2*(Integral(A*sqrt(-c*sin(e + f*x) + c), x) + Integral(2*A*sqrt(-c*sin(e
 + f*x) + c)*sin(e + f*x), x) + Integral(A*sqrt(-c*sin(e + f*x) + c)*sin(e
+ f*x)**2, x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Int
egral(2*B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(B*sqrt(-
c*sin(e + f*x) + c)*sin(e + f*x)**3, x))
```

3.93 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=161

$$-\frac{2a^2c(A+B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{4a^2(A+B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}+\frac{4\sqrt{2}\,a^2(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f}-\frac{2a^2Bc^2\cos^5(}{5f(c-c\sin(e}$$

```
[Out] -2/5*a^2*B*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)-2/3*a^2*(A+B)*c*cos(f*
x+e)^3/f/(c-c*sin(f*x+e))^(3/2)+4*a^2*(A+B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*
2^(1/2)/(c-c*sin(f*x+e))^(1/2))*2^(1/2)/f/c^(1/2)-4*a^2*(A+B)*cos(f*x+e)/f/
(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2967, 2860, 2679, 2649, 206}

$$-\frac{2a^2c(A+B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{4a^2(A+B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}+\frac{4\sqrt{2}\,a^2(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f}-\frac{2a^2Bc^2\cos^5(}{5f(c-c\sin(e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] (4*Sqrt[2]*a^2*(A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*S
in[e + f*x]])])/(Sqrt[c]*f) - (2*a^2*B*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[
e + f*x])^(5/2)) - (2*a^2*(A + B)*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x
])^(3/2)) - (4*a^2*(A + B)*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$$

$$= -\frac{2a^2Bc^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} + \left(a^2(A+B)c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx)}$$

$$= -\frac{2a^2Bc^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{2a^2(A+B)c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} + \left(2a^2(A\right.$$

$$= -\frac{2a^2Bc^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{2a^2(A+B)c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \frac{4a^2(A}{f\sqrt{c}}$$

$$= -\frac{2a^2Bc^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{2a^2(A+B)c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \frac{4a^2(A}{f\sqrt{c}}$$

$$= \frac{4\sqrt{2}\,a^2(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f} - \frac{2a^2Bc^2\cos^5(e+f}{5f(c-c\sin(e+fx}$$

Mathematica [C] time = 1.18, size = 175, normalized size = 1.09

$$-\frac{a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(2(5A+11B)\sin\right.}{15f\sqrt{c-c\sin(e+fx)}\,\left(\sin\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e +
f*x]],x]
```

```
[Out] -1/15*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*((120
 + 120*I)*(-1)^(1/4)*(A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*
x)/4])] + (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(70*A + 79*B - 3*B*Cos[2*(e
 + f*x)] + 2*(5*A + 11*B)*Sin[e + f*x])))/(f*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^4*Sqrt[c - c*Sin[e + f*x]])
```

fricas [B] time = 0.45, size = 310, normalized size = 1.93

$$2\left(\frac{15\sqrt{2}\left((A+B)a^2c\cos\left(fx+e\right)-(A+B)a^2c\sin\left(fx+e\right)+(A+B)a^2c\right)\log\left(-\frac{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)+\frac{2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)}{\sqrt{c}}+}{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right.}{\sqrt{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/15*(15*sqrt(2)*((A + B)*a^2*c*cos(f*x + e) - (A + B)*a^2*c*sin(f*x + e) +
 (A + B)*a^2*c)*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) + 2*
sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(c)
 + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) -
cos(f*x + e) - 2))/sqrt(c) + (3*B*a^2*cos(f*x + e)^3 + (5*A + 14*B)*a^2*cos
(f*x + e)^2 - (35*A + 41*B)*a^2*cos(f*x + e) - 4*(10*A + 13*B)*a^2 + (3*B*a
^2*cos(f*x + e)^2 - (5*A + 11*B)*a^2*cos(f*x + e) - 4*(10*A + 13*B)*a^2)*si
n(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c*f*cos(f*x + e) - c*f*sin(f*x + e)
 + c*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)2/f*(2/sqrt(c*tan((f*x+exp(1))/2)^2+c)/(c*tan((f*x+exp(1))/2)^2+
c)^2*(tan((f*x+exp(1))/2)*(tan((f*x+exp(1))/2)*(tan((f*x+exp(1))/2)*(tan((f
```

```
*x+exp(1))/2)*(1/3600*tan((f*x+exp(1))/2)*(4200*A*a^2*c^2*sign(tan((f*x+exp
(1))/2)-1)+4560*B*a^2*c^2*sign(tan((f*x+exp(1))/2)-1))+1/3600*(5400*A*a^2*c
^2*sign(tan((f*x+exp(1))/2)-1)+7200*B*a^2*c^2*sign(tan((f*x+exp(1))/2)-1)))
+1/3600*(9600*A*a^2*c^2*sign(tan((f*x+exp(1))/2)-1)+13200*B*a^2*c^2*sign(ta
n((f*x+exp(1))/2)-1)))+1/3600*(9600*A*a^2*c^2*sign(tan((f*x+exp(1))/2)-1)+1
3200*B*a^2*c^2*sign(tan((f*x+exp(1))/2)-1)))+1/3600*(5400*A*a^2*c^2*sign(ta
n((f*x+exp(1))/2)-1)+7200*B*a^2*c^2*sign(tan((f*x+exp(1))/2)-1)))+1/3600*(4
200*A*a^2*c^2*sign(tan((f*x+exp(1))/2)-1)+4560*B*a^2*c^2*sign(tan((f*x+exp(
1))/2)-1)))+sqrt(2)*(4*A*a^2+4*B*a^2)*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sq
rt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(-c)/sign(tan(
(f*x+exp(1))/2)-1)+(-60*A*a^2*c*sqrt(2)*atan(sqrt(c)/sqrt(-c))-40*A*a^2*sqr
t(-c)*sqrt(2)*sqrt(c)-60*B*a^2*c*sqrt(2)*atan(sqrt(c)/sqrt(-c))-52*B*a^2*sq
rt(-c)*sqrt(2)*sqrt(c))/15/c/sqrt(-c)*sign(tan((f*x+exp(1))/2)-1))
```

maple [A] time = 1.49, size = 197, normalized size = 1.22

$$\frac{2\left(\sin\left(fx+e\right)-1\right)\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,a^{2}\left(-30c^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{2}}{2\sqrt{c}}\right)A-30c^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{}}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/15*(sin(f*x+e)-1)*(c*(1+sin(f*x+e)))^(1/2)*a^2*(-30*c^(5/2)*2^(1/2)*arcta
nh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*A-30*c^(5/2)*2^(1/2)*arcta
nh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*B+3*B*(c*(1+sin(f*x+e)))^(
5/2)+5*A*(c*(1+sin(f*x+e)))^(3/2)*c+5*B*(c*(1+sin(f*x+e)))^(3/2)*c+30*A*c^2
*(c*(1+sin(f*x+e)))^(1/2)+30*B*c^2*(c*(1+sin(f*x+e)))^(1/2))/c^3/cos(f*x+e)
/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{2}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/sqrt(-c*sin(f*x + e)
+ c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^2}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(1/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(1/2
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{A}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int \frac{2A\sin\left(e+fx\right)}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int \frac{A\sin^2\left(e+fx\right)}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int \frac{B\sin\left(e+f\right.}{\sqrt{-c\sin\left(e+f\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a**2*(Integral(A/sqrt(-c*sin(e + f*x) + c), x) + Integral(2*A*sin(e + f*x)/
sqrt(-c*sin(e + f*x) + c), x) + Integral(A*sin(e + f*x)**2/sqrt(-c*sin(e +
f*x) + c), x) + Integral(B*sin(e + f*x)/sqrt(-c*sin(e + f*x) + c), x) + Int
egral(2*B*sin(e + f*x)**2/sqrt(-c*sin(e + f*x) + c), x) + Integral(B*sin(e
+ f*x)**3/sqrt(-c*sin(e + f*x) + c), x))
```

3.94 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=176

$$-\frac{\sqrt{2}\,a^2(3A+7B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}+\frac{a^2c^2(A+B)\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}+\frac{a^2(3A+7B)\cos^3(e+fx)}{6f(c-c\sin(e+fx))^{3/2}}+\frac{a^2(3A+7}{cf\sqrt{c-c}}$$

```
[Out] 1/2*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(7/2)+1/6*a^2*(3*A+7*B)*c
os(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)-a^2*(3*A+7*B)*arctanh(1/2*cos(f*x+e)*c
^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))*2^(1/2)/c^(3/2)/f+a^2*(3*A+7*B)*cos(
f*x+e)/c/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2967, 2859, 2679, 2649, 206}

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{\sqrt{2}\,a^2(3A+7B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}+\frac{a^2(3A+7B)\cos^3(e+fx)}{6f(c-c\sin(e+fx))^{3/2}}+\frac{a^2(3A+7B)}{cf\sqrt{c-c\,\mathrm{si}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2
),x]
```

```
[Out] -((Sqrt[2]*a^2*(3*A + 7*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c -
 c*Sin[e + f*x]])])/(c^(3/2)*f)) + (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(2*f*(c
 - c*Sin[e + f*x])^(7/2)) + (a^2*(3*A + 7*B)*Cos[e + f*x]^3)/(6*f*(c - c*Si
n[e + f*x])^(3/2)) + (a^2*(3*A + 7*B)*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e +
 f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}} - \frac{1}{4}\left(a^2(3A+7B)c\right)\int \frac{\cos^4(e+}{(c-c\sin(e+}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}} + \frac{a^2(3A+7B)\cos^3(e+fx)}{6f(c-c\sin(e+fx))^{3/2}} - \frac{1}{2}\left(a^2(\right.$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}} + \frac{a^2(3A+7B)\cos^3(e+fx)}{6f(c-c\sin(e+fx))^{3/2}} + \frac{a^2(3A}{cf\sqrt{}}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}} + \frac{a^2(3A+7B)\cos^3(e+fx)}{6f(c-c\sin(e+fx))^{3/2}} + \frac{a^2(3A}{cf\sqrt{}}$$

$$= -\frac{\sqrt{2}\,a^2(3A+7B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f} + \frac{a^2(A+B)c^2\cos}{2f(c-c\sin(e+}$$

Mathematica [C] time = 0.90, size = 355, normalized size = 2.02

$$\frac{a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(12(A+B)\sin\left(\frac{1}{2}(e+fx)\right)+3(2A+7B)\cos\left(\frac{1}{2}(e+f\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(6*(A + B)*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + (6 + 6*I)*(-1)^(1/4)*(3*A + 7*B)*Ar
cTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^2 + 3*(2*A + 7*B)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])^2 - B*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^2 + 12*(A + B)*Sin[(e + f*x)/2] + 3*(2*A + 7*B)*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])^2*Sin[(e + f*x)/2] + B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
2*Sin[(3*(e + f*x))/2]))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c -
c*Sin[e + f*x])^(3/2))
```

fricas [B] time = 0.45, size = 385, normalized size = 2.19

$$\frac{3\sqrt{2}\left((3A+7B)a^2c\cos\left(fx+e\right)^2-(3A+7B)a^2c\cos\left(fx+e\right)-2\left(3A+7B\right)a^2c+\left((3A+7B)a^2c\cos\left(fx+e\right)+2\left(3A+7B\right)a^2c\right)\sin\left(fx+e\right)\right)\log\left(-\frac{\cos\left(fx+e\right)^2+\left(\cos\right.}{}\right.}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/6*(3*sqrt(2)*((3*A + 7*B)*a^2*c*cos(f*x + e)^2 - (3*A + 7*B)*a^2*c*cos(f*
x + e) - 2*(3*A + 7*B)*a^2*c + ((3*A + 7*B)*a^2*c*cos(f*x + e) + 2*(3*A + 7
*B)*a^2*c)*sin(f*x + e))*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x
+ e) - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1
)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*
x + e) - cos(f*x + e) - 2))/sqrt(c) - 4*(B*a^2*cos(f*x + e)^3 + (3*A + 10*B
)*a^2*cos(f*x + e)^2 + 6*(A + 2*B)*a^2*cos(f*x + e) + 3*(A + B)*a^2 + (B*a^
2*cos(f*x + e)^2 - 3*(A + 3*B)*a^2*cos(f*x + e) + 3*(A + B)*a^2)*sin(f*x +
e))*sqrt(-c*sin(f*x + e) + c))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) -
 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2/sqrt(c*t
an((f*x+exp(1))/2)^2+c)/(c*tan((f*x+exp(1))/2)^2+c)*(tan((f*x+exp(1))/2)*(t
an((f*x+exp(1))/2)*(-1/144*tan((f*x+exp(1))/2)*(72*A*a^2*sign(tan((f*x+exp(
1))/2)-1)+240*B*a^2*sign(tan((f*x+exp(1))/2)-1))-1/144*(72*A*a^2*sign(tan((
f*x+exp(1))/2)-1)+288*B*a^2*sign(tan((f*x+exp(1))/2)-1)))-1/144*(72*A*a^2*s
ign(tan((f*x+exp(1))/2)-1)+288*B*a^2*sign(tan((f*x+exp(1))/2)-1)))-1/144*(7
```

```
2*A*a^2*sign(tan((f*x+exp(1))/2)-1)+240*B*a^2*sign(tan((f*x+exp(1))/2)-1)))
+2*((-3*A*a^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c)
)^3-3*B*a^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^
3+A*a^2*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-A*
a^2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^
2+B*a^2*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-B*
a^2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^
2-A*a^2*sqrt(c)*c-B*a^2*sqrt(c)*c)/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*t
an((f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*t
an((f*x+exp(1))/2)^2+c))+c)^2/c/sign(tan((f*x+exp(1))/2)-1)+1/2*(-6*A*a^2-1
4*B*a^2)*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))
/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/sqrt(-c)/c/sign(tan((f*x+exp(1))/2)-1))
)
```

maple [A] time = 1.46, size = 282, normalized size = 1.60

$$\frac{a^2\left(\sin\left(fx+e\right)\left(9A\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c+c\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{c}}\right)c^2-6A\sqrt{c+c\sin\left(fx+e\right)}\ c^{\frac{3}{2}}+21B\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(}}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/3*a^2*(sin(f*x+e)*(9*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)
/c^(1/2))*c^2-6*A*(c+c*sin(f*x+e))^(1/2)*c^(3/2)+21*B*2^(1/2)*arctanh(1/2*(
c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2-18*B*(c+c*sin(f*x+e))^(1/2)*c^(3
/2)-2*B*(c+c*sin(f*x+e))^(3/2)*c^(1/2))-9*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*
x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+12*A*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-21*B*2
^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+24*B*(c+c*si
n(f*x+e))^(1/2)*c^(3/2)+2*B*(c+c*sin(f*x+e))^(3/2)*c^(1/2))*(c*(1+sin(f*x+e
)))^(1/2)/c^(7/2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^2}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)
^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^2}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(3/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(3/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.95 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=175

$$\frac{3a^2(A+9B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}c^{5/2}f}+\frac{a^2c^2(A+B)\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{3a^2(A+9B)\cos(e+fx)}{8c^2f\sqrt{c-c\sin(e+fx)}}-\frac{a^2(A+9B)\cos^3(\ldots}{8f(c-c\sin(e+\ldots}$$

```
[Out] 1/4*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(9/2)-1/8*a^2*(A+9*B)*cos
(f*x+e)^3/f/(c-c*sin(f*x+e))^(5/2)+3/8*a^2*(A+9*B)*arctanh(1/2*cos(f*x+e)*c
^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(5/2)/f*2^(1/2)-3/8*a^2*(A+9*B)*co
s(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2967, 2859, 2680, 2679, 2649, 206}

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{3a^2(A+9B)\cos(e+fx)}{8c^2f\sqrt{c-c\sin(e+fx)}}+\frac{3a^2(A+9B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}c^{5/2}f}-\frac{a^2(A+9B)\cos^3(\ldots}{8f(c-c\sin(e+\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/2
),x]
```

```
[Out] (3*a^2*(A + 9*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e +
 f*x]])])/(4*Sqrt[2]*c^(5/2)*f) + (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(4*f*(c
- c*Sin[e + f*x])^(9/2)) - (a^2*(A + 9*B)*Cos[e + f*x]^3)/(8*f*(c - c*Sin[e
 + f*x])^(5/2)) - (3*a^2*(A + 9*B)*Cos[e + f*x])/(8*c^2*f*Sqrt[c - c*Sin[e
+ f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}} - \frac{1}{8}\left(a^2(A+9B)c\right)\int \frac{\cos^4(e+f}{(c-c\sin(e+f}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}} - \frac{a^2(A+9B)\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{5/2}} + \frac{\left(3a^2(A\right.}{}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}} - \frac{a^2(A+9B)\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{5/2}} - \frac{3a^2(A}{8c^2f\sqrt{}}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}} - \frac{a^2(A+9B)\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{5/2}} - \frac{3a^2(A}{8c^2f\sqrt{}}$$

$$= \frac{3a^2(A+9B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}c^{5/2}f} + \frac{a^2(A+B)c^2\cos^5(e+}{4f(c-c\sin(e+fx))}$$

Mathematica [C] time = 1.17, size = 344, normalized size = 1.97

$$\frac{a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(8(A+B)\sin\left(\frac{1}{2}(e+fx)\right)-(5A+13B)\left(\cos\left(\frac{1}{2}(e+f\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(4*(A + B)*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2]) - (5*A + 13*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 - (
3 + 3*I)*(-1)^(1/4)*(A + 9*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f
*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 8*B*Cos[(e + f*x)/2]*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 8*(A + B)*Sin[(e + f*x)/2] - 2*(5*A
+ 13*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] - 8*B*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)
/(4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(5/2))
```

fricas [B] time = 0.45, size = 449, normalized size = 2.57

$$\frac{3\sqrt{2}\left((A+9B)a^2\cos\left(fx+e\right)^3+3(A+9B)a^2\cos\left(fx+e\right)^2-2(A+9B)a^2\cos\left(fx+e\right)-4(A+9B)a^2-\left((A\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] 1/16*(3*sqrt(2)*((A + 9*B)*a^2*cos(f*x + e)^3 + 3*(A + 9*B)*a^2*cos(f*x + e
)^2 - 2*(A + 9*B)*a^2*cos(f*x + e) - 4*(A + 9*B)*a^2 - ((A + 9*B)*a^2*cos(f
*x + e)^2 - 2*(A + 9*B)*a^2*cos(f*x + e) - 4*(A + 9*B)*a^2)*sin(f*x + e))*s
qrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)
*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2
*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) -
 cos(f*x + e) - 2)) - 4*(8*B*a^2*cos(f*x + e)^3 - (5*A + 21*B)*a^2*cos(f*x
+ e)^2 - (A + 25*B)*a^2*cos(f*x + e) + 4*(A + B)*a^2 + (8*B*a^2*cos(f*x + e
)^2 + (5*A + 29*B)*a^2*cos(f*x + e) + 4*(A + B)*a^2)*sin(f*x + e))*sqrt(-c*
sin(f*x + e) + c))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f
*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*
c^3*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)2/f*(2*sqrt(c*tan((f*x+exp(1))/2)^2+c)*(1/2*B*a^2/c^2/
sign(tan((f*x+exp(1))/2)-1)+1/2*B*a^2*tan((f*x+exp(1))/2)/c^2/sign(tan((f*x
+exp(1))/2)-1))/(c*tan((f*x+exp(1))/2)^2+c)+2*(1/8*(-5*A*a^2*(-sqrt(c)*tan(
(f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7+19*B*a^2*(-sqrt(c)*tan((
f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7+29*A*a^2*sqrt(c)*(-sqrt(c
)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6+133*B*a^2*sqrt(c)*
(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6+17*A*a^2*c
*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5+89*B*a^2*
c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5-13*A*a^2
```

```
*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4
-117*B*a^2*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/
2)^2+c))^4+9*A*a^2*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1)
)/2)^2+c))^3+17*B*a^2*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp
(1))/2)^2+c))^3-13*A*a^2*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+
exp(1))/2)^2+c))-9*A*a^2*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*t
an((f*x+exp(1))/2)^2+c))^2-53*B*a^2*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(
c*tan((f*x+exp(1))/2)^2+c))+47*B*a^2*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))
/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+A*a^2*sqrt(c)*c^3+9*B*a^2*sqrt(c)*c^
3)/c^2/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2
*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^
4/sign(tan((f*x+exp(1))/2)-1)+1/8*(3*A*a^2+27*B*a^2)*atan((-sqrt(c)*tan((f*
x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqr
t(2)/c^2/sqrt(-c)/sign(tan((f*x+exp(1))/2)-1)))
```

maple [B] time = 1.60, size = 386, normalized size = 2.21

$$-\frac{a^2\left(3A\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\left(\sin^2\left(fx+e\right)\right)c^2+27B\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\left(\sin^2\left(fx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/8/c^(9/2)*a^2*(3*A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/
c^(1/2))*sin(f*x+e)^2*c^2+27*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)
*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^2-6*A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)
))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^2-16*B*(c*(1+sin(f*x+e)))^(1/2)*c^(3
/2)*sin(f*x+e)^2-54*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/
c^(1/2))*sin(f*x+e)*c^2+10*A*(c*(1+sin(f*x+e)))^(3/2)*c^(1/2)+3*A*2^(1/2)*a
rctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^2+26*B*(c*(1+sin(f*x
+e)))^(3/2)*c^(1/2)+32*B*c^(3/2)*(c*(1+sin(f*x+e)))^(1/2)*sin(f*x+e)+27*B*2
^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^2-12*A*(c*(1
+sin(f*x+e)))^(1/2)*c^(3/2)-60*B*(c*(1+sin(f*x+e)))^(1/2)*c^(3/2))*(c*(1+si
n(f*x+e)))^(1/2)/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^2}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)
^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(5/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(5/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.96 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=175

$$\frac{a^2(A-11B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}c^{7/2}f}+\frac{a^2c^2(A+B)\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}}-\frac{a^2(A-11B)\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{3/2}}+\frac{a^2(A-11B)\cos}{24f(c-c\sin(e}$$

```
[Out] 1/6*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(11/2)+1/24*a^2*(A-11*B)*
cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(7/2)-1/16*a^2*(A-11*B)*cos(f*x+e)/c^2/f/(c
-c*sin(f*x+e))^(3/2)+1/32*a^2*(A-11*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/
2)/(c-c*sin(f*x+e))^(1/2))/c^(7/2)/f*2^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2967, 2859, 2680, 2649, 206}

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}}-\frac{a^2(A-11B)\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{3/2}}+\frac{a^2(A-11B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}c^{7/2}f}+\frac{a^2(A-11B)\cos}{24f(c-c\sin(e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/2
),x]
```

```
[Out] (a^2*(A - 11*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e +
f*x]])])/(16*Sqrt[2]*c^(7/2)*f) + (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(6*f*(c
- c*Sin[e + f*x])^(11/2)) + (a^2*(A - 11*B)*Cos[e + f*x]^3)/(24*f*(c - c*Si
n[e + f*x])^(7/2)) - (a^2*(A - 11*B)*Cos[e + f*x])/(16*c^2*f*(c - c*Sin[e +
 f*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}} + \frac{1}{12}\left(a^2(A-11B)c\right)\int \frac{\cos^4(e+}{(c-c\sin(e+}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}} + \frac{a^2(A-11B)\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{7/2}} - \frac{\left(a^2(A\right.}{}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}} + \frac{a^2(A-11B)\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{7/2}} - \frac{a^2(A}{16c^2f}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}} + \frac{a^2(A-11B)\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{7/2}} - \frac{a^2(A}{16c^2f}$$

$$= \frac{a^2(A-11B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}c^{7/2}f} + \frac{a^2(A+B)c^2\cos^5(e+}{6f(c-c\sin(e+fx))^1}$$

Mathematica [C] time = 1.77, size = 342, normalized size = 1.95

$$\frac{a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(64(A+B)\sin\left(\frac{1}{2}(e+fx)\right)+3(A+21B)\left(\cos\left(\frac{1}{2}(e+f\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(7/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(32*(A + B)*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]) - 4*(7*A + 19*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3
+ 3*(A + 21*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 - (3 + 3*I)*(-1)^(1/
4)*(A - 11*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^6 + 64*(A + B)*Sin[(e + f*x)/2] - 8*(7*A + 1
9*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 6*(A + 21*B
)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2])*(1 + Sin[e + f*
x])^2)/(48*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(
7/2))
```

fricas [B] time = 0.46, size = 521, normalized size = 2.98

$$-\frac{3\sqrt{2}\left((A-11\,B)a^2\cos\left(fx+e\right)^4-3\,(A-11\,B)a^2\cos\left(fx+e\right)^3-8\,(A-11\,B)a^2\cos\left(fx+e\right)^2+4\,(A-11\,B)a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] -1/192*(3*sqrt(2)*((A - 11*B)*a^2*cos(f*x + e)^4 - 3*(A - 11*B)*a^2*cos(f*x
 + e)^3 - 8*(A - 11*B)*a^2*cos(f*x + e)^2 + 4*(A - 11*B)*a^2*cos(f*x + e) +
 8*(A - 11*B)*a^2 + ((A - 11*B)*a^2*cos(f*x + e)^3 + 4*(A - 11*B)*a^2*cos(f
*x + e)^2 - 4*(A - 11*B)*a^2*cos(f*x + e) - 8*(A - 11*B)*a^2)*sin(f*x + e))
*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(
c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) -
 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e)
 - cos(f*x + e) - 2)) + 4*(3*(A + 21*B)*a^2*cos(f*x + e)^3 + (25*A + 13*B)*
a^2*cos(f*x + e)^2 - 2*(5*A + 41*B)*a^2*cos(f*x + e) - 32*(A + B)*a^2 + (3*
(A + 21*B)*a^2*cos(f*x + e)^2 - 2*(11*A - 25*B)*a^2*cos(f*x + e) - 32*(A +
B)*a^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^4*f*cos(f*x + e)^4 - 3*
c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^
4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e)
 - 8*c^4*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*2*(1/96*(-93*A*a^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*
x+exp(1))/2)^2+c))^11-33*B*a^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*
x+exp(1))/2)^2+c))^11-63*A*a^2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c
*tan((f*x+exp(1))/2)^2+c))^10-171*B*a^2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/
2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^10-477*A*a^2*c*(-sqrt(c)*tan((f*x+exp(1
))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^9-1953*B*a^2*c*(-sqrt(c)*tan((f*x+ex
p(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^9+93*A*a^2*sqrt(c)*c*(-sqrt(c)*ta
n((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^8-1503*B*a^2*sqrt(c)*c*(
```

```
-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^8+654*A*a^2*c
^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7+2214*B*
a^2*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7+26
6*A*a^2*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2
)^2+c))^6+722*B*a^2*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f
*x+exp(1))/2)^2+c))^6-522*A*a^2*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*ta
n((f*x+exp(1))/2)^2+c))^5-978*B*a^2*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(
c*tan((f*x+exp(1))/2)^2+c))^5-198*A*a^2*sqrt(c)*c^3*(-sqrt(c)*tan((f*x+exp(
1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4+1218*B*a^2*sqrt(c)*c^3*(-sqrt(c)*
tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4+223*A*a^2*c^4*(-sqrt
(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-53*B*a^2*c^4*(-s
qrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-9*A*a^2*c^5*(
-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-315*A*a^2*sqr
t(c)*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+1
95*B*a^2*c^5*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))
-855*B*a^2*sqrt(c)*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1)
)/2)^2+c))^2-7*A*a^2*sqrt(c)*c^5-19*B*a^2*sqrt(c)*c^5)/c^3/(-(-sqrt(c)*tan(
(f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan(
(f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^6/sign(tan((f*x+exp(1))
/2)-1)+1/32*(A*a^2-11*B*a^2)*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqr
t(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/c^3/sqrt(-c)/sign(t
an((f*x+exp(1))/2)-1))
```

maple [B] time = 1.69, size = 354, normalized size = 2.02

$$-\frac{a^{2}\left(-3\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c+c\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{c}}\right)c^{3}\left(A-11B\right)\sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)+12\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c+c\sin\left(fx+e\right.}}{2\sqrt{c}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] -1/96*a^2*(-3*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c
^3*(A-11*B)*sin(f*x+e)*cos(f*x+e)^2+12*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))
^(1/2)*2^(1/2)/c^(1/2))*c^3*(A-11*B)*sin(f*x+e)+9*2^(1/2)*arctanh(1/2*(c+c*
sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3*(A-11*B)*cos(f*x+e)^2+24*A*(c+c*sin(
f*x+e))^(1/2)*c^(5/2)-32*A*(c+c*sin(f*x+e))^(3/2)*c^(3/2)-6*A*(c+c*sin(f*x+
e))^(5/2)*c^(1/2)-264*B*(c+c*sin(f*x+e))^(1/2)*c^(5/2)+352*B*(c+c*sin(f*x+e
))^(3/2)*c^(3/2)-126*B*(c+c*sin(f*x+e))^(5/2)*c^(1/2)-12*A*2^(1/2)*arctanh(
1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3+132*B*2^(1/2)*arctanh(1/2*(
c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3)*(c*(1+sin(f*x+e)))^(1/2)/c^(13/
2)/(sin(f*x+e)-1)^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)
^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(7/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(7/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.97 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=222

$$\frac{a^2(3A-13B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{256\sqrt{2}\,c^{9/2}f}+\frac{a^2(3A-13B)\cos(e+fx)}{256c^3f(c-c\sin(e+fx))^{3/2}}+\frac{a^2c^2(A+B)\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}}-\frac{a^2(3A-13B\ldots}{64c^2f(c-c\,\mathrm{s}\ldots}$$

```
[Out] 1/8*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(13/2)+1/48*a^2*(3*A-13*B
)*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(9/2)-1/64*a^2*(3*A-13*B)*cos(f*x+e)/c^2/
f/(c-c*sin(f*x+e))^(5/2)+1/256*a^2*(3*A-13*B)*cos(f*x+e)/c^3/f/(c-c*sin(f*x
+e))^(3/2)+1/512*a^2*(3*A-13*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c
*sin(f*x+e))^(1/2))/c^(9/2)/f*2^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2967, 2859, 2680, 2650, 2649, 206}

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}}+\frac{a^2(3A-13B)\cos(e+fx)}{256c^3f(c-c\sin(e+fx))^{3/2}}-\frac{a^2(3A-13B)\cos(e+fx)}{64c^2f(c-c\sin(e+fx))^{5/2}}+\frac{a^2(3A-13B)\tanh^{-1}\left(\frac{\ldots}{\sqrt{2}\ldots}\right.}{256\sqrt{2}\,c^{9/2}f\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(9/2
),x]
```

```
[Out] (a^2*(3*A - 13*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e
+ f*x]])])/(256*Sqrt[2]*c^(9/2)*f) + (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(8*f*
(c - c*Sin[e + f*x])^(13/2)) + (a^2*(3*A - 13*B)*Cos[e + f*x]^3)/(48*f*(c -
 c*Sin[e + f*x])^(9/2)) - (a^2*(3*A - 13*B)*Cos[e + f*x])/(64*c^2*f*(c - c*
Sin[e + f*x])^(5/2)) + (a^2*(3*A - 13*B)*Cos[e + f*x])/(256*c^3*f*(c - c*Si
n[e + f*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}} + \frac{1}{16}\left(a^2(3A-13B)c\right)\int \frac{\cos^4(e}{(c-c\sin(e}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}} + \frac{a^2(3A-13B)\cos^3(e+fx)}{48f(c-c\sin(e+fx))^{9/2}} - \frac{\left(a^2(3\right.}{}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}} + \frac{a^2(3A-13B)\cos^3(e+fx)}{48f(c-c\sin(e+fx))^{9/2}} - \frac{a^2(3}{64c^2}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}} + \frac{a^2(3A-13B)\cos^3(e+fx)}{48f(c-c\sin(e+fx))^{9/2}} - \frac{a^2(3}{64c^2}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}} + \frac{a^2(3A-13B)\cos^3(e+fx)}{48f(c-c\sin(e+fx))^{9/2}} - \frac{a^2(3}{64c^2}$$

$$= \frac{a^2(3A-13B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{256\sqrt{2}\,c^{9/2}f} + \frac{a^2(A+B)c^2\cos^5(e+}{8f(c-c\sin(e+fx)}$$

Mathematica [C] time = 2.69, size = 357, normalized size = 1.61

$$\frac{a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((-24-24i)\sqrt[4]{-1}\,(3A-13B)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\,\left(\tan\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(9/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(2013*A*Cos
[(e + f*x)/2] + 1517*B*Cos[(e + f*x)/2] - 999*A*Cos[(3*(e + f*x))/2] - 791*
B*Cos[(3*(e + f*x))/2] - 69*A*Cos[(5*(e + f*x))/2] - 725*B*Cos[(5*(e + f*x)
)/2] - 9*A*Cos[(7*(e + f*x))/2] + 39*B*Cos[(7*(e + f*x))/2] - (24 + 24*I)*(
-1)^(1/4)*(3*A - 13*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])
]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8 + 2013*A*Sin[(e + f*x)/2] + 1517*
B*Sin[(e + f*x)/2] + 999*A*Sin[(3*(e + f*x))/2] + 791*B*Sin[(3*(e + f*x))/2
] - 69*A*Sin[(5*(e + f*x))/2] - 725*B*Sin[(5*(e + f*x))/2] + 9*A*Sin[(7*(e
```

```
+ f*x))/2] - 39*B*Sin[(7*(e + f*x))/2]))/(6144*f*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])^4*(c - c*Sin[e + f*x])^(9/2))
```

fricas [B] time = 0.49, size = 654, normalized size = 2.95

$$-\frac{3\sqrt{2}\left((3A-13B)a^2\cos\left(fx+e\right)^5+5(3A-13B)a^2\cos\left(fx+e\right)^4-8(3A-13B)a^2\cos\left(fx+e\right)^3-20(3A-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="fricas")
```

```
[Out] -1/3072*(3*sqrt(2)*((3*A - 13*B)*a^2*cos(f*x + e)^5 + 5*(3*A - 13*B)*a^2*co
s(f*x + e)^4 - 8*(3*A - 13*B)*a^2*cos(f*x + e)^3 - 20*(3*A - 13*B)*a^2*cos(
f*x + e)^2 + 8*(3*A - 13*B)*a^2*cos(f*x + e) + 16*(3*A - 13*B)*a^2 - ((3*A
- 13*B)*a^2*cos(f*x + e)^4 - 4*(3*A - 13*B)*a^2*cos(f*x + e)^3 - 12*(3*A -
13*B)*a^2*cos(f*x + e)^2 + 8*(3*A - 13*B)*a^2*cos(f*x + e) + 16*(3*A - 13*B
)*a^2)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin
(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e)
 + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x +
e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(3*(3*A - 13*B)*a^2*cos(f*x +
 e)^4 + (39*A + 343*B)*a^2*cos(f*x + e)^3 + 2*(129*A + 209*B)*a^2*cos(f*x +
 e)^2 - 12*(13*A + 29*B)*a^2*cos(f*x + e) - 384*(A + B)*a^2 - (3*(3*A - 13*
B)*a^2*cos(f*x + e)^3 - 2*(15*A + 191*B)*a^2*cos(f*x + e)^2 + 12*(19*A + 3*
B)*a^2*cos(f*x + e) + 384*(A + B)*a^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) +
 c))/(c^5*f*cos(f*x + e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^
3 - 20*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(
f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos
(f*x + e) + 16*c^5*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*2*(1/1536*(-1527*A*a^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan
((f*x+exp(1))/2)^2+c))^15-39*B*a^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan
```

```
((f*x+exp(1))/2)^2+c))^15-4473*A*a^2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+
sqrt(c*tan((f*x+exp(1))/2)^2+c))^14+2487*B*a^2*sqrt(c)*(-sqrt(c)*tan((f*x+e
xp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^14-22233*A*a^2*c*(-sqrt(c)*tan((
f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^13-7593*B*a^2*c*(-sqrt(c)*t
an((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^13-23811*A*a^2*sqrt(c)*
c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^12+1293*B*
a^2*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c)
)^12+2133*A*a^2*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2
)^2+c))^11-1563*B*a^2*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp
(1))/2)^2+c))^11+68019*A*a^2*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt
(c*tan((f*x+exp(1))/2)^2+c))^10-10589*B*a^2*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+
exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^10+25371*A*a^2*c^3*(-sqrt(c)*ta
n((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^9+9355*B*a^2*c^3*(-sqrt(
c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^9-71487*A*a^2*sqrt(
c)*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^8-305
5*B*a^2*sqrt(c)*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2
)^2+c))^8-25173*A*a^2*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp
(1))/2)^2+c))^7+7195*B*a^2*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*
x+exp(1))/2)^2+c))^7+56469*A*a^2*sqrt(c)*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+
sqrt(c*tan((f*x+exp(1))/2)^2+c))^6+15909*B*a^2*sqrt(c)*c^4*(-sqrt(c)*tan((f
*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6-10971*A*a^2*c^5*(-sqrt(c)*
tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5-2123*B*a^2*c^5*(-sqr
t(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5-31881*A*a^2*sqr
t(c)*c^5*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4-3
673*B*a^2*sqrt(c)*c^5*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))
/2)^2+c))^4+17079*A*a^2*c^6*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+e
xp(1))/2)^2+c))^3-5913*B*a^2*c^6*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((
f*x+exp(1))/2)^2+c))^3+345*A*a^2*c^7*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*t
an((f*x+exp(1))/2)^2+c))-7695*A*a^2*sqrt(c)*c^6*(-sqrt(c)*tan((f*x+exp(1))/
2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+41*B*a^2*c^7*(-sqrt(c)*tan((f*x+exp(1
))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-3007*B*a^2*sqrt(c)*c^6*(-sqrt(c)*tan
((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-117*A*a^2*sqrt(c)*c^7-5
*B*a^2*sqrt(c)*c^7)/c^4/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp
(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp
(1))/2)^2+c))+c)^8/sign(tan((f*x+exp(1))/2)-1)+1/512*(3*A*a^2-13*B*a^2)*ata
n((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sq
rt(2)/sqrt(-c))/sqrt(2)/c^4/sqrt(-c)/sign(tan((f*x+exp(1))/2)-1))
```

maple [B] time = 1.70, size = 440, normalized size = 1.98

$$a^2\left(-12\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\ c^5\left(3A-13B\right)\sin\left(fx+e\right)\left(\cos^2\left(fx+e\right)\right)+24\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin\left(fx+e\right)}\ \sqrt{}}{2\sqrt{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] 1/1536/c^(19/2)*a^2*(-12*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2)
)*2^(1/2)*c^5*(3*A-13*B)*sin(f*x+e)*cos(f*x+e)^2+24*arctanh(1/2*(c+c*sin(f*
x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^5*(3*A-13*B)*sin(f*x+e)-3*arctanh(1/
2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^5*(3*A-13*B)*cos(f*x+e)
^4+24*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^5*(3*A-
13*B)*cos(f*x+e)^2+144*A*(c+c*sin(f*x+e))^(1/2)*c^(9/2)-264*A*(c+c*sin(f*x+
e))^(3/2)*c^(7/2)-132*A*(c+c*sin(f*x+e))^(5/2)*c^(5/2)+18*A*(c+c*sin(f*x+e)
)^(7/2)*c^(3/2)-624*B*(c+c*sin(f*x+e))^(1/2)*c^(9/2)+1144*B*(c+c*sin(f*x+e)
)^(3/2)*c^(7/2)-452*B*(c+c*sin(f*x+e))^(5/2)*c^(5/2)-78*B*(c+c*sin(f*x+e))^
(7/2)*c^(3/2)-72*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/
2))*c^5+312*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c
^5)*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)^3/cos(f*x+e)/(c-c*sin(f*x+e))^(
1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)
^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(9/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(9/2
), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),x)
[Out] Timed out
```

3.98 $\int (a + a\sin(e + fx))^3(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}\,dx$

Optimal. Leaf size=210

$$\frac{256a^3c^7(15A-B)\cos^7(e+fx)}{45045f(c-c\sin(e+fx))^{7/2}}+\frac{64a^3c^6(15A-B)\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^{5/2}}+\frac{8a^3c^5(15A-B)\cos^7(e+fx)}{715f(c-c\sin(e+fx))^{3/2}}+\frac{2a^3c^4(15A-B)\cos^7(e+fx)}{195f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 256/45045*a^3*(15*A-B)*c^7*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)+64/6435*a^
3*(15*A-B)*c^6*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(5/2)+8/715*a^3*(15*A-B)*c^5
*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(3/2)+2/195*a^3*(15*A-B)*c^4*cos(f*x+e)^7/
f/(c-c*sin(f*x+e))^(1/2)-2/15*a^3*B*c^3*cos(f*x+e)^7*(c-c*sin(f*x+e))^(1/2)
/f
```

Rubi [A] time = 0.53, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2856, 2674, 2673}

$$\frac{2a^3c^4(15A-B)\cos^7(e+fx)}{195f\sqrt{c-c\sin(e+fx)}}+\frac{8a^3c^5(15A-B)\cos^7(e+fx)}{715f(c-c\sin(e+fx))^{3/2}}+\frac{64a^3c^6(15A-B)\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^{5/2}}+\frac{256a^3c^7(15A-B)\cos^7(e+fx)}{45045f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2),
x]
```

```
[Out] (256*a^3*(15*A - B)*c^7*Cos[e + f*x]^7)/(45045*f*(c - c*Sin[e + f*x])^(7/2)
) + (64*a^3*(15*A - B)*c^6*Cos[e + f*x]^7)/(6435*f*(c - c*Sin[e + f*x])^(5/
2)) + (8*a^3*(15*A - B)*c^5*Cos[e + f*x]^7)/(715*f*(c - c*Sin[e + f*x])^(3/
2)) + (2*a^3*(15*A - B)*c^4*Cos[e + f*x]^7)/(195*f*Sqrt[c - c*Sin[e + f*x]]
) - (2*a^3*B*c^3*Cos[e + f*x]^7*Sqrt[c - c*Sin[e + f*x]])/(15*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
```

```
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(A+B\sin(e+fx))\sqrt{}$$

$$= -\frac{2a^3Bc^3\cos^7(e+fx)\sqrt{c-c\sin(e+fx)}}{15f} +$$

$$= \frac{2a^3(15A-B)c^4\cos^7(e+fx)}{195f\sqrt{c-c\sin(e+fx)}} - \frac{2a^3Bc^3\cos^7}{}$$

$$= \frac{8a^3(15A-B)c^5\cos^7(e+fx)}{715f(c-c\sin(e+fx))^{3/2}} + \frac{2a^3(15A-}{195f\sqrt{c}}$$

$$= \frac{64a^3(15A-B)c^6\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^{5/2}} + \frac{8a^3(15A-}{715f(c-}$$

$$= \frac{256a^3(15A-B)c^7\cos^7(e+fx)}{45045f(c-c\sin(e+fx))^{7/2}} + \frac{64a^3(15A}{6435f(}$$

Mathematica [B] time = 6.83, size = 1569, normalized size = 7.47

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] (5*(8*A - B)*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(
7/2))/(64*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^6) - (5*(6*A + B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^
3*(c - c*Sin[e + f*x])^(7/2))/(192*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (3*(10*A - 3*B)*Cos[(5*(e + f*
x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(320*f*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (3
*(4*A + 3*B)*Cos[(7*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x
])^(7/2))/(448*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^6) + ((12*A - 5*B)*Cos[(9*(e + f*x))/2]*(a + a*Sin[e +
f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(576*f*(Cos[(e + f*x)/2] - Sin[(e + f*x
)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - ((2*A + 5*B)*Cos[(11*(e
+ f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(704*f*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
+ ((2*A - B)*Cos[(13*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*
x])^(7/2))/(832*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^6) - (B*Cos[(15*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(
c - c*Sin[e + f*x])^(7/2))/(960*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (5*(8*A - B)*Sin[(e + f*x)/2]*(a
+ a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(64*f*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (5*(6*A + B)*
(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2)*Sin[(3*(e + f*x))/2])/(19
2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])^6) + (3*(10*A - 3*B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/
2)*Sin[(5*(e + f*x))/2])/(320*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (3*(4*A + 3*B)*(a + a*Sin[e + f*x])
^3*(c - c*Sin[e + f*x])^(7/2)*Sin[(7*(e + f*x))/2])/(448*f*(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((12*A -
 5*B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2)*Sin[(9*(e + f*x))/2
])/(576*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^6) + ((2*A + 5*B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^
(7/2)*Sin[(11*(e + f*x))/2])/(704*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((2*A - B)*(a + a*Sin[e + f*x])
^3*(c - c*Sin[e + f*x])^(7/2)*Sin[(13*(e + f*x))/2])/(832*f*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (B*(a +
 a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2)*Sin[(15*(e + f*x))/2])/(960*f
```

```
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^6)
```

fricas [B] time = 0.46, size = 405, normalized size = 1.93

$$-\frac{2\left(3003\,Ba^3c^3\cos\left(fx+e\right)^8-231\,(15\,A-14\,B)a^3c^3\cos\left(fx+e\right)^7+21\,(15\,A-B)a^3c^3\cos\left(fx+e\right)^6-28\,(15\,A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] -2/45045*(3003*B*a^3*c^3*cos(f*x + e)^8 - 231*(15*A - 14*B)*a^3*c^3*cos(f*x
 + e)^7 + 21*(15*A - B)*a^3*c^3*cos(f*x + e)^6 - 28*(15*A - B)*a^3*c^3*cos(
f*x + e)^5 + 40*(15*A - B)*a^3*c^3*cos(f*x + e)^4 - 64*(15*A - B)*a^3*c^3*c
os(f*x + e)^3 + 128*(15*A - B)*a^3*c^3*cos(f*x + e)^2 - 512*(15*A - B)*a^3*
c^3*cos(f*x + e) - 1024*(15*A - B)*a^3*c^3 - (3003*B*a^3*c^3*cos(f*x + e)^7
 + 231*(15*A - B)*a^3*c^3*cos(f*x + e)^6 + 252*(15*A - B)*a^3*c^3*cos(f*x +
 e)^5 + 280*(15*A - B)*a^3*c^3*cos(f*x + e)^4 + 320*(15*A - B)*a^3*c^3*cos(
f*x + e)^3 + 384*(15*A - B)*a^3*c^3*cos(f*x + e)^2 + 512*(15*A - B)*a^3*c^3
*cos(f*x + e) + 1024*(15*A - B)*a^3*c^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e)
 + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*c)*(5760*A*a^
3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(6*f*x+6*exp(1)+pi))/(19
2*f)^2-9600*A*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(10*f*x+
```

```
10*exp(1)-pi))/(320*f)^2+1344*A*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi)
)*cos(1/4*(14*f*x+14*exp(1)+pi))/(224*f)^2-1728*A*a^3*c^3*f*sign(sin(1/2*(f
*x+exp(1))-1/4*pi))*cos(1/4*(18*f*x+18*exp(1)-pi))/(288*f)^2+1408*A*a^3*c^3
*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(22*f*x+22*exp(1)+pi))/(704*f
)^2-1664*A*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(26*f*x+26*
exp(1)-pi))/(832*f)^2-160*A*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*co
s(1/4*(2*f*x-pi)+1/2*exp(1))/(16*f)^2+1280*B*a^3*c^3*f*sign(sin(1/2*(f*x+ex
p(1))-1/4*pi))*sin(1/4*(2*f*x+2*exp(1)+pi))/(128*f)^2-3840*B*a^3*c^3*f*sign
(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(6*f*x+6*exp(1)-pi))/(384*f)^2+11520
*B*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(10*f*x+10*exp(1)+p
i))/(640*f)^2-16128*B*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*
(14*f*x+14*exp(1)-pi))/(896*f)^2+11520*B*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1)
)-1/4*pi))*sin(1/4*(18*f*x+18*exp(1)+pi))/(1152*f)^2-14080*B*a^3*c^3*f*sign
(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(22*f*x+22*exp(1)-pi))/(1408*f)^2+33
28*B*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(26*f*x+26*exp(1)
+pi))/(1664*f)^2-3840*B*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/
4*(30*f*x+30*exp(1)-pi))/(1920*f)^2)
```

maple [A] time = 1.37, size = 121, normalized size = 0.58

$$\frac{2\left(\sin\left(fx+e\right)-1\right)c^4\left(1+\sin\left(fx+e\right)\right)^4a^3\left(\left(-3465A+12243B\right)\sin\left(fx+e\right)\left(\cos^2\left(fx+e\right)\right)+\left(24780A-2567\right.\right.}{45045\cos\left(fx+e\right)\sqrt{c-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 2/45045*(sin(f*x+e)-1)*c^4*(1+sin(f*x+e))^4*a^3*((-3465*A+12243*B)*sin(f*x+
e)*cos(f*x+e)^2+(24780*A-25676*B)*sin(f*x+e)+3003*B*cos(f*x+e)^4+(14175*A-2
4969*B)*cos(f*x+e)^2-26700*A+25804*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^3\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)
^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^3\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(7/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(7/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.99 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=161

$$\frac{64a^3c^6(13A+B)\cos^7(e+fx)}{9009f(c-c\sin(e+fx))^{7/2}}+\frac{16a^3c^5(13A+B)\cos^7(e+fx)}{1287f(c-c\sin(e+fx))^{5/2}}+\frac{2a^3c^4(13A+B)\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}}-\frac{2a^3Bc^3\cos^7(e+}{13f\sqrt{c-c\sin(e+}}$$

```
[Out] 64/9009*a^3*(13*A+B)*c^6*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)+16/1287*a^3*
(13*A+B)*c^5*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(5/2)+2/143*a^3*(13*A+B)*c^4*c
os(f*x+e)^7/f/(c-c*sin(f*x+e))^(3/2)-2/13*a^3*B*c^3*cos(f*x+e)^7/f/(c-c*sin
(f*x+e))^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2856, 2674, 2673}

$$\frac{2a^3c^4(13A+B)\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}}+\frac{16a^3c^5(13A+B)\cos^7(e+fx)}{1287f(c-c\sin(e+fx))^{5/2}}+\frac{64a^3c^6(13A+B)\cos^7(e+fx)}{9009f(c-c\sin(e+fx))^{7/2}}-\frac{2a^3Bc^3\cos^7(e+}{13f\sqrt{c-c\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (64*a^3*(13*A + B)*c^6*Cos[e + f*x]^7)/(9009*f*(c - c*Sin[e + f*x])^(7/2))
+ (16*a^3*(13*A + B)*c^5*Cos[e + f*x]^7)/(1287*f*(c - c*Sin[e + f*x])^(5/2)
) + (2*a^3*(13*A + B)*c^4*Cos[e + f*x]^7)/(143*f*(c - c*Sin[e + f*x])^(3/2)
) - (2*a^3*B*c^3*Cos[e + f*x]^7)/(13*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
```

```
[m + p, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,d$$

$$= -\frac{2a^3Bc^3\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}} + \frac{1}{13}\left(a^3(13A+B\right.$$

$$= \frac{2a^3(13A+B)c^4\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}} - \frac{2a^3Bc^3\cos}{13f\sqrt{c-c\sin}}$$

$$= \frac{16a^3(13A+B)c^5\cos^7(e+fx)}{1287f(c-c\sin(e+fx))^{5/2}} + \frac{2a^3(13A+}{143f(c-}$$

$$= \frac{64a^3(13A+B)c^6\cos^7(e+fx)}{9009f(c-c\sin(e+fx))^{7/2}} + \frac{16a^3(13A}{1287f(c}$$

Mathematica [B] time = 6.71, size = 1351, normalized size = 8.39

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (5*A*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(8
*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^6) - (5*(4*A + B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*
Sin[e + f*x])^(5/2))/(96*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^6) + ((2*A - B)*Cos[(5*(e + f*x))/2]*(a + a*S
in[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(32*f*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - ((5*A + 2*B)*Cos[(
7*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(112*f*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^6) + ((A - 2*B)*Cos[(9*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e
+ f*x])^(5/2))/(144*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^6) - ((2*A + B)*Cos[(11*(e + f*x))/2]*(a + a*Sin[e
 + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(352*f*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (B*Cos[(13*(e + f*x))
/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(416*f*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (5*A*
Sin[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(8*f*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^6) + (5*(4*A + B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2)*Sin[(3
*(e + f*x))/2])/(96*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^6) + ((2*A - B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[
e + f*x])^(5/2)*Sin[(5*(e + f*x))/2])/(32*f*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((5*A + 2*B)*(a + a*Sin
[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2)*Sin[(7*(e + f*x))/2])/(112*f*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
+ ((A - 2*B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2)*Sin[(9*(e +
f*x))/2])/(144*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^6) + ((2*A + B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f
*x])^(5/2)*Sin[(11*(e + f*x))/2])/(352*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (B*(a + a*Sin[e + f*x])^3*
(c - c*Sin[e + f*x])^(5/2)*Sin[(13*(e + f*x))/2])/(416*f*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

fricas [B] time = 0.44, size = 334, normalized size = 2.07

$$-\frac{2\left(693\,Ba^3c^2\cos\left(fx+e\right)^7+63\,(13\,A+12\,B)a^3c^2\cos\left(fx+e\right)^6-7\,(13\,A+B)a^3c^2\cos\left(fx+e\right)^5+10\,(13\,A+B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] -2/9009*(693*B*a^3*c^2*cos(f*x + e)^7 + 63*(13*A + 12*B)*a^3*c^2*cos(f*x +
e)^6 - 7*(13*A + B)*a^3*c^2*cos(f*x + e)^5 + 10*(13*A + B)*a^3*c^2*cos(f*x
+ e)^4 - 16*(13*A + B)*a^3*c^2*cos(f*x + e)^3 + 32*(13*A + B)*a^3*c^2*cos(f
*x + e)^2 - 128*(13*A + B)*a^3*c^2*cos(f*x + e) - 256*(13*A + B)*a^3*c^2 +
(693*B*a^3*c^2*cos(f*x + e)^6 - 63*(13*A + B)*a^3*c^2*cos(f*x + e)^5 - 70*(
13*A + B)*a^3*c^2*cos(f*x + e)^4 - 80*(13*A + B)*a^3*c^2*cos(f*x + e)^3 - 9
6*(13*A + B)*a^3*c^2*cos(f*x + e)^2 - 128*(13*A + B)*a^3*c^2*cos(f*x + e) -
 256*(13*A + B)*a^3*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x
 + e) - f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)sqrt(2*c)*(16*f*(2*A*a^3*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))+2*B*a^3*c^
2*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(2*f*x+2*exp(1)+pi))/(16*f)^2
+288*f*(2*A*a^3*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))+2*B*a^3*c^2*sign(sin
(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(18*f*x+18*exp(1)+pi))/(288*f)^2-288*f*
(4*A*a^3*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))-2*B*a^3*c^2*sign(sin(1/2*(f
*x+exp(1))-1/4*pi)))*cos(1/4*(18*f*x+18*exp(1)-pi))/(288*f)^2+160*f*(6*A*a^
3*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))+6*B*a^3*c^2*sign(sin(1/2*(f*x+exp(
1))-1/4*pi)))*sin(1/4*(10*f*x+10*exp(1)+pi))/(160*f)^2-16*f*(12*A*a^3*c^2*s
ign(sin(1/2*(f*x+exp(1))-1/4*pi))+2*B*a^3*c^2*sign(sin(1/2*(f*x+exp(1))-1/4
*pi)))*cos(1/4*(2*f*x-pi)+1/2*exp(1))/(16*f)^2-320*f*(32*A*a^3*c^2*sign(sin
(1/2*(f*x+exp(1))-1/4*pi))+2*B*a^3*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*
cos(1/4*(10*f*x+10*exp(1)-pi))/(320*f)^2+48*f*(-2*A*a^3*c^2*sign(sin(1/2*(f
```

```
*x+exp(1))-1/4*pi))-2*B*a^3*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4
*(6*f*x+6*exp(1)-pi))/(48*f)^2+352*f*(-2*A*a^3*c^2*sign(sin(1/2*(f*x+exp(1)
)-1/4*pi))-2*B*a^3*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(22*f*x+
22*exp(1)-pi))/(352*f)^2-224*f*(-4*A*a^3*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*
pi))+2*B*a^3*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(14*f*x+14*exp
(1)+pi))/(224*f)^2+224*f*(-6*A*a^3*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))-6
*B*a^3*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(14*f*x+14*exp(1)-pi
))/(224*f)^2-192*f*(-32*A*a^3*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))-2*B*a^
3*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(6*f*x+6*exp(1)+pi))/(192
*f)^2-1408*B*a^3*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(22*f*x+2
2*exp(1)+pi))/(704*f)^2+1664*B*a^3*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))
*cos(1/4*(26*f*x+26*exp(1)-pi))/(832*f)^2)
```

maple [A] time = 1.34, size = 105, normalized size = 0.65

$$-\frac{2\left(\sin\left(fx+e\right)-1\right)c^3\left(1+\sin\left(fx+e\right)\right)^4 a^3\left(-693B\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+\left(-2366A+2590B\right)\sin\left(fx\right.\right.}{9009\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/9009*(sin(f*x+e)-1)*c^3*(1+sin(f*x+e))^4*a^3*(-693*B*cos(f*x+e)^2*sin(f*
x+e)+(-2366*A+2590*B)*sin(f*x+e)+(-819*A+2016*B)*cos(f*x+e)^2+2782*A-2558*B
)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^3\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)
^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^3\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.100 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=124

$$\frac{8a^3c^5(11A+3B)\cos^7(e+fx)}{693f(c-c\sin(e+fx))^{7/2}}+\frac{2a^3c^4(11A+3B)\cos^7(e+fx)}{99f(c-c\sin(e+fx))^{5/2}}-\frac{2a^3Bc^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 8/693*a^3*(11*A+3*B)*c^5*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)+2/99*a^3*(11
*A+3*B)*c^4*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(5/2)-2/11*a^3*B*c^3*cos(f*x+e)
^7/f/(c-c*sin(f*x+e))^(3/2)
```

Rubi [A] time = 0.41, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2856, 2674, 2673}

$$\frac{2a^3c^4(11A+3B)\cos^7(e+fx)}{99f(c-c\sin(e+fx))^{5/2}}+\frac{8a^3c^5(11A+3B)\cos^7(e+fx)}{693f(c-c\sin(e+fx))^{7/2}}-\frac{2a^3Bc^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (8*a^3*(11*A + 3*B)*c^5*Cos[e + f*x]^7)/(693*f*(c - c*Sin[e + f*x])^(7/2))
+ (2*a^3*(11*A + 3*B)*c^4*Cos[e + f*x]^7)/(99*f*(c - c*Sin[e + f*x])^(5/2))
 - (2*a^3*B*c^3*Cos[e + f*x]^7)/(11*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}$$

$$= -\frac{2a^3Bc^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}} + \frac{1}{11}\left(a^3(11A+\right.$$

$$= \frac{2a^3(11A+3B)c^4\cos^7(e+fx)}{99f(c-c\sin(e+fx))^{5/2}} - \frac{2a^3Bc^3}{11f(c-c}$$

$$= \frac{8a^3(11A+3B)c^5\cos^7(e+fx)}{693f(c-c\sin(e+fx))^{7/2}} + \frac{2a^3(11A+}{99f(c-}$$

Mathematica [B] time = 6.50, size = 1157, normalized size = 9.33

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] ((6*A + B)*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/
2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])^6) - ((8*A + 3*B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(
c - c*Sin[e + f*x])^(3/2))/(24*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (B*Cos[(5*(e + f*x))/2]*(a + a*Sin
```

```
[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))/(16*f*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - ((6*A + B)*Cos[(7*(e
 + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))/(112*f*(Cos[
(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
 - ((2*A + 3*B)*Cos[(9*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e +
f*x])^(3/2))/(144*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^6) + (B*Cos[(11*(e + f*x))/2]*(a + a*Sin[e + f*x])^3
*(c - c*Sin[e + f*x])^(3/2))/(176*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((6*A + B)*Sin[(e + f*x)/2]*(a
+ a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))/(8*f*(Cos[(e + f*x)/2] - Si
n[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((8*A + 3*B)*(
a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)*Sin[(3*(e + f*x))/2])/(24*
f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])^6) - (B*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)*Sin[(5*(e +
f*x))/2])/(16*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^6) + ((6*A + B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*
x])^(3/2)*Sin[(7*(e + f*x))/2])/(112*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - ((2*A + 3*B)*(a + a*Sin[e +
f*x])^3*(c - c*Sin[e + f*x])^(3/2)*Sin[(9*(e + f*x))/2])/(144*f*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (B*
(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)*Sin[(11*(e + f*x))/2])/(1
76*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^6)
```

fricas [B] time = 0.45, size = 287, normalized size = 2.31

$$\frac{2\left(63\,Ba^3c\cos\left(fx+e\right)^6-7\,(11\,A+12\,B)a^3c\cos\left(fx+e\right)^5-(187\,A+177\,B)a^3c\cos\left(fx+e\right)^4+2\,(11\,A+3\,B)a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 2/693*(63*B*a^3*c*cos(f*x + e)^6 - 7*(11*A + 12*B)*a^3*c*cos(f*x + e)^5 - (
187*A + 177*B)*a^3*c*cos(f*x + e)^4 + 2*(11*A + 3*B)*a^3*c*cos(f*x + e)^3 -
 4*(11*A + 3*B)*a^3*c*cos(f*x + e)^2 + 16*(11*A + 3*B)*a^3*c*cos(f*x + e) +
 32*(11*A + 3*B)*a^3*c - (63*B*a^3*c*cos(f*x + e)^5 + 7*(11*A + 21*B)*a^3*c
*cos(f*x + e)^4 - 10*(11*A + 3*B)*a^3*c*cos(f*x + e)^3 - 12*(11*A + 3*B)*a^
3*c*cos(f*x + e)^2 - 16*(11*A + 3*B)*a^3*c*cos(f*x + e) - 32*(11*A + 3*B)*a
^3*c)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x +
 e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*c)*(-112*f*(2*A*a^3*c*sign(sin(
1/2*(f*x+exp(1))-1/4*pi))+4*B*a^3*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*cos
(1/4*(14*f*x+14*exp(1)+pi))/(112*f)^2+16*f*(8*A*a^3*c*sign(sin(1/2*(f*x+exp
(1))-1/4*pi))+6*B*a^3*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(2*f*x+
2*exp(1)+pi))/(16*f)^2-8*f*(10*A*a^3*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))+4
*B*a^3*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(2*f*x-pi)+1/2*exp(1))
/(8*f)^2+160*f*(16*A*a^3*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))+10*B*a^3*c*si
gn(sin(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(10*f*x+10*exp(1)+pi))/(160*f)^2-
144*f*(-2*A*a^3*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))-4*B*a^3*c*sign(sin(1/2
*(f*x+exp(1))-1/4*pi)))*cos(1/4*(18*f*x+18*exp(1)-pi))/(144*f)^2+48*f*(-8*A
*a^3*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))-6*B*a^3*c*sign(sin(1/2*(f*x+exp(1
))-1/4*pi)))*sin(1/4*(6*f*x+6*exp(1)-pi))/(48*f)^2+224*f*(-16*A*a^3*c*sign(
sin(1/2*(f*x+exp(1))-1/4*pi))-10*B*a^3*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))
)*sin(1/4*(14*f*x+14*exp(1)-pi))/(224*f)^2+24*A*a^3*c*f*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*cos(1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2-40*A*a^3*c*f*sign(sin
(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(10*f*x+10*exp(1)-pi))/(20*f)^2-576*B*a^
3*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(18*f*x+18*exp(1)+pi))/(28
8*f)^2+704*B*a^3*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(22*f*x+22*
exp(1)-pi))/(352*f)^2)
```

maple [A] time = 1.29, size = 83, normalized size = 0.67

$$\frac{2\left(\sin\left(fx+e\right)-1\right)c^{2}\left(1+\sin\left(fx+e\right)\right)^{4}a^{3}\left(\sin\left(fx+e\right)\left(77A-105B\right)-63B\left(\cos^{2}\left(fx+e\right)\right)-121A+93B\right)}{693\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/693*(sin(f*x+e)-1)*c^2*(1+sin(f*x+e))^4*a^3*(sin(f*x+e)*(77*A-105*B)-63*B
*cos(f*x+e)^2-121*A+93*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{3}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)
^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.101 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx}$

Optimal. Leaf size=81

$$\frac{2a^3c^4(9A+5B)\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}}-\frac{2a^3Bc^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 2/63*a^3*(9*A+5*B)*c^4*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)-2/9*a^3*B*c^3*
cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(5/2)
```

Rubi [A] time = 0.30, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2967, 2856, 2673}

$$\frac{2a^3c^4(9A+5B)\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}}-\frac{2a^3Bc^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a^3*(9*A + 5*B)*c^4*Cos[e + f*x]^7)/(63*f*(c - c*Sin[e + f*x])^(7/2)) -
(2*a^3*B*c^3*Cos[e + f*x]^7)/(9*f*(c - c*Sin[e + f*x])^(5/2))
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
```

```
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$$
$$= -\frac{2a^3Bc^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}} + \frac{1}{9}\left(a^3(9A+5B)c^3\right)$$
$$= \frac{2a^3(9A+5B)c^4\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}} - \frac{2a^3Bc^3\cos^7(}{9f(c-c\sin(e}$$

Mathematica [A] time = 1.01, size = 89, normalized size = 1.10

$$\frac{2a^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7(9A+7B\sin(e+fx)-2B)}{63f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (2*a^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(9*A - 2*B + 7*B*Sin[e + f*x
])*Sqrt[c - c*Sin[e + f*x]])/(63*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

fricas [B] time = 0.44, size = 232, normalized size = 2.86

$$\frac{2\left(7\,Ba^3\cos\left(fx+e\right)^5+(9\,A+26\,B)a^3\cos\left(fx+e\right)^4-(27\,A+29\,B)a^3\cos\left(fx+e\right)^3-4\,(18\,A+17\,B)a^3\cos\left(fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/63*(7*B*a^3*cos(f*x + e)^5 + (9*A + 26*B)*a^3*cos(f*x + e)^4 - (27*A + 29
*B)*a^3*cos(f*x + e)^3 - 4*(18*A + 17*B)*a^3*cos(f*x + e)^2 + 4*(9*A + 5*B)
*a^3*cos(f*x + e) + 8*(9*A + 5*B)*a^3 + (7*B*a^3*cos(f*x + e)^4 - (9*A + 19
*B)*a^3*cos(f*x + e)^3 - 12*(3*A + 4*B)*a^3*cos(f*x + e)^2 + 4*(9*A + 5*B)*
```

```
a^3*cos(f*x + e) + 8*(9*A + 5*B)*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) +
c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*c)*(56*f*(2*A*a^3*sign(sin(1/2*(f*x+exp(1))-
1/4*pi))+6*B*a^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(14*f*x+14*exp
(1)-pi))/(56*f)^2-12*f*(6*A*a^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))+8*B*a^3*
sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2+8
*f*(30*A*a^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))+26*B*a^3*sign(sin(1/2*(f*x+
exp(1))-1/4*pi)))*sin(1/4*(2*f*x+2*exp(1)+pi))/(8*f)^2-8*f*(40*A*a^3*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))+30*B*a^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*c
os(1/4*(2*f*x-pi)+1/2*exp(1))/(8*f)^2+40*f*(-2*A*a^3*sign(sin(1/2*(f*x+exp(
1))-1/4*pi))-6*B*a^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(10*f*x+10
*exp(1)+pi))/(40*f)^2-20*f*(-6*A*a^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))-8*B
*a^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(10*f*x+10*exp(1)-pi))/(20
*f)^2+24*f*(-30*A*a^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))-26*B*a^3*sign(sin(
1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(6*f*x+6*exp(1)-pi))/(24*f)^2+224*B*a^3*
f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(14*f*x+14*exp(1)+pi))/(112*f)
^2-288*B*a^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(18*f*x+18*exp(1)
-pi))/(144*f)^2)
```

maple [A] time = 1.24, size = 65, normalized size = 0.80

$$-\frac{2\left(\sin\left(fx+e\right)-1\right)c\left(1+\sin\left(fx+e\right)\right)^{4}a^{3}\left(7B\sin\left(fx+e\right)+9A-2B\right)}{63\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/63*(sin(f*x+e)-1)*c*(1+sin(f*x+e))^4*a^3*(7*B*sin(f*x+e)+9*A-2*B)/cos(f*
x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{3}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*sqrt(-c*sin(f*x + e)
+ c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int A\sqrt{-c\sin\left(e+fx\right)+c}\,dx+\int 3A\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)dx+\int 3A\sqrt{-c\sin\left(e+fx\right)+c}\sin^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a**3*(Integral(A*sqrt(-c*sin(e + f*x) + c), x) + Integral(3*A*sqrt(-c*sin(e
 + f*x) + c)*sin(e + f*x), x) + Integral(3*A*sqrt(-c*sin(e + f*x) + c)*sin(
e + f*x)**2, x) + Integral(A*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**3, x)
+ Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(3*B*sqrt
(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(3*B*sqrt(-c*sin(e + f*
x) + c)*sin(e + f*x)**3, x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e +
f*x)**4, x))
```

3.102 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=200

$$-\frac{2a^3c^2(A+B)\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}-\frac{4a^3c(A+B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{8a^3(A+B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}+\frac{8\sqrt{2}\,a^3(A+B)\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{2}\sqrt{}}\right.}{\sqrt{c}\,f}$$

```
[Out] -2/7*a^3*B*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)-2/5*a^3*(A+B)*c^2*cos(
f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)-4/3*a^3*(A+B)*c*cos(f*x+e)^3/f/(c-c*sin(f
*x+e))^(3/2)+8*a^3*(A+B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*
x+e))^(1/2))*2^(1/2)/f/c^(1/2)-8*a^3*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(1
/2)
```

Rubi [A] time = 0.52, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2967, 2860, 2679, 2649, 206}

$$-\frac{2a^3c^2(A+B)\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}-\frac{4a^3c(A+B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{8a^3(A+B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}+\frac{8\sqrt{2}\,a^3(A+B)\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{2}\sqrt{}}\right.}{\sqrt{c}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] (8*Sqrt[2]*a^3*(A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*S
in[e + f*x]])])/(Sqrt[c]*f) - (2*a^3*B*c^3*Cos[e + f*x]^7)/(7*f*(c - c*Sin[
e + f*x])^(7/2)) - (2*a^3*(A + B)*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f
*x])^(5/2)) - (4*a^3*(A + B)*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^(3
/2)) - (8*a^3*(A + B)*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$$

$$= -\frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}} + \left(a^3(A+B)c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx)}$$

$$= -\frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}} - \frac{2a^3(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} + \left(2a^3(\right.$$

$$= -\frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}} - \frac{2a^3(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{4a^3(A}{3f(c}$$

$$= -\frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}} - \frac{2a^3(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{4a^3(A}{3f(c}$$

$$= -\frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}} - \frac{2a^3(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{4a^3(A}{3f(c}$$

$$= \frac{8\sqrt{2}\,a^3(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}\,f} - \frac{2a^3Bc^3\cos^7(e+f}{7f(c-c\sin(e+fx}$$

Mathematica [C] time = 1.35, size = 193, normalized size = 0.96

$$-\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(-(448A+6}{420}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e +
f*x]],x]
```

```
[Out] -1/420*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*((67
20 + 6720*I)*(-1)^(1/4)*(A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e +
 f*x)/4])] - 2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-2086*A - 2236*B + 6*
(7*A + 22*B)*Cos[2*(e + f*x)] - (448*A + 673*B)*Sin[e + f*x] + 15*B*Sin[3*(
e + f*x)])))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*Sqrt[c - c*Sin[e +
f*x]])
```

fricas [A] time = 0.45, size = 353, normalized size = 1.76

$$2\left(\frac{210\,\sqrt{2}\left((A+B)a^{3}c\cos\left(fx+e\right)-(A+B)a^{3}c\sin\left(fx+e\right)+(A+B)a^{3}c\right)\log\left(-\frac{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)+\frac{2\,\sqrt{2}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)}{\sqrt{c}}}{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right.}{\sqrt{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/105*(210*sqrt(2)*((A + B)*a^3*c*cos(f*x + e) - (A + B)*a^3*c*sin(f*x + e)
 + (A + B)*a^3*c)*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) +
2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(
c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e)
- cos(f*x + e) - 2))/sqrt(c) - (15*B*a^3*cos(f*x + e)^4 - 3*(7*A + 22*B)*a^
3*cos(f*x + e)^3 - (133*A + 253*B)*a^3*cos(f*x + e)^2 + 4*(133*A + 148*B)*a
^3*cos(f*x + e) + 4*(161*A + 191*B)*a^3 - (15*B*a^3*cos(f*x + e)^3 + 3*(7*A
 + 27*B)*a^3*cos(f*x + e)^2 - 4*(28*A + 43*B)*a^3*cos(f*x + e) - 4*(161*A +
 191*B)*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c*f*cos(f*x + e) - c
*f*sin(f*x + e) + c*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
```

```
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2/sqrt(c*tan((f*x+exp(1))/2)^2+c)
/(c*tan((f*x+exp(1))/2)^2+c)^3*(tan((f*x+exp(1))/2)*(tan((f*x+exp(1))/2)*(t
an((f*x+exp(1))/2)*(tan((f*x+exp(1))/2)*(tan((f*x+exp(1))/2)*(tan((f*x+exp(
1))/2)*(-1/2822400*tan((f*x+exp(1))/2)*(-6867840*A*a^3*c^3*sign(tan((f*x+ex
p(1))/2)-1)-7069440*B*a^3*c^3*sign(tan((f*x+exp(1))/2)-1))-1/2822400*(-9878
400*A*a^3*c^3*sign(tan((f*x+exp(1))/2)-1)-11289600*B*a^3*c^3*sign(tan((f*x+
exp(1))/2)-1)))-1/2822400*(-24743040*A*a^3*c^3*sign(tan((f*x+exp(1))/2)-1)-
28976640*B*a^3*c^3*sign(tan((f*x+exp(1))/2)-1)))-1/2822400*(-27753600*A*a^3
*c^3*sign(tan((f*x+exp(1))/2)-1)-34809600*B*a^3*c^3*sign(tan((f*x+exp(1))/2
)-1)))-1/2822400*(-27753600*A*a^3*c^3*sign(tan((f*x+exp(1))/2)-1)-34809600*
B*a^3*c^3*sign(tan((f*x+exp(1))/2)-1)))-1/2822400*(-24743040*A*a^3*c^3*sign
(tan((f*x+exp(1))/2)-1)-28976640*B*a^3*c^3*sign(tan((f*x+exp(1))/2)-1)))-1/
2822400*(-9878400*A*a^3*c^3*sign(tan((f*x+exp(1))/2)-1)-11289600*B*a^3*c^3*
sign(tan((f*x+exp(1))/2)-1)))-1/2822400*(-6867840*A*a^3*c^3*sign(tan((f*x+e
xp(1))/2)-1)-7069440*B*a^3*c^3*sign(tan((f*x+exp(1))/2)-1)))+sqrt(2)*(8*A*a
^3+8*B*a^3)*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(
1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(-c)/sign(tan((f*x+exp(1))/2)-1)+(-840*A
*a^3*c*sqrt(2)*atan(sqrt(c)/sqrt(-c))-644*A*a^3*sqrt(-c)*sqrt(2)*sqrt(c)-84
0*B*a^3*c*sqrt(2)*atan(sqrt(c)/sqrt(-c))-764*B*a^3*sqrt(-c)*sqrt(2)*sqrt(c)
)/105/c/sqrt(-c)*sign(tan((f*x+exp(1))/2)-1))
```

maple [A] time = 1.56, size = 233, normalized size = 1.16

$$-\frac{2\left(\sin\left(fx+e\right)-1\right)\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\, a^3\left(420c^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{2}}{2\sqrt{c}}\right)A+420c^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/105*(sin(f*x+e)-1)*(c*(1+sin(f*x+e)))^(1/2)*a^3*(420*c^(7/2)*2^(1/2)*arc
tanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*A+420*c^(7/2)*2^(1/2)*ar
ctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*B-15*B*(c*(1+sin(f*x+e)
))^(7/2)-21*A*(c*(1+sin(f*x+e)))^(5/2)*c-21*B*(c*(1+sin(f*x+e)))^(5/2)*c-70
*A*(c*(1+sin(f*x+e)))^(3/2)*c^2-70*B*(c*(1+sin(f*x+e)))^(3/2)*c^2-420*A*c^3
*(c*(1+sin(f*x+e)))^(1/2)-420*B*c^3*(c*(1+sin(f*x+e)))^(1/2))/c^4/cos(f*x+e
)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{3}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/sqrt(-c*sin(f*x + e)
+ c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(1/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(1/2
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int \frac{A}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int \frac{3A\sin\left(e+fx\right)}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int \frac{3A\sin^{2}\left(e+fx\right)}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int \frac{A\sin^{3}\left(e+\right.}{\sqrt{-c\sin\left(e+f\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a**3*(Integral(A/sqrt(-c*sin(e + f*x) + c), x) + Integral(3*A*sin(e + f*x)/
sqrt(-c*sin(e + f*x) + c), x) + Integral(3*A*sin(e + f*x)**2/sqrt(-c*sin(e
+ f*x) + c), x) + Integral(A*sin(e + f*x)**3/sqrt(-c*sin(e + f*x) + c), x)
+ Integral(B*sin(e + f*x)/sqrt(-c*sin(e + f*x) + c), x) + Integral(3*B*sin(
e + f*x)**2/sqrt(-c*sin(e + f*x) + c), x) + Integral(3*B*sin(e + f*x)**3/sq
rt(-c*sin(e + f*x) + c), x) + Integral(B*sin(e + f*x)**4/sqrt(-c*sin(e + f*
x) + c), x))
```

3.103 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=218

$$-\frac{2\sqrt{2}\,a^3(5A+9B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}+\frac{a^3c(5A+9B)\cos^5(e+fx)}{10f(c-c\sin(e+fx))^{5/2}}+\frac{a^3(5A+}{3f(c-}$$

```
[Out] 1/2*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(9/2)+1/10*a^3*(5*A+9*B)*
c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+1/3*a^3*(5*A+9*B)*cos(f*x+e)^3/f/(c
-c*sin(f*x+e))^(3/2)-2*a^3*(5*A+9*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)
/(c-c*sin(f*x+e))^(1/2))*2^(1/2)/c^(3/2)/f+2*a^3*(5*A+9*B)*cos(f*x+e)/c/f/(
c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2967, 2859, 2679, 2649, 206}

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{2\sqrt{2}\,a^3(5A+9B)\tanh^{-1}\left(\frac{\sqrt{c}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}+\frac{a^3c(5A+9B)\cos^5(e+fx)}{10f(c-c\sin(e+fx))^{5/2}}+\frac{a^3(5A+9}{3f(c-c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2
),x]
```

```
[Out] (-2*Sqrt[2]*a^3*(5*A + 9*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c
- c*Sin[e + f*x]])])/(c^(3/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(2*f*(c
 - c*Sin[e + f*x])^(9/2)) + (a^3*(5*A + 9*B)*c*Cos[e + f*x]^5)/(10*f*(c - c
*Sin[e + f*x])^(5/2)) + (a^3*(5*A + 9*B)*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e
+ f*x])^(3/2)) + (2*a^3*(5*A + 9*B)*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f
*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} - \frac{1}{4}\left(a^3(5A+9B)c^2\right)\int \frac{\cos^6(e+}{(c-c\sin(e+}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} + \frac{a^3(5A+9B)c\cos^5(e+fx)}{10f(c-c\sin(e+fx))^{5/2}} - \frac{1}{2}\left(a^3\right.$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} + \frac{a^3(5A+9B)c\cos^5(e+fx)}{10f(c-c\sin(e+fx))^{5/2}} + \frac{a^3(5}{3f(}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} + \frac{a^3(5A+9B)c\cos^5(e+fx)}{10f(c-c\sin(e+fx))^{5/2}} + \frac{a^3(5}{3f(}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} + \frac{a^3(5A+9B)c\cos^5(e+fx)}{10f(c-c\sin(e+fx))^{5/2}} + \frac{a^3(5}{3f(}$$

$$= -\frac{2\sqrt{2}\,a^3(5A+9B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f} + \frac{a^3(A+B)c^3\cos}{2f(c-c\sin(e}$$

Mathematica [C] time = 1.69, size = 444, normalized size = 2.04

$$\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(240(A+B)\sin\left(\frac{1}{2}(e+fx)\right)+30(9A+20B)\cos\left(\frac{1}{2}(e\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(120*(A + B
)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + (120 + 120*I)*(-1)^(1/4)*(5*A + 9
*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^2 + 30*(9*A + 20*B)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^2 - 5*(2*A + 9*B)*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^2 - 3*B*Cos[(5*(e + f*x))/2]*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^2 + 240*(A + B)*Sin[(e + f*x)/2] + 30*(9*A + 20*B)*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 5*(2*A + 9*B)*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])^2*Sin[(3*(e + f*x))/2] - 3*B*(Cos[(e + f*x)/2] -
```

```
 Sin[(e + f*x)/2])^2*Sin[(5*(e + f*x))/2]))/(30*f*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^6*(c - c*Sin[e + f*x])^(3/2))
```

fricas [B] time = 0.46, size = 430, normalized size = 1.97

$$\frac{\frac{15\sqrt{2}\left((5A+9B)a^3c\cos\left(fx+e\right)^2-(5A+9B)a^3c\cos\left(fx+e\right)-2(5A+9B)a^3c+\left((5A+9B)a^3c\cos\left(fx+e\right)+2(5A+9B)a^3c\right)\sin\left(fx+e\right)\right)\log\left(-\frac{\cos\left(fx+e\right)^2+(\mathrm{cc}}{}\right.}{\sqrt{c}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/15*(15*sqrt(2)*((5*A + 9*B)*a^3*c*cos(f*x + e)^2 - (5*A + 9*B)*a^3*c*cos(
f*x + e) - 2*(5*A + 9*B)*a^3*c + ((5*A + 9*B)*a^3*c*cos(f*x + e) + 2*(5*A +
 9*B)*a^3*c)*sin(f*x + e))*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*
x + e) - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) +
 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(
f*x + e) - cos(f*x + e) - 2))/sqrt(c) + 2*(3*B*a^3*cos(f*x + e)^4 - (5*A +
18*B)*a^3*cos(f*x + e)^3 - (65*A + 141*B)*a^3*cos(f*x + e)^2 - 30*(3*A + 5*
B)*a^3*cos(f*x + e) - 30*(A + B)*a^3 - (3*B*a^3*cos(f*x + e)^3 + (5*A + 21*
B)*a^3*cos(f*x + e)^2 - 60*(A + 2*B)*a^3*cos(f*x + e) + 30*(A + B)*a^3)*sin
(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x
 + e) - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
```

```
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)2/f*(2/sqrt(c*tan((f*x+exp(1))/2)^2+c)/(c*tan((f*x+exp(1))/2)^2+
c)^2*(tan((f*x+exp(1))/2)*(tan((f*x+exp(1))/2)*(tan((f*x+exp(1))/2)*(tan((f
*x+exp(1))/2)*(1/3600*tan((f*x+exp(1))/2)*(-7800*A*a^3*c*sign(tan((f*x+exp(
1))/2)-1)-16560*B*a^3*c*sign(tan((f*x+exp(1))/2)-1))+1/3600*(-9000*A*a^3*c*
sign(tan((f*x+exp(1))/2)-1)-21600*B*a^3*c*sign(tan((f*x+exp(1))/2)-1)))+1/3
600*(-16800*A*a^3*c*sign(tan((f*x+exp(1))/2)-1)-39600*B*a^3*c*sign(tan((f*x
+exp(1))/2)-1)))+1/3600*(-16800*A*a^3*c*sign(tan((f*x+exp(1))/2)-1)-39600*B
*a^3*c*sign(tan((f*x+exp(1))/2)-1)))+1/3600*(-9000*A*a^3*c*sign(tan((f*x+ex
p(1))/2)-1)-21600*B*a^3*c*sign(tan((f*x+exp(1))/2)-1)))+1/3600*(-7800*A*a^3
*c*sign(tan((f*x+exp(1))/2)-1)-16560*B*a^3*c*sign(tan((f*x+exp(1))/2)-1)))+
2*((-6*A*a^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))
^3-6*B*a^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3
+2*A*a^3*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-2
*A*a^3*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c
))^2+2*B*a^3*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c
))-2*B*a^3*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)
^2+c))^2-2*A*a^3*sqrt(c)*c-2*B*a^3*sqrt(c)*c)/(-(-sqrt(c)*tan((f*x+exp(1))/
2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/
2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^2/c/sign(tan((f*x+exp(1))/2)-1)+1/2*
(-20*A*a^3-36*B*a^3)*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan(
(f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/sqrt(-c)/c/sign(tan((f*x+ex
p(1))/2)-1)))
```

maple [A] time = 1.47, size = 354, normalized size = 1.62

$$\frac{2a^3\left(\sin\left(fx+e\right)\left(-60A\sqrt{c+c\sin\left(fx+e\right)}\;c^{\frac{5}{2}}-5A\left(c+c\sin\left(fx+e\right)\right)^{\frac{3}{2}}c^{\frac{3}{2}}-120B\sqrt{c+c\sin\left(fx+e\right)}\;c^{\frac{5}{2}}-15\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/15*a^3*(sin(f*x+e)*(-60*A*(c+c*sin(f*x+e))^(1/2)*c^(5/2)-5*A*(c+c*sin(f*x
+e))^(3/2)*c^(3/2)-120*B*(c+c*sin(f*x+e))^(1/2)*c^(5/2)-15*B*(c+c*sin(f*x+e
))^(3/2)*c^(3/2)-3*B*(c+c*sin(f*x+e))^(5/2)*c^(1/2)+75*A*2^(1/2)*arctanh(1/
2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3+135*B*2^(1/2)*arctanh(1/2*(c+
c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3)+90*A*(c+c*sin(f*x+e))^(1/2)*c^(5/
2)+5*A*(c+c*sin(f*x+e))^(3/2)*c^(3/2)+150*B*(c+c*sin(f*x+e))^(1/2)*c^(5/2)+
15*B*(c+c*sin(f*x+e))^(3/2)*c^(3/2)+3*B*(c+c*sin(f*x+e))^(5/2)*c^(1/2)-75*A
*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3-135*B*2^(1
/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3)*(c*(1+sin(f*x+
e)))^(1/2)/c^(9/2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{3}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)
^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^3}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(3/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(3/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.104 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=225

$$\frac{5a^3(3A+11B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{5a^3(3A+11B)\cos(e+fx)}{4c^2f\sqrt{c-c\sin(e+fx)}}-\frac{a^3c(3A+11\ldots}{8f(c-c\sin\ldots}$$

```
[Out] 1/4*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(11/2)-1/8*a^3*(3*A+11*B)
*c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(7/2)-5/24*a^3*(3*A+11*B)*cos(f*x+e)^3/c
/f/(c-c*sin(f*x+e))^(3/2)+5/4*a^3*(3*A+11*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)
*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(5/2)/f*2^(1/2)-5/4*a^3*(3*A+11*B)*cos(f
*x+e)/c^2/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2967, 2859, 2680, 2679, 2649, 206}

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{5a^3(3A+11B)\cos(e+fx)}{4c^2f\sqrt{c-c\sin(e+fx)}}+\frac{5a^3(3A+11B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f}-\frac{a^3c(3A+11\ldots}{8f(c-c\sin\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/2
),x]
```

```
[Out] (5*a^3*(3*A + 11*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[
e + f*x]])])/(2*Sqrt[2]*c^(5/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(4*f*
(c - c*Sin[e + f*x])^(11/2)) - (a^3*(3*A + 11*B)*c*Cos[e + f*x]^5)/(8*f*(c
- c*Sin[e + f*x])^(7/2)) - (5*a^3*(3*A + 11*B)*Cos[e + f*x]^3)/(24*c*f*(c -
 c*Sin[e + f*x])^(3/2)) - (5*a^3*(3*A + 11*B)*Cos[e + f*x])/(4*c^2*f*Sqrt[c
 - c*Sin[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{1}{8}\left(a^3(3A+11B)c^2\right)\int \frac{\cos^6(e}{(c-c\sin(e}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^3(3A+11B)c\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{7/2}} + \frac{1}{16}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^3(3A+11B)c\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{7/2}} - \frac{5a^3}{24}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^3(3A+11B)c\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{7/2}} - \frac{5a^3}{24}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^3(3A+11B)c\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{7/2}} - \frac{5a^3}{24}$$

$$= \frac{5a^3(3A+11B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f} + \frac{a^3(A+B)c^3\cos^7(e}{4f(c-c\sin(e+fx}$$

Mathematica [C] time = 2.21, size = 434, normalized size = 1.93

$$\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(24(A+B)\sin\left(\frac{1}{2}(e+fx)\right)-6(2A+11B)\cos\left(\frac{1}{2}(e+f\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(12*(A + B)
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) - 3*(9*A + 17*B)*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^3 - (15 + 15*I)*(-1)^(1/4)*(3*A + 11*B)*ArcTan[(1/2 + I/
2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^4 - 6*(2*A + 11*B)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
4 + 2*B*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 24*(
A + B)*Sin[(e + f*x)/2] - 6*(9*A + 17*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^2*Sin[(e + f*x)/2] - 6*(2*A + 11*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
```

```
])^4*Sin[(e + f*x)/2] - 2*B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(3*
(e + f*x))/2]))/(6*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e +
 f*x])^(5/2))
```

fricas [B] time = 0.46, size = 505, normalized size = 2.24

$$15\sqrt{2}\left((3A+11B)a^3\cos\left(fx+e\right)^3+3(3A+11B)a^3\cos\left(fx+e\right)^2-2(3A+11B)a^3\cos\left(fx+e\right)-4(3A+11$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] 1/24*(15*sqrt(2)*((3*A + 11*B)*a^3*cos(f*x + e)^3 + 3*(3*A + 11*B)*a^3*cos(
f*x + e)^2 - 2*(3*A + 11*B)*a^3*cos(f*x + e) - 4*(3*A + 11*B)*a^3 - ((3*A +
 11*B)*a^3*cos(f*x + e)^2 - 2*(3*A + 11*B)*a^3*cos(f*x + e) - 4*(3*A + 11*B
)*a^3)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin
(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e)
 + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x +
e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(4*B*a^3*cos(f*x + e)^4 - 4*(
3*A + 14*B)*a^3*cos(f*x + e)^3 + 3*(13*A + 37*B)*a^3*cos(f*x + e)^2 + 3*(13
*A + 53*B)*a^3*cos(f*x + e) - 12*(A + B)*a^3 - (4*B*a^3*cos(f*x + e)^3 + 12
*(A + 5*B)*a^3*cos(f*x + e)^2 + 3*(17*A + 57*B)*a^3*cos(f*x + e) + 12*(A +
B)*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^3*f*cos(f*x + e)^3 + 3*
c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)
^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2/sqrt(c*t
```

```
an((f*x+exp(1))/2)^2+c)/(c*tan((f*x+exp(1))/2)^2+c)*(tan((f*x+exp(1))/2)*(t
an((f*x+exp(1))/2)*(-1/144*tan((f*x+exp(1))/2)*(-72*A*a^3*c^3*sign(tan((f*x
+exp(1))/2)-1)-384*B*a^3*c^3*sign(tan((f*x+exp(1))/2)-1))/c^4-1/144*(-72*A*
a^3*c^3*sign(tan((f*x+exp(1))/2)-1)-432*B*a^3*c^3*sign(tan((f*x+exp(1))/2)-
1))/c^4)-1/144*(-72*A*a^3*c^3*sign(tan((f*x+exp(1))/2)-1)-432*B*a^3*c^3*sig
n(tan((f*x+exp(1))/2)-1))/c^4)-1/144*(-72*A*a^3*c^3*sign(tan((f*x+exp(1))/2
)-1)-384*B*a^3*c^3*sign(tan((f*x+exp(1))/2)-1))/c^4)+2*(1/4*(7*A*a^3*(-sqrt
(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7+31*B*a^3*(-sqrt(
c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7+81*A*a^3*sqrt(c)*
(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6+185*B*a^3*
sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6+53
*A*a^3*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5+1
25*B*a^3*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5
-65*A*a^3*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2
)^2+c))^4-169*B*a^3*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x
+exp(1))/2)^2+c))^4+13*A*a^3*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((
f*x+exp(1))/2)^2+c))^3+21*B*a^3*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*ta
n((f*x+exp(1))/2)^2+c))^3-33*A*a^3*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c
*tan((f*x+exp(1))/2)^2+c))+19*A*a^3*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/
2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-73*B*a^3*c^3*(-sqrt(c)*tan((f*x+exp(1
))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+75*B*a^3*sqrt(c)*c^2*(-sqrt(c)*tan((
f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+5*A*a^3*sqrt(c)*c^3+13*B*
a^3*sqrt(c)*c^3)/c^2/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1)
)/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1)
)/2)^2+c))+c)^4/sign(tan((f*x+exp(1))/2)-1)+1/4*(15*A*a^3+55*B*a^3)*atan((-
sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2
)/sqrt(-c))/sqrt(2)/c^2/sqrt(-c)/sign(tan((f*x+exp(1))/2)-1)))
```

maple [B] time = 1.99, size = 434, normalized size = 1.93

$$-\frac{a^3\left(\sin\left(fx+e\right)\left(-90A\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c+c\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{c}}\right)c^2+48A\sqrt{c+c\sin\left(fx+e\right)}\ c^{\frac{3}{2}}-330B\sqrt{2}\ \operatorname{arctanh}\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/12/c^(9/2)*a^3*(sin(f*x+e)*(-90*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(
1/2)*2^(1/2)/c^(1/2))*c^2+48*A*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-330*B*2^(1/2)
*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+240*B*(c+c*sin(f*x
+e))^(1/2)*c^(3/2)+16*B*(c+c*sin(f*x+e))^(3/2)*c^(1/2))+(-45*A*2^(1/2)*arct
anh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+24*A*(c+c*sin(f*x+e))^(
1/2)*c^(3/2)-165*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/
```

```
2))*c^2+120*B*(c+c*sin(f*x+e))^(1/2)*c^(3/2)+8*B*(c+c*sin(f*x+e))^(3/2)*c^(
1/2))*cos(f*x+e)^2+90*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/
c^(1/2))*c^2-132*A*(c+c*sin(f*x+e))^(1/2)*c^(3/2)+54*A*(c+c*sin(f*x+e))^(3/
2)*c^(1/2)+330*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2)
)*c^2-420*B*(c+c*sin(f*x+e))^(1/2)*c^(3/2)+86*B*(c+c*sin(f*x+e))^(3/2)*c^(1
/2))*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1
/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{3}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)
^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(5/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(5/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.105 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=217

$$-\frac{5a^3(A+13B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{7/2}f}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}+\frac{5a^3(A+13B)\cos(e+fx)}{16c^3f\sqrt{c-c\sin(e+fx)}}-\frac{a^3c(A+13B)}{24f(c-c\sin}$$

```
[Out] 1/6*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(13/2)-1/24*a^3*(A+13*B)*
c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(9/2)+5/48*a^3*(A+13*B)*cos(f*x+e)^3/c/f/
(c-c*sin(f*x+e))^(5/2)-5/16*a^3*(A+13*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(
1/2)/(c-c*sin(f*x+e))^(1/2))/c^(7/2)/f*2^(1/2)+5/16*a^3*(A+13*B)*cos(f*x+e)
/c^3/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2967, 2859, 2680, 2679, 2649, 206}

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}+\frac{5a^3(A+13B)\cos(e+fx)}{16c^3f\sqrt{c-c\sin(e+fx)}}-\frac{5a^3(A+13B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{7/2}f}-\frac{a^3c(A+13B)c}{24f(c-c\sin(}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/2
),x]
```

```
[Out] (-5*a^3*(A + 13*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e
 + f*x]])])/(8*Sqrt[2]*c^(7/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(6*f*(
c - c*Sin[e + f*x])^(13/2)) - (a^3*(A + 13*B)*c*Cos[e + f*x]^5)/(24*f*(c -
c*Sin[e + f*x])^(9/2)) + (5*a^3*(A + 13*B)*Cos[e + f*x]^3)/(48*c*f*(c - c*S
in[e + f*x])^(5/2)) + (5*a^3*(A + 13*B)*Cos[e + f*x])/(16*c^3*f*Sqrt[c - c*
Sin[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}} - \frac{1}{12}\left(a^3(A+13B)c^2\right)\int \frac{\cos^6(e}{(c-c\sin(e}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(A+13B)c\cos^5(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} + \frac{1}{48}\left(5\right.$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(A+13B)c\cos^5(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} + \frac{5a^3(}{48c}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(A+13B)c\cos^5(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} + \frac{5a^3(}{48c}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(A+13B)c\cos^5(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} + \frac{5a^3(}{48c}$$

$$= -\frac{5a^3(A+13B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{7/2}f} + \frac{a^3(A+B)c^3\cos^7(e}{6f(c-c\sin(e+f}$$

Mathematica [C] time = 3.09, size = 422, normalized size = 1.94

$$\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(64(A+B)\sin\left(\frac{1}{2}(e+fx)\right)+3(11A+47B)\left(\cos\left(\frac{1}{2}(e\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(7/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(32*(A + B)*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]) - 4*(13*A + 25*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3
 + 3*(11*A + 47*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + (15 + 15*I)*(-
1)^(1/4)*(A + 13*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6 + 48*B*Cos[(e + f*x)/2]*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])^6 + 64*(A + B)*Sin[(e + f*x)/2] - 8*(13*A + 25*B)
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 6*(11*A + 47*B)
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2] + 48*B*(Cos[(e +
```

```
f*x)/2] - Sin[(e + f*x)/2])^6*Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3)/(24*f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^(7/2))
```

fricas [B] time = 0.47, size = 554, normalized size = 2.55

$$15\sqrt{2}\left((A+13\,B)a^3\cos\left(fx+e\right)^4-3\,(A+13\,B)a^3\cos\left(fx+e\right)^3-8\,(A+13\,B)a^3\cos\left(fx+e\right)^2+4\,(A+13\,B)a^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] 1/96*(15*sqrt(2)*((A + 13*B)*a^3*cos(f*x + e)^4 - 3*(A + 13*B)*a^3*cos(f*x
+ e)^3 - 8*(A + 13*B)*a^3*cos(f*x + e)^2 + 4*(A + 13*B)*a^3*cos(f*x + e) +
8*(A + 13*B)*a^3 + ((A + 13*B)*a^3*cos(f*x + e)^3 + 4*(A + 13*B)*a^3*cos(f*
x + e)^2 - 4*(A + 13*B)*a^3*cos(f*x + e) - 8*(A + 13*B)*a^3)*sin(f*x + e))*
sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c
)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) -
2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e)
- cos(f*x + e) - 2)) - 4*(48*B*a^3*cos(f*x + e)^4 + 3*(11*A + 95*B)*a^3*cos
(f*x + e)^3 + (19*A - 137*B)*a^3*cos(f*x + e)^2 - 2*(23*A + 203*B)*a^3*cos(
f*x + e) - 32*(A + B)*a^3 - (48*B*a^3*cos(f*x + e)^3 - 3*(11*A + 79*B)*a^3*
cos(f*x + e)^2 - 2*(7*A + 187*B)*a^3*cos(f*x + e) + 32*(A + B)*a^3)*sin(f*x
 + e))*sqrt(-c*sin(f*x + e) + c))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x +
 e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*co
s(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin
(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
```

```
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integrat
ion of abs or sign assumes constant sign by intervals (correct if the argum
ent is real):Check [abs(sin((f*t_nostep+exp(1))/2-pi/4))]Unable to check si
gn: (8*pi/t_nostep/2)>(-8*pi/t_nostep/2)Discontinuities at zeroes of sin((f
*t_nostep+exp(1))/2-pi/4) were not checkedUnable to check sign: (4*pi/t_nos
tep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes cons
tant sign by intervals (correct if the argument is real):Check [abs(t_noste
p-1)]Evaluation time: 2.23Not invertible Error: Bad Argument Value
```

maple [B] time = 1.98, size = 524, normalized size = 2.41

$$\frac{a^3\left(15A\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\left(\sin^3\left(fx+e\right)\right)c^3+195B\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\left(\sin^3\left(fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/48/c^(13/2)*a^3*(15*A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2
)/c^(1/2))*sin(f*x+e)^3*c^3+195*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1
/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^3*c^3-45*A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*
x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^3-96*B*(c*(1+sin(f*x+e)))^(1/2
)*c^(5/2)*sin(f*x+e)^3-585*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2
^(1/2)/c^(1/2))*sin(f*x+e)^2*c^3+66*A*(c*(1+sin(f*x+e)))^(5/2)*c^(1/2)+45*A
*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c
^3+282*B*(c*(1+sin(f*x+e)))^(5/2)*c^(1/2)+288*B*(c*(1+sin(f*x+e)))^(1/2)*c^
(5/2)*sin(f*x+e)^2+585*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/
2)/c^(1/2))*sin(f*x+e)*c^3-160*A*(c*(1+sin(f*x+e)))^(3/2)*c^(3/2)-15*A*2^(1
/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^3-928*B*(c*(1+s
in(f*x+e)))^(3/2)*c^(3/2)-288*B*c^(5/2)*(c*(1+sin(f*x+e)))^(1/2)*sin(f*x+e)
-195*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^3+12
0*A*(c*(1+sin(f*x+e)))^(1/2)*c^(5/2)+888*B*(c*(1+sin(f*x+e)))^(1/2)*c^(5/2)
)*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/
2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{3}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)
^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^3}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(7/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(7/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.106 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=217

$$-\frac{5a^3(A-15B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}c^{9/2}f}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}}+\frac{5a^3(A-15B)\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{3/2}}+\frac{a^3c(A-15I}{48f(c-cs}$$

```
[Out] 1/8*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(15/2)+1/48*a^3*(A-15*B)*
c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(11/2)-5/192*a^3*(A-15*B)*cos(f*x+e)^3/c/
f/(c-c*sin(f*x+e))^(7/2)+5/128*a^3*(A-15*B)*cos(f*x+e)/c^3/f/(c-c*sin(f*x+e
))^(3/2)-5/256*a^3*(A-15*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin
(f*x+e))^(1/2))/c^(9/2)/f*2^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2967, 2859, 2680, 2649, 206}

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}}+\frac{5a^3(A-15B)\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{3/2}}-\frac{5a^3(A-15B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}c^{9/2}f}+\frac{a^3c(A-15B)}{48f(c-c\sin}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(9/2
),x]
```

```
[Out] (-5*a^3*(A - 15*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e
 + f*x]])])/(128*Sqrt[2]*c^(9/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(8*f
*(c - c*Sin[e + f*x])^(15/2)) + (a^3*(A - 15*B)*c*Cos[e + f*x]^5)/(48*f*(c
- c*Sin[e + f*x])^(11/2)) - (5*a^3*(A - 15*B)*Cos[e + f*x]^3)/(192*c*f*(c -
 c*Sin[e + f*x])^(7/2)) + (5*a^3*(A - 15*B)*Cos[e + f*x])/(128*c^3*f*(c - c
*Sin[e + f*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}} + \frac{1}{16}\left(a^3(A-15B)c^2\right)\int \frac{\cos^6(e}{(c-c\sin(e}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}} + \frac{a^3(A-15B)c\cos^5(e+fx)}{48f(c-c\sin(e+fx))^{11/2}} - \frac{1}{96}\left(5\right.$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}} + \frac{a^3(A-15B)c\cos^5(e+fx)}{48f(c-c\sin(e+fx))^{11/2}} - \frac{5a^3(}{192c}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}} + \frac{a^3(A-15B)c\cos^5(e+fx)}{48f(c-c\sin(e+fx))^{11/2}} - \frac{5a^3(}{192c}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}} + \frac{a^3(A-15B)c\cos^5(e+fx)}{48f(c-c\sin(e+fx))^{11/2}} - \frac{5a^3(}{192c}$$

$$= -\frac{5a^3(A-15B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}c^{9/2}f} + \frac{a^3(A+B)c^3\cos^7(e}{8f(c-c\sin(e+f}$$

Mathematica [C] time = 4.38, size = 355, normalized size = 1.64

$$\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((120+120i)\sqrt[4]{-1}(A-15B)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(\tan\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(9/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(1765*A*Cos
[(e + f*x)/2] + 405*B*Cos[(e + f*x)/2] - 895*A*Cos[(3*(e + f*x))/2] - 2703*
B*Cos[(3*(e + f*x))/2] - 397*A*Cos[(5*(e + f*x))/2] + 579*B*Cos[(5*(e + f*x
))/2] + 15*A*Cos[(7*(e + f*x))/2] + 543*B*Cos[(7*(e + f*x))/2] + (120 + 120
*I)*(-1)^(1/4)*(A - 15*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/
4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8 + 1765*A*Sin[(e + f*x)/2] + 40
5*B*Sin[(e + f*x)/2] + 895*A*Sin[(3*(e + f*x))/2] + 2703*B*Sin[(3*(e + f*x)
)/2] - 397*A*Sin[(5*(e + f*x))/2] + 579*B*Sin[(5*(e + f*x))/2] - 15*A*Sin[(
```

```
7*(e + f*x))/2] - 543*B*Sin[(7*(e + f*x))/2]))/(3072*f*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^(9/2))
```

fricas [B] time = 0.48, size = 633, normalized size = 2.92

$$-\frac{15\sqrt{2}\left((A-15B)a^3\cos\left(fx+e\right)^5+5(A-15B)a^3\cos\left(fx+e\right)^4-8(A-15B)a^3\cos\left(fx+e\right)^3-20(A-15B)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="fricas")
```

```
[Out] -1/1536*(15*sqrt(2)*((A - 15*B)*a^3*cos(f*x + e)^5 + 5*(A - 15*B)*a^3*cos(f
*x + e)^4 - 8*(A - 15*B)*a^3*cos(f*x + e)^3 - 20*(A - 15*B)*a^3*cos(f*x + e
)^2 + 8*(A - 15*B)*a^3*cos(f*x + e) + 16*(A - 15*B)*a^3 - ((A - 15*B)*a^3*c
os(f*x + e)^4 - 4*(A - 15*B)*a^3*cos(f*x + e)^3 - 12*(A - 15*B)*a^3*cos(f*x
 + e)^2 + 8*(A - 15*B)*a^3*cos(f*x + e) + 16*(A - 15*B)*a^3)*sin(f*x + e))*
sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c
)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) -
2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e)
- cos(f*x + e) - 2)) - 4*(3*(5*A + 181*B)*a^3*cos(f*x + e)^4 - (191*A - 561
*B)*a^3*cos(f*x + e)^3 - 2*(169*A + 537*B)*a^3*cos(f*x + e)^2 + 12*(21*A -
59*B)*a^3*cos(f*x + e) + 384*(A + B)*a^3 - (3*(5*A + 181*B)*a^3*cos(f*x + e
)^3 + 2*(103*A - 9*B)*a^3*cos(f*x + e)^2 - 12*(11*A + 91*B)*a^3*cos(f*x + e
) - 384*(A + B)*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^5*f*cos(f*
x + e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f
*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5
*f*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5
*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*2*(1/768*(-783*A*a^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((
f*x+exp(1))/2)^2+c))^15+225*B*a^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan(
```

```
(f*x+exp(1))/2)^2+c))^15-993*A*a^3*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sq
rt(c*tan((f*x+exp(1))/2)^2+c))^14+4911*B*a^3*sqrt(c)*(-sqrt(c)*tan((f*x+exp
(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^14-14913*A*a^3*c*(-sqrt(c)*tan((f*
x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^13+14031*B*a^3*c*(-sqrt(c)*ta
n((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^13-11259*A*a^3*sqrt(c)*c
*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^12+77493*B*
a^3*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c)
)^12+285*A*a^3*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)
^2+c))^11+54861*B*a^3*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp
(1))/2)^2+c))^11+28715*A*a^3*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt
(c*tan((f*x+exp(1))/2)^2+c))^10-124293*B*a^3*sqrt(c)*c^2*(-sqrt(c)*tan((f*x
+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^10+17363*A*a^3*c^3*(-sqrt(c)*t
an((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^9-73821*B*a^3*c^3*(-sqr
t(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^9-37271*A*a^3*sqr
t(c)*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^8+8
9817*B*a^3*sqrt(c)*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1)
)/2)^2+c))^8-8989*A*a^3*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+e
xp(1))/2)^2+c))^7-10317*B*a^3*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan(
(f*x+exp(1))/2)^2+c))^7+36189*A*a^3*sqrt(c)*c^4*(-sqrt(c)*tan((f*x+exp(1))/
2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6-32115*B*a^3*sqrt(c)*c^4*(-sqrt(c)*tan
((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6-6547*A*a^3*c^5*(-sqrt(c
)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5+71325*B*a^3*c^5*(-
sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5-17777*A*a^3*
sqrt(c)*c^5*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^
4-7521*B*a^3*sqrt(c)*c^5*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(
1))/2)^2+c))^4+5583*A*a^3*c^6*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x
+exp(1))/2)^2+c))^3-35361*B*a^3*c^6*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*ta
n((f*x+exp(1))/2)^2+c))^3+193*A*a^3*c^7*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(
c*tan((f*x+exp(1))/2)^2+c))-5351*A*a^3*sqrt(c)*c^6*(-sqrt(c)*tan((f*x+exp(1
))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2127*B*a^3*c^7*(-sqrt(c)*tan((f*x+
exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+10377*B*a^3*sqrt(c)*c^6*(-sqrt(
c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-61*A*a^3*sqrt(c)*
c^7+147*B*a^3*sqrt(c)*c^7)/c^4/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((
f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((
f*x+exp(1))/2)^2+c))+c)^8/sign(tan((f*x+exp(1))/2)-1)+1/256*(-5*A*a^3+75*B*
a^3)*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^
2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/c^4/sqrt(-c)/sign(tan((f*x+exp(1))/2)-1))
```

maple [B] time = 1.88, size = 432, normalized size = 1.99

$$a^{3}\left(60\,\mathrm{arctanh}\left(\frac{\sqrt{c+c\sin\left(fx+e\right)}\;\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\;c^{4}\left(A-15B\right)\sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)-120\,\mathrm{arctanh}\left(\frac{\sqrt{c+c\sin\left(fx+e\right)}\;\sqrt{2}}{2\sqrt{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] 1/768*a^3*(60*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c
^4*(A-15*B)*sin(f*x+e)*cos(f*x+e)^2-120*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*
2^(1/2)/c^(1/2))*2^(1/2)*c^4*(A-15*B)*sin(f*x+e)+15*arctanh(1/2*(c+c*sin(f*
x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^4*(A-15*B)*cos(f*x+e)^4-120*arctanh(
1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^4*(A-15*B)*cos(f*x+e)
^2-30*A*(c+c*sin(f*x+e))^(7/2)*c^(1/2)-292*A*(c+c*sin(f*x+e))^(5/2)*c^(3/2)
+440*A*(c+c*sin(f*x+e))^(3/2)*c^(5/2)-240*A*(c+c*sin(f*x+e))^(1/2)*c^(7/2)-
1086*B*(c+c*sin(f*x+e))^(7/2)*c^(1/2)+4380*B*(c+c*sin(f*x+e))^(5/2)*c^(3/2)
-6600*B*(c+c*sin(f*x+e))^(3/2)*c^(5/2)+3600*B*(c+c*sin(f*x+e))^(1/2)*c^(7/2
)+120*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4-180
0*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4)*(c*(1+
sin(f*x+e)))^(1/2)/c^(17/2)/(sin(f*x+e)-1)^3/cos(f*x+e)/(c-c*sin(f*x+e))^(1
/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{3}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)
^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^3}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(9/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(9/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),x)
[Out] Timed out
```

3.107 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=266

$$-\frac{a^3(3A-17B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{512\sqrt{2}\,c^{11/2}f}-\frac{a^3(3A-17B)\cos(e+fx)}{512c^4f(c-c\sin(e+fx))^{3/2}}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}}+\frac{a^3(3A-17B}{128c^3f(c-c}$$

```
[Out] 1/10*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(17/2)+1/80*a^3*(3*A-17*
B)*c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(13/2)-1/96*a^3*(3*A-17*B)*cos(f*x+e)^
3/c/f/(c-c*sin(f*x+e))^(9/2)+1/128*a^3*(3*A-17*B)*cos(f*x+e)/c^3/f/(c-c*sin
(f*x+e))^(5/2)-1/512*a^3*(3*A-17*B)*cos(f*x+e)/c^4/f/(c-c*sin(f*x+e))^(3/2)
-1/1024*a^3*(3*A-17*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+
e))^(1/2))/c^(11/2)/f*2^(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2967, 2859, 2680, 2650, 2649, 206}

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}}-\frac{a^3(3A-17B)\cos(e+fx)}{512c^4f(c-c\sin(e+fx))^{3/2}}+\frac{a^3(3A-17B)\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{5/2}}-\frac{a^3(3A-17B)\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{2}}\right.}{512\sqrt{2}\,c^{11/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(11/
2),x]
```

```
[Out] -(a^3*(3*A - 17*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e
 + f*x]])])/(512*Sqrt[2]*c^(11/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(10
*f*(c - c*Sin[e + f*x])^(17/2)) + (a^3*(3*A - 17*B)*c*Cos[e + f*x]^5)/(80*f
*(c - c*Sin[e + f*x])^(13/2)) - (a^3*(3*A - 17*B)*Cos[e + f*x]^3)/(96*c*f*(
c - c*Sin[e + f*x])^(9/2)) + (a^3*(3*A - 17*B)*Cos[e + f*x])/(128*c^3*f*(c
- c*Sin[e + f*x])^(5/2)) - (a^3*(3*A - 17*B)*Cos[e + f*x])/(512*c^4*f*(c -
c*Sin[e + f*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}} + \frac{1}{20}\left(a^3(3A-17B)c^2\right)\int \frac{\cos^6(e}{(c-c\sin(e}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}} + \frac{a^3(3A-17B)c\cos^5(e+fx)}{80f(c-c\sin(e+fx))^{13/2}} - \frac{1}{32}\left(a\right.$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}} + \frac{a^3(3A-17B)c\cos^5(e+fx)}{80f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(3A}{96cf}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}} + \frac{a^3(3A-17B)c\cos^5(e+fx)}{80f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(3A}{96cf}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}} + \frac{a^3(3A-17B)c\cos^5(e+fx)}{80f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(3A}{96cf}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}} + \frac{a^3(3A-17B)c\cos^5(e+fx)}{80f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(3A}{96cf}$$

$$= -\frac{a^3(3A-17B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{512\sqrt{2}\,c^{11/2}f} + \frac{a^3(A+B)c^3\cos^7(e+}{10f(c-c\sin(e+fx}$$

Mathematica [C] time = 6.83, size = 485, normalized size = 1.82

$$\frac{(a\sin(e+fx)+a)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(56370A\sin\left(\frac{1}{2}(e+fx)\right)+31140A\sin\left(\frac{3}{2}(e+fx)\right)-104\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(11/2),x]
```

```
[Out] ((1/512 + I/512)*(-1)^(1/4)*(3*A - 17*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*Sec[
(e + f*x)/4]*(Cos[(e + f*x)/4] + Sin[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^11*(a + a*Sin[e + f*x])^3)/(f*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^6*(c - c*Sin[e + f*x])^(11/2)) + ((Cos[(e + f*x)/2] - Sin[(e + f*x)
/2])*(a + a*Sin[e + f*x])^3*(56370*A*Cos[(e + f*x)/2] + 38970*B*Cos[(e + f*
x)/2] - 31140*A*Cos[(3*(e + f*x))/2] - 38580*B*Cos[(3*(e + f*x))/2] - 10404
```

```
*A*Cos[(5*(e + f*x))/2] - 12724*B*Cos[(5*(e + f*x))/2] + 435*A*Cos[(7*(e +
f*x))/2] + 7775*B*Cos[(7*(e + f*x))/2] - 45*A*Cos[(9*(e + f*x))/2] + 255*B*
Cos[(9*(e + f*x))/2] + 56370*A*Sin[(e + f*x)/2] + 38970*B*Sin[(e + f*x)/2]
+ 31140*A*Sin[(3*(e + f*x))/2] + 38580*B*Sin[(3*(e + f*x))/2] - 10404*A*Sin
[(5*(e + f*x))/2] - 12724*B*Sin[(5*(e + f*x))/2] - 435*A*Sin[(7*(e + f*x))/
2] - 7775*B*Sin[(7*(e + f*x))/2] - 45*A*Sin[(9*(e + f*x))/2] + 255*B*Sin[(9
*(e + f*x))/2]))/(122880*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*S
in[e + f*x])^(11/2))
```

fricas [B] time = 0.49, size = 760, normalized size = 2.86

$$-\frac{15\sqrt{2}\Big((3A-17B)a^3\cos\big(fx+e\big)^6-5(3A-17B)a^3\cos\big(fx+e\big)^5-18(3A-17B)a^3\cos\big(fx+e\big)^4+20(3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="fricas")
```

```
[Out] -1/30720*(15*sqrt(2)*((3*A - 17*B)*a^3*cos(f*x + e)^6 - 5*(3*A - 17*B)*a^3*
cos(f*x + e)^5 - 18*(3*A - 17*B)*a^3*cos(f*x + e)^4 + 20*(3*A - 17*B)*a^3*c
os(f*x + e)^3 + 48*(3*A - 17*B)*a^3*cos(f*x + e)^2 - 16*(3*A - 17*B)*a^3*co
s(f*x + e) - 32*(3*A - 17*B)*a^3 + ((3*A - 17*B)*a^3*cos(f*x + e)^5 + 6*(3*
A - 17*B)*a^3*cos(f*x + e)^4 - 12*(3*A - 17*B)*a^3*cos(f*x + e)^3 - 32*(3*A
 - 17*B)*a^3*cos(f*x + e)^2 + 16*(3*A - 17*B)*a^3*cos(f*x + e) + 32*(3*A -
17*B)*a^3)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c
*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x
+ e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*
x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(15*(3*A - 17*B)*a^3*cos(
f*x + e)^5 - 5*(39*A + 803*B)*a^3*cos(f*x + e)^4 + 4*(609*A + 389*B)*a^3*co
s(f*x + e)^3 + 12*(449*A + 869*B)*a^3*cos(f*x + e)^2 - 24*(143*A + 43*B)*a^
3*cos(f*x + e) - 6144*(A + B)*a^3 + (15*(3*A - 17*B)*a^3*cos(f*x + e)^4 + 8
0*(3*A + 47*B)*a^3*cos(f*x + e)^3 + 12*(223*A + 443*B)*a^3*cos(f*x + e)^2 -
 24*(113*A + 213*B)*a^3*cos(f*x + e) - 6144*(A + B)*a^3)*sin(f*x + e))*sqrt
(-c*sin(f*x + e) + c))/(c^6*f*cos(f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*
c^6*f*cos(f*x + e)^4 + 20*c^6*f*cos(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 -
16*c^6*f*cos(f*x + e) - 32*c^6*f + (c^6*f*cos(f*x + e)^5 + 6*c^6*f*cos(f*x
+ e)^4 - 12*c^6*f*cos(f*x + e)^3 - 32*c^6*f*cos(f*x + e)^2 + 16*c^6*f*cos(f
*x + e) + 32*c^6*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*2*(1/15360*(-15405*A*a^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*t
an((f*x+exp(1))/2)^2+c))^19+255*B*a^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*
tan((f*x+exp(1))/2)^2+c))^19-46935*A*a^3*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))
/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^18+35565*B*a^3*sqrt(c)*(-sqrt(c)*tan((
f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^18-452265*A*a^3*c*(-sqrt(c)
*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^17-43245*B*a^3*c*(-sq
rt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^17-812175*A*a^3*
sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^16
+583125*B*a^3*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1
))/2)^2+c))^16-1157268*A*a^3*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((
f*x+exp(1))/2)^2+c))^15+270492*B*a^3*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt
(c*tan((f*x+exp(1))/2)^2+c))^15+999780*A*a^3*sqrt(c)*c^2*(-sqrt(c)*tan((f*x
+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^14-320140*B*a^3*sqrt(c)*c^2*(-
sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^14+2949420*A*a
^3*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^13-30
8900*B*a^3*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c
))^13+775380*A*a^3*sqrt(c)*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*
x+exp(1))/2)^2+c))^12-768860*B*a^3*sqrt(c)*c^3*(-sqrt(c)*tan((f*x+exp(1))/2
)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^12-3937830*A*a^3*c^4*(-sqrt(c)*tan((f*x+
exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^11+820610*B*a^3*c^4*(-sqrt(c)*t
an((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^11-1793586*A*a^3*sqrt(c
)*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^10+507
334*B*a^3*sqrt(c)*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))
/2)^2+c))^10+4240770*A*a^3*c^5*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*
x+exp(1))/2)^2+c))^9-1492790*B*a^3*c^5*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c
*tan((f*x+exp(1))/2)^2+c))^9+850830*A*a^3*sqrt(c)*c^5*(-sqrt(c)*tan((f*x+ex
p(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^8+308870*B*a^3*sqrt(c)*c^5*(-sqrt
(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^8-3213060*A*a^3*c^
6*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7+1434220*
B*a^3*c^6*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7+
712020*A*a^3*sqrt(c)*c^6*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(
1))/2)^2+c))^6-164060*B*a^3*sqrt(c)*c^6*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(
c*tan((f*x+exp(1))/2)^2+c))^6+805596*A*a^3*c^7*(-sqrt(c)*tan((f*x+exp(1))/2
)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5-284724*B*a^3*c^7*(-sqrt(c)*tan((f*x+ex
p(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5-1056540*A*a^3*sqrt(c)*c^7*(-sqr
t(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4-54540*B*a^3*sqr
t(c)*c^7*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4+2
89515*A*a^3*c^8*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+
```

```
c))^3-253065*B*a^3*c^8*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1)
)/2)^2+c))^3+3615*A*a^3*c^9*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+e
xp(1))/2)^2+c))-114735*A*a^3*sqrt(c)*c^8*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt
(c*tan((f*x+exp(1))/2)^2+c))^2-5125*B*a^3*c^9*(-sqrt(c)*tan((f*x+exp(1))/2)
+sqrt(c*tan((f*x+exp(1))/2)^2+c))+10165*B*a^3*sqrt(c)*c^8*(-sqrt(c)*tan((f*
x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-951*A*a^3*sqrt(c)*c^9+269*B
*a^3*sqrt(c)*c^9)/c^5/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1
))/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1
))/2)^2+c))+c)^10/sign(tan((f*x+exp(1))/2)-1)+1/1024*(-3*A*a^3+17*B*a^3)*at
an((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/s
qrt(2)/sqrt(-c))/sqrt(2)/c^5/sqrt(-c)/sign(tan((f*x+exp(1))/2)-1))
```

maple [B] time = 2.05, size = 526, normalized size = 1.98

$$\frac{a^3\left(15\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c+c\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{c}}\right)c^6\left(3A-17B\right)\sin\left(fx+e\right)\left(\cos^4\left(fx+e\right)\right)-180\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c+c\sin\left(fx+\right.}}{2\sqrt{c}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x)
```

```
[Out] 1/15360*a^3*(15*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))
*c^6*(3*A-17*B)*sin(f*x+e)*cos(f*x+e)^4-180*2^(1/2)*arctanh(1/2*(c+c*sin(f*
x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6*(3*A-17*B)*cos(f*x+e)^2*sin(f*x+e)+240*2^(
1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6*(3*A-17*B)*sin
(f*x+e)-75*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6*
(3*A-17*B)*cos(f*x+e)^4+300*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1
/2)/c^(1/2))*c^6*(3*A-17*B)*cos(f*x+e)^2-90*A*(c+c*sin(f*x+e))^(9/2)*c^(3/2
)+840*A*(c+c*sin(f*x+e))^(7/2)*c^(5/2)+3072*A*(c+c*sin(f*x+e))^(5/2)*c^(7/2
)-3360*A*(c+c*sin(f*x+e))^(3/2)*c^(9/2)+1440*A*(c+c*sin(f*x+e))^(1/2)*c^(11
/2)+510*B*(c+c*sin(f*x+e))^(9/2)*c^(3/2)+5480*B*(c+c*sin(f*x+e))^(7/2)*c^(5
/2)-17408*B*(c+c*sin(f*x+e))^(5/2)*c^(7/2)+19040*B*(c+c*sin(f*x+e))^(3/2)*c
^(9/2)-8160*B*(c+c*sin(f*x+e))^(1/2)*c^(11/2)-720*A*2^(1/2)*arctanh(1/2*(c+
c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6+4080*B*2^(1/2)*arctanh(1/2*(c+c*si
n(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6)*(c*(1+sin(f*x+e)))^(1/2)/c^(23/2)/(si
n(f*x+e)-1)^4/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^3}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)
^(11/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(11/
2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(11/
2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

3.108 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=200

$$-\frac{128c^4(7A-9B)\cos(e+fx)}{35af\sqrt{c-c\sin(e+fx)}}-\frac{32c^3(7A-9B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{35af}-\frac{12c^2(7A-9B)\cos(e+fx)(c-c}{35af}$$

```
[Out] -12/35*(7*A-9*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f-1/7*(7*A-9*B)*c*
cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a/f-(A-B)*sec(f*x+e)*(c-c*sin(f*x+e))^(9/
2)/a/c/f-128/35*(7*A-9*B)*c^4*cos(f*x+e)/a/f/(c-c*sin(f*x+e))^(1/2)-32/35*(
7*A-9*B)*c^3*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f
```

Rubi [A] time = 0.38, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2855, 2647, 2646}

$$-\frac{12c^2(7A-9B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{35af}-\frac{32c^3(7A-9B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{35af}-\frac{128c^4(7A-9}{35af\sqrt{c-c}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x]),
x]
```

```
[Out] (-128*(7*A - 9*B)*c^4*Cos[e + f*x])/(35*a*f*Sqrt[c - c*Sin[e + f*x]]) - (32
*(7*A - 9*B)*c^3*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(35*a*f) - (12*(7*A
 - 9*B)*c^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(35*a*f) - ((7*A - 9*B
)*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(7*a*f) - ((A - B)*Sec[e + f*x
]*(c - c*Sin[e + f*x])^(9/2))/(a*c*f)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx = \frac{\int \sec^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx}{ac}$$
$$= -\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{acf} - \frac{(7A-9B)\int(c-c\,\mathrm{s}}{2a}$$
$$= -\frac{(7A-9B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{7af} - \frac{(A-B)\sec(e+}{}$$
$$= -\frac{12(7A-9B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{35af} - \frac{(7A-9B)c}{}$$
$$= -\frac{32(7A-9B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{35af} - \frac{12(7A-9B)c}{}$$
$$= -\frac{128(7A-9B)c^4\cos(e+fx)}{35af\sqrt{c-c\sin(e+fx)}} - \frac{32(7A-9B)c^3\cos(e+fx)\sqrt{c-}}{35af}$$

Mathematica [A] time = 5.59, size = 157, normalized size = 0.78

$$-\frac{c^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(196(A-2B)\cos(2(e+fx))+2450A\sin(e+fx)-14}{140af(\sin(e+fx)+1)\left(\cos\left(\frac{1}{2}(e+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
f*x]),x]
```

```
[Out] -1/140*(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*
(4900*A - 6125*B + 196*(A - 2*B)*Cos[2*(e + f*x)] + 5*B*Cos[4*(e + f*x)] +
2450*A*Sin[e + f*x] - 3430*B*Sin[e + f*x] - 14*A*Sin[3*(e + f*x)] + 58*B*Si
n[3*(e + f*x)]))/(a*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*
x]))
```

fricas [A] time = 0.44, size = 115, normalized size = 0.58

$$-\frac{2\left(5\,Bc^3\cos\left(fx+e\right)^4+(49\,A-103\,B)c^3\cos\left(fx+e\right)^2+4\,(147\,A-179\,B)c^3-\left((7\,A-29\,B)c^3\cos\left(fx+e\right)^2\right.}{35\,af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] -2/35*(5*B*c^3*cos(f*x + e)^4 + (49*A - 103*B)*c^3*cos(f*x + e)^2 + 4*(147*
A - 179*B)*c^3 - ((7*A - 29*B)*c^3*cos(f*x + e)^2 - 4*(77*A - 109*B)*c^3)*s
in(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.17, size = 111, normalized size = 0.56

$$\frac{2c^4\left(\sin\left(fx+e\right)-1\right)\left(\left(-7A+29B\right)\sin\left(fx+e\right)\left(\cos^2\left(fx+e\right)\right)+\left(308A-436B\right)\sin\left(fx+e\right)+5B\left(\cos^4\left(fx\right.\right.\right.}{35a\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\;f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] 2/35*c^4/a*(sin(f*x+e)-1)*((-7*A+29*B)*sin(f*x+e)*cos(f*x+e)^2+(308*A-436*B
)*sin(f*x+e)+5*B*cos(f*x+e)^4+(49*A-103*B)*cos(f*x+e)^2+588*A-716*B)/cos(f*
x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [B] time = 0.50, size = 478, normalized size = 2.39

$$2\left(\frac{7\left(91\,c^{\frac{7}{2}}+\frac{86\,c^{\frac{7}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{336\,c^{\frac{7}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{266\,c^{\frac{7}{2}}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{490\,c^{\frac{7}{2}}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{266\,c^{\frac{7}{2}}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+\frac{336\,c^{\frac{7}{2}}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}+\frac{86\,c^{\frac{7}{2}}\sin\left(fx+e\right)^{7}}{\left(\cos\left(fx+e\right)+1\right)^{7}}+\frac{91\,c^{\frac{7}{2}}\sin\left(fx+e\right.}{\left(\cos\left(fx+e\right)+1\right)}\right.}{\left(a+\frac{a\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] 2/35*(7*(91*c^(7/2) + 86*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 336*c^(7
/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 266*c^(7/2)*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 490*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 266*c^(7
/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 336*c^(7/2)*sin(f*x + e)^6/(cos(f
*x + e) + 1)^6 + 86*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 91*c^(7/2
)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8)*A/((a + a*sin(f*x + e)/(cos(f*x + e)
 + 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2)) - 2*(407*c^(7/2) +
407*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 1442*c^(7/2)*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 1337*c^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 20
30*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1337*c^(7/2)*sin(f*x + e)^
5/(cos(f*x + e) + 1)^5 + 1442*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 +
 407*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 407*c^(7/2)*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8)*B/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*(sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}}{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x)),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x)),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.109 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=159

$$-\frac{32c^3(5A-7B)\cos(e+fx)}{15af\sqrt{c-c\sin(e+fx)}}-\frac{8c^2(5A-7B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{15af}-\frac{c(5A-7B)\cos(e+fx)(c-c\sin(e+}{5af}$$

```
[Out] -1/5*(5*A-7*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f-(A-B)*sec(f*x+e)*(c-
c*sin(f*x+e))^(7/2)/a/c/f-32/15*(5*A-7*B)*c^3*cos(f*x+e)/a/f/(c-c*sin(f*x+e
))^(1/2)-8/15*(5*A-7*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f
```

Rubi [A] time = 0.35, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2855, 2647, 2646}

$$-\frac{8c^2(5A-7B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{15af}-\frac{32c^3(5A-7B)\cos(e+fx)}{15af\sqrt{c-c\sin(e+fx)}}-\frac{c(5A-7B)\cos(e+fx)(c-c\sin(e+}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x]),
x]
```

```
[Out] (-32*(5*A - 7*B)*c^3*Cos[e + f*x])/(15*a*f*Sqrt[c - c*Sin[e + f*x]]) - (8*(
5*A - 7*B)*c^2*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(15*a*f) - ((5*A - 7*
B)*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(5*a*f) - ((A - B)*Sec[e + f*
x]*(c - c*Sin[e + f*x])^(7/2))/(a*c*f)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
```

```
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx = \frac{\int \sec^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx}{ac}$$

$$= -\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{acf} - \frac{(5A-7B)\int(c-}{2}$$

$$= -\frac{(5A-7B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5af} - \frac{(A-B)\sec(e}{}$$

$$= -\frac{8(5A-7B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{15af} - \frac{(5A-7B)c\,c}{}$$

$$= -\frac{32(5A-7B)c^3\cos(e+fx)}{15af\sqrt{c-c\sin(e+fx)}} - \frac{8(5A-7B)c^2\cos(e+fx)\sqrt{c-}}{15af}$$

Mathematica [A] time = 1.76, size = 134, normalized size = 0.84

$$-\frac{c^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(25(8A-13B)\sin(e+fx)+2(5A-16B)\cos(2(e+}{30af(\sin(e+fx)+1)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
f*x]),x]
```

```
[Out] -1/30*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(
450*A - 600*B + 2*(5*A - 16*B)*Cos[2*(e + f*x)] + 25*(8*A - 13*B)*Sin[e + f
```

```
*x] + 3*B*Sin[3*(e + f*x)]))/(a*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1
+ Sin[e + f*x]))
```

fricas [A] time = 0.43, size = 95, normalized size = 0.60

$$-\frac{2\left((5A-16B)c^2\cos\left(fx+e\right)^2+2(55A-71B)c^2+\left(3Bc^2\cos\left(fx+e\right)^2+2(25A-41B)c^2\right)\sin\left(fx+e\right)\right)\sqrt{-}}{15af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] -2/15*((5*A - 16*B)*c^2*cos(f*x + e)^2 + 2*(55*A - 71*B)*c^2 + (3*B*c^2*cos
(f*x + e)^2 + 2*(25*A - 41*B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/
(a*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.10, size = 95, normalized size = 0.60

$$-\frac{2c^3\left(\sin\left(fx+e\right)-1\right)\left(-3B\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+(-50A+82B)\sin\left(fx+e\right)+(-5A+16B)\left(\cos^2\left(fx\right.\right.\right.}{15a\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\ f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] -2/15*c^3/a*(sin(f*x+e)-1)*(-3*B*cos(f*x+e)^2*sin(f*x+e)+(-50*A+82*B)*sin(f
*x+e)+(-5*A+16*B)*cos(f*x+e)^2-110*A+142*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/
2)/f
```

maxima [B] time = 0.46, size = 386, normalized size = 2.43

$$\frac{2\left(\frac{5\left(23\,c^{\frac{5}{2}}+\frac{20\,c^{\frac{5}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{65\,c^{\frac{5}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{40\,c^{\frac{5}{2}}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{65\,c^{\frac{5}{2}}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{20\,c^{\frac{5}{2}}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+\frac{23\,c^{\frac{5}{2}}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}\right)A}{\left(a+\frac{a\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)^{\frac{5}{2}}}-\frac{2\left(79\,c^{\frac{5}{2}}+\frac{79\,c^{\frac{5}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{20 \cdots}{\cdots}\right.}{\cdots}\right.}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] 2/15*(5*(23*c^(5/2) + 20*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 65*c^(5/
2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 40*c^(5/2)*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 65*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 20*c^(5/2)*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 23*c^(5/2)*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6)*A/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*(sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 1)^(5/2)) - 2*(79*c^(5/2) + 79*c^(5/2)*sin(f*x + e)/(cos(
f*x + e) + 1) + 205*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 170*c^(5/
2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 205*c^(5/2)*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 + 79*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 79*c^(5/2)
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)*B/((a + a*sin(f*x + e)/(cos(f*x + e)
+ 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x)),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x)),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.110 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=118

$$-\frac{4c^2(3A-5B)\cos(e+fx)}{3af\sqrt{c-c\sin(e+fx)}}-\frac{c(3A-5B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3af}-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{acf}$$

```
[Out] -(A-B)*sec(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a/c/f-4/3*(3*A-5*B)*c^2*cos(f*x+e)
/a/f/(c-c*sin(f*x+e))^(1/2)-1/3*(3*A-5*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(1/
2)/a/f
```

Rubi [A] time = 0.32, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2855, 2647, 2646}

$$-\frac{4c^2(3A-5B)\cos(e+fx)}{3af\sqrt{c-c\sin(e+fx)}}-\frac{c(3A-5B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3af}-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{acf}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x]),
x]
```

```
[Out] (-4*(3*A - 5*B)*c^2*Cos[e + f*x])/(3*a*f*Sqrt[c - c*Sin[e + f*x]]) - ((3*A
- 5*B)*c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*a*f) - ((A - B)*Sec[e +
f*x]*(c - c*Sin[e + f*x])^(5/2))/(a*c*f)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
```

```
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx = \frac{\int \sec^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx}{ac}$$
$$= -\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{acf} - \frac{(3A-5B)\int(c-}{2}$$
$$= -\frac{(3A-5B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3af} - \frac{(A-B)\sec(e+}{}$$
$$= -\frac{4(3A-5B)c^2\cos(e+fx)}{3af\sqrt{c-c\sin(e+fx)}} - \frac{(3A-5B)c\cos(e+fx)\sqrt{c-c\,\mathrm{si}}}{3af}$$

Mathematica [A] time = 0.60, size = 113, normalized size = 0.96

$$\frac{c\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)((14B-6A)\sin(e+fx)-18A+B\cos(2(e+fx))+27}{3af(\sin(e+fx)+1)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
f*x]),x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-18*A + 27*B + B*Cos[2*(e + f*x)]
 + (-6*A + 14*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(3*a*f*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x]))
```

fricas [A] time = 0.43, size = 67, normalized size = 0.57

$$\frac{2 \left(Bc \cos \left(f x + e\right)^{2} - (3 \, A - 7 \, B)c \sin \left(f x + e\right) - (9 \, A - 13 \, B)c\right) \sqrt{-c \sin \left(f x + e\right) + c}}{3 \, a f \cos \left(f x + e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] 2/3*(B*c*cos(f*x + e)^2 - (3*A - 7*B)*c*sin(f*x + e) - (9*A - 13*B)*c)*sqrt
(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.12, size = 73, normalized size = 0.62

$$\frac{2c^{2} \left(\sin \left(f x + e\right) - 1\right) \left(\sin \left(f x + e\right) (3A - 7B) - B \left(\cos^{2} \left(f x + e\right)\right) + 9A - 13B\right)}{3a \cos \left(f x + e\right) \sqrt{c - c \sin \left(f x + e\right)}\; f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] 2/3*c^2/a*(sin(f*x+e)-1)*(sin(f*x+e)*(3*A-7*B)-B*cos(f*x+e)^2+9*A-13*B)/cos
(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [B] time = 0.46, size = 294, normalized size = 2.49

$$\frac{2 \left(\frac{3 \left(3 c^{\frac{3}{2}} + \frac{2 c^{\frac{3}{2}} \sin\left(fx+e\right)}{\cos\left(fx+e\right)+1} + \frac{6 c^{\frac{3}{2}} \sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}} + \frac{2 c^{\frac{3}{2}} \sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}} + \frac{3 c^{\frac{3}{2}} \sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}\right) A}{\left(a + \frac{a \sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right) \left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}} + 1\right)^{\frac{3}{2}}} - \frac{2 \left(7 c^{\frac{3}{2}} + \frac{7 c^{\frac{3}{2}} \sin\left(fx+e\right)}{\cos\left(fx+e\right)+1} + \frac{12 c^{\frac{3}{2}} \sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}} + \frac{7 c^{\frac{3}{2}} \sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}} + \frac{7 c^{\frac{3}{2}} \sin\left(fx+e\right.}{\left(\cos\left(fx+e\right)+1\right.}}{\left(a + \frac{a \sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right) \left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}} + 1\right)^{\frac{3}{2}}}}{3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] 2/3*(3*(3*c^(3/2) + 2*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 6*c^(3/2)*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 3*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*A/((a + a*sin(f*x
+ e)/(cos(f*x + e) + 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)) -
 2*(7*c^(3/2) + 7*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 12*c^(3/2)*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 7*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*B/((a + a*sin(f*x + e
)/(cos(f*x + e) + 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x)),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x)),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{Ac\sqrt{-c\sin\left(e+fx\right)+c}}{\sin\left(e+fx\right)+1}\,dx+\int\left(-\frac{Ac\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)}{\sin\left(e+fx\right)+1}\right)dx+\int\frac{Bc\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)}{\sin\left(e+fx\right)+1}\,dx+\int\left(-\frac{Bc\sqrt{-c\sin\left(e+f\right.}}{\sin\left(e-\right.}\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] (Integral(A*c*sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x) + 1), x) + Integral(-
A*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x) + 1), x) + Integra
l(B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x) + 1), x) + Integ
ral(-B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2/(sin(e + f*x) + 1), x))/
a
```

3.111 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=73

$$-\frac{c(A-3B)\cos(e+fx)}{af\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{acf}$$

```
[Out] -(A-B)*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/c/f-(A-3*B)*c*cos(f*x+e)/a/f/(c-
c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2967, 2855, 2646}

$$-\frac{c(A-3B)\cos(e+fx)}{af\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{acf}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x]),x]
```

```
[Out] -(((A - 3*B)*c*Cos[e + f*x])/(a*f*Sqrt[c - c*Sin[e + f*x]])) - ((A - B)*Sec
[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(a*c*f)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
```

```
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx = \frac{\int \sec^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx}{ac}$$
$$= -\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{acf} - \frac{(A-3B)\int \sqrt{c-c\,s}}{2a}$$
$$= -\frac{(A-3B)c\cos(e+fx)}{af\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec(e+fx)(c-c\sin(e+fx)}{acf}$$

Mathematica [A] time = 0.20, size = 44, normalized size = 0.60

$$\frac{2\sec(e+fx)\sqrt{c-c\sin(e+fx)}\,(-A+B\sin(e+fx)+2B)}{af}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*
x]),x]
```

```
[Out] (2*Sec[e + f*x]*(-A + 2*B + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a*f)
```

fricas [A] time = 0.43, size = 44, normalized size = 0.60

$$\frac{2\left(B\sin\left(fx+e\right)-A+2B\right)\sqrt{-c\sin\left(fx+e\right)+c}}{af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] 2*(B*sin(f*x + e) - A + 2*B)*sqrt(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
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(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
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to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
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eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
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 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
```

gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)Un
able to check sign: (8*pi/x/2)>(-8*pi/x/2)Unable to check sign: (8*pi/x/2)>
(-8*pi/x/2)4*sqrt(2*c)*((70368744177664*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))
-1/4*pi))-140737488355328*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))+7036
8744177664*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*
exp(1)))^2+70368744177664*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(
1/2*(1/2*f*x+2*exp(1)))^3-422212465065984*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1
))-1/4*pi))*tan(1/2*exp(1))^2+703687441776640*sqrt(2)*A*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*tan(1/2*exp(1))^3+633318697598976*sqrt(2)*A*sign(sin(1/2*(f
*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^4-211106232532992*sqrt(2)*A*sign(sin(1/
2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^5-1055531162664960*sqrt(2)*A*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^2+1055531162664960*sqrt(2)*A*s
ign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^4-70368744177664*sqrt(2)*
A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^6+70368744177664*sqrt(
2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))-2111062
32532992*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))-14073
7488355328*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*
exp(1)))^2+211106232532992*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan
(1/2*exp(1))^2-2111062325329920*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi)
)*tan(1/2*exp(1))^3-844424930131968*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4
*pi))*tan(1/2*exp(1))^4+633318697598976*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))
-1/4*pi))*tan(1/2*exp(1))^5-70368744177664*sqrt(2)*B*sign(sin(1/2*(f*x+exp(
1))-1/4*pi))*tan(1/2*exp(1))^6+2111062325329920*sqrt(2)*B*sign(sin(1/2*(f*x
+exp(1))-1/4*pi))*tan(1/4*exp(1))^2-1407374883553280*sqrt(2)*B*sign(sin(1/2
*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^3-2111062325329920*sqrt(2)*B*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^4+422212465065984*sqrt(2)*B*sig
n(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^5+140737488355328*sqrt(2)*B
sign(sin(1/2(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^6-281474976710656*sqrt(
2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))+6333186
97598976*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))+42221
2465065984*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))-422
212465065984*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+
2*exp(1)))^2*tan(1/2*exp(1))^2+703687441776640*sqrt(2)*A*sign(sin(1/2*(f*x+
exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^3+6333186975
98976*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1
)))^2*tan(1/2*exp(1))^4-211106232532992*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))
-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^5-1055531162664960*
sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*

tan(1/4*exp(1))^2+1055531162664960*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*
pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/4*exp(1))^4-70368744177664*sqrt(2)
*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/4
*exp(1))^6-211106232532992*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan
(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))-422212465065984*sqrt(2)*A*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^
2-703687441776640*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2
*f*x+2*exp(1)))^3*tan(1/2*exp(1))^3+633318697598976*sqrt(2)*A*sign(sin(1/2*
(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^4+21110
6232532992*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*
exp(1)))^3*tan(1/2*exp(1))^5-1055531162664960*sqrt(2)*A*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/4*exp(1))^2+10555311626
64960*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1
)))^3*tan(1/4*exp(1))^4-70368744177664*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-
1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/4*exp(1))^6+211106232532992*sq
rt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*ta
n(1/2*exp(1))+6333186975989760*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))
*tan(1/2*exp(1))^2*tan(1/4*exp(1))^2+12666373951979520*sqrt(2)*A*sign(sin(1
/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/4*exp(1))^3-63331869759897
60*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/4*e
xp(1))^4-3799912185593856*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(
1/2*exp(1))^2*tan(1/4*exp(1))^5+422212465065984*sqrt(2)*A*sign(sin(1/2*(f*x
+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/4*exp(1))^6-3799912185593856*sqrt
(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/4*exp(1))-
10555311626649600*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(
1))^3*tan(1/4*exp(1))^2+14073748835532800*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1
))-1/4*pi))*tan(1/2*exp(1))^3*tan(1/4*exp(1))^3+10555311626649600*sqrt(2)*A
sign(sin(1/2(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^3*tan(1/4*exp(1))^4-422
2124650659840*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^
3*tan(1/4*exp(1))^5-703687441776640*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4
*pi))*tan(1/2*exp(1))^3*tan(1/4*exp(1))^6-4222124650659840*sqrt(2)*A*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^3*tan(1/4*exp(1))-949978046398
4640*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^4*tan(1/4
*exp(1))^2-8444249301319680*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*ta
n(1/2*exp(1))^4*tan(1/4*exp(1))^3+9499780463984640*sqrt(2)*A*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^4*tan(1/4*exp(1))^4+2533274790395904*s
qrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^4*tan(1/4*exp(1
))^5-633318697598976*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*e
xp(1))^4*tan(1/4*exp(1))^6+2533274790395904*sqrt(2)*A*sign(sin(1/2*(f*x+exp
(1))-1/4*pi))*tan(1/2*exp(1))^4*tan(1/4*exp(1))+3166593487994880*sqrt(2)*A*
sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^5*tan(1/4*exp(1))^2-4222
124650659840*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^5
*tan(1/4*exp(1))^3-3166593487994880*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4
*pi))*tan(1/2*exp(1))^5*tan(1/4*exp(1))^4+1266637395197952*sqrt(2)*A*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^5*tan(1/4*exp(1))^5+2111062325

32992*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^5*tan(1/
4*exp(1))^6+1266637395197952*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*t
an(1/2*exp(1))^5*tan(1/4*exp(1))+1407374883553280*sqrt(2)*A*sign(sin(1/2*(f
*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^6*tan(1/4*exp(1))^3-422212465065984*sqr
t(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^6*tan(1/4*exp(1))
^5-422212465065984*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp
(1))^6*tan(1/4*exp(1))-422212465065984*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-
1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^2-703687441776640*sqrt
(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/
2*exp(1))^3+633318697598976*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*ta
n(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^4+211106232532992*sqrt(2)*A*sign(
sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^5
-1055531162664960*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2
*f*x+2*exp(1)))*tan(1/4*exp(1))^2+1055531162664960*sqrt(2)*A*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/4*exp(1))^4-70368744
177664*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(
1)))*tan(1/4*exp(1))^6+211106232532992*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-
1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))+3166593487994880*sqrt(
2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/4*exp(1))^2-4
222124650659840*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1)
)*tan(1/4*exp(1))^3-3166593487994880*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/
4*pi))*tan(1/2*exp(1))*tan(1/4*exp(1))^4+1266637395197952*sqrt(2)*A*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/4*exp(1))^5+2111062325329
92*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/4*exp
(1))^6+1266637395197952*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/
2*exp(1))*tan(1/4*exp(1))+2744381022928896*sqrt(2)*B*sign(sin(1/2*(f*x+exp(
1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^2+70368744177664
0*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^
2*tan(1/2*exp(1))^3-2533274790395904*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/
4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^4-211106232532992*sqrt
(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(
1/2*exp(1))^5+211106232532992*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*
tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^6+2111062325329920*sqrt(2)*B*
sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/4*ex
p(1))^2+4222124650659840*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1
/2*(1/2*f*x+2*exp(1)))^2*tan(1/4*exp(1))^3-2111062325329920*sqrt(2)*B*sign(
sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/4*exp(1))
^4-1266637395197952*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1
/2*f*x+2*exp(1)))^2*tan(1/4*exp(1))^5+140737488355328*sqrt(2)*B*sign(sin(1/
2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/4*exp(1))^6-211
106232532992*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+
2*exp(1)))^2*tan(1/2*exp(1))-1266637395197952*sqrt(2)*B*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/4*exp(1))+6333186975989
76*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))
^3*tan(1/2*exp(1))^2+2111062325329920*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1

/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^3-422212465065984*sqr
t(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan
(1/2*exp(1))^4-633318697598976*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))
tan(1/2(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^5+70368744177664*sqrt(2)*B*s
ign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp
(1))^6-1407374883553280*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/
2*(1/2*f*x+2*exp(1)))^3*tan(1/4*exp(1))^3+422212465065984*sqrt(2)*B*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/4*exp(1))^5
-633318697598976*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*
f*x+2*exp(1)))^3*tan(1/2*exp(1))+422212465065984*sqrt(2)*B*sign(sin(1/2*(f*
x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/4*exp(1))-3166593487
994880*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1
/4*exp(1))^2-16888498602639360*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))
*tan(1/2*exp(1))^2*tan(1/4*exp(1))^3+3166593487994880*sqrt(2)*B*sign(sin(1/
2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/4*exp(1))^4+506654958079180
8*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/4*ex
p(1))^5-211106232532992*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/
2*exp(1))^2*tan(1/4*exp(1))^6+5066549580791808*sqrt(2)*B*sign(sin(1/2*(f*x+
exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/4*exp(1))+31665934879948800*sqrt(2
)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^3*tan(1/4*exp(1))^2-
42221246506598400*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(
1))^3*tan(1/4*exp(1))^3-31665934879948800*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1
))-1/4*pi))*tan(1/2*exp(1))^3*tan(1/4*exp(1))^4+12666373951979520*sqrt(2)*B
sign(sin(1/2(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^3*tan(1/4*exp(1))^5+211
1062325329920*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^
3*tan(1/4*exp(1))^6+12666373951979520*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1
/4*pi))*tan(1/2*exp(1))^3*tan(1/4*exp(1))+12666373951979520*sqrt(2)*B*sign(
sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^4*tan(1/4*exp(1))^2+422212465
0659840*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^4*tan(
1/4*exp(1))^3-12666373951979520*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi)
)*tan(1/2*exp(1))^4*tan(1/4*exp(1))^4-1266637395197952*sqrt(2)*B*sign(sin(1
/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^4*tan(1/4*exp(1))^5+84442493013196
8*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^4*tan(1/4*ex
p(1))^6-1266637395197952*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1
/2*exp(1))^4*tan(1/4*exp(1))-9499780463984640*sqrt(2)*B*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*tan(1/2*exp(1))^5*tan(1/4*exp(1))^2+12666373951979520*sqrt(
2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^5*tan(1/4*exp(1))^3
+9499780463984640*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(
1))^5*tan(1/4*exp(1))^4-3799912185593856*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1)
)-1/4*pi))*tan(1/2*exp(1))^5*tan(1/4*exp(1))^5-633318697598976*sqrt(2)*B*si
gn(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^5*tan(1/4*exp(1))^6-379991
2185593856*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^5*t
an(1/4*exp(1))+1055531162664960*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi)
)*tan(1/2*exp(1))^6*tan(1/4*exp(1))^2-2814749767106560*sqrt(2)*B*sign(sin(1
/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^6*tan(1/4*exp(1))^3-10555311626649

60*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^6*tan(1/4*e
xp(1))^4+844424930131968*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1
/2*exp(1))^6*tan(1/4*exp(1))^5+70368744177664*sqrt(2)*B*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*tan(1/2*exp(1))^6*tan(1/4*exp(1))^6+844424930131968*sqrt(2)
*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^6*tan(1/4*exp(1))+232
2168557862912*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x
+2*exp(1)))*tan(1/2*exp(1))^2-703687441776640*sqrt(2)*B*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^3-2955487255461
888*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1))
)*tan(1/2*exp(1))^4+211106232532992*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4
*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^5+70368744177664*sqrt(2)*
B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*ex
p(1))^6+4222124650659840*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1
/2*(1/2*f*x+2*exp(1)))*tan(1/4*exp(1))^2-1407374883553280*sqrt(2)*B*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/4*exp(1))^3-4
222124650659840*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f
*x+2*exp(1)))*tan(1/4*exp(1))^4+422212465065984*sqrt(2)*B*sign(sin(1/2*(f*x
+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/4*exp(1))^5+28147497671
0656*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)
))*tan(1/4*exp(1))^6+211106232532992*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/
4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))+422212465065984*sqrt(2)*
B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/4*ex
p(1))-9499780463984640*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2
*exp(1))*tan(1/4*exp(1))^2+12666373951979520*sqrt(2)*B*sign(sin(1/2*(f*x+ex
p(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/4*exp(1))^3+9499780463984640*sqrt(2)*B
sign(sin(1/2(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/4*exp(1))^4-37999
12185593856*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*ta
n(1/4*exp(1))^5-633318697598976*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi)
)*tan(1/2*exp(1))*tan(1/4*exp(1))^6-3799912185593856*sqrt(2)*B*sign(sin(1/2
*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/4*exp(1))+6333186975989760*sqr
t(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan
(1/2*exp(1))^2*tan(1/4*exp(1))^2+12666373951979520*sqrt(2)*A*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^2*tan(1/
4*exp(1))^3-6333186975989760*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*t
an(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^2*tan(1/4*exp(1))^4-3799912185
593856*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(
1)))^2*tan(1/2*exp(1))^2*tan(1/4*exp(1))^5+422212465065984*sqrt(2)*A*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^
2*tan(1/4*exp(1))^6-3799912185593856*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/
4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^2*tan(1/4*exp(1))-1055
5311626649600*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x
+2*exp(1)))^2*tan(1/2*exp(1))^3*tan(1/4*exp(1))^2+14073748835532800*sqrt(2)
*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2
*exp(1))^3*tan(1/4*exp(1))^3+10555311626649600*sqrt(2)*A*sign(sin(1/2*(f*x+
exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^3*tan(1/4*ex

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p(1))^4-4222124650659840*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1
/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^3*tan(1/4*exp(1))^5-70368744177664
0*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^
2*tan(1/2*exp(1))^3*tan(1/4*exp(1))^6-4222124650659840*sqrt(2)*A*sign(sin(1
/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^3*ta
n(1/4*exp(1))-9499780463984640*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))
*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^4*tan(1/4*exp(1))^2-84442493
01319680*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*ex
p(1)))^2*tan(1/2*exp(1))^4*tan(1/4*exp(1))^3+9499780463984640*sqrt(2)*A*sig
n(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1
))^4*tan(1/4*exp(1))^4+2533274790395904*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))
-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^4*tan(1/4*exp(1))^5
-633318697598976*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*
f*x+2*exp(1)))^2*tan(1/2*exp(1))^4*tan(1/4*exp(1))^6+2533274790395904*sqrt(
2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1
/2*exp(1))^4*tan(1/4*exp(1))+3166593487994880*sqrt(2)*A*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^5*tan(1/4*exp
(1))^2-4222124650659840*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/
2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^5*tan(1/4*exp(1))^3-316659348799488
0*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^
2*tan(1/2*exp(1))^5*tan(1/4*exp(1))^4+1266637395197952*sqrt(2)*A*sign(sin(1
/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^5*ta
n(1/4*exp(1))^5+211106232532992*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi)
)*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^5*tan(1/4*exp(1))^6+1266637
395197952*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*e
xp(1)))^2*tan(1/2*exp(1))^5*tan(1/4*exp(1))+1407374883553280*sqrt(2)*A*sign
(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1)
)^6*tan(1/4*exp(1))^3-422212465065984*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1
/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^6*tan(1/4*exp(1))^5-4
22212465065984*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*
x+2*exp(1)))^2*tan(1/2*exp(1))^6*tan(1/4*exp(1))+3166593487994880*sqrt(2)*A
*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*e
xp(1))*tan(1/4*exp(1))^2-4222124650659840*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1
))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))*tan(1/4*exp(1))^3
-3166593487994880*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2
*f*x+2*exp(1)))^2*tan(1/2*exp(1))*tan(1/4*exp(1))^4+1266637395197952*sqrt(2
)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/
2*exp(1))*tan(1/4*exp(1))^5+211106232532992*sqrt(2)*A*sign(sin(1/2*(f*x+exp
(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))*tan(1/4*exp(1))
^6+1266637395197952*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1
/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))*tan(1/4*exp(1))+6333186975989760*sqrt(2
)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/
2*exp(1))^2*tan(1/4*exp(1))^2-12666373951979520*sqrt(2)*A*sign(sin(1/2*(f*x
+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^2*tan(1/4*e
xp(1))^3-6333186975989760*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(
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1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^2*tan(1/4*exp(1))^4+3799912185593
856*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1))
)^3*tan(1/2*exp(1))^2*tan(1/4*exp(1))^5+422212465065984*sqrt(2)*A*sign(sin(
1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^2*t
an(1/4*exp(1))^6+3799912185593856*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*p
i))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^2*tan(1/4*exp(1))+1055531
1626649600*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*
exp(1)))^3*tan(1/2*exp(1))^3*tan(1/4*exp(1))^2+14073748835532800*sqrt(2)*A*
sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*ex
p(1))^3*tan(1/4*exp(1))^3-10555311626649600*sqrt(2)*A*sign(sin(1/2*(f*x+exp
(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^3*tan(1/4*exp(1
))^4-4222124650659840*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*
(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^3*tan(1/4*exp(1))^5+703687441776640*s
qrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*t
an(1/2*exp(1))^3*tan(1/4*exp(1))^6-4222124650659840*sqrt(2)*A*sign(sin(1/2*
(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^3*tan(1
/4*exp(1))-9499780463984640*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*ta
n(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^4*tan(1/4*exp(1))^2+84442493013
19680*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1
)))^3*tan(1/2*exp(1))^4*tan(1/4*exp(1))^3+9499780463984640*sqrt(2)*A*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^
4*tan(1/4*exp(1))^4-2533274790395904*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/
4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^4*tan(1/4*exp(1))^5-63
3318697598976*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x
+2*exp(1)))^3*tan(1/2*exp(1))^4*tan(1/4*exp(1))^6-2533274790395904*sqrt(2)*
A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*
exp(1))^4*tan(1/4*exp(1))-3166593487994880*sqrt(2)*A*sign(sin(1/2*(f*x+exp(
1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^5*tan(1/4*exp(1)
)^2-4222124650659840*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(
1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^5*tan(1/4*exp(1))^3+3166593487994880*s
qrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*t
an(1/2*exp(1))^5*tan(1/4*exp(1))^4+1266637395197952*sqrt(2)*A*sign(sin(1/2*
(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^5*tan(1
/4*exp(1))^5-211106232532992*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*t
an(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^5*tan(1/4*exp(1))^6+1266637395
197952*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(
1)))^3*tan(1/2*exp(1))^5*tan(1/4*exp(1))-1407374883553280*sqrt(2)*A*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^6
*tan(1/4*exp(1))^3+422212465065984*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*
pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^6*tan(1/4*exp(1))^5+4222
12465065984*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2
*exp(1)))^3*tan(1/2*exp(1))^6*tan(1/4*exp(1))-3166593487994880*sqrt(2)*A*si
gn(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(
1))*tan(1/4*exp(1))^2-4222124650659840*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-
1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))*tan(1/4*exp(1))^3+31
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66593487994880*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))*tan(1/4*exp(1))^4+1266637395197952*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))*tan(1/4*exp(1))^5-211106232532992*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))*tan(1/4*exp(1))^6+1266637395197952*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))*tan(1/4*exp(1))+6333186975989760*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^2*tan(1/4*exp(1))^2-12666373951979520*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^2*tan(1/4*exp(1))^3-6333186975989760*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^2*tan(1/4*exp(1))^4+3799912185593856*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^2*tan(1/4*exp(1))^5+422212465065984*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^2*tan(1/4*exp(1))^6+3799912185593856*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^2*tan(1/4*exp(1))+10555311626649600*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^3*tan(1/4*exp(1))^2+14073748835532800*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^3*tan(1/4*exp(1))^3-10555311626649600*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^3*tan(1/4*exp(1))^4-4222124650659840*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^3*tan(1/4*exp(1))^5+703687441776640*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^3*tan(1/4*exp(1))^6-4222124650659840*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^3*tan(1/4*exp(1))-9499780463984640*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^4*tan(1/4*exp(1))^2+8444249301319680*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^4*tan(1/4*exp(1))^3+9499780463984640*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^4*tan(1/4*exp(1))^4-2533274790395904*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^4*tan(1/4*exp(1))^5-633318697598976*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^4*tan(1/4*exp(1))^6-2533274790395904*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^4*tan(1/4*exp(1))-3166593487994880*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^5*tan(1/4*exp(1))^2-4222124650659840*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^5*tan(1/4*exp(1))^3+3166593487994880*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^5*tan(1/4*exp(1))^4+1266637395197952*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^5*tan(1/4*exp(1))^5-211106232532992*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^5*ta

n(1/4*exp(1))^6+1266637395197952*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi
))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^5*tan(1/4*exp(1))-1407374883
553280*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(
1)))*tan(1/2*exp(1))^6*tan(1/4*exp(1))^3+422212465065984*sqrt(2)*A*sign(sin
(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^6*ta
n(1/4*exp(1))^5+422212465065984*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi)
)*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^6*tan(1/4*exp(1))-31665934879
94880*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1
)))*tan(1/2*exp(1))*tan(1/4*exp(1))^2-4222124650659840*sqrt(2)*A*sign(sin(1
/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))*tan(1/
4*exp(1))^3+3166593487994880*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*t
an(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))*tan(1/4*exp(1))^4+12666373951979
52*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))
*tan(1/2*exp(1))*tan(1/4*exp(1))^5-211106232532992*sqrt(2)*A*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))*tan(1/4*ex
p(1))^6+1266637395197952*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1
/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))*tan(1/4*exp(1))-41165715343933440*sq
rt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*ta
n(1/2*exp(1))^2*tan(1/4*exp(1))^2-50665495807918080*sqrt(2)*B*sign(sin(1/2*
(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^2*tan(1
/4*exp(1))^3+41165715343933440*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))
tan(1/2(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^2*tan(1/4*exp(1))^4+15199648
742375424*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*e
xp(1)))^2*tan(1/2*exp(1))^2*tan(1/4*exp(1))^5-2744381022928896*sqrt(2)*B*si
gn(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(
1))^2*tan(1/4*exp(1))^6+15199648742375424*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1
))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^2*tan(1/4*exp(1))
-10555311626649600*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/
2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^3*tan(1/4*exp(1))^2+14073748835532800*sq
rt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*ta
n(1/2*exp(1))^3*tan(1/4*exp(1))^3+10555311626649600*sqrt(2)*B*sign(sin(1/2*
(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^3*tan(1
/4*exp(1))^4-4222124650659840*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*
tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^3*tan(1/4*exp(1))^5-703687441
776640*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(
1)))^2*tan(1/2*exp(1))^3*tan(1/4*exp(1))^6-4222124650659840*sqrt(2)*B*sign(
sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))
^3*tan(1/4*exp(1))+37999121855938560*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/
4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^4*tan(1/4*exp(1))^2+54
887620458577920*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f
*x+2*exp(1)))^2*tan(1/2*exp(1))^4*tan(1/4*exp(1))^3-37999121855938560*sqrt(
2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1
/2*exp(1))^4*tan(1/4*exp(1))^4-16466286137573376*sqrt(2)*B*sign(sin(1/2*(f*
x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^4*tan(1/4*
exp(1))^5+2533274790395904*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan

(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^4*tan(1/4*exp(1))^6-164662861375
73376*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1
)))^2*tan(1/2*exp(1))^4*tan(1/4*exp(1))+3166593487994880*sqrt(2)*B*sign(sin
(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^5*
tan(1/4*exp(1))^2-4222124650659840*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*
pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^5*tan(1/4*exp(1))^3-3166
593487994880*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+
2*exp(1)))^2*tan(1/2*exp(1))^5*tan(1/4*exp(1))^4+1266637395197952*sqrt(2)*B
sign(sin(1/2(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*e
xp(1))^5*tan(1/4*exp(1))^5+211106232532992*sqrt(2)*B*sign(sin(1/2*(f*x+exp(
1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^5*tan(1/4*exp(1)
)^6+1266637395197952*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(
1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^5*tan(1/4*exp(1))-3166593487994880*sqr
t(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan
(1/2*exp(1))^6*tan(1/4*exp(1))^2-2814749767106560*sqrt(2)*B*sign(sin(1/2*(f
*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^6*tan(1/4
*exp(1))^3+3166593487994880*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*ta
n(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^6*tan(1/4*exp(1))^4+84442493013
1968*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)
))^2*tan(1/2*exp(1))^6*tan(1/4*exp(1))^5-211106232532992*sqrt(2)*B*sign(sin
(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^6*
tan(1/4*exp(1))^6+844424930131968*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*p
i))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))^6*tan(1/4*exp(1))+3166593
487994880*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*e
xp(1)))^2*tan(1/2*exp(1))*tan(1/4*exp(1))^2-4222124650659840*sqrt(2)*B*sign
(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1)
)*tan(1/4*exp(1))^3-3166593487994880*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/
4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))*tan(1/4*exp(1))^4+1266
637395197952*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+
2*exp(1)))^2*tan(1/2*exp(1))*tan(1/4*exp(1))^5+211106232532992*sqrt(2)*B*si
gn(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(
1))*tan(1/4*exp(1))^6+1266637395197952*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-
1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^2*tan(1/2*exp(1))*tan(1/4*exp(1))-9499
780463984640*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+
2*exp(1)))^3*tan(1/2*exp(1))^2*tan(1/4*exp(1))^2+8444249301319680*sqrt(2)*B
sign(sin(1/2(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*e
xp(1))^2*tan(1/4*exp(1))^3+9499780463984640*sqrt(2)*B*sign(sin(1/2*(f*x+exp
(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^2*tan(1/4*exp(1
))^4-2533274790395904*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*
(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^2*tan(1/4*exp(1))^5-633318697598976*s
qrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*t
an(1/2*exp(1))^2*tan(1/4*exp(1))^6-2533274790395904*sqrt(2)*B*sign(sin(1/2*
(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^2*tan(1
/4*exp(1))-31665934879948800*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*t
an(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^3*tan(1/4*exp(1))^2-4222124650

6598400*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp
(1)))^3*tan(1/2*exp(1))^3*tan(1/4*exp(1))^3+31665934879948800*sqrt(2)*B*sig
n(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1
))^3*tan(1/4*exp(1))^4+12666373951979520*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1)
)-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^3*tan(1/4*exp(1))^
5-2111062325329920*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/
2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^3*tan(1/4*exp(1))^6+12666373951979520*sq
rt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*ta
n(1/2*exp(1))^3*tan(1/4*exp(1))+6333186975989760*sqrt(2)*B*sign(sin(1/2*(f*
x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^4*tan(1/4*
exp(1))^2-12666373951979520*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*ta
n(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^4*tan(1/4*exp(1))^3-63331869759
89760*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1
)))^3*tan(1/2*exp(1))^4*tan(1/4*exp(1))^4+3799912185593856*sqrt(2)*B*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^
4*tan(1/4*exp(1))^5+422212465065984*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4
*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^4*tan(1/4*exp(1))^6+379
9912185593856*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x
+2*exp(1)))^3*tan(1/2*exp(1))^4*tan(1/4*exp(1))+9499780463984640*sqrt(2)*B*
sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*ex
p(1))^5*tan(1/4*exp(1))^2+12666373951979520*sqrt(2)*B*sign(sin(1/2*(f*x+exp
(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^5*tan(1/4*exp(1
))^3-9499780463984640*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*
(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^5*tan(1/4*exp(1))^4-3799912185593856*
sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*
tan(1/2*exp(1))^5*tan(1/4*exp(1))^5+633318697598976*sqrt(2)*B*sign(sin(1/2*
(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^5*tan(1
/4*exp(1))^6-3799912185593856*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*
tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^5*tan(1/4*exp(1))-10555311626
64960*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1
)))^3*tan(1/2*exp(1))^6*tan(1/4*exp(1))^2+1055531162664960*sqrt(2)*B*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^
6*tan(1/4*exp(1))^4-70368744177664*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*
pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^6*tan(1/4*exp(1))^6+9499
780463984640*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+
2*exp(1)))^3*tan(1/2*exp(1))*tan(1/4*exp(1))^2+12666373951979520*sqrt(2)*B*
sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*ex
p(1))*tan(1/4*exp(1))^3-9499780463984640*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1)
)-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))*tan(1/4*exp(1))^4-
3799912185593856*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*
f*x+2*exp(1)))^3*tan(1/2*exp(1))*tan(1/4*exp(1))^5+633318697598976*sqrt(2)*
B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*
exp(1))*tan(1/4*exp(1))^6-3799912185593856*sqrt(2)*B*sign(sin(1/2*(f*x+exp(
1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))*tan(1/4*exp(1))-
3483252836794368O*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2

*f*x+2*exp(1)))*tan(1/2*exp(1))^2*tan(1/4*exp(1))^2+5910974510923776 0*sqrt(
2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2
*exp(1))^2*tan(1/4*exp(1))^3+3483252836794368 0*sqrt(2)*B*sign(sin(1/2*(f*x+
exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^2*tan(1/4*exp(
1))^4-17732923532771328*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/
2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^2*tan(1/4*exp(1))^5-2322168557862912*
sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*ta
n(1/2*exp(1))^2*tan(1/4*exp(1))^6-17732923532771328*sqrt(2)*B*sign(sin(1/2*
(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^2*tan(1/4
*exp(1))+10555311626649600*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan
(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^3*tan(1/4*exp(1))^2+14073748835532
800*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1))
)*tan(1/2*exp(1))^3*tan(1/4*exp(1))^3-10555311626649600*sqrt(2)*B*sign(sin(
1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^3*tan
(1/4*exp(1))^4-4222124650659840*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi)
)*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^3*tan(1/4*exp(1))^5+703687441
776640*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(
1)))*tan(1/2*exp(1))^3*tan(1/4*exp(1))^6-4222124650659840*sqrt(2)*B*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^3*t
an(1/4*exp(1))+44332308831928320*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi
))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^4*tan(1/4*exp(1))^2-46443371
157258240*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*e
xp(1)))*tan(1/2*exp(1))^4*tan(1/4*exp(1))^3-44332308831928320*sqrt(2)*B*sig
n(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))
^4*tan(1/4*exp(1))^4+13933011347177472*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-
1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^4*tan(1/4*exp(1))^5+29
55487255461888*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*
x+2*exp(1)))*tan(1/2*exp(1))^4*tan(1/4*exp(1))^6+13933011347177472*sqrt(2)*
B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*ex
p(1))^4*tan(1/4*exp(1))-3166593487994880*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1)
)-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^5*tan(1/4*exp(1))^2-
4222124650659840*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*
f*x+2*exp(1)))*tan(1/2*exp(1))^5*tan(1/4*exp(1))^3+3166593487994880*sqrt(2)
*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*e
xp(1))^5*tan(1/4*exp(1))^4+1266637395197952*sqrt(2)*B*sign(sin(1/2*(f*x+exp
(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^5*tan(1/4*exp(1))
^5-211106232532992*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/
2*f*x+2*exp(1)))*tan(1/2*exp(1))^5*tan(1/4*exp(1))^6+1266637395197952*sqrt(
2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2
*exp(1))^5*tan(1/4*exp(1))-1055531162664960*sqrt(2)*B*sign(sin(1/2*(f*x+exp
(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^6*tan(1/4*exp(1))
^2+5629499534213120*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1
/2*f*x+2*exp(1)))*tan(1/2*exp(1))^6*tan(1/4*exp(1))^3+1055531162664960*sqrt
(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/
2*exp(1))^6*tan(1/4*exp(1))^4-1688849860263936*sqrt(2)*B*sign(sin(1/2*(f*x+

exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^6*tan(1/4*exp(
1))^5-70368744177664*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(
1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^6*tan(1/4*exp(1))^6-1688849860263936*sqr
t(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1
/2*exp(1))^6*tan(1/4*exp(1))-3166593487994880*sqrt(2)*B*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))*tan(1/4*exp(1))
^2-4222124650659840*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1
/2*f*x+2*exp(1)))*tan(1/2*exp(1))*tan(1/4*exp(1))^3+3166593487994880*sqrt(2
)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*
exp(1))*tan(1/4*exp(1))^4+1266637395197952*sqrt(2)*B*sign(sin(1/2*(f*x+exp(
1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))*tan(1/4*exp(1))^5-
211106232532992*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f
*x+2*exp(1)))*tan(1/2*exp(1))*tan(1/4*exp(1))^6+1266637395197952*sqrt(2)*B*
sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(
1))*tan(1/4*exp(1)))/(422212465065984*a*tan(1/2*exp(1))-140737488355328*a*t
an(1/2*exp(1))^3*tan(1/4*exp(1))^6+422212465065984*a*tan(1/2*exp(1))^2*tan(
1/4*exp(1))^6-422212465065984*a*tan(1/2*exp(1))^3*tan(1/4*exp(1))^4-1407374
88355328*a*tan(1/4*exp(1))^6+1266637395197952*a*tan(1/2*exp(1))^2*tan(1/4*e
xp(1))^4-422212465065984*a*tan(1/2*exp(1))^3*tan(1/4*exp(1))^2-422212465065
984*a*tan(1/4*exp(1))^4+1266637395197952*a*tan(1/2*exp(1))^2*tan(1/4*exp(1)
)^2-140737488355328*a*tan(1/2*exp(1))^3+422212465065984*a*tan(1/2*exp(1))^2
-422212465065984*a*tan(1/4*exp(1))^2+1266637395197952*a*tan(1/2*exp(1))*tan
(1/4*exp(1))^2+1266637395197952*a*tan(1/2*exp(1))*tan(1/4*exp(1))^4+4222124
65065984*a*tan(1/2*exp(1))*tan(1/4*exp(1))^6-140737488355328*a)/(-2*tan(1/2
*(1/2*f*x+2*exp(1)))^3-6*tan(1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))+tan(
1/2*(1/2*f*x+2*exp(1)))^4*tan(1/2*exp(1))^3+2*tan(1/2*(1/2*f*x+2*exp(1)))^3
*tan(1/2*exp(1))^3-3*tan(1/2*(1/2*f*x+2*exp(1)))^4*tan(1/2*exp(1))^2+6*tan(
1/2*(1/2*f*x+2*exp(1)))^3*tan(1/2*exp(1))^2+tan(1/2*(1/2*f*x+2*exp(1)))^4-t
an(1/2*exp(1))^3+3*tan(1/2*exp(1))^2-3*tan(1/2*(1/2*f*x+2*exp(1)))^4*tan(1/
2*exp(1))+6*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^2+2*tan(1/2*(1/2*f*
x+2*exp(1)))*tan(1/2*exp(1))^3-6*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1)
)-2*tan(1/2*(1/2*f*x+2*exp(1)))+3*tan(1/2*exp(1))-1)+1/2*(79228162514264337
593543950336*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^3
-1584563250285286751870879006720*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi
))*tan(1/4*exp(1))^3+4753689750855860255612637020160*sqrt(2)*A*sign(sin(1/2*
(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^5-237684487542793012780631851008*sqrt
(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))+475368975085586025
561263702016*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))-7
9228162514264337593543950336*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*t
an(1/2*exp(1))^3+1584563250285286751870879006720*sqrt(2)*B*sign(sin(1/2*(f*
x+exp(1))-1/4*pi))*tan(1/4*exp(1))^3-4753689750855860255612637020160*sqrt(2)
*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^5+2376844875427930127
80631851008*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))-47
5368975085586025561263702016*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*t
an(1/4*exp(1))+4753689750855860255612637020160*sqrt(2)*A*sign(sin(1/2*(f*x+

```
exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/4*exp(1))^3-1426106925256758076683
791106048*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*ta
n(1/4*exp(1))^5-1426106925256758076683791106048*sqrt(2)*A*sign(sin(1/2*(f*x
+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/4*exp(1))-11884224377139650639031
59255040*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^3*tan
(1/4*exp(1))^2+1188422437713965063903159255040*sqrt(2)*A*sign(sin(1/2*(f*x+
exp(1))-1/4*pi))*tan(1/2*exp(1))^3*tan(1/4*exp(1))^4-7922816251426433759354
3950336*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^3*tan(
1/4*exp(1))^6+3565267313141895191709477765120*sqrt(2)*A*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/4*exp(1))^2-3565267313141895191709477
765120*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/4
*exp(1))^4+237684487542793012780631851008*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1
))-1/4*pi))*tan(1/2*exp(1))*tan(1/4*exp(1))^6-47536897508558602556126370201
60*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/4*e
xp(1))^3+1426106925256758076683791106048*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1)
)-1/4*pi))*tan(1/2*exp(1))^2*tan(1/4*exp(1))^5+1426106925256758076683791106
048*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/4*
exp(1))+1188422437713965063903159255040*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))
-1/4*pi))*tan(1/2*exp(1))^3*tan(1/4*exp(1))^2-11884224377139650639031592550
40*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^3*tan(1/4*e
xp(1))^4+79228162514264337593543950336*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-
1/4*pi))*tan(1/2*exp(1))^3*tan(1/4*exp(1))^6-356526731314189519170947776512
0*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/4*exp(
1))^2+3565267313141895191709477765120*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1
/4*pi))*tan(1/2*exp(1))*tan(1/4*exp(1))^4-237684487542793012780631851008*sq
rt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/4*exp(1))^
6)*ln(abs(2*tan(1/2*exp(1))^3-2*sqrt(2)*sqrt(tan(1/2*exp(1))^2+1)*(tan(1/2*
exp(1))^2+1)+6*tan(1/2*exp(1))^2-6*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(
1))^2+2*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^3-6*tan(1/2*(1/2*f*x+2*
exp(1)))*tan(1/2*exp(1))+2*tan(1/2*(1/2*f*x+2*exp(1)))-6*tan(1/2*exp(1))-2)
/abs(2*tan(1/2*exp(1))^3+2*sqrt(2)*sqrt(tan(1/2*exp(1))^2+1)*(tan(1/2*exp(1
))^2+1)+6*tan(1/2*exp(1))^2-6*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^2
+2*tan(1/2*(1/2*f*x+2*exp(1)))*tan(1/2*exp(1))^3-6*tan(1/2*(1/2*f*x+2*exp(1
)))*tan(1/2*exp(1))+2*tan(1/2*(1/2*f*x+2*exp(1)))-6*tan(1/2*exp(1))-2))/sqr
t(2)/sqrt(tan(1/2*exp(1))^2+1)/(tan(1/2*exp(1))^2+1)/(158456325028528675187
087900672*a*tan(1/4*exp(1))^6+475368975085586025561263702016*a*tan(1/4*exp(
1))^4+475368975085586025561263702016*a*tan(1/4*exp(1))^2+158456325028528675
187087900672*a))/f
```

maple [A] time = 1.02, size = 53, normalized size = 0.73

$$\frac{2c\left(\sin\left(fx+e\right)-1\right)\left(-B\sin\left(fx+e\right)+A-2B\right)}{a\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] 2*c/a*(sin(f*x+e)-1)*(-B*sin(f*x+e)+A-2*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2
)/f
```

maxima [B] time = 0.45, size = 174, normalized size = 2.38

$$-\frac{2\left(\frac{2\,B\left(\sqrt{c}+\frac{\sqrt{c}\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sqrt{c}\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}}-\frac{A\left(\sqrt{c}+\frac{\sqrt{c}\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] -2*(2*B*(sqrt(c) + sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + sqrt(c)*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2)/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*sqr
t(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)) - A*(sqrt(c) + sqrt(c)*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2)/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*sqrt(
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)))/f
```

mupad [B] time = 13.15, size = 128, normalized size = 1.75

$$\frac{2\sqrt{-c\left(\sin\left(e+f\,x\right)-1\right)}\,\left(2\,B\,\sin\left(2\,e+2\,f\,x\right)-2\,A\,\sin\left(2\,e+2\,f\,x\right)-4\,A\,\left(2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)+7\,B\,\left(2\sin\right.\right.}{a\,f\,\left(4\sin\left(e+f\,x\right)^2+\sin\left(e+f\,x\right)+\sin\left(3\,e+3\,f\,x\right)-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x)),
x)
```

```
[Out] (2*(-c*(sin(e + f*x) - 1))^(1/2)*(2*B*sin(2*e + 2*f*x) - 2*A*sin(2*e + 2*f*
x) - 4*A*(2*sin(e/2 + (f*x)/2)^2 - 1) + 7*B*(2*sin(e/2 + (f*x)/2)^2 - 1) +
B*(2*sin((3*e)/2 + (3*f*x)/2)^2 - 1)))/(a*f*(sin(e + f*x) + sin(3*e + 3*f*x
) + 4*sin(e + f*x)^2 - 4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sqrt{-c\sin\left(e+fx\right)+c}}{\sin\left(e+fx\right)+1}\,dx+\int \frac{B\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)}{\sin\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] (Integral(A*sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x) + 1), x) + Integral(B*s
qrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x) + 1), x))/a
```

3.112 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=91

$$\frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}a\sqrt{c}f}-\frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf}$$

```
[Out] 1/2*(A+B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a/
f*2^(1/2)/c^(1/2)-(A-B)*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/c/f
```

Rubi [A] time = 0.28, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2855, 2649, 206}

$$\frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}a\sqrt{c}f}-\frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] ((A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]
)/(Sqrt[2]*a*Sqrt[c]*f) - ((A - B)*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(
a*c*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
```

```
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\int \sec^2(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx}{ac}$$

$$= -\frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf} + \frac{(A+B)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}}{2a}$$

$$= -\frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf} - \frac{(A+B)\,\mathrm{Subst}\left(\int \frac{1}{2c-x^2}\right.}{af}$$

$$= \frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}\,a\sqrt{c}\,f} - \frac{(A-B)\sec(e+fx)\sqrt{c-c\sin}}{acf}$$

Mathematica [C] time = 0.45, size = 140, normalized size = 1.54

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-(1+i)\sqrt[4]{-1}\,(A+B)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-}\right.\right.}{af(\sin(e+fx)+1)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x
]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-A + B - (1 + I)*(-1)^(1/4)*(A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + T
```

```
an[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))/(a*f*(1 + Sin[e +
 f*x])*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.43, size = 162, normalized size = 1.78

$$\frac{\sqrt{2}\,(A+B)\sqrt{c}\,\cos\left(fx+e\right)\log\left(-\frac{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)+\frac{2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)}{\sqrt{c}}+3\cos\left(fx+e\right)+2}{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right)}{4\,acf\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(A + B)*sqrt(c)*cos(f*x + e)*log(-(cos(f*x + e)^2 + (cos(f*x +
 e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) +
 sin(f*x + e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x
 + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*sqrt(-c*sin(f*x + e) + c)*
(A - B))/(a*c*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2
*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign by
 intervals (correct if the argument is real):Check [abs(sin((f*t_nostep+exp
(1))/2-pi/4))]Unable to check sign: (8*pi/t_nostep/2)>(-8*pi/t_nostep/2)Dis
continuities at zeroes of sin((f*t_nostep+exp(1))/2-pi/4) were not checkedU
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
```

```
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*p
i/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(t_nostep-1)]inde
x.cc index_m i_lex_is_greater Error: Bad Argument Value
```

maple [A] time = 1.29, size = 130, normalized size = 1.43

$$-\frac{\left(\sin\left(fx+e\right)-1\right)\left(\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ A+\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right.\right.}{2a\sqrt{c}\ \cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\ f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/2/a*(sin(f*x+e)-1)*(2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)
/c^(1/2))*(c*(1+sin(f*x+e)))^(1/2)*A+2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))
^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(1/2)*B-2*c^(1/2)*A+2*c^(1/2)*B)
/c^(1/2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
 c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{\left(a+a\,\sin\left(e+f\,x\right)\right)\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2)),
x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2)),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)+\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int \frac{B\sin\left(e+fx\right)}{\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)+\sqrt{-c\sin\left(e+fx\right)+c}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] (Integral(A/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + sqrt(-c*sin(e + f*x)
+ c)), x) + Integral(B*sin(e + f*x)/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)
 + sqrt(-c*sin(e + f*x) + c)), x))/a
```

3.113 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=136

$$\frac{(3A-B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}\,ac^{3/2}f}+\frac{(3A-B)\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}}-\frac{(A-B)\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/4*(3*A-B)*cos(f*x+e)/a/f/(c-c*sin(f*x+e))^(3/2)+1/8*(3*A-B)*arctanh(1/2*c
os(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a/c^(3/2)/f*2^(1/2)-(A-B)
*sec(f*x+e)/a/c/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2967, 2855, 2650, 2649, 206}

$$\frac{(3A-B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}\,ac^{3/2}f}+\frac{(3A-B)\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}}-\frac{(A-B)\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2)),
x]
```

```
[Out] ((3*A - B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]]
)])/(4*Sqrt[2]*a*c^(3/2)*f) + ((3*A - B)*Cos[e + f*x])/(4*a*f*(c - c*Sin[e
+ f*x])^(3/2)) - ((A - B)*Sec[e + f*x])/(a*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx}{ac}$$

$$= -\frac{(A-B)\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}} + \frac{(3A-B)\int \frac{1}{(c-c\sin(e+fx))^{3/2}}\,dx}{2a}$$

$$= \frac{(3A-B)\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}} - \frac{(A-B)\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}} + \frac{(3A-B)}{}$$

$$= \frac{(3A-B)\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}} - \frac{(A-B)\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}} - \frac{(3A-B)}{}$$

$$= \frac{(3A-B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}\,ac^{3/2}f} + \frac{(3A-B)\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}}$$

Mathematica [C] time = 0.54, size = 284, normalized size = 2.09

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2(B-A)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(2*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + (A + B)*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - (1 + I)*
(-1)^(1/4)*(3*A - B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
]) + 2*(A + B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))/(4*
a*f*(1 + Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))
```

fricas [A] time = 0.45, size = 231, normalized size = 1.70

$$-\frac{\sqrt{2}\left((3A-B)\cos\left(fx+e\right)\sin\left(fx+e\right)-(3A-B)\cos\left(fx+e\right)\right)\sqrt{c}\log\left(-\frac{c\cos\left(fx+e\right)^{2}-2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\sqrt{c}\left(\cos\left(\right.\right.}{\cos\left(fx+e\right)^{2}+\left(c\right.}\right.}{16\left(ac^{2}f\cos\left(fx+e\right)\sin\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*((3*A - B)*cos(f*x + e)*sin(f*x + e) - (3*A - B)*cos(f*x + e
))*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqr
t(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e)
 - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x +
e) - cos(f*x + e) - 2)) + 4*((3*A - B)*sin(f*x + e) - A + 3*B)*sqrt(-c*sin(
f*x + e) + c))/(a*c^2*f*cos(f*x + e)*sin(f*x + e) - a*c^2*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(-1/4*(-A*(-sqrt(
c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+B*(-sqrt(c)*tan((f*
x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-A*sqrt(c)+B*sqrt(c))/a/c/(-(-
sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+2*sqrt(c)*(-
sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)/sign(tan((f
*x+exp(1))/2)-1)+1/8*(-3*A*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+ex
p(1))/2)^2+c))^3-3*B*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/
2)^2+c))^3+A*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c
))-A*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))
^2+B*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-B*sqr
t(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-A*sqr
t(c)*c-B*sqrt(c)*c)/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))
/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))
/2)^2+c))+c)^2/a/c/sign(tan((f*x+exp(1))/2)-1)+1/8*(3*A-B)*atan((-sqrt(c)*t
an((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c
))/sqrt(2)/sqrt(-c)/a/c/sign(tan((f*x+exp(1))/2)-1))
```

maple [A] time = 1.28, size = 225, normalized size = 1.65

$$-\frac{\sin\left(fx+e\right)\left(3A\sqrt{c+c\sin\left(fx+e\right)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{c}}\right)c-B\sqrt{c+c\sin\left(fx+e\right)}\,\sqrt{2}\,\operatorname{arctanh}\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/8/c^(5/2)/a*(sin(f*x+e)*(3*A*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*
(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c-B*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*
arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c-6*A*c^(3/2)+2*B*c^(3/
2))-3*A*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2
^(1/2)/c^(1/2))*c+B*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*sin(f*x
+e))^(1/2)*2^(1/2)/c^(1/2))*c+2*A*c^(3/2)-6*B*c^(3/2))/cos(f*x+e)/(c-c*sin(
f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^
(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{\left(a+a\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2)),
x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2)),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{-c\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^2\left(e+fx\right)+c\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int \frac{B\sin\left(e+fx\right)}{-c\sqrt{-c\sin\left(e+fx\right)+c}\,\sin^2\left(e+fx\right)+c\sqrt{-c\sin\left(e+fx\right)+c}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] (Integral(A/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2 + c*sqrt(-c*sin(e
 + f*x) + c)), x) + Integral(B*sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x) + c)*s
in(e + f*x)**2 + c*sqrt(-c*sin(e + f*x) + c)), x))/a
```

3.114 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=180

$$\frac{3(5A-3B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}\,ac^{5/2}f}-\frac{(5A-3B)\sec(e+fx)}{8ac^2f\sqrt{c-c\sin(e+fx)}}+\frac{3(5A-3B)\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}}+\frac{(A+B)\sec(e}{4acf(c-c\sin(e}$$

```
[Out] 3/32*(5*A-3*B)*cos(f*x+e)/a/c/f/(c-c*sin(f*x+e))^(3/2)+1/4*(A+B)*sec(f*x+e)
/a/c/f/(c-c*sin(f*x+e))^(3/2)+3/64*(5*A-3*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)
*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a/c^(5/2)/f*2^(1/2)-1/8*(5*A-3*B)*sec(f*x+
e)/a/c^2/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2967, 2859, 2687, 2650, 2649, 206}

$$-\frac{(5A-3B)\sec(e+fx)}{8ac^2f\sqrt{c-c\sin(e+fx)}}+\frac{3(5A-3B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}\,ac^{5/2}f}+\frac{3(5A-3B)\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}}+\frac{(A+B)\sec(}{4acf(c-c\sin(}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2)),
x]
```

```
[Out] (3*(5*A - 3*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f
*x]])])/(32*Sqrt[2]*a*c^(5/2)*f) + (3*(5*A - 3*B)*Cos[e + f*x])/(32*a*c*f*(
c - c*Sin[e + f*x])^(3/2)) + ((A + B)*Sec[e + f*x])/(4*a*c*f*(c - c*Sin[e +
 f*x])^(3/2)) - ((5*A - 3*B)*Sec[e + f*x])/(8*a*c^2*f*Sqrt[c - c*Sin[e + f*
x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= \frac{(A+B)\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} + \frac{(5A-3B)\int \frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx}{8ac^2}$$

$$= \frac{(A+B)\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} - \frac{(5A-3B)\sec(e+fx)}{8ac^2f\sqrt{c-c\sin(e+fx)}} + \frac{(3(5A}{}$$

$$= \frac{3(5A-3B)\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}} + \frac{(A+B)\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} - \frac{(5A}{8ac^2}$$

$$= \frac{3(5A-3B)\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}} + \frac{(A+B)\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} - \frac{(5A}{8ac^2}$$

$$= \frac{3(5A-3B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}\,ac^{5/2}f} + \frac{3(5A-3B)\cos(e+f}{32acf(c-c\sin(e+fx}$$

Mathematica [C] time = 0.85, size = 404, normalized size = 2.24

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(8(B-A)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(8*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 4*(A + B)*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (7*A -
 B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2]) - (3 + 3*I)*(-1)^(1/4)*(5*A - 3*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1
+ Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]) + 8*(A + B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2]) + 2*(7*A - B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(
e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))/(32*a*f*(1 + Sin[e + f*
x])*(c - c*Sin[e + f*x])^(5/2))
```

fricas [A] time = 0.44, size = 282, normalized size = 1.57

$$-\frac{3\sqrt{2}\left((5A-3B)\cos\left(fx+e\right)^{3}+2(5A-3B)\cos\left(fx+e\right)\sin\left(fx+e\right)-2(5A-3B)\cos\left(fx+e\right)\right)\sqrt{c}\log\left(-\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/128*(3*sqrt(2)*((5*A - 3*B)*cos(f*x + e)^3 + 2*(5*A - 3*B)*cos(f*x + e)*
sin(f*x + e) - 2*(5*A - 3*B)*cos(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 -
 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) +
 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*
x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(3*(5*A
 - 3*B)*cos(f*x + e)^2 + 4*(5*A - 3*B)*sin(f*x + e) - 12*A + 20*B)*sqrt(-c*
sin(f*x + e) + c))/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(f*x + e)*sin(f*x
 + e) - 2*a*c^3*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(-1/8*(-A*(-sqrt(
c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+B*(-sqrt(c)*tan((f*
x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-A*sqrt(c)+B*sqrt(c))/c^2/a/(-
(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+2*sqrt(c)*
(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)/sign(tan(
(f*x+exp(1))/2)-1)+1/64*(-41*A*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*
x+exp(1))/2)^2+c))^7-17*B*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp
(1))/2)^2+c))^7-127*A*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x
+exp(1))/2)^2+c))^6-23*B*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((
f*x+exp(1))/2)^2+c))^6-91*A*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x
+exp(1))/2)^2+c))^5-19*B*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+ex
p(1))/2)^2+c))^5+143*A*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((
f*x+exp(1))/2)^2+c))^4+39*B*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*
tan((f*x+exp(1))/2)^2+c))^4-3*A*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*ta
```

```
n((f*x+exp(1))/2)^2+c))^3+5*B*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan(
(f*x+exp(1))/2)^2+c))^3+47*A*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((
f*x+exp(1))/2)^2+c))-93*A*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*
tan((f*x+exp(1))/2)^2+c))^2+7*B*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*ta
n((f*x+exp(1))/2)^2+c))-37*B*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt
(c*tan((f*x+exp(1))/2)^2+c))^2-11*A*sqrt(c)*c^3-3*B*sqrt(c)*c^3)/c^2/(-(-sq
rt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sq
rt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^4/a/sign(tan(
(f*x+exp(1))/2)-1)+1/64*(15*A-9*B)*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(
c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/c^2/sqrt(-c)/
a/sign(tan((f*x+exp(1))/2)-1))
```

maple [B] time = 1.78, size = 350, normalized size = 1.94

$$-\frac{\sin\left(fx+e\right)\left(-30A\sqrt{c+c\sin\left(fx+e\right)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{c}}\right)c^{2}+40A\,c^{\frac{5}{2}}+18B\sqrt{c+c\sin\left(fx+e\right)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/64/c^(9/2)/a*(sin(f*x+e)*(-30*A*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1
/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+40*A*c^(5/2)+18*B*(c+c*sin(f
*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^
2-24*B*c^(5/2))+(-15*A*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*sin(
f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+30*A*c^(5/2)+9*B*(c+c*sin(f*x+e))^(1/2)*
2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2-18*B*c^(5/2
))*cos(f*x+e)^2+30*A*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*sin(f*
x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2-24*A*c^(5/2)-18*B*(c+c*sin(f*x+e))^(1/2)*2
^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+40*B*c^(5/2)
)/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^
(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{\left(a+a\,\sin\left(e+f\,x\right)\right)\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2)),
x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2)),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.115 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=242

$$\frac{2048c^4(7A-13B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{105a^2f}-\frac{512c^3(7A-13B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{105a^2f}-\frac{(A-B)\,\text{s}}{}$$

```
[Out] -512/105*(7*A-13*B)*c^3*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f-64/105*(7*A
-13*B)*c^2*sec(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a^2/f-16/105*(7*A-13*B)*c*sec(
f*x+e)*(c-c*sin(f*x+e))^(7/2)/a^2/f-1/21*(7*A-13*B)*sec(f*x+e)*(c-c*sin(f*x
+e))^(9/2)/a^2/f-1/3*(A-B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(13/2)/a^2/c^2/f+2
048/105*(7*A-13*B)*c^4*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f
```

Rubi [A] time = 0.65, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2855, 2674, 2673}

$$-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^2c^2f}-\frac{64c^2(7A-13B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{105a^2f}-\frac{512c^3(7A-13B}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])^
2,x]
```

```
[Out] (2048*(7*A - 13*B)*c^4*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(105*a^2*f) -
 (512*(7*A - 13*B)*c^3*Sec[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(105*a^2*f)
 - (64*(7*A - 13*B)*c^2*Sec[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(105*a^2*f
) - (16*(7*A - 13*B)*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(105*a^2*f)
 - ((7*A - 13*B)*Sec[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(21*a^2*f) - ((A
- B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(13/2))/(3*a^2*c^2*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
```

```
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{13/2}\,dx}{a^2c^2}$$

$$= -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^2c^2f} - \frac{(7A-13B)\int \sec$$

$$= -\frac{(7A-13B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{21a^2f} - \frac{(A-B)\sec^3(e$$

$$= -\frac{16(7A-13B)c\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{105a^2f} - \frac{(7A-13B)$$

$$= -\frac{64(7A-13B)c^2\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{105a^2f} - \frac{16(7A-1$$

$$= -\frac{512(7A-13B)c^3\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{105a^2f} - \frac{64(7A-$$

$$= \frac{2048(7A-13B)c^4\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{105a^2f} - \frac{512(7A-1$$

Mathematica [B] time = 6.82, size = 953, normalized size = 3.94

$$-\frac{(26A-83B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\sin\left(\frac{3}{2}(e+fx)\right)(c-c\sin(e+fx))^{9/2}}{12f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(\sin(e+fx)a+a)^2}-\frac{(2A-13B)\left(\cos\left(\frac{1}{2}(e\right.\right.}{20f\left(\cos\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e +
f*x])^2,x]
```

```
[Out] (-32*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(9/
2))/(3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a + a*Sin[e + f*x])^2) +
(32*(2*A - 3*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e + f*x]
)^(9/2))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a + a*Sin[e + f*x])^2)
 + ((164*A - 351*B)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
4*(c - c*Sin[e + f*x])^(9/2))/(4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*
(a + a*Sin[e + f*x])^2) + ((26*A - 83*B)*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(9/2))/(12*f*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^9*(a + a*Sin[e + f*x])^2) - ((2*A - 13*B)*Cos[(5*(e
 + f*x))/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(9
/2))/(20*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a + a*Sin[e + f*x])^2)
+ (B*Cos[(7*(e + f*x))/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Si
n[e + f*x])^(9/2))/(28*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a + a*Sin
[e + f*x])^2) + ((164*A - 351*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^4*(c - c*Sin[e + f*x])^(9/2))/(4*f*(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2])^9*(a + a*Sin[e + f*x])^2) - ((26*A - 83*B)*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(9/2)*Sin[(3*(e + f*x))/2])/(12*f*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a + a*Sin[e + f*x])^2) - ((2*A - 13
*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(9/2)*Sin[
(5*(e + f*x))/2])/(20*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a + a*Sin[
e + f*x])^2) - (B*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*
x])^(9/2)*Sin[(7*(e + f*x))/2])/(28*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^9*(a + a*Sin[e + f*x])^2)
```

fricas [A] time = 0.44, size = 153, normalized size = 0.63

$$\frac{2\left(3\,(7\,A-38\,B)c^4\cos\left(fx+e\right)^4-12\,(154\,A-311\,B)c^4\cos\left(fx+e\right)^2+24\,(287\,A-543\,B)c^4+\left(15\,Bc^4\cos\left(fx\right.\right.\right.}{105\left(a^2f\cos\left(fx+e\right)\sin\left(fx+e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] 2/105*(3*(7*A - 38*B)*c^4*cos(f*x + e)^4 - 12*(154*A - 311*B)*c^4*cos(f*x +
 e)^2 + 24*(287*A - 543*B)*c^4 + (15*B*c^4*cos(f*x + e)^4 + 4*(49*A - 136*B
)*c^4*cos(f*x + e)^2 + 8*(931*A - 1699*B)*c^4)*sin(f*x + e))*sqrt(-c*sin(f*
x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.36, size = 143, normalized size = 0.59

$$-\frac{2c^5\left(\sin\left(fx+e\right)-1\right)\left(15B\sin\left(fx+e\right)\left(\cos^4\left(fx+e\right)\right)+\left(196A-544B\right)\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+(7448A}{105a^2\left(1+\sin\left(fx+e\right)\right)\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x)
```

```
[Out] -2/105*c^5/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(15*B*sin(f*x+e)*cos(f*x+e)^4+
(196*A-544*B)*cos(f*x+e)^2*sin(f*x+e)+(7448*A-13592*B)*sin(f*x+e)+(21*A-114
*B)*cos(f*x+e)^4+(-1848*A+3732*B)*cos(f*x+e)^2+6888*A-13032*B)/cos(f*x+e)/(
c-c*sin(f*x+e))^(1/2)/f
```

maxima [B] time = 0.46, size = 762, normalized size = 3.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] -2/105*(7*(723*c^(9/2) + 2184*c^(9/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 537
0*c^(9/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10696*c^(9/2)*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 15021*c^(9/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
+ 21168*c^(9/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20748*c^(9/2)*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 21168*c^(9/2)*sin(f*x + e)^7/(cos(f*x + e) +
 1)^7 + 15021*c^(9/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 10696*c^(9/2)*s
in(f*x + e)^9/(cos(f*x + e) + 1)^9 + 5370*c^(9/2)*sin(f*x + e)^10/(cos(f*x
```

```
+ e) + 1)^10 + 2184*c^(9/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 723*c^(
9/2)*sin(f*x + e)^12/(cos(f*x + e) + 1)^12)*A/((a^2 + 3*a^2*sin(f*x + e)/(c
os(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(9/2
)) - 2*(4707*c^(9/2) + 14121*c^(9/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 3525
0*c^(9/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 68549*c^(9/2)*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 99549*c^(9/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
+ 134802*c^(9/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 138012*c^(9/2)*sin(f
*x + e)^6/(cos(f*x + e) + 1)^6 + 134802*c^(9/2)*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 + 99549*c^(9/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 68549*c^(9/2
)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 35250*c^(9/2)*sin(f*x + e)^10/(cos(
f*x + e) + 1)^10 + 14121*c^(9/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 47
07*c^(9/2)*sin(f*x + e)^12/(cos(f*x + e) + 1)^12)*B/((a^2 + 3*a^2*sin(f*x +
 e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1
)^(9/2)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{9/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^
2,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^
2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.116 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=201

$$\frac{128c^3(5A-11B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{15a^2f}-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{3a^2c^2f}-\frac{32c^2(5A-11B)\sec}{}$$

```
[Out] -32/15*(5*A-11*B)*c^2*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f-4/15*(5*A-11*
B)*c*sec(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a^2/f-1/15*(5*A-11*B)*sec(f*x+e)*(c-
c*sin(f*x+e))^(7/2)/a^2/f-1/3*(A-B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(11/2)/a^
2/c^2/f+128/15*(5*A-11*B)*c^3*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f
```

Rubi [A] time = 0.56, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2855, 2674, 2673}

$$-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{3a^2c^2f}-\frac{32c^2(5A-11B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^2f}+\frac{128c^3(5A-11B)}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^
2,x]
```

```
[Out] (128*(5*A - 11*B)*c^3*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(15*a^2*f) - (
32*(5*A - 11*B)*c^2*Sec[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(15*a^2*f) - (
4*(5*A - 11*B)*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(15*a^2*f) - ((5*
A - 11*B)*Sec[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(15*a^2*f) - ((A - B)*Se
c[e + f*x]^3*(c - c*Sin[e + f*x])^(11/2))/(3*a^2*c^2*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
```

```
[m + p, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{11/2}\,dx}{a^2c^2}$$

$$= -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{3a^2c^2f} - \frac{(5A-11B)\int \mathrm{se}}{}$$

$$= -\frac{(5A-11B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{15a^2f} - \frac{(A-B)\sec^3(}{}$$

$$= -\frac{4(5A-11B)c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^2f} - \frac{(5A-11B)}{}$$

$$= -\frac{32(5A-11B)c^2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^2f} - \frac{4(5A-1}{}$$

$$= \frac{128(5A-11B)c^3\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{15a^2f} - \frac{32(5A-11}{}$$

Mathematica [A] time = 2.83, size = 159, normalized size = 0.79

$$-\frac{c^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(12(25A-62B)\cos(2(e+fx))-2730A\sin(e+fx)-}{60a^2f(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
f*x])^2,x]
```

```
[Out] -1/60*(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(
-2100*A + 4725*B + 12*(25*A - 62*B)*Cos[2*(e + f*x)] + 3*B*Cos[4*(e + f*x)]
 - 2730*A*Sin[e + f*x] + 5838*B*Sin[e + f*x] - 10*A*Sin[3*(e + f*x)] + 46*B
*Sin[3*(e + f*x)]))/(a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e
 + f*x])^2)
```

fricas [A] time = 0.43, size = 133, normalized size = 0.66

$$-\frac{2\left(3\,Bc^3\cos\left(fx+e\right)^4+3\,(25\,A-63\,B)c^3\cos\left(fx+e\right)^2-12\,(25\,A-57\,B)c^3-\left((5\,A-23\,B)c^3\cos\left(fx+e\right)^2+\right.\right.}{15\left(a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] -2/15*(3*B*c^3*cos(f*x + e)^4 + 3*(25*A - 63*B)*c^3*cos(f*x + e)^2 - 12*(25
*A - 57*B)*c^3 - ((5*A - 23*B)*c^3*cos(f*x + e)^2 + 4*(85*A - 181*B)*c^3)*s
in(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a
^2*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.51, size = 121, normalized size = 0.60

$$\frac{2c^4\left(\sin\left(fx+e\right)-1\right)\left((-5A+23B)\sin\left(fx+e\right)\left(\cos^2\left(fx+e\right)\right)+(-340A+724B)\sin\left(fx+e\right)+3B\left(\cos^4\left(fx\right.\right.\right.}{15a^2\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\;f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 2/15*c^4/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*((-5*A+23*B)*sin(f*x+e)*cos(f*x+
e)^2+(-340*A+724*B)*sin(f*x+e)+3*B*cos(f*x+e)^4+(75*A-189*B)*cos(f*x+e)^2-3
00*A+684*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [B] time = 0.53, size = 670, normalized size = 3.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] -2/15*(5*(45*c^(7/2) + 138*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 285*c^
(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 544*c^(7/2)*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 630*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 812*c^
(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 630*c^(7/2)*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 + 544*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 285*c^
(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 138*c^(7/2)*sin(f*x + e)^9/(cos
(f*x + e) + 1)^9 + 45*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)*A/((a^
2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 1)^(7/2)) - 2*(249*c^(7/2) + 747*c^(7/2)*sin(f*x + e)/(co
s(f*x + e) + 1) + 1611*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2896*c
^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3612*c^(7/2)*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 4298*c^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 361
2*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 2896*c^(7/2)*sin(f*x + e)^7
/(cos(f*x + e) + 1)^7 + 1611*c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 +
747*c^(7/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 249*c^(7/2)*sin(f*x + e)^
10/(cos(f*x + e) + 1)^10)*B/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) +
 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^
2,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^
2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.117 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=154

$$-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3a^2c^2f}+\frac{32c^2(A-3B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f}-\frac{(A-3B)\sec(e+fx)}{3a}$$

```
[Out] -8/3*(A-3*B)*c*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f-1/3*(A-3*B)*sec(f*x+
e)*(c-c*sin(f*x+e))^(5/2)/a^2/f-1/3*(A-B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(9/
2)/a^2/c^2/f+32/3*(A-3*B)*c^2*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f
```

Rubi [A] time = 0.48, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2855, 2674, 2673}

$$-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3a^2c^2f}+\frac{32c^2(A-3B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f}-\frac{(A-3B)\sec(e+fx)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^
2,x]
```

```
[Out] (32*(A - 3*B)*c^2*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f) - (8*(A
- 3*B)*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*f) - ((A - 3*B)*Se
c[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*a^2*f) - ((A - B)*Sec[e + f*x]^3*
(c - c*Sin[e + f*x])^(9/2))/(3*a^2*c^2*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx}{a^2c^2}$$
$$= -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3a^2c^2f} - \frac{(A-3B)\int \sec^2(e}{}$$
$$= -\frac{(A-3B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f} - \frac{(A-B)\sec^3(e+}{}$$
$$= -\frac{8(A-3B)c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f} - \frac{(A-3B)\sec(e}{}$$
$$= \frac{32(A-3B)c^2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f} - \frac{8(A-3B)c\sec(}{}$$

Mathematica [A] time = 1.16, size = 130, normalized size = 0.84

$$-\frac{c^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)((201B-72A)\sin(e+fx)+6(A-4B)\cos(2(e+fx)}{6a^2f(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
f*x])^2,x]
```

```
[Out] -1/6*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(-
50*A + 160*B + 6*(A - 4*B)*Cos[2*(e + f*x)] + (-72*A + 201*B)*Sin[e + f*x]
+ B*Sin[3*(e + f*x)]))/(a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Si
n[e + f*x])^2)
```

fricas [A] time = 0.44, size = 110, normalized size = 0.71

$$-\frac{2\left(3\,(A-4\,B)c^2\cos\left(fx+e\right)^2-2\,(7\,A-23\,B)c^2+\left(Bc^2\cos\left(fx+e\right)^2-2\,(9\,A-25\,B)c^2\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] -2/3*(3*(A - 4*B)*c^2*cos(f*x + e)^2 - 2*(7*A - 23*B)*c^2 + (B*c^2*cos(f*x
+ e)^2 - 2*(9*A - 25*B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^2*f
*cos(f*x + e)*sin(f*x + e) + a^2*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.27, size = 105, normalized size = 0.68

$$-\frac{2c^3\left(\sin\left(fx+e\right)-1\right)\left(-B\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+(18A-50B)\sin\left(fx+e\right)+(-3A+12B)\left(\cos^2\left(fx+e\right)\right)+14A-46B\right)}{3a^2\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\;f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x)
```

```
[Out] -2/3*c^3/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(-B*cos(f*x+e)^2*sin(f*x+e)+(18*
A-50*B)*sin(f*x+e)+(-3*A+12*B)*cos(f*x+e)^2+14*A-46*B)/cos(f*x+e)/(c-c*sin(
f*x+e))^(1/2)/f
```

maxima [B] time = 0.46, size = 577, normalized size = 3.75

$$-\frac{2\left(\frac{\left(11\,c^{\frac{5}{2}}+\frac{36\,c^{\frac{5}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{56\,c^{\frac{5}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{108\,c^{\frac{5}{2}}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{90\,c^{\frac{5}{2}}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{108\,c^{\frac{5}{2}}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+\frac{56\,c^{\frac{5}{2}}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}+\frac{36\,c^{\frac{5}{2}}\sin\left(fx+e\right)^{7}}{\left(\cos\left(fx+e\right)+1\right)^{7}}+\frac{11\,c^{\frac{5}{2}}\sin\left(fx+e\right)^{8}}{\left(\cos\left(fx+e\right)+1\right)^{8}}\right)}{\left(a^{2}+\frac{3\,a^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\,a^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a^{2}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}\right)\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)^{\frac{5}{2}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] -2/3*((11*c^(5/2) + 36*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 56*c^(5/2)
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 108*c^(5/2)*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 90*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 108*c^(5/2)*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 56*c^(5/2)*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 + 36*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 11*c^(5/2)*sin
(f*x + e)^8/(cos(f*x + e) + 1)^8)*A/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e
) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2)) - 2*(17
*c^(5/2) + 51*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 92*c^(5/2)*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 149*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 150*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 149*c^(5/2)*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5 + 92*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^
6 + 51*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 17*c^(5/2)*sin(f*x + e
)^8/(cos(f*x + e) + 1)^8)*B/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) +
 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^
2,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^
2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**2,x)
[Out] Timed out
```

3.118 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=115

$$-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^2c^2f}-\frac{(A-7B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}+\frac{4c(A-7B)\sec(e+fx)\sqrt{}}{3a^2f}$$

```
[Out] -1/3*(A-7*B)*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f-1/3*(A-B)*sec(f*x+e)^3
*(c-c*sin(f*x+e))^(7/2)/a^2/c^2/f+4/3*(A-7*B)*c*sec(f*x+e)*(c-c*sin(f*x+e))
^(1/2)/a^2/f
```

Rubi [A] time = 0.41, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2855, 2674, 2673}

$$-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^2c^2f}-\frac{(A-7B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}+\frac{4c(A-7B)\sec(e+fx)\sqrt{}}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^
2,x]
```

```
[Out] (4*(A - 7*B)*c*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f) - ((A - 7*B
)*Sec[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*f) - ((A - B)*Sec[e + f*x
]^3*(c - c*Sin[e + f*x])^(7/2))/(3*a^2*c^2*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx}{a^2c^2}$$

$$= -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^2c^2f} - \frac{(A-7B)\int \sec^2$$

$$= -\frac{(A-7B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f} - \frac{(A-B)\sec^3(e+$$

$$= \frac{4(A-7B)c\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f} - \frac{(A-7B)\sec(e+$$

Mathematica [A] time = 0.66, size = 113, normalized size = 0.98

$$\frac{c\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(6(A-5B)\sin(e+fx)+2A+3B\cos(2(e+fx))-23B}{3a^2f(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
f*x])^2,x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*A - 23*B + 3*B*Cos[2*(e + f*x)]
 + 6*(A - 5*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)
```

fricas [A] time = 0.43, size = 80, normalized size = 0.70

$$\frac{2\left(3\,Bc\cos\left(fx+e\right)^{2}+3\,(A-5\,B)c\sin\left(fx+e\right)+(A-13\,B)c\right)\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(a^{2}f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^{2}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] 2/3*(3*B*c*cos(f*x + e)^2 + 3*(A - 5*B)*c*sin(f*x + e) + (A - 13*B)*c)*sqrt
(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f*cos(f*x + e)
)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.42, size = 81, normalized size = 0.70

$$-\frac{2c^{2}\left(\sin\left(fx+e\right)-1\right)\left(\sin\left(fx+e\right)(3A-15B)+3B\left(\cos^{2}\left(fx+e\right)\right)+A-13B\right)}{3a^{2}\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x)
```

```
[Out] -2/3*c^2/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(sin(f*x+e)*(3*A-15*B)+3*B*cos(f
*x+e)^2+A-13*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [B] time = 0.58, size = 482, normalized size = 4.19

$$-\frac{2\left(\frac{\left(c^{\frac{3}{2}}+\frac{6\,c^{\frac{3}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\,c^{\frac{3}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{12\,c^{\frac{3}{2}}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{3\,c^{\frac{3}{2}}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{6\,c^{\frac{3}{2}}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+\frac{c^{\frac{3}{2}}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}\right)A}{\left(a^{2}+\frac{3\,a^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\,a^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a^{2}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}\right)\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)^{\frac{3}{2}}}-\frac{2\left(5\,c^{\frac{3}{2}}+\frac{15\,c^{\frac{3}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{21\,c^{\frac{3}{2}}\sin\left(fx+e\right.}{\left(\cos\left(fx+e\right)+1\right.}\right.}{\left(a^{2}+\frac{3\,a^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right.}\right.}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] -2/3*((c^(3/2) + 6*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^(3/2)*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 12*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 3*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 6*c^(3/2)*sin(f*x +
 e)^5/(cos(f*x + e) + 1)^5 + c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)*A
/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 1)^(3/2)) - 2*(5*c^(3/2) + 15*c^(3/2)*sin(f*x + e)/(
cos(f*x + e) + 1) + 21*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*c^(
3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 21*c^(3/2)*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 15*c^(3/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*c^(3/2)
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)*B/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x
 + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)))/f
```

mupad [B] time = 17.18, size = 492, normalized size = 4.28

$$-\frac{\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\left(\frac{2\,B\,c}{a^2\,f}-\frac{B\,c\,e^{e\,1i+f\,x\,1i}\,2i}{a^2\,f}\right)}{e^{e\,1i+f\,x\,1i}-i}+\frac{e^{e\,1i+f\,x\,1i}\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\left(\frac{2\,B\,c}{3\,a^2\,f}-\frac{c(\cdots}{\cdots}\right.}{\left(e^{e\,1i+f\,x\,1i}-i\right)\left(e^{\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^
2,x)
```

```
[Out] (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1
i)*1i)/2))^(1/2)*((2*B*c)/(3*a^2*f) - (c*(2*A - 3*B))/(3*a^2*f) - (2*c*(3*A
 - 2*B))/(3*a^2*f) + (c*(A*2i - B*3i)*1i)/(3*a^2*f) + (c*(A*3i - B*2i)*2i)/
(3*a^2*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^3) - ((c -
 c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((2*B*c
)/(a^2*f) - (B*c*exp(e*1i + f*x*1i)*2i)/(a^2*f)))/(exp(e*1i + f*x*1i) - 1i)
 - (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*
x*1i)*1i)/2))^(1/2)*((c*(A - B)*4i)/(a^2*f) + (c*(A*1i - B*2i))/(a^2*f) + (
c*(A*1i + B*2i))/(3*a^2*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i)
 + 1i)^2) - (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(
e*1i + f*x*1i)*1i)/2))^(1/2)*((4*B*c)/(a^2*f) + (c*(A*1i - B*2i)*4i)/(a^2*f
)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.119 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=78

$$-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2c^2f}-\frac{(A+5B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f}$$

```
[Out] -1/3*(A-B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(5/2)/a^2/c^2/f-1/3*(A+5*B)*sec(f*
x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f
```

Rubi [A] time = 0.31, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2967, 2855, 2673}

$$-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2c^2f}-\frac{(A+5B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^2,
x]
```

```
[Out] -((A + 5*B)*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f) - ((A - B)*Sec
[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(3*a^2*c^2*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
```

```
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx}{a^2c^2}$$
$$= -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2c^2f} + \frac{(A+5B)\int \sec^2(e-$$
$$= -\frac{(A+5B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f} - \frac{(A-B)\sec^3(e+fx)}{3a^2}$$

Mathematica [A] time = 0.28, size = 87, normalized size = 1.12

$$-\frac{2\sqrt{c-c\sin(e+fx)}\,(A+3B\sin(e+fx)+2B)}{3a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*
x])^2,x]
```

```
[Out] (-2*(A + 2*B + 3*B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

fricas [A] time = 0.43, size = 60, normalized size = 0.77

$$-\frac{2\left(3\,B\sin\left(fx+e\right)+A+2\,B\right)\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] -2/3*(3*B*sin(f*x + e) + A + 2*B)*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x
+ e)*sin(f*x + e) + a^2*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.31, size = 63, normalized size = 0.81

$$\frac{2c\left(\sin\left(fx+e\right)-1\right)\left(3B\sin\left(fx+e\right)+A+2B\right)}{3a^{2}\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 2/3*c/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(3*B*sin(f*x+e)+A+2*B)/cos(f*x+e)/(
c-c*sin(f*x+e))^(1/2)/f
```

maxima [B] time = 0.46, size = 343, normalized size = 4.40

$$\frac{2\left(\frac{2B\left(\sqrt{c}+\frac{3\sqrt{c}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{2\sqrt{c}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{3\sqrt{c}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{\sqrt{c}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}\right)}{\left(a^{2}+\frac{3a^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3a^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a^{2}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}\right)\sqrt{\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1}}+\frac{A\left(\sqrt{c}+\frac{2\sqrt{c}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{\sqrt{c}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}\right)}{\left(a^{2}+\frac{3a^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3a^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a^{2}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}\right)\sqrt{\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] 2/3*(2*B*(sqrt(c) + 3*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + 2*sqrt(c)*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sqrt(c)*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/((a^2 + 3*a^2*sin(f*
x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 1)) + A*(sqrt(c) + 2*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + sq
rt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/((a^2 + 3*a^2*sin(f*x + e)/(cos(
f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3)*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)))/f
```

mupad [B] time = 17.51, size = 137, normalized size = 1.76

$$\frac{4\,{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\sqrt{c-c\,\left(\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\,\left(B\,3\mathrm{i}+2\,A\,{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+4\,B\,{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-B\,{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,3\mathrm{i}\right)}{3\,a^2\,f\,{\left({\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)}^3\,\left(1+{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^
2,x)
```

```
[Out] (4*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x
*1i)*1i)/2))^(1/2)*(B*3i + 2*A*exp(e*1i + f*x*1i) + 4*B*exp(e*1i + f*x*1i)
- B*exp(e*2i + f*x*2i)*3i))/(3*a^2*f*(exp(e*1i + f*x*1i) + 1i)^3*(exp(e*1i
+ f*x*1i)*1i + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sqrt{-c\sin\left(e+fx\right)+c}}{\sin^2\left(e+fx\right)+2\sin\left(e+fx\right)+1}\,dx+\int \frac{B\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)}{\sin^2\left(e+fx\right)+2\sin\left(e+fx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] (Integral(A*sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1
), x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x)**2
+ 2*sin(e + f*x) + 1), x))/a**2
```

3.120 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=135

$$-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2c^2f}-\frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf}+\frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e}{\sqrt{2}\sqrt{c-c\sin}}\right.}{2\sqrt{2}\,a^2\sqrt{c}\,f}$$

```
[Out] -1/3*(A-B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^2/c^2/f+1/4*(A+B)*arctanh(
1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^2/f*2^(1/2)/c^(1/2
)-1/2*(A+B)*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/c/f
```

Rubi [A] time = 0.35, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2967, 2855, 2675, 2649, 206}

$$-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2c^2f}-\frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf}+\frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e}{\sqrt{2}\sqrt{c-c\sin}}\right.}{2\sqrt{2}\,a^2\sqrt{c}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]]),
x]
```

```
[Out] ((A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]
)/(2*Sqrt[2]*a^2*Sqrt[c]*f) - ((A + B)*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]
])/(2*a^2*c*f) - ((A - B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*a^2
*c^2*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx}{a^2c^2}$$

$$= -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2c^2f} + \frac{(A+B)\int \sec^2(e+$$

$$= -\frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf} - \frac{(A-B)\sec^3(e+fx)(c}{3a^2c^2}$$

$$= -\frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf} - \frac{(A-B)\sec^3(e+fx)(c}{3a^2c^2}$$

$$= \frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}\,a^2\sqrt{c}\,f} - \frac{(A+B)\sec(e+fx)\sqrt{c-c\sin}}{2a^2cf}$$

Mathematica [C] time = 0.51, size = 176, normalized size = 1.30

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-3(A+B)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e\right.\right.\right.}{6a^2f(\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f
*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(2*(-A + B) - 3*(A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (3 + 3*
I)*(-1)^(1/4)*(A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3))/(6*a^2*f*(1 + Sin[e + f*x])^2*Sq
rt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.45, size = 217, normalized size = 1.61

$$\frac{3\sqrt{2}\left((A+B)\cos\left(fx+e\right)\sin\left(fx+e\right)+(A+B)\cos\left(fx+e\right)\right)\sqrt{c}\,\log\left(-\frac{c\cos\left(fx+e\right)^2+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\,\sqrt{c}\left(\cos\left(fx\right.\right.}{\cos\left(fx+e\right)^2+\left(\cos\right.}\right.}{24\left(a^2cf\cos\left(fx+e\right)\sin\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/24*(3*sqrt(2)*((A + B)*cos(f*x + e)*sin(f*x + e) + (A + B)*cos(f*x + e))*
sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c
)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) -
2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e)
- cos(f*x + e) - 2)) - 4*(3*(A + B)*sin(f*x + e) + 5*A + B)*sqrt(-c*sin(f*x
 + e) + c))/(a^2*c*f*cos(f*x + e)*sin(f*x + e) + a^2*c*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
```

```
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes con
stant sign by intervals (correct if the argument is real):Check [abs(sin((f
*t_nostep+exp(1))/2-pi/4))]Unable to check sign: (8*pi/t_nostep/2)>(-8*pi/t
_nostep/2)Discontinuities at zeroes of sin((f*t_nostep+exp(1))/2-pi/4) were
 not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep-1)]index.cc index_m i_lex_is_greater Error: Bad Argument Value
```

maple [A] time = 1.78, size = 168, normalized size = 1.24

$$\frac{\left(\sin\left(fx+e\right)-1\right)\left(-3\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\left(c\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}cA-3\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right.}}{2\sqrt{c}}\right.}{12a^{2}c^{\frac{5}{2}}\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/12/a^2*(sin(f*x+e)-1)/c^(5/2)/(1+sin(f*x+e))*(-3*2^(1/2)*arctanh(1/2*(c*(
1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(3/2)*c*A-3*2^(1/2
)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^
(3/2)*c*B+10*A*c^(5/2)+6*A*c^(5/2)*sin(f*x+e)+2*B*c^(5/2)+6*B*c^(5/2)*sin(f
*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{2}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^2*sqrt(-c*sin(f*x + e)
 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\sin\left(e+fx\right)}{{\left(a+a\sin\left(e+fx\right)\right)}^2\sqrt{c-c\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2)
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2)
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.121 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=175

$$\frac{(5A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}\,a^2c^{3/2}f}-\frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f}+\frac{(5A+B)\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}}-\frac{(5A+\ldots}{6a^2cf\sqrt{\ldots}}$$

```
[Out] 1/8*(5*A+B)*cos(f*x+e)/a^2/f/(c-c*sin(f*x+e))^(3/2)+1/16*(5*A+B)*arctanh(1/
2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^2/c^(3/2)/f*2^(1/2)-
1/6*(5*A+B)*sec(f*x+e)/a^2/c/f/(c-c*sin(f*x+e))^(1/2)-1/3*(A-B)*sec(f*x+e)^
3*(c-c*sin(f*x+e))^(1/2)/a^2/c^2/f
```

Rubi [A] time = 0.39, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2967, 2855, 2687, 2650, 2649, 206}

$$-\frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f}+\frac{(5A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}\,a^2c^{3/2}f}+\frac{(5A+B)\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}}-\frac{(5A\ldots}{6a^2cf\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(3/2)
),x]
```

```
[Out] ((5*A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]]
)])/(8*Sqrt[2]*a^2*c^(3/2)*f) + ((5*A + B)*Cos[e + f*x])/(8*a^2*f*(c - c*Si
n[e + f*x])^(3/2)) - ((5*A + B)*Sec[e + f*x])/(6*a^2*c*f*Sqrt[c - c*Sin[e +
 f*x]]) - ((A - B)*Sec[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*c^2*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx}{a^2c^2}$$

$$= -\frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f} + \frac{(5A+B)\int \frac{\sec^2(e+}{\sqrt{c-c\sin(}}}{6a^2c}$$

$$= -\frac{(5A+B)\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+}}{3a^2c^2f}$$

$$= \frac{(5A+B)\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}} - \frac{(5A+B)\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}} - \frac{(A-B}{}$$

$$= \frac{(5A+B)\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}} - \frac{(5A+B)\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}} - \frac{(A-B}{}$$

$$= \frac{(5A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}\,a^2c^{3/2}f} + \frac{(5A+B)\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}}$$

Mathematica [C] time = 0.86, size = 300, normalized size = 1.71

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(3(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-12*A*Cos[e + f*x]^2 + 4*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
2 + 3*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^3 - (3 + 3*I)*(-1)^(1/4)*(5*A + B)*ArcTan[(1/2 + I/2)*(-1)^(
1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2])^3 + 6*(A + B)*Sin[(e + f*x)/2]*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])^3))/(24*a^2*f*(1 + Sin[e + f*x])^2*(c - c*Sin[e +
 f*x])^(3/2))
```

fricas [A] time = 0.45, size = 206, normalized size = 1.18

$$\frac{3\sqrt{2}(5A+B)\sqrt{c}\cos\left(fx+e\right)^{3}\log\left(-\frac{c\cos\left(fx+e\right)^{2}+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\sqrt{c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)+3c\cos\left(fx+e\right)+\left(c\cos\left(fx+\right.\right.}{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right.}{96a^{2}c^{2}f\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/96*(3*sqrt(2)*(5*A + B)*sqrt(c)*cos(f*x + e)^3*log(-(c*cos(f*x + e)^2 + 2
*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1
) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x
+ e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(3*(5*A +
 B)*cos(f*x + e)^2 - 2*(5*A + B)*sin(f*x + e) - 2*A - 10*B)*sqrt(-c*sin(f*x
 + e) + c))/(a^2*c^2*f*cos(f*x + e)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(1/16*(-3*A*(-sqr
t(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-3*B*(-sqrt(c)*t
an((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3+A*c*(-sqrt(c)*tan((f*
x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-A*sqrt(c)*(-sqrt(c)*tan((f*x+
exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+B*c*(-sqrt(c)*tan((f*x+exp(1)
)/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-B*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/
2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-A*sqrt(c)*c-B*sqrt(c)*c)/a^2/(-(-sqrt
(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt
(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^2/c/sign(tan((f
*x+exp(1))/2)-1)-1/12*(-6*A*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+e
xp(1))/2)^2+c))^5+3*B*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))
/2)^2+c))^5+12*A*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(
1))/2)^2+c))^4-3*B*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+ex
p(1))/2)^2+c))^4+8*A*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1)
)/2)^2+c))^3-2*B*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)
^2+c))^3-18*A*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^
```

```
2+c))-24*A*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/
2)^2+c))^2+3*B*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)
^2+c))+6*B*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/
2)^2+c))^2-4*A*sqrt(c)*c^2+B*sqrt(c)*c^2)/a^2/(-(-sqrt(c)*tan((f*x+exp(1))/
2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/
2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^3/c/sign(tan((f*x+exp(1))/2)-1)+1/16
*(5*A+B)*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))
/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/a^2/sqrt(-c)/c/sign(tan((f*x+exp(1))/2)
-1))
```

maple [A] time = 1.56, size = 258, normalized size = 1.47

$$-\frac{\sin\left(fx+e\right)\left(15\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c+c\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\left(c+c\sin\left(fx+e\right)\right)^{\frac{3}{2}}cA+3\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c+c\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\left(c+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/48/c^(7/2)/a^2*(sin(f*x+e)*(15*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2
)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(3/2)*c*A+3*2^(1/2)*arctanh(1/2*(c+c*si
n(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(3/2)*c*B-20*A*c^(5/2)-4*
B*c^(5/2))+(30*A*c^(5/2)+6*B*c^(5/2))*cos(f*x+e)^2-15*2^(1/2)*arctanh(1/2*(
c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(3/2)*c*A-3*2^(1/2)
*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(3/2)
*c*B-4*A*c^(5/2)-20*B*c^(5/2))/(1+sin(f*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(
1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2)
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2)
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.122 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=225

$$\frac{5(7A-B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{64\sqrt{2}\,a^2c^{5/2}f}-\frac{(A-B)\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}}-\frac{5(7A-B)\sec(e+fx)}{48a^2c^2f\sqrt{c-c\sin(e+fx)}}+\frac{5(7A-B)\cos(e}{64a^2cf(c-c\sin(e}$$

```
[Out] 5/64*(7*A-B)*cos(f*x+e)/a^2/c/f/(c-c*sin(f*x+e))^(3/2)+1/24*(7*A-B)*sec(f*x
+e)/a^2/c/f/(c-c*sin(f*x+e))^(3/2)+5/128*(7*A-B)*arctanh(1/2*cos(f*x+e)*c^(
1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^2/c^(5/2)/f*2^(1/2)-5/48*(7*A-B)*sec
(f*x+e)/a^2/c^2/f/(c-c*sin(f*x+e))^(1/2)-1/3*(A-B)*sec(f*x+e)^3/a^2/c^2/f/(
c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {2967, 2855, 2681, 2687, 2650, 2649, 206}

$$-\frac{(A-B)\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}}-\frac{5(7A-B)\sec(e+fx)}{48a^2c^2f\sqrt{c-c\sin(e+fx)}}+\frac{5(7A-B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{64\sqrt{2}\,a^2c^{5/2}f}+\frac{5(7A-B)\cos}{64a^2cf(c-c\sin}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)
),x]
```

```
[Out] (5*(7*A - B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x
]])])/(64*Sqrt[2]*a^2*c^(5/2)*f) + (5*(7*A - B)*Cos[e + f*x])/(64*a^2*c*f*(
c - c*Sin[e + f*x])^(3/2)) + ((7*A - B)*Sec[e + f*x])/(24*a^2*c*f*(c - c*Si
n[e + f*x])^(3/2)) - (5*(7*A - B)*Sec[e + f*x])/(48*a^2*c^2*f*Sqrt[c - c*Si
n[e + f*x]]) - ((A - B)*Sec[e + f*x]^3)/(3*a^2*c^2*f*Sqrt[c - c*Sin[e + f*x
]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx}{a^2c^2}$$

$$= -\frac{(A-B)\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}} + \frac{(7A-B)\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx}{6a^2c}$$

$$= \frac{(7A-B)\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} - \frac{(A-B)\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}} + \frac{(5(7}{}$$

$$= \frac{(7A-B)\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} - \frac{5(7A-B)\sec(e+fx)}{48a^2c^2f\sqrt{c-c\sin(e+fx)}} - \frac{}{3a}$$

$$= \frac{5(7A-B)\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}} + \frac{(7A-B)\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} - \frac{}{48}$$

$$= \frac{5(7A-B)\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}} + \frac{(7A-B)\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} - \frac{}{48}$$

$$= \frac{5(7A-B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{64\sqrt{2}\,a^2c^{5/2}f} + \frac{5(7A-B)\cos(e+fx}{64a^2cf(c-c\sin(e+fx)}$$

Mathematica [C] time = 1.41, size = 430, normalized size = 1.91

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(3(11A+3B)\cos^3(e+fx)+24(B-3A)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(3*(11*A + 3*B)*Cos[e + f*x]^3 + 16*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^4 + 24*(-3*A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^2 + 12*(A + B)*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - (15 + 15*I)*(-1)^(1/4)*(
```

```
7*A - B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 24*(A
 + B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 6*(11*A +
3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2]*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^3))/(192*a^2*f*(1 + Sin[e + f*x])^2*(c - c*Sin[e +
 f*x])^(5/2))
```

fricas [A] time = 0.46, size = 279, normalized size = 1.24

$$-\frac{15\sqrt{2}\left((7A-B)\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)-(7A-B)\cos\left(fx+e\right)^{3}\right)\sqrt{c}\log\left(-\frac{c\cos\left(fx+e\right)^{2}-2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\cdots}{\cos\left(fx\cdots\right.}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] -1/768*(15*sqrt(2)*((7*A - B)*cos(f*x + e)^3*sin(f*x + e) - (7*A - B)*cos(f
*x + e)^3)*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e)
+ c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(
f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*si
n(f*x + e) - cos(f*x + e) - 2)) - 4*(5*(7*A - B)*cos(f*x + e)^2 - (15*(7*A
- B)*cos(f*x + e)^2 + 56*A - 8*B)*sin(f*x + e) + 8*A - 56*B)*sqrt(-c*sin(f*
x + e) + c))/(a^2*c^3*f*cos(f*x + e)^3*sin(f*x + e) - a^2*c^3*f*cos(f*x + e
)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(-1/48*(-15*A*(-s
qrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5+9*B*(-sqrt(c)
*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5+33*A*sqrt(c)*(-sqrt
(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4-15*B*sqrt(c)*(-s
qrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4+22*A*c*(-sqrt
(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-10*B*c*(-sqrt(c)
*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-51*A*c^2*(-sqrt(c)*
```

```
tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-66*A*sqrt(c)*c*(-sqrt(
c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+21*B*c^2*(-sqrt(c
)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+30*B*sqrt(c)*c*(-sqr
t(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-11*A*sqrt(c)*c^
2+5*B*sqrt(c)*c^2)/a^2/c^2/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+
exp(1))/2)^2+c))^2+2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+
exp(1))/2)^2+c))+c)^3/sign(tan((f*x+exp(1))/2)-1)+1/128*(-53*A*(-sqrt(c)*ta
n((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7-29*B*(-sqrt(c)*tan((f*
x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7-179*A*sqrt(c)*(-sqrt(c)*tan
((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6-75*B*sqrt(c)*(-sqrt(c)*
tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6-127*A*c*(-sqrt(c)*ta
n((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5-55*B*c*(-sqrt(c)*tan((
f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5+195*A*sqrt(c)*c*(-sqrt(c)
*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4+91*B*sqrt(c)*c*(-sq
rt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4-7*A*c^2*(-sqrt
(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3+B*c^2*(-sqrt(c)*
tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3+67*A*c^3*(-sqrt(c)*t
an((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-121*A*sqrt(c)*c^2*(-sqr
t(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+27*B*c^3*(-sqrt
(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-65*B*sqrt(c)*c^2*(
-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-15*A*sqrt(c
)*c^3-7*B*sqrt(c)*c^3)/a^2/c^2/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((
f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((
f*x+exp(1))/2)^2+c))+c)^4/sign(tan((f*x+exp(1))/2)-1)+1/128*(35*A-5*B)*atan
((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqr
t(2)/sqrt(-c))/sqrt(2)/a^2/c^2/sqrt(-c)/sign(tan((f*x+exp(1))/2)-1))
```

maple [B] time = 1.74, size = 426, normalized size = 1.89

$$-\frac{-86A\,c^{\frac{7}{2}}+122B\,c^{\frac{7}{2}}+70A\,c^{\frac{7}{2}}\left(\sin^2\left(fx+e\right)\right)-10B\,c^{\frac{7}{2}}\left(\sin^2\left(fx+e\right)\right)+322A\,c^{\frac{7}{2}}\sin\left(fx+e\right)-46B\,c^{\frac{7}{2}}\sin\left(fx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/384/c^(11/2)/a^2*(-86*A*c^(7/2)+122*B*c^(7/2)+70*A*c^(7/2)*sin(f*x+e)^2-
10*B*c^(7/2)*sin(f*x+e)^2+322*A*c^(7/2)*sin(f*x+e)-46*B*c^(7/2)*sin(f*x+e)-
210*A*c^(7/2)*sin(f*x+e)^3+30*B*c^(7/2)*sin(f*x+e)^3+105*A*(c*(1+sin(f*x+e)
))^(3/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(
f*x+e)^2*c^2-15*B*(c*(1+sin(f*x+e)))^(3/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*
x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^2-210*A*(c*(1+sin(f*x+e)))^(3/
2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)
```

```
*c^2+30*B*(c*(1+sin(f*x+e)))^(3/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(
1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^2+105*A*(c*(1+sin(f*x+e)))^(3/2)*2^(1/2)
*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^2-15*B*(c*(1+sin(f
*x+e)))^(3/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))
*c^2)/(1+sin(f*x+e))/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2)
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2)
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.123 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=242

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{15/2}}{5a^3c^3f}-\frac{2048c^3(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f}+\frac{512c^2(A-3B)\,\mathrm{s}}{}$$

```
[Out] -2048/15*(A-3*B)*c^3*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/f+512/5*(A-3*B
)*c^2*sec(f*x+e)^3*(c-c*sin(f*x+e))^(5/2)/a^3/f-64/5*(A-3*B)*c*sec(f*x+e)^3
*(c-c*sin(f*x+e))^(7/2)/a^3/f-16/15*(A-3*B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(
9/2)/a^3/f-1/5*(A-3*B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(11/2)/a^3/c/f-1/5*(A-
B)*sec(f*x+e)^5*(c-c*sin(f*x+e))^(15/2)/a^3/c^3/f
```

Rubi [A] time = 0.65, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2855, 2674, 2673}

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{15/2}}{5a^3c^3f}+\frac{512c^2(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f}-\frac{2048c^3(A-3B)\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])^
3,x]
```

```
[Out] (-2048*(A - 3*B)*c^3*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^3*f)
+ (512*(A - 3*B)*c^2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*f) -
 (64*(A - 3*B)*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(7/2))/(5*a^3*f) - (16
*(A - 3*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(9/2))/(15*a^3*f) - ((A - 3*
B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(11/2))/(5*a^3*c*f) - ((A - B)*Sec[e
 + f*x]^5*(c - c*Sin[e + f*x])^(15/2))/(5*a^3*c^3*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
```

```
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{15/2}\,dx}{a^3c^3}$$

$$= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{15/2}}{5a^3c^3f} - \frac{(A-3B)\int \sec$$

$$= -\frac{(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3cf} - \frac{(A-B)\sec^5(e$$

$$= -\frac{16(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^3f} - \frac{(A-3B)\sec$$

$$= -\frac{64(A-3B)c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3f} - \frac{16(A-3B)$$

$$= \frac{512(A-3B)c^2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f} - \frac{64(A-3B$$

$$= -\frac{2048(A-3B)c^3\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f} + \frac{512(A-$$

Mathematica [A] time = 4.19, size = 176, normalized size = 0.73

$$-\frac{c^4(\sin(e+fx)-1)^4\sqrt{c-c\sin(e+fx)}\,(-40(137A-402B)\cos(2(e+fx))-10(A-6B)\cos(4(e+fx))+15600}{120a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e +
f*x])^3,x]
```

```
[Out] -1/120*(c^4*(-1 + Sin[e + f*x])^4*Sqrt[c - c*Sin[e + f*x]]*(11298*A - 33516
*B - 40*(137*A - 402*B)*Cos[2*(e + f*x)] - 10*(A - 6*B)*Cos[4*(e + f*x)] +
15600*A*Sin[e + f*x] - 47430*B*Sin[e + f*x] - 400*A*Sin[3*(e + f*x)] + 1335
*B*Sin[3*(e + f*x)] - 3*B*Sin[5*(e + f*x)]))/(a^3*f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

fricas [A] time = 0.45, size = 166, normalized size = 0.69

$$-\frac{2\left(5\,(A-6\,B)c^4\cos\left(fx+e\right)^4+20\,(34\,A-99\,B)c^4\cos\left(fx+e\right)^2-8\,(131\,A-387\,B)c^4+\left(3\,Bc^4\cos\left(fx+e\right)^4+\right.\right.}{15\left(a^3f\cos\left(fx+e\right)^3-2\,a^3f\cos\left(fx+e\right)\sin\left(f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] -2/15*(5*(A - 6*B)*c^4*cos(f*x + e)^4 + 20*(34*A - 99*B)*c^4*cos(f*x + e)^2
 - 8*(131*A - 387*B)*c^4 + (3*B*c^4*cos(f*x + e)^4 + 4*(25*A - 84*B)*c^4*co
s(f*x + e)^2 - 8*(125*A - 381*B)*c^4)*sin(f*x + e))*sqrt(-c*sin(f*x + e) +
c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(
f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.47, size = 143, normalized size = 0.59

$$-\frac{2c^5\left(\sin\left(fx+e\right)-1\right)\left(3B\sin\left(fx+e\right)\left(\cos^4\left(fx+e\right)\right)+\left(100A-336B\right)\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+\left(-1000\right.\right.}{15a^3\left(1+\sin\left(fx+e\right)\right)^2\cos\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x)
```

```
[Out] -2/15*c^5/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(3*B*sin(f*x+e)*cos(f*x+e)^4+
(100*A-336*B)*cos(f*x+e)^2*sin(f*x+e)+(-1000*A+3048*B)*sin(f*x+e)+(5*A-30*B
)*cos(f*x+e)^4+(680*A-1980*B)*cos(f*x+e)^2-1048*A+3096*B)/cos(f*x+e)/(c-c*s
in(f*x+e))^(1/2)/f
```

maxima [B] time = 0.49, size = 945, normalized size = 3.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] 2/15*((363*c^(9/2) + 1800*c^(9/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 5301*c^
(9/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 11600*c^(9/2)*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 21343*c^(9/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 30
200*c^(9/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 40065*c^(9/2)*sin(f*x + e
)^6/(cos(f*x + e) + 1)^6 + 40800*c^(9/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^
7 + 40065*c^(9/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 30200*c^(9/2)*sin(f
*x + e)^9/(cos(f*x + e) + 1)^9 + 21343*c^(9/2)*sin(f*x + e)^10/(cos(f*x + e
) + 1)^10 + 11600*c^(9/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 5301*c^(9
/2)*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 1800*c^(9/2)*sin(f*x + e)^13/(c
os(f*x + e) + 1)^13 + 363*c^(9/2)*sin(f*x + e)^14/(cos(f*x + e) + 1)^14)*A/
((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*(s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(9/2)) - 6*(181*c^(9/2) + 905*c^(9/
2)*sin(f*x + e)/(cos(f*x + e) + 1) + 2627*c^(9/2)*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 5870*c^(9/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 10521*c^(9/
2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 15351*c^(9/2)*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5 + 19695*c^(9/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 20772
*c^(9/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 19695*c^(9/2)*sin(f*x + e)^8
/(cos(f*x + e) + 1)^8 + 15351*c^(9/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 +
 10521*c^(9/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 5870*c^(9/2)*sin(f*x
 + e)^11/(cos(f*x + e) + 1)^11 + 2627*c^(9/2)*sin(f*x + e)^12/(cos(f*x + e)
```

```
 + 1)^12 + 905*c^(9/2)*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 181*c^(9/2)*
sin(f*x + e)^14/(cos(f*x + e) + 1)^14)*B/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*
x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^
3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 1)^(9/2)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\sin\left(e+fx\right)\right)\left(c-c\sin\left(e+fx\right)\right)^{9/2}}{\left(a+a\sin\left(e+fx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^
3,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^
3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.124 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=209

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{5a^3c^3f}-\frac{128c^2(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f}-\frac{(3A-13B)\text{se}}{}$$

```
[Out] -128/15*(3*A-13*B)*c^2*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/f+32/5*(3*A-
13*B)*c*sec(f*x+e)^3*(c-c*sin(f*x+e))^(5/2)/a^3/f-4/5*(3*A-13*B)*sec(f*x+e)
^3*(c-c*sin(f*x+e))^(7/2)/a^3/f-1/15*(3*A-13*B)*sec(f*x+e)^3*(c-c*sin(f*x+e
))^(9/2)/a^3/c/f-1/5*(A-B)*sec(f*x+e)^5*(c-c*sin(f*x+e))^(13/2)/a^3/c^3/f
```

Rubi [A] time = 0.57, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2855, 2674, 2673}

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{5a^3c^3f}-\frac{128c^2(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f}-\frac{(3A-13B)\text{se}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^
3,x]
```

```
[Out] (-128*(3*A - 13*B)*c^2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^3*f
) + (32*(3*A - 13*B)*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*f)
 - (4*(3*A - 13*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(7/2))/(5*a^3*f) - (
(3*A - 13*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(9/2))/(15*a^3*c*f) - ((A
- B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(13/2))/(5*a^3*c^3*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
```

```
[m + p, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{13/2}\,dx}{a^3c^3}$$

$$= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{5a^3c^3f} - \frac{(3A-13B)\int \sec\ldots}{\ldots}$$

$$= -\frac{(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^3cf} - \frac{(A-B)\sec^5(\ldots}{\ldots}$$

$$= -\frac{4(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3f} - \frac{(3A-13B)\,\mathrm{s}\ldots}{\ldots}$$

$$= \frac{32(3A-13B)c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f} - \frac{4(3A-13B\ldots}{\ldots}$$

$$= -\frac{128(3A-13B)c^2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f} + \frac{32(3A-\ldots}{\ldots}$$

Mathematica [A] time = 2.63, size = 158, normalized size = 0.76

$$-\frac{c^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)((2200B-540A)\cos(2(e+fx))+1410A\sin(e+fx)\ldots}{60a^3f(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e\ldots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
f*x])^3,x]
```

```
[Out] -1/60*(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(
1092*A - 4557*B + (-540*A + 2200*B)*Cos[2*(e + f*x)] + 5*B*Cos[4*(e + f*x)]
 + 1410*A*Sin[e + f*x] - 6390*B*Sin[e + f*x] - 30*A*Sin[3*(e + f*x)] + 170*
B*Sin[3*(e + f*x)]))/(a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[
e + f*x])^3)
```

fricas [A] time = 0.44, size = 148, normalized size = 0.71

$$\frac{2\left(5\,Bc^{3}\cos\left(fx+e\right)^{4}-5\,(27\,A-109\,B)c^{3}\cos\left(fx+e\right)^{2}+4\,(51\,A-211\,B)c^{3}-5\left((3\,A-17\,B)c^{3}\cos\left(fx+e\right)^{2}\right.\right.}{15\left(a^{3}f\cos\left(fx+e\right)^{3}-2\,a^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^{3}f\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] 2/15*(5*B*c^3*cos(f*x + e)^4 - 5*(27*A - 109*B)*c^3*cos(f*x + e)^2 + 4*(51*
A - 211*B)*c^3 - 5*((3*A - 17*B)*c^3*cos(f*x + e)^2 - 4*(9*A - 41*B)*c^3)*s
in(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(
f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.60, size = 121, normalized size = 0.58

$$\frac{2c^{4}\left(\sin\left(fx+e\right)-1\right)\left(\left(-15A+85B\right)\sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)+\left(180A-820B\right)\sin\left(fx+e\right)+5B\left(\cos^{4}\left(fx\right.\right.\right.}{15a^{3}\left(1+\sin\left(fx+e\right)\right)^{2}\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/15*c^4/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*((-15*A+85*B)*sin(f*x+e)*cos(f
*x+e)^2+(180*A-820*B)*sin(f*x+e)+5*B*cos(f*x+e)^4+(-135*A+545*B)*cos(f*x+e)
^2+204*A-844*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [B] time = 1.25, size = 854, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] 2/15*(3*(23*c^(7/2) + 110*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 318*c^(
7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 590*c^(7/2)*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 1065*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1220*c
^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1540*c^(7/2)*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 1220*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 106
5*c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 590*c^(7/2)*sin(f*x + e)^9/
(cos(f*x + e) + 1)^9 + 318*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 +
110*c^(7/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 23*c^(7/2)*sin(f*x + e)
^12/(cos(f*x + e) + 1)^12)*A/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1)
+ 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2))
 - 2*(147*c^(7/2) + 735*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 1992*c^(7
/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4015*c^(7/2)*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 6605*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 8370*c
^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9520*c^(7/2)*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 8370*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 660
5*c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 4015*c^(7/2)*sin(f*x + e)^9
/(cos(f*x + e) + 1)^9 + 1992*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10
+ 735*c^(7/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 147*c^(7/2)*sin(f*x +
 e)^12/(cos(f*x + e) + 1)^12)*B/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) +
1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/
2)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^
3,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^
3, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.125 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=160

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3c^3f}-\frac{(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3cf}+\frac{8(A-11B)\sec^3(e+f}{5a}$$

```
[Out] -32/15*(A-11*B)*c*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/f+8/5*(A-11*B)*se
c(f*x+e)^3*(c-c*sin(f*x+e))^(5/2)/a^3/f-1/5*(A-11*B)*sec(f*x+e)^3*(c-c*sin(
f*x+e))^(7/2)/a^3/c/f-1/5*(A-B)*sec(f*x+e)^5*(c-c*sin(f*x+e))^(11/2)/a^3/c^
3/f
```

Rubi [A] time = 0.48, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2855, 2674, 2673}

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3c^3f}-\frac{(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3cf}+\frac{8(A-11B)\sec^3(e+f}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^
3,x]
```

```
[Out] (-32*(A - 11*B)*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^3*f) + (
8*(A - 11*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*f) - ((A - 1
1*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(7/2))/(5*a^3*c*f) - ((A - B)*Sec[
e + f*x]^5*(c - c*Sin[e + f*x])^(11/2))/(5*a^3*c^3*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{11/2}\,dx}{a^3c^3}$$
$$= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3c^3f} - \frac{(A-11B)\int \sec$$
$$= -\frac{(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3cf} - \frac{(A-B)\sec^5(e}{}$$
$$= \frac{8(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f} - \frac{(A-11B)\sec^3}{}$$
$$= -\frac{32(A-11B)c\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f} + \frac{8(A-11B}{}$$

Mathematica [A] time = 1.21, size = 132, normalized size = 0.82

$$-\frac{c^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(5(8A-133B)\sin(e+fx)-30(A-8B)\cos(2(e+f}{30a^3f(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
f*x])^3,x]
```

```
[Out] -1/30*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(
58*A - 488*B - 30*(A - 8*B)*Cos[2*(e + f*x)] + 5*(8*A - 133*B)*Sin[e + f*x]
 + 15*B*Sin[3*(e + f*x)]))/(a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1
+ Sin[e + f*x])^3)
```

fricas [A] time = 0.44, size = 125, normalized size = 0.78

$$-\frac{2\left(15\,(A-8\,B)c^2\cos\left(fx+e\right)^2-2\,(11\,A-91\,B)c^2-5\left(3\,Bc^2\cos\left(fx+e\right)^2+2\,(A-17\,B)c^2\right)\sin\left(fx+e\right)\right)\sqrt{-c}}{15\left(a^3f\cos\left(fx+e\right)^3-2\,a^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^3f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] -2/15*(15*(A - 8*B)*c^2*cos(f*x + e)^2 - 2*(11*A - 91*B)*c^2 - 5*(3*B*c^2*c
os(f*x + e)^2 + 2*(A - 17*B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(
a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x
+ e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.60, size = 105, normalized size = 0.66

$$\frac{2c^3\left(\sin\left(fx+e\right)-1\right)\left(15B\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+(10A-170B)\sin\left(fx+e\right)+(-15A+120B)\left(\cos^2\left(fx\right.\right.\right.}{15a^3\left(1+\sin\left(fx+e\right)\right)^2\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\;f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/15*c^3/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(15*B*cos(f*x+e)^2*sin(f*x+e)+
(10*A-170*B)*sin(f*x+e)+(-15*A+120*B)*cos(f*x+e)^2+22*A-182*B)/cos(f*x+e)/(
c-c*sin(f*x+e))^(1/2)/f
```

maxima [B] time = 0.48, size = 761, normalized size = 4.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] 2/15*((7*c^(5/2) + 20*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 95*c^(5/2)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 80*c^(5/2)*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 250*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 120*c^(5/2)*s
in(f*x + e)^5/(cos(f*x + e) + 1)^5 + 250*c^(5/2)*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 + 80*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 95*c^(5/2)*sin
(f*x + e)^8/(cos(f*x + e) + 1)^8 + 20*c^(5/2)*sin(f*x + e)^9/(cos(f*x + e)
+ 1)^9 + 7*c^(5/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)*A/((a^3 + 5*a^3*s
in(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 1)^(5/2)) - 2*(31*c^(5/2) + 155*c^(5/2)*sin(f*x + e)/
(cos(f*x + e) + 1) + 395*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 680*
c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1030*c^(5/2)*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 1050*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 10
30*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 680*c^(5/2)*sin(f*x + e)^7
/(cos(f*x + e) + 1)^7 + 395*c^(5/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1
55*c^(5/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 31*c^(5/2)*sin(f*x + e)^10
/(cos(f*x + e) + 1)^10)*B/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 1
0*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^
5/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2)))/f
```

mupad [B] time = 22.79, size = 904, normalized size = 5.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^
3,x)
```

```
[Out] ((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((
2*B*c^2)/(a^3*f) - (B*c^2*exp(e*1i + f*x*1i)*2i)/(a^3*f)))/(exp(e*1i + f*x*
1i) - 1i) - (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(
e*1i + f*x*1i)*1i)/2))^(1/2)*((c^2*(A*2i - B*7i)*1i)/(3*a^3*f) - (2*c^2*(7*
A - 12*B))/(3*a^3*f) + (c^2*(A*23i - B*28i)*2i)/(3*a^3*f) - (c^2*(42*A - 67
*B))/(15*a^3*f) + (2*B*c^2)/(3*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1
```

```
i + f*x*1i) + 1i)^3) + (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i
)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((c^2*(A*1i - B*4i)*4i)/(a^3*f) + (
4*B*c^2)/(a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)) -
(exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1
i)*1i)/2))^(1/2)*((8*c^2*(A*1i - B*1i))/(a^3*f) + (c^2*(A*1i - B*3i))/(2*a^
3*f) + (c^2*(A*11i - B*1i))/(10*a^3*f) + (c^2*(12*A - 17*B)*1i)/(4*a^3*f) +
 (c^2*(52*A - 47*B)*1i)/(4*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i +
f*x*1i) + 1i)^4) + (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2
- (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((c^2*(A*1i - B*4i))/(a^3*f) + (c^2*(A*
5i - B*4i))/(3*a^3*f) + (c^2*(A - 2*B)*8i)/(a^3*f)))/((exp(e*1i + f*x*1i) -
 1i)*(exp(e*1i + f*x*1i) + 1i)^2) + (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i
 - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((c^2*(A*2i - B*5i)*1i
)/(5*a^3*f) - (c^2*(4*A - 3*B))/(a^3*f) - (c^2*(2*A - 5*B))/(5*a^3*f) + (c^
2*(A*4i - B*3i)*1i)/(a^3*f) - (c^2*(10*A - 11*B))/(5*a^3*f) + (c^2*(A*10i -
 B*11i)*1i)/(5*a^3*f) + (2*B*c^2)/(5*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(e
xp(e*1i + f*x*1i) + 1i)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.126 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=121

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{5a^3c^3f}-\frac{(A+9B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3cf}+\frac{4(A+9B)\sec^3(e+fx)\ldots}{15a\ldots}$$

```
[Out] 4/15*(A+9*B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/f-1/5*(A+9*B)*sec(f*x+
e)^3*(c-c*sin(f*x+e))^(5/2)/a^3/c/f-1/5*(A-B)*sec(f*x+e)^5*(c-c*sin(f*x+e))
^(9/2)/a^3/c^3/f
```

Rubi [A] time = 0.41, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2855, 2674, 2673}

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{5a^3c^3f}-\frac{(A+9B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3cf}+\frac{4(A+9B)\sec^3(e+fx)\ldots}{15a\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^
3,x]
```

```
[Out] (4*(A + 9*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^3*f) - ((A +
9*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*c*f) - ((A - B)*Sec[
e + f*x]^5*(c - c*Sin[e + f*x])^(9/2))/(5*a^3*c^3*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx}{a^3c^3}$$

$$= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{5a^3c^3f} + \frac{(A+9B)\int \sec^4(e$$

$$= -\frac{(A+9B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3cf} - \frac{(A-B)\sec^5(e+$$

$$= \frac{4(A+9B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f} - \frac{(A+9B)\sec^3(e$$

Mathematica [A] time = 0.70, size = 113, normalized size = 0.93

$$\frac{c\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(10(A+3B)\sin(e+fx)-2A-15B\cos(2(e+fx))+27}{15a^3f(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
f*x])^3,x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-2*A + 27*B - 15*B*Cos[2*(e + f*x
)] + 10*(A + 3*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(15*a^3*f*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3)
```

fricas [A] time = 0.43, size = 95, normalized size = 0.79

$$\frac{2\left(15\,Bc\cos\left(fx+e\right)^{2}-5\,(A+3\,B)c\sin\left(fx+e\right)+(A-21\,B)c\right)\sqrt{-c\sin\left(fx+e\right)+c}}{15\left(a^{3}f\cos\left(fx+e\right)^{3}-2\,a^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^{3}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] 2/15*(15*B*c*cos(f*x + e)^2 - 5*(A + 3*B)*c*sin(f*x + e) + (A - 21*B)*c)*sq
rt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*
x + e) - 2*a^3*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.46, size = 83, normalized size = 0.69

$$-\frac{2c^{2}\left(\sin\left(fx+e\right)-1\right)\left(\sin\left(fx+e\right)(5A+15B)-15B\left(\cos^{2}\left(fx+e\right)\right)-A+21B\right)}{15a^{3}\left(1+\sin\left(fx+e\right)\right)^{2}\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x)
```

```
[Out] -2/15*c^2/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(sin(f*x+e)*(5*A+15*B)-15*B*c
os(f*x+e)^2-A+21*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [B] time = 0.48, size = 663, normalized size = 5.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] 2/15*((c^(3/2) - 10*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 4*c^(3/2)*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 - 30*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 6*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 30*c^(3/2)*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5 + 4*c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^
6 - 10*c^(3/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + c^(3/2)*sin(f*x + e)^8
/(cos(f*x + e) + 1)^8)*A/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10
*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)) - 6
*(c^(3/2) + 5*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 14*c^(3/2)*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 15*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3 + 26*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 15*c^(3/2)*sin(f*x + e
)^5/(cos(f*x + e) + 1)^5 + 14*c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 +
 5*c^(3/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + c^(3/2)*sin(f*x + e)^8/(co
s(f*x + e) + 1)^8)*B/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)))/f
```

mupad [B] time = 19.16, size = 683, normalized size = 5.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^
3,x)
```

```
[Out] (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1
i)*1i)/2))^(1/2)*((4*B*c)/(5*a^3*f) - (2*c*(2*A - 3*B))/(5*a^3*f) - (4*c*(3
*A - 2*B))/(5*a^3*f) + (c*(A*2i - B*3i)*2i)/(5*a^3*f) + (c*(A*3i - B*2i)*4i
)/(5*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^5) - (ex
p(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*
1i)/2))^(1/2)*((2*B*c)/(3*a^3*f) + (c*(A*2i - B*5i)*2i)/(3*a^3*f) - (2*c*(1
0*A - 13*B))/(3*a^3*f) + (c*(A*8i - B*13i)*2i)/(15*a^3*f)))/((exp(e*1i + f*
x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^3) + (exp(e*1i + f*x*1i)*(c - c*((exp
(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((c*(2*A - 3*B)
*1i)/(3*a^3*f) - (B*c*1i)/(a^3*f) + (2*c*(A*1i - B*3i))/(a^3*f)))/((exp(e*1
i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^2) - (exp(e*1i + f*x*1i)*(c - c
*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((c*(A -
B)*4i)/(a^3*f) - (B*c*1i)/(2*a^3*f) + (c*(8*A - 3*B)*1i)/(10*a^3*f) + (c*(A
*1i - B*2i))/(a^3*f) + (c*(A*7i - B*6i))/(a^3*f)))/((exp(e*1i + f*x*1i) - 1
i)*(exp(e*1i + f*x*1i) + 1i)^4) + (4*B*c*exp(e*1i + f*x*1i)*(c - c*((exp(-
e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(a^3*f*(exp(e*1i
+ f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**3,x)
[Out] Timed out
```

3.127 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=85

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3c^3f}-\frac{(3A+7B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3cf}$$

```
[Out] -1/15*(3*A+7*B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/c/f-1/5*(A-B)*sec(f
*x+e)^5*(c-c*sin(f*x+e))^(7/2)/a^3/c^3/f
```

Rubi [A] time = 0.32, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2967, 2855, 2673}

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3c^3f}-\frac{(3A+7B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3cf}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^3,
x]
```

```
[Out] -((3*A + 7*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^3*c*f) - ((A
 - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(7/2))/(5*a^3*c^3*f)
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
```

```
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx}{a^3c^3}$$

$$= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3c^3f} + \frac{(3A+7B)\int \sec^4(}{}$$

$$= -\frac{(3A+7B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3cf} - \frac{(A-B)\sec^5(e}{}$$

Mathematica [A] time = 0.30, size = 89, normalized size = 1.05

$$-\frac{2\sqrt{c-c\sin(e+fx)}\,(3A+5B\sin(e+fx)+2B)}{15a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*
x])^3,x]
```

```
[Out] (-2*(3*A + 2*B + 5*B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(15*a^3*f*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

fricas [A] time = 0.44, size = 77, normalized size = 0.91

$$\frac{2\left(5B\sin\left(fx+e\right)+3A+2B\right)\sqrt{-c\sin\left(fx+e\right)+c}}{15\left(a^3f\cos\left(fx+e\right)^3-2a^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-2a^3f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] 2/15*(5*B*sin(f*x + e) + 3*A + 2*B)*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*
x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.41, size = 65, normalized size = 0.76

$$\frac{2c\left(\sin\left(fx+e\right)-1\right)\left(5B\sin\left(fx+e\right)+3A+2B\right)}{15a^{3}\left(1+\sin\left(fx+e\right)\right)^{2}\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/15*c/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(5*B*sin(f*x+e)+3*A+2*B)/cos(f*x
+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [B] time = 0.51, size = 505, normalized size = 5.94

$$\frac{2\left(\frac{2B\left(\sqrt{c}+\frac{5\sqrt{c}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\sqrt{c}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{10\sqrt{c}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{3\sqrt{c}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{5\sqrt{c}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+\frac{\sqrt{c}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}\right)}{\left(a^{3}+\frac{5a^{3}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{10a^{3}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{10a^{3}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{5a^{3}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{a^{3}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}\right)\sqrt{\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1}}+\frac{3A\left(\sqrt{c}+\frac{3\cdots}{(\cdots}\right.}{\left(a^{3}+\frac{5a^{3}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{10a^{3}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\cdots\right.}\right.}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] 2/15*(2*B*(sqrt(c) + 5*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sqrt(c)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*sqrt(c)*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*sqrt(c)*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + sqrt(c)*sin(f*x + e)^6/(cos(f*x + e) + 1)^
6)/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)) + 3*A*(sqrt(c) + 3*sqrt(c)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sqrt(c)*sin(f*x + e)^4/(cos(f*x + e
```

```
) + 1)^4 + sqrt(c)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)/((a^3 + 5*a^3*sin(f
*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1
0*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*sqrt(sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 1)))/f
```

mupad [B] time = 17.54, size = 479, normalized size = 5.64

$$\frac{e^{e\,1i+f\,x\,1i}\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}\left(\frac{8\,B}{5\,a^3\,f}-\frac{16\,A-8\,B}{10\,a^3\,f}+\frac{(A\,16i-B\,8i)\,1i}{10\,a^3\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-i\right)\left(e^{e\,1i+f\,x\,1i}+1i\right)^5}-\frac{e^{e\,1i+f\,x\,1i}\sqrt{c-c\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+}}{}\right.}}{\left(e^{e\,1i+f\,x\,1i}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^
3,x)
```

```
[Out] (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1
i)*1i)/2))^(1/2)*((8*B)/(5*a^3*f) - (16*A - 8*B)/(10*a^3*f) + ((A*16i - B*8
i)*1i)/(10*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^5)
 - (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*
x*1i)*1i)/2))^(1/2)*((4*B)/(3*a^3*f) - (16*A - 16*B)/(30*a^3*f) + ((A*80i -
 B*120i)*1i)/(30*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) +
1i)^3) - (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1
i + f*x*1i)*1i)/2))^(1/2)*((A*16i - B*16i)/(40*a^3*f) - (B*1i)/(a^3*f) + (A
*80i - B*80i)/(40*a^3*f) + ((160*A - 120*B)*1i)/(40*a^3*f)))/((exp(e*1i + f
*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^4) - (B*exp(e*1i + f*x*1i)*(c - c*((
exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*8i)/(3*a^3*f
*(exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.128 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=174

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^3f}-\frac{(A+B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{6a^3c^2f}-\frac{(A+B)\sec(e+fx)\sqrt{c-}}{4a^3cf}$$

```
[Out] -1/6*(A+B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/c^2/f-1/5*(A-B)*sec(f*x+
e)^5*(c-c*sin(f*x+e))^(5/2)/a^3/c^3/f+1/8*(A+B)*arctanh(1/2*cos(f*x+e)*c^(1
/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^3/f*2^(1/2)/c^(1/2)-1/4*(A+B)*sec(f*x
+e)*(c-c*sin(f*x+e))^(1/2)/a^3/c/f
```

Rubi [A] time = 0.44, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2967, 2855, 2675, 2649, 206}

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^3f}-\frac{(A+B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{6a^3c^2f}-\frac{(A+B)\sec(e+fx)\sqrt{c-}}{4a^3cf}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]]),
x]
```

```
[Out] ((A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]
)/(4*Sqrt[2]*a^3*Sqrt[c]*f) - ((A + B)*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]
])/(4*a^3*c*f) - ((A + B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(6*a^3
*c^2*f) - ((A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*c^3*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx}{a^3c^3}$$

$$= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^3f} + \frac{(A+B)\int \sec^4(e+$$

$$= -\frac{(A+B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{6a^3c^2f} - \frac{(A-B)\sec^5(e+fx}{5a$$

$$= -\frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf} - \frac{(A+B)\sec^3(e+fx)(}{6a^3c^2}$$

$$= -\frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf} - \frac{(A+B)\sec^3(e+fx)(}{6a^3c^2}$$

$$= \frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}\,a^3\sqrt{c}\,f} - \frac{(A+B)\sec(e+fx)\sqrt{c-c\,\mathrm{si}}}{4a^3cf}$$

Mathematica [C] time = 0.77, size = 204, normalized size = 1.17

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right) - \sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-15(A+B)\left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f
*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(12*(-A + B) - 10*(A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 15*(A
 + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - (15 + 15*I)*(-1)^(1/4)*(A +
 B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^5))/(60*a^3*f*(1 + Sin[e + f*x])^3*Sqrt[c - c*Sin[e +
f*x]])
```

fricas [A] time = 0.44, size = 262, normalized size = 1.51

$$\frac{15\sqrt{2}\left((A+B)\cos\left(fx+e\right)^3 - 2\,(A+B)\cos\left(fx+e\right)\sin\left(fx+e\right) - 2\,(A+B)\cos\left(fx+e\right)\right)\sqrt{c}\,\log\left(-\frac{c\cos\left(fx+e\right)^2}{}\right.}{240\left(a^3cf\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/240*(15*sqrt(2)*((A + B)*cos(f*x + e)^3 - 2*(A + B)*cos(f*x + e)*sin(f*x
+ e) - 2*(A + B)*cos(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*s
qrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*co
s(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (
cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(15*(A + B)*cos(f*x
 + e)^2 - 40*(A + B)*sin(f*x + e) - 52*A - 28*B)*sqrt(-c*sin(f*x + e) + c))
/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*c*f*
cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
ep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constan
t sign by intervals (correct if the argument is real):Check [abs(sin((f*t_n
ostep+exp(1))/2-pi/4))]Unable to check sign: (8*pi/t_nostep/2)>(-8*pi/t_nos
tep/2)Discontinuities at zeroes of sin((f*t_nostep+exp(1))/2-pi/4) were not
 checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes cons
tant sign by intervals (correct if the argument is real):Check [abs(t_noste
```

```
p-1)]Evaluation time: 0.43index.cc index_m i_lex_is_greater Error: Bad Argu
ment Value
```

maple [A] time = 1.94, size = 200, normalized size = 1.15

$$\frac{\left(\sin\left(fx+e\right)-1\right)\left(74c^{\frac{9}{2}}A+80A\,c^{\frac{9}{2}}\sin\left(fx+e\right)+30A\,c^{\frac{9}{2}}\left(\sin^{2}\left(fx+e\right)\right)-15\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\left(\right.}{120a^{3}c^{\frac{9}{2}}\left(1+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/120/a^3*(sin(f*x+e)-1)/c^(9/2)/(1+sin(f*x+e))^2*(74*c^(9/2)*A+80*A*c^(9/2
)*sin(f*x+e)+30*A*c^(9/2)*sin(f*x+e)^2-15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x
+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(5/2)*c^2*A+26*c^(9/2)*B+80
*B*c^(9/2)*sin(f*x+e)+30*B*c^(9/2)*sin(f*x+e)^2-15*2^(1/2)*arctanh(1/2*(c*(
1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(5/2)*c^2*B)/cos(f
*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2)
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2)
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.129 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=224

$$\frac{(7A+3B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}\,a^3c^{3/2}f}-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5a^3c^3f}-\frac{(7A+3B)\sec^3(e+fx)\sqrt{c-c\sin}}{30a^3c^2f}$$

```
[Out] 1/16*(7*A+3*B)*cos(f*x+e)/a^3/f/(c-c*sin(f*x+e))^(3/2)-1/5*(A-B)*sec(f*x+e)
^5*(c-c*sin(f*x+e))^(3/2)/a^3/c^3/f+1/32*(7*A+3*B)*arctanh(1/2*cos(f*x+e)*c
^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^3/c^(3/2)/f*2^(1/2)-1/12*(7*A+3*B)
*sec(f*x+e)/a^3/c/f/(c-c*sin(f*x+e))^(1/2)-1/30*(7*A+3*B)*sec(f*x+e)^3*(c-c
*sin(f*x+e))^(1/2)/a^3/c^2/f
```

Rubi [A] time = 0.48, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {2967, 2855, 2675, 2687, 2650, 2649, 206}

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5a^3c^3f}-\frac{(7A+3B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{30a^3c^2f}+\frac{(7A+3B)\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{2}\sqrt{}}\right)}{16\sqrt{2}\,a^3c^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)
),x]
```

```
[Out] ((7*A + 3*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x
]])])/(16*Sqrt[2]*a^3*c^(3/2)*f) + ((7*A + 3*B)*Cos[e + f*x])/(16*a^3*f*(c
- c*Sin[e + f*x])^(3/2)) - ((7*A + 3*B)*Sec[e + f*x])/(12*a^3*c*f*Sqrt[c -
c*Sin[e + f*x]]) - ((7*A + 3*B)*Sec[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(3
0*a^3*c^2*f) - ((A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(3/2))/(5*a^3*c
^3*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx}{a^3c^3}$$

$$= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5a^3c^3f} + \frac{(7A+3B)\int \sec^4$$

$$= -\frac{(7A+3B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{30a^3c^2f} - \frac{(A-B)\sec^5(e+$$

$$= -\frac{(7A+3B)\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}} - \frac{(7A+3B)\sec^3(e+fx)\sqrt{c-c\sin}}{30a^3c^2f}$$

$$= \frac{(7A+3B)\cos(e+fx)}{16a^3f(c-c\sin(e+fx))^{3/2}} - \frac{(7A+3B)\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}} - \frac{(7A}{}$$

$$= \frac{(7A+3B)\cos(e+fx)}{16a^3f(c-c\sin(e+fx))^{3/2}} - \frac{(7A+3B)\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}} - \frac{(7A}{}$$

$$= \frac{(7A+3B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}\,a^3c^{3/2}f} + \frac{(7A+3B)\cos(e+fx)}{16a^3f(c-c\sin(e+fx))}$$

Mathematica [C] time = 1.36, size = 357, normalized size = 1.59

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(15(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-40*A*Cos[e + f*x]^2 + 24*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^2 - 30*(3*A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^4 + 15*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - (15 + 15*I)*(-1)^(1/4)*(7*A + 3*B)
*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + 30*(A + B)*Si
```

```
n[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5))/(240*a^3*f*(1 + Si
n[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))
```

fricas [A] time = 0.46, size = 277, normalized size = 1.24

$$15\sqrt{2}\left((7A+3B)\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)+(7A+3B)\cos\left(fx+e\right)^{3}\right)\sqrt{c}\log\left(-\frac{c\cos\left(fx+e\right)^{2}+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)}\cdots}{\cos\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/960*(15*sqrt(2)*((7*A + 3*B)*cos(f*x + e)^3*sin(f*x + e) + (7*A + 3*B)*co
s(f*x + e)^3)*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x +
e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*c
os(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)
*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(25*(7*A + 3*B)*cos(f*x + e)^2 + 3*(
5*(7*A + 3*B)*cos(f*x + e)^2 - 28*A - 12*B)*sin(f*x + e) - 36*A - 84*B)*sqr
t(-c*sin(f*x + e) + c))/(a^3*c^2*f*cos(f*x + e)^3*sin(f*x + e) + a^3*c^2*f*
cos(f*x + e)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(1/32*(-3*A*(-sqr
t(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-3*B*(-sqrt(c)*t
an((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3+A*c*(-sqrt(c)*tan((f*
x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-A*sqrt(c)*(-sqrt(c)*tan((f*x+
exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+B*c*(-sqrt(c)*tan((f*x+exp(1)
)/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-B*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/
2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-A*sqrt(c)*c-B*sqrt(c)*c)/a^3/(-(-sqrt
(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt
(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^2/c/sign(tan((f
*x+exp(1))/2)-1)-1/240*(-165*A*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*
x+exp(1))/2)^2+c))^9+45*B*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp
```

```
(1))/2)^2+c))^9+795*A*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x
+exp(1))/2)^2+c))^8-75*B*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((
f*x+exp(1))/2)^2+c))^8-1100*A*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f
*x+exp(1))/2)^2+c))^7+60*B*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+
exp(1))/2)^2+c))^7-1180*A*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*ta
n((f*x+exp(1))/2)^2+c))^6+60*B*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt
(c*tan((f*x+exp(1))/2)^2+c))^6+2618*A*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqr
t(c*tan((f*x+exp(1))/2)^2+c))^5+102*B*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqr
t(c*tan((f*x+exp(1))/2)^2+c))^5+1130*A*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1
))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4+150*B*sqrt(c)*c^2*(-sqrt(c)*tan((f
*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4-1980*A*c^3*(-sqrt(c)*tan((
f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-180*B*c^3*(-sqrt(c)*tan((
f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-605*A*c^4*(-sqrt(c)*tan((
f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-1900*A*sqrt(c)*c^3*(-sqrt(c
)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-75*B*c^4*(-sqrt(c)
*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-180*B*sqrt(c)*c^3*(-s
qrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-77*A*sqrt(c)*
c^4-3*B*sqrt(c)*c^4)/a^3/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+ex
p(1))/2)^2+c))^2+2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+ex
p(1))/2)^2+c))+c)^5/c/sign(tan((f*x+exp(1))/2)-1)+1/32*(7*A+3*B)*atan((-sqr
t(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/s
qrt(-c))/sqrt(2)/a^3/sqrt(-c)/c/sign(tan((f*x+exp(1))/2)-1))
```

maple [A] time = 1.69, size = 308, normalized size = 1.38

$$-\frac{278A\,c^{\frac{7}{2}}-18B\,c^{\frac{7}{2}}-350A\,c^{\frac{7}{2}}\left(\sin^{2}\left(fx+e\right)\right)-150B\,c^{\frac{7}{2}}\left(\sin^{2}\left(fx+e\right)\right)+42A\,c^{\frac{7}{2}}\sin\left(fx+e\right)+18B\,c^{\frac{7}{2}}\sin\left(fx-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/480/c^(9/2)/a^3*(278*A*c^(7/2)-18*B*c^(7/2)-350*A*c^(7/2)*sin(f*x+e)^2-1
50*B*c^(7/2)*sin(f*x+e)^2+42*A*c^(7/2)*sin(f*x+e)+18*B*c^(7/2)*sin(f*x+e)-2
10*A*c^(7/2)*sin(f*x+e)^3-90*B*c^(7/2)*sin(f*x+e)^3+45*B*(c*(1+sin(f*x+e)))
^(5/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*
x+e)*c-105*A*(c*(1+sin(f*x+e)))^(5/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e))
)^(1/2)*2^(1/2)/c^(1/2))*c-45*B*(c*(1+sin(f*x+e)))^(5/2)*2^(1/2)*arctanh(1/
2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c+105*A*(c*(1+sin(f*x+e)))^(5/2
)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*
c)/(1+sin(f*x+e))^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2)
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2)
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.130 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=258

$$\frac{7(9A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}\,a^3c^{5/2}f}-\frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f}-\frac{(9A+B)\sec^3(e+fx)}{30a^3c^2f\sqrt{c-c\sin(e+fx)}}-\frac{7(9}{96a^3}$$

```
[Out] 7/128*(9*A+B)*cos(f*x+e)/a^3/c/f/(c-c*sin(f*x+e))^(3/2)+7/240*(9*A+B)*sec(f
*x+e)/a^3/c/f/(c-c*sin(f*x+e))^(3/2)+7/256*(9*A+B)*arctanh(1/2*cos(f*x+e)*c
^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^3/c^(5/2)/f*2^(1/2)-7/96*(9*A+B)*s
ec(f*x+e)/a^3/c^2/f/(c-c*sin(f*x+e))^(1/2)-1/30*(9*A+B)*sec(f*x+e)^3/a^3/c^
2/f/(c-c*sin(f*x+e))^(1/2)-1/5*(A-B)*sec(f*x+e)^5*(c-c*sin(f*x+e))^(1/2)/a^
3/c^3/f
```

Rubi [A] time = 0.56, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {2967, 2855, 2687, 2681, 2650, 2649, 206}

$$-\frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f}-\frac{(9A+B)\sec^3(e+fx)}{30a^3c^2f\sqrt{c-c\sin(e+fx)}}-\frac{7(9A+B)\sec(e+fx)}{96a^3c^2f\sqrt{c-c\sin(e+fx)}}+\frac{7(9A+B)\,\mathrm{ta}}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2)
),x]
```

```
[Out] (7*(9*A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x
]])])/(128*Sqrt[2]*a^3*c^(5/2)*f) + (7*(9*A + B)*Cos[e + f*x])/(128*a^3*c*f
*(c - c*Sin[e + f*x])^(3/2)) + (7*(9*A + B)*Sec[e + f*x])/(240*a^3*c*f*(c -
 c*Sin[e + f*x])^(3/2)) - (7*(9*A + B)*Sec[e + f*x])/(96*a^3*c^2*f*Sqrt[c -
 c*Sin[e + f*x]]) - ((9*A + B)*Sec[e + f*x]^3)/(30*a^3*c^2*f*Sqrt[c - c*Sin
[e + f*x]]) - ((A - B)*Sec[e + f*x]^5*Sqrt[c - c*Sin[e + f*x]])/(5*a^3*c^3*
f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
```

```
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx}{a^3c^3}$$

$$= -\frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f} + \frac{(9A+B)\int \frac{\sec^4(e+}{\sqrt{c-c\sin(}}}{10a^3c^2}$$

$$= -\frac{(9A+B)\sec^3(e+fx)}{30a^3c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(}}{5a^3c^3f}$$

$$= \frac{7(9A+B)\sec(e+fx)}{240a^3cf(c-c\sin(e+fx))^{3/2}} - \frac{(9A+B)\sec^3(e+fx)}{30a^3c^2f\sqrt{c-c\sin(e+fx)}} - $$

$$= \frac{7(9A+B)\sec(e+fx)}{240a^3cf(c-c\sin(e+fx))^{3/2}} - \frac{7(9A+B)\sec(e+fx)}{96a^3c^2f\sqrt{c-c\sin(e+fx)}} - $$

$$= \frac{7(9A+B)\cos(e+fx)}{128a^3cf(c-c\sin(e+fx))^{3/2}} + \frac{7(9A+B)\sec(e+fx)}{240a^3cf(c-c\sin(e+fx))^{3/2}} - $$

$$= \frac{7(9A+B)\cos(e+fx)}{128a^3cf(c-c\sin(e+fx))^{3/2}} + \frac{7(9A+B)\sec(e+fx)}{240a^3cf(c-c\sin(e+fx))^{3/2}} - $$

$$= \frac{7(9A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}\,a^3c^{5/2}f} + \frac{7(9A+B)\cos(e+f}{128a^3cf(c-c\sin(e+f}$$

Mathematica [C] time = 2.29, size = 479, normalized size = 1.86

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(15(15A+7B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-720*A*Cos[e + f*x]^4 + 96*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^4 + 80*(-3*A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^2 + 60*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + 15*(15*A + 7*B)*(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - (105 + 10
5*I)*(-1)^(1/4)*(9*A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/
4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^5 + 120*(A + B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^5 + 30*(15*A + 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*
x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5))/(1920*a^3*f*(1 + Sin[e + f*
x])^3*(c - c*Sin[e + f*x])^(5/2))
```

fricas [A] time = 0.49, size = 238, normalized size = 0.92

$$105\sqrt{2}\,(9A+B)\sqrt{c}\,\cos\left(fx+e\right)^5\log\left(-\frac{c\cos\left(fx+e\right)^2+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\,\sqrt{c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)+3c\cos\left(fx+e\right)+\left(c\cos\left(\cdots\right.\right.}{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] 1/7680*(105*sqrt(2)*(9*A + B)*sqrt(c)*cos(f*x + e)^5*log(-(c*cos(f*x + e)^2
 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e)
 + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(
f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(105*
(9*A + B)*cos(f*x + e)^4 - 14*(9*A + B)*cos(f*x + e)^2 - 2*(35*(9*A + B)*co
s(f*x + e)^2 + 216*A + 24*B)*sin(f*x + e) - 48*A - 432*B)*sqrt(-c*sin(f*x +
 e) + c))/(a^3*c^3*f*cos(f*x + e)^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(1/256*(-65*A*(-s
qrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7-41*B*(-sqrt(c
```

```
)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7-231*A*sqrt(c)*(-sq
rt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6-127*B*sqrt(c)*
(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6-163*A*c*(-
sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5-91*B*c*(-sqr
t(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5+247*A*sqrt(c)*c
*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4+143*B*sqr
t(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4-11*
A*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-3*B*
c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3+87*A*c
^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-149*A*sqr
t(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+4
7*B*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-93*B
*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))
^2-19*A*sqrt(c)*c^3-11*B*sqrt(c)*c^3)/a^3/c^2/(-(-sqrt(c)*tan((f*x+exp(1))/
2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/
2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^4/sign(tan((f*x+exp(1))/2)-1)-1/240*
(-120*A*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^9+45
*B*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^9+630*A*s
qrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^8-165
*B*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^8
-900*A*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7+2
00*B*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7-960
*A*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))
^6+220*B*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)
^2+c))^6+2244*A*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2
)^2+c))^5-374*B*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2
)^2+c))^5+1020*A*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+
exp(1))/2)^2+c))^4-110*B*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*t
an((f*x+exp(1))/2)^2+c))^4-1740*A*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*
tan((f*x+exp(1))/2)^2+c))^3+240*B*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*
tan((f*x+exp(1))/2)^2+c))^3-540*A*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*
tan((f*x+exp(1))/2)^2+c))-1680*A*sqrt(c)*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+
sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+65*B*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+s
qrt(c*tan((f*x+exp(1))/2)^2+c))+220*B*sqrt(c)*c^3*(-sqrt(c)*tan((f*x+exp(1)
)/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-66*A*sqrt(c)*c^4+11*B*sqrt(c)*c^4)/
a^3/c^2/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+
2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)
^5/sign(tan((f*x+exp(1))/2)-1)+1/256*(63*A+7*B)*atan((-sqrt(c)*tan((f*x+exp
(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/
a^3/c^2/sqrt(-c)/sign(tan((f*x+exp(1))/2)-1))
```

maple [A] time = 2.06, size = 410, normalized size = 1.59

$$-\frac{\left(1260c^{\frac{9}{2}}A+140c^{\frac{9}{2}}B\right)\sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)+\left(-1890\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{c+c\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{c}}\right)\left(c+c\sin\left(fx+e\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/3840/c^(13/2)/a^3*((1260*c^(9/2)*A+140*c^(9/2)*B)*sin(f*x+e)*cos(f*x+e)^
2+(-1890*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*s
in(f*x+e))^(5/2)*c^2*A+864*c^(9/2)*A-210*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e
))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(5/2)*c^2*B+96*c^(9/2)*B)*sin(f*
x+e)+(-1890*c^(9/2)*A-210*c^(9/2)*B)*cos(f*x+e)^4+(-945*2^(1/2)*arctanh(1/2
*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(5/2)*c^2*A+252*c
^(9/2)*A-105*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c
+c*sin(f*x+e))^(5/2)*c^2*B+28*c^(9/2)*B)*cos(f*x+e)^2+1890*2^(1/2)*arctanh(
1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(5/2)*c^2*A+96
*c^(9/2)*A+210*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*
(c+c*sin(f*x+e))^(5/2)*c^2*B+864*c^(9/2)*B)/(1+sin(f*x+e))^2/(sin(f*x+e)-1)
/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2)
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2)
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.131 $\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=94

$$\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{5cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -1/4*a*(A+B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1/2)+1/5
*a*B*cos(f*x+e)*(c-c*sin(f*x+e))^(9/2)/c/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2971, 2738}

$$\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{5cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2
),x]
```

```
[Out] -(a*(A + B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(4*f*Sqrt[a + a*Sin[e
+ f*x]]) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(5*c*f*Sqrt[a + a*
Sin[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = (A+B)\int \sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+$$
$$= -\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 1.00, size = 118, normalized size = 1.26

$$-\frac{c^3\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}\,(4(23B-60A)\sin(e+fx)+4\cos(2(e+fx))(4(5A-6B)}{160f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]
)^(7/2),x]
```

```
[Out] -1/160*(c^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]
]*(4*(-60*A + 23*B)*Sin[e + f*x] + 4*Cos[2*(e + f*x)]*(-35*A + 25*B + 4*(5*
A - 6*B)*Sin[e + f*x]) + Cos[4*(e + f*x)]*(5*A - 15*B + 4*B*Sin[e + f*x])))
/f
```

fricas [A] time = 0.45, size = 140, normalized size = 1.49

$$-\frac{\left(5\,(A-3\,B)c^3\cos\left(fx+e\right)^4-40\,(A-B)c^3\cos\left(fx+e\right)^2+5\,(7\,A-5\,B)c^3+4\left(Bc^3\cos\left(fx+e\right)^4+(5\,A-7\,B)\right.\right.}{20\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] -1/20*(5*(A - 3*B)*c^3*cos(f*x + e)^4 - 40*(A - B)*c^3*cos(f*x + e)^2 + 5*(
7*A - 5*B)*c^3 + 4*(B*c^3*cos(f*x + e)^4 + (5*A - 7*B)*c^3*cos(f*x + e)^2 -
 2*(5*A - 3*B)*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sq
rt(2*c)*(32*f*(A*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi))-3*B*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*
x+exp(1))-1/4*pi)))*cos(-2*f*x-2*exp(1))/(-32*f)^2+96*f*(12*A*c^3*sign(sin(
1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-15*B*c^3*sign(
sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(3*f*x
+3*exp(1))/(96*f)^2-32*f*(15*A*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(
cos(1/2*(f*x+exp(1))-1/4*pi))-13*B*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*s
ign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(2*f*x+2*exp(1))/(32*f)^2-64*f*(-A*c^
3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+3*B
*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))
*cos(4*f*x+4*exp(1))/(64*f)^2+16*f*(-14*A*c^3*sign(sin(1/2*(f*x+exp(1))-1/4
*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+7*B*c^3*sign(sin(1/2*(f*x+exp(1))-
1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(f*x+exp(1))/(16*f)^2+160*B
*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)
)*sin(5*f*x+5*exp(1))/(160*f)^2)
```

maple [B] time = 0.87, size = 174, normalized size = 1.85

$$\frac{\left(-4B\left(\cos^4\left(fx+e\right)\right)+5A\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)-15B\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)-20A\left(\cos^2\left(fx+e\right.\right.}{20f\left(\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/20/f*(-4*B*cos(f*x+e)^4+5*A*cos(f*x+e)^2*sin(f*x+e)-15*B*cos(f*x+e)^2*sin
(f*x+e)-20*A*cos(f*x+e)^2+28*B*cos(f*x+e)^2-35*A*sin(f*x+e)+25*B*sin(f*x+e)
```

```
+40*A-24*B)*(-c*(sin(f*x+e)-1))^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)/(
cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-4*sin(f*x+e)+4)/cos(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
c)^(7/2), x)
```

mupad [B] time = 16.17, size = 173, normalized size = 1.84

$$-\frac{c^3\sqrt{a\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\left(\sin\left(e+f\,x\right)-1\right)}\,\left(100\,B\,\cos\left(e+f\,x\right)-140\,A\,\cos\left(e+f\,x\right)-135\,A\,\cos\left(3\,e\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(7
/2),x)
```

```
[Out] -(c^3*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(100*B*cos
(e + f*x) - 140*A*cos(e + f*x) - 135*A*cos(3*e + 3*f*x) + 5*A*cos(5*e + 5*f
*x) + 85*B*cos(3*e + 3*f*x) - 15*B*cos(5*e + 5*f*x) - 240*A*sin(2*e + 2*f*x
) + 40*A*sin(4*e + 4*f*x) + 90*B*sin(2*e + 2*f*x) - 48*B*sin(4*e + 4*f*x) +
 2*B*sin(6*e + 6*f*x)))/(160*f*(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)*(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

3.132 $\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=94

$$\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -1/3*a*(A+B)*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(1/2)+1/4
*a*B*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/c/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2971, 2738}

$$\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2
),x]
```

```
[Out] -(a*(A + B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*f*Sqrt[a + a*Sin[e
+ f*x]]) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(4*c*f*Sqrt[a + a*
Sin[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = (A+B)\int \sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+$$

$$= -\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 0.82, size = 102, normalized size = 1.09

$$\frac{c^2\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}\,(16(7A-2B)\sin(e+fx)-4\cos(2(e+fx))(4(A-2B)\sin(}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] (c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(3*B*
Cos[4*(e + f*x)] + 16*(7*A - 2*B)*Sin[e + f*x] - 4*Cos[2*(e + f*x)]*(-12*A
+ 9*B + 4*(A - 2*B)*Sin[e + f*x])))/(96*f)
```

fricas [A] time = 0.44, size = 121, normalized size = 1.29

$$\frac{\left(3\,Bc^2\cos\left(fx+e\right)^4+12\,(A-B)c^2\cos\left(fx+e\right)^2-3\,(4\,A-3\,B)c^2-4\left((A-2\,B)c^2\cos\left(fx+e\right)^2-2\,(2\,A-B)c^2\right.}{12\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/12*(3*B*c^2*cos(f*x + e)^4 + 12*(A - B)*c^2*cos(f*x + e)^2 - 3*(4*A - 3*B
)*c^2 - 4*((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(2*A - B)*c^2)*sin(f*x + e))*sq
rt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(24*f*(A*c^2*sign(sin(1/2*(f*x+
exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-2*B*c^2*sign(sin(1/2*(f
*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(3*f*x+3*exp(1))
/(24*f)^2-32*f*(8*A*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*
x+exp(1))-1/4*pi))-7*B*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*
(f*x+exp(1))-1/4*pi)))*cos(2*f*x+2*exp(1))/(32*f)^2+8*f*(-5*A*c^2*sign(sin(
1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+2*B*c^2*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(f*x+ex
p(1))/(8*f)^2-64*B*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f
*x+exp(1))-1/4*pi))*cos(4*f*x+4*exp(1))/(64*f)^2-32*B*c^2*f*sign(sin(1/2*(f
*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-2*f*x-2*exp(1))
/(-32*f)^2)
```

maple [A] time = 0.85, size = 129, normalized size = 1.37

$$\frac{\left(3B\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+4A\left(\cos^2\left(fx+e\right)\right)-8B\left(\cos^2\left(fx+e\right)\right)+12A\sin\left(fx+e\right)-9B\sin\left(fx+e\right.\right.}{12f\left(2\sin\left(fx+e\right)+\cos^2\left(fx+e\right)-2\right)\cos\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/12/f*(3*B*cos(f*x+e)^2*sin(f*x+e)+4*A*cos(f*x+e)^2-8*B*cos(f*x+e)^2+12*A*
sin(f*x+e)-9*B*sin(f*x+e)-16*A+8*B)*(-c*(sin(f*x+e)-1))^(5/2)*sin(f*x+e)*(a
*(1+sin(f*x+e)))^(1/2)/(2*sin(f*x+e)+cos(f*x+e)^2-2)/cos(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
c)^(5/2), x)
```

mupad [B] time = 14.96, size = 149, normalized size = 1.59

$$\frac{c^2\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(48\,A\,\cos\left(e+f\,x\right)-36\,B\,\cos\left(e+f\,x\right)+48\,A\,\cos\left(3\,e+3\,f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(5
/2),x)
```

```
[Out] (c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(48*A*cos(e
 + f*x) - 36*B*cos(e + f*x) + 48*A*cos(3*e + 3*f*x) - 33*B*cos(3*e + 3*f*x)
 + 3*B*cos(5*e + 5*f*x) + 112*A*sin(2*e + 2*f*x) - 8*A*sin(4*e + 4*f*x) - 3
2*B*sin(2*e + 2*f*x) + 16*B*sin(4*e + 4*f*x)))/(96*f*(cos(2*e + 2*f*x) + 1)
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)*(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

3.133 $\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=94

$$\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -1/2*a*(A+B)*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)+1/3
*a*B*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/c/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2971, 2738}

$$\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2
),x]
```

```
[Out] -(a*(A + B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*f*Sqrt[a + a*Sin[e
+ f*x]]) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*c*f*Sqrt[a + a*
Sin[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = (A+B)\int \sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+$$

$$= -\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 0.56, size = 84, normalized size = 0.89

$$\frac{c\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}\,(2(6A-B)\sin(e+fx)+\cos(2(e+fx))(3A+2B\sin(e+fx}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (c*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(2*(6*A
 - B)*Sin[e + f*x] + Cos[2*(e + f*x)]*(3*A - 3*B + 2*B*Sin[e + f*x])))/(12*
f)
```

fricas [A] time = 0.45, size = 92, normalized size = 0.98

$$\frac{\left(3\,(A-B)c\cos\left(fx+e\right)^2-3\,(A-B)c+2\left(Bc\cos\left(fx+e\right)^2+(3\,A-B)c\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c}}{6\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/6*(3*(A - B)*c*cos(f*x + e)^2 - 3*(A - B)*c + 2*(B*c*cos(f*x + e)^2 + (3*
A - B)*c)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/
(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(
2*c)*(-8*f*(A*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1)
)-1/4*pi))-B*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))
-1/4*pi)))*cos(2*f*x+2*exp(1))/(8*f)^2+8*f*(-4*A*c*sign(sin(1/2*(f*x+exp(1)
)-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+B*c*sign(sin(1/2*(f*x+exp(1))
-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(f*x+exp(1))/(8*f)^2-24*B*
c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*s
in(3*f*x+3*exp(1))/(24*f)^2)
```

maple [A] time = 0.82, size = 91, normalized size = 0.97

$$\frac{\left(-2B\left(\cos^{2}\left(fx+e\right)\right)+3A\sin\left(fx+e\right)-3B\sin\left(fx+e\right)-6A+2B\right)\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)\sqrt{a\left(\right.}}{6f\left(\sin\left(fx+e\right)-1\right)\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/6/f*(-2*B*cos(f*x+e)^2+3*A*sin(f*x+e)-3*B*sin(f*x+e)-6*A+2*B)*(-c*(sin(f*
x+e)-1))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/cos(f*x+e
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
c)^(3/2), x)
```

mupad [B] time = 1.79, size = 122, normalized size = 1.30

$$\frac{c\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\left(3A\cos\left(e+fx\right)-3B\cos\left(e+fx\right)+3A\cos\left(3e+3fx\right)\right.}{12f\left(\cos\left(2e+2fx\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(3
/2),x)
```

```
[Out] (c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(3*A*cos(e +
f*x) - 3*B*cos(e + f*x) + 3*A*cos(3*e + 3*f*x) - 3*B*cos(3*e + 3*f*x) + 12*
A*sin(2*e + 2*f*x) - 2*B*sin(2*e + 2*f*x) + B*sin(4*e + 4*f*x)))/(12*f*(cos
(2*e + 2*f*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x) - 1))**(3/2)*(A + B*s
in(e + f*x)), x)
```

3.134 $\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=92

$$\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] 1/2*a*B*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/c/f/(a+a*sin(f*x+e))^(1/2)-a*(A+B
)*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2971, 2738}

$$\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] -((a*(A + B)*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f
*x]])) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*c*f*Sqrt[a + a*Si
n[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = (A+B)\int \sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+f$$

$$= -\frac{a(A+B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{a}{}$$

Mathematica [A] time = 0.17, size = 63, normalized size = 0.68

$$\frac{\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}\,(4A\sin(e+fx)-B\cos(2(e+fx)))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e +
f*x]],x]
```

```
[Out] (Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(-(B*Cos[2*(e + f*x)]) + 4*A*Sin[e
 + f*x])*Sqrt[c - c*Sin[e + f*x]])/(4*f)
```

fricas [A] time = 0.44, size = 61, normalized size = 0.66

$$-\frac{\left(B\cos\left(fx+e\right)^{2}-2\,A\sin\left(fx+e\right)-B\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{2\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] -1/2*(B*cos(f*x + e)^2 - 2*A*sin(f*x + e) - B)*sqrt(a*sin(f*x + e) + a)*sqr
t(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
```

```
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-2*A*f*sign(sin(1/2*(f*x+exp(1))-
1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1))/(2*f)^2+8*B*f*s
ign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(2*
f*x+2*exp(1))/(8*f)^2)
```

maple [A] time = 0.80, size = 57, normalized size = 0.62

$$\frac{\left(B\sin\left(fx+e\right)+2A\right)\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\,\sin\left(fx+e\right)\sqrt{a\left(1+\sin\left(fx+e\right)\right)}}{2f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*(B*sin(f*x+e)+2*A)*(-c*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x
+e)))^(1/2)/cos(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c), x)
```

mupad [B] time = 0.94, size = 75, normalized size = 0.82

$$-\frac{\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(B\,\cos\left(e+f\,x\right)+B\,\cos\left(3\,e+3\,f\,x\right)-4\,A\,\sin\left(2\,e+2\,f\,x\right)\right)}{4\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1
/2),x)
```

```
[Out] -((a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(B*cos(e + f*x
) + B*cos(3*e + 3*f*x) - 4*A*sin(2*e + 2*f*x)))/(4*f*(cos(2*e + 2*f*x) + 1)
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)*(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(
e + f*x)), x)
```

3.135 $\int \frac{\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=100

$$\frac{aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -a*(A+B)*cos(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+
e))^(1/2)+a*B*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/c/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2971, 2738, 2737, 2667, 31}

$$\frac{aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] -((a*(A + B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c - c*Sin[e + f*x]])) + (a*B*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/
(c*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = (A+B)\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx - \frac{B\int \sqrt{a+a\sin(e+fx)}\,\sqrt{c-\cdots}}{c}$$

$$= \frac{aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a+a\sin(e+fx)}} + \frac{(a(A+B)c\cos(e+fx))\int \cdots}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\cdots}}$$

$$= \frac{aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a+a\sin(e+fx)}} - \frac{(a(A+B)\cos(e+fx))\,\mathrm{Subs}\cdots}{f\sqrt{a+a\sin(e+f\cdots}}$$

$$= -\frac{a(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}} + \frac{aB\cos(e+fx)\sqrt{c\cdots}}{cf\sqrt{a+a\sin\cdots}}$$

Mathematica [C] time = 1.13, size = 120, normalized size = 1.20

$$-\frac{\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(B\sin(e+fx)+(A+B)\left(2\log\left(i-e^{i(e+fx)}\right)-ifx\right)\right)}{f\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e
+ f*x]],x]
```

```
[Out] -(((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*((A + B
)*((-I)*f*x + 2*Log[I - E^(I*(e + f*x))]) + B*Sin[e + f*x]))/(f*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]))
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{c\sin\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)/(c*sin(f*x + e) - c), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (8*pi/x/2)>(-8*pi/x/2)4*sqrt(2*a)*(A*sqrt(c*tan(1/2*exp(1)
)^2+c)*(-12582912*tan(1/2*exp(1))^5-37748736*tan(1/2*exp(1))^4+41943040*tan
(1/2*exp(1))^3+25165824*tan(1/2*exp(1))^2-12582912*tan(1/2*exp(1))-4194304)
+B*sqrt(c*tan(1/2*exp(1))^2+c)*(-12582912*tan(1/2*exp(1))^5-37748736*tan(1/
2*exp(1))^4+41943040*tan(1/2*exp(1))^3+25165824*tan(1/2*exp(1))^2-12582912*
tan(1/2*exp(1))-4194304)+A*sqrt(c*tan(1/2*exp(1))^2+c)*(12582912*tan(1/2*ex
p(1))^5+37748736*tan(1/2*exp(1))^4-41943040*tan(1/2*exp(1))^3-25165824*tan(
1/2*exp(1))^2+12582912*tan(1/2*exp(1))+4194304)*tan(1/4*exp(1))^6+A*sqrt(c*
tan(1/2*exp(1))^2+c)*(83886080*tan(1/2*exp(1))^6+251658240*tan(1/2*exp(1))^
5-503316480*tan(1/2*exp(1))^4-838860800*tan(1/2*exp(1))^3+754974720*tan(1/2
*exp(1))^2+251658240*tan(1/2*exp(1)))*tan(1/4*exp(1))^3+A*sqrt(c*tan(1/2*ex
p(1))^2+c)*(188743680*tan(1/2*exp(1))^5+566231040*tan(1/2*exp(1))^4-6291456
00*tan(1/2*exp(1))^3-377487360*tan(1/2*exp(1))^2+188743680*tan(1/2*exp(1))+
62914560)*tan(1/4*exp(1))^2+A*sqrt(c*tan(1/2*exp(1))^2+c)*(-25165824*tan(1/
2*exp(1))^6-75497472*tan(1/2*exp(1))^5+150994944*tan(1/2*exp(1))^4+25165824
0*tan(1/2*exp(1))^3-226492416*tan(1/2*exp(1))^2-75497472*tan(1/2*exp(1)))*t
```

```
an(1/4*exp(1))^5+A*sqrt(c*tan(1/2*exp(1))^2+c)*(-25165824*tan(1/2*exp(1))^6
-75497472*tan(1/2*exp(1))^5+150994944*tan(1/2*exp(1))^4+251658240*tan(1/2*e
xp(1))^3-226492416*tan(1/2*exp(1))^2-75497472*tan(1/2*exp(1)))*tan(1/4*exp(
1))+A*sqrt(c*tan(1/2*exp(1))^2+c)*(-188743680*tan(1/2*exp(1))^5-566231040*t
an(1/2*exp(1))^4+629145600*tan(1/2*exp(1))^3+377487360*tan(1/2*exp(1))^2-18
8743680*tan(1/2*exp(1))-62914560)*tan(1/4*exp(1))^4+B*sqrt(c*tan(1/2*exp(1)
)^2+c)*(12582912*tan(1/2*exp(1))^5+37748736*tan(1/2*exp(1))^4-41943040*tan(
1/2*exp(1))^3-25165824*tan(1/2*exp(1))^2+12582912*tan(1/2*exp(1))+4194304)*
tan(1/4*exp(1))^6+B*sqrt(c*tan(1/2*exp(1))^2+c)*(83886080*tan(1/2*exp(1))^6
+251658240*tan(1/2*exp(1))^5-503316480*tan(1/2*exp(1))^4-838860800*tan(1/2*
exp(1))^3+754974720*tan(1/2*exp(1))^2+251658240*tan(1/2*exp(1)))*tan(1/4*ex
p(1))^3+B*sqrt(c*tan(1/2*exp(1))^2+c)*(188743680*tan(1/2*exp(1))^5+56623104
0*tan(1/2*exp(1))^4-629145600*tan(1/2*exp(1))^3-377487360*tan(1/2*exp(1))^2
+188743680*tan(1/2*exp(1))+62914560)*tan(1/4*exp(1))^2+B*sqrt(c*tan(1/2*exp
(1))^2+c)*(-25165824*tan(1/2*exp(1))^6-75497472*tan(1/2*exp(1))^5+150994944
*tan(1/2*exp(1))^4+251658240*tan(1/2*exp(1))^3-226492416*tan(1/2*exp(1))^2-
75497472*tan(1/2*exp(1)))*tan(1/4*exp(1))^5+B*sqrt(c*tan(1/2*exp(1))^2+c)*(
-25165824*tan(1/2*exp(1))^6-75497472*tan(1/2*exp(1))^5+150994944*tan(1/2*ex
p(1))^4+251658240*tan(1/2*exp(1))^3-226492416*tan(1/2*exp(1))^2-75497472*ta
n(1/2*exp(1)))*tan(1/4*exp(1))+B*sqrt(c*tan(1/2*exp(1))^2+c)*(-188743680*ta
n(1/2*exp(1))^5-566231040*tan(1/2*exp(1))^4+629145600*tan(1/2*exp(1))^3+377
487360*tan(1/2*exp(1))^2-188743680*tan(1/2*exp(1))-62914560)*tan(1/4*exp(1)
)^4)*ln(abs(-2*tan(1/2*exp(1))^3+6*tan(1/2*exp(1))^2+(tan(1/2*(1/2*f*x+2*ex
p(1)))-1/tan(1/2*(1/2*f*x+2*exp(1))))*(tan(1/2*exp(1))^3+3*tan(1/2*exp(1))^
2-3*tan(1/2*exp(1))-1)+6*tan(1/2*exp(1))-2))/f/(-8388608*sqrt(2)*c*tan(1/2*
exp(1))^7+8388608*sqrt(2)*c+(-8388608*sqrt(2)*c*tan(1/2*exp(1))^7-25165824*
sqrt(2)*c*tan(1/2*exp(1))^6+8388608*sqrt(2)*c*tan(1/2*exp(1))^5-41943040*sq
rt(2)*c*tan(1/2*exp(1))^4+41943040*sqrt(2)*c*tan(1/2*exp(1))^3-8388608*sqrt
(2)*c*tan(1/2*exp(1))^2+8388608*sqrt(2)*c+25165824*sqrt(2)*c*tan(1/2*exp(1)
))*tan(1/4*exp(1))^6+(-25165824*sqrt(2)*c*tan(1/2*exp(1))^7-75497472*sqrt(2
)*c*tan(1/2*exp(1))^6+25165824*sqrt(2)*c*tan(1/2*exp(1))^5-125829120*sqrt(2
)*c*tan(1/2*exp(1))^4+125829120*sqrt(2)*c*tan(1/2*exp(1))^3-25165824*sqrt(2
)*c*tan(1/2*exp(1))^2+25165824*sqrt(2)*c+75497472*sqrt(2)*c*tan(1/2*exp(1))
)*tan(1/4*exp(1))^2+(-25165824*sqrt(2)*c*tan(1/2*exp(1))^7-75497472*sqrt(2)
*c*tan(1/2*exp(1))^6+25165824*sqrt(2)*c*tan(1/2*exp(1))^5-125829120*sqrt(2)
*c*tan(1/2*exp(1))^4+125829120*sqrt(2)*c*tan(1/2*exp(1))^3-25165824*sqrt(2)
*c*tan(1/2*exp(1))^2+25165824*sqrt(2)*c+75497472*sqrt(2)*c*tan(1/2*exp(1)))
*tan(1/4*exp(1))^4-25165824*sqrt(2)*c*tan(1/2*exp(1))^6+8388608*sqrt(2)*c*t
an(1/2*exp(1))^5-41943040*sqrt(2)*c*tan(1/2*exp(1))^4+41943040*sqrt(2)*c*ta
n(1/2*exp(1))^3-8388608*sqrt(2)*c*tan(1/2*exp(1))^2+25165824*sqrt(2)*c*tan(
1/2*exp(1)))
```

maple [B] time = 0.73, size = 395, normalized size = 3.95

$$-\frac{\left(2A\sin\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-A\sin\left(fx+e\right)\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)+2A\cos\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx\right.}{\sin}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(2*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*sin(f*x+e
)*ln(2/(cos(f*x+e)+1))+2*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))-A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+B*sin(f*x+e)*cos(f*x+e)+2*B*sin(f*x
+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*sin(f*x+e)*ln(2/(cos(f*x+e
)+1))-B*cos(f*x+e)^2+2*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))-B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-2*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))+A*ln(2/(cos(f*x+e)+1))-B*sin(f*x+e)-2*B*ln(-(-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))+B*ln(2/(cos(f*x+e)+1))+B)*(a*(1+sin(f*x+e)))^(1/2)/(1-cos
(f*x+e)+sin(f*x+e))/(-c*(sin(f*x+e)-1))^(1/2)
```

maxima [A] time = 0.55, size = 175, normalized size = 1.75

$$\frac{B\left(\frac{2\sqrt{a}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{\sqrt{c}}-\frac{\sqrt{a}\log\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)}{\sqrt{c}}+\frac{2\sqrt{a}\sqrt{c}\sin\left(fx+e\right)}{\left(c+\frac{c\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)\left(\cos\left(fx+e\right)+1\right)}\right)+A\left(\frac{2\sqrt{a}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{\sqrt{c}}-\frac{\sqrt{a}\log\left(\cdots\right.}{}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] (B*(2*sqrt(a)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(c) - sqrt(a)*lo
g(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(c) + 2*sqrt(a)*sqrt(c)*sin(
f*x + e)/((c + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)))
+ A*(2*sqrt(a)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(c) - sqrt(a)*l
og(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(c)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^
(1/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^
(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(A+B\sin\left(e+fx\right)\right)}{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))/sqrt(-c*(sin(e + f
*x) - 1)), x)
```

3.136 $\int \frac{\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=99

$$\frac{a(A+B)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{aB\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

```
[Out] a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+a*B*cos(
f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2971, 2737, 2667, 31, 2738}

$$\frac{a(A+B)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{aB\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] (a*(A + B)*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
3/2)) + (a*B*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*
x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=(A+B)\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx-\frac{B\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}$$

$$=\frac{a(A+B)\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}-\frac{(aB\cos(e+fx))\,\int}{\sqrt{a+a\sin(e+fx)}\,\sqrt{}}$$

$$=\frac{a(A+B)\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}+\frac{(aB\cos(e+fx))\,\mathrm{Sul}}{cf\sqrt{a+a\sin(e}}$$

$$=\frac{a(A+B)\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}+\frac{aB\cos(e+fx)\log}{cf\sqrt{a+a\sin(e+fx}}$$

Mathematica [C] time = 1.16, size = 147, normalized size = 1.48

$$\frac{\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(A+2B\log\left(i-e^{i(e+fx)}\right)+iB\left(fx+2i\log\left(i-e^{i(e+fx)}\right)\right)\right.}{f(c-c\sin(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*
x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(A + B -
I*B*f*x + 2*B*Log[I - E^(I*(e + f*x))] + I*B*(f*x + (2*I)*Log[I - E^(I*(e +
 f*x))])*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin
[e + f*x])^(3/2))
```

fricas [F] time = 1.17, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{c^2\cos\left(fx+e\right)^2+2\,c^2\sin\left(fx+e\right)-2\,c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.69, size = 403, normalized size = 4.07

$$\frac{\left(B\left(\cos^2\left(fx+e\right)\right)\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)-2B\left(\cos^2\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-B\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)\sin\left(fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/f*(B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-2*B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))-B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)+2*B*
ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)+A*cos(f*x+
```

```
e)^2-A*sin(f*x+e)*cos(f*x+e)+B*cos(f*x+e)^2-B*sin(f*x+e)*cos(f*x+e)+B*cos(f
*x+e)*ln(2/(cos(f*x+e)+1))-2*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))+2*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-4*B*sin(f*x+e)*ln(-(-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))+A*sin(f*x+e)+B*sin(f*x+e)-2*B*ln(2/(cos(f*x+e)
+1))+4*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A-B)*(a*(1+sin(f*x+e)))
^(1/2)/(-1+cos(f*x+e)-sin(f*x+e))/(-c*(sin(f*x+e)-1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) +
c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^
(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\left(A+B\sin\left(e+fx\right)\right)}{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))/(-c*(sin(e + f*x)
- 1))**(3/2), x)
```

3.137 $\int \frac{\sqrt{a+a \sin(e+fx)}\,(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=92

$$\frac{a(A+B)\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}-\frac{aB\cos(e+fx)}{cf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 1/2*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)-a*B*
cos(f*x+e)/c/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2971, 2738}

$$\frac{a(A+B)\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}-\frac{aB\cos(e+fx)}{cf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] (a*(A + B)*Cos[e + f*x])/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])
^(5/2)) - (a*B*Cos[e + f*x])/(c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(3/2))
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = (A+B)\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{5/2}}\,dx - \frac{B\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx}{c}$$
$$= \frac{a(A+B)\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} - \frac{aB\,\mathrm{co}}{cf\sqrt{a+a\sin(e+f}}$$

Mathematica [A] time = 0.54, size = 101, normalized size = 1.10

$$\frac{\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}\,(A+2B\sin(e+fx)-B)}{2c^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*
x])^(5/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[e + f*x])]*(A - B + 2*B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f
*x]])/(2*c^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2]))
```

fricas [A] time = 0.43, size = 87, normalized size = 0.95

$$-\frac{\left(2\,B\sin\left(fx+e\right)+A-B\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{2\left(c^3f\cos\left(fx+e\right)^3+2\,c^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,c^3f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] -1/2*(2*B*sin(f*x + e) + A - B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
e) + c)/(c^3*f*cos(f*x + e)^3 + 2*c^3*f*cos(f*x + e)*sin(f*x + e) - 2*c^3*f
*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.71, size = 137, normalized size = 1.49

$$-\frac{\left(A\left(\cos^2\left(fx+e\right)\right)-A\sin\left(fx+e\right)\cos\left(fx+e\right)-B\left(\cos^2\left(fx+e\right)\right)+B\sin\left(fx+e\right)\cos\left(fx+e\right)+2A\cos\left(\right.\right.}{2f\left(-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)\right)\left(-c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/2/f*(A*cos(f*x+e)^2-A*sin(f*x+e)*cos(f*x+e)-B*cos(f*x+e)^2+B*sin(f*x+e)*
cos(f*x+e)+2*A*cos(f*x+e)+3*A*sin(f*x+e)-B*sin(f*x+e)-3*A+B)*sin(f*x+e)*(a*
(1+sin(f*x+e)))^(1/2)/(-1+cos(f*x+e)-sin(f*x+e))/(-c*(sin(f*x+e)-1))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) +
c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^
(5/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^
(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\left(A+B\sin\left(e+fx\right)\right)}{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))/(-c*(sin(e + f*x)
- 1))**(5/2), x)
```

3.138 $\int \frac{\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=94

$$\frac{a(A+B)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}-\frac{aB\cos(e+fx)}{2cf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 1/3*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2)-1/2*
a*B*cos(f*x+e)/c/f/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2971, 2738}

$$\frac{a(A+B)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}-\frac{aB\cos(e+fx)}{2cf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] (a*(A + B)*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])
^(7/2)) - (a*B*Cos[e + f*x])/(2*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e +
 f*x])^(5/2))
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = (A+B)\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}\,dx - \frac{B\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{5/2}}\,dx}{c}$$
$$= \frac{a(A+B)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}} - \frac{aB\,\mathrm{co}}{2cf\sqrt{a+a\sin(e+}}$$

Mathematica [A] time = 0.60, size = 103, normalized size = 1.10

$$\frac{\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}\,(2A+3B\sin(e+fx)-B)}{6c^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*
x])^(7/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[e + f*x])]*(2*A - B + 3*B*Sin[e + f*x])*Sqrt[c - c*Sin[e +
 f*x]])/(6*c^4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]))
```

fricas [A] time = 0.44, size = 106, normalized size = 1.13

$$-\frac{\left(3\,B\sin\left(fx+e\right)+2\,A-B\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{6\left(3\,c^4f\cos\left(fx+e\right)^3-4\,c^4f\cos\left(fx+e\right)-\left(c^4f\cos\left(fx+e\right)^3-4\,c^4f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="fricas")
```

```
[Out] -1/6*(3*B*sin(f*x + e) + 2*A - B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c)/(3*c^4*f*cos(f*x + e)^3 - 4*c^4*f*cos(f*x + e) - (c^4*f*cos(f*x +
 e)^3 - 4*c^4*f*cos(f*x + e))*sin(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.72, size = 205, normalized size = 2.18

$$-\frac{\sin\left(fx+e\right)\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\ \left(2A\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)+2A\left(\cos^{3}\left(fx+e\right)\right)-B\left(\cos^{2}\left(fx+e\right)\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] -1/6/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)*(2*A*cos(f*x+e)^2*sin(f*x+e)+2*A
*cos(f*x+e)^3-B*cos(f*x+e)^2*sin(f*x+e)-B*cos(f*x+e)^3+6*A*sin(f*x+e)*cos(f
*x+e)-8*A*cos(f*x+e)^2-3*B*sin(f*x+e)*cos(f*x+e)+4*B*cos(f*x+e)^2-14*A*sin(
f*x+e)-8*A*cos(f*x+e)+4*B*sin(f*x+e)+B*cos(f*x+e)+14*A-4*B)/(-c*(sin(f*x+e)
-1))^(7/2)/(1-cos(f*x+e)+sin(f*x+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) +
c)^(7/2), x)
```

mupad [B] time = 17.63, size = 153, normalized size = 1.63

$$-\frac{2\,A\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}-B\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}+3\,B\,\sin\left(e+f\right.}{\frac{9\,c^4\,f\,\cos\left(3\,e+3\,f\,x\right)}{2}+\frac{21\,c^4\,f\,\sin\left(2\,e+2\,f\,x\right)}{2}-\frac{3\,c^4\,f\,\sin\left(4\,e+4\,f\,x\right)}{4}-\frac{21\,c^4\,f\,\cos\left(e+f\right.}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^
(7/2),x)
```

```
[Out] -(2*A*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2) - B*(a + a*sin(
e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2) + 3*B*sin(e + f*x)*(a + a*sin(e
+ f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2))/((9*c^4*f*cos(3*e + 3*f*x))/2 + (
21*c^4*f*sin(2*e + 2*f*x))/2 - (3*c^4*f*sin(4*e + 4*f*x))/4 - (21*c^4*f*cos
(e + f*x))/2)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Timed out
```

3.139 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=146

$$-\frac{a^2(3A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{30f\sqrt{a\sin(e+fx)+a}}-\frac{a(3A-B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}{15f}-$$

```
[Out] -1/6*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/f-1/30*a^2*
(3*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1/2)-1/15*a*(
3*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.36, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2973, 2740, 2738}

$$-\frac{a^2(3A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{30f\sqrt{a\sin(e+fx)+a}}-\frac{a(3A-B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}{15f}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] -(a^2*(3*A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(30*f*Sqrt[a + a*S
in[e + f*x]]) - (a*(3*A - B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*S
in[e + f*x])^(7/2))/(15*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c
- c*Sin[e + f*x])^(7/2))/(6*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
```

```
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\,\mathrm{s}}{6f}$$

$$= -\frac{a(3A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f}$$

$$= -\frac{a^2(3A-B)\cos(e+fx)(c-c\sin(e+fx))}{30f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 1.49, size = 205, normalized size = 1.40

$$-\frac{c^3(\sin(e+fx)-1)^3(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}\,(15(16A-11B)\cos(2(e+fx))+30(2A-B)\cos(4}{960f\left(\cos\left(\frac{1}{2}(e+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(7/2),x]
```

```
[Out] -1/960*(c^3*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*S
in[e + f*x]]*(15*(16*A - 11*B)*Cos[2*(e + f*x)] + 30*(2*A - B)*Cos[4*(e + f
*x)] + 5*B*Cos[6*(e + f*x)] + 840*A*Sin[e + f*x] - 240*B*Sin[e + f*x] + 60*
A*Sin[3*(e + f*x)] + 40*B*Sin[3*(e + f*x)] - 12*A*Sin[5*(e + f*x)] + 24*B*S
in[5*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^3)
```

fricas [A] time = 0.45, size = 148, normalized size = 1.01

$$\frac{\left(5\,Bac^{3}\cos\left(fx+e\right)^{6}+15\,(A-B)ac^{3}\cos\left(fx+e\right)^{4}-5\,(3\,A-2\,B)ac^{3}-2\left(3\,(A-2\,B)ac^{3}\cos\left(fx+e\right)^{4}-2\,(3\,A\right.\right.}{30\,f\cos\left(fx+e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="fricas")
```

```
[Out] 1/30*(5*B*a*c^3*cos(f*x + e)^6 + 15*(A - B)*a*c^3*cos(f*x + e)^4 - 5*(3*A -
 2*B)*a*c^3 - 2*(3*(A - 2*B)*a*c^3*cos(f*x + e)^4 - 2*(3*A - B)*a*c^3*cos(f
*x + e)^2 - 4*(3*A - B)*a*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(
-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
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to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(160*f*(A*a*c^3*sign(sin(1/2
*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-2*B*a*c^3*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(5*f*x+5
*exp(1))/(160*f)^2-64*f*(4*A*a*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(
```

```
cos(1/2*(f*x+exp(1))-1/4*pi))-3*B*a*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*
sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(2*f*x+2*exp(1))/(64*f)^2-256*f*(8*A
*a*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)
)-5*B*a*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/
4*pi)))*cos(4*f*x+4*exp(1))/(256*f)^2+96*f*(-3*A*a*c^3*sign(sin(1/2*(f*x+ex
p(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-2*B*a*c^3*sign(sin(1/2*(f
*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(3*f*x+3*exp(1))
/(96*f)^2+16*f*(-7*A*a*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*
(f*x+exp(1))-1/4*pi))+2*B*a*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos
(1/2*(f*x+exp(1))-1/4*pi)))*sin(f*x+exp(1))/(16*f)^2+128*f*(-8*A*a*c^3*sign
(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+5*B*a*c^3
*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos
(-2*f*x-2*exp(1))/(-128*f)^2-384*B*a*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi
))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(6*f*x+6*exp(1))/(384*f)^2-256*B*a
*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)
)*cos(-4*f*x-4*exp(1))/(-256*f)^2)
```

maple [A] time = 0.84, size = 185, normalized size = 1.27

$$\frac{\left(5B\sin\left(fx+e\right)\left(\cos^4\left(fx+e\right)\right)+6A\left(\cos^4\left(fx+e\right)\right)-12B\left(\cos^4\left(fx+e\right)\right)+15A\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/30/f*(5*B*sin(f*x+e)*cos(f*x+e)^4+6*A*cos(f*x+e)^4-12*B*cos(f*x+e)^4+15*A
*cos(f*x+e)^2*sin(f*x+e)-10*B*cos(f*x+e)^2*sin(f*x+e)-12*A*cos(f*x+e)^2+4*B
*cos(f*x+e)^2+15*A*sin(f*x+e)-10*B*sin(f*x+e)-24*A+8*B)*(-c*(sin(f*x+e)-1))
^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)/(2*sin(f*x+e)+cos(f*x+e)^2-2)/co
s(f*x+e)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
+ c)^(7/2), x)
```

mupad [B] time = 17.53, size = 323, normalized size = 2.21

$$\frac{e^{-e\,6i-f\,x\,6i}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{B\,a\,c^3\,e^{e\,6i+f\,x\,6i}\,\cos\left(6\,e+6\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{96\,f}-\frac{a\,c^3\,e^{e\,6i+f\,x\,6i}\,\sin\left(e+f\,x\right)\,\left(A\,7i-B\,2i\right)\,\sqrt{a+a\,\sin\left(e+\right.}}{4\,f}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(7
/2),x)
```

```
[Out] (exp(- e*6i - f*x*6i)*(c - c*sin(e + f*x))^(1/2)*((B*a*c^3*exp(e*6i + f*x*6
i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*x))^(1/2))/(96*f) - (a*c^3*exp(e*6i +
f*x*6i)*sin(e + f*x)*(A*7i - B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)/(4*f) + (
a*c^3*exp(e*6i + f*x*6i)*cos(4*e + 4*f*x)*(2*A - B)*(a + a*sin(e + f*x))^(1
/2))/(16*f) + (a*c^3*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(16*A - 11*B)*(a +
 a*sin(e + f*x))^(1/2))/(32*f) - (a*c^3*exp(e*6i + f*x*6i)*sin(3*e + 3*f*x)
*(A*3i + B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)/(24*f) + (a*c^3*exp(e*6i + f*
x*6i)*sin(5*e + 5*f*x)*(A*1i - B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)/(40*f))
)/(2*cos(e + f*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Timed out
```

3.140 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=146

$$-\frac{a^2(5A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{30f\sqrt{a\sin(e+fx)+a}}-\frac{a(5A-B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}{20f}-\frac{B$$

```
[Out] -1/5*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/f-1/30*a^2*
(5*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(1/2)-1/20*a*(
5*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.36, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2973, 2740, 2738}

$$-\frac{a^2(5A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{30f\sqrt{a\sin(e+fx)+a}}-\frac{a(5A-B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}{20f}-\frac{B$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] -(a^2*(5*A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(30*f*Sqrt[a + a*S
in[e + f*x]]) - (a*(5*A - B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*S
in[e + f*x])^(5/2))/(20*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c
- c*Sin[e + f*x])^(5/2))/(5*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
```

```
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c}{5f}$$

$$= -\frac{a(5A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx}}{20f}$$

$$= -\frac{a^2(5A-B)\cos(e+fx)(c-c\sin(e+fx)}{30f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 1.61, size = 172, normalized size = 1.18

$$\frac{c^2(\sin(e+fx)-1)^2(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}\,(4(100A-11B)\sin(e+fx)+3\cos(4(e+fx))(5A}{480f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\sin\left(\frac{1}{2}(e+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(5/2),x]
```

```
[Out] (c^2*(-1 + Sin[e + f*x])^2*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e +
f*x]]*(4*(100*A - 11*B)*Sin[e + f*x] + 3*Cos[4*(e + f*x)]*(5*A - 5*B + 4*B*
Sin[e + f*x]) + 4*Cos[2*(e + f*x)]*(15*(A - B) + 4*(5*A + 2*B)*Sin[e + f*x]
)))/(480*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^3)
```

fricas [A] time = 0.45, size = 126, normalized size = 0.86

$$\frac{\left(15\,(A-B)ac^2\cos\left(fx+e\right)^4-15\,(A-B)ac^2+4\left(3\,Bac^2\cos\left(fx+e\right)^4+(5\,A-B)ac^2\cos\left(fx+e\right)^2+2\,(5\,A-\right.\right.}{60\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] 1/60*(15*(A - B)*a*c^2*cos(f*x + e)^4 - 15*(A - B)*a*c^2 + 4*(3*B*a*c^2*cos
(f*x + e)^4 + (5*A - B)*a*c^2*cos(f*x + e)^2 + 2*(5*A - B)*a*c^2)*sin(f*x +
 e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sq
rt(2*c)*(-32*f*(A*a*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*
x+exp(1))-1/4*pi))-B*a*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*
(f*x+exp(1))-1/4*pi)))*cos(2*f*x+2*exp(1))/(32*f)^2-64*f*(A*a*c^2*sign(sin(
1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-B*a*c^2*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(4*f*x+
4*exp(1))/(64*f)^2+32*f*(-A*a*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(c
os(1/2*(f*x+exp(1))-1/4*pi))+B*a*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sig
n(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(-2*f*x-2*exp(1))/(-32*f)^2+96*f*(-4*A*
a*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))
-B*a*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*p
i)))*sin(3*f*x+3*exp(1))/(96*f)^2+16*f*(-6*A*a*c^2*sign(sin(1/2*(f*x+exp(1)
)-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+B*a*c^2*sign(sin(1/2*(f*x+exp
(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(f*x+exp(1))/(16*f)^2-
160*B*a*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-
1/4*pi))*sin(5*f*x+5*exp(1))/(160*f)^2)
```

maple [A] time = 0.79, size = 147, normalized size = 1.01

$$\frac{\left(-12B\left(\cos^4\left(fx+e\right)\right)+15A\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)-15B\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)-20A\left(\cos^2\left(fx+\right.\right.\right.}{60f\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/60/f*(-12*B*cos(f*x+e)^4+15*A*cos(f*x+e)^2*sin(f*x+e)-15*B*cos(f*x+e)^2*s
in(f*x+e)-20*A*cos(f*x+e)^2+4*B*cos(f*x+e)^2+15*A*sin(f*x+e)-15*B*sin(f*x+e
)-40*A+8*B)*(-c*(sin(f*x+e)-1))^(5/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)/(
sin(f*x+e)-1)/cos(f*x+e)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
+ c)^(5/2), x)
```

mupad [B] time = 16.57, size = 174, normalized size = 1.19

$$\frac{a\,c^2\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(60\,A\,\cos\left(e+f\,x\right)-60\,B\,\cos\left(e+f\,x\right)+75\,A\,\cos\left(3\,e+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5
/2),x)
```

```
[Out] (a*c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(60*A*cos
(e + f*x) - 60*B*cos(e + f*x) + 75*A*cos(3*e + 3*f*x) + 15*A*cos(5*e + 5*f*
x) - 75*B*cos(3*e + 3*f*x) - 15*B*cos(5*e + 5*f*x) + 400*A*sin(2*e + 2*f*x)
 + 40*A*sin(4*e + 4*f*x) - 50*B*sin(2*e + 2*f*x) + 16*B*sin(4*e + 4*f*x) +
6*B*sin(6*e + 6*f*x)))/(480*f*(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

3.141 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=134

$$-\frac{a^2A\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3f\sqrt{a\sin(e+fx)+a}}-\frac{aA\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}{3f}-\frac{B\cos(e+fx)(\cdots}{\cdots}$$

```
[Out] -1/4*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/f-1/3*a^2*A
*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)-1/3*a*A*cos(f*x
+e)*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.35, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2973, 2740, 2738}

$$-\frac{a^2A\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3f\sqrt{a\sin(e+fx)+a}}-\frac{aA\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}{3f}-\frac{B\cos(e+fx)(\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] -(a^2*A*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(3*f*Sqrt[a + a*Sin[e + f*
x]]) - (a*A*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2
))/(3*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^
(3/2))/(4*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
```

```
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\,\mathrm{s}}{4f}$$

$$= -\frac{aA\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\,\mathrm{si}}{3f}$$

$$= -\frac{a^2A\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3f\sqrt{a+a\sin(e+fx)}} - \frac{aA}{}$$

Mathematica [A] time = 0.75, size = 96, normalized size = 0.72

$$-\frac{c(\sin(e+fx)-1)\sec^3(e+fx)(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}\,(8A(9\sin(e+fx)+\sin(3(e+fx)))-12}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(3/2),x]
```

```
[Out] -1/96*(c*Sec[e + f*x]^3*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(3/2)*Sq
rt[c - c*Sin[e + f*x]]*(-12*B*Cos[2*(e + f*x)] - 3*B*Cos[4*(e + f*x)] + 8*A
*(9*Sin[e + f*x] + Sin[3*(e + f*x)])))/f
```

fricas [A] time = 0.44, size = 83, normalized size = 0.62

$$-\frac{\left(3\,Bac\cos\left(fx+e\right)^4-3\,Bac-4\left(Aac\cos\left(fx+e\right)^2+2\,Aac\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right.}}{12\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] -1/12*(3*B*a*c*cos(f*x + e)^4 - 3*B*a*c - 4*(A*a*c*cos(f*x + e)^2 + 2*A*a*c
)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f
*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(
2*c)*(-24*A*a*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(
1))-1/4*pi))*sin(f*x+exp(1))/(8*f)^2-24*A*a*c*f*sign(sin(1/2*(f*x+exp(1))-1
/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(3*f*x+3*exp(1))/(24*f)^2+32*
B*a*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi
))*cos(2*f*x+2*exp(1))/(32*f)^2+64*B*a*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi
))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(4*f*x+4*exp(1))/(64*f)^2+32*B*a*c
*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*co
s(-2*f*x-2*exp(1))/(-32*f)^2)
```

maple [A] time = 0.75, size = 86, normalized size = 0.64

$$\frac{\left(3B\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)+4A\left(\cos^{2}\left(fx+e\right)\right)+3B\sin\left(fx+e\right)+8A\right)\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{3}{2}}\sin\left(fx\right.}{12f\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/12/f*(3*B*cos(f*x+e)^2*sin(f*x+e)+4*A*cos(f*x+e)^2+3*B*sin(f*x+e)+8*A)*(-
c*(sin(f*x+e)-1))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)/cos(f*x+e)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
+ c)^(3/2), x)
```

mupad [B] time = 1.84, size = 103, normalized size = 0.77

$$-\frac{a\,c\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(12\,B\,\cos\left(e+f\,x\right)+15\,B\,\cos\left(3\,e+3\,f\,x\right)+3\,B\,\cos\left(5\,e+\right.\right.}{96\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3
/2),x)
```

```
[Out] -(a*c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(12*B*cos(
e + f*x) + 15*B*cos(3*e + 3*f*x) + 3*B*cos(5*e + 5*f*x) - 80*A*sin(2*e + 2*
f*x) - 8*A*sin(4*e + 4*f*x)))/(96*f*(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

3.142 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+f}$

Optimal. Leaf size=96

$$\frac{c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/2*(A-B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(1/2)+1/3*
B*c*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/a/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2971, 2738}

$$\frac{c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] ((A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*Sqrt[c - c*Sin[e +
 f*x]]) + (B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*a*f*Sqrt[c - c*S
in[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{B\int (a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}\,d}{a}$$
$$= \frac{(A-B)c\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} +$$

Mathematica [A] time = 0.56, size = 81, normalized size = 0.84

$$-\frac{a\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(\cos(2(e+fx))(3(A+B)+2B\sin(e+fx))-2(6A+B)\sin}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e
+ f*x]],x]
```

```
[Out] -1/12*(a*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
-2*(6*A + B)*Sin[e + f*x] + Cos[2*(e + f*x)]*(3*(A + B) + 2*B*Sin[e + f*x])
))/f
```

fricas [A] time = 0.45, size = 87, normalized size = 0.91

$$-\frac{\left(3\,(A+B)a\cos\left(fx+e\right)^2-3\,(A+B)a+2\left(Ba\cos\left(fx+e\right)^2-(3\,A+B)a\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{}}{6\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] -1/6*(3*(A + B)*a*cos(f*x + e)^2 - 3*(A + B)*a + 2*(B*a*cos(f*x + e)^2 - (3
*A + B)*a)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)
/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(
2*c)*(-8*f*(-A*a*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1
))-1/4*pi))-B*a*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1)
)-1/4*pi)))*cos(2*f*x+2*exp(1))/(8*f)^2+8*f*(-4*A*a*sign(sin(1/2*(f*x+exp(1
))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-B*a*sign(sin(1/2*(f*x+exp(1)
)-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(f*x+exp(1))/(8*f)^2+24*B
*a*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*
sin(3*f*x+3*exp(1))/(24*f)^2)
```

maple [A] time = 0.80, size = 91, normalized size = 0.95

$$\frac{\left(-2B\left(\cos^{2}\left(fx+e\right)\right)+3A\sin\left(fx+e\right)+3B\sin\left(fx+e\right)+6A+2B\right)\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\ \sin\left(fx+e\right)\left(a\left(1\right.\right.}{6f\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/6/f*(-2*B*cos(f*x+e)^2+3*A*sin(f*x+e)+3*B*sin(f*x+e)+6*A+2*B)*(-c*(sin(f*
x+e)-1))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)/(1+sin(f*x+e))/cos(f*x+e
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x +
 e) + c), x)
```

mupad [B] time = 14.29, size = 122, normalized size = 1.27

$$-\frac{a\sqrt{a\left(\sin\left(e+f\,x\right)+1\right)}\ \sqrt{-c\left(\sin\left(e+f\,x\right)-1\right)}\ \left(3\,A\,\cos\left(e+f\,x\right)+3\,B\,\cos\left(e+f\,x\right)+3\,A\,\cos\left(3\,e+3\,f\,x\right.\right.}{12\,f\left(\cos\left(2\,e+2\,f\,x\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1
/2),x)
```

```
[Out] -(a*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(3*A*cos(e +
 f*x) + 3*B*cos(e + f*x) + 3*A*cos(3*e + 3*f*x) + 3*B*cos(3*e + 3*f*x) - 12
*A*sin(2*e + 2*f*x) - 2*B*sin(2*e + 2*f*x) + B*sin(4*e + 4*f*x)))/(12*f*(co
s(2*e + 2*f*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*sqrt(-c*(sin(e + f*x) - 1))*(A + B*s
in(e + f*x)), x)
```

3.143 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=145

$$-\frac{2a^2(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a(A+B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}-\frac{B\cos(e+fx)(a\sin(e}{2f\sqrt{c-c\sin(e}$$

```
[Out] -1/2*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(1/2)-2*a^2*(A+
B)*cos(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1
/2)-a*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2973, 2740, 2737, 2667, 31}

$$-\frac{2a^2(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a(A+B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}-\frac{B\cos(e+fx)(a\sin(e}{2f\sqrt{c-c\sin(e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (-2*a^2*(A + B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[c - c*Sin[e + f*x]]) - (a*(A + B)*Cos[e + f*x]*Sqrt[a + a*Sin[e +
 f*x]])/(f*Sqrt[c - c*Sin[e + f*x]]) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])
^(3/2))/(2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
```

```
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} + (A+B)\int \frac{(a+a\sin(e}{\sqrt{c-c\sin(}}$$

$$= -\frac{a(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)(a+}{2f\sqrt{c-c\,\mathrm{si}}}$$

$$= -\frac{a(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)(a+}{2f\sqrt{c-c\,\mathrm{si}}}$$

$$= -\frac{a(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)(a+}{2f\sqrt{c-c\,\mathrm{si}}}$$

$$= -\frac{2a^2(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{a(A+B)\cos(}{f\sqrt{c}}$$

Mathematica [A] time = 0.69, size = 136, normalized size = 0.94

$$-\frac{(a(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(4(A+2B)\sin(e+fx)+16(A+B)\log\left(\cos\left(\frac{1}{2}(e-\right.\right.\right.}{4f\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[
e + f*x]],x]
```

```
[Out] -1/4*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2)*(-
(B*Cos[2*(e + f*x)]) + 16*(A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]
+ 4*(A + 2*B)*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqr
t[c - c*Sin[e + f*x]])
```

fricas [F] time = 2.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Ba\cos\left(fx+e\right)^2-(A+B)a\sin\left(fx+e\right)-(A+B)a\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{c\sin\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*s
in(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e) - c), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (8*pi/x/2)>(-8*pi/x/2)4*sqrt(2*a)*(A*a*sqrt(c*tan(1/2*exp(
1))^2+c)*(-27021597764222976*tan(1/2*exp(1))^5-81064793292668928*tan(1/2*ex
p(1))^4+90071992547409920*tan(1/2*exp(1))^3+54043195528445952*tan(1/2*exp(1
))^2-27021597764222976*tan(1/2*exp(1))-9007199254740992)+B*a*sqrt(c*tan(1/2
*exp(1))^2+c)*(-27021597764222976*tan(1/2*exp(1))^5-81064793292668928*tan(1
```

/2*exp(1))^4+90071992547409920*tan(1/2*exp(1))^3+54043195528445952*tan(1/2*
exp(1))^2-27021597764222976*tan(1/2*exp(1))-9007199254740992)+A*a*sqrt(c*ta
n(1/2*exp(1))^2+c)*(27021597764222976*tan(1/2*exp(1))^5+81064793292668928*t
an(1/2*exp(1))^4-90071992547409920*tan(1/2*exp(1))^3-54043195528445952*tan(
1/2*exp(1))^2+27021597764222976*tan(1/2*exp(1))+9007199254740992)*tan(1/4*e
xp(1))^6+A*a*sqrt(c*tan(1/2*exp(1))^2+c)*(180143985094819840*tan(1/2*exp(1)
)^6+540431955284459520*tan(1/2*exp(1))^5-1080863910568919040*tan(1/2*exp(1)
)^4-1801439850948198400*tan(1/2*exp(1))^3+1621295865853378560*tan(1/2*exp(1
))^2+540431955284459520*tan(1/2*exp(1)))*tan(1/4*exp(1))^3+A*a*sqrt(c*tan(1
/2*exp(1))^2+c)*(405323966463344640*tan(1/2*exp(1))^5+1215971899390033920*t
an(1/2*exp(1))^4-1351079888211148800*tan(1/2*exp(1))^3-810647932926689280*t
an(1/2*exp(1))^2+405323966463344640*tan(1/2*exp(1))+135107988821114880)*tan
(1/4*exp(1))^2+A*a*sqrt(c*tan(1/2*exp(1))^2+c)*(-54043195528445952*tan(1/2*
exp(1))^6-162129586585337856*tan(1/2*exp(1))^5+324259173170675712*tan(1/2*e
xp(1))^4+540431955284459520*tan(1/2*exp(1))^3-486388759756013568*tan(1/2*ex
p(1))^2-162129586585337856*tan(1/2*exp(1)))*tan(1/4*exp(1))^5+A*a*sqrt(c*ta
n(1/2*exp(1))^2+c)*(-54043195528445952*tan(1/2*exp(1))^6-162129586585337856
*tan(1/2*exp(1))^5+324259173170675712*tan(1/2*exp(1))^4+540431955284459520*
tan(1/2*exp(1))^3-486388759756013568*tan(1/2*exp(1))^2-162129586585337856*t
an(1/2*exp(1)))*tan(1/4*exp(1))+A*a*sqrt(c*tan(1/2*exp(1))^2+c)*(-405323966
463344640*tan(1/2*exp(1))^5-1215971899390033920*tan(1/2*exp(1))^4+135107988
8211148800*tan(1/2*exp(1))^3+810647932926689280*tan(1/2*exp(1))^2-405323966
463344640*tan(1/2*exp(1))-135107988821114880)*tan(1/4*exp(1))^4+B*a*sqrt(c*
tan(1/2*exp(1))^2+c)*(27021597764222976*tan(1/2*exp(1))^5+81064793292668928
*tan(1/2*exp(1))^4-90071992547409920*tan(1/2*exp(1))^3-54043195528445952*ta
n(1/2*exp(1))^2+27021597764222976*tan(1/2*exp(1))+9007199254740992)*tan(1/4
*exp(1))^6+B*a*sqrt(c*tan(1/2*exp(1))^2+c)*(180143985094819840*tan(1/2*exp(
1))^6+540431955284459520*tan(1/2*exp(1))^5-1080863910568919040*tan(1/2*exp(
1))^4-1801439850948198400*tan(1/2*exp(1))^3+1621295865853378560*tan(1/2*exp
(1))^2+540431955284459520*tan(1/2*exp(1)))*tan(1/4*exp(1))^3+B*a*sqrt(c*tan
(1/2*exp(1))^2+c)*(405323966463344640*tan(1/2*exp(1))^5+1215971899390033920
*tan(1/2*exp(1))^4-1351079888211148800*tan(1/2*exp(1))^3-810647932926689280
*tan(1/2*exp(1))^2+405323966463344640*tan(1/2*exp(1))+135107988821114880)*t
an(1/4*exp(1))^2+B*a*sqrt(c*tan(1/2*exp(1))^2+c)*(-54043195528445952*tan(1/
2*exp(1))^6-162129586585337856*tan(1/2*exp(1))^5+324259173170675712*tan(1/2
*exp(1))^4+540431955284459520*tan(1/2*exp(1))^3-486388759756013568*tan(1/2*
exp(1))^2-162129586585337856*tan(1/2*exp(1)))*tan(1/4*exp(1))^5+B*a*sqrt(c*
tan(1/2*exp(1))^2+c)*(-54043195528445952*tan(1/2*exp(1))^6-1621295865853378
56*tan(1/2*exp(1))^5+324259173170675712*tan(1/2*exp(1))^4+54043195528445952
0*tan(1/2*exp(1))^3-486388759756013568*tan(1/2*exp(1))^2-162129586585337856
*tan(1/2*exp(1)))*tan(1/4*exp(1))+B*a*sqrt(c*tan(1/2*exp(1))^2+c)*(-4053239
66463344640*tan(1/2*exp(1))^5-1215971899390033920*tan(1/2*exp(1))^4+1351079
888211148800*tan(1/2*exp(1))^3+810647932926689280*tan(1/2*exp(1))^2-4053239
66463344640*tan(1/2*exp(1))-135107988821114880)*tan(1/4*exp(1))^4)*ln(abs(-
2*tan(1/2*exp(1))^3+6*tan(1/2*exp(1))^2+(tan(1/2*(1/2*f*x+2*exp(1)))-1/tan(

```
1/2*(1/2*f*x+2*exp(1))))*(tan(1/2*exp(1))^3+3*tan(1/2*exp(1))^2-3*tan(1/2*e
xp(1))-1)+6*tan(1/2*exp(1))-2))/f/(-9007199254740992*sqrt(2)*c*tan(1/2*exp(
1))^7+9007199254740992*sqrt(2)*c+(-9007199254740992*sqrt(2)*c*tan(1/2*exp(1
))^7-27021597764222976*sqrt(2)*c*tan(1/2*exp(1))^6+9007199254740992*sqrt(2)
*c*tan(1/2*exp(1))^5-45035996273704960*sqrt(2)*c*tan(1/2*exp(1))^4+45035996
273704960*sqrt(2)*c*tan(1/2*exp(1))^3-9007199254740992*sqrt(2)*c*tan(1/2*ex
p(1))^2+9007199254740992*sqrt(2)*c+27021597764222976*sqrt(2)*c*tan(1/2*exp(
1)))*tan(1/4*exp(1))^6+(-27021597764222976*sqrt(2)*c*tan(1/2*exp(1))^7-8106
4793292668928*sqrt(2)*c*tan(1/2*exp(1))^6+27021597764222976*sqrt(2)*c*tan(1
/2*exp(1))^5-135107988821114880*sqrt(2)*c*tan(1/2*exp(1))^4+135107988821114
880*sqrt(2)*c*tan(1/2*exp(1))^3-27021597764222976*sqrt(2)*c*tan(1/2*exp(1))
^2+27021597764222976*sqrt(2)*c+81064793292668928*sqrt(2)*c*tan(1/2*exp(1)))
*tan(1/4*exp(1))^2+(-27021597764222976*sqrt(2)*c*tan(1/2*exp(1))^7-81064793
292668928*sqrt(2)*c*tan(1/2*exp(1))^6+27021597764222976*sqrt(2)*c*tan(1/2*e
xp(1))^5-135107988821114880*sqrt(2)*c*tan(1/2*exp(1))^4+135107988821114880*
sqrt(2)*c*tan(1/2*exp(1))^3-27021597764222976*sqrt(2)*c*tan(1/2*exp(1))^2+2
7021597764222976*sqrt(2)*c+81064793292668928*sqrt(2)*c*tan(1/2*exp(1)))*tan
(1/4*exp(1))^4-27021597764222976*sqrt(2)*c*tan(1/2*exp(1))^6+90071992547409
92*sqrt(2)*c*tan(1/2*exp(1))^5-45035996273704960*sqrt(2)*c*tan(1/2*exp(1))^
4+45035996273704960*sqrt(2)*c*tan(1/2*exp(1))^3-9007199254740992*sqrt(2)*c*
tan(1/2*exp(1))^2+27021597764222976*sqrt(2)*c*tan(1/2*exp(1)))
```

maple [B] time = 0.70, size = 494, normalized size = 3.41

$$-\frac{\left(B\left(\cos^3\left(fx+e\right)\right)+B\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+2A\left(\cos^2\left(fx+e\right)\right)-2A\sin\left(fx+e\right)\cos\left(fx+e\right)-8A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*(B*cos(f*x+e)^3+B*cos(f*x+e)^2*sin(f*x+e)+2*A*cos(f*x+e)^2-2*A*sin(f
*x+e)*cos(f*x+e)-8*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+
4*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-8*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))+4*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+3*B*cos(f*x+e)^2-4*B
*sin(f*x+e)*cos(f*x+e)-8*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))+4*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-8*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))+4*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+2*A*sin(f*x+e)
+8*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-4*A*ln(2/(cos(f*x+e)+1))-B*
cos(f*x+e)+3*B*sin(f*x+e)+8*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-4*
B*ln(2/(cos(f*x+e)+1))-2*A-3*B)*(a*(1+sin(f*x+e)))^(3/2)/(cos(f*x+e)^2+sin(
f*x+e)*cos(f*x+e)+cos(f*x+e)-2*sin(f*x+e)-2)/(-c*(sin(f*x+e)-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/sqrt(-c*sin(f*x +
 e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(1/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(A+B\sin\left(e+fx\right)\right)}{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))/sqrt(-c*(sin(e
+ f*x) - 1)), x)
```

3.144 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=158

$$\frac{a^2(A+3B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{a(A+3B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2cf\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a}{2f(c-c\sin(}$$

```
[Out] 1/2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(3/2)+a^2*(A
+3*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e
))^(1/2)+1/2*a*(A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e
))^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2972, 2740, 2737, 2667, 31}

$$\frac{a^2(A+3B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{a(A+3B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2cf\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a}{2f(c-c\sin(}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*(c - c*Sin[e + f*x])
^(3/2)) + (a^2*(A + 3*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a +
a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (a*(A + 3*B)*Cos[e + f*x]*Sqrt[
a + a*Sin[e + f*x]])/(2*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(A+3B)\int \frac{(a+a\sin}{\sqrt{c-c}}}{2c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a(A+3B)\cos(e}{2cf\sqrt{c}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a(A+3B)\cos(e}{2cf\sqrt{c}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a(A+3B)\cos(e}{2cf\sqrt{c}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a^2(A+3B)\cos(e}{cf\sqrt{a+a\sin(e}}$$

Mathematica [A] time = 0.86, size = 210, normalized size = 1.33

$$-\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-2\sin(e+fx)\left(2(A+3B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\mathrm{s}\right.\right.\right.}{2cf(\sin(e+fx)-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(3/2),x]
```

```
[Out] -1/2*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(4
*A + 3*B + B*Cos[2*(e + f*x)] + 4*A*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
] + 12*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 2*(-B + 2*(A + 3*B)*Log
[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])*Sin[e + f*x]))/(c*f*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 6.01, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(Ba\cos\left(fx+e\right)^2-(A+B)a\sin\left(fx+e\right)-(A+B)a\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{c^2\cos\left(fx+e\right)^2+2\,c^2\sin\left(fx+e\right)-2\,c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*s
in(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(
f*x + e) - 2*c^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.67, size = 748, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/f*(-6*B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*A*sin(f
*x+e)-2*A*sin(f*x+e)*cos(f*x+e)-4*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))+3*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-B*cos(f*x+e)^2*sin(f*x
+e)-2*A-4*B+A*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-2*A*cos(f*x+e)^2*ln(-(-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-2*A*co
s(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+A*cos(f*x+e)*ln(2/(cos(
f*x+e)+1))-3*B*sin(f*x+e)*cos(f*x+e)-12*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))+6*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-6*B*cos(f*x+e)*ln(
-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+3*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))
-A*cos(f*x+e)*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+2*A*cos(f*x+e)*sin(f*x+e)*ln(
-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-3*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)
*cos(f*x+e)+6*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f
*x+e)+4*B*cos(f*x+e)^2-B*cos(f*x+e)^3+4*B*sin(f*x+e)+B*cos(f*x+e)+2*A*cos(f
*x+e)^2+4*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*A*ln(2/(cos(f*x+e)
+1))+12*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-6*B*ln(2/(cos(f*x+e)+1
)))*(a*(1+sin(f*x+e)))^(3/2)/(cos(f*x+e)^2+sin(f*x+e)*cos(f*x+e)+cos(f*x+e)
-2*sin(f*x+e)-2)/(-c*(sin(f*x+e)-1))^(3/2)
```

maxima [B] time = 0.57, size = 366, normalized size = 2.32

$$-\frac{B\left(\frac{6a^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{\frac{3}{2}}}-\frac{3a^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{c^{\frac{3}{2}}}+\frac{2\left(\frac{3a^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}-\frac{2a^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3a^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{c^{\frac{3}{2}}-\frac{2c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{2c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{2c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}\right)+A\left(\frac{2a^{\frac{3}{2}}\log\cdots}{\cdots}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] -(B*(6*a^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(3/2) - 3*a^(3/2)
*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(3/2) + 2*(3*a^(3/2)*sin(f*
x + e)/(cos(f*x + e) + 1) - 2*a^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 3*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(c^(3/2) - 2*c^(3/2)*sin(f*
x + e)/(cos(f*x + e) + 1) + 2*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 -
 2*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + c^(3/2)*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4)) + A*(2*a^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)
/c^(3/2) - a^(3/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(3/2) + 4
*a^(3/2)*sqrt(c)*sin(f*x + e)/((c^2 - 2*c^2*sin(f*x + e)/(cos(f*x + e) + 1)
 + c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(A+B\sin\left(e+fx\right)\right)}{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))/(-c*(sin(e + f*
x) - 1))**(3/2), x)
```

3.145 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{a^2B\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{aB\cos(e+fx)\sqrt{a\sin(e}{cf(c-c\sin(e+f}$$

```
[Out] 1/4*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(5/2)-a*B*co
s(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(3/2)-a^2*B*cos(f*x+e)
*ln(1-sin(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2972, 2739, 2737, 2667, 31}

$$-\frac{a^2B\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{aB\cos(e+fx)\sqrt{a\sin(e}{cf(c-c\sin(e+f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*f*(c - c*Sin[e + f*x])
^(5/2)) - (a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f*(c - c*Sin[e + f
*x])^(3/2)) - (a^2*B*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
```

```
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{B\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}}{c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{aB\cos(e+fx)\sqrt{a}}{cf(c-c\sin(e}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{aB\cos(e+fx)\sqrt{a}}{cf(c-c\sin(e}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{aB\cos(e+fx)\sqrt{a}}{cf(c-c\sin(e}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{aB\cos(e+fx)\sqrt{a}}{cf(c-c\sin(e}$$

Mathematica [A] time = 0.94, size = 198, normalized size = 1.33

$$\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin(e+fx)\left(A+4B\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e\right.\right.\right.\right.}{c^2f(\sin(e+fx)-1)^2\sqrt{c-c\sin}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(5/2),x]
```

```
[Out] (a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(B*Cos[
2*(e + f*x)]*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - B*(2 + 3*Log[Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2]]) + (A + 3*B + 4*B*Log[Cos[(e + f*x)/2] - Sin
[(e + f*x)/2]])*Sin[e + f*x]))/(c^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 2.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Ba\cos\left(fx+e\right)^2-(A+B)a\sin\left(fx+e\right)-(A+B)a\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{3c^3\cos\left(fx+e\right)^2-4c^3-\left(c^3\cos\left(fx+e\right)^2-4c^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*s
in(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3 -
(c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.66, size = 594, normalized size = 3.99

$$\frac{\left(A\sin\left(fx+e\right)-A\sin\left(fx+e\right)\cos\left(fx+e\right)+2B\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)-A+3B+B\sin\left(fx+e\right)\cos\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/f*(6*B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+A*sin(f*x+
e)-A*sin(f*x+e)*cos(f*x+e)-3*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+2*B*cos(f*
x+e)^2*sin(f*x+e)-A+3*B-2*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(
f*x+e)^3+B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^3+B*sin(f*x+e)*cos(f*x+e)+8*B*si
n(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-4*B*sin(f*x+e)*ln(2/(co
s(f*x+e)+1))+4*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*B*
cos(f*x+e)*ln(2/(cos(f*x+e)+1))-2*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))*cos(f*x+e)^2*sin(f*x+e)+B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^2*sin(f*x+e)+
2*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-4*B*ln(-(-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)-3*B*cos(f*x+e)^2+2*B*cos(f*x+e)^3-
3*B*sin(f*x+e)-2*B*cos(f*x+e)+A*cos(f*x+e)^2-8*B*ln(-(-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))+4*B*ln(2/(cos(f*x+e)+1)))*(a*(1+sin(f*x+e)))^(3/2)/(cos(f*
x+e)^2+sin(f*x+e)*cos(f*x+e)+cos(f*x+e)-2*sin(f*x+e)-2)/(-c*(sin(f*x+e)-1))
^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
+ c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(5/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

3.146 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=96

$$\frac{(A-5B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{24cf(c-c\sin(e+fx))^{5/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

```
[Out] 1/6*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(7/2)+1/24*(
A-5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(5/2)
```

Rubi [A] time = 0.27, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2972, 2742}

$$\frac{(A-5B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{24cf(c-c\sin(e+fx))^{5/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(6*f*(c - c*Sin[e + f*x])
^(7/2)) + ((A - 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(24*c*f*(c -
c*Sin[e + f*x])^(5/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{7/2}} + \frac{(A-5B)\int \frac{(a+a\sin}{(c-c\sin}}{6c}$$
$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{7/2}} + \frac{(A-5B)\cos(e+}{24cf(c-}$$

Mathematica [A] time = 0.99, size = 125, normalized size = 1.30

$$-\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(3(A-B)\sin(e+fx)+A-3B\cos(2(e+fx))+4B}{6c^3f(\sin(e+fx)-1)^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(7/2),x]
```

```
[Out] -1/6*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(A
 + 4*B - 3*B*Cos[2*(e + f*x)] + 3*(A - B)*Sin[e + f*x]))/(c^3*f*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.45, size = 125, normalized size = 1.30

$$\frac{\left(6\,Ba\cos\left(fx+e\right)^2-3\,(A-B)a\sin\left(fx+e\right)-(A+7\,B)a\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{6\left(3\,c^4f\cos\left(fx+e\right)^3-4\,c^4f\cos\left(fx+e\right)-\left(c^4f\cos\left(fx+e\right)^3-4\,c^4f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="fricas")
```

```
[Out] 1/6*(6*B*a*cos(f*x + e)^2 - 3*(A - B)*a*sin(f*x + e) - (A + 7*B)*a)*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^4*f*cos(f*x + e)^3 - 4*c^4
*f*cos(f*x + e) - (c^4*f*cos(f*x + e)^3 - 4*c^4*f*cos(f*x + e))*sin(f*x + e
))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.66, size = 223, normalized size = 2.32

$$\frac{\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\left(A\left(\cos^3\left(fx+e\right)\right)+A\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+B\left(\cos^3\left(fx+e\right)\right)+B\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/6/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)*(A*cos(f*x+e)^3+A*cos(f*x+e)^2*si
n(f*x+e)+B*cos(f*x+e)^3+B*cos(f*x+e)^2*sin(f*x+e)-4*A*cos(f*x+e)^2+3*A*sin(
f*x+e)*cos(f*x+e)+2*B*cos(f*x+e)^2-3*B*sin(f*x+e)*cos(f*x+e)-7*A*cos(f*x+e)
-10*A*sin(f*x+e)-B*cos(f*x+e)+2*B*sin(f*x+e)+10*A-2*B)/(-c*(sin(f*x+e)-1))^
(7/2)/(cos(f*x+e)^2+sin(f*x+e)*cos(f*x+e)+cos(f*x+e)-2*sin(f*x+e)-2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
+ c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(7/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Timed out
```

3.147 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=146

$$\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{96c^2f(c-c\sin(e+fx))^{5/2}}+\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{24cf(c-c\sin(e+fx))^{7/2}}+\frac{(A+B)\cos(e+fx)(a\sin}{8f(c-c\sin(e+}$$

```
[Out] 1/8*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(9/2)+1/24*(
A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(7/2)+1/96*(A
-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(5/2)
```

Rubi [A] time = 0.38, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2972, 2743, 2742}

$$\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{96c^2f(c-c\sin(e+fx))^{5/2}}+\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{24cf(c-c\sin(e+fx))^{7/2}}+\frac{(A+B)\cos(e+fx)(a\sin}{8f(c-c\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(9/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(8*f*(c - c*Sin[e + f*x])
^(9/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(24*c*f*(c -
c*Sin[e + f*x])^(7/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)
)/(96*c^2*f*(c - c*Sin[e + f*x])^(5/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
```

```
SumSimplerQ[n, 1])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{(A-3B)\int \frac{(a+a\sin}{(c-c\sin}}{4c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{(A-3B)\cos(e+}{24cf(c-}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{(A-3B)\cos(e+}{24cf(c-}$$

Mathematica [A] time = 1.35, size = 123, normalized size = 0.84

$$\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(4A\sin(e+fx)+2A-3B\cos(2(e+fx))+3B)}{12c^4f(\sin(e+fx)-1)^4\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(9/2),x]
```

```
[Out] (a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(2*A +
3*B - 3*B*Cos[2*(e + f*x)] + 4*A*Sin[e + f*x]))/(12*c^4*f*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^4*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.47, size = 134, normalized size = 0.92

$$-\frac{\left(3\,Ba\cos\left(fx+e\right)^{2}-2\,Aa\sin\left(fx+e\right)-(A+3\,B)a\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{6\left(c^{5}f\cos\left(fx+e\right)^{5}-8\,c^{5}f\cos\left(fx+e\right)^{3}+8\,c^{5}f\cos\left(fx+e\right)+4\left(c^{5}f\cos\left(fx+e\right)^{3}-2\,c^{5}f\cos\left(fx+e\right)\right)\sin\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="fricas")
```

```
[Out] -1/6*(3*B*a*cos(f*x + e)^2 - 2*A*a*sin(f*x + e) - (A + 3*B)*a)*sqrt(a*sin(f
*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^5*f*cos(f*x + e)^5 - 8*c^5*f*cos(
f*x + e)^3 + 8*c^5*f*cos(f*x + e) + 4*(c^5*f*cos(f*x + e)^3 - 2*c^5*f*cos(f
*x + e))*sin(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.68, size = 217, normalized size = 1.49

$$\frac{\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\left(A\left(\cos^{4}\left(fx+e\right)\right)-A\left(\cos^{3}\left(fx+e\right)\right)\sin\left(fx+e\right)+4A\left(\cos^{3}\left(fx+e\right)\right)+5\right.}{6f\left(-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] 1/6/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)*(A*cos(f*x+e)^4-A*cos(f*x+e)^3*si
n(f*x+e)+4*A*cos(f*x+e)^3+5*A*cos(f*x+e)^2*sin(f*x+e)-12*A*cos(f*x+e)^2+7*A
*sin(f*x+e)*cos(f*x+e)+3*B*cos(f*x+e)^2-3*B*sin(f*x+e)*cos(f*x+e)-10*A*cos(
f*x+e)-17*A*sin(f*x+e)+3*B*sin(f*x+e)+17*A-3*B)/(-c*(sin(f*x+e)-1))^(9/2)/(
cos(f*x+e)^2+sin(f*x+e)*cos(f*x+e)+cos(f*x+e)-2*sin(f*x+e)-2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
+ c)^(9/2), x)
```

mupad [B] time = 18.99, size = 245, normalized size = 1.68

$$\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{8\,a\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\left(2\,A+3\,B\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{3\,c^5\,f}+\frac{32\,A\,a\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{3\,c^5\,f}-\frac{8\,B\,a\,\mathrm{e}^{e\,5\mathrm{i}+}}{}\right.}{84\,\cos\left(e+f\,x\right)\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}-54\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)+2\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)-96\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(2\,e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(9/2),x)
```

```
[Out] ((c - c*sin(e + f*x))^(1/2)*((8*a*exp(e*5i + f*x*5i)*(2*A + 3*B)*(a + a*sin
(e + f*x))^(1/2))/(3*c^5*f) + (32*A*a*exp(e*5i + f*x*5i)*sin(e + f*x)*(a +
a*sin(e + f*x))^(1/2))/(3*c^5*f) - (8*B*a*exp(e*5i + f*x*5i)*cos(2*e + 2*f*
x)*(a + a*sin(e + f*x))^(1/2))/(c^5*f)))/(84*cos(e + f*x)*exp(e*5i + f*x*5i
) - 54*exp(e*5i + f*x*5i)*cos(3*e + 3*f*x) + 2*exp(e*5i + f*x*5i)*cos(5*e +
 5*f*x) - 96*exp(e*5i + f*x*5i)*sin(2*e + 2*f*x) + 16*exp(e*5i + f*x*5i)*si
n(4*e + 4*f*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),
x)
```

```
[Out] Timed out
```

3.148 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=154

$$-\frac{a^2(3A-7B)\cos(e+fx)}{120c^2f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}+\frac{a(3A-7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{40cf(c-c\sin(e+fx))^{9/2}}+\frac{(A+B)\cos(e+f}{10f(c-c}$$

```
[Out] 1/10*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(11/2)-1/12
0*a^2*(3*A-7*B)*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1
/2)+1/40*a*(3*A-7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))
^(9/2)
```

Rubi [A] time = 0.37, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2972, 2739, 2738}

$$-\frac{a^2(3A-7B)\cos(e+fx)}{120c^2f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}+\frac{a(3A-7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{40cf(c-c\sin(e+fx))^{9/2}}+\frac{(A+B)\cos(e+f}{10f(c-c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(11/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(10*f*(c - c*Sin[e + f*x]
)^(11/2)) + (a*(3*A - 7*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(40*c*f*(
c - c*Sin[e + f*x])^(9/2)) - (a^2*(3*A - 7*B)*Cos[e + f*x])/(120*c^2*f*Sqrt
[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2))
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
```

```
 + 1, 0])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(3A-7B)\int \frac{(a+a}{(c-c}}{10c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{a(3A-7B)\cos(e}{40cf(c-}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{a(3A-7B)\cos(e}{40cf(c-}$$

Mathematica [A] time = 1.95, size = 126, normalized size = 0.82

$$-\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(5(3A+B)\sin(e+fx)+9(A+B)-10B\cos(2(e+}{60c^5f(\sin(e+fx)-1)^5\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(11/2),x]

[Out] -1/60*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(
9*(A + B) - 10*B*Cos[2*(e + f*x)] + 5*(3*A + B)*Sin[e + f*x]))/(c^5*f*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^5*Sqrt[c - c*Sin[e + f
*x]])
```

fricas [A] time = 0.46, size = 155, normalized size = 1.01

$$-\frac{\left(20\,Ba\cos\left(fx+e\right)^{2}-5\,(3\,A+B)a\sin\left(fx+e\right)-(9\,A+19\,B)a\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin}}{60\left(5\,c^{6}f\cos\left(fx+e\right)^{5}-20\,c^{6}f\cos\left(fx+e\right)^{3}+16\,c^{6}f\cos\left(fx+e\right)-\left(c^{6}f\cos\left(fx+e\right)^{5}-12\,c^{6}f\cos\left(fx+e\right)^{3}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="fricas")
```

```
[Out] -1/60*(20*B*a*cos(f*x + e)^2 - 5*(3*A + B)*a*sin(f*x + e) - (9*A + 19*B)*a)
*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^6*f*cos(f*x + e)^5
 - 20*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e) - (c^6*f*cos(f*x + e)^5
- 12*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e))*sin(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.69, size = 339, normalized size = 2.20

$$-\frac{\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\left(9A\left(\cos^{5}\left(fx+e\right)\right)+9A\left(\cos^{4}\left(fx+e\right)\right)\sin\left(fx+e\right)-B\left(\cos^{5}\left(fx+e\right)\right)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x)
```

```
[Out] -1/60/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)*(9*A*cos(f*x+e)^5+9*A*cos(f*x+e
)^4*sin(f*x+e)-B*cos(f*x+e)^5-B*sin(f*x+e)*cos(f*x+e)^4-54*A*cos(f*x+e)^4+4
5*A*cos(f*x+e)^3*sin(f*x+e)+6*B*cos(f*x+e)^4-5*B*cos(f*x+e)^3*sin(f*x+e)-10
8*A*cos(f*x+e)^3-153*A*cos(f*x+e)^2*sin(f*x+e)+12*B*cos(f*x+e)^3+17*B*cos(f
*x+e)^2*sin(f*x+e)+288*A*cos(f*x+e)^2-135*A*sin(f*x+e)*cos(f*x+e)-52*B*cos(
f*x+e)^2+35*B*sin(f*x+e)*cos(f*x+e)+159*A*cos(f*x+e)+294*A*sin(f*x+e)-11*B*
cos(f*x+e)-46*B*sin(f*x+e)-294*A+46*B)/(-c*(sin(f*x+e)-1))^(11/2)/(cos(f*x+
e)^2+sin(f*x+e)*cos(f*x+e)+cos(f*x+e)-2*sin(f*x+e)-2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
+ c)^(11/2), x)
```

mupad [B] time = 20.23, size = 279, normalized size = 1.81

$$\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{a\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\left(A+B\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,48{}\mathrm{i}}{5\,c^6\,f}-\frac{B\,a\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+a\,\sin\left(\right.}}{3\,c^6\,f}}{\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,264{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)\,220{}\mathrm{i}+{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)\,20{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^
(11/2),x)
```

```
[Out] ((c - c*sin(e + f*x))^(1/2)*((a*exp(e*6i + f*x*6i)*(A + B)*(a + a*sin(e + f
*x))^(1/2)*48i)/(5*c^6*f) - (B*a*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(a + a
*sin(e + f*x))^(1/2)*32i)/(3*c^6*f) + (16*a*exp(e*6i + f*x*6i)*sin(e + f*x)
*(A*3i + B*1i)*(a + a*sin(e + f*x))^(1/2))/(3*c^6*f)))/(cos(e + f*x)*exp(e*
6i + f*x*6i)*264i - exp(e*6i + f*x*6i)*cos(3*e + 3*f*x)*220i + exp(e*6i + f
*x*6i)*cos(5*e + 5*f*x)*20i - exp(e*6i + f*x*6i)*sin(2*e + 2*f*x)*330i + ex
p(e*6i + f*x*6i)*sin(4*e + 4*f*x)*88i - exp(e*6i + f*x*6i)*sin(6*e + 6*f*x)
*2i)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(11/2)
,x)
```

```
[Out] Timed out
```

3.149 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=198

$$-\frac{a^3(7A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{105f\sqrt{a\sin(e+fx)+a}}-\frac{2a^2(7A-B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}{105f}-$$

```
[Out] -1/42*a*(7*A-B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/f-
1/7*B*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2)/f-1/105*a^3*
(7*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1/2)-2/105*a^
2*(7*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.48, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2973, 2740, 2738}

$$-\frac{2a^2(7A-B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}{105f}-\frac{a^3(7A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{105f\sqrt{a\sin(e+fx)+a}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] -(a^3*(7*A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(105*f*Sqrt[a + a*
Sin[e + f*x]]) - (2*a^2*(7*A - B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c
- c*Sin[e + f*x])^(7/2))/(105*f) - (a*(7*A - B)*Cos[e + f*x]*(a + a*Sin[e +
 f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2))/(42*f) - (B*Cos[e + f*x]*(a + a*Si
n[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(7/2))/(7*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
```

```
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c}{7f}$$

$$= -\frac{a(7A-B)\cos(e+fx)(a+a\sin(e+fx)}{42f}$$

$$= -\frac{2a^2(7A-B)\cos(e+fx)\sqrt{a+a\sin(e+}}{105f}$$

$$= -\frac{a^3(7A-B)\cos(e+fx)(c-c\sin(e+fx)}{105f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 2.65, size = 223, normalized size = 1.13

$$-\frac{c^3(\sin(e+fx)-1)^3(a(\sin(e+fx)+1))^{5/2}\sqrt{c-c\sin(e+fx)}\,(525(A-B)\cos(2(e+fx))+210(A-B)\cos(4(e}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(7/2),x]
```

```
[Out] -1/6720*(c^3*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*
Sin[e + f*x]]*(525*(A - B)*Cos[2*(e + f*x)] + 210*(A - B)*Cos[4*(e + f*x)]
+ 35*A*Cos[6*(e + f*x)] - 35*B*Cos[6*(e + f*x)] + 4200*A*Sin[e + f*x] - 525
*B*Sin[e + f*x] + 700*A*Sin[3*(e + f*x)] + 35*B*Sin[3*(e + f*x)] + 84*A*Sin
```

```
[5*(e + f*x)] + 63*B*Sin[5*(e + f*x)] + 15*B*Sin[7*(e + f*x)]))/(f*(Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

fricas [A] time = 0.48, size = 160, normalized size = 0.81

$$\frac{\left(35\,(A-B)a^2c^3\cos\left(fx+e\right)^6-35\,(A-B)a^2c^3+2\left(15\,Ba^2c^3\cos\left(fx+e\right)^6+3\,(7\,A-B)a^2c^3\cos\left(fx+e\right)^4+4\,(7\right.\right.}{210\,f\cos\left(fx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="fricas")
```

```
[Out] 1/210*(35*(A - B)*a^2*c^3*cos(f*x + e)^6 - 35*(A - B)*a^2*c^3 + 2*(15*B*a^2
*c^3*cos(f*x + e)^6 + 3*(7*A - B)*a^2*c^3*cos(f*x + e)^4 + 4*(7*A - B)*a^2*
c^3*cos(f*x + e)^2 + 8*(7*A - B)*a^2*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e)
 + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
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(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
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to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
```

```
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)
*sqrt(2*c)*(-64*f*(A*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/
2*(f*x+exp(1))-1/4*pi))-B*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(c
os(1/2*(f*x+exp(1))-1/4*pi)))*cos(2*f*x+2*exp(1))/(64*f)^2-384*f*(A*a^2*c^3
*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-B*a^
2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))
)*cos(6*f*x+6*exp(1))/(384*f)^2-256*f*(3*A*a^2*c^3*sign(sin(1/2*(f*x+exp(1)
)-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-3*B*a^2*c^3*sign(sin(1/2*(f*x
+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(4*f*x+4*exp(1))/(
256*f)^2+256*f*(-A*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*
(f*x+exp(1))-1/4*pi))+B*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos
(1/2*(f*x+exp(1))-1/4*pi)))*cos(-4*f*x-4*exp(1))/(-256*f)^2+128*f*(-3*A*a^2
*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+
3*B*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/
4*pi)))*cos(-2*f*x-2*exp(1))/(-128*f)^2+640*f*(-4*A*a^2*c^3*sign(sin(1/2*(f
*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-3*B*a^2*c^3*sign(sin
(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(5*f*x+5*
exp(1))/(640*f)^2+384*f*(-20*A*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*s
ign(cos(1/2*(f*x+exp(1))-1/4*pi))-B*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*p
i))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(3*f*x+3*exp(1))/(384*f)^2+128*f
*(-40*A*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1)
)-1/4*pi))+5*B*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x
+exp(1))-1/4*pi)))*sin(f*x+exp(1))/(128*f)^2-896*B*a^2*c^3*f*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(7*f*x+7*exp(1))
/(896*f)^2)
```

maple [A] time = 0.90, size = 203, normalized size = 1.03

$$\frac{\left(-30B\left(\cos^{6}\left(fx+e\right)\right)+35A\left(\cos^{4}\left(fx+e\right)\right)\sin\left(fx+e\right)-35B\sin\left(fx+e\right)\left(\cos^{4}\left(fx+e\right)\right)-42A\left(\cos^{4}\left(fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/210/f*(-30*B*cos(f*x+e)^6+35*A*cos(f*x+e)^4*sin(f*x+e)-35*B*sin(f*x+e)*co
s(f*x+e)^4-42*A*cos(f*x+e)^4+6*B*cos(f*x+e)^4+35*A*cos(f*x+e)^2*sin(f*x+e)-
35*B*cos(f*x+e)^2*sin(f*x+e)-56*A*cos(f*x+e)^2+8*B*cos(f*x+e)^2+35*A*sin(f*
x+e)-35*B*sin(f*x+e)-112*A+16*B)*(-c*(sin(f*x+e)-1))^(7/2)*sin(f*x+e)*(a*(1
+sin(f*x+e)))^(5/2)/(sin(f*x+e)-1)/cos(f*x+e)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
+ c)^(7/2), x)
```

mupad [B] time = 18.17, size = 383, normalized size = 1.93

$$\frac{e^{-e\,7\mathrm{i}-f\,x\,7\mathrm{i}}\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(-\frac{a^2\,c^3\,e^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\left(A\,1\mathrm{i}-B\,1\mathrm{i}\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,5\mathrm{i}}{32\,f}-\frac{a^2\,c^3\,e^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos\left(4\,e+4\,f\,x\right)\,\left(A\,1\mathrm{i}-B\right.}{16\,f}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(7
/2),x)
```

```
[Out] (exp(- e*7i - f*x*7i)*(c - c*sin(e + f*x))^(1/2)*((a^2*c^3*exp(e*7i + f*x*7
i)*sin(5*e + 5*f*x)*(4*A + 3*B)*(a + a*sin(e + f*x))^(1/2))/(160*f) - (a^2*
c^3*exp(e*7i + f*x*7i)*cos(4*e + 4*f*x)*(A*1i - B*1i)*(a + a*sin(e + f*x))^
(1/2)*1i)/(16*f) - (a^2*c^3*exp(e*7i + f*x*7i)*cos(6*e + 6*f*x)*(A*1i - B*1
i)*(a + a*sin(e + f*x))^(1/2)*1i)/(96*f) - (a^2*c^3*exp(e*7i + f*x*7i)*cos(
2*e + 2*f*x)*(A*1i - B*1i)*(a + a*sin(e + f*x))^(1/2)*5i)/(32*f) + (5*a^2*c
^3*exp(e*7i + f*x*7i)*sin(e + f*x)*(8*A - B)*(a + a*sin(e + f*x))^(1/2))/(3
2*f) + (a^2*c^3*exp(e*7i + f*x*7i)*sin(3*e + 3*f*x)*(20*A + B)*(a + a*sin(e
 + f*x))^(1/2))/(96*f) + (B*a^2*c^3*exp(e*7i + f*x*7i)*sin(7*e + 7*f*x)*(a
+ a*sin(e + f*x))^(1/2))/(224*f)))/(2*cos(e + f*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Timed out
```

3.150 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=180

$$-\frac{2a^3A\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15f\sqrt{a\sin(e+fx)+a}}-\frac{a^2A\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}{5f}-\frac{aA\cos(e+ \dots}{}$$

```
[Out] -1/5*a*A*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/f-1/6*B*c
os(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2)/f-2/15*a^3*A*cos(f*
x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(1/2)-1/5*a^2*A*cos(f*x+e)*(
c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.47, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2973, 2740, 2738}

$$-\frac{2a^3A\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15f\sqrt{a\sin(e+fx)+a}}-\frac{a^2A\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}{5f}-\frac{aA\cos(e+ \dots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] (-2*a^3*A*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(15*f*Sqrt[a + a*Sin[e +
 f*x]]) - (a^2*A*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])
^(5/2))/(5*f) - (a*A*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
 f*x])^(5/2))/(5*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin
[e + f*x])^(5/2))/(6*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
```

```
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\,\mathrm{s}}{6f}$$

$$= -\frac{aA\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c}{5f}$$

$$= -\frac{a^2A\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\,\mathrm{s}}{5f}$$

$$= -\frac{2a^3A\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15f\sqrt{a+a\sin(e+fx)}} -$$

Mathematica [A] time = 0.80, size = 113, normalized size = 0.63

$$\frac{a^2c^2\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}\,(600A\sin(e+fx)+100A\sin(3(e+fx))+12A\sin(5(e+}{960f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(5/2),x]
```

```
[Out] (a^2*c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
-75*B*Cos[2*(e + f*x)] - 30*B*Cos[4*(e + f*x)] - 5*B*Cos[6*(e + f*x)] + 600
*A*Sin[e + f*x] + 100*A*Sin[3*(e + f*x)] + 12*A*Sin[5*(e + f*x)]))/(960*f)
```

fricas [A] time = 0.46, size = 117, normalized size = 0.65

$$-\frac{\left(5\,Ba^2c^2\cos\left(fx+e\right)^6-5\,Ba^2c^2-2\left(3\,Aa^2c^2\cos\left(fx+e\right)^4+4\,Aa^2c^2\cos\left(fx+e\right)^2+8\,Aa^2c^2\right)\sin\left(fx+e\right)\right)\sqrt{}}{30\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] -1/30*(5*B*a^2*c^2*cos(f*x + e)^6 - 5*B*a^2*c^2 - 2*(3*A*a^2*c^2*cos(f*x +
e)^4 + 4*A*a^2*c^2*cos(f*x + e)^2 + 8*A*a^2*c^2)*sin(f*x + e))*sqrt(a*sin(f
*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-80*A*a^2*c^2*f*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1))/(16
*f)^2-480*A*a^2*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+
exp(1))-1/4*pi))*sin(3*f*x+3*exp(1))/(96*f)^2-160*A*a^2*c^2*f*sign(sin(1/2*
(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(5*f*x+5*exp(1)
)/(160*f)^2+64*B*a^2*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*
(f*x+exp(1))-1/4*pi))*cos(2*f*x+2*exp(1))/(64*f)^2+768*B*a^2*c^2*f*sign(sin
(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(4*f*x+4*e
xp(1))/(256*f)^2+384*B*a^2*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(co
s(1/2*(f*x+exp(1))-1/4*pi))*cos(6*f*x+6*exp(1))/(384*f)^2+384*B*a^2*c^2*f*s
ign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-2
```

```
*f*x-2*exp(1))/(-128*f)^2+256*B*a^2*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi)
)*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-4*f*x-4*exp(1))/(-256*f)^2)
```

maple [A] time = 0.77, size = 114, normalized size = 0.63

$$\frac{\left(5B\sin\left(fx+e\right)\left(\cos^4\left(fx+e\right)\right)+6A\left(\cos^4\left(fx+e\right)\right)+5B\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+8A\left(\cos^2\left(fx+e\right)\right)+5}{30f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/30/f*(5*B*sin(f*x+e)*cos(f*x+e)^4+6*A*cos(f*x+e)^4+5*B*cos(f*x+e)^2*sin(f
*x+e)+8*A*cos(f*x+e)^2+5*B*sin(f*x+e)+16*A)*(-c*(sin(f*x+e)-1))^(5/2)*sin(f
*x+e)*(a*(1+sin(f*x+e)))^(5/2)/cos(f*x+e)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
+ c)^(5/2), x)
```

mupad [B] time = 16.00, size = 131, normalized size = 0.73

$$-\frac{a^2\,c^2\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(75\,B\,\cos\left(e+f\,x\right)+105\,B\,\cos\left(3\,e+3\,f\,x\right)+35\,B\,\cos\left(5\right.\right.}{960\,f\,\left(\cos\left(2\,e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(5
/2),x)
```

```
[Out] -(a^2*c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(75*B*
cos(e + f*x) + 105*B*cos(3*e + 3*f*x) + 35*B*cos(5*e + 5*f*x) + 5*B*cos(7*e
 + 7*f*x) - 700*A*sin(2*e + 2*f*x) - 112*A*sin(4*e + 4*f*x) - 12*A*sin(6*e
+ 6*f*x)))/(960*f*(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

3.151 $\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2}\,dx$

Optimal. Leaf size=142

$$\frac{c^2(5A + B)\cos(e + fx)(a\sin(e + fx) + a)^{5/2}}{30f\sqrt{c - c\sin(e + fx)}} + \frac{c(5A + B)\cos(e + fx)(a\sin(e + fx) + a)^{5/2}\sqrt{c - c\sin(e + fx)}}{20f} - B\,c$$

```
[Out] -1/5*B*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/f+1/30*(5*A
+B)*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(1/2)+1/20*(5*
A+B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.36, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2973, 2740, 2738}

$$\frac{c^2(5A + B)\cos(e + fx)(a\sin(e + fx) + a)^{5/2}}{30f\sqrt{c - c\sin(e + fx)}} + \frac{c(5A + B)\cos(e + fx)(a\sin(e + fx) + a)^{5/2}\sqrt{c - c\sin(e + fx)}}{20f} - B\,c$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] ((5*A + B)*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(30*f*Sqrt[c - c*Si
n[e + f*x]]) + ((5*A + B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c
- c*Sin[e + f*x]])/(20*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c -
 c*Sin[e + f*x])^(3/2))/(5*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
```

```
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a + a\sin(e+fx))^{5/2}(A + B\sin(e+fx))(c - c\sin(e+fx))^{3/2}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c}{5f}$$

$$= \frac{(5A+B)c\cos(e+fx)(a+a\sin(e+fx))^{5}}{20f}$$

$$= \frac{(5A+B)c^2\cos(e+fx)(a+a\sin(e+fx))}{30f\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 1.80, size = 165, normalized size = 1.16

$$-\frac{c(\sin(e+fx)-1)(a(\sin(e+fx)+1))^{5/2}\sqrt{c-c\sin(e+fx)}\,(4(100A+11B)\sin(e+fx)+4\cos(2(e+fx))(4(5A}{480f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\sin\left(\frac{1}{2}(e+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(3/2),x]
```

```
[Out] -1/480*(c*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e
 + f*x]]*(4*(100*A + 11*B)*Sin[e + f*x] + 4*Cos[2*(e + f*x)]*(-15*(A + B) +
 4*(5*A - 2*B)*Sin[e + f*x]) - 3*Cos[4*(e + f*x)]*(5*(A + B) + 4*B*Sin[e +
f*x])))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^5)
```

fricas [A] time = 0.46, size = 119, normalized size = 0.84

$$-\frac{\left(15\,(A+B)a^2c\cos\left(fx+e\right)^4-15\,(A+B)a^2c+4\left(3\,Ba^2c\cos\left(fx+e\right)^4-(5\,A+B)a^2c\cos\left(fx+e\right)^2-2\,(5\,A\right.\right.}{60\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] -1/60*(15*(A + B)*a^2*c*cos(f*x + e)^4 - 15*(A + B)*a^2*c + 4*(3*B*a^2*c*co
s(f*x + e)^4 - (5*A + B)*a^2*c*cos(f*x + e)^2 - 2*(5*A + B)*a^2*c)*sin(f*x
+ e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
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able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sq
rt(2*c)*(32*f*(A*a^2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x
+exp(1))-1/4*pi))+B*a^2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(
f*x+exp(1))-1/4*pi)))*cos(-2*f*x-2*exp(1))/(-32*f)^2-32*f*(-A*a^2*c*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-B*a^2*c*sign
(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(2*f*
x+2*exp(1))/(32*f)^2-64*f*(-A*a^2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign
(cos(1/2*(f*x+exp(1))-1/4*pi))-B*a^2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*s
ign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(4*f*x+4*exp(1))/(64*f)^2+96*f*(-4*A*
a^2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))
+B*a^2*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*p
i)))*sin(3*f*x+3*exp(1))/(96*f)^2+16*f*(-6*A*a^2*c*sign(sin(1/2*(f*x+exp(1)
)-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-B*a^2*c*sign(sin(1/2*(f*x+exp
(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(f*x+exp(1))/(16*f)^2+
160*B*a^2*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-
1/4*pi))*sin(5*f*x+5*exp(1))/(160*f)^2)
```

maple [A] time = 0.79, size = 147, normalized size = 1.04

$$\frac{\left(-12B\left(\cos^4\left(fx+e\right)\right)+15A\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+15B\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+20A\left(\cos^2\left(fx+\right.\right.\right.}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/60/f*(-12*B*cos(f*x+e)^4+15*A*cos(f*x+e)^2*sin(f*x+e)+15*B*cos(f*x+e)^2*s
in(f*x+e)+20*A*cos(f*x+e)^2+4*B*cos(f*x+e)^2+15*A*sin(f*x+e)+15*B*sin(f*x+e
)+40*A+8*B)*(-c*(sin(f*x+e)-1))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)/(
1+sin(f*x+e))/cos(f*x+e)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
+ c)^(3/2), x)
```

mupad [B] time = 16.32, size = 174, normalized size = 1.23

$$-\frac{a^2\,c\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(60\,A\,\cos\left(e+f\,x\right)+60\,B\,\cos\left(e+f\,x\right)+75\,A\,\cos\left(3\,e\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(3
/2),x)
```

```
[Out] -(a^2*c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(60*A*co
s(e + f*x) + 60*B*cos(e + f*x) + 75*A*cos(3*e + 3*f*x) + 15*A*cos(5*e + 5*f
*x) + 75*B*cos(3*e + 3*f*x) + 15*B*cos(5*e + 5*f*x) - 400*A*sin(2*e + 2*f*x
) - 40*A*sin(4*e + 4*f*x) - 50*B*sin(2*e + 2*f*x) + 16*B*sin(4*e + 4*f*x) +
 6*B*sin(6*e + 6*f*x)))/(480*f*(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

3.152 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+f$

Optimal. Leaf size=96

$$\frac{c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/3*(A-B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(1/2)+1/4*
B*c*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/a/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2971, 2738}

$$\frac{c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] ((A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*f*Sqrt[c - c*Sin[e +
 f*x]]) + (B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*a*f*Sqrt[c - c*S
in[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{B\int (a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}\,d}{a}$$
$$= \frac{(A-B)c\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}} +$$

Mathematica [A] time = 0.83, size = 102, normalized size = 1.06

$$\frac{a^2\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}\,(16(7A+2B)\sin(e+fx)-4\cos(2(e+fx))(4(A+2B)\sin(}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e
+ f*x]],x]
```

```
[Out] (a^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(3*B*
Cos[4*(e + f*x)] + 16*(7*A + 2*B)*Sin[e + f*x] - 4*Cos[2*(e + f*x)]*(12*A +
 9*B + 4*(A + 2*B)*Sin[e + f*x])))/(96*f)
```

fricas [A] time = 0.45, size = 117, normalized size = 1.22

$$\frac{\left(3\,Ba^2\cos\left(fx+e\right)^4-12\,(A+B)a^2\cos\left(fx+e\right)^2+3\,(4\,A+3\,B)a^2-4\left((A+2\,B)a^2\cos\left(fx+e\right)^2-2\,(2\,A+B)a\right.\right.}{12\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/12*(3*B*a^2*cos(f*x + e)^4 - 12*(A + B)*a^2*cos(f*x + e)^2 + 3*(4*A + 3*B
)*a^2 - 4*((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(2*A + B)*a^2)*sin(f*x + e))*sq
rt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(24*f*(A*a^2*sign(sin(1/2*(f*x+
exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+2*B*a^2*sign(sin(1/2*(f
*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(3*f*x+3*exp(1))
/(24*f)^2+8*f*(-5*A*a^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*
x+exp(1))-1/4*pi))-2*B*a^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*
(f*x+exp(1))-1/4*pi)))*sin(f*x+exp(1))/(8*f)^2-32*f*(-8*A*a^2*sign(sin(1/2*
(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-7*B*a^2*sign(sin(1
/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(2*f*x+2*ex
p(1))/(32*f)^2-64*B*a^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(
f*x+exp(1))-1/4*pi))*cos(4*f*x+4*exp(1))/(64*f)^2-32*B*a^2*f*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-2*f*x-2*exp(1)
)/(-32*f)^2)
```

maple [A] time = 0.80, size = 129, normalized size = 1.34

$$\frac{\left(3B\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+4A\left(\cos^2\left(fx+e\right)\right)+8B\left(\cos^2\left(fx+e\right)\right)-12A\sin\left(fx+e\right)-9B\sin\left(fx+e\right.\right.}{12f\left(\cos^2\left(fx+e\right)-2\sin\left(fx+e\right)-2\right)\cos\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/12/f*(3*B*cos(f*x+e)^2*sin(f*x+e)+4*A*cos(f*x+e)^2+8*B*cos(f*x+e)^2-12*A*
sin(f*x+e)-9*B*sin(f*x+e)-16*A-8*B)*(-c*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)*(a
*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^2-2*sin(f*x+e)-2)/cos(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x +
 e) + c), x)
```

mupad [B] time = 2.70, size = 149, normalized size = 1.55

$$-\frac{a^2\sqrt{a\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\left(\sin\left(e+f\,x\right)-1\right)}\,\left(48\,A\,\cos\left(e+f\,x\right)+36\,B\,\cos\left(e+f\,x\right)+48\,A\,\cos\left(3\,e+3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(1
/2),x)
```

```
[Out] -(a^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(48*A*cos(
e + f*x) + 36*B*cos(e + f*x) + 48*A*cos(3*e + 3*f*x) + 33*B*cos(3*e + 3*f*x
) - 3*B*cos(5*e + 5*f*x) - 112*A*sin(2*e + 2*f*x) + 8*A*sin(4*e + 4*f*x) -
32*B*sin(2*e + 2*f*x) + 16*B*sin(4*e + 4*f*x)))/(96*f*(cos(2*e + 2*f*x) + 1
))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

3.153 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=193

$$-\frac{4a^3(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{2a^2(A+B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}-\frac{a(A+B)\cos(e+f}{2f\sqrt{c-c}}$$

```
[Out] -1/2*a*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(1/2)-1/3
*B*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(1/2)-4*a^3*(A+B)*c
os(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-
2*a^2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2973, 2740, 2737, 2667, 31}

$$-\frac{2a^2(A+B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}-\frac{4a^3(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{a(A+B)\cos(e+f}{2f\sqrt{c-c}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (-4*a^3*(A + B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[c - c*Sin[e + f*x]]) - (2*a^2*(A + B)*Cos[e + f*x]*Sqrt[a + a*Sin
[e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]]) - (a*(A + B)*Cos[e + f*x]*(a + a*S
in[e + f*x])^(3/2))/(2*f*Sqrt[c - c*Sin[e + f*x]]) - (B*Cos[e + f*x]*(a + a
*Sin[e + f*x])^(5/2))/(3*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}} + (A+B)\int \frac{(a+a\sin(}{\sqrt{c-c\sin}}$$

$$= -\frac{a(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)(}{3f\sqrt{c-}}$$

$$= -\frac{2a^2(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a(A+B)\cos(}{2f\sqrt{}}$$

$$= -\frac{2a^2(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a(A+B)\cos(}{2f\sqrt{}}$$

$$= -\frac{2a^2(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a(A+B)\cos(}{2f\sqrt{}}$$

$$= -\frac{4a^3(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2a^2(A+B)\,c}{f}$$

Mathematica [A] time = 1.50, size = 177, normalized size = 0.92

$$-\frac{(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((36A+51B)\sin(e+fx)-3(A+3B)\cos(2(e+f\right.}{12f\sqrt{c-c\sin(e+fx)}\left(\sin\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[
e + f*x]],x]
```

```
[Out] -1/12*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2)*(
-3*(A + 3*B)*Cos[2*(e + f*x)] + 96*A*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2
]] + 96*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (36*A + 51*B)*Sin[e +
f*x] - B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sqrt
[c - c*Sin[e + f*x]])
```

fricas [F] time = 4.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left((A+2\,B)a^2\cos\left(fx+e\right)^2-2\,(A+B)a^2+\left(Ba^2\cos\left(fx+e\right)^2-2\,(A+B)a^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(f\right.}}{c\sin\left(fx+e\right)-c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x + e
)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(c*sin(f*x + e) - c), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.72, size = 590, normalized size = 3.06

$$\frac{\left(15A\sin\left(fx+e\right)-18A\sin\left(fx+e\right)\cos\left(fx+e\right)+7B\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)-48A\sin\left(fx+e\right)\ln\left(-\frac{-1+c}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/6/f*(2*B*cos(f*x+e)^3*sin(f*x+e)+15*A*sin(f*x+e)-18*A*sin(f*x+e)*cos(f*x+
e)-48*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+3*A*cos(f*x+e
)^2*sin(f*x+e)+7*B*cos(f*x+e)^2*sin(f*x+e)-2*B*cos(f*x+e)^4-15*A-17*B+3*A*c
os(f*x+e)^3+24*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-48*A*cos(f*x+e)*ln(-(-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))+24*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-26*B*
sin(f*x+e)*cos(f*x+e)-48*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))+24*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-48*B*cos(f*x+e)*ln(-(-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))+24*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+19*B*cos(f*
x+e)^2+9*B*cos(f*x+e)^3+17*B*sin(f*x+e)-3*A*cos(f*x+e)-9*B*cos(f*x+e)+15*A*
cos(f*x+e)^2+48*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-24*A*ln(2/(cos
(f*x+e)+1))+48*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-24*B*ln(2/(cos(
f*x+e)+1)))*(a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^3-cos(f*x+e)^2*sin(f*x+e)-
3*cos(f*x+e)^2-2*sin(f*x+e)*cos(f*x+e)-2*cos(f*x+e)+4*sin(f*x+e)+4)/(-c*(si
n(f*x+e)-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/sqrt(-c*sin(f*x +
 e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(1/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

3.154 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=210

$$\frac{4a^3(A+2B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{2a^2(A+2B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}+\frac{a(A+2B)\cos(e+}{2cf\sqrt{c-}}$$

```
[Out] 1/2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(3/2)+1/2*a*
(A+2*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(1/2)+4*a^3*
(A+2*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x
+e))^(1/2)+2*a^2*(A+2*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x
+e))^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2972, 2740, 2737, 2667, 31}

$$\frac{2a^2(A+2B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}+\frac{4a^3(A+2B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{a(A+2B)\cos(e+}{2cf\sqrt{c-}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(2*f*(c - c*Sin[e + f*x])
^(3/2)) + (4*a^3*(A + 2*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a
+ a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (2*a^2*(A + 2*B)*Cos[e + f*x]
*Sqrt[a + a*Sin[e + f*x]])/(c*f*Sqrt[c - c*Sin[e + f*x]]) + (a*(A + 2*B)*Co
s[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a + a\sin(e+fx))^{5/2}(A + B\sin(e+fx))}{(c - c\sin(e+fx))^{3/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(A+2B)\int \frac{(a+a\sin(}{\sqrt{c-c\sin}}}{c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a(A+2B)\cos(e+}{2cf\sqrt{c-}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2(A+2B)\cos(e}{cf\sqrt{c-}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2(A+2B)\cos(e}{cf\sqrt{c-}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2(A+2B)\cos(e}{cf\sqrt{c-}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{4a^3(A+2B)\cos(e}{cf\sqrt{a+a\sin(e}}$$

Mathematica [A] time = 1.64, size = 231, normalized size = 1.10

$$-\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(2A+7B)\cos(2(e+fx))+\sin(e+fx)\left(-64(A+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(3/2),x]
```

```
[Out] -1/8*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*
(28*A + 16*B + 2*(2*A + 7*B)*Cos[2*(e + f*x)] + 64*A*Log[Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]] + 128*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (8*A
+ 31*B - 64*(A + 2*B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])*Sin[e + f*x
] + B*Sin[3*(e + f*x)]))/(c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + S
in[e + f*x])*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 10.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\left(A+2\,B\right)a^2\cos\left(fx+e\right)^2-2\,(A+B)a^2+\left(Ba^2\cos\left(fx+e\right)^2-2\,(A+B)a^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx\right.}}{c^2\cos\left(fx+e\right)^2+2\,c^2\sin\left(fx+e\right)-2\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x + e
)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.64, size = 846, normalized size = 4.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/2/f*(32*B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*cos(
f*x+e)^3*sin(f*x+e)-12*A*sin(f*x+e)+10*A*sin(f*x+e)*cos(f*x+e)+32*A*sin(f*x
+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-16*B*cos(f*x+e)^2*ln(2/(cos(
f*x+e)+1))+2*A*cos(f*x+e)^2*sin(f*x+e)+6*B*cos(f*x+e)^2*sin(f*x+e)-B*cos(f*
x+e)^4+12*A+22*B+2*A*cos(f*x+e)^3-8*A*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+16*
A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-16*A*sin(f*x+e)*l
n(2/(cos(f*x+e)+1))+16*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))-8*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+15*B*sin(f*x+e)*cos(f*x+e)+64*B*sin
(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-32*B*sin(f*x+e)*ln(2/(co
s(f*x+e)+1))+32*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-16*
B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+8*A*cos(f*x+e)*sin(f*x+e)*ln(2/(cos(f*x+e
)+1))-16*A*cos(f*x+e)*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
+16*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-32*B*ln(-(-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)-21*B*cos(f*x+e)^2+7*B*cos(f*x+e
)^3-22*B*sin(f*x+e)-2*A*cos(f*x+e)-7*B*cos(f*x+e)-12*A*cos(f*x+e)^2-32*A*ln
(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+16*A*ln(2/(cos(f*x+e)+1))-64*B*ln(
-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+32*B*ln(2/(cos(f*x+e)+1)))*(a*(1+si
n(f*x+e)))^(5/2)/(cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^3+2*sin(f*x+e)*cos(f*x
+e)+3*cos(f*x+e)^2-4*sin(f*x+e)+2*cos(f*x+e)-4)/(-c*(sin(f*x+e)-1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
+ c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

3.155 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=212

$$-\frac{a^3(A+5B)\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a^2(A+5B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2c^2f\sqrt{c-c\sin(e+fx)}}-\frac{a(A+5B)\cos(e+\ldots}{4cf(c-c\ldots}$$

```
[Out] 1/4*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(5/2)-1/4*a*
(A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(3/2)-a^3*(A
+5*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x
+e))^(1/2)-1/2*a^2*(A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c*sin
(f*x+e))^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2972, 2739, 2740, 2737, 2667, 31}

$$-\frac{a^2(A+5B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2c^2f\sqrt{c-c\sin(e+fx)}}-\frac{a^3(A+5B)\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a(A+5B)\cos(e+\ldots}{4cf(c-c\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(4*f*(c - c*Sin[e + f*x])
^(5/2)) - (a*(A + 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*c*f*(c -
 c*Sin[e + f*x])^(3/2)) - (a^3*(A + 5*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]]
)/(c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (a^2*(A + 5*B
)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(2*c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{(A+5B)\int \frac{(a+a\,\mathrm{si}}{(c-c\,\mathrm{si}}}{4c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+5B)\cos(e}{4cf(c-}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+5B)\cos(e}{4cf(c-}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+5B)\cos(e}{4cf(c-}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+5B)\cos(e}{4cf(c-}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+5B)\cos(e}{4cf(c-}$$

Mathematica [A] time = 1.16, size = 207, normalized size = 0.98

$$\frac{(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-4(A+2B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2 -}{f(c-c\,\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2)*(2*(A +
 B) - 4*(A + 2*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 - 2*(A + 5*B)*Log
[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^4 - B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(f*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(5/2))
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left((A+2\,B)a^2\cos\left(fx+e\right)^2-2\,(A+B)a^2+\left(Ba^2\cos\left(fx+e\right)^2-2\,(A+B)a^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(f\right.}}{3\,c^3\cos\left(fx+e\right)^2-4\,c^3-\left(c^3\cos\left(fx+e\right)^2-4\,c^3\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral(((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x + e
)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin
(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.65, size = 1092, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/f*(30*B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*cos(f*
x+e)^3*sin(f*x+e)-2*A*sin(f*x+e)+8*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))-15*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+2*A*cos(f*x+e)^2*sin
(f*x+e)+9*B*cos(f*x+e)^2*sin(f*x+e)+B*cos(f*x+e)^4+2*A+14*B+2*A*cos(f*x+e)^
3-10*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^3+5*B*ln(2/(co
s(f*x+e)+1))*cos(f*x+e)^3-3*A*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+6*A*cos(f*x
+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-4*A*sin(f*x+e)*ln(2/(cos(f
*x+e)+1))+4*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*A*cos
(f*x+e)*ln(2/(cos(f*x+e)+1))+6*B*sin(f*x+e)*cos(f*x+e)+40*B*sin(f*x+e)*ln(-
(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-20*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))
+20*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-10*B*cos(f*x+e)
*ln(2/(cos(f*x+e)+1))+2*A*cos(f*x+e)*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-4*A*co
s(f*x+e)*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-10*B*ln(-(-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+5*B*ln(2/(cos(f
*x+e)+1))*cos(f*x+e)^2*sin(f*x+e)+10*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(
f*x+e)-20*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e
)-15*B*cos(f*x+e)^2-2*A*cos(f*x+e)^3*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))+A*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))+A*cos(f*x+e)^2*sin(f*x+e)*ln(2/(co
```

```
s(f*x+e)+1))-2*A*cos(f*x+e)^2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))+8*B*cos(f*x+e)^3-14*B*sin(f*x+e)-2*A*cos(f*x+e)-8*B*cos(f*x+e)-2*A
*cos(f*x+e)^2-8*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+4*A*ln(2/(cos(
f*x+e)+1))-40*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+20*B*ln(2/(cos(f
*x+e)+1)))*(a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^3-cos(f*x+e)^2*sin(f*x+e)-3
*cos(f*x+e)^2-2*sin(f*x+e)*cos(f*x+e)-2*cos(f*x+e)+4*sin(f*x+e)+4)/(-c*(sin
(f*x+e)-1))^(5/2)
```

maxima [B] time = 0.48, size = 506, normalized size = 2.39

$$-\frac{\left(\frac{8\,a^{\frac{5}{2}}\sqrt{c}\,\sin\left(fx+e\right)^{2}}{\left(c^{3}-\frac{4\,c^{3}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{6\,c^{3}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{4\,c^{3}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{c^{3}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}\right)\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{2\,a^{\frac{5}{2}}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{c^{\frac{5}{2}}}+\frac{a^{\frac{5}{2}}\log\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)}{c^{\frac{5}{2}}}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] -((8*a^(5/2)*sqrt(c)*sin(f*x + e)^2/((c^3 - 4*c^3*sin(f*x + e)/(cos(f*x + e
) + 1) + 6*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^3*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3 + c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*(cos(f*x + e
) + 1)^2) - 2*a^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(5/2) + a^
(5/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(5/2))*A - B*(10*a^(5/
2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(5/2) - 5*a^(5/2)*log(sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(5/2) + 2*(5*a^(5/2)*sin(f*x + e)/(cos(
f*x + e) + 1) - 16*a^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 14*a^(5/2)
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 16*a^(5/2)*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 5*a^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(c^(5/2) - 4*c^
(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 7*c^(5/2)*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 8*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*c^(5/2)*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 - 4*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) +
1)^5 + c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(5/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

3.156 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=196

$$\frac{a^3B\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{a^2B\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f(c-c\sin(e+fx))^{3/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+\ldots}{6f(c-c\sin(e+fx))^{7\ldots}}$$

```
[Out] 1/6*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(7/2)-1/2*a*
B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(5/2)+a^2*B*cos(f*
x+e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c*sin(f*x+e))^(3/2)+a^3*B*cos(f*x+e)*l
n(1-sin(f*x+e))/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2972, 2739, 2737, 2667, 31}

$$\frac{a^2B\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f(c-c\sin(e+fx))^{3/2}}+\frac{a^3B\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+\ldots}{6f(c-c\sin(e+fx))^{7\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*f*(c - c*Sin[e + f*x])
^(7/2)) - (a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*(c - c*Sin[e
 + f*x])^(5/2)) + (a^2*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^2*f*(c -
 c*Sin[e + f*x])^(3/2)) + (a^3*B*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^3*f
*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{B\int \frac{(a+a\sin(e+fx))^{5/}}{(c-c\sin(e+fx))^{5/}}}{c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e+fx)(a}{2cf(c-c\,\mathrm{si}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e+fx)(a}{2cf(c-c\,\mathrm{si}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e+fx)(a}{2cf(c-c\,\mathrm{si}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e+fx)(a}{2cf(c-c\,\mathrm{si}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e+fx)(a}{2cf(c-c\,\mathrm{si}}$$

Mathematica [A] time = 1.19, size = 204, normalized size = 1.04

$$\frac{(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(3(A+5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-6}{3f(c-c\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(7/2),x]
```

```
[Out] ((4*(A + B) - 6*(A + 2*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + 3*(A +
5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 6*B*Log[Cos[(e + f*x)/2] - S
in[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6)*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2))/(3*f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(7/2))
```

fricas [F] time = 6.12, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left((A+2\,B)a^2\cos\left(fx+e\right)^2-2\,(A+B)a^2+\left(Ba^2\cos\left(fx+e\right)^2-2\,(A+B)a^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin(}}{c^4\cos\left(fx+e\right)^4-8\,c^4\cos\left(fx+e\right)^2+8\,c^4+4\left(c^4\cos\left(fx+e\right)^2-2\,c^4\right)\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="fricas")
```

```
[Out] integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*
x + e) + c)/(c^4*cos(f*x + e)^4 - 8*c^4*cos(f*x + e)^2 + 8*c^4 + 4*(c^4*cos
(f*x + e)^2 - 2*c^4)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.69, size = 832, normalized size = 4.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/3/f*(48*B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-7*B*cos
(f*x+e)^3*sin(f*x+e)-4*A*sin(f*x+e)+4*A*sin(f*x+e)*cos(f*x+e)-A*cos(f*x+e)^
3*sin(f*x+e)-24*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+A*cos(f*x+e)^2*sin(f*x+
e)+13*B*cos(f*x+e)^2*sin(f*x+e)+7*B*cos(f*x+e)^4+4*A+16*B-18*B*ln(-(-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^3+9*B*ln(2/(cos(f*x+e)+1))*cos(f*
x+e)^3+10*B*sin(f*x+e)*cos(f*x+e)+48*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))-24*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+24*B*cos(f*x+e)*ln(-
(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-12*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))
-24*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+12
*B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^2*sin(f*x+e)+12*B*ln(2/(cos(f*x+e)+1))*s
in(f*x+e)*cos(f*x+e)-24*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*
x+e)*cos(f*x+e)+3*B*cos(f*x+e)^4*ln(2/(cos(f*x+e)+1))-6*B*cos(f*x+e)^4*ln(-
(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-23*B*cos(f*x+e)^2+A*cos(f*x+e)^4+6*B
*cos(f*x+e)^3-16*B*sin(f*x+e)+6*B*cos(f*x+e)^3*sin(f*x+e)*ln(-(-1+cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))-3*B*cos(f*x+e)^3*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-
6*B*cos(f*x+e)-5*A*cos(f*x+e)^2-48*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))+24*B*ln(2/(cos(f*x+e)+1)))*(a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^3-cos(
```

```
f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-2*sin(f*x+e)*cos(f*x+e)-2*cos(f*x+e)+4*s
in(f*x+e)+4)/(-c*(sin(f*x+e)-1))^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
+ c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(7/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Timed out
```

3.157 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=96

$$\frac{(A-7B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{48cf(c-c\sin(e+fx))^{7/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

```
[Out] 1/8*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(9/2)+1/48*(
A-7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(7/2)
```

Rubi [A] time = 0.28, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2972, 2742}

$$\frac{(A-7B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{48cf(c-c\sin(e+fx))^{7/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(9/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(8*f*(c - c*Sin[e + f*x])
^(9/2)) + ((A - 7*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(48*c*f*(c -
c*Sin[e + f*x])^(7/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{(A-7B)\int \frac{(a+a\,\mathrm{si}}{(c-c\,\mathrm{si}}}{8c}$$
$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{(A-7B)\cos(e+}{48cf(c-}$$

Mathematica [A] time = 2.96, size = 145, normalized size = 1.51

$$\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)((4A+17B)\sin(e+fx)-3(A-B)\cos(2(e+fx))+}{12c^4f(\sin(e+fx)-1)^4\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(9/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(5*A
- 5*B - 3*(A - B)*Cos[2*(e + f*x)] + (4*A + 17*B)*Sin[e + f*x] - 3*B*Sin[3*
(e + f*x)]))/(12*c^4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e +
f*x])^4*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.45, size = 165, normalized size = 1.72

$$-\frac{\left(3(A-B)a^2\cos\left(fx+e\right)^2-4(A-B)a^2+2\left(3Ba^2\cos\left(fx+e\right)^2-(A+5B)a^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)}}{6\left(c^5f\cos\left(fx+e\right)^5-8c^5f\cos\left(fx+e\right)^3+8c^5f\cos\left(fx+e\right)+4\left(c^5f\cos\left(fx+e\right)^3-2c^5f\cos\left(fx\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="fricas")
```

```
[Out] -1/6*(3*(A - B)*a^2*cos(f*x + e)^2 - 4*(A - B)*a^2 + 2*(3*B*a^2*cos(f*x + e
)^2 - (A + 5*B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(c^5*f*cos(f*x + e)^5 - 8*c^5*f*cos(f*x + e)^3 + 8*c^5*f*cos(f*x
 + e) + 4*(c^5*f*cos(f*x + e)^3 - 2*c^5*f*cos(f*x + e))*sin(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.68, size = 309, normalized size = 3.22

$$-\frac{\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}\left(A\left(\cos^{4}\left(fx+e\right)\right)-A\left(\cos^{3}\left(fx+e\right)\right)\sin\left(fx+e\right)-B\left(\cos^{4}\left(fx+e\right)\right)+B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] -1/6/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(A*cos(f*x+e)^4-A*cos(f*x+e)^3*s
in(f*x+e)-B*cos(f*x+e)^4+B*cos(f*x+e)^3*sin(f*x+e)+4*A*cos(f*x+e)^3+5*A*cos
(f*x+e)^2*sin(f*x+e)+2*B*cos(f*x+e)^3+B*cos(f*x+e)^2*sin(f*x+e)-9*A*cos(f*x
+e)^2+4*A*sin(f*x+e)*cos(f*x+e)+3*B*cos(f*x+e)^2-4*B*sin(f*x+e)*cos(f*x+e)-
10*A*cos(f*x+e)-14*A*sin(f*x+e)-2*B*cos(f*x+e)+2*B*sin(f*x+e)+14*A-2*B)/(-c
*(sin(f*x+e)-1))^(9/2)/(cos(f*x+e)^3-cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2
-2*sin(f*x+e)*cos(f*x+e)-2*cos(f*x+e)+4*sin(f*x+e)+4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
+ c)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(9/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),
x)
```

```
[Out] Timed out
```

3.158 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=146

$$\frac{(A-4B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{240c^2f(c-c\sin(e+fx))^{7/2}}+\frac{(A-4B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{40cf(c-c\sin(e+fx))^{9/2}}+\frac{(A+B)\cos(e+fx)(a\sin}{10f(c-c\sin(e+}$$

```
[Out] 1/10*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(11/2)+1/40
*(A-4*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(9/2)+1/240
*(A-4*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c^2/f/(c-c*sin(f*x+e))^(7/2)
```

Rubi [A] time = 0.38, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2972, 2743, 2742}

$$\frac{(A-4B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{240c^2f(c-c\sin(e+fx))^{7/2}}+\frac{(A-4B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{40cf(c-c\sin(e+fx))^{9/2}}+\frac{(A+B)\cos(e+fx)(a\sin}{10f(c-c\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(11/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(10*f*(c - c*Sin[e + f*x]
)^(11/2)) + ((A - 4*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(40*c*f*(c
- c*Sin[e + f*x])^(9/2)) + ((A - 4*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/
2))/(240*c^2*f*(c - c*Sin[e + f*x])^(7/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
```

```
SumSimplerQ[n, 1])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(A-4B)\int \frac{(a+a\,\mathrm{si}}{(c-c\,\mathrm{si}}}{5c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(A-4B)\cos(e+}{40cf(c-}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(A-4B)\cos(e+}{40cf(c-}$$

Mathematica [A] time = 4.08, size = 146, normalized size = 1.00

$$\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(-5(8A+13B)\sin(e+fx)+10(2A+B)\cos(2(e+}{120c^5f(\sin(e+fx)-1)^5\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+f\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(11/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-36*
A - 6*B + 10*(2*A + B)*Cos[2*(e + f*x)] - 5*(8*A + 13*B)*Sin[e + f*x] + 15*
B*Sin[3*(e + f*x)]))/(120*c^5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 +
 Sin[e + f*x])^5*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.46, size = 182, normalized size = 1.25

$$-\frac{\left(5\,(2\,A+B)a^2\cos\left(fx+e\right)^2-2\,(7\,A+2\,B)a^2+5\left(3\,Ba^2\cos\left(fx+e\right)^2-2\,(A+2\,B)a^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin(}}{30\left(5\,c^6f\cos\left(fx+e\right)^5-20\,c^6f\cos\left(fx+e\right)^3+16\,c^6f\cos\left(fx+e\right)-\left(c^6f\cos\left(fx+e\right)^5-12\,c^6f\cos\left(fx+e\right)^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="fricas")
```

```
[Out] -1/30*(5*(2*A + B)*a^2*cos(f*x + e)^2 - 2*(7*A + 2*B)*a^2 + 5*(3*B*a^2*cos(
f*x + e)^2 - 2*(A + 2*B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-
c*sin(f*x + e) + c)/(5*c^6*f*cos(f*x + e)^5 - 20*c^6*f*cos(f*x + e)^3 + 16*
c^6*f*cos(f*x + e) - (c^6*f*cos(f*x + e)^5 - 12*c^6*f*cos(f*x + e)^3 + 16*c
^6*f*cos(f*x + e))*sin(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.70, size = 368, normalized size = 2.52

$$\frac{\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}\left(4A\left(\cos^5\left(fx+e\right)\right)+4A\left(\cos^4\left(fx+e\right)\right)\sin\left(fx+e\right)-B\left(\cos^5\left(fx+e\right)\right)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x)
```

```
[Out] 1/30/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(4*A*cos(f*x+e)^5+4*A*cos(f*x+e)
^4*sin(f*x+e)-B*cos(f*x+e)^5-B*sin(f*x+e)*cos(f*x+e)^4-24*A*cos(f*x+e)^4+20
*A*cos(f*x+e)^3*sin(f*x+e)+6*B*cos(f*x+e)^4-5*B*cos(f*x+e)^3*sin(f*x+e)-48*
A*cos(f*x+e)^3-68*A*cos(f*x+e)^2*sin(f*x+e)-3*B*cos(f*x+e)^3+2*B*cos(f*x+e)
^2*sin(f*x+e)+118*A*cos(f*x+e)^2-50*A*sin(f*x+e)*cos(f*x+e)-22*B*cos(f*x+e)
^2+20*B*sin(f*x+e)*cos(f*x+e)+74*A*cos(f*x+e)+124*A*sin(f*x+e)+4*B*cos(f*x+
e)-16*B*sin(f*x+e)-124*A+16*B)/(-c*(sin(f*x+e)-1))^(11/2)/(cos(f*x+e)^3-cos
```

```
(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-2*sin(f*x+e)*cos(f*x+e)-2*cos(f*x+e)+4*
sin(f*x+e)+4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
+ c)^(11/2), x)
```

mupad [B] time = 20.76, size = 341, normalized size = 2.34

$$\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{16\,a^2\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\left(A\,6{}\mathrm{i}+B\,1{}\mathrm{i}\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{5\,c^6\,f}-\frac{16\,a^2\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\left(A\,2{}\mathrm{i}+B\,1{}\mathrm{i}\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{3\,c^6\,f}+\frac{a^2\,\mathrm{e}^{\cdots}}{\cdots}\right.}{\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,264{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)\,220{}\mathrm{i}+{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)\,20{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(11/2),x)
```

```
[Out] ((c - c*sin(e + f*x))^(1/2)*((16*a^2*exp(e*6i + f*x*6i)*(A*6i + B*1i)*(a +
a*sin(e + f*x))^(1/2))/(5*c^6*f) - (16*a^2*exp(e*6i + f*x*6i)*cos(2*e + 2*f
*x)*(A*2i + B*1i)*(a + a*sin(e + f*x))^(1/2))/(3*c^6*f) + (a^2*exp(e*6i + f
*x*6i)*sin(e + f*x)*(8*A + 13*B)*(a + a*sin(e + f*x))^(1/2)*8i)/(3*c^6*f) -
 (B*a^2*exp(e*6i + f*x*6i)*sin(3*e + 3*f*x)*(a + a*sin(e + f*x))^(1/2)*8i)/
(c^6*f)))/(cos(e + f*x)*exp(e*6i + f*x*6i)*264i - exp(e*6i + f*x*6i)*cos(3*
e + 3*f*x)*220i + exp(e*6i + f*x*6i)*cos(5*e + 5*f*x)*20i - exp(e*6i + f*x*
6i)*sin(2*e + 2*f*x)*330i + exp(e*6i + f*x*6i)*sin(4*e + 4*f*x)*88i - exp(e
*6i + f*x*6i)*sin(6*e + 6*f*x)*2i)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(11/2)
,x)
```

```
[Out] Timed out
```

3.159 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx$

Optimal. Leaf size=196

$$\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{960c^3f(c-c\sin(e+fx))^{7/2}}+\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{160c^2f(c-c\sin(e+fx))^{9/2}}+\frac{(A-3B)\cos(e+fx)(a\,\mathrm{s}}{40cf(c-c\sin(e}$$

```
[Out] 1/12*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(13/2)+1/40
*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(11/2)+1/16
0*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c^2/f/(c-c*sin(f*x+e))^(9/2)+1/
960*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c^3/f/(c-c*sin(f*x+e))^(7/2)
```

Rubi [A] time = 0.48, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2972, 2743, 2742}

$$\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{960c^3f(c-c\sin(e+fx))^{7/2}}+\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{160c^2f(c-c\sin(e+fx))^{9/2}}+\frac{(A-3B)\cos(e+fx)(a\,\mathrm{s}}{40cf(c-c\sin(e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(13/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(12*f*(c - c*Sin[e + f*x]
)^(13/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(40*c*f*(c
- c*Sin[e + f*x])^(11/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5
/2))/(160*c^2*f*(c - c*Sin[e + f*x])^(9/2)) + ((A - 3*B)*Cos[e + f*x]*(a +
a*Sin[e + f*x])^(5/2))/(960*c^3*f*(c - c*Sin[e + f*x])^(7/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
```

```
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-3B)\int \frac{(a+a\,\mathrm{si}}{(c-c\,\mathrm{sin}}}{4c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-3B)\cos(e+}{40cf(c-}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-3B)\cos(e+}{40cf(c-}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-3B)\cos(e+}{40cf(c-}$$

Mathematica [A] time = 5.62, size = 144, normalized size = 0.73

$$\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(6(6A+7B)\sin(e+fx)-15(A+B)\cos(2(e+fx))}{120c^6f(\sin(e+fx)-1)^6\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(13/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(29*A
 + 13*B - 15*(A + B)*Cos[2*(e + f*x)] + 6*(6*A + 7*B)*Sin[e + f*x] - 10*B*S
```

```
in[3*(e + f*x)]))/(120*c^6*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Si
n[e + f*x])^6*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.47, size = 196, normalized size = 1.00

$$\frac{\left(15\,(A+B)a^2\cos\left(fx+e\right)^2-2\,(11\,A+7\,B)a^2+2\left(10\,Ba^2\cos\left(fx+e\right)^2-(9\,A+13\,B)a^2\right)\sin\left(fx+\right.}{60\left(c^7f\cos\left(fx+e\right)^7-18\,c^7f\cos\left(fx+e\right)^5+48\,c^7f\cos\left(fx+e\right)^3-32\,c^7f\cos\left(fx+e\right)+2\left(3\,c^7f\cos\left(fx+e\right)^5\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x
, algorithm="fricas")
```

```
[Out] 1/60*(15*(A + B)*a^2*cos(f*x + e)^2 - 2*(11*A + 7*B)*a^2 + 2*(10*B*a^2*cos(
f*x + e)^2 - (9*A + 13*B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(
-c*sin(f*x + e) + c)/(c^7*f*cos(f*x + e)^7 - 18*c^7*f*cos(f*x + e)^5 + 48*c
^7*f*cos(f*x + e)^3 - 32*c^7*f*cos(f*x + e) + 2*(3*c^7*f*cos(f*x + e)^5 - 1
6*c^7*f*cos(f*x + e)^3 + 16*c^7*f*cos(f*x + e))*sin(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.72, size = 423, normalized size = 2.16

$$\frac{\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}\left(444A\sin\left(fx+e\right)-202A\sin\left(fx+e\right)\cos\left(fx+e\right)+29B\left(\cos^2\left(fx+e\right)\right)\mathrm{s}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x)
```

```
[Out] 1/60/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(-17*B*cos(f*x+e)^3*sin(f*x+e)+4
44*A*sin(f*x+e)-202*A*sin(f*x+e)*cos(f*x+e)+119*A*cos(f*x+e)^3*sin(f*x+e)+4
9*A*cos(f*x+e)^4*sin(f*x+e)-7*B*sin(f*x+e)*cos(f*x+e)^4-343*A*cos(f*x+e)^2*
sin(f*x+e)+29*B*cos(f*x+e)^2*sin(f*x+e)+24*B*cos(f*x+e)^4-444*A+52*B-224*A*
cos(f*x+e)^3+46*B*sin(f*x+e)*cos(f*x+e)+42*A*cos(f*x+e)^5-6*B*cos(f*x+e)^5-
```

```
B*cos(f*x+e)^6+B*cos(f*x+e)^5*sin(f*x+e)-7*A*cos(f*x+e)^5*sin(f*x+e)-75*B*c
os(f*x+e)^2-168*A*cos(f*x+e)^4+12*B*cos(f*x+e)^3-52*B*sin(f*x+e)+242*A*cos(
f*x+e)-6*B*cos(f*x+e)+7*A*cos(f*x+e)^6+545*A*cos(f*x+e)^2)/(-c*(sin(f*x+e)-
1))^(13/2)/(cos(f*x+e)^3-cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-2*sin(f*x+e
)*cos(f*x+e)-2*cos(f*x+e)+4*sin(f*x+e)+4)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x
, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 20.71, size = 357, normalized size = 1.82

$$\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{a^2\,{\mathrm{e}}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\left(A\,29\mathrm{i}+B\,13\mathrm{i}\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,16\mathrm{i}}{15\,c^7\,f}-\frac{a^2\,{\mathrm{e}}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\left(A\,1\mathrm{i}+B\,1\mathrm{i}\right)\,\sqrt{a+a\,\mathrm{si}}}{c^7\,f}\right.}{-858\,\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}+858\,{\mathrm{e}}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)-130\,{\mathrm{e}}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)+2\,{\mathrm{e}}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\mathrm{cos}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^
(13/2),x)
```

```
[Out] ((c - c*sin(e + f*x))^(1/2)*((a^2*exp(e*7i + f*x*7i)*(A*29i + B*13i)*(a + a
*sin(e + f*x))^(1/2)*16i)/(15*c^7*f) - (a^2*exp(e*7i + f*x*7i)*cos(2*e + 2*
f*x)*(A*1i + B*1i)*(a + a*sin(e + f*x))^(1/2)*16i)/(c^7*f) - (32*a^2*exp(e*
7i + f*x*7i)*sin(e + f*x)*(6*A + 7*B)*(a + a*sin(e + f*x))^(1/2))/(5*c^7*f)
 + (32*B*a^2*exp(e*7i + f*x*7i)*sin(3*e + 3*f*x)*(a + a*sin(e + f*x))^(1/2)
)/(3*c^7*f)))/(858*exp(e*7i + f*x*7i)*cos(3*e + 3*f*x) - 858*cos(e + f*x)*e
xp(e*7i + f*x*7i) - 130*exp(e*7i + f*x*7i)*cos(5*e + 5*f*x) + 2*exp(e*7i +
f*x*7i)*cos(7*e + 7*f*x) + 1144*exp(e*7i + f*x*7i)*sin(2*e + 2*f*x) - 416*e
xp(e*7i + f*x*7i)*sin(4*e + 4*f*x) + 24*exp(e*7i + f*x*7i)*sin(6*e + 6*f*x)
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(13/2)
,x)
```

```
[Out] Timed out
```

3.160 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx$

Optimal. Leaf size=250

$$-\frac{a^4(9A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{315f\sqrt{a\sin(e+fx)+a}}-\frac{a^3(9A-B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{9/2}}{126f}-$$

```
[Out] -1/84*a^2*(9*A-B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(9/2)/
f-1/72*a*(9*A-B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(9/2)/f
-1/9*B*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2)/f-1/315*a^4
*(9*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(9/2)/f/(a+a*sin(f*x+e))^(1/2)-1/126*a
^3*(9*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(9/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.57, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2973, 2740, 2738}

$$-\frac{a^2(9A-B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{9/2}}{84f}-\frac{a^3(9A-B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c}{126f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9
/2),x]
```

```
[Out] -(a^4*(9*A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(315*f*Sqrt[a + a*
Sin[e + f*x]]) - (a^3*(9*A - B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c -
c*Sin[e + f*x])^(9/2))/(126*f) - (a^2*(9*A - B)*Cos[e + f*x]*(a + a*Sin[e +
 f*x])^(3/2)*(c - c*Sin[e + f*x])^(9/2))/(84*f) - (a*(9*A - B)*Cos[e + f*x]
*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(9/2))/(72*f) - (B*Cos[e +
 f*x]*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(9/2))/(9*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
```

```
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx=-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c}{9f}$$

$$=-\frac{a(9A-B)\cos(e+fx)(a+a\sin(e+fx)}{72f}$$

$$=-\frac{a^2(9A-B)\cos(e+fx)(a+a\sin(e+fx}{84f}$$

$$=-\frac{a^3(9A-B)\cos(e+fx)\sqrt{a+a\sin(e+f}}{126f}$$

$$=-\frac{a^4(9A-B)\cos(e+fx)(c-c\sin(e+fx)}{315f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 6.85, size = 269, normalized size = 1.08

$$\frac{a^3c^4(\sin(e+fx)-1)^4(\sin(e+fx)+1)^3\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}\,(17640(A-B)\cos(2(e+fx))+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(9/2),x]
```

```
[Out] (a^3*c^4*(-1 + Sin[e + f*x])^4*(1 + Sin[e + f*x])^3*Sqrt[a*(1 + Sin[e + f*x
])]*Sqrt[c - c*Sin[e + f*x]]*(17640*(A - B)*Cos[2*(e + f*x)] + 8820*(A - B)
*Cos[4*(e + f*x)] + 2520*A*Cos[6*(e + f*x)] - 2520*B*Cos[6*(e + f*x)] + 315
*A*Cos[8*(e + f*x)] - 315*B*Cos[8*(e + f*x)] + 176400*A*Sin[e + f*x] - 1764
0*B*Sin[e + f*x] + 35280*A*Sin[3*(e + f*x)] + 7056*A*Sin[5*(e + f*x)] + 201
6*B*Sin[5*(e + f*x)] + 720*A*Sin[7*(e + f*x)] + 900*B*Sin[7*(e + f*x)] + 14
0*B*Sin[9*(e + f*x)]))/(322560*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^7)
```

fricas [A] time = 0.53, size = 183, normalized size = 0.73

$$\frac{\left(315\,(A-B)a^3c^4\cos\left(fx+e\right)^8 - 315\,(A-B)a^3c^4 + 8\left(35\,Ba^3c^4\cos\left(fx+e\right)^8 + 5\,(9\,A-B)a^3c^4\cos\left(fx+e\right)^6 + 6\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x,
 algorithm="fricas")
```

```
[Out] 1/2520*(315*(A - B)*a^3*c^4*cos(f*x + e)^8 - 315*(A - B)*a^3*c^4 + 8*(35*B*
a^3*c^4*cos(f*x + e)^8 + 5*(9*A - B)*a^3*c^4*cos(f*x + e)^6 + 6*(9*A - B)*a
^3*c^4*cos(f*x + e)^4 + 8*(9*A - B)*a^3*c^4*cos(f*x + e)^2 + 16*(9*A - B)*a
^3*c^4)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f
*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
```

to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
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2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
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le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)
*(-2048*f*(A*a^3*c^4*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi))-B*a^3*c^4*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(
f*x+exp(1))-1/4*pi)))*cos(8*f*x+8*exp(1))/(2048*f)^2-512*f*(5*A*a^3*c^4*sig
n(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-5*B*a^3*
c^4*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*
cos(2*f*x+2*exp(1))/(512*f)^2-1536*f*(5*A*a^3*c^4*sign(sin(1/2*(f*x+exp(1))
-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-5*B*a^3*c^4*sign(sin(1/2*(f*x+
exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(6*f*x+6*exp(1))/(1
536*f)^2-1024*f*(9*A*a^3*c^4*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/
2*(f*x+exp(1))-1/4*pi))-9*B*a^3*c^4*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign
(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(4*f*x+4*exp(1))/(1024*f)^2+1536*f*(-A*a
^3*c^4*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)
)+B*a^3*c^4*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/
4*pi)))*cos(-6*f*x-6*exp(1))/(-1536*f)^2+3584*f*(-4*A*a^3*c^4*sign(sin(1/2*
(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-5*B*a^3*c^4*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(7*f*x+
7*exp(1))/(3584*f)^2+1024*f*(-5*A*a^3*c^4*sign(sin(1/2*(f*x+exp(1))-1/4*pi)
)*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+5*B*a^3*c^4*sign(sin(1/2*(f*x+exp(1))-
1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(-4*f*x-4*exp(1))/(-1024*f)
^2+640*f*(-7*A*a^3*c^4*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x
+exp(1))-1/4*pi))-2*B*a^3*c^4*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1
/2*(f*x+exp(1))-1/4*pi)))*sin(5*f*x+5*exp(1))/(640*f)^2+512*f*(-9*A*a^3*c^4
sign(sin(1/2(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+9*B*
a^3*c^4*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi
)))*cos(-2*f*x-2*exp(1))/(-512*f)^2+256*f*(-70*A*a^3*c^4*sign(sin(1/2*(f*x+
exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+7*B*a^3*c^4*sign(sin(1/
2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(f*x+exp(1))
/(256*f)^2-8064*A*a^3*c^4*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))*sin(3*f*x+3*exp(1))/(384*f)^2-4608*B*a^3*c^4*f*sign(

```
sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(9*f*x+
9*exp(1))/(4608*f)^2)
```

maple [A] time = 0.83, size = 259, normalized size = 1.04

$$\frac{\left(-280B\left(\cos^8\left(fx+e\right)\right)+315A\left(\cos^6\left(fx+e\right)\right)\sin\left(fx+e\right)-315B\left(\cos^6\left(fx+e\right)\right)\sin\left(fx+e\right)-360A\left(\cos^6\left(f\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] 1/2520/f*(-280*B*cos(f*x+e)^8+315*A*cos(f*x+e)^6*sin(f*x+e)-315*B*cos(f*x+e
)^6*sin(f*x+e)-360*A*cos(f*x+e)^6+40*B*cos(f*x+e)^6+315*A*cos(f*x+e)^4*sin(
f*x+e)-315*B*sin(f*x+e)*cos(f*x+e)^4-432*A*cos(f*x+e)^4+48*B*cos(f*x+e)^4+3
15*A*cos(f*x+e)^2*sin(f*x+e)-315*B*cos(f*x+e)^2*sin(f*x+e)-576*A*cos(f*x+e)
^2+64*B*cos(f*x+e)^2+315*A*sin(f*x+e)-315*B*sin(f*x+e)-1152*A+128*B)*(-c*(s
in(f*x+e)-1))^(9/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)-1)/cos(
f*x+e)^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e)
+ c)^(9/2), x)
```

mupad [B] time = 20.03, size = 482, normalized size = 1.93

$$\frac{e^{-e\,9i-f\,x\,9i}\sqrt{c-c\,\sin\left(e+f\,x\right)}\left(-\frac{a^3\,c^4\,e^{e\,9i+f\,x\,9i}\,\cos\left(2\,e+2\,f\,x\right)\left(A\,1i-B\,1i\right)\sqrt{a+a\,\sin\left(e+f\,x\right)}\,7i}{64\,f}-\frac{a^3\,c^4\,e^{e\,9i+f\,x\,9i}\,\cos\left(4\,e+4\,f\,x\right)\left(A\,1i-B\right.}{128\,f}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(9
/2),x)
```

```
[Out] (exp(- e*9i - f*x*9i)*(c - c*sin(e + f*x))^(1/2)*((a^3*c^4*exp(e*9i + f*x*9
i)*sin(5*e + 5*f*x)*(7*A + 2*B)*(a + a*sin(e + f*x))^(1/2))/(160*f) - (a^3*
```

```
c^4*exp(e*9i + f*x*9i)*cos(4*e + 4*f*x)*(A*1i - B*1i)*(a + a*sin(e + f*x))^
(1/2)*7i)/(128*f) - (a^3*c^4*exp(e*9i + f*x*9i)*cos(6*e + 6*f*x)*(A*1i - B*
1i)*(a + a*sin(e + f*x))^(1/2)*1i)/(64*f) - (a^3*c^4*exp(e*9i + f*x*9i)*cos
(8*e + 8*f*x)*(A*1i - B*1i)*(a + a*sin(e + f*x))^(1/2)*1i)/(512*f) - (a^3*c
^4*exp(e*9i + f*x*9i)*cos(2*e + 2*f*x)*(A*1i - B*1i)*(a + a*sin(e + f*x))^(
1/2)*7i)/(64*f) + (a^3*c^4*exp(e*9i + f*x*9i)*sin(7*e + 7*f*x)*(4*A + 5*B)*
(a + a*sin(e + f*x))^(1/2))/(896*f) + (7*A*a^3*c^4*exp(e*9i + f*x*9i)*sin(3
*e + 3*f*x)*(a + a*sin(e + f*x))^(1/2))/(32*f) + (7*a^3*c^4*exp(e*9i + f*x*
9i)*sin(e + f*x)*(10*A - B)*(a + a*sin(e + f*x))^(1/2))/(64*f) + (B*a^3*c^4
*exp(e*9i + f*x*9i)*sin(9*e + 9*f*x)*(a + a*sin(e + f*x))^(1/2))/(1152*f)))
/(2*cos(e + f*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2),
x)

[Out] Timed out
```

3.161 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=226

$$-\frac{2a^4A\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35f\sqrt{a\sin(e+fx)+a}}-\frac{4a^3A\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}{35f}-\frac{a^2A\cos(e+$$

```
[Out] -1/7*a^2*A*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/f-1/7*a
*A*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2)/f-1/8*B*cos(f*x
+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2)/f-2/35*a^4*A*cos(f*x+e)*(
c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1/2)-4/35*a^3*A*cos(f*x+e)*(c-c*s
in(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.56, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2973, 2740, 2738}

$$-\frac{2a^4A\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35f\sqrt{a\sin(e+fx)+a}}-\frac{4a^3A\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}{35f}-\frac{a^2A\cos(e+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] (-2*a^4*A*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(35*f*Sqrt[a + a*Sin[e +
 f*x]]) - (4*a^3*A*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x
])^(7/2))/(35*f) - (a^2*A*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Si
n[e + f*x])^(7/2))/(7*f) - (a*A*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c
- c*Sin[e + f*x])^(7/2))/(7*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)
*(c - c*Sin[e + f*x])^(7/2))/(8*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
```

```
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c}{8f}$$

$$= -\frac{aA\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-}{7f}$$

$$= -\frac{a^2A\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-}{7f}$$

$$= -\frac{4a^3A\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-}{35f}$$

$$= -\frac{2a^4A\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35f\sqrt{a+a\sin(e+fx)}} -$$

Mathematica [A] time = 1.58, size = 135, normalized size = 0.60

$$\frac{a^3c^3\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}\,(19600A\sin(e+fx)+3920A\sin(3(e+fx))+784A\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(7/2),x]
```

```
[Out] (a^3*c^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
-1960*B*Cos[2*(e + f*x)] - 980*B*Cos[4*(e + f*x)] - 280*B*Cos[6*(e + f*x)]
- 35*B*Cos[8*(e + f*x)] + 19600*A*Sin[e + f*x] + 3920*A*Sin[3*(e + f*x)] +
784*A*Sin[5*(e + f*x)] + 80*A*Sin[7*(e + f*x)]))/(35840*f)
```

fricas [A] time = 0.48, size = 134, normalized size = 0.59

$$-\frac{\left(35\,Ba^3c^3\cos\left(fx+e\right)^8-35\,Ba^3c^3-8\left(5\,Aa^3c^3\cos\left(fx+e\right)^6+6\,Aa^3c^3\cos\left(fx+e\right)^4+8\,Aa^3c^3\cos\left(fx+e\right)^2\right.\right.}{280\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="fricas")
```

```
[Out] -1/280*(35*B*a^3*c^3*cos(f*x + e)^8 - 35*B*a^3*c^3 - 8*(5*A*a^3*c^3*cos(f*x
 + e)^6 + 6*A*a^3*c^3*cos(f*x + e)^4 + 8*A*a^3*c^3*cos(f*x + e)^2 + 16*A*a^
3*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*
cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
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able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
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to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sq
rt(2*c)*(-4480*A*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*
(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1))/(128*f)^2-8064*A*a^3*c^3*f*sign(sin(1
```

```
/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(3*f*x+3*exp
(1))/(384*f)^2-4480*A*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos
(1/2*(f*x+exp(1))-1/4*pi))*sin(5*f*x+5*exp(1))/(640*f)^2-896*A*a^3*c^3*f*si
gn(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(7*f
*x+7*exp(1))/(896*f)^2+2560*B*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*
sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(2*f*x+2*exp(1))/(512*f)^2+9216*B*a^3
*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)
)*cos(4*f*x+4*exp(1))/(1024*f)^2+7680*B*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))
-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(6*f*x+6*exp(1))/(1536*f)^2
+2048*B*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(
1))-1/4*pi))*cos(8*f*x+8*exp(1))/(2048*f)^2+4608*B*a^3*c^3*f*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-2*f*x-2*exp(1)
)/(-512*f)^2+5120*B*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1
/2*(f*x+exp(1))-1/4*pi))*cos(-4*f*x-4*exp(1))/(-1024*f)^2+1536*B*a^3*c^3*f*
sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-
6*f*x-6*exp(1))/(-1536*f)^2)
```

maple [A] time = 0.84, size = 142, normalized size = 0.63

$$\frac{\left(35B\left(\cos^{6}\left(fx+e\right)\right)\sin\left(fx+e\right)+40A\left(\cos^{6}\left(fx+e\right)\right)+35B\sin\left(fx+e\right)\left(\cos^{4}\left(fx+e\right)\right)+48A\left(\cos^{4}\left(fx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/280/f*(35*B*cos(f*x+e)^6*sin(f*x+e)+40*A*cos(f*x+e)^6+35*B*sin(f*x+e)*cos
(f*x+e)^4+48*A*cos(f*x+e)^4+35*B*cos(f*x+e)^2*sin(f*x+e)+64*A*cos(f*x+e)^2+
35*B*sin(f*x+e)+128*A)*(-c*(sin(f*x+e)-1))^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e
)))^(7/2)/cos(f*x+e)^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e)
+ c)^(7/2), x)
```

mupad [B] time = 17.65, size = 384, normalized size = 1.70

$$\frac{e^{-e\,8i-f\,x\,8i}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{35\,A\,a^3\,c^3\,e^{e\,8i+f\,x\,8i}\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{32\,f}-\frac{7\,B\,a^3\,c^3\,e^{e\,8i+f\,x\,8i}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{64\,f}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(7
/2),x)
```

```
[Out] (exp(- e*8i - f*x*8i)*(c - c*sin(e + f*x))^(1/2)*((35*A*a^3*c^3*exp(e*8i +
f*x*8i)*sin(e + f*x)*(a + a*sin(e + f*x))^(1/2))/(32*f) - (7*B*a^3*c^3*exp(
e*8i + f*x*8i)*cos(2*e + 2*f*x)*(a + a*sin(e + f*x))^(1/2))/(64*f) - (7*B*a
^3*c^3*exp(e*8i + f*x*8i)*cos(4*e + 4*f*x)*(a + a*sin(e + f*x))^(1/2))/(128
*f) - (B*a^3*c^3*exp(e*8i + f*x*8i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*x))^(
1/2))/(64*f) - (B*a^3*c^3*exp(e*8i + f*x*8i)*cos(8*e + 8*f*x)*(a + a*sin(e
+ f*x))^(1/2))/(512*f) + (7*A*a^3*c^3*exp(e*8i + f*x*8i)*sin(3*e + 3*f*x)*(
a + a*sin(e + f*x))^(1/2))/(32*f) + (7*A*a^3*c^3*exp(e*8i + f*x*8i)*sin(5*e
 + 5*f*x)*(a + a*sin(e + f*x))^(1/2))/(160*f) + (A*a^3*c^3*exp(e*8i + f*x*8
i)*sin(7*e + 7*f*x)*(a + a*sin(e + f*x))^(1/2))/(224*f)))/(2*cos(e + f*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Timed out
```

3.162 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=192

$$\frac{c^3(7A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{105f\sqrt{c-c\sin(e+fx)}}+\frac{2c^2(7A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{105f}$$

```
[Out] 1/42*(7*A+B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2)/f-1
/7*B*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2)/f+1/105*(7*A+
B)*c^3*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(1/2)+2/105*(7*
A+B)*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.46, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2973, 2740, 2738}

$$\frac{2c^2(7A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{105f}+\frac{c^3(7A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{105f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] ((7*A + B)*c^3*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(105*f*Sqrt[c - c*S
in[e + f*x]]) + (2*(7*A + B)*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sq
rt[c - c*Sin[e + f*x]])/(105*f) + ((7*A + B)*c*Cos[e + f*x]*(a + a*Sin[e +
f*x])^(7/2)*(c - c*Sin[e + f*x])^(3/2))/(42*f) - (B*Cos[e + f*x]*(a + a*Sin
[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(5/2))/(7*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
```

```
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\,\mathrm{s}}{7f}$$

$$= \frac{(7A+B)c\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{42f}$$

$$= \frac{2(7A+B)c^2\cos(e+fx)(a+a\sin(e+fx))}{105f}$$

$$= \frac{(7A+B)c^3\cos(e+fx)(a+a\sin(e+fx))^{7/}}{105f\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 2.07, size = 232, normalized size = 1.21

$$\frac{a^3c^2(\sin(e+fx)-1)^2(\sin(e+fx)+1)^3\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(-525(A+B)\cos(2(e+fx))-21}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(5/2),x]
```

```
[Out] (a^3*c^2*(-1 + Sin[e + f*x])^2*(1 + Sin[e + f*x])^3*Sqrt[a*(1 + Sin[e + f*x
])]*Sqrt[c - c*Sin[e + f*x]]*(-525*(A + B)*Cos[2*(e + f*x)] - 210*(A + B)*C
os[4*(e + f*x)] - 35*A*Cos[6*(e + f*x)] - 35*B*Cos[6*(e + f*x)] + 4200*A*Si
n[e + f*x] + 525*B*Sin[e + f*x] + 700*A*Sin[3*(e + f*x)] - 35*B*Sin[3*(e +
```

```
f*x)] + 84*A*Sin[5*(e + f*x)] - 63*B*Sin[5*(e + f*x)] - 15*B*Sin[7*(e + f*x
)]))/(6720*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^7)
```

fricas [A] time = 0.48, size = 150, normalized size = 0.78

$$-\frac{\left(35\,(A+B)a^3c^2\cos\left(fx+e\right)^6-35\,(A+B)a^3c^2+2\left(15\,Ba^3c^2\cos\left(fx+e\right)^6-3\,(7\,A+B)a^3c^2\cos\left(fx+e\right)^4-\right.\right.}{210\,f\cos\left(f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] -1/210*(35*(A + B)*a^3*c^2*cos(f*x + e)^6 - 35*(A + B)*a^3*c^2 + 2*(15*B*a^
3*c^2*cos(f*x + e)^6 - 3*(7*A + B)*a^3*c^2*cos(f*x + e)^4 - 4*(7*A + B)*a^3
*c^2*cos(f*x + e)^2 - 8*(7*A + B)*a^3*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e
) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
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```

```
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(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)
*sqrt(2*c)*(256*f*(A*a^3*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/
2*(f*x+exp(1))-1/4*pi))+B*a^3*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(c
os(1/2*(f*x+exp(1))-1/4*pi)))*cos(-4*f*x-4*exp(1))/(-256*f)^2+128*f*(3*A*a^
3*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))
+3*B*a^3*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1
/4*pi)))*cos(-2*f*x-2*exp(1))/(-128*f)^2-64*f*(-A*a^3*c^2*sign(sin(1/2*(f*x
+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-B*a^3*c^2*sign(sin(1/2
*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(2*f*x+2*exp(
1))/(64*f)^2-384*f*(-A*a^3*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(
1/2*(f*x+exp(1))-1/4*pi))-B*a^3*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign
(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(6*f*x+6*exp(1))/(384*f)^2-256*f*(-3*A*a
^3*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)
)-3*B*a^3*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-
1/4*pi)))*cos(4*f*x+4*exp(1))/(256*f)^2+640*f*(-4*A*a^3*c^2*sign(sin(1/2*(f
*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+3*B*a^3*c^2*sign(sin
(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(5*f*x+5*
exp(1))/(640*f)^2+384*f*(-20*A*a^3*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*s
ign(cos(1/2*(f*x+exp(1))-1/4*pi))+B*a^3*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*p
i))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(3*f*x+3*exp(1))/(384*f)^2+128*f
*(-40*A*a^3*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1)
)-1/4*pi))-5*B*a^3*c^2*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x
+exp(1))-1/4*pi)))*sin(f*x+exp(1))/(128*f)^2+896*B*a^3*c^2*f*sign(sin(1/2*(
f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(7*f*x+7*exp(1))
/(896*f)^2)
```

maple [A] time = 0.85, size = 203, normalized size = 1.06

$$\frac{\left(-30B\left(\cos^{6}\left(fx+e\right)\right)+35A\left(\cos^{4}\left(fx+e\right)\right)\sin\left(fx+e\right)+35B\sin\left(fx+e\right)\left(\cos^{4}\left(fx+e\right)\right)+42A\left(\cos^{4}\left(fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/210/f*(-30*B*cos(f*x+e)^6+35*A*cos(f*x+e)^4*sin(f*x+e)+35*B*sin(f*x+e)*co
s(f*x+e)^4+42*A*cos(f*x+e)^4+6*B*cos(f*x+e)^4+35*A*cos(f*x+e)^2*sin(f*x+e)+
35*B*cos(f*x+e)^2*sin(f*x+e)+56*A*cos(f*x+e)^2+8*B*cos(f*x+e)^2+35*A*sin(f*
x+e)+35*B*sin(f*x+e)+112*A+16*B)*(-c*(sin(f*x+e)-1))^(5/2)*sin(f*x+e)*(a*(1
+sin(f*x+e)))^(7/2)/(1+sin(f*x+e))/cos(f*x+e)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e)
+ c)^(5/2), x)
```

mupad [B] time = 18.43, size = 383, normalized size = 1.99

$$\frac{e^{-e\,7\mathrm{i}-f\,x\,7\mathrm{i}}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{a^3\,c^2\,e^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\left(A\,1\mathrm{i}+B\,1\mathrm{i}\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,5\mathrm{i}}{32\,f}+\frac{a^3\,c^2\,e^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos\left(4\,e+4\,f\,x\right)\,\left(A\,1\mathrm{i}+B\right.}{16\,f}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(5
/2),x)
```

```
[Out] (exp(- e*7i - f*x*7i)*(c - c*sin(e + f*x))^(1/2)*((a^3*c^2*exp(e*7i + f*x*7
i)*cos(2*e + 2*f*x)*(A*1i + B*1i)*(a + a*sin(e + f*x))^(1/2)*5i)/(32*f) + (
a^3*c^2*exp(e*7i + f*x*7i)*cos(4*e + 4*f*x)*(A*1i + B*1i)*(a + a*sin(e + f*
x))^(1/2)*1i)/(16*f) + (a^3*c^2*exp(e*7i + f*x*7i)*cos(6*e + 6*f*x)*(A*1i +
 B*1i)*(a + a*sin(e + f*x))^(1/2)*1i)/(96*f) + (a^3*c^2*exp(e*7i + f*x*7i)*
sin(5*e + 5*f*x)*(4*A - 3*B)*(a + a*sin(e + f*x))^(1/2))/(160*f) + (a^3*c^2
*exp(e*7i + f*x*7i)*sin(3*e + 3*f*x)*(20*A - B)*(a + a*sin(e + f*x))^(1/2))
/(96*f) + (5*a^3*c^2*exp(e*7i + f*x*7i)*sin(e + f*x)*(8*A + B)*(a + a*sin(e
 + f*x))^(1/2))/(32*f) - (B*a^3*c^2*exp(e*7i + f*x*7i)*sin(7*e + 7*f*x)*(a
+ a*sin(e + f*x))^(1/2))/(224*f)))/(2*cos(e + f*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

3.163 $\int (a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2}\,dx$

Optimal. Leaf size=142

$$\frac{c^2(3A + B)\cos(e + fx)(a\sin(e + fx) + a)^{7/2}}{30f\sqrt{c - c\sin(e + fx)}} + \frac{c(3A + B)\cos(e + fx)(a\sin(e + fx) + a)^{7/2}\sqrt{c - c\sin(e + fx)}}{15f} - \frac{B c}{}$$

```
[Out] -1/6*B*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2)/f+1/30*(3*A
+B)*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(1/2)+1/15*(3*
A+B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.36, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2973, 2740, 2738}

$$\frac{c^2(3A + B)\cos(e + fx)(a\sin(e + fx) + a)^{7/2}}{30f\sqrt{c - c\sin(e + fx)}} + \frac{c(3A + B)\cos(e + fx)(a\sin(e + fx) + a)^{7/2}\sqrt{c - c\sin(e + fx)}}{15f} - \frac{B c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] ((3*A + B)*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(30*f*Sqrt[c - c*Si
n[e + f*x]]) + ((3*A + B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c
- c*Sin[e + f*x]])/(15*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*(c -
 c*Sin[e + f*x])^(3/2))/(6*f)
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
```

```
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c}{6f}$$

$$= \frac{(3A+B)c\cos(e+fx)(a+a\sin(e+fx))^{7}}{15f}$$

$$= \frac{(3A+B)c^2\cos(e+fx)(a+a\sin(e+fx))}{30f\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 1.85, size = 212, normalized size = 1.49

$$-\frac{a^3c(\sin(e+fx)-1)(\sin(e+fx)+1)^3\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}\,(-15(16A+11B)\cos(2(e+fx))}{960f\Big(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(3/2),x]
```

```
[Out] -1/960*(a^3*c*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])^3*Sqrt[a*(1 + Sin[e +
f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-15*(16*A + 11*B)*Cos[2*(e + f*x)] - 30*(2
*A + B)*Cos[4*(e + f*x)] + 5*B*Cos[6*(e + f*x)] + 840*A*Sin[e + f*x] + 240*
B*Sin[e + f*x] + 60*A*Sin[3*(e + f*x)] - 40*B*Sin[3*(e + f*x)] - 12*A*Sin[5
*(e + f*x)] - 24*B*Sin[5*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7)
```

fricas [A] time = 0.46, size = 142, normalized size = 1.00

$$\frac{\left(5\,Ba^3c\cos\left(fx+e\right)^6-15\,(A+B)a^3c\cos\left(fx+e\right)^4+5\,(3\,A+2\,B)a^3c-2\left(3\,(A+2\,B)a^3c\cos\left(fx+e\right)^4-2\,(3\,A\right.}{30\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] 1/30*(5*B*a^3*c*cos(f*x + e)^6 - 15*(A + B)*a^3*c*cos(f*x + e)^4 + 5*(3*A +
 2*B)*a^3*c - 2*(3*(A + 2*B)*a^3*c*cos(f*x + e)^4 - 2*(3*A + B)*a^3*c*cos(f
*x + e)^2 - 4*(3*A + B)*a^3*c)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(
-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
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able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
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to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(160*f*(A*a^3*c*sign(sin(1/2
*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+2*B*a^3*c*sign(si
n(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(5*f*x+5
*exp(1))/(160*f)^2+128*f*(8*A*a^3*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign
```

```
(cos(1/2*(f*x+exp(1))-1/4*pi))+5*B*a^3*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))
*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(-2*f*x-2*exp(1))/(-128*f)^2+96*f*(
-3*A*a^3*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4
*pi))+2*B*a^3*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1)
)-1/4*pi)))*sin(3*f*x+3*exp(1))/(96*f)^2-64*f*(-4*A*a^3*c*sign(sin(1/2*(f*x
+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-3*B*a^3*c*sign(sin(1/2
*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(2*f*x+2*exp(
1))/(64*f)^2+16*f*(-7*A*a^3*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1
/2*(f*x+exp(1))-1/4*pi))-2*B*a^3*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(
cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(f*x+exp(1))/(16*f)^2-256*f*(-8*A*a^3*c*s
ign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-5*B*a^
3*c*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*
cos(4*f*x+4*exp(1))/(256*f)^2-384*B*a^3*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*p
i))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(6*f*x+6*exp(1))/(384*f)^2-256*B*
a^3*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi
))*cos(-4*f*x-4*exp(1))/(-256*f)^2)
```

maple [A] time = 0.80, size = 185, normalized size = 1.30

$$\frac{\left(5B\sin\left(fx+e\right)\left(\cos^4\left(fx+e\right)\right)+6A\left(\cos^4\left(fx+e\right)\right)+12B\left(\cos^4\left(fx+e\right)\right)-15A\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/30/f*(5*B*sin(f*x+e)*cos(f*x+e)^4+6*A*cos(f*x+e)^4+12*B*cos(f*x+e)^4-15*A
*cos(f*x+e)^2*sin(f*x+e)-10*B*cos(f*x+e)^2*sin(f*x+e)-12*A*cos(f*x+e)^2-4*B
*cos(f*x+e)^2-15*A*sin(f*x+e)-10*B*sin(f*x+e)-24*A-8*B)*(-c*(sin(f*x+e)-1))
^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)/(cos(f*x+e)^2-2*sin(f*x+e)-2)/co
s(f*x+e)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e)
+ c)^(3/2), x)
```

mupad [B] time = 18.17, size = 321, normalized size = 2.26

$$-\frac{e^{-e\,6i-f\,x\,6i}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{a^3\,c\,e^{e\,6i+f\,x\,6i}\,\cos\left(4\,e+4\,f\,x\right)\,\left(2\,A+B\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{16\,f}-\frac{B\,a^3\,c\,e^{e\,6i+f\,x\,6i}\,\cos\left(6\,e+6\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+\right.}}{96\,f}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(3
/2),x)
```

```
[Out] -(exp(- e*6i - f*x*6i)*(c - c*sin(e + f*x))^(1/2)*((a^3*c*exp(e*6i + f*x*6i
)*cos(4*e + 4*f*x)*(2*A + B)*(a + a*sin(e + f*x))^(1/2))/(16*f) - (B*a^3*c*
exp(e*6i + f*x*6i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*x))^(1/2))/(96*f) + (a
^3*c*exp(e*6i + f*x*6i)*sin(e + f*x)*(A*7i + B*2i)*(a + a*sin(e + f*x))^(1/
2)*1i)/(4*f) + (a^3*c*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(16*A + 11*B)*(a
+ a*sin(e + f*x))^(1/2))/(32*f) + (a^3*c*exp(e*6i + f*x*6i)*sin(3*e + 3*f*x
)*(A*3i - B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)/(24*f) - (a^3*c*exp(e*6i + f
*x*6i)*sin(5*e + 5*f*x)*(A*1i + B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)/(40*f)
))/(2*cos(e + f*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

3.164 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+f$

Optimal. Leaf size=96

$$\frac{c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{5af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/4*(A-B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(1/2)+1/5*
B*c*cos(f*x+e)*(a+a*sin(f*x+e))^(9/2)/a/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2971, 2738}

$$\frac{c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{5af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] ((A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*f*Sqrt[c - c*Sin[e +
 f*x]]) + (B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(5*a*f*Sqrt[c - c*S
in[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{B\int (a+a\sin(e+fx))^{9/2}\sqrt{c-c\sin(e+fx)}\,d}{a}$$
$$= \frac{(A-B)c\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f\sqrt{c-c\sin(e+fx)}} +$$

Mathematica [A] time = 0.97, size = 121, normalized size = 1.26

$$\frac{a^3\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}\,(4(60A+23B)\sin(e+fx)+\cos(4(e+fx))(5A+4B\sin(e}{160f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e
+ f*x]],x]
```

```
[Out] (a^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(4*(6
0*A + 23*B)*Sin[e + f*x] + Cos[4*(e + f*x)]*(5*A + 15*B + 4*B*Sin[e + f*x])
 - 4*Cos[2*(e + f*x)]*(5*(7*A + 5*B) + 4*(5*A + 6*B)*Sin[e + f*x])))/(160*f
)
```

fricas [A] time = 0.45, size = 139, normalized size = 1.45

$$\frac{\left(5\,(A+3\,B)a^3\cos\left(fx+e\right)^4-40\,(A+B)a^3\cos\left(fx+e\right)^2+5\,(7\,A+5\,B)a^3+4\left(Ba^3\cos\left(fx+e\right)^4-(5\,A+7\,B)a\right.\right.}{20\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/20*(5*(A + 3*B)*a^3*cos(f*x + e)^4 - 40*(A + B)*a^3*cos(f*x + e)^2 + 5*(7
*A + 5*B)*a^3 + 4*(B*a^3*cos(f*x + e)^4 - (5*A + 7*B)*a^3*cos(f*x + e)^2 +
2*(5*A + 3*B)*a^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
 e) + c)/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sq
rt(2*c)*(-64*f*(A*a^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+
exp(1))-1/4*pi))+3*B*a^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f
*x+exp(1))-1/4*pi)))*cos(4*f*x+4*exp(1))/(64*f)^2+96*f*(12*A*a^3*sign(sin(1
/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+15*B*a^3*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(3*f*x+
3*exp(1))/(96*f)^2+32*f*(-A*a^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos
(1/2*(f*x+exp(1))-1/4*pi))-3*B*a^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(
cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(-2*f*x-2*exp(1))/(-32*f)^2+16*f*(-14*A*a
^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-7*
B*a^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))
)*sin(f*x+exp(1))/(16*f)^2-32*f*(-15*A*a^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi
))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-13*B*a^3*sign(sin(1/2*(f*x+exp(1))-1/
4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(2*f*x+2*exp(1))/(32*f)^2-160
*B*a^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*p
i))*sin(5*f*x+5*exp(1))/(160*f)^2)
```

maple [B] time = 0.82, size = 174, normalized size = 1.81

$$\frac{\left(-4B\left(\cos^4\left(fx+e\right)\right)+5A\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+15B\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+20A\left(\cos^2\left(fx+e\right.\right.\right.}{20f\left(\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/20/f*(-4*B*cos(f*x+e)^4+5*A*cos(f*x+e)^2*sin(f*x+e)+15*B*cos(f*x+e)^2*sin
(f*x+e)+20*A*cos(f*x+e)^2+28*B*cos(f*x+e)^2-35*A*sin(f*x+e)-25*B*sin(f*x+e)
```

```
-40*A-24*B)*(-c*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)/(
cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^2-4*sin(f*x+e)-4)/cos(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*sqrt(-c*sin(f*x +
 e) + c), x)
```

mupad [B] time = 16.68, size = 173, normalized size = 1.80

$$-\frac{a^3\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(140\,A\,\cos\left(e+f\,x\right)+100\,B\,\cos\left(e+f\,x\right)+135\,A\,\cos\left(3\,e\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(1
/2),x)
```

```
[Out] -(a^3*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(140*A*cos
(e + f*x) + 100*B*cos(e + f*x) + 135*A*cos(3*e + 3*f*x) - 5*A*cos(5*e + 5*f
*x) + 85*B*cos(3*e + 3*f*x) - 15*B*cos(5*e + 5*f*x) - 240*A*sin(2*e + 2*f*x
) + 40*A*sin(4*e + 4*f*x) - 90*B*sin(2*e + 2*f*x) + 48*B*sin(4*e + 4*f*x) -
 2*B*sin(6*e + 6*f*x)))/(160*f*(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

3.165 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=239

$$-\frac{8a^4(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{4a^3(A+B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}-\frac{a^2(A+B)\cos(e+}{f\sqrt{c-c}}$$

```
[Out] -a^2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(1/2)-1/3*a
*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(1/2)-1/4*B*cos
(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(1/2)-8*a^4*(A+B)*cos(f*x
+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-4*a^3*
(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [**A**] time = 0.57, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2973, 2740, 2737, 2667, 31}

$$-\frac{4a^3(A+B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}-\frac{a^2(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f\sqrt{c-c\sin(e+fx)}}-\frac{8a^4(A+B)\cos(e+}{f\sqrt{a\sin(e+fx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (-8*a^4*(A + B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[c - c*Sin[e + f*x]]) - (4*a^3*(A + B)*Cos[e + f*x]*Sqrt[a + a*Sin
[e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]]) - (a^2*(A + B)*Cos[e + f*x]*(a + a
*Sin[e + f*x])^(3/2))/(f*Sqrt[c - c*Sin[e + f*x]]) - (a*(A + B)*Cos[e + f*x
]*(a + a*Sin[e + f*x])^(5/2))/(3*f*Sqrt[c - c*Sin[e + f*x]]) - (B*Cos[e + f
*x]*(a + a*Sin[e + f*x])^(7/2))/(4*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

```
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f\sqrt{c-c\sin(e+fx)}} + (A+B)\int \frac{(a+a\sin(}{\sqrt{c-c\sin}}$$

$$= -\frac{a(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)(}{4f\sqrt{c-}}$$

$$= -\frac{a^2(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a(A+B)\cos(}{3f}$$

$$= -\frac{4a^3(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a^2(A+B)\cos}{f\sqrt{}}$$

$$= -\frac{4a^3(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a^2(A+B)\cos}{f\sqrt{}}$$

$$= -\frac{4a^3(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a^2(A+B)\cos}{f\sqrt{}}$$

$$= -\frac{8a^4(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{4a^3(A+B)\,c}{f}$$

Mathematica [A] time = 2.84, size = 183, normalized size = 0.77

$$-\frac{a^3(\sin(e+fx)+1)^3\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(24(29A+36B)\sin(e+fx)-8(\right.}{96f\sqrt{c-c\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[
e + f*x]],x]
```

```
[Out] -1/96*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*Sqrt[
a*(1 + Sin[e + f*x])]*(-12*(8*A + 15*B)*Cos[2*(e + f*x)] + 3*B*Cos[4*(e + f
*x)] + 1536*(A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 24*(29*A + 3
6*B)*Sin[e + f*x] - 8*(A + 4*B)*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^7*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(Ba^3\cos\left(fx+e\right)^4-(3A+5B)a^3\cos\left(fx+e\right)^2+4(A+B)a^3-\left((A+3B)a^3\cos\left(fx+e\right)^2-4(A\right.\right.}{c\sin\left(fx+e\right)-c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B
)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(a
*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e) - c), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.69, size = 671, normalized size = 2.81

$$-\frac{\left(-64A\sin\left(fx+e\right)+88A\sin\left(fx+e\right)\cos\left(fx+e\right)-32B\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)+192A\sin\left(fx+e\right)\ln\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/12/f*(-16*B*cos(f*x+e)^3*sin(f*x+e)-64*A*sin(f*x+e)+88*A*sin(f*x+e)*cos(
f*x+e)-4*A*cos(f*x+e)^3*sin(f*x+e)+3*B*sin(f*x+e)*cos(f*x+e)^4+192*A*sin(f*
x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-20*A*cos(f*x+e)^2*sin(f*x+e
)-32*B*cos(f*x+e)^2*sin(f*x+e)+13*B*cos(f*x+e)^4+64*A+67*B-24*A*cos(f*x+e)^
3-96*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+192*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))-96*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+112*B*sin(f*x+
e)*cos(f*x+e)+192*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-9
6*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+192*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))-96*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+3*B*cos(f*x+e)^5-
80*B*cos(f*x+e)^2+4*A*cos(f*x+e)^4-48*B*cos(f*x+e)^3-67*B*sin(f*x+e)+24*A*c
os(f*x+e)+45*B*cos(f*x+e)-68*A*cos(f*x+e)^2-192*A*ln(-(-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))+96*A*ln(2/(cos(f*x+e)+1))-192*B*ln(-(-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))+96*B*ln(2/(cos(f*x+e)+1)))*(a*(1+sin(f*x+e)))^(7/2)/(cos(
f*x+e)^4+sin(f*x+e)*cos(f*x+e)^3+3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f*x+e)-8
*cos(f*x+e)^2-4*sin(f*x+e)*cos(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)/(-c*(sin
(f*x+e)-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/sqrt(-c*sin(f*x +
 e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(1/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

3.166 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=271

$$\frac{4a^4(3A+5B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2a^3(3A+5B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}+\frac{a^2(3A+5B)\cos(}{2cf\sqrt{}}$$

```
[Out] 1/2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(3/2)+1/2*a^
2*(3*A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(1/2)+1/
6*a*(3*A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(1/2)+
4*a^4*(3*A+5*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c
*sin(f*x+e))^(1/2)+2*a^3*(3*A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c
-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2972, 2740, 2737, 2667, 31}

$$\frac{2a^3(3A+5B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}+\frac{a^2(3A+5B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}}+\frac{4a^4(3A+5B)\cos}{cf\sqrt{a\sin(e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(2*f*(c - c*Sin[e + f*x])
^(3/2)) + (4*a^4*(3*A + 5*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[
a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (2*a^3*(3*A + 5*B)*Cos[e +
f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f*Sqrt[c - c*Sin[e + f*x]]) + (a^2*(3*A +
 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*Sqrt[c - c*Sin[e + f*
x]]) + (a*(3*A + 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*c*f*Sqrt[
c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
```

```
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(3A+5B)\int \frac{(a+a\sin}{\sqrt{c-c\,s}}}{2c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a(3A+5B)\cos(e}{6cf\sqrt{c}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a^2(3A+5B)\cos(e}{2cf\sqrt{c}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{2a^3(3A+5B)\cos(}{cf\sqrt{c-}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{2a^3(3A+5B)\cos(}{cf\sqrt{c-}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{2a^3(3A+5B)\cos(}{cf\sqrt{c-}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{4a^4(3A+5B)\cos(}{cf\sqrt{a+a\sin(e}}$$

Mathematica [A] time = 3.50, size = 292, normalized size = 1.08

$$\frac{a^3\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-2(27A+59B)\cos(2(e+fx))-117A\sin(e+fx)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(3/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-132
*A - 45*B - 2*(27*A + 59*B)*Cos[2*(e + f*x)] + B*Cos[4*(e + f*x)] - 576*A*L
og[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 960*B*Log[Cos[(e + f*x)/2] - Sin[
(e + f*x)/2]] - 117*A*Sin[e + f*x] - 279*B*Sin[e + f*x] + 576*A*Log[Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] + 960*B*Log[Cos[(e + f*x)/2] - S
in[(e + f*x)/2]]*Sin[e + f*x] - 3*A*Sin[3*(e + f*x)] - 13*B*Sin[3*(e + f*x)
]))/(24*c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*Sqrt[
c - c*Sin[e + f*x]])
```

fricas [F] time = 1.17, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(Ba^3\cos\left(fx+e\right)^4-(3A+5B)a^3\cos\left(fx+e\right)^2+4(A+B)a^3-\left((A+3B)a^3\cos\left(fx+e\right)^2-4(A\right.\right.}{c^2\cos\left(fx+e\right)^2+2c^2\sin\left(fx+e\right)-2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B
)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(a
*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*si
n(f*x + e) - 2*c^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.65, size = 927, normalized size = 3.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/6/f*(240*B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+13*B*c
os(f*x+e)^3*sin(f*x+e)-102*A*sin(f*x+e)+75*A*sin(f*x+e)*cos(f*x+e)+3*A*cos(
f*x+e)^3*sin(f*x+e)-2*B*sin(f*x+e)*cos(f*x+e)^4+288*A*sin(f*x+e)*ln(-(-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))-120*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+24
*A*cos(f*x+e)^2*sin(f*x+e)+48*B*cos(f*x+e)^2*sin(f*x+e)-11*B*cos(f*x+e)^4+1
02*A+166*B+27*A*cos(f*x+e)^3-72*A*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+144*A*c
os(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-144*A*sin(f*x+e)*ln(
2/(cos(f*x+e)+1))+144*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))-72*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+107*B*sin(f*x+e)*cos(f*x+e)+480*B*s
in(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-240*B*sin(f*x+e)*ln(2/
(cos(f*x+e)+1))+240*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
```

```
-120*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+72*A*cos(f*x+e)*sin(f*x+e)*ln(2/(cos
(f*x+e)+1))-144*A*cos(f*x+e)*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))-2*B*cos(f*x+e)^5+120*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-2
40*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)-155*B
*cos(f*x+e)^2-3*A*cos(f*x+e)^4+61*B*cos(f*x+e)^3-166*B*sin(f*x+e)-27*A*cos(
f*x+e)-59*B*cos(f*x+e)-99*A*cos(f*x+e)^2-288*A*ln(-(-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))+144*A*ln(2/(cos(f*x+e)+1))-480*B*ln(-(-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))+240*B*ln(2/(cos(f*x+e)+1)))*(a*(1+sin(f*x+e)))^(7/2)/(cos(f
*x+e)^4+sin(f*x+e)*cos(f*x+e)^3+3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f*x+e)-8*
cos(f*x+e)^2-4*sin(f*x+e)*cos(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)/(-c*(sin(
f*x+e)-1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

3.167 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=263

$$-\frac{6a^4(A+3B)\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{3a^3(A+3B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f\sqrt{c-c\sin(e+fx)}}-\frac{3a^2(A+3B)\cos(e}{4c^2f\sqrt{c}}$$

```
[Out] 1/4*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(5/2)-1/2*a*
(A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(3/2)-3/4*a^
2*(A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(1/2)-6*
a^4*(A+3*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*s
in(f*x+e))^(1/2)-3*a^3*(A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c
*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2972, 2739, 2740, 2737, 2667, 31}

$$-\frac{3a^3(A+3B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f\sqrt{c-c\sin(e+fx)}}-\frac{3a^2(A+3B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4c^2f\sqrt{c-c\sin(e+fx)}}-\frac{6a^4(A+3B)\cos(e}{c^2f\sqrt{a\sin(e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*f*(c - c*Sin[e + f*x])
^(5/2)) - (a*(A + 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(2*c*f*(c -
 c*Sin[e + f*x])^(3/2)) - (6*a^4*(A + 3*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x
]])/(c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (3*a^3*(A +
 3*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^2*f*Sqrt[c - c*Sin[e + f*x]
]) - (3*a^2*(A + 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*c^2*f*Sqr
t[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
```

```
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{(A+3B)\int \frac{(a+a\sin(}{(c-c\sin(}}{2c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+3B)\cos(e+}{2cf(c-c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+3B)\cos(e+}{2cf(c-c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+3B)\cos(e+}{2cf(c-c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+3B)\cos(e+}{2cf(c-c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+3B)\cos(e+}{2cf(c-c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+3B)\cos(e+}{2cf(c-c}$$

Mathematica [A] time = 2.57, size = 251, normalized size = 0.95

$$\frac{(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-4(A+6B)\sin(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2)*(16*(A
+ B) - 16*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + B*Cos[2*(e
+ f*x)]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 48*(A + 3*B)*Log[Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 4*(A
 + 6*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(4*f*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(5/2))
```

fricas [F] time = 25.36, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(Ba^3\cos\left(fx+e\right)^4-(3A+5B)a^3\cos\left(fx+e\right)^2+4(A+B)a^3-\left((A+3B)a^3\cos\left(fx+e\right)^2-4(A\right.\right.}{3c^3\cos\left(fx+e\right)^2-4c^3-\left(c^3\cos\left(fx+e\right)^2-4c^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B
)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(a
*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3
- (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.65, size = 1191, normalized size = 4.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/2/f*(-216*B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+10*B*
cos(f*x+e)^3*sin(f*x+e)+32*A*sin(f*x+e)-12*A*sin(f*x+e)*cos(f*x+e)+2*A*cos(
f*x+e)^3*sin(f*x+e)-B*sin(f*x+e)*cos(f*x+e)^4-96*A*sin(f*x+e)*ln(-(-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))+108*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-22*A*
cos(f*x+e)^2*sin(f*x+e)-63*B*cos(f*x+e)^2*sin(f*x+e)-9*B*cos(f*x+e)^4-32*A-
100*B-20*A*cos(f*x+e)^3+72*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos
(f*x+e)^3-36*B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^3+36*A*cos(f*x+e)^2*ln(2/(co
s(f*x+e)+1))-72*A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+4
8*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-48*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))+24*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-46*B*sin(f*x+e)*co
s(f*x+e)-288*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+144*B*
```

```
sin(f*x+e)*ln(2/(cos(f*x+e)+1))-144*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))+72*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-24*A*cos(f*x+e)*sin(f
*x+e)*ln(2/(cos(f*x+e)+1))+48*A*cos(f*x+e)*sin(f*x+e)*ln(-(-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))-B*cos(f*x+e)^5+72*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-36*B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^2*si
n(f*x+e)-72*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)+144*B*ln(-(-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)+109*B*cos(f*x+e)^2+24*
A*cos(f*x+e)^3*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-12*A*cos(f*x+e)^3
*ln(2/(cos(f*x+e)+1))-2*A*cos(f*x+e)^4-12*A*cos(f*x+e)^2*sin(f*x+e)*ln(2/(c
os(f*x+e)+1))+24*A*cos(f*x+e)^2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))-53*B*cos(f*x+e)^3+100*B*sin(f*x+e)+20*A*cos(f*x+e)+54*B*cos(f*x+
e)+34*A*cos(f*x+e)^2+96*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-48*A*l
n(2/(cos(f*x+e)+1))+288*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-144*B*
ln(2/(cos(f*x+e)+1)))*(a*(1+sin(f*x+e)))^(7/2)/(cos(f*x+e)^4+sin(f*x+e)*cos
(f*x+e)^3+3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2-4*sin(f*x
+e)*cos(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(5/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

3.168 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=264

$$\frac{a^4(A+7B)\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{a^3(A+7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2c^3f\sqrt{c-c\sin(e+fx)}}+\frac{a^2(A+7B)\cos(e+f}{4c^2f(c-c\sin}$$

```
[Out] 1/6*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(7/2)-1/12*a
*(A+7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(5/2)+1/4*a
^2*(A+7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(3/2)+a
^4*(A+7*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-c*si
n(f*x+e))^(1/2)+1/2*a^3*(A+7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^3/f/(c-
c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.61, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2972, 2739, 2740, 2737, 2667, 31}

$$\frac{a^3(A+7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2c^3f\sqrt{c-c\sin(e+fx)}}+\frac{a^2(A+7B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4c^2f(c-c\sin(e+fx))^{3/2}}+\frac{a^4(A+7B)\cos(e+f}{c^3f\sqrt{a\sin(e+fx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(6*f*(c - c*Sin[e + f*x])
^(7/2)) - (a*(A + 7*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(12*c*f*(c
- c*Sin[e + f*x])^(5/2)) + (a^2*(A + 7*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])
^(3/2))/(4*c^2*f*(c - c*Sin[e + f*x])^(3/2)) + (a^4*(A + 7*B)*Cos[e + f*x]*
Log[1 - Sin[e + f*x]])/(c^3*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f
*x]]) + (a^3*(A + 7*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(2*c^3*f*Sqrt
[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
```

```
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{(A+7B)\int \frac{(a+a\sin(}{(c-c\sin(}}{6c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+7B)\cos(e+}{12cf(c-c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+7B)\cos(e+}{12cf(c-c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+7B)\cos(e+}{12cf(c-c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+7B)\cos(e+}{12cf(c-c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+7B)\cos(e+}{12cf(c-c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+7B)\cos(e+}{12cf(c-c}$$

Mathematica [A] time = 2.90, size = 244, normalized size = 0.92

$$\frac{(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(18(A+3B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-6(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(7/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2)*(8*(A +
 B) - 6*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + 18*(A + 3*B)*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 6*(A + 7*B)*Log[Cos[(e + f*x)/2]
- Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6 + 3*B*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^6*Sin[e + f*x]))/(3*f*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(7/2))
```

fricas [F] time = 44.32, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Ba^3\cos\left(fx+e\right)^4-(3\,A+5\,B)a^3\cos\left(fx+e\right)^2+4\,(A+B)a^3-\left((A+3\,B)a^3\cos\left(fx+e\right)^2-4\,(A+\right.\right.}{c^4\cos\left(fx+e\right)^4-8\,c^4\cos\left(fx+e\right)^2+8\,c^4+4\left(c^4\cos\left(fx+e\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B)
*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^4*cos(f*x + e)^4 - 8*c^4*cos
(f*x + e)^2 + 8*c^4 + 4*(c^4*cos(f*x + e)^2 - 2*c^4)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.66, size = 1455, normalized size = 5.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/3/f*(336*B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-41*B*c
os(f*x+e)^3*sin(f*x+e)-20*A*sin(f*x+e)+14*A*sin(f*x+e)*cos(f*x+e)-8*A*cos(f
*x+e)^3*sin(f*x+e)-3*B*sin(f*x+e)*cos(f*x+e)^4+48*A*sin(f*x+e)*ln(-(-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))-168*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+14*A
*cos(f*x+e)^2*sin(f*x+e)+98*B*cos(f*x+e)^2*sin(f*x+e)+44*B*cos(f*x+e)^4+20*
A+116*B+6*A*cos(f*x+e)^3-126*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*c
os(f*x+e)^3+63*B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^3-24*A*cos(f*x+e)^2*ln(2/(
cos(f*x+e)+1))+48*A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
-24*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+24*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))-12*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+62*B*sin(f*x+e)*
cos(f*x+e)+336*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-168*
```

```
B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+168*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))-84*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+12*A*cos(f*x+e)*sin
(f*x+e)*ln(2/(cos(f*x+e)+1))-24*A*cos(f*x+e)*sin(f*x+e)*ln(-(-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))-3*B*cos(f*x+e)^5-168*B*ln(-(-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+84*B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)
^2*sin(f*x+e)+84*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-168*B*ln(-(-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)-6*A*cos(f*x+e)^4*
ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+3*A*cos(f*x+e)^4*ln(2/(cos(f*x+e
)+1))+21*B*cos(f*x+e)^4*ln(2/(cos(f*x+e)+1))-42*B*cos(f*x+e)^4*ln(-(-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))-160*B*cos(f*x+e)^2-18*A*cos(f*x+e)^3*ln(-(-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+9*A*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))+8
*A*cos(f*x+e)^4+12*A*cos(f*x+e)^2*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-24*A*cos(
f*x+e)^2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+57*B*cos(f*x
+e)^3-116*B*sin(f*x+e)-3*A*sin(f*x+e)*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))+6*A
*sin(f*x+e)*cos(f*x+e)^3*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+42*B*co
s(f*x+e)^3*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-21*B*cos(f
*x+e)^3*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-6*A*cos(f*x+e)-54*B*cos(f*x+e)-28*A
*cos(f*x+e)^2-48*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+24*A*ln(2/(co
s(f*x+e)+1))-336*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+168*B*ln(2/(c
os(f*x+e)+1)))*(a*(1+sin(f*x+e)))^(7/2)/(cos(f*x+e)^4+sin(f*x+e)*cos(f*x+e)
^3+3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2-4*sin(f*x+e)*cos
(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(7/2)
```

maxima [B] time = 1.19, size = 749, normalized size = 2.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="maxima")
```

```
[Out] -1/3*(B*(42*a^(7/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(7/2) - 21*a
^(7/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(7/2) + 2*(21*a^(7/2)
*sin(f*x + e)/(cos(f*x + e) + 1) - 102*a^(7/2)*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 227*a^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 228*a^(7/2)*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + 227*a^(7/2)*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5 - 102*a^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 21*a^(7/2)*sin(
f*x + e)^7/(cos(f*x + e) + 1)^7)/(c^(7/2) - 6*c^(7/2)*sin(f*x + e)/(cos(f*x
 + e) + 1) + 16*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 26*c^(7/2)*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 30*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 - 26*c^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 16*c^(7/2)*sin(f
*x + e)^6/(cos(f*x + e) + 1)^6 - 6*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1
)^7 + c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8)) + A*(6*a^(7/2)*log(sin(
f*x + e)/(cos(f*x + e) + 1) - 1)/c^(7/2) - 3*a^(7/2)*log(sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 1)/c^(7/2) + 4*(3*a^(7/2)*sqrt(c)*sin(f*x + e)/(cos(f*x
```

```
 + e) + 1) - 6*a^(7/2)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 22*a^(
7/2)*sqrt(c)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6*a^(7/2)*sqrt(c)*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 3*a^(7/2)*sqrt(c)*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5)/(c^4 - 6*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 15*c^4*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 - 20*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1
5*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 6*c^4*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5 + c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(7/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Timed out
```

3.169 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=247

$$-\frac{a^4B\cos(e+fx)\log(1-\sin(e+fx))}{c^4f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{a^3B\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^3f(c-c\sin(e+fx))^{3/2}}+\frac{a^2B\cos(e+fx)(a\sin(e+fx)\cdots}{2c^2f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 1/8*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(9/2)-1/3*a*
B*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(7/2)+1/2*a^2*B*co
s(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(5/2)-a^3*B*cos(f*x+
e)*(a+a*sin(f*x+e))^(1/2)/c^3/f/(c-c*sin(f*x+e))^(3/2)-a^4*B*cos(f*x+e)*ln(
1-sin(f*x+e))/c^4/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2972, 2739, 2737, 2667, 31}

$$-\frac{a^3B\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^3f(c-c\sin(e+fx))^{3/2}}+\frac{a^2B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}}-\frac{a^4B\cos(e+fx)\log(1-\sin(e+\cdots}{c^4f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(9/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(8*f*(c - c*Sin[e + f*x])
^(9/2)) - (a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*c*f*(c - c*Sin[e
 + f*x])^(7/2)) + (a^2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c^2*f*
(c - c*Sin[e + f*x])^(5/2)) - (a^3*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])
/(c^3*f*(c - c*Sin[e + f*x])^(3/2)) - (a^4*B*Cos[e + f*x]*Log[1 - Sin[e + f
*x]])/(c^4*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{B\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}}{c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{aB\cos(e+fx)(a+}{3cf(c-c\sin}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{aB\cos(e+fx)(a+}{3cf(c-c\sin}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{aB\cos(e+fx)(a+}{3cf(c-c\sin}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{aB\cos(e+fx)(a+}{3cf(c-c\sin}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{aB\cos(e+fx)(a+}{3cf(c-c\sin}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{aB\cos(e+fx)(a+}{3cf(c-c\sin}$$

Mathematica [A] time = 2.68, size = 238, normalized size = 0.96

$$\frac{(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-3(A+7B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^6+9\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(9/2),x]
```

```
[Out] ((6*(A + B) - 4*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + 9*(A
+ 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 3*(A + 7*B)*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^6 - 6*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^8)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
7*(c - c*Sin[e + f*x])^(9/2))
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Ba^3\cos\left(fx+e\right)^4-(3\,A+5\,B)a^3\cos\left(fx+e\right)^2+4\,(A+B)a^3-\left((A+3\,B)a^3\cos\left(fx+e\right)^2-4\,(A+\right.\right.}{5\,c^5\cos\left(fx+e\right)^4-20\,c^5\cos\left(fx+e\right)^2+16\,c^5-\left(c^5\cos\left(fx+e\right)^4-12\,c^5\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B)
*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^5*cos(f*x + e)^4 - 20*c^5*
cos(f*x + e)^2 + 16*c^5 - (c^5*cos(f*x + e)^4 - 12*c^5*cos(f*x + e)^2 + 16*
c^5)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.72, size = 1019, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] -1/3/f*(120*B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-11*B*
cos(f*x+e)^3*sin(f*x+e)-3*B*cos(f*x+e)^5*ln(2/(cos(f*x+e)+1))+6*A*sin(f*x+e
)-6*A*sin(f*x+e)*cos(f*x+e)+3*A*cos(f*x+e)^3*sin(f*x+e)-8*B*sin(f*x+e)*cos(
f*x+e)^4-60*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-3*A*cos(f*x+e)^2*sin(f*x+e)
+39*B*cos(f*x+e)^2*sin(f*x+e)+19*B*cos(f*x+e)^4-6*A+34*B-48*B*ln(-(-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^3+24*B*ln(2/(cos(f*x+e)+1))*cos(f*
x+e)^3+14*B*sin(f*x+e)*cos(f*x+e)+96*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))-48*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+48*B*cos(f*x+e)*ln(-
(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-24*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))
+6*B*cos(f*x+e)^5*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-8*B*cos(f*x+e)
```

```
^5-72*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+
36*B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^2*sin(f*x+e)+24*B*ln(2/(cos(f*x+e)+1))
*sin(f*x+e)*cos(f*x+e)-48*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(
f*x+e)*cos(f*x+e)+15*B*cos(f*x+e)^4*ln(2/(cos(f*x+e)+1))-30*B*cos(f*x+e)^4*
ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-53*B*cos(f*x+e)^2-3*A*cos(f*x+e)
^4+6*B*sin(f*x+e)*cos(f*x+e)^4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-3
*B*sin(f*x+e)*cos(f*x+e)^4*ln(2/(cos(f*x+e)+1))+28*B*cos(f*x+e)^3-34*B*sin(
f*x+e)+24*B*cos(f*x+e)^3*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))-12*B*cos(f*x+e)^3*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-20*B*cos(f*x+e)+9*A*c
os(f*x+e)^2-96*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+48*B*ln(2/(cos(
f*x+e)+1)))*(a*(1+sin(f*x+e)))^(7/2)/(cos(f*x+e)^4+sin(f*x+e)*cos(f*x+e)^3+
3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2-4*sin(f*x+e)*cos(f*
x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(9/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),
x)
```

```
[Out] Timed out
```

3.170 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=96

$$\frac{(A-9B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

```
[Out] 1/10*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(11/2)+1/80
*(A-9*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(9/2)
```

Rubi [A] time = 0.27, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2972, 2742}

$$\frac{(A-9B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(11/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*f*(c - c*Sin[e + f*x]
)^(11/2)) + ((A - 9*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(80*c*f*(c
- c*Sin[e + f*x])^(9/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(A-9B)\int \frac{(a+a\,\mathrm{si}}{(c-c\,\mathrm{si}}}{10c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(A-9B)\cos(e+}{80cf(c-}$$

Mathematica [B] time = 6.94, size = 434, normalized size = 4.52

$$\frac{(-A-7B)(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}{2f(c-c\sin(e+fx))^{11/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7} + \frac{2(A+3B)(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\right.\right.}{f(c-c\sin(e+fx))^{11/2}\left(\sin\left(\frac{1}{2}(e+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(11/2),x]
```

```
[Out] (8*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/
2))/(5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2
)) + ((-3*A - 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e +
f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x
])^(11/2)) + (2*(A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + S
in[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[
e + f*x])^(11/2)) + ((-A - 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*
(1 + Sin[e + f*x]))^(7/2))/(2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c
- c*Sin[e + f*x])^(11/2)) + (B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(
1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c
*Sin[e + f*x])^(11/2))
```

fricas [B] time = 0.48, size = 199, normalized size = 2.07

$$\frac{\left(10\,Ba^3\cos\left(fx+e\right)^4 - 5\,(A+7\,B)a^3\cos\left(fx+e\right)^2 + 2\,(3\,A+13\,B)a^3 - 5\left((A-B)a^3\cos\left(fx+e\right)^2 - 2\,(A-B)\right.\right.}{10\left(5\,c^6f\cos\left(fx+e\right)^5 - 20\,c^6f\cos\left(fx+e\right)^3 + 16\,c^6f\cos\left(fx+e\right) - \left(c^6f\cos\left(fx+e\right)^5 - 12\,c^6f\,\mathrm{co}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="fricas")
```

```
[Out] 1/10*(10*B*a^3*cos(f*x + e)^4 - 5*(A + 7*B)*a^3*cos(f*x + e)^2 + 2*(3*A + 1
3*B)*a^3 - 5*((A - B)*a^3*cos(f*x + e)^2 - 2*(A - B)*a^3)*sin(f*x + e))*sqr
t(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^6*f*cos(f*x + e)^5 - 2
0*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e) - (c^6*f*cos(f*x + e)^5 - 12
*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e))*sin(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.70, size = 389, normalized size = 4.05

$$\frac{\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{7}{2}}\left(A\left(\cos^{5}\left(fx+e\right)\right)+A\left(\cos^{4}\left(fx+e\right)\right)\sin\left(fx+e\right)+B\left(\cos^{5}\left(fx+e\right)\right)+B\,\mathrm{s}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x)
```

```
[Out] 1/10/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(A*cos(f*x+e)^5+A*cos(f*x+e)^4*s
in(f*x+e)+B*cos(f*x+e)^5+B*sin(f*x+e)*cos(f*x+e)^4-6*A*cos(f*x+e)^4+5*A*cos
(f*x+e)^3*sin(f*x+e)+4*B*cos(f*x+e)^4-5*B*cos(f*x+e)^3*sin(f*x+e)-17*A*cos(
f*x+e)^3-22*A*cos(f*x+e)^2*sin(f*x+e)-7*B*cos(f*x+e)^3-2*B*cos(f*x+e)^2*sin
(f*x+e)+32*A*cos(f*x+e)^2-10*A*sin(f*x+e)*cos(f*x+e)-8*B*cos(f*x+e)^2+10*B*
sin(f*x+e)*cos(f*x+e)+26*A*cos(f*x+e)+36*A*sin(f*x+e)+6*B*cos(f*x+e)-4*B*si
n(f*x+e)-36*A+4*B)/(-c*(sin(f*x+e)-1))^(11/2)/(cos(f*x+e)^4+sin(f*x+e)*cos(
f*x+e)^3+3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2-4*sin(f*x+
e)*cos(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(11/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\sin\left(e+fx\right)\right)\left(a+a\sin\left(e+fx\right)\right)^{7/2}}{\left(c-c\sin\left(e+fx\right)\right)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(11/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(11/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(11/2)
,x)
```

```
[Out] Timed out
```

3.171 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx$

Optimal. Leaf size=146

$$\frac{(A-5B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{480c^2f(c-c\sin(e+fx))^{9/2}}+\frac{(A-5B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{60cf(c-c\sin(e+fx))^{11/2}}+\frac{(A+B)\cos(e+fx)(a\sin}{12f(c-c\sin(e+}$$

```
[Out] 1/12*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(13/2)+1/60
*(A-5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(11/2)+1/48
0*(A-5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^2/f/(c-c*sin(f*x+e))^(9/2)
```

Rubi [A] time = 0.38, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2972, 2743, 2742}

$$\frac{(A-5B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{480c^2f(c-c\sin(e+fx))^{9/2}}+\frac{(A-5B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{60cf(c-c\sin(e+fx))^{11/2}}+\frac{(A+B)\cos(e+fx)(a\sin}{12f(c-c\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(13/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(12*f*(c - c*Sin[e + f*x]
)^(13/2)) + ((A - 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(60*c*f*(c
- c*Sin[e + f*x])^(11/2)) + ((A - 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7
/2))/(480*c^2*f*(c - c*Sin[e + f*x])^(9/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
```

```
SumSimplerQ[n, 1])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-5B)\int \frac{(a+a\sin}{(c-c\sin}}{6c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-5B)\cos(e+}{60cf(c-}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-5B)\cos(e+}{60cf(c-}$$

Mathematica [B] time = 6.99, size = 442, normalized size = 3.03

$$\frac{(-A-7B)(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}{3f(c-c\sin(e+fx))^{13/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7}+\frac{3(A+3B)(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\right.\right.}{2f(c-c\sin(e+fx))^{13/2}\left(\sin\left(\frac{1}{2}(e+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(13/2),x]
```

```
[Out] (4*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/
2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2
)) - (4*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e +
 f*x]))^(7/2))/(5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e +
f*x])^(13/2)) + (3*(A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1
+ Sin[e + f*x]))^(7/2))/(2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c
```

```
*Sin[e + f*x])^(13/2)) + ((-A - 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
7*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
7*(c - c*Sin[e + f*x])^(13/2)) + (B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9
*(a*(1 + Sin[e + f*x]))^(7/2))/(2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7
*(c - c*Sin[e + f*x])^(13/2))
```

fricas [A] time = 0.50, size = 214, normalized size = 1.47

$$-\frac{\left(15\,Ba^3\cos\left(fx+e\right)^4-15\,(A+3\,B)a^3\cos\left(fx+e\right)^2+6\,(3\,A+5\,B)a^3-2\left(5\,(A+B)a^3\cos\left(fx+e\right)^2-(11\,A+\right.}{30\left(c^7f\cos\left(fx+e\right)^7-18\,c^7f\cos\left(fx+e\right)^5+48\,c^7f\cos\left(fx+e\right)^3-32\,c^7f\cos\left(fx+e\right)+2\left(3\,c^7f\cos\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x
, algorithm="fricas")
```

```
[Out] -1/30*(15*B*a^3*cos(f*x + e)^4 - 15*(A + 3*B)*a^3*cos(f*x + e)^2 + 6*(3*A +
 5*B)*a^3 - 2*(5*(A + B)*a^3*cos(f*x + e)^2 - (11*A + 5*B)*a^3)*sin(f*x + e
))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^7*f*cos(f*x + e)^7
 - 18*c^7*f*cos(f*x + e)^5 + 48*c^7*f*cos(f*x + e)^3 - 32*c^7*f*cos(f*x + e
) + 2*(3*c^7*f*cos(f*x + e)^5 - 16*c^7*f*cos(f*x + e)^3 + 16*c^7*f*cos(f*x
+ e))*sin(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.73, size = 393, normalized size = 2.69

$$-\frac{\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{7}{2}}\left(3A\left(\cos^5\left(fx+e\right)\right)\sin\left(fx+e\right)-3A\left(\cos^6\left(fx+e\right)\right)-21A\left(\cos^4\left(fx+e\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x)
```

```
[Out] -1/30/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(3*A*cos(f*x+e)^5*sin(f*x+e)-3*
A*cos(f*x+e)^6-21*A*cos(f*x+e)^4*sin(f*x+e)-18*A*cos(f*x+e)^5-51*A*cos(f*x+
```

```
e)^3*sin(f*x+e)+72*A*cos(f*x+e)^4+15*B*cos(f*x+e)^3*sin(f*x+e)-15*B*cos(f*x
+e)^4+157*A*cos(f*x+e)^2*sin(f*x+e)+106*A*cos(f*x+e)^3-5*B*cos(f*x+e)^2*sin
(f*x+e)+10*B*cos(f*x+e)^3+78*A*sin(f*x+e)*cos(f*x+e)-235*A*cos(f*x+e)^2-30*
B*sin(f*x+e)*cos(f*x+e)+35*B*cos(f*x+e)^2-196*A*sin(f*x+e)-118*A*cos(f*x+e)
+20*B*sin(f*x+e)-10*B*cos(f*x+e)+196*A-20*B)/(-c*(sin(f*x+e)-1))^(13/2)/(co
s(f*x+e)^4+sin(f*x+e)*cos(f*x+e)^3+3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f*x+e)
-8*cos(f*x+e)^2-4*sin(f*x+e)*cos(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x
, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 22.85, size = 406, normalized size = 2.78

$$-\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{56\,a^3\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\left(4\,A+5\,B\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{5\,c^7\,f}+\frac{a^3\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,\left(A\,1{}\mathrm{i}+B\,1{}\mathrm{i}\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,32{}\mathrm{i}}{3\,c^7\,f}-\frac{32\,\dots}{}\right.}{-858\,\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}+858\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)-130\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)+2\,{\mathrm{e}}^{e\dots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(13/2),x)
```

```
[Out] -((c - c*sin(e + f*x))^(1/2)*((56*a^3*exp(e*7i + f*x*7i)*(4*A + 5*B)*(a + a
*sin(e + f*x))^(1/2))/(5*c^7*f) + (a^3*exp(e*7i + f*x*7i)*sin(3*e + 3*f*x)*
(A*1i + B*1i)*(a + a*sin(e + f*x))^(1/2)*32i)/(3*c^7*f) - (32*a^3*exp(e*7i
+ f*x*7i)*cos(2*e + 2*f*x)*(A + 2*B)*(a + a*sin(e + f*x))^(1/2))/(c^7*f) +
(8*B*a^3*exp(e*7i + f*x*7i)*cos(4*e + 4*f*x)*(a + a*sin(e + f*x))^(1/2))/(c
^7*f) - (a^3*exp(e*7i + f*x*7i)*sin(e + f*x)*(A*13i + B*5i)*(a + a*sin(e +
f*x))^(1/2)*32i)/(5*c^7*f)))/(858*exp(e*7i + f*x*7i)*cos(3*e + 3*f*x) - 858
*cos(e + f*x)*exp(e*7i + f*x*7i) - 130*exp(e*7i + f*x*7i)*cos(5*e + 5*f*x)
+ 2*exp(e*7i + f*x*7i)*cos(7*e + 7*f*x) + 1144*exp(e*7i + f*x*7i)*sin(2*e +
 2*f*x) - 416*exp(e*7i + f*x*7i)*sin(4*e + 4*f*x) + 24*exp(e*7i + f*x*7i)*s
in(6*e + 6*f*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(13/2)
,x)
```

```
[Out] Timed out
```

3.172 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx$

Optimal. Leaf size=202

$$\frac{(3A-11B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6720c^3f(c-c\sin(e+fx))^{9/2}}+\frac{(3A-11B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{840c^2f(c-c\sin(e+fx))^{11/2}}+\frac{(3A-11B)\cos(e+}{168cf(c-}$$

```
[Out] 1/14*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(15/2)+1/16
8*(3*A-11*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(13/2)+
1/840*(3*A-11*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^2/f/(c-c*sin(f*x+e))^(
11/2)+1/6720*(3*A-11*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^3/f/(c-c*sin(f*
x+e))^(9/2)
```

Rubi [A] time = 0.49, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2972, 2743, 2742}

$$\frac{(3A-11B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6720c^3f(c-c\sin(e+fx))^{9/2}}+\frac{(3A-11B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{840c^2f(c-c\sin(e+fx))^{11/2}}+\frac{(3A-11B)\cos(e+}{168cf(c-}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(15/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(14*f*(c - c*Sin[e + f*x]
)^(15/2)) + ((3*A - 11*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(168*c*f
*(c - c*Sin[e + f*x])^(13/2)) + ((3*A - 11*B)*Cos[e + f*x]*(a + a*Sin[e + f
*x])^(7/2))/(840*c^2*f*(c - c*Sin[e + f*x])^(11/2)) + ((3*A - 11*B)*Cos[e +
 f*x]*(a + a*Sin[e + f*x])^(7/2))/(6720*c^3*f*(c - c*Sin[e + f*x])^(9/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}} + \frac{(3A-11B)\int \frac{(a+a\,\mathrm{s}}{(c-c\,\mathrm{si}}}{14c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}} + \frac{(3A-11B)\cos(e+}{168cf(c-}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}} + \frac{(3A-11B)\cos(e+}{168cf(c-}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}} + \frac{(3A-11B)\cos(e+}{168cf(c-}$$

Mathematica [B] time = 7.11, size = 442, normalized size = 2.19

$$\frac{(-A-7B)(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}{4f(c-c\sin(e+fx))^{15/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7}+\frac{6(A+3B)(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(\right.\right.}{5f(c-c\sin(e+fx))^{15/2}\left(\sin\left(\frac{1}{2}(e+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(15/2),x]
```

```
[Out] (8*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/
2))/(7*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2
)) - (2*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e +
 f*x]))^(7/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e +
f*x])^(15/2)) + (6*(A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1
+ Sin[e + f*x]))^(7/2))/(5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c
*Sin[e + f*x])^(15/2)) + ((-A - 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
7*(a*(1 + Sin[e + f*x]))^(7/2))/(4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
7*(c - c*Sin[e + f*x])^(15/2)) + (B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9
*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7
*(c - c*Sin[e + f*x])^(15/2))
```

fricas [A] time = 0.54, size = 234, normalized size = 1.16

$$-\frac{\left(140\,Ba^3\cos\left(fx+e\right)^4-7\,(27\,A+61\,B)a^3\cos\left(fx+e\right)^2+4\,(57\,A+71\,B)a^3-7\left(5\,(3\,A+5\,B)a^3\cos\left(fx\right.\right.}{420\left(7\,c^8f\cos\left(fx+e\right)^7-56\,c^8f\cos\left(fx+e\right)^5+112\,c^8f\cos\left(fx+e\right)^3-64\,c^8f\cos\left(fx+e\right)-\left(c^8f\cos\left(fx+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),x
, algorithm="fricas")
```

```
[Out] -1/420*(140*B*a^3*cos(f*x + e)^4 - 7*(27*A + 61*B)*a^3*cos(f*x + e)^2 + 4*(
57*A + 71*B)*a^3 - 7*(5*(3*A + 5*B)*a^3*cos(f*x + e)^2 - 4*(9*A + 7*B)*a^3)
*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(7*c^8*f*
cos(f*x + e)^7 - 56*c^8*f*cos(f*x + e)^5 + 112*c^8*f*cos(f*x + e)^3 - 64*c^
8*f*cos(f*x + e) - (c^8*f*cos(f*x + e)^7 - 24*c^8*f*cos(f*x + e)^5 + 80*c^8
*f*cos(f*x + e)^3 - 64*c^8*f*cos(f*x + e))*sin(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.76, size = 505, normalized size = 2.50

$$-\frac{\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{7}{2}}\left(-5016A\sin\left(fx+e\right)+2268A\sin\left(fx+e\right)\cos\left(fx+e\right)-352B\left(\cos^2\left(fx\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),x)
```

```
[Out] -1/420/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(287*B*cos(f*x+e)^3*sin(f*x+e)
-5016*A*sin(f*x+e)+2268*A*sin(f*x+e)*cos(f*x+e)+39*A*cos(f*x+e)^6*sin(f*x+e
)-1911*A*cos(f*x+e)^3*sin(f*x+e)-1209*A*cos(f*x+e)^4*sin(f*x+e)+93*B*sin(f*
x+e)*cos(f*x+e)^4+5136*A*cos(f*x+e)^2*sin(f*x+e)-352*B*cos(f*x+e)^2*sin(f*x
+e)-380*B*cos(f*x+e)^4+5016*A-472*B+3225*A*cos(f*x+e)^3-476*B*sin(f*x+e)*co
s(f*x+e)-3*B*cos(f*x+e)^6*sin(f*x+e)-936*A*cos(f*x+e)^5+72*B*cos(f*x+e)^5+2
4*B*cos(f*x+e)^6-21*B*cos(f*x+e)^5*sin(f*x+e)+273*A*cos(f*x+e)^5*sin(f*x+e)
+828*B*cos(f*x+e)^2+3120*A*cos(f*x+e)^4-65*B*cos(f*x+e)^3+472*B*sin(f*x+e)-
2748*A*cos(f*x+e)-4*B*cos(f*x+e)-312*A*cos(f*x+e)^6-7404*A*cos(f*x+e)^2+39*
A*cos(f*x+e)^7-3*B*cos(f*x+e)^7)/(-c*(sin(f*x+e)-1))^(15/2)/(cos(f*x+e)^4+s
in(f*x+e)*cos(f*x+e)^3+3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e
)^2-4*sin(f*x+e)*cos(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),x
, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 25.26, size = 827, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(15/2),x)
```

```
[Out] -((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(
(B*a^3*exp(e*4i + f*x*4i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i +
 f*x*1i)*1i)/2))^(1/2)*16i)/(3*c^8*f) + (B*a^3*exp(e*12i + f*x*12i)*(a + a*
((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*16i)/(3*c^
8*f) - (a^3*exp(e*5i + f*x*5i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e
*1i + f*x*1i)*1i)/2))^(1/2)*(A*3i + B*5i)*8i)/(3*c^8*f) + (a^3*exp(e*11i +
f*x*11i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^
(1/2)*(A*3i + B*5i)*8i)/(3*c^8*f) - (a^3*exp(e*6i + f*x*6i)*(a + a*((exp(-
e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(27*A + 41*B)*16i)
/(15*c^8*f) - (a^3*exp(e*10i + f*x*10i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2
```

```
 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(27*A + 41*B)*16i)/(15*c^8*f) + (a^3*e
xp(e*7i + f*x*7i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)
*1i)/2))^(1/2)*(A*43i + B*29i)*8i)/(5*c^8*f) - (a^3*exp(e*9i + f*x*9i)*(a +
 a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(A*43i
+ B*29i)*8i)/(5*c^8*f) + (a^3*exp(e*8i + f*x*8i)*(a + a*((exp(- e*1i - f*x*
1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(89*A + 82*B)*32i)/(35*c^8*f)
))/(exp(e*1i + f*x*1i)*14i - 90*exp(e*2i + f*x*2i) - exp(e*3i + f*x*3i)*350
i + 910*exp(e*4i + f*x*4i) + exp(e*5i + f*x*5i)*1638i - 2002*exp(e*6i + f*x
*6i) - exp(e*7i + f*x*7i)*1430i - exp(e*9i + f*x*9i)*1430i + 2002*exp(e*10i
 + f*x*10i) + exp(e*11i + f*x*11i)*1638i - 910*exp(e*12i + f*x*12i) - exp(e
*13i + f*x*13i)*350i + 90*exp(e*14i + f*x*14i) + exp(e*15i + f*x*15i)*14i -
 exp(e*16i + f*x*16i) + 1)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(15/2)
,x)
```

```
[Out] Timed out
```

3.173 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx$

Optimal. Leaf size=246

$$\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8960c^4f(c-c\sin(e+fx))^{9/2}}+\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{1120c^3f(c-c\sin(e+fx))^{11/2}}+\frac{(A-3B)\cos(e+fx)(a\,\mathrm{s}}{224c^2f(c-c\sin(}$$

```
[Out] 1/16*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(17/2)+1/56
*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(15/2)+1/22
4*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^2/f/(c-c*sin(f*x+e))^(13/2)+1
/1120*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^3/f/(c-c*sin(f*x+e))^(11/
2)+1/8960*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^4/f/(c-c*sin(f*x+e))^
(9/2)
```

Rubi [A] time = 0.59, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2972, 2743, 2742}

$$\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8960c^4f(c-c\sin(e+fx))^{9/2}}+\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{1120c^3f(c-c\sin(e+fx))^{11/2}}+\frac{(A-3B)\cos(e+fx)(a\,\mathrm{s}}{224c^2f(c-c\sin(}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(17/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(16*f*(c - c*Sin[e + f*x]
)^(17/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(56*c*f*(c
- c*Sin[e + f*x])^(15/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7
/2))/(224*c^2*f*(c - c*Sin[e + f*x])^(13/2)) + ((A - 3*B)*Cos[e + f*x]*(a +
 a*Sin[e + f*x])^(7/2))/(1120*c^3*f*(c - c*Sin[e + f*x])^(11/2)) + ((A - 3*
B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(8960*c^4*f*(c - c*Sin[e + f*x]
)^(9/2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{16f(c-c\sin(e+fx))^{17/2}} + \frac{(A-3B)\int \frac{(a+a\sin}{(c-c\sin}}{4c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{16f(c-c\sin(e+fx))^{17/2}} + \frac{(A-3B)\cos(e+}{56cf(c-}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{16f(c-c\sin(e+fx))^{17/2}} + \frac{(A-3B)\cos(e+}{56cf(c-}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{16f(c-c\sin(e+fx))^{17/2}} + \frac{(A-3B)\cos(e+}{56cf(c-}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{16f(c-c\sin(e+fx))^{17/2}} + \frac{(A-3B)\cos(e+}{56cf(c-}$$

Mathematica [A] time = 7.14, size = 436, normalized size = 1.77

$$\frac{(-A-7B)(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}{5f(c-c\sin(e+fx))^{17/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7} + \frac{(A+3B)(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}\right.\right.}{f(c-c\sin(e+fx))^{17/2}\left(\sin\left(\frac{1}{2}(e+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(17/2),x]
```

```
[Out] ((A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2)
)/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2)) -
 (4*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x
]))^(7/2))/(7*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x]
)^(17/2)) + ((A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[
e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e +
 f*x])^(17/2)) + ((-A - 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1
+ Sin[e + f*x]))^(7/2))/(5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c
*Sin[e + f*x])^(17/2)) + (B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 +
 Sin[e + f*x]))^(7/2))/(4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*
Sin[e + f*x])^(17/2))
```

fricas [A] time = 0.54, size = 243, normalized size = 0.99

$$\frac{\left(35\,Ba^{3}\cos\left(fx+e\right)^{4}-56\,(A+2\,B)a^{3}\cos\left(fx+e\right)^{2}+4\,(17\,A+19\,B)a^{3}-4\left(7\,(A+2\,B)a^{3}\right.}{140\left(c^{9}f\cos\left(fx+e\right)^{9}-32\,c^{9}f\cos\left(fx+e\right)^{7}+160\,c^{9}f\cos\left(fx+e\right)^{5}-256\,c^{9}f\cos\left(fx+e\right)^{3}+128\,c^{9}f\cos\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),x
, algorithm="fricas")
```

```
[Out] 1/140*(35*B*a^3*cos(f*x + e)^4 - 56*(A + 2*B)*a^3*cos(f*x + e)^2 + 4*(17*A
+ 19*B)*a^3 - 4*(7*(A + 2*B)*a^3*cos(f*x + e)^2 - 2*(9*A + 8*B)*a^3)*sin(f*
x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^9*f*cos(f*x +
 e)^9 - 32*c^9*f*cos(f*x + e)^7 + 160*c^9*f*cos(f*x + e)^5 - 256*c^9*f*cos(
f*x + e)^3 + 128*c^9*f*cos(f*x + e) + 8*(c^9*f*cos(f*x + e)^7 - 10*c^9*f*co
s(f*x + e)^5 + 24*c^9*f*cos(f*x + e)^3 - 16*c^9*f*cos(f*x + e))*sin(f*x + e
))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.81, size = 560, normalized size = 2.28

$$-\frac{\sin\left(fx+e\right)\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{7}{2}}\left(-3076A\sin\left(fx+e\right)+1468A\sin\left(fx+e\right)\cos\left(fx+e\right)-300B\left(\cos^{2}\left(fx\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),x)
```

```
[Out] -1/140/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(164*B*cos(f*x+e)^3*sin(f*x+e)
-3076*A*sin(f*x+e)+1468*A*sin(f*x+e)*cos(f*x+e)+108*A*cos(f*x+e)^6*sin(f*x+
e)-1548*A*cos(f*x+e)^3*sin(f*x+e)-1332*A*cos(f*x+e)^4*sin(f*x+e)+111*B*sin(
f*x+e)*cos(f*x+e)^4+3880*A*cos(f*x+e)^2*sin(f*x+e)-300*B*cos(f*x+e)^2*sin(f
*x+e)-275*B*cos(f*x+e)^4+3076*A-268*B+2332*A*cos(f*x+e)^3-204*B*sin(f*x+e)*
cos(f*x+e)-9*B*cos(f*x+e)^6*sin(f*x+e)-960*A*cos(f*x+e)^5+80*B*cos(f*x+e)^5
+40*B*cos(f*x+e)^6-31*B*cos(f*x+e)^5*sin(f*x+e)+372*A*cos(f*x+e)^5*sin(f*x+
e)-B*cos(f*x+e)^8+504*B*cos(f*x+e)^2+2880*A*cos(f*x+e)^4-136*B*cos(f*x+e)^3
+268*B*sin(f*x+e)-1608*A*cos(f*x+e)-12*A*cos(f*x+e)^7*sin(f*x+e)+B*cos(f*x+
e)^7*sin(f*x+e)+64*B*cos(f*x+e)-480*A*cos(f*x+e)^6-5348*A*cos(f*x+e)^2+96*A
*cos(f*x+e)^7-8*B*cos(f*x+e)^7+12*A*cos(f*x+e)^8)/(-c*(sin(f*x+e)-1))^(17/2
)/(cos(f*x+e)^4+sin(f*x+e)*cos(f*x+e)^3+3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f
*x+e)-8*cos(f*x+e)^2-4*sin(f*x+e)*cos(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),x
, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 28.34, size = 841, normalized size = 3.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^
(17/2),x)
```

```
[Out] ((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((
8*B*a^3*exp(e*5i + f*x*5i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i
```

```
+ f*x*1i)*1i)/2))^(1/2))/(c^9*f) + (8*B*a^3*exp(e*13i + f*x*13i)*(a + a*((e
xp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(c^9*f) - (6
4*a^3*exp(e*6i + f*x*6i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i +
f*x*1i)*1i)/2))^(1/2)*(A*1i + B*2i))/(5*c^9*f) - (32*a^3*exp(e*7i + f*x*7i)
*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(8
*A + 11*B))/(5*c^9*f) + (64*a^3*exp(e*12i + f*x*12i)*(a + a*((exp(- e*1i -
f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(A*1i + B*2i))/(5*c^9*f)
- (32*a^3*exp(e*11i + f*x*11i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e
*1i + f*x*1i)*1i)/2))^(1/2)*(8*A + 11*B))/(5*c^9*f) + (64*a^3*exp(e*8i + f*
x*8i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/
2)*(A*13i + B*10i))/(7*c^9*f) - (64*a^3*exp(e*10i + f*x*10i)*(a + a*((exp(-
 e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(A*13i + B*10i))/
(7*c^9*f) + (16*a^3*exp(e*9i + f*x*9i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2
- (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(64*A + 53*B))/(7*c^9*f)))/(exp(e*1i +
f*x*1i)*16i - 119*exp(e*2i + f*x*2i) - exp(e*3i + f*x*3i)*544i + 1700*exp(e
*4i + f*x*4i) + exp(e*5i + f*x*5i)*3808i - 6188*exp(e*6i + f*x*6i) - exp(e*
7i + f*x*7i)*7072i + 4862*exp(e*8i + f*x*8i) + 4862*exp(e*10i + f*x*10i) +
exp(e*11i + f*x*11i)*7072i - 6188*exp(e*12i + f*x*12i) - exp(e*13i + f*x*13
i)*3808i + 1700*exp(e*14i + f*x*14i) + exp(e*15i + f*x*15i)*544i - 119*exp(
e*16i + f*x*16i) - exp(e*17i + f*x*17i)*16i + exp(e*18i + f*x*18i) + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(17/2)
,x)
```

```
[Out] Timed out
```

3.174 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=197

$$\frac{4c^3(A-B)\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2c^2(A-B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}+\frac{c(A-B)\cos(e+fx)(}{2f\sqrt{a\sin(e}$$

```
[Out] 1/2*(A-B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)-1/3*
B*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(1/2)+4*(A-B)*c^3*co
s(f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+2
*(A-B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2973, 2740, 2737, 2667, 31}

$$\frac{2c^2(A-B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}+\frac{4c^3(A-B)\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c(A-B)\cos(e+fx)(}{2f\sqrt{a\sin(e}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/Sqrt[a + a*Sin[e + f*
x]],x]
```

```
[Out] (4*(A - B)*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*
x]]*Sqrt[c - c*Sin[e + f*x]]) + (2*(A - B)*c^2*Cos[e + f*x]*Sqrt[c - c*Sin[
e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]) + ((A - B)*c*Cos[e + f*x]*(c - c*Si
n[e + f*x])^(3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]]) - (B*Cos[e + f*x]*(c - c*
Sin[e + f*x])^(5/2))/(3*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a+a\sin(e+fx)}} + (A-B)\int \frac{(c-c\sin(e\cdots}{\sqrt{a+a\sin(\cdots}}$$

$$= \frac{(A-B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} - \frac{B\cos(e+fx)(c-\cdots}{3f\sqrt{a+a\,\mathrm{s}\cdots}}$$

$$= \frac{2(A-B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{(A-B)c\cos(e+\cdots}{2f\sqrt{a\cdots}}$$

$$= \frac{2(A-B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{(A-B)c\cos(e+\cdots}{2f\sqrt{a\cdots}}$$

$$= \frac{2(A-B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{(A-B)c\cos(e+\cdots}{2f\sqrt{a\cdots}}$$

$$= \frac{4(A-B)c^3\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2(A-B)c^2\cos\cdots}{f\sqrt{a\cdots}}$$

Mathematica [A] time = 1.31, size = 185, normalized size = 0.94

$$-\frac{c^2(\sin(e+fx)-1)^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left((36A-51B)\sin(e+fx)+3(A-\cdots\right.}{12f\sqrt{a(\sin(e+fx)\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/Sqrt[a + a*Sin[
e + f*x]],x]
```

```
[Out] -1/12*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*Sqrt
[c - c*Sin[e + f*x]]*(3*(A - 3*B)*Cos[2*(e + f*x)] - 96*A*Log[Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]] + 96*B*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (
36*A - 51*B)*Sin[e + f*x] + B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^5*Sqrt[a*(1 + Sin[e + f*x])])
```

fricas [F] time = 4.08, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left((A-2B)c^2\cos\left(fx+e\right)^2-2(A-B)c^2+\left(Bc^2\cos\left(fx+e\right)^2+2(A-B)c^2\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(\cdots\right.}}{\sqrt{a\sin\left(fx+e\right)+a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(-((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x +
e)^2 + 2*(A - B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(f*
x + e) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.72, size = 595, normalized size = 3.02

$$-\frac{\left(48A\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-48B\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-15A\sin\left(fx+e\right)+18A\sin\left(fx+e\right)\cos\left(fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/6/f*(2*B*cos(f*x+e)^3*sin(f*x+e)+48*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f
*x+e))-48*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-15*A*sin(f*x+e)+18*A*s
in(f*x+e)*cos(f*x+e)-3*A*cos(f*x+e)^2*sin(f*x+e)+7*B*cos(f*x+e)^2*sin(f*x+e
)+2*B*cos(f*x+e)^4-15*A+17*B+3*A*cos(f*x+e)^3-24*A*sin(f*x+e)*ln(2/(cos(f*x
+e)+1))+24*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-26*B*sin(f*x+e)*cos(f*x+e)+24*
B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-24*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-48*A
*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+48*B*cos(f*x+e)*ln((1-
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+48*A*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))-48*B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-1
9*B*cos(f*x+e)^2-9*B*cos(f*x+e)^3+17*B*sin(f*x+e)-3*A*cos(f*x+e)+9*B*cos(f*
x+e)+15*A*cos(f*x+e)^2-24*A*ln(2/(cos(f*x+e)+1))+24*B*ln(2/(cos(f*x+e)+1)))
*(-c*(sin(f*x+e)-1))^(5/2)/(cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^3+2*sin(f*x+
e)*cos(f*x+e)-3*cos(f*x+e)^2-4*sin(f*x+e)-2*cos(f*x+e)+4)/(a*(1+sin(f*x+e))
)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(5/2)/sqrt(a*sin(f*x +
 e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^
(1/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^
(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

3.175 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=146

$$\frac{2c^2(A-B)\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c(A-B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}-\frac{B\cos(e+fx)(c-c\sin(e}{2f\sqrt{a\sin(e+fx)+}}$$

```
[Out] -1/2*B*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)+2*(A-B)*c
^2*cos(f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1
/2)+(A-B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2973, 2740, 2737, 2667, 31}

$$\frac{2c^2(A-B)\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c(A-B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}-\frac{B\cos(e+fx)(c-c\sin(e}{2f\sqrt{a\sin(e+fx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/Sqrt[a + a*Sin[e + f*
x]],x]
```

```
[Out] (2*(A - B)*c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*
x]]*Sqrt[c - c*Sin[e + f*x]]) + ((A - B)*c*Cos[e + f*x]*Sqrt[c - c*Sin[e +
f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]) - (B*Cos[e + f*x]*(c - c*Sin[e + f*x])^
(3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
```

```
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{B\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} + (A-B)\int \frac{(c-c\sin(e}{\sqrt{a+a\sin(}}$$

$$= \frac{(A-B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{B\cos(e+fx)(c-c}{2f\sqrt{a+a\sin}}$$

$$= \frac{(A-B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{B\cos(e+fx)(c-c}{2f\sqrt{a+a\sin}}$$

$$= \frac{(A-B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{B\cos(e+fx)(c-c}{2f\sqrt{a+a\sin}}$$

$$= \frac{2(A-B)c^2\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{(A-B)c\cos(e}{f\sqrt{a}}$$

Mathematica [A] time = 0.66, size = 146, normalized size = 1.00

$$-\frac{c(\sin(e+fx)-1)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(B\cos(2(e+fx))-4\left((A-2B)\sin(e\right.\right.}{4f\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/Sqrt[a + a*Sin[
e + f*x]],x]
```

```
[Out] -1/4*(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*Sqrt[c -
c*Sin[e + f*x]]*(B*Cos[2*(e + f*x)] - 4*(4*(-A + B)*Log[Cos[(e + f*x)/2] +
Sin[(e + f*x)/2]] + (A - 2*B)*Sin[e + f*x])))/(f*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])])
```

fricas [F] time = 2.82, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(Bc\cos\left(fx+e\right)^{2}-(A-B)c\sin\left(fx+e\right)+(A-B)c\right)\sqrt{-c\sin\left(fx+e\right)+c}}{\sqrt{a\sin\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*sqrt(-c*
sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (8*pi/x/2)>(-8*pi/x/2)4*sqrt(2*c)*(A*c*sqrt(a*tan(1/2*exp(
1))^2+a)*(27021597764222976*tan(1/2*exp(1))^5-81064793292668928*tan(1/2*exp
(1))^4-90071992547409920*tan(1/2*exp(1))^3+54043195528445952*tan(1/2*exp(1)
)^2+27021597764222976*tan(1/2*exp(1))-9007199254740992)+B*c*sqrt(a*tan(1/2*
exp(1))^2+a)*(-27021597764222976*tan(1/2*exp(1))^5+81064793292668928*tan(1/
```

2*exp(1))^4+90071992547409920*tan(1/2*exp(1))^3-54043195528445952*tan(1/2*e
xp(1))^2-27021597764222976*tan(1/2*exp(1))+9007199254740992)+A*c*sqrt(a*tan
(1/2*exp(1))^2+a)*(54043195528445952*tan(1/2*exp(1))^6-162129586585337856*t
an(1/2*exp(1))^5-324259173170675712*tan(1/2*exp(1))^4+540431955284459520*ta
n(1/2*exp(1))^3+486388759756013568*tan(1/2*exp(1))^2-162129586585337856*tan
(1/2*exp(1)))*tan(1/4*exp(1))^5+A*c*sqrt(a*tan(1/2*exp(1))^2+a)*(5404319552
8445952*tan(1/2*exp(1))^6-162129586585337856*tan(1/2*exp(1))^5-324259173170
675712*tan(1/2*exp(1))^4+540431955284459520*tan(1/2*exp(1))^3+4863887597560
13568*tan(1/2*exp(1))^2-162129586585337856*tan(1/2*exp(1)))*tan(1/4*exp(1))
+A*c*sqrt(a*tan(1/2*exp(1))^2+a)*(405323966463344640*tan(1/2*exp(1))^5-1215
971899390033920*tan(1/2*exp(1))^4-1351079888211148800*tan(1/2*exp(1))^3+810
647932926689280*tan(1/2*exp(1))^2+405323966463344640*tan(1/2*exp(1))-135107
988821114880)*tan(1/4*exp(1))^4+A*c*sqrt(a*tan(1/2*exp(1))^2+a)*(-270215977
64222976*tan(1/2*exp(1))^5+81064793292668928*tan(1/2*exp(1))^4+900719925474
09920*tan(1/2*exp(1))^3-54043195528445952*tan(1/2*exp(1))^2-270215977642229
76*tan(1/2*exp(1))+9007199254740992)*tan(1/4*exp(1))^6+A*c*sqrt(a*tan(1/2*e
xp(1))^2+a)*(-180143985094819840*tan(1/2*exp(1))^6+540431955284459520*tan(1
/2*exp(1))^5+1080863910568919040*tan(1/2*exp(1))^4-1801439850948198400*tan(
1/2*exp(1))^3-1621295865853378560*tan(1/2*exp(1))^2+540431955284459520*tan(
1/2*exp(1)))*tan(1/4*exp(1))^3+A*c*sqrt(a*tan(1/2*exp(1))^2+a)*(-4053239664
63344640*tan(1/2*exp(1))^5+1215971899390033920*tan(1/2*exp(1))^4+1351079888
211148800*tan(1/2*exp(1))^3-810647932926689280*tan(1/2*exp(1))^2-4053239664
63344640*tan(1/2*exp(1))+135107988821114880)*tan(1/4*exp(1))^2+B*c*sqrt(a*t
an(1/2*exp(1))^2+a)*(27021597764222976*tan(1/2*exp(1))^5-81064793292668928*
tan(1/2*exp(1))^4-90071992547409920*tan(1/2*exp(1))^3+54043195528445952*tan
(1/2*exp(1))^2+27021597764222976*tan(1/2*exp(1))-9007199254740992)*tan(1/4*
exp(1))^6+B*c*sqrt(a*tan(1/2*exp(1))^2+a)*(180143985094819840*tan(1/2*exp(1
))^6-540431955284459520*tan(1/2*exp(1))^5-1080863910568919040*tan(1/2*exp(1
))^4+1801439850948198400*tan(1/2*exp(1))^3+1621295865853378560*tan(1/2*exp(
1))^2-540431955284459520*tan(1/2*exp(1)))*tan(1/4*exp(1))^3+B*c*sqrt(a*tan(
1/2*exp(1))^2+a)*(405323966463344640*tan(1/2*exp(1))^5-1215971899390033920*
tan(1/2*exp(1))^4-1351079888211148800*tan(1/2*exp(1))^3+810647932926689280*
tan(1/2*exp(1))^2+405323966463344640*tan(1/2*exp(1))-135107988821114880)*ta
n(1/4*exp(1))^2+B*c*sqrt(a*tan(1/2*exp(1))^2+a)*(-54043195528445952*tan(1/2
*exp(1))^6+162129586585337856*tan(1/2*exp(1))^5+324259173170675712*tan(1/2*
exp(1))^4-540431955284459520*tan(1/2*exp(1))^3-486388759756013568*tan(1/2*e
xp(1))^2+162129586585337856*tan(1/2*exp(1)))*tan(1/4*exp(1))^5+B*c*sqrt(a*t
an(1/2*exp(1))^2+a)*(-54043195528445952*tan(1/2*exp(1))^6+16212958658533785
6*tan(1/2*exp(1))^5+324259173170675712*tan(1/2*exp(1))^4-540431955284459520
*tan(1/2*exp(1))^3-486388759756013568*tan(1/2*exp(1))^2+162129586585337856*
tan(1/2*exp(1)))*tan(1/4*exp(1))+B*c*sqrt(a*tan(1/2*exp(1))^2+a)*(-40532396
6463344640*tan(1/2*exp(1))^5+1215971899390033920*tan(1/2*exp(1))^4+13510798
88211148800*tan(1/2*exp(1))^3-810647932926689280*tan(1/2*exp(1))^2-40532396
6463344640*tan(1/2*exp(1))+135107988821114880)*tan(1/4*exp(1))^4)*ln(abs(2*
tan(1/2*exp(1))^3+6*tan(1/2*exp(1))^2+(tan(1/2*(1/2*f*x+2*exp(1)))-1/tan(1/

```
2*(1/2*f*x+2*exp(1))))*(tan(1/2*exp(1))^3-3*tan(1/2*exp(1))^2-3*tan(1/2*exp
(1))+1)-6*tan(1/2*exp(1))-2))/f/(-9007199254740992*sqrt(2)*a*tan(1/2*exp(1)
)^7-9007199254740992*sqrt(2)*a+(-9007199254740992*sqrt(2)*a*tan(1/2*exp(1))
^7+27021597764222976*sqrt(2)*a*tan(1/2*exp(1))^6+9007199254740992*sqrt(2)*a
*tan(1/2*exp(1))^5+45035996273704960*sqrt(2)*a*tan(1/2*exp(1))^4+4503599627
3704960*sqrt(2)*a*tan(1/2*exp(1))^3+9007199254740992*sqrt(2)*a*tan(1/2*exp(
1))^2-9007199254740992*sqrt(2)*a+27021597764222976*sqrt(2)*a*tan(1/2*exp(1)
))*tan(1/4*exp(1))^6+(-27021597764222976*sqrt(2)*a*tan(1/2*exp(1))^7+810647
93292668928*sqrt(2)*a*tan(1/2*exp(1))^6+27021597764222976*sqrt(2)*a*tan(1/2
*exp(1))^5+135107988821114880*sqrt(2)*a*tan(1/2*exp(1))^4+13510798882111488
0*sqrt(2)*a*tan(1/2*exp(1))^3+27021597764222976*sqrt(2)*a*tan(1/2*exp(1))^2
-27021597764222976*sqrt(2)*a+81064793292668928*sqrt(2)*a*tan(1/2*exp(1)))*t
an(1/4*exp(1))^2+(-27021597764222976*sqrt(2)*a*tan(1/2*exp(1))^7+8106479329
2668928*sqrt(2)*a*tan(1/2*exp(1))^6+27021597764222976*sqrt(2)*a*tan(1/2*exp
(1))^5+135107988821114880*sqrt(2)*a*tan(1/2*exp(1))^4+135107988821114880*sq
rt(2)*a*tan(1/2*exp(1))^3+27021597764222976*sqrt(2)*a*tan(1/2*exp(1))^2-270
21597764222976*sqrt(2)*a+81064793292668928*sqrt(2)*a*tan(1/2*exp(1)))*tan(1
/4*exp(1))^4+27021597764222976*sqrt(2)*a*tan(1/2*exp(1))^6+9007199254740992
*sqrt(2)*a*tan(1/2*exp(1))^5+45035996273704960*sqrt(2)*a*tan(1/2*exp(1))^4+
45035996273704960*sqrt(2)*a*tan(1/2*exp(1))^3+9007199254740992*sqrt(2)*a*ta
n(1/2*exp(1))^2+27021597764222976*sqrt(2)*a*tan(1/2*exp(1)))
```

maple [B] time = 0.68, size = 501, normalized size = 3.43

$$\frac{\left(-B\left(\cos^3\left(fx+e\right)\right)+B\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+2A\left(\cos^2\left(fx+e\right)\right)+2A\sin\left(fx+e\right)\cos\left(fx+e\right)+4A\,\mathrm{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*(-B*cos(f*x+e)^3+B*cos(f*x+e)^2*sin(f*x+e)+2*A*cos(f*x+e)^2+2*A*sin(f
*x+e)*cos(f*x+e)+4*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-8*A*cos(f*x+e)*ln((1-c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))-4*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+8*A*s
in(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-3*B*cos(f*x+e)^2-4*B*sin
(f*x+e)*cos(f*x+e)-4*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+8*B*cos(f*x+e)*ln((1
-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+4*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-8*B
*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*A*sin(f*x+e)-4*A*ln(
2/(cos(f*x+e)+1))+8*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*cos(f*x+e)
+3*B*sin(f*x+e)+4*B*ln(2/(cos(f*x+e)+1))-8*B*ln((1-cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))-2*A+3*B)*(-c*(sin(f*x+e)-1))^(3/2)/(cos(f*x+e)^2-sin(f*x+e)*cos(
f*x+e)+cos(f*x+e)+2*sin(f*x+e)-2)/(a*(1+sin(f*x+e)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(3/2)/sqrt(a*sin(f*x +
 e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^
(1/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^
(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}\left(A+B\sin\left(e+fx\right)\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((-c*(sin(e + f*x) - 1))**(3/2)*(A + B*sin(e + f*x))/sqrt(a*(sin(e
+ f*x) + 1)), x)
```

3.176 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=96

$$\frac{c(A-B)\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] (A-B)*c*cos(f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e
))^(1/2)-B*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2971, 2738, 2737, 2667, 31}

$$\frac{c(A-B)\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[e + f*x]
],x]
```

```
[Out] ((A - B)*c*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*
Sqrt[c - c*Sin[e + f*x]]) - (B*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sq
rt[a + a*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{B\int \sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}\,dx}{a} - (-A+B)\int \frac{\sqrt{c}}{\sqrt{a}}$$

$$= -\frac{B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{(a(-A+B)c\cos(e+fx))\int}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c-}}$$

$$= -\frac{B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{((-A+B)c\cos(e+fx))\,\mathrm{Su}}{f\sqrt{a+a\sin(e+}}$$

$$= \frac{(A-B)c\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)\sqrt{c-}}{f\sqrt{a+a\sin(}}$$

Mathematica [C] time = 1.13, size = 119, normalized size = 1.24

$$\frac{\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(B\sin(e+fx)+(A-B)\left(2\log\left(e^{i(e+fx)}+i\right)-ifx\right)\right)}{f\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[e
+ f*x]],x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((A - B)*((-I)*f*x + 2*Log[I + E^(I*
(e + f*x))]) + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(f*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])])
```

fricas [F] time = 1.36, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{-c\sin\left(fx+e\right)+c}}{\sqrt{a\sin\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(f*x + e)
 + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (8*pi/x/2)>(-8*pi/x/2)4*sqrt(2*c)*(A*sqrt(a*tan(1/2*exp(1)
)^2+a)*(12582912*tan(1/2*exp(1))^5-37748736*tan(1/2*exp(1))^4-41943040*tan(
1/2*exp(1))^3+25165824*tan(1/2*exp(1))^2+12582912*tan(1/2*exp(1))-4194304)+
B*sqrt(a*tan(1/2*exp(1))^2+a)*(-12582912*tan(1/2*exp(1))^5+37748736*tan(1/2
*exp(1))^4+41943040*tan(1/2*exp(1))^3-25165824*tan(1/2*exp(1))^2-12582912*t
an(1/2*exp(1))+4194304)+A*sqrt(a*tan(1/2*exp(1))^2+a)*(25165824*tan(1/2*exp
(1))^6-75497472*tan(1/2*exp(1))^5-150994944*tan(1/2*exp(1))^4+251658240*tan
(1/2*exp(1))^3+226492416*tan(1/2*exp(1))^2-75497472*tan(1/2*exp(1)))*tan(1/
4*exp(1))^5+A*sqrt(a*tan(1/2*exp(1))^2+a)*(25165824*tan(1/2*exp(1))^6-75497
472*tan(1/2*exp(1))^5-150994944*tan(1/2*exp(1))^4+251658240*tan(1/2*exp(1))
^3+226492416*tan(1/2*exp(1))^2-75497472*tan(1/2*exp(1)))*tan(1/4*exp(1))+A*
sqrt(a*tan(1/2*exp(1))^2+a)*(188743680*tan(1/2*exp(1))^5-566231040*tan(1/2*
exp(1))^4-629145600*tan(1/2*exp(1))^3+377487360*tan(1/2*exp(1))^2+188743680
*tan(1/2*exp(1))-62914560)*tan(1/4*exp(1))^4+A*sqrt(a*tan(1/2*exp(1))^2+a)*
(-12582912*tan(1/2*exp(1))^5+37748736*tan(1/2*exp(1))^4+41943040*tan(1/2*ex
p(1))^3-25165824*tan(1/2*exp(1))^2-12582912*tan(1/2*exp(1))+4194304)*tan(1/
```

```
4*exp(1))^6+A*sqrt(a*tan(1/2*exp(1))^2+a)*(-83886080*tan(1/2*exp(1))^6+2516
58240*tan(1/2*exp(1))^5+503316480*tan(1/2*exp(1))^4-838860800*tan(1/2*exp(1
))^3-754974720*tan(1/2*exp(1))^2+251658240*tan(1/2*exp(1)))*tan(1/4*exp(1))
^3+A*sqrt(a*tan(1/2*exp(1))^2+a)*(-188743680*tan(1/2*exp(1))^5+566231040*ta
n(1/2*exp(1))^4+629145600*tan(1/2*exp(1))^3-377487360*tan(1/2*exp(1))^2-188
743680*tan(1/2*exp(1))+62914560)*tan(1/4*exp(1))^2+B*sqrt(a*tan(1/2*exp(1))
^2+a)*(12582912*tan(1/2*exp(1))^5-37748736*tan(1/2*exp(1))^4-41943040*tan(1
/2*exp(1))^3+25165824*tan(1/2*exp(1))^2+12582912*tan(1/2*exp(1))-4194304)*t
an(1/4*exp(1))^6+B*sqrt(a*tan(1/2*exp(1))^2+a)*(83886080*tan(1/2*exp(1))^6-
251658240*tan(1/2*exp(1))^5-503316480*tan(1/2*exp(1))^4+838860800*tan(1/2*e
xp(1))^3+754974720*tan(1/2*exp(1))^2-251658240*tan(1/2*exp(1)))*tan(1/4*exp
(1))^3+B*sqrt(a*tan(1/2*exp(1))^2+a)*(188743680*tan(1/2*exp(1))^5-566231040
*tan(1/2*exp(1))^4-629145600*tan(1/2*exp(1))^3+377487360*tan(1/2*exp(1))^2+
188743680*tan(1/2*exp(1))-62914560)*tan(1/4*exp(1))^2+B*sqrt(a*tan(1/2*exp(
1))^2+a)*(-25165824*tan(1/2*exp(1))^6+75497472*tan(1/2*exp(1))^5+150994944*
tan(1/2*exp(1))^4-251658240*tan(1/2*exp(1))^3-226492416*tan(1/2*exp(1))^2+7
5497472*tan(1/2*exp(1)))*tan(1/4*exp(1))^5+B*sqrt(a*tan(1/2*exp(1))^2+a)*(-
25165824*tan(1/2*exp(1))^6+75497472*tan(1/2*exp(1))^5+150994944*tan(1/2*exp
(1))^4-251658240*tan(1/2*exp(1))^3-226492416*tan(1/2*exp(1))^2+75497472*tan
(1/2*exp(1)))*tan(1/4*exp(1))+B*sqrt(a*tan(1/2*exp(1))^2+a)*(-188743680*tan
(1/2*exp(1))^5+566231040*tan(1/2*exp(1))^4+629145600*tan(1/2*exp(1))^3-3774
87360*tan(1/2*exp(1))^2-188743680*tan(1/2*exp(1))+62914560)*tan(1/4*exp(1))
^4)*ln(abs(2*tan(1/2*exp(1))^3+6*tan(1/2*exp(1))^2+(tan(1/2*(1/2*f*x+2*exp(
1)))-1/tan(1/2*(1/2*f*x+2*exp(1))))*(tan(1/2*exp(1))^3-3*tan(1/2*exp(1))^2-
3*tan(1/2*exp(1))+1)-6*tan(1/2*exp(1))-2))/f/(-8388608*sqrt(2)*a*tan(1/2*ex
p(1))^7-8388608*sqrt(2)*a+(-8388608*sqrt(2)*a*tan(1/2*exp(1))^7+25165824*sq
rt(2)*a*tan(1/2*exp(1))^6+8388608*sqrt(2)*a*tan(1/2*exp(1))^5+41943040*sqrt
(2)*a*tan(1/2*exp(1))^4+41943040*sqrt(2)*a*tan(1/2*exp(1))^3+8388608*sqrt(2
)*a*tan(1/2*exp(1))^2-8388608*sqrt(2)*a+25165824*sqrt(2)*a*tan(1/2*exp(1)))
*tan(1/4*exp(1))^6+(-25165824*sqrt(2)*a*tan(1/2*exp(1))^7+75497472*sqrt(2)*
a*tan(1/2*exp(1))^6+25165824*sqrt(2)*a*tan(1/2*exp(1))^5+125829120*sqrt(2)*
a*tan(1/2*exp(1))^4+125829120*sqrt(2)*a*tan(1/2*exp(1))^3+25165824*sqrt(2)*
a*tan(1/2*exp(1))^2-25165824*sqrt(2)*a+75497472*sqrt(2)*a*tan(1/2*exp(1)))*
tan(1/4*exp(1))^2+(-25165824*sqrt(2)*a*tan(1/2*exp(1))^7+75497472*sqrt(2)*a
*tan(1/2*exp(1))^6+25165824*sqrt(2)*a*tan(1/2*exp(1))^5+125829120*sqrt(2)*a
*tan(1/2*exp(1))^4+125829120*sqrt(2)*a*tan(1/2*exp(1))^3+25165824*sqrt(2)*a
*tan(1/2*exp(1))^2-25165824*sqrt(2)*a+75497472*sqrt(2)*a*tan(1/2*exp(1)))*t
an(1/4*exp(1))^4+25165824*sqrt(2)*a*tan(1/2*exp(1))^6+8388608*sqrt(2)*a*tan
(1/2*exp(1))^5+41943040*sqrt(2)*a*tan(1/2*exp(1))^4+41943040*sqrt(2)*a*tan(
1/2*exp(1))^3+8388608*sqrt(2)*a*tan(1/2*exp(1))^2+25165824*sqrt(2)*a*tan(1/
2*exp(1)))
```

maple [B] time = 0.68, size = 399, normalized size = 4.16

$$-\frac{\left(2A\cos\left(fx+e\right)\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-A\cos\left(fx+e\right)\ln\left(\frac{2}{\cos(fx+e)+1}\right)-2A\sin\left(fx+e\right)\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(2*A*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*cos(f*x+e)*
ln(2/(cos(f*x+e)+1))-2*A*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)+A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+B*cos(f*x+e)^2+B*sin(f*x+e)*cos(f*x+e)-
2*B*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*cos(f*x+e)*ln(2/(
cos(f*x+e)+1))+2*B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*si
n(f*x+e)*ln(2/(cos(f*x+e)+1))-2*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+
A*ln(2/(cos(f*x+e)+1))-B*sin(f*x+e)+2*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))-B*ln(2/(cos(f*x+e)+1))-B)*(-c*(sin(f*x+e)-1))^(1/2)/(-1+cos(f*x+e)+si
n(f*x+e))/(a*(1+sin(f*x+e)))^(1/2)
```

maxima [A] time = 0.79, size = 176, normalized size = 1.83

$$\frac{B\left(\frac{2\sqrt{c}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{a}}-\frac{\sqrt{c}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{a}}-\frac{2\sqrt{a}\sqrt{c}\sin(fx+e)}{\left(a+\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right)-A\left(\frac{2\sqrt{c}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{a}}-\frac{\sqrt{c}\log\left(\frac{\sin}{(\cos}\right.}{}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] (B*(2*sqrt(c)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(a) - sqrt(c)*lo
g(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(a) - 2*sqrt(a)*sqrt(c)*sin(
f*x + e)/((a + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)))
- A*(2*sqrt(c)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(a) - sqrt(c)*l
og(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(a)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^
(1/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^
(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\left(A+B\sin\left(e+fx\right)\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(e + f*x))/sqrt(a*(sin(e + f
*x) + 1)), x)
```

3.177 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=113

$$\frac{(A-B)\cos(e+fx)\log(\sin(e+fx)+1)}{2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/2*(A+B)*cos(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*
x+e))^(1/2)+1/2*(A-B)*cos(f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/
(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2969, 2737, 2667, 31}

$$\frac{(A-B)\cos(e+fx)\log(\sin(e+fx)+1)}{2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]
),x]
```

```
[Out] -((A + B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(2*f*Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c - c*Sin[e + f*x]]) + ((A - B)*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(
2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2969

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist
[(A*b + a*B)/(2*a*b), Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] + Dist[(B*c + A*d)/(2*c*d), Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a +
b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}\,dx = \frac{(A+B)\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{2a} + \frac{(A-B)\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx}{2c}$$

$$= \frac{(a(A-B)\cos(e+fx))\int \frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{2\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}} + \frac{((A+B)c\cos(e+fx}{2\sqrt{a+a\sin(e+fx)}\,\cdots}$$

$$= \frac{((A-B)\cos(e+fx))\,\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\sin(e+fx)\right)}{2f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}} - \frac{((A+\cdots}{\cdots}$$

$$= -\frac{(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{2f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}} + \frac{(A-B)\cos(e+fx\cdots}{2f\sqrt{a+a\sin(e+\cdots}}$$

Mathematica [A] time = 0.34, size = 97, normalized size = 0.86

$$-\frac{\cos(e+fx)\left((A+B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+(B-A)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\cdots\right.\right.}{f\sqrt{a(\sin(e+fx)+1)}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e +
 f*x]]),x]
```

```
[Out] -((Cos[e + f*x]*((A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (-A + B
)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]))/(f*Sqrt[a*(1 + Sin[e + f*x])]*
Sqrt[c - c*Sin[e + f*x]]))
```

fricas [F] time = 1.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{ac\cos\left(fx+e\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)/(a*c*cos(f*x + e)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
e) + c)), x)
```

maple [A] time = 0.59, size = 165, normalized size = 1.46

$$-\frac{\left(A\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-A\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+B\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-B\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)}{f\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*ln((1-cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))+B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*ln(2/(cos
(f*x+e)+1))+B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)))*cos(f*x+e)/(a*(1+si
n(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
1/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\sin\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))/(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e +
f*x) - 1))), x)
```

3.178 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=103

$$\frac{(A+B)\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{(A-B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/2*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/2*(A
-B)*arctanh(sin(f*x+e))*cos(f*x+e)/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+
e))^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2972, 2741, 3770}

$$\frac{(A+B)\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{(A-B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/
2)),x]
```

```
[Out] ((A + B)*Cos[e + f*x])/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
3/2)) + ((A - B)*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*c*f*Sqrt[a + a*Sin[
e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(A+B)\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} + \frac{(A-B)\int \frac{}{\sqrt{a+a\,\mathrm{si}}}}{}$$

$$= \frac{(A+B)\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} + \frac{((A-B)\cos(e}{2c\sqrt{a+a\sin(e+}}$$

$$= \frac{(A+B)\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} + \frac{(A-B)\tanh^{-1}(}{2cf\sqrt{a+a\sin(e}}$$

Mathematica [A] time = 0.54, size = 191, normalized size = 1.85

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left((B-A)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.\right.}{2f\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x
])^(3/2)),x]
```

```
[Out] ((A + B + (-A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^2 + (A - B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)
/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(2*f*Sqrt[a*(1 + Sin[e + f*x])]
*(c - c*Sin[e + f*x])^(3/2))
```

fricas [A] time = 0.52, size = 337, normalized size = 3.27

$$\left[-\frac{\left((A-B)\cos\left(fx+e\right)\sin\left(fx+e\right)-(A-B)\cos\left(fx+e\right)\right)\sqrt{ac}\,\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)+2\,\sqrt{ac}\,\sqrt{a\sin\left(fx+e\right)}}{\cos\left(fx+e\right)^{3}}\right.}{4\left(ac^{2}f\cos\left(fx+e\right)\sin\left(fx+e\right)-ac^{2}f\,\mathrm{c}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] [-1/4*(((A - B)*cos(f*x + e)*sin(f*x + e) - (A - B)*cos(f*x + e))*sqrt(a*c)
*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) + 2*sqrt(a*c)*sqrt(a*sin(f*x
 + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) + 2*sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A + B))/(a*c^2*f*cos(f*x +
e)*sin(f*x + e) - a*c^2*f*cos(f*x + e)), -1/2*(((A - B)*cos(f*x + e)*sin(f*
x + e) - (A - B)*cos(f*x + e))*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x
+ e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) + sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A + B))/(a*c^2*f*cos(f*x +
e)*sin(f*x + e) - a*c^2*f*cos(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
 c)^(3/2)), x)
```

maple [B] time = 0.60, size = 302, normalized size = 2.93

$$\frac{\left(A\sin\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-A\sin\left(fx+e\right)\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-B\sin\left(fx+e\right)\ln\left(-\frac{-1+\cos}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*(A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*sin(f*x+e)
*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))+B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)-A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+A*ln((1-cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))+A*sin(f*x+e)+B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*
ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*sin(f*x+e))*cos(f*x+e)/(a*(1+sin
(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
 c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
3/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\sin\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))/(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x)
 - 1))**(3/2)), x)
```

3.179 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=153

$$\frac{(A-B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{4c^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{(A-B)\cos(e+fx)}{4cf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{(A+B)\cos}{4f\sqrt{a\sin(e+fx)+a}\,(}$$

```
[Out] 1/4*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)+1/4*(A
-B)*cos(f*x+e)/c/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/4*(A-B)*
arctanh(sin(f*x+e))*cos(f*x+e)/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e)
)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2972, 2743, 2741, 3770}

$$\frac{(A-B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{4c^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{(A-B)\cos(e+fx)}{4cf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{(A+B)\cos}{4f\sqrt{a\sin(e+fx)+a}\,(}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/
2)),x]
```

```
[Out] ((A + B)*Cos[e + f*x])/(4*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
5/2)) + ((A - B)*Cos[e + f*x])/(4*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
 + f*x])^(3/2)) + ((A - B)*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(4*c^2*f*Sqr
t[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
```

```
SumSimplerQ[n, 1])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}\,dx = \frac{(A+B)\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} + \frac{(A-B)\int \frac{}{\sqrt{a+a\,\mathrm{si}}}}{}$$

$$= \frac{(A+B)\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} + \frac{(A-}{4cf\sqrt{a+a\sin(e}}$$

$$= \frac{(A+B)\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} + \frac{(A-}{4cf\sqrt{a+a\sin(e}}$$

$$= \frac{(A+B)\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} + \frac{(A-}{4cf\sqrt{a+a\sin(e}}$$

Mathematica [A] time = 0.63, size = 222, normalized size = 1.45

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left((A-B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x
])^(5/2)),x]
```

```
[Out] ((A + B + (A - B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + (-A + B)*Log[Co
s[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4
+ (A - B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])^4)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2]))/(4*f*Sqrt[a*(1 + Sin[e + f*x])]*(c - c*Sin[e + f*x])^(5/
2))
```

fricas [A] time = 0.54, size = 424, normalized size = 2.77

$$\left[-\frac{\left((A-B)\cos\left(fx+e\right)^{3}+2\,(A-B)\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,(A-B)\cos\left(fx+e\right)\right)\sqrt{ac}\,\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\cdots}{\cdots}\right.}{8\left(ac^{3}f\cos\left(fx+e\right)^{3}+2\,ac^{3}f\,\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] [-1/8*(((A - B)*cos(f*x + e)^3 + 2*(A - B)*cos(f*x + e)*sin(f*x + e) - 2*(A
 - B)*cos(f*x + e))*sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e)
 + 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x +
 e))/cos(f*x + e)^3) - 2*((A - B)*sin(f*x + e) - 2*A)*sqrt(a*sin(f*x + e) +
 a)*sqrt(-c*sin(f*x + e) + c))/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(f*x
+ e)*sin(f*x + e) - 2*a*c^3*f*cos(f*x + e)), -1/4*(((A - B)*cos(f*x + e)^3
+ 2*(A - B)*cos(f*x + e)*sin(f*x + e) - 2*(A - B)*cos(f*x + e))*sqrt(-a*c)*
arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*c
os(f*x + e)*sin(f*x + e))) - ((A - B)*sin(f*x + e) - 2*A)*sqrt(a*sin(f*x +
e) + a)*sqrt(-c*sin(f*x + e) + c))/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(
f*x + e)*sin(f*x + e) - 2*a*c^3*f*cos(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
 c)^(5/2)), x)
```

maple [B] time = 0.62, size = 465, normalized size = 3.04

$$\frac{\left(A\left(\cos^2\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-A\left(\cos^2\left(fx+e\right)\right)\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-B\left(\cos^2\left(fx+e\right)\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/4/f*(A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*cos(f*x+
e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*cos(f*x+e)^2*ln(-(-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))+B*cos(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))+2*A*cos(f*x+e)^2+2*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))-2*A*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*B*sin(f
*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*B*sin(f*x+e)*ln((1-cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))+3*A*sin(f*x+e)-2*A*ln(-(-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))+2*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*sin(f*x+e)+
2*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*B*ln((1-cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))-2*A)*cos(f*x+e)/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1
))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
 c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
5/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

3.180 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=271

$$-\frac{4c^4(3A-5B)\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2c^3(3A-5B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}-\frac{c^2(3A-5B)\cos}{2af}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(3/2)-1/2*(
3*A-5*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f/(a+a*sin(f*x+e))^(1/2)-1
/6*(3*A-5*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a/f/(a+a*sin(f*x+e))^(1/2)
-4*(3*A-5*B)*c^4*cos(f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-
c*sin(f*x+e))^(1/2)-2*(3*A-5*B)*c^3*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(
a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2972, 2740, 2737, 2667, 31}

$$-\frac{2c^3(3A-5B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}-\frac{c^2(3A-5B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a\sin(e+fx)+a}}-\frac{4c^4(3A-5B)\cos}{af\sqrt{a\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (-4*(3*A - 5*B)*c^4*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin
[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (2*(3*A - 5*B)*c^3*Cos[e + f*x]*Sqrt
[c - c*Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]) - ((3*A - 5*B)*c^2*Cos
[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*a*f*Sqrt[a + a*Sin[e + f*x]]) - ((
3*A - 5*B)*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(6*a*f*Sqrt[a + a*Sin
[e + f*x]]) - ((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(2*f*(a + a
*Sin[e + f*x])^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
```

```
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{(3A-5B)\int \frac{(c-c\,s}{\sqrt{a+}}}{2a}$$

$$= -\frac{(3A-5B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e}{2f(a}$$

$$= -\frac{(3A-5B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(3A-5B)c\,c}{6a}$$

$$= -\frac{2(3A-5B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{(3A-5B)c^2}{2a}$$

$$= -\frac{2(3A-5B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{(3A-5B)c^2}{2a}$$

$$= -\frac{2(3A-5B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{(3A-5B)c^2}{2a}$$

$$= -\frac{4(3A-5B)c^4\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2(3A-5B}{}$$

Mathematica [A] time = 3.48, size = 271, normalized size = 1.00

$$-\frac{c^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2(27A-59B)\cos(2(e+fx))-117A\sin(e+fx)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
f*x])^(3/2),x]
```

```
[Out] -1/24*(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(
132*A - 45*B + 2*(27*A - 59*B)*Cos[2*(e + f*x)] + B*Cos[4*(e + f*x)] + 576*
A*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 960*B*Log[Cos[(e + f*x)/2] + S
in[(e + f*x)/2]] - 117*A*Sin[e + f*x] + 279*B*Sin[e + f*x] + 576*A*Log[Cos[
(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] - 960*B*Log[Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]]*Sin[e + f*x] - 3*A*Sin[3*(e + f*x)] + 13*B*Sin[3*(e + f
*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2
))
```

fricas [F] time = 14.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bc^3\cos\left(fx+e\right)^4+(3A-5B)c^3\cos\left(fx+e\right)^2-4(A-B)c^3-\left((A-3B)c^3\cos\left(fx+e\right)^2-4(A-B)\right.\right.}{a^2\cos\left(fx+e\right)^2-2a^2\sin\left(fx+e\right)-2a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*c^3*cos(f*x + e)^4 + (3*A - 5*B)*c^3*cos(f*x + e)^2 - 4*(A - B)
*c^3 - ((A - 3*B)*c^3*cos(f*x + e)^2 - 4*(A - B)*c^3)*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin
(f*x + e) - 2*a^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.66, size = 939, normalized size = 3.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/6/f*(13*B*cos(f*x+e)^3*sin(f*x+e)-288*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))+480*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+102*A*sin(f*x+e)-75*
A*sin(f*x+e)*cos(f*x+e)-3*A*cos(f*x+e)^3*sin(f*x+e)-2*B*sin(f*x+e)*cos(f*x+
e)^4+120*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-24*A*cos(f*x+e)^2*sin(f*x+e)+4
8*B*cos(f*x+e)^2*sin(f*x+e)+11*B*cos(f*x+e)^4+102*A-166*B+27*A*cos(f*x+e)^3
-72*A*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+144*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1
))-72*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+107*B*sin(f*x+e)*cos(f*x+e)-240*B*s
in(f*x+e)*ln(2/(cos(f*x+e)+1))+120*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+144*A*
cos(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-240*B*cos(f*x+e)^2*ln
((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-72*A*cos(f*x+e)*sin(f*x+e)*ln(2/(cos
(f*x+e)+1))+2*B*cos(f*x+e)^5+120*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+
```

```
e)+144*A*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-240*B*cos(f*x+
e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-288*A*sin(f*x+e)*ln((1-cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))+480*B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))+155*B*cos(f*x+e)^2-3*A*cos(f*x+e)^4-61*B*cos(f*x+e)^3-166*B*sin(f
*x+e)+144*A*sin(f*x+e)*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-
240*B*sin(f*x+e)*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-27*A*c
os(f*x+e)+59*B*cos(f*x+e)-99*A*cos(f*x+e)^2+144*A*ln(2/(cos(f*x+e)+1))-240*
B*ln(2/(cos(f*x+e)+1)))*(-c*(sin(f*x+e)-1))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3-
cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+
e)+8*cos(f*x+e)^2+8*sin(f*x+e)+4*cos(f*x+e)-8)/(a*(1+sin(f*x+e)))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e)
+ a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^
(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

3.181 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=210

$$-\frac{4c^3(A-2B)\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2c^2(A-2B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}-\frac{c(A-2B)\cos(e+\ldots}{2af\sqrt{a\sin\ldots}}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(3/2)-1/2*(
A-2*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f/(a+a*sin(f*x+e))^(1/2)-4*(A-
2*B)*c^3*cos(f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*
x+e))^(1/2)-2*(A-2*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*
x+e))^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2972, 2740, 2737, 2667, 31}

$$-\frac{2c^2(A-2B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}-\frac{4c^3(A-2B)\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{c(A-2B)\cos(e+\ldots}{2af\sqrt{a\sin\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (-4*(A - 2*B)*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e
 + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (2*(A - 2*B)*c^2*Cos[e + f*x]*Sqrt[c -
 c*Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]) - ((A - 2*B)*c*Cos[e + f*x
]*(c - c*Sin[e + f*x])^(3/2))/(2*a*f*Sqrt[a + a*Sin[e + f*x]]) - ((A - B)*C
os[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{(A-2B)\int \frac{(c-c\sin(}{\sqrt{a+a\sin}}}{a}$$

$$= -\frac{(A-2B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+}{2f(a+}$$

$$= -\frac{2(A-2B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{(A-2B)c\cos(e}{2af\sqrt{}}$$

$$= -\frac{2(A-2B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{(A-2B)c\cos(e}{2af\sqrt{}}$$

$$= -\frac{2(A-2B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{(A-2B)c\cos(e}{2af\sqrt{}}$$

$$= -\frac{4(A-2B)c^3\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2(A-2B)c^2 c}{af}$$

Mathematica [A] time = 1.60, size = 212, normalized size = 1.01

$$-\frac{c^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2(2A-7B)\cos(2(e+fx))+\sin(e+fx)\left(64(A-2B\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
f*x])^(3/2),x]
```

```
[Out] -1/8*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(2
8*A - 16*B + 2*(2*A - 7*B)*Cos[2*(e + f*x)] + 64*A*Log[Cos[(e + f*x)/2] + S
in[(e + f*x)/2]] - 128*B*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (-8*A +
 31*B + 64*(A - 2*B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]
 + B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Si
n[e + f*x]))^(3/2))
```

fricas [F] time = 8.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\left(A-2\,B\right)c^2\cos\left(fx+e\right)^2-2\,\left(A-B\right)c^2+\left(Bc^2\cos\left(fx+e\right)^2+2\,\left(A-B\right)c^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx-\right.}}{a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x + e
)^2 + 2*(A - B)*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.63, size = 853, normalized size = 4.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/2/f*(B*cos(f*x+e)^3*sin(f*x+e)-32*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))+64*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+12*A*sin(f*x+e)-10*A*sin
(f*x+e)*cos(f*x+e)+16*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-2*A*cos(f*x+e)^2*
sin(f*x+e)+6*B*cos(f*x+e)^2*sin(f*x+e)+B*cos(f*x+e)^4+12*A-22*B+2*A*cos(f*x
+e)^3-8*A*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+16*A*sin(f*x+e)*ln(2/(cos(f*x+e
)+1))-8*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+15*B*sin(f*x+e)*cos(f*x+e)-32*B*s
in(f*x+e)*ln(2/(cos(f*x+e)+1))+16*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+16*A*co
s(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-32*B*cos(f*x+e)^2*ln((1
-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-8*A*cos(f*x+e)*sin(f*x+e)*ln(2/(cos(f*x
+e)+1))+16*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)+16*A*cos(f*x+e)*ln(
(1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-32*B*cos(f*x+e)*ln((1-cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))-32*A*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)+64*B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+21*B*cos(f*x+e)^
2-7*B*cos(f*x+e)^3-22*B*sin(f*x+e)+16*A*sin(f*x+e)*cos(f*x+e)*ln((1-cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))-32*B*sin(f*x+e)*cos(f*x+e)*ln((1-cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))-2*A*cos(f*x+e)+7*B*cos(f*x+e)-12*A*cos(f*x+e)^2+16*A*l
n(2/(cos(f*x+e)+1))-32*B*ln(2/(cos(f*x+e)+1)))*(-c*(sin(f*x+e)-1))^(5/2)/(c
os(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^3+2*sin(f*x+e)*cos(f*x+e)-3*cos(f*x+e)^2-
4*sin(f*x+e)-2*cos(f*x+e)+4)/(a*(1+sin(f*x+e)))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e)
+ a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^
(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

3.182 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=159

$$-\frac{c^2(A-3B)\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{c(A-3B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2af\sqrt{a\sin(e+fx)+a}}-\frac{(A-B)\cos(e+fx)(c}{2f(a\sin(e+}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(3/2)-(A-3*
B)*c^2*cos(f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+
e))^(1/2)-1/2*(A-3*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+
e))^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2972, 2740, 2737, 2667, 31}

$$-\frac{c^2(A-3B)\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{c(A-3B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2af\sqrt{a\sin(e+fx)+a}}-\frac{(A-B)\cos(e+fx)(c}{2f(a\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^
(3/2),x]
```

```
[Out] -(((A - 3*B)*c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e
+ f*x]]*Sqrt[c - c*Sin[e + f*x]])) - ((A - 3*B)*c*Cos[e + f*x]*Sqrt[c - c*S
in[e + f*x]])/(2*a*f*Sqrt[a + a*Sin[e + f*x]]) - ((A - B)*Cos[e + f*x]*(c -
 c*Sin[e + f*x])^(3/2))/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{(A-3B)\int \frac{(c-c\sin}{\sqrt{a+a}}}{2a}$$

$$= -\frac{(A-3B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+}{2f(a+a}$$

$$= -\frac{(A-3B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+}{2f(a+a}$$

$$= -\frac{(A-3B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+}{2f(a+a}$$

$$= -\frac{(A-3B)c^2\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{(A-3B)c\cos}{2af}$$

Mathematica [A] time = 0.86, size = 190, normalized size = 1.19

$$-\frac{c\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2\sin(e+fx)\left(2(A-3B)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\right.\right.\right.}{2f(a(\sin(e+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
f*x])^(3/2),x]
```

```
[Out] -1/2*(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(4*A
 - 3*B - B*Cos[2*(e + f*x)] + 4*A*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]
- 12*B*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 2*(B + 2*(A - 3*B)*Log[Co
s[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(Bc\cos\left(fx+e\right)^2-(A-B)c\sin\left(fx+e\right)+(A-B)c\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin
(f*x + e) - 2*a^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.64, size = 759, normalized size = 4.77

$$-\frac{\left(4A\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-12B\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-2A\sin\left(fx+e\right)+2A\sin\left(fx+e\right)\cos\left(fx+e\right)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/f*(4*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-12*B*ln((1-cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))-2*A*sin(f*x+e)+2*A*sin(f*x+e)*cos(f*x+e)-3*B*cos(f*x+
e)^2*ln(2/(cos(f*x+e)+1))-B*cos(f*x+e)^2*sin(f*x+e)-2*A+4*B+A*cos(f*x+e)^2*
ln(2/(cos(f*x+e)+1))-2*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+A*cos(f*x+e)*ln(2/
(cos(f*x+e)+1))-3*B*sin(f*x+e)*cos(f*x+e)+6*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1
))-3*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-2*A*cos(f*x+e)^2*ln((1-cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))+6*B*cos(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))+A*cos(f*x+e)*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-3*B*ln(2/(cos(f*x+e)+1))*
sin(f*x+e)*cos(f*x+e)-2*A*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))+6*B*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+4*A*sin(f*x+e)*l
n((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-12*B*sin(f*x+e)*ln((1-cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))-4*B*cos(f*x+e)^2+B*cos(f*x+e)^3+4*B*sin(f*x+e)-2*A*si
n(f*x+e)*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+6*B*sin(f*x+e)
*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*cos(f*x+e)+2*A*cos(f
*x+e)^2-2*A*ln(2/(cos(f*x+e)+1))+6*B*ln(2/(cos(f*x+e)+1)))*(-c*(sin(f*x+e)-
1))^(3/2)/(cos(f*x+e)^2-sin(f*x+e)*cos(f*x+e)+cos(f*x+e)+2*sin(f*x+e)-2)/(a
*(1+sin(f*x+e)))^(3/2)
```

maxima [B] time = 0.55, size = 367, normalized size = 2.31

$$-\frac{B\left(\frac{6\,c^{\frac{3}{2}}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{a^{\frac{3}{2}}}-\frac{3\,c^{\frac{3}{2}}\log\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)}{a^{\frac{3}{2}}}-\frac{2\left(\frac{3\,c^{\frac{3}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{2\,c^{\frac{3}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{3\,c^{\frac{3}{2}}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}\right)}{a^{\frac{3}{2}}+\frac{2\,a^{\frac{3}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{2\,a^{\frac{3}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{2\,a^{\frac{3}{2}}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{a^{\frac{3}{2}}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}}\right)-A\left(\frac{2\,c^{\frac{3}{2}}\,\mathrm{lo}\cdots}{\cdots}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] -(B*(6*c^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(3/2) - 3*c^(3/2)
*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(3/2) - 2*(3*c^(3/2)*sin(f*
x + e)/(cos(f*x + e) + 1) + 2*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 3*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^(3/2) + 2*a^(3/2)*sin(f*
x + e)/(cos(f*x + e) + 1) + 2*a^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 2*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a^(3/2)*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4)) - A*(2*c^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)
/a^(3/2) - c^(3/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(3/2) - 4
*sqrt(a)*c^(3/2)*sin(f*x + e)/((a^2 + 2*a^2*sin(f*x + e)/(cos(f*x + e) + 1)
 + a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^
(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(- c \left(\sin{\left(e + f x \right)} - 1\right)\right)^{\frac{3}{2}} \left(A + B \sin{\left(e + f x \right)}\right)}{\left(a \left(\sin{\left(e + f x \right)} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2),
x)
```

```
[Out] Integral((-c*(sin(e + f*x) - 1))**(3/2)*(A + B*sin(e + f*x))/(a*(sin(e + f*
x) + 1))**(3/2), x)
```

3.183 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{Bc\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{c(A-B)\cos(e+fx)}{f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -(A-B)*c*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)+B*c*cos
(f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2971, 2737, 2667, 31, 2738}

$$\frac{Bc\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{c(A-B)\cos(e+fx)}{f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^(3
/2),x]
```

```
[Out] -(((A - B)*c*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e +
 f*x]])) + (B*c*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e +
 f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{B\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a} - (-A+B)\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx$$

$$= -\frac{(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{(Bc\cos(e+fx))}{\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{(Bc\cos(e+fx))\,\mathrm{Su}}{af\sqrt{a+a\sin(}}$$

$$= -\frac{(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{Bc\cos(e+fx)\,\mathrm{lo}}{af\sqrt{a+a\sin(e+f}}$$

Mathematica [C] time = 1.18, size = 143, normalized size = 1.43

$$\frac{\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-A+2B\log\left(e^{i(e+fx)}+i\right)+B\left(2\log\left(e^{i(e+fx)}+i\right)-ifx\right)\right.}{f(a(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*
x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(-A + B - I
*B*f*x + 2*B*Log[I + E^(I*(e + f*x))] + B*((-I)*f*x + 2*Log[I + E^(I*(e + f
*x))])*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[
e + f*x]))^(3/2))
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.65, size = 408, normalized size = 4.08

$$-\frac{\left(2B\left(\cos^2\left(fx+e\right)\right)\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-B\left(\cos^2\left(fx+e\right)\right)\ln\left(\frac{2}{\cos\left(fx+e\right)+1}\right)+2B\sin\left(fx+e\right)\cos\left(fx+e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/f*(2*B*cos(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*cos(f*x+e
)^2*ln(2/(cos(f*x+e)+1))+2*B*sin(f*x+e)*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))-B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)+A*cos(f*x+e)^
```

```
2+A*sin(f*x+e)*cos(f*x+e)-B*cos(f*x+e)^2-B*sin(f*x+e)*cos(f*x+e)+2*B*cos(f*
x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*cos(f*x+e)*ln(2/(cos(f*x+e)
+1))-4*B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*B*sin(f*x+e)
*ln(2/(cos(f*x+e)+1))-A*sin(f*x+e)+B*sin(f*x+e)-4*B*ln((1-cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))+2*B*ln(2/(cos(f*x+e)+1))-A+B)*(-c*(sin(f*x+e)-1))^(1/2)/(
-1+cos(f*x+e)+sin(f*x+e))/(a*(1+sin(f*x+e)))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{-c\sin\left(fx+e\right)+c}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e) +
a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^
(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\,\left(A+B\sin\left(e+fx\right)\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(3/2),
x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(e + f*x))/(a*(sin(e + f*x)
+ 1))**(3/2), x)
```

3.184 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=103

$$\frac{(A+B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2af\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)+1/2*(
A+B)*arctanh(sin(f*x+e))*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x
+e))^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2972, 2741, 3770}

$$\frac{(A+B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2af\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x
]]),x]
```

```
[Out] -((A - B)*Cos[e + f*x])/(2*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e +
f*x]]) + ((A + B)*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a*f*Sqrt[a + a*Sin
[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\int \frac{}{\sqrt{a+a\,\mathrm{s}}}}{}$$
$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{((A+B)\cos(e}{2a\sqrt{a+a\sin(e}}$$
$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\tanh^{-1}(}{2af\sqrt{a+a\sin(e}}$$

Mathematica [A] time = 0.56, size = 186, normalized size = 1.81

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-\left((A+B)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e\right.\right.\right.\right.}{2f(a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e
 + f*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-A + B - (A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^2 + (A + B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2))/(2*f*(a*(1 + Sin[e + f*x]))^(3/
2)*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.52, size = 329, normalized size = 3.19

$$\left[\frac{\left((A+B)\cos\left(fx+e\right)\sin\left(fx+e\right)+(A+B)\cos\left(fx+e\right)\right)\sqrt{ac}\,\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\,\sqrt{a\sin\left(fx+e\right)+a}}{\cos\left(fx+e\right)^{3}}\right.}{4\left(a^{2}cf\cos\left(fx+e\right)\sin\left(fx+e\right)+a^{2}cf\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] [1/4*(((A + B)*cos(f*x + e)*sin(f*x + e) + (A + B)*cos(f*x + e))*sqrt(a*c)*
log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x
+ e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) - 2*sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A - B))/(a^2*c*f*cos(f*x + e
)*sin(f*x + e) + a^2*c*f*cos(f*x + e)), -1/2*(((A + B)*cos(f*x + e)*sin(f*x
 + e) + (A + B)*cos(f*x + e))*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x +
 e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) + sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A - B))/(a^2*c*f*cos(f*x + e
)*sin(f*x + e) + a^2*c*f*cos(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin\left(fx+e\right) + A}{\left(a \sin\left(fx+e\right) + a\right)^{\frac{3}{2}} \sqrt{-c \sin\left(fx+e\right) + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x
+ e) + c)), x)
```

maple [B] time = 0.60, size = 303, normalized size = 2.94

$$\frac{\left(A \sin\left(fx+e\right) \ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right) - A \sin\left(fx+e\right) \ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right) + B \sin\left(fx+e\right) \ln\left(\frac{1-\cos(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*(A*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*sin(f*x+e)*l
n(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*sin(f*x+e)*ln((1-cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))-B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)+A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*ln(-(-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))+A*sin(f*x+e)+B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*ln
(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*sin(f*x+e))*cos(f*x+e)/(a*(1+sin
(f*x+e)))^(3/2)/(-c*(sin(f*x+e)-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x
+ e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(
1/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(
1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\sin\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))/((a*(sin(e + f*x) + 1))**(3/2)*sqrt(-c*(sin(e
 + f*x) - 1))), x)
```

3.185 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=150

$$-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}+\frac{A\cos(e+fx)}{2af\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{A\cos(e+fx)}{2acf\sqrt{a\sin(e+f}}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)+1/2*A
*cos(f*x+e)/a/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/2*A*arctanh
(sin(f*x+e))*cos(f*x+e)/a/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2972, 2743, 2741, 3770}

$$-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}+\frac{A\cos(e+fx)}{2af\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{A\cos(e+fx)}{2acf\sqrt{a\sin(e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(
3/2)),x]
```

```
[Out] -((A - B)*Cos[e + f*x])/(2*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x]
)^(3/2)) + (A*Cos[e + f*x])/(2*a*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e +
f*x])^(3/2)) + (A*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a*c*f*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{A\int \frac{}{\sqrt{a+a\sin(e}}}{}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{}{2af\sqrt{a+a\sin}}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{}{2af\sqrt{a+a\sin}}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{}{2af\sqrt{a+a\sin}}$$

Mathematica [A] time = 0.67, size = 178, normalized size = 1.19

$$-\frac{\cos(e+fx)\left(2A\sin(e+fx)-A\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+A\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+\right.\right.\right.}{4cf(\sin(e+fx)-1)(a(\sin(e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f
*x])^(3/2)),x]
```

```
[Out] -1/4*(Cos[e + f*x]*(2*B - A*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + A*Lo
g[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + A*Cos[2*(e + f*x)]*(-Log[Cos[(e +
```

```
f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 2
*A*Sin[e + f*x]))/(c*f*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(3/2)*Sqr
t[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.51, size = 272, normalized size = 1.81

$$\left[\frac{\sqrt{ac}\,A\cos\left(fx+e\right)^{3}\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)^{3}}\right)+2\left(A\sin\left(fx+\right.\right.}{4\,a^{2}c^{2}f\cos\left(fx+e\right)^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a*c)*A*cos(f*x + e)^3*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x +
 e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*
x + e))/cos(f*x + e)^3) + 2*(A*sin(f*x + e) + B)*sqrt(a*sin(f*x + e) + a)*s
qrt(-c*sin(f*x + e) + c))/(a^2*c^2*f*cos(f*x + e)^3), -1/2*(sqrt(-a*c)*A*ar
ctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos
(f*x + e)*sin(f*x + e)))*cos(f*x + e)^3 - (A*sin(f*x + e) + B)*sqrt(a*sin(f
*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^2*f*cos(f*x + e)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
 + c)^(3/2)), x)
```

maple [A] time = 0.56, size = 130, normalized size = 0.87

$$\frac{\left(A\left(\cos^{2}\left(fx+e\right)\right)\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-A\left(\cos^{2}\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-B\left(\cos^{2}\left(fx+e\right)\right)\right.}{2f\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\left(-c\left(\sin\left(fx+e\right)-1\right)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/2/f*(A*cos(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*cos(f*x+e)
^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*cos(f*x+e)^2+A*sin(f*x+e)+B
)*cos(f*x+e)/(a*(1+sin(f*x+e)))^(3/2)/(-c*(sin(f*x+e)-1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
 + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(
3/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(
3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\sin\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))/((a*(sin(e + f*x) + 1))**(3/2)*(-c*(sin(e + f
*x) - 1))**(3/2)), x)
```

3.186 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=217

$$\frac{(3A-B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8ac^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{(3A-B)\cos(e+fx)}{8acf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{(3A-B}{8af\sqrt{a\sin(e+fx)}}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)+1/8*(
3*A-B)*cos(f*x+e)/a/f/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)+1/8*(3*
A-B)*cos(f*x+e)/a/c/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/8*(3*
A-B)*arctanh(sin(f*x+e))*cos(f*x+e)/a/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin
(f*x+e))^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2972, 2743, 2741, 3770}

$$\frac{(3A-B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8ac^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{(3A-B)\cos(e+fx)}{8acf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{(3A-B}{8af\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(
5/2)),x]
```

```
[Out] -((A - B)*Cos[e + f*x])/(2*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x]
)^(5/2)) + ((3*A - B)*Cos[e + f*x])/(8*a*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*
Sin[e + f*x])^(5/2)) + ((3*A - B)*Cos[e + f*x])/(8*a*c*f*Sqrt[a + a*Sin[e +
 f*x]]*(c - c*Sin[e + f*x])^(3/2)) + ((3*A - B)*ArcTanh[Sin[e + f*x]]*Cos[e
 + f*x])/(8*a*c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{(3A-B)\int -\ldots}{\sqrt{\ldots}}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{(3\ldots}{8af\sqrt{a+a\,\mathrm{si}\ldots}}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{(3\ldots}{8af\sqrt{a+a\,\mathrm{si}\ldots}}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{(3\ldots}{8af\sqrt{a+a\,\mathrm{si}\ldots}}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{(3\ldots}{8af\sqrt{a+a\,\mathrm{si}\ldots}}$$

Mathematica [A] time = 0.95, size = 306, normalized size = 1.41

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left((B-A)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+f\ldots\right.\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f
*x])^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(2*A*Cos[e + f*x]^2 + (-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 +
(A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (-3*A + B)*Log[Cos[(e + f
*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^2 + (3*A - B)*Log[Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^2))/(8*f*(a*(1 + Sin[e + f*x]))^(3/2)*(c - c*Sin[e + f*x])^(5/
2))
```

fricas [A] time = 0.53, size = 437, normalized size = 2.01

$$\left[-\frac{\left((3A-B)\cos\left(fx+e\right)^3\sin\left(fx+e\right)-(3A-B)\cos\left(fx+e\right)^3\right)\sqrt{ac}\,\log\left(-\frac{ac\cos\left(fx+e\right)^3-2\,ac\cos\left(fx+e\right)+2\,\sqrt{ac}\,\sqrt{a\sin\cdots}}{\cos\left(fx+\cdots\right.}\right.}{16\left(a^2c^3f\cos\left(\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] [-1/16*(((3*A - B)*cos(f*x + e)^3*sin(f*x + e) - (3*A - B)*cos(f*x + e)^3)*
sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) + 2*sqrt(a*c)*sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3)
 + 2*((3*A - B)*cos(f*x + e)^2 + (3*A - B)*sin(f*x + e) - A + 3*B)*sqrt(a*s
in(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^3*f*cos(f*x + e)^3*sin(f
*x + e) - a^2*c^3*f*cos(f*x + e)^3), -1/8*(((3*A - B)*cos(f*x + e)^3*sin(f*
x + e) - (3*A - B)*cos(f*x + e)^3)*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(
f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) +
((3*A - B)*cos(f*x + e)^2 + (3*A - B)*sin(f*x + e) - A + 3*B)*sqrt(a*sin(f*
x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^3*f*cos(f*x + e)^3*sin(f*x +
e) - a^2*c^3*f*cos(f*x + e)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
 + c)^(5/2)), x)
```

maple [B] time = 0.55, size = 429, normalized size = 1.98

$$-\frac{\left(3A\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)-3A\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin}{\sin\left(fx+e\right)}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/8/f*(3*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)^
2-3*A*cos(f*x+e)^2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*
ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)^2+B*ln(-(-1+
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-3*A*cos(f*x+e)^2
*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*A*cos(f*x+e)^2*sin(f*x+e)+3*A*c
os(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*cos(f*x+e)^2*ln((1
-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*B*cos(f*x+e)^2*sin(f*x+e)-B*cos(f*x+e
)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*cos(f*x+e)^2+3*B*cos(f*x+e
)^2-3*A*sin(f*x+e)+B*sin(f*x+e)+A-3*B)*cos(f*x+e)/(a*(1+sin(f*x+e)))^(3/2)/
(-c*(sin(f*x+e)-1))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
 + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(
5/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(
5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

3.187 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=323

$$\frac{8c^5(3A-7B)\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{4c^4(3A-7B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{c^3(3A-7B)\cos(e}{a^2f\sqrt{a}}$$

```
[Out] 1/4*(3*A-7*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/a/f/(a+a*sin(f*x+e))^(3/2
)-1/4*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(9/2)/f/(a+a*sin(f*x+e))^(5/2)+(3*A
-7*B)*c^3*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)+1/
3*(3*A-7*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a^2/f/(a+a*sin(f*x+e))^(1
/2)+8*(3*A-7*B)*c^5*cos(f*x+e)*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2
)/(c-c*sin(f*x+e))^(1/2)+4*(3*A-7*B)*c^4*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/
a^2/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.71, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2972, 2739, 2740, 2737, 2667, 31}

$$\frac{4c^4(3A-7B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{c^3(3A-7B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{c^2(3A-7B)\cos(e+}{3a^2f\sqrt{a\,s}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (8*(3*A - 7*B)*c^5*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Si
n[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (4*(3*A - 7*B)*c^4*Cos[e + f*x]*Sqr
t[c - c*Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) + ((3*A - 7*B)*c^3*
Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) +
 ((3*A - 7*B)*c^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*a^2*f*Sqrt[a
+ a*Sin[e + f*x]]) + ((3*A - 7*B)*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2)
)/(4*a*f*(a + a*Sin[e + f*x])^(3/2)) - ((A - B)*Cos[e + f*x]*(c - c*Sin[e +
 f*x])^(9/2))/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3A-7B)\int \frac{(c-c\sin}{(a+a\sin}}{4a}$$

$$= \frac{(3A-7B)c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+}{4f(a+a}$$

$$= \frac{(3A-7B)c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)c\cos}{4af(a}$$

$$= \frac{(3A-7B)c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)c^2\cos}{3a^2f}$$

$$= \frac{4(3A-7B)c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)c^3\cos}{a^2f\sqrt{}}$$

$$= \frac{4(3A-7B)c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)c^3\cos}{a^2f\sqrt{}}$$

$$= \frac{4(3A-7B)c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)c^3\cos}{a^2f\sqrt{}}$$

$$= \frac{8(3A-7B)c^5\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{4(3A-7B)c^4}{a^2f}$$

Mathematica [A] time = 7.01, size = 573, normalized size = 1.77

$$-\frac{(28A-97B)\sin(e+fx)(c-c\sin(e+fx))^{9/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}{4f(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9} - \frac{(A-7B)\cos(2(e+fx))(c-}{4f(a(\sin(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e +
f*x])^(5/2),x]
```

```
[Out] (-8*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(9/2
))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2))
 + (16*(2*A - 3*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e + f
```

```
*x])^(9/2))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]
))^(5/2)) - ((A - 7*B)*Cos[2*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^5*(c - c*Sin[e + f*x])^(9/2))/(4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^9*(a*(1 + Sin[e + f*x]))^(5/2)) + (16*(3*A - 7*B)*Log[Cos[(e + f*x)/2] + S
in[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x
])^(9/2))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))
^(5/2)) - ((28*A - 97*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sin[e + f*
x]*(c - c*Sin[e + f*x])^(9/2))/(4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9
*(a*(1 + Sin[e + f*x]))^(5/2)) - (B*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5
*(c - c*Sin[e + f*x])^(9/2)*Sin[3*(e + f*x)])/(12*f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [F] time = 45.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left((A-4\,B)c^4\cos\left(fx+e\right)^4-4\,(2\,A-3\,B)c^4\cos\left(fx+e\right)^2+8\,(A-B)c^4+\left(Bc^4\cos\left(fx+e\right)^4+4\,(A\right.\right.}{3\,a^3\cos\left(fx+e\right)^2-4\,a^3+\left(a^3\cos\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral(-((A - 4*B)*c^4*cos(f*x + e)^4 - 4*(2*A - 3*B)*c^4*cos(f*x + e)^2
+ 8*(A - B)*c^4 + (B*c^4*cos(f*x + e)^4 + 4*(A - 2*B)*c^4*cos(f*x + e)^2 -
8*(A - B)*c^4)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
+ c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x +
 e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.68, size = 1287, normalized size = 3.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/6/f*(-108*B*cos(f*x+e)^3*sin(f*x+e)-1152*A*ln((1-cos(f*x+e)+sin(f*x+e))/
sin(f*x+e))+2688*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+396*A*sin(f*x+e
)-174*A*sin(f*x+e)*cos(f*x+e)+36*A*cos(f*x+e)^3*sin(f*x+e)-3*A*cos(f*x+e)^4
*sin(f*x+e)+15*B*sin(f*x+e)*cos(f*x+e)^4+1008*B*cos(f*x+e)^2*ln(2/(cos(f*x+
e)+1))-255*A*cos(f*x+e)^2*sin(f*x+e)+581*B*cos(f*x+e)^2*sin(f*x+e)-93*B*cos
(f*x+e)^4+396*A-932*B+219*A*cos(f*x+e)^3-336*B*ln(2/(cos(f*x+e)+1))*cos(f*x
+e)^3-432*A*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+576*A*sin(f*x+e)*ln(2/(cos(f*
x+e)+1))-288*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+442*B*sin(f*x+e)*cos(f*x+e)-
1344*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+672*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1)
)+864*A*cos(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2016*B*cos(f*
x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-288*A*cos(f*x+e)*sin(f*x+e)
*ln(2/(cos(f*x+e)+1))+3*A*cos(f*x+e)^5-17*B*cos(f*x+e)^5+2*B*cos(f*x+e)^6+3
36*B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^2*sin(f*x+e)+672*B*ln(2/(cos(f*x+e)+1)
)*sin(f*x+e)*cos(f*x+e)+2*B*cos(f*x+e)^5*sin(f*x+e)+576*A*cos(f*x+e)*ln((1-
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-1344*B*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))-1152*A*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))+2688*B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+1023*B*cos(f*
x+e)^2+144*A*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))+33*A*cos(f*x+e)^4-144*A*cos(
f*x+e)^2*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-288*A*cos(f*x+e)^3*ln((1-cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))+672*B*cos(f*x+e)^3*ln((1-cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))-473*B*cos(f*x+e)^3-932*B*sin(f*x+e)+576*A*sin(f*x+e)*cos(f*x+e)*
ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-1344*B*sin(f*x+e)*cos(f*x+e)*ln((1
-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+288*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))*sin(f*x+e)*cos(f*x+e)^2-672*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))*sin(f*x+e)*cos(f*x+e)^2-222*A*cos(f*x+e)+490*B*cos(f*x+e)-429*A*cos(f*x
+e)^2+576*A*ln(2/(cos(f*x+e)+1))-1344*B*ln(2/(cos(f*x+e)+1)))*(-c*(sin(f*x+
e)-1))^(9/2)/(sin(f*x+e)*cos(f*x+e)^4+cos(f*x+e)^5+4*sin(f*x+e)*cos(f*x+e)^
3-5*cos(f*x+e)^4-12*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^3-8*sin(f*x+e)*cos
(f*x+e)+20*cos(f*x+e)^2+16*sin(f*x+e)+8*cos(f*x+e)-16)/(a*(1+sin(f*x+e)))^(
5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(9/2)/(a*sin(f*x + e)
+ a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{9/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^
(5/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^
(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

3.188 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=263

$$\frac{6c^4(A-3B)\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{3c^3(A-3B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{3c^2(A-3B)\cos(e+}{4a^2f\sqrt{a\sin}}$$

```
[Out] 1/2*(A-3*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a/f/(a+a*sin(f*x+e))^(3/2)-
1/4*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(5/2)+3/4*(A
-3*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)+6*
(A-3*B)*c^4*cos(f*x+e)*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*s
in(f*x+e))^(1/2)+3*(A-3*B)*c^3*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f/(a+a
*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.61, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2972, 2739, 2740, 2737, 2667, 31}

$$\frac{3c^3(A-3B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{3c^2(A-3B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4a^2f\sqrt{a\sin(e+fx)+a}}+\frac{6c^4(A-3B)\cos(e+}{a^2f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (6*(A - 3*B)*c^4*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[
e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (3*(A - 3*B)*c^3*Cos[e + f*x]*Sqrt[c
- c*Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (3*(A - 3*B)*c^2*Cos[
e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(4*a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (
(A - 3*B)*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(2*a*f*(a + a*Sin[e +
f*x])^(3/2)) - ((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(4*f*(a +
a*Sin[e + f*x])^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
```

```
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(A-3B)\int \frac{(c-c\sin(}{(a+a\sin(}}{2a}$$

$$= \frac{(A-3B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+f}{4f(a+a}$$

$$= \frac{3(A-3B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(A-3B)c\cos(}{2af(a}$$

$$= \frac{3(A-3B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3(A-3B)c^2\cos(}{4a^2f\sqrt{}}$$

$$= \frac{3(A-3B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3(A-3B)c^2\cos(}{4a^2f\sqrt{}}$$

$$= \frac{3(A-3B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3(A-3B)c^2\cos(}{4a^2f\sqrt{}}$$

$$= \frac{6(A-3B)c^4\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{3(A-3B)c^3\cos}{a^2f\sqrt{}}$$

Mathematica [A] time = 2.54, size = 243, normalized size = 0.92

$$\frac{(c-c\sin(e+fx))^{7/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-4(A-6B)\sin(e+fx)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(7/2)*(-16*(A -
 B) + 16*(3*A - 5*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + B*Cos[2*(e +
 f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + 48*(A - 3*B)*Log[Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 4*(A
- 6*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(4*f*(Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [F] time = 1.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bc^3\cos\left(fx+e\right)^4+(3A-5B)c^3\cos\left(fx+e\right)^2-4(A-B)c^3-\left((A-3B)c^3\cos\left(fx+e\right)^2-4(A-\right.\right.}{3a^3\cos\left(fx+e\right)^2-4a^3+\left(a^3\cos\left(fx+e\right)^2-4a^3\right)s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*c^3*cos(f*x + e)^4 + (3*A - 5*B)*c^3*cos(f*x + e)^2 - 4*(A - B)
*c^3 - ((A - 3*B)*c^3*cos(f*x + e)^2 - 4*(A - B)*c^3)*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 +
 (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.64, size = 1206, normalized size = 4.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/2/f*(-10*B*cos(f*x+e)^3*sin(f*x+e)-96*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))+288*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+32*A*sin(f*x+e)-12*
A*sin(f*x+e)*cos(f*x+e)+2*A*cos(f*x+e)^3*sin(f*x+e)+B*sin(f*x+e)*cos(f*x+e)
^4+108*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-22*A*cos(f*x+e)^2*sin(f*x+e)+63*
B*cos(f*x+e)^2*sin(f*x+e)-9*B*cos(f*x+e)^4+32*A-100*B+20*A*cos(f*x+e)^3-36*
B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^3-36*A*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+
48*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-24*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+4
6*B*sin(f*x+e)*cos(f*x+e)-144*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+72*B*cos(f*
x+e)*ln(2/(cos(f*x+e)+1))+72*A*cos(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))-216*B*cos(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-24*A*
cos(f*x+e)*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-B*cos(f*x+e)^5+36*B*ln(2/(cos(f*
```

```
x+e)+1))*cos(f*x+e)^2*sin(f*x+e)+72*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f
*x+e)+48*A*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-144*B*cos(f*
x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-96*A*sin(f*x+e)*ln((1-cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))+288*B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))+109*B*cos(f*x+e)^2+12*A*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))+2*A*co
s(f*x+e)^4-12*A*cos(f*x+e)^2*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-24*A*cos(f*x+e
)^3*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+72*B*cos(f*x+e)^3*ln((1-cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))-53*B*cos(f*x+e)^3-100*B*sin(f*x+e)+48*A*sin(f*
x+e)*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-144*B*sin(f*x+e)*c
os(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+24*A*ln((1-cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)^2-72*B*ln((1-cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)^2-20*A*cos(f*x+e)+54*B*cos(f*x+e)-34
*A*cos(f*x+e)^2+48*A*ln(2/(cos(f*x+e)+1))-144*B*ln(2/(cos(f*x+e)+1)))*(-c*(
sin(f*x+e)-1))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3-cos(f*x+e)^4-4*cos(f*x+e)^2*s
in(f*x+e)-3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)+8*cos(f*x+e)^2+8*sin(f*x+e
)+4*cos(f*x+e)-8)/(a*(1+sin(f*x+e)))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e)
+ a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^
(5/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^
(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

3.189 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=211

$$\frac{c^3(A-5B)\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c^2(A-5B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2f\sqrt{a\sin(e+fx)+a}}+\frac{c(A-5B)\cos(e+fx)(}{4af(a\sin(e+}$$

```
[Out] 1/4*(A-5*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f/(a+a*sin(f*x+e))^(3/2)-
1/4*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(5/2)+(A-5*B
)*c^3*cos(f*x+e)*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x
+e))^(1/2)+1/2*(A-5*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f/(a+a*sin
(f*x+e))^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2972, 2739, 2740, 2737, 2667, 31}

$$\frac{c^2(A-5B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2f\sqrt{a\sin(e+fx)+a}}+\frac{c^3(A-5B)\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c(A-5B)\cos(e+fx)(}{4af(a\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] ((A - 5*B)*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e
+ f*x]]*Sqrt[c - c*Sin[e + f*x]]) + ((A - 5*B)*c^2*Cos[e + f*x]*Sqrt[c - c*
Sin[e + f*x]])/(2*a^2*f*Sqrt[a + a*Sin[e + f*x]]) + ((A - 5*B)*c*Cos[e + f*
x]*(c - c*Sin[e + f*x])^(3/2))/(4*a*f*(a + a*Sin[e + f*x])^(3/2)) - ((A - B
)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}dx = -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(A-5B)\int \frac{(c-c\sin(}{(a+a\sin(}}{4a}$$

$$= \frac{(A-5B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+f}{4f(a+a}$$

$$= \frac{(A-5B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(A-5B)c\cos(e+}{4af(a+a}$$

$$= \frac{(A-5B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(A-5B)c\cos(e+}{4af(a+a}$$

$$= \frac{(A-5B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(A-5B)c\cos(e+}{4af(a+a}$$

$$= \frac{(A-5B)c^3\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{(A-5B)c^2\cos(}{2a^2f\sqrt{a}}$$

Mathematica [A] time = 1.15, size = 199, normalized size = 0.94

$$\frac{(c-c\sin(e+fx))^{5/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(4(A-2B)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2+2(A}{f(a(\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(5/2)*(-2*A + 2
*B + 4*(A - 2*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 2*(A - 5*B)*Log[
Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
4 + B*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(f*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [F] time = 1.13, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left((A-2B)c^2\cos\left(fx+e\right)^2-2(A-B)c^2+\left(Bc^2\cos\left(fx+e\right)^2+2(A-B)c^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx-\right.}}{3a^3\cos\left(fx+e\right)^2-4a^3+\left(a^3\cos\left(fx+e\right)^2-4a^3\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral(((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x + e
)^2 + 2*(A - B)*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin
(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.63, size = 1106, normalized size = 5.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/f*(-B*cos(f*x+e)^3*sin(f*x+e)-8*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)+40*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*A*sin(f*x+e)+15*B*cos(f*x
+e)^2*ln(2/(cos(f*x+e)+1))-2*A*cos(f*x+e)^2*sin(f*x+e)+9*B*cos(f*x+e)^2*sin
(f*x+e)-B*cos(f*x+e)^4+2*A-14*B+2*A*cos(f*x+e)^3-5*B*ln(2/(cos(f*x+e)+1))*c
os(f*x+e)^3-3*A*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+4*A*sin(f*x+e)*ln(2/(cos(
f*x+e)+1))-2*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+6*B*sin(f*x+e)*cos(f*x+e)-20
*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+10*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+6*A
*cos(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-30*B*cos(f*x+e)^2*ln
((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*A*cos(f*x+e)*sin(f*x+e)*ln(2/(cos(
f*x+e)+1))+5*B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^2*sin(f*x+e)+10*B*ln(2/(cos(
f*x+e)+1))*sin(f*x+e)*cos(f*x+e)+4*A*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))-20*B*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-8*A*
sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+40*B*sin(f*x+e)*ln((1-c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))+15*B*cos(f*x+e)^2+A*cos(f*x+e)^3*ln(2/(co
s(f*x+e)+1))-A*cos(f*x+e)^2*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-2*A*cos(f*x+e)^
3*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+10*B*cos(f*x+e)^3*ln((1-cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))-8*B*cos(f*x+e)^3-14*B*sin(f*x+e)+4*A*sin(f*x+e)*
cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-20*B*sin(f*x+e)*cos(f*x
```

```
+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*A*ln((1-cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)^2-10*B*ln((1-cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))*sin(f*x+e)*cos(f*x+e)^2-2*A*cos(f*x+e)+8*B*cos(f*x+e)-2*A*cos(f*x
+e)^2+4*A*ln(2/(cos(f*x+e)+1))-20*B*ln(2/(cos(f*x+e)+1)))*(-c*(sin(f*x+e)-1
))^(5/2)/(cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^3+2*sin(f*x+e)*cos(f*x+e)-3*co
s(f*x+e)^2-4*sin(f*x+e)-2*cos(f*x+e)+4)/(a*(1+sin(f*x+e)))^(5/2)
```

maxima [B] time = 0.49, size = 504, normalized size = 2.39

$$\left(\left(\frac{8\,\sqrt{a}\,c^{\frac{5}{2}}\sin\left(fx+e\right)^2}{\left(a^3+\frac{4\,a^3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{6\,a^3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{4\,a^3\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{a^3\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}\right)\left(\cos\left(fx+e\right)+1\right)^2}-\frac{2\,c^{\frac{5}{2}}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{a^{\frac{5}{2}}}+\frac{c^{\frac{5}{2}}\log\left(\frac{\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+1\right)}{a^{\frac{5}{2}}}\right)A+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] ((8*sqrt(a)*c^(5/2)*sin(f*x + e)^2/((a^3 + 4*a^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 6*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^3*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*(cos(f*x + e)
 + 1)^2) - 2*c^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(5/2) + c^(
5/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(5/2))*A + B*(10*c^(5/2
)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(5/2) - 5*c^(5/2)*log(sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(5/2) - 2*(5*c^(5/2)*sin(f*x + e)/(cos(f
*x + e) + 1) + 16*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 14*c^(5/2)*
sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 16*c^(5/2)*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 + 5*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^(5/2) + 4*a^(
5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 7*a^(5/2)*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 8*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*a^(5/2)*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 4*a^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1
)^5 + a^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^
(5/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^
(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

3.190 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{Bc^2\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}}-\frac{Bc\cos(e+fx)\sqrt{c-c\sin(}{af(a\sin(e+fx)+a}$$

```
[Out] -1/4*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(5/2)-B*c^2
*cos(f*x+e)*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^
(1/2)-B*c*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^(3/2)
```

Rubi [A] time = 0.39, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2972, 2739, 2737, 2667, 31}

$$-\frac{Bc^2\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}}-\frac{Bc\cos(e+fx)\sqrt{c-c\sin(}{af(a\sin(e+fx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] -((B*c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c - c*Sin[e + f*x]])) - (B*c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/
(a*f*(a + a*Sin[e + f*x])^(3/2)) - ((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x
])^(3/2))/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
```

```
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a+a\sin(e+fx))^{5/2}} + \frac{B\int \frac{(c-c\sin(e+fx))^{3}}{(a+a\sin(e+fx))^{3}}}{a}$$

$$= -\frac{Bc\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-}{4f(a+a\sin(e+}$$

$$= -\frac{Bc\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-}{4f(a+a\sin(e+}$$

$$= -\frac{Bc\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-}{4f(a+a\sin(e+}$$

$$= -\frac{Bc^2\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{Bc\cos(e+fx)\sqrt{}}{af(a+a\,\mathrm{si}}$$

Mathematica [A] time = 0.98, size = 179, normalized size = 1.20

$$\frac{c\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin(e+fx)\left(A-4B\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+\right.\right.\right.\right.}{f(a(\sin(e+fx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
f*x])^(5/2),x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(B*Cos[2*
(e + f*x)]*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - B*(2 + 3*Log[Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2]]) + (A - 3*B - 4*B*Log[Cos[(e + f*x)/2] + Sin[(
e + f*x)/2]])*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1
 + Sin[e + f*x]))^(5/2))
```

fricas [F] time = 2.71, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(Bc\cos\left(fx+e\right)^{2}-(A-B)c\sin\left(fx+e\right)+(A-B)c\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{3\,a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 +
 (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.64, size = 606, normalized size = 4.07

$$\frac{\left(8B\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-A\sin\left(fx+e\right)+A\sin\left(fx+e\right)\cos\left(fx+e\right)+2B\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/f*(8*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*sin(f*x+e)+A*sin(f*x+e)
*cos(f*x+e)+3*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+2*B*cos(f*x+e)^2*sin(f*x+
e)-A-3*B-B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^3+B*sin(f*x+e)*cos(f*x+e)-4*B*si
n(f*x+e)*ln(2/(cos(f*x+e)+1))+2*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-6*B*cos(f
*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*ln(2/(cos(f*x+e)+1))*cos
(f*x+e)^2*sin(f*x+e)+2*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-4*B*cos
(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+8*B*sin(f*x+e)*ln((1-cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))+3*B*cos(f*x+e)^2+2*B*cos(f*x+e)^3*ln((1-cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))-2*B*cos(f*x+e)^3-3*B*sin(f*x+e)-4*B*sin(f*x+e
)*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*B*ln((1-cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)^2+2*B*cos(f*x+e)+A*cos(f*x+e)
^2-4*B*ln(2/(cos(f*x+e)+1)))*(-c*(sin(f*x+e)-1))^(3/2)/(sin(f*x+e)*cos(f*x+
e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)/(a*(1+sin(f*x+e)))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e)
+ a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^
(5/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^
(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

3.191 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{c(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}-\frac{Bc\cos(e+fx)}{af(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/2*(A-B)*c*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2)-B*c
*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi **[A]** time = 0.33, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2971, 2738}

$$-\frac{c(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}-\frac{Bc\cos(e+fx)}{af(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^(5
/2),x]
```

```
[Out] -((A - B)*c*Cos[e + f*x])/(2*f*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e
+ f*x]]) - (B*c*Cos[e + f*x])/(a*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Si
n[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{B\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx}{a} - (-A+B)\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx$$
$$= -\frac{(A-B)c\cos(e+fx)}{2f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{Bc\,\mathrm{c}}{af(a+a\sin(e+f}$$

Mathematica [**A**] time = 0.51, size = 99, normalized size = 1.05

$$-\frac{\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}\,(A+2B\sin(e+fx)+B)}{2a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*
x])^(5/2),x]
```

```
[Out] -1/2*(Sqrt[a*(1 + Sin[e + f*x])]*(A + B + 2*B*Sin[e + f*x])*Sqrt[c - c*Sin[
e + f*x]])/(a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^5)
```

fricas [**A**] time = 0.43, size = 85, normalized size = 0.90

$$\frac{\left(2B\sin\left(fx+e\right)+A+B\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{2\left(a^3f\cos\left(fx+e\right)^3-2a^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-2a^3f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] 1/2*(2*B*sin(f*x + e) + A + B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*
cos(f*x + e))
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.66, size = 135, normalized size = 1.44

$$-\frac{\sin\left(fx+e\right)\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\,\left(A\left(\cos^2\left(fx+e\right)\right)+A\sin\left(fx+e\right)\cos\left(fx+e\right)+B\left(\cos^2\left(fx+e\right)\right)+B\right.}{2f\left(a\left(1+\sin\left(fx+e\right)\right)\right)^{\frac{5}{2}}\left(-1+\cos\left(fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/2/f*sin(f*x+e)*(-c*(sin(f*x+e)-1))^(1/2)*(A*cos(f*x+e)^2+A*sin(f*x+e)*co
s(f*x+e)+B*cos(f*x+e)^2+B*sin(f*x+e)*cos(f*x+e)+2*A*cos(f*x+e)-3*A*sin(f*x+
e)-B*sin(f*x+e)-3*A-B)/(a*(1+sin(f*x+e)))^(5/2)/(-1+cos(f*x+e)+sin(f*x+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{-c\sin\left(fx+e\right)+c}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e) +
a)^(5/2), x)
```

mupad [B] time = 14.86, size = 156, normalized size = 1.66

$$-\frac{2\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(A\,\sin\left(2\,e+2\,f\,x\right)+3\,B\,\sin\left(2\,e+2\,f\,x\right)-2\,A\,\left(2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)-3\,B\,\left(2\sin\right.\right.}{a^2\,f\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\left(-8\sin\left(e+f\,x\right)^2+4\,\sin\left(e+f\,x\right)+2\sin\left(2\,e+2\,f\,x\right)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^
(5/2),x)
```

```
[Out] -(2*(-c*(sin(e + f*x) - 1))^(1/2)*(A*sin(2*e + 2*f*x) + 3*B*sin(2*e + 2*f*x
) - 2*A*(2*sin(e/2 + (f*x)/2)^2 - 1) - 3*B*(2*sin(e/2 + (f*x)/2)^2 - 1) + B
```

```
*(2*sin((3*e)/2 + (3*f*x)/2)^2 - 1)))/(a^2*f*(a*(sin(e + f*x) + 1))^(1/2)*(
4*sin(e + f*x) + 4*sin(3*e + 3*f*x) + 2*sin(2*e + 2*f*x)^2 - 8*sin(e + f*x)
^2 + 8))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\left(A+B\sin\left(e+fx\right)\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(5/2),
x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(e + f*x))/(a*(sin(e + f*x)
+ 1))**(5/2), x)
```

3.192 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=151

$$\frac{(A+B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{4a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{(A+B)\cos(e+fx)}{4af(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\text{c}}{4f(a\sin(e+fx)+a)}$$

```
[Out] -1/4*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2)-1/4*(
A+B)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)+1/4*(A+B)
*arctanh(sin(f*x+e))*cos(f*x+e)/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e
))^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2972, 2743, 2741, 3770}

$$\frac{(A+B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{4a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{(A+B)\cos(e+fx)}{4af(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\text{c}}{4f(a\sin(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x
]]),x]
```

```
[Out] -((A - B)*Cos[e + f*x])/(4*f*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e +
f*x]]) - ((A + B)*Cos[e + f*x])/(4*a*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c -
c*Sin[e + f*x]]) + ((A + B)*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(4*a^2*f*Sq
rt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
```

```
SumSimplerQ[n, 1])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\int \frac{}{(a+a\sin}}{}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{(A+}{4af(a+a\sin(e-}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{(A+}{4af(a+a\sin(e-}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{(A+}{4af(a+a\sin(e-}$$

Mathematica [A] time = 0.67, size = 214, normalized size = 1.42

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-(A+B)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e
 + f*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-A + B - (A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (A + B)*Log[C
os[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4
 + (A + B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^4))/(4*f*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e + f*x
]])
```

fricas [A] time = 0.53, size = 408, normalized size = 2.70

$$\left[\frac{\left((A+B)\cos\left(fx+e\right)^3 - 2\,(A+B)\cos\left(fx+e\right)\sin\left(fx+e\right) - 2\,(A+B)\cos\left(fx+e\right)\right)\sqrt{ac}\,\log\left(-\frac{ac\cos\left(fx+e\right)^3-2}{}\right.}{8\left(a^3cf\cos\left(fx+e\right)^3 - 2\,a^3cf\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] [1/8*(((A + B)*cos(f*x + e)^3 - 2*(A + B)*cos(f*x + e)*sin(f*x + e) - 2*(A
+ B)*cos(f*x + e))*sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e)
- 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x +
e))/cos(f*x + e)^3) + 2*((A + B)*sin(f*x + e) + 2*A)*sqrt(a*sin(f*x + e) +
a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f*x +
 e)*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e)), -1/4*(((A + B)*cos(f*x + e)^3 -
 2*(A + B)*cos(f*x + e)*sin(f*x + e) - 2*(A + B)*cos(f*x + e))*sqrt(-a*c)*a
rctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*co
s(f*x + e)*sin(f*x + e))) - ((A + B)*sin(f*x + e) + 2*A)*sqrt(a*sin(f*x + e
) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f
*x + e)*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x
+ e) + c)), x)
```

maple [B] time = 0.63, size = 466, normalized size = 3.09

$$\frac{\left(-A\left(\cos^2\left(fx+e\right)\right)\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+A\left(\cos^2\left(fx+e\right)\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-B\left(\cos^2\left(fx+e\right)\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/4/f*(-A*cos(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+A*cos(f*x+e
)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*cos(f*x+e)^2*ln((1-cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))+B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))+2*A*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*A*sin(
f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*A*cos(f*x+e)^2+2*B*sin(
f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*B*sin(f*x+e)*ln(-(-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))+2*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
+3*A*sin(f*x+e)-2*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*B*ln((1-co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*sin(f*x+e)-2*B*ln(-(-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))+2*A)*cos(f*x+e)/(a*(1+sin(f*x+e)))^(5/2)/(-c*(sin(f*x+e)-
1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x
+ e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(
1/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(
1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sin\left(e + fx\right)}{\left(a\left(\sin\left(e + fx\right) + 1\right)\right)^{\frac{5}{2}}\sqrt{-c\left(\sin\left(e + fx\right) - 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))/((a*(sin(e + f*x) + 1))**(5/2)*sqrt(-c*(sin(e
 + f*x) - 1))), x)
```

3.193 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=208

$$\frac{(3A+B)\cos(e+fx)}{8a^2f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{(3A+B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8a^2cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{(3A+B}{8af(a\sin(e+fx)+}$$

```
[Out] -1/4*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2)-1/8*(
3*A+B)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)+1/8*(3*
A+B)*cos(f*x+e)/a^2/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/8*(3*
A+B)*arctanh(sin(f*x+e))*cos(f*x+e)/a^2/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin
(f*x+e))^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2972, 2743, 2741, 3770}

$$\frac{(3A+B)\cos(e+fx)}{8a^2f\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{(3A+B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8a^2cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{(3A+B}{8af(a\sin(e+fx)+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(
3/2)),x]
```

```
[Out] -((A - B)*Cos[e + f*x])/(4*f*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x]
)^(3/2)) - ((3*A + B)*Cos[e + f*x])/(8*a*f*(a + a*Sin[e + f*x])^(3/2)*(c -
c*Sin[e + f*x])^(3/2)) + ((3*A + B)*Cos[e + f*x])/(8*a^2*f*Sqrt[a + a*Sin[e
 + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + ((3*A + B)*ArcTanh[Sin[e + f*x]]*Cos
[e + f*x])/(8*a^2*c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}+\frac{(3A+B)\int}{}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}-\frac{}{8af(a+a\sin}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}-\frac{}{8af(a+a\sin}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}-\frac{}{8af(a+a\sin}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}-\frac{}{8af(a+a\sin}$$

Mathematica [A] time = 0.96, size = 305, normalized size = 1.47

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left((A+B)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f
*x])^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-2*A*Cos[e + f*x]^2 + (-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 +
 (A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - (3*A + B)*Log[Cos[(e + f
*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^4 + (3*A + B)*Log[Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^4))/(8*f*(a*(1 + Sin[e + f*x]))^(5/2)*(c - c*Sin[e + f*x])^(3/
2))
```

fricas [A] time = 0.55, size = 419, normalized size = 2.01

$$\left[\frac{\left((3\,A+B)\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)+(3\,A+B)\cos\left(fx+e\right)^{3}\right)\sqrt{ac}\,\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\,\sqrt{a\sin\left(fx\right.}}{\cos\left(fx+e\right)^{3}}\right.}{16\left(a^{3}c^{2}f\cos\left(fx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] [1/16*(((3*A + B)*cos(f*x + e)^3*sin(f*x + e) + (3*A + B)*cos(f*x + e)^3)*s
qrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a
*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3)
- 2*((3*A + B)*cos(f*x + e)^2 - (3*A + B)*sin(f*x + e) - A - 3*B)*sqrt(a*si
n(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^2*f*cos(f*x + e)^3*sin(f*
x + e) + a^3*c^2*f*cos(f*x + e)^3), -1/8*(((3*A + B)*cos(f*x + e)^3*sin(f*x
 + e) + (3*A + B)*cos(f*x + e)^3)*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f
*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) + (
(3*A + B)*cos(f*x + e)^2 - (3*A + B)*sin(f*x + e) - A - 3*B)*sqrt(a*sin(f*x
 + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^2*f*cos(f*x + e)^3*sin(f*x + e
) + a^3*c^2*f*cos(f*x + e)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
 + c)^(3/2)), x)
```

maple [B] time = 0.57, size = 431, normalized size = 2.07

$$-\frac{\left(3A\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-3A\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)\left(\cos^2\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/8/f*(3*A*cos(f*x+e)^2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))-3*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)^2+B*
ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-B*ln((1-
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)^2-2*A*cos(f*x+e)^2
*sin(f*x+e)+3*A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-3*A
*cos(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*B*cos(f*x+e)^2*sin
(f*x+e)+B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*cos(f*x
+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+A*cos(f*x+e)^2+3*B*cos(f*x+e
)^2-3*A*sin(f*x+e)-B*sin(f*x+e)-A-3*B)*cos(f*x+e)/(a*(1+sin(f*x+e)))^(5/2)/
(-c*(sin(f*x+e)-1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
 + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(
3/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(
3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

3.194 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=245

$$\frac{3A\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8a^2c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{3A\cos(e+fx)}{8a^2cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{3A}{8a^2f\sqrt{a\sin(e+}}$$

```
[Out] -1/4*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2)-1/2*A
*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)+3/8*A*cos(f*x
+e)/a^2/f/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)+3/8*A*cos(f*x+e)/a^
2/c/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+3/8*A*arctanh(sin(f*x+e
))*cos(f*x+e)/a^2/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2972, 2743, 2741, 3770}

$$\frac{3A\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8a^2c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{3A\cos(e+fx)}{8a^2cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{3A}{8a^2f\sqrt{a\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(
5/2)),x]
```

```
[Out] -((A - B)*Cos[e + f*x])/(4*f*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x]
)^(5/2)) - (A*Cos[e + f*x])/(2*a*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e
+ f*x])^(5/2)) + (3*A*Cos[e + f*x])/(8*a^2*f*Sqrt[a + a*Sin[e + f*x]]*(c -
c*Sin[e + f*x])^(5/2)) + (3*A*Cos[e + f*x])/(8*a^2*c*f*Sqrt[a + a*Sin[e + f
*x]]*(c - c*Sin[e + f*x])^(3/2)) + (3*A*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])
/(8*a^2*c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
```

```
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}+\frac{A\int\frac{}{(a+a\sin(e}}{}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}-\frac{}{2af(a+a\sin}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}-\frac{}{2af(a+a\sin}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}-\frac{}{2af(a+a\sin}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}-\frac{}{2af(a+a\sin}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}-\frac{}{2af(a+a\sin}$$

Mathematica [A] time = 0.94, size = 246, normalized size = 1.00

$$\frac{\sec^3(e+fx)\left(22A\sin(e+fx)+6A\sin(3(e+fx))-9A\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-12A\cos(2(e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f
*x])^(5/2)),x]
```

```
[Out] (Sec[e + f*x]^3*(16*B - 9*A*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 12*A
*Cos[2*(e + f*x)]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]]) - 3*A*Cos[4*(e + f*x)]*(Log[Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 9*A*Log[Co
s[(e + f*x)/2] + Sin[(e + f*x)/2]] + 22*A*Sin[e + f*x] + 6*A*Sin[3*(e + f*x
)]))/(64*a^2*c^2*f*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.56, size = 306, normalized size = 1.25

$$\left[\frac{3\sqrt{ac}\,A\cos\left(fx+e\right)^5\log\left(-\frac{ac\cos\left(fx+e\right)^3-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)^3}\right)+2\left(\left(3\,A\cos\left(f\right.\right.\right.}{16\,a^3c^3f\cos\left(fx+e\right)^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(a*c)*A*cos(f*x + e)^5*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*
x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin
(f*x + e))/cos(f*x + e)^3) + 2*((3*A*cos(f*x + e)^2 + 2*A)*sin(f*x + e) + 2
*B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^3*f*cos(f*x
+ e)^5), -1/8*(3*sqrt(-a*c)*A*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sq
rt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e)^5 - (
(3*A*cos(f*x + e)^2 + 2*A)*sin(f*x + e) + 2*B)*sqrt(a*sin(f*x + e) + a)*sqr
t(-c*sin(f*x + e) + c))/(a^3*c^3*f*cos(f*x + e)^5)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
 + c)^(5/2)), x)
```

maple [A] time = 0.60, size = 151, normalized size = 0.62

$$\frac{\Big(3A\big(\cos^4(fx+e)\big)\ln\Big(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\Big)-3A\big(\cos^4(fx+e)\big)\ln\Big(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\Big)-2B\big(\cos^4(fx+e}{8f\big(a(1+\sin(fx+e))\big)^{\frac{5}{2}}\big(-c(\sin(fx+e)-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/8/f*(3*A*cos(f*x+e)^4*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-3*A*cos(f*
x+e)^4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*B*cos(f*x+e)^4+3*A*cos(
f*x+e)^2*sin(f*x+e)+2*A*sin(f*x+e)+2*B)*cos(f*x+e)/(a*(1+sin(f*x+e)))^(5/2)
/(-c*(sin(f*x+e)-1))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin(fx+e)+A}{(a\sin(fx+e)+a)^{\frac{5}{2}}(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
 + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\sin(e+f\,x)}{{(a+a\,\sin(e+f\,x))}^{5/2}\,{(c-c\,\sin(e+f\,x))}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(
5/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(
5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

3.195 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=174

$$\frac{c2^{n+\frac{1}{2}}(A(m+n+1)+B(m-n))\cos(e+fx)(1-\sin(e+fx))^{\frac{1}{2}-n}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}\,{}_2F_1\Big(\tfrac{1}{2}}{f(2m+1)(m+n+1)}$$

```
[Out] 2^(1/2+n)*c*(B*(m-n)+A*(1+m+n))*cos(f*x+e)*hypergeom([1/2-n, 1/2+m],[3/2+m]
,1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x
+e))^(-1+n)/f/(1+2*m)/(1+m+n)-B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+
e))^n/f/(1+m+n)
```

Rubi [A] time = 0.31, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2973, 2745, 2689, 70, 69}

$$\frac{c2^{n+\frac{1}{2}}(A(m+n+1)+B(m-n))\cos(e+fx)(1-\sin(e+fx))^{\frac{1}{2}-n}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}\,{}_2F_1\Big(\tfrac{1}{2}}{f(2m+1)(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + n)*c*(B*(m - n) + A*(1 + m + n))*Cos[e + f*x]*Hypergeometric2F1[(
1 + 2*m)/2, (1 - 2*n)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*
x])^(1/2 - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 + n))/(f*(1 +
 2*m)*(1 + m + n)) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
f*x])^n)/(f*(1 + m + n))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
```

```
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^n\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e}{f(1+m+n)}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e}{f(1+m+n)}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e}{f(1+m+n)}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e}{f(1+m+n)}$$

$$= \frac{2^{\frac{1}{2}+n}c\left(A+\frac{B(m-n)}{1+m+n}\right)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(1+2m\right.}{}$$

Mathematica [C] time = 14.24, size = 2903, normalized size = 16.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
n,x]
```

```
[Out] (4*(8*B*AppellF1[1/2 + n, -2*m, 2*(1 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*
x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - (A + B)*AppellF1[1/2 + n, -2*m, 1 +
 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^
2] - 8*B*AppellF1[1/2 + n, -2*m, 3 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f
*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*(m
+ n))*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n*(A*Cos[(-e + Pi/2 - f*x
)/2]^(2*m)*Sin[(-e + Pi/2 - f*x)/2]^(2*n) + B*Cos[(-e + Pi/2 - f*x)/2]^(2*m
)*Sin[(-e + Pi/2 - f*x)/2]^(2*n)*Sin[e + f*x])*Tan[(-e + Pi/2 - f*x)/4])/(f
*(1 + 2*n)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)*((-4*m*(8*B*AppellF1[1/2
+ n, -2*m, 2*(1 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + P
i/2 - f*x)/4]^2] - (A + B)*AppellF1[1/2 + n, -2*m, 1 + 2*(m + n), 3/2 + n,
Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 8*B*AppellF1[1/2
 + n, -2*m, 3 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
Pi/2 - f*x)/4]^2])*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*(Sec[(-e + Pi/2 - f*x)/4]
^2)^(1 + 2*(m + n))*Sin[(-e + Pi/2 - f*x)/2]^(2*n)*Tan[(-e + Pi/2 - f*x)/4]
^2*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(-1 - 2*m))/(1 + 2*n) - ((8*B*AppellF1[
1/2 + n, -2*m, 2*(1 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
```

```
 + Pi/2 - f*x)/4]^2] - (A + B)*AppellF1[1/2 + n, -2*m, 1 + 2*(m + n), 3/2 +
 n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 8*B*AppellF1
[1/2 + n, -2*m, 3 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-
e + Pi/2 - f*x)/4]^2])*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*(Sec[(-e + Pi/2 - f*x
)/4]^2)^(1 + 2*(m + n))*Sin[(-e + Pi/2 - f*x)/2]^(2*n))/((1 + 2*n)*(1 - Tan
[(-e + Pi/2 - f*x)/4]^2)^(2*m)) - (4*n*(8*B*AppellF1[1/2 + n, -2*m, 2*(1 +
m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] -
 (A + B)*AppellF1[1/2 + n, -2*m, 1 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f
*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 8*B*AppellF1[1/2 + n, -2*m, 3 + 2*
(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])
*Cos[(-e + Pi/2 - f*x)/2]^(1 + 2*m)*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*(m + n)
)*Sin[(-e + Pi/2 - f*x)/2]^(-1 + 2*n)*Tan[(-e + Pi/2 - f*x)/4])/((1 + 2*n)*
(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) + (4*m*(8*B*AppellF1[1/2 + n, -2*m,
 2*(1 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)
/4]^2] - (A + B)*AppellF1[1/2 + n, -2*m, 1 + 2*(m + n), 3/2 + n, Tan[(-e +
Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 8*B*AppellF1[1/2 + n, -2*m
, 3 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x
)/4]^2])*Cos[(-e + Pi/2 - f*x)/2]^(-1 + 2*m)*(Sec[(-e + Pi/2 - f*x)/4]^2)^(
2*(m + n))*Sin[(-e + Pi/2 - f*x)/2]^(1 + 2*n)*Tan[(-e + Pi/2 - f*x)/4])/((1
 + 2*n)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) - (4*(m + n)*(8*B*AppellF1[
1/2 + n, -2*m, 2*(1 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
 + Pi/2 - f*x)/4]^2] - (A + B)*AppellF1[1/2 + n, -2*m, 1 + 2*(m + n), 3/2 +
 n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 8*B*AppellF1
[1/2 + n, -2*m, 3 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-
e + Pi/2 - f*x)/4]^2])*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*(Sec[(-e + Pi/2 - f*x
)/4]^2)^(2*(m + n))*Sin[(-e + Pi/2 - f*x)/2]^(2*n)*Tan[(-e + Pi/2 - f*x)/4]
^2)/((1 + 2*n)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) - (4*Cos[(-e + Pi/2
- f*x)/2]^(2*m)*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*(m + n))*Sin[(-e + Pi/2 - f
*x)/2]^(2*n)*Tan[(-e + Pi/2 - f*x)/4]*(-((A + B)*(-((m*(1/2 + n)*AppellF1[3
/2 + n, 1 - 2*m, 1 + 2*(m + n), 5/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(
-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])
/(3/2 + n)) - ((1/2 + n)*(1 + 2*(m + n))*AppellF1[3/2 + n, -2*m, 2 + 2*(m +
 n), 5/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[
(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(2*(3/2 + n)))) - 8*B*(-((
m*(1/2 + n)*AppellF1[3/2 + n, 1 - 2*m, 3 + 2*(m + n), 5/2 + n, Tan[(-e + Pi
/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan
[(-e + Pi/2 - f*x)/4])/(3/2 + n)) - ((1/2 + n)*(3 + 2*(m + n))*AppellF1[3/2
 + n, -2*m, 4 + 2*(m + n), 5/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(2*(
3/2 + n))) + 8*B*(-((m*(1/2 + n)*AppellF1[3/2 + n, 1 - 2*m, 2*(1 + m + n),
5/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e +
 Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 + n)) - ((1/2 + n)*(1 + m
+ n)*AppellF1[3/2 + n, -2*m, 1 + 2*(1 + m + n), 5/2 + n, Tan[(-e + Pi/2 - f
*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e +
 Pi/2 - f*x)/4])/(3/2 + n))))/((1 + 2*n)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(
```

```
2*m))))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^n, x)
```

maple [F] time = 5.77, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(c-c\,\sin\left(e+f\,x\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n,x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^m\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^n\left(A+B\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**n,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**n*(A + B*sin(e
+ f*x)), x)
```

3.196 $\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^3\, dx$

Optimal. Leaf size=145

$$\frac{a^4c^3 2^{m+\frac{1}{2}}(B(3-m) - A(m+4))\cos^7(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-4}\,{}_2F_1\left(\frac{7}{2}, \frac{1}{2}-m; \frac{9}{2}; \frac{1}{2}(1-\mathrm{si}\right.}{7f(m+4)}$$

```
[Out] 1/7*2^(1/2+m)*a^4*c^3*(B*(3-m)-A*(4+m))*cos(f*x+e)^7*hypergeom([7/2, 1/2-m]
,[9/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(-4+m)/f
/(4+m)-a^3*B*c^3*cos(f*x+e)^7*(a+a*sin(f*x+e))^(-3+m)/f/(4+m)
```

Rubi [A] time = 0.34, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2860, 2689, 70, 69}

$$\frac{a^4c^3 2^{m+\frac{1}{2}}(B(3-m) - A(m+4))\cos^7(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-4}\,{}_2F_1\left(\frac{7}{2}, \frac{1}{2}-m; \frac{9}{2}; \frac{1}{2}(1-\mathrm{si}\right.}{7f(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (2^(1/2 + m)*a^4*c^3*(B*(3 - m) - A*(4 + m))*Cos[e + f*x]^7*Hypergeometric2
F1[7/2, 1/2 - m, 9/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a
 + a*Sin[e + f*x])^(-4 + m))/(7*f*(4 + m)) - (a^3*B*c^3*Cos[e + f*x]^7*(a +
 a*Sin[e + f*x])^(-3 + m))/(f*(4 + m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(a+a\sin(e+fx))^{-3+m}$$

$$= -\frac{a^3Bc^3\cos^7(e+fx)(a+a\sin(e+fx))^{-3+m}}{f(4+m)}$$

$$= -\frac{a^3Bc^3\cos^7(e+fx)(a+a\sin(e+fx))^{-3+m}}{f(4+m)}$$

$$= -\frac{a^3Bc^3\cos^7(e+fx)(a+a\sin(e+fx))^{-3+m}}{f(4+m)}$$

$$= -\frac{2^{\frac{1}{2}+m}a^4c^3\left(A-\frac{B(3-m)}{4+m}\right)\cos^7(e+fx)\,{}_2F_1\left(\frac{7}{2}\right.}{}$$

Mathematica [F] time = 180.06, size = 0, normalized size = 0.00

$$\$\text{Aborted}$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
3,x]
```

```
[Out] $Aborted
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(Bc^3\cos\left(fx+e\right)^4+(3A-5B)c^3\cos\left(fx+e\right)^2-4(A-B)c^3-\left((A-3B)c^3\cos\left(fx+e\right)^2-4(A-B\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] integral(-(B*c^3*cos(f*x + e)^4 + (3*A - 5*B)*c^3*cos(f*x + e)^2 - 4*(A - B
)*c^3 - ((A - 3*B)*c^3*cos(f*x + e)^2 - 4*(A - B)*c^3)*sin(f*x + e))*(a*sin
(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(B\sin\left(fx+e\right)+A\right)\left(c\sin\left(fx+e\right)-c\right)^3\left(a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] integrate(-(B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^3*(a*sin(f*x + e) + a)
^m, x)
```

maple [F] time = 8.10, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^m\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(B\sin\left(fx+e\right)+A\right)\left(c\sin\left(fx+e\right)-c\right)^{3}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] -integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^3*(a*sin(f*x + e) + a)
^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^3,x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.197 $\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^2 \, dx$

Optimal. Leaf size=145

$$\frac{a^3 c^2 2^{m+\frac{1}{2}}(B(2-m) - A(m+3))\cos^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{5}{2}, \frac{1}{2}-m; \frac{7}{2}; \frac{1}{2}(1-\mathrm{si}\right.}{5f(m+3)}$$

```
[Out] 1/5*2^(1/2+m)*a^3*c^2*(B*(2-m)-A*(3+m))*cos(f*x+e)^5*hypergeom([5/2, 1/2-m]
,[7/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(-3+m)/f
/(3+m)-a^2*B*c^2*cos(f*x+e)^5*(a+a*sin(f*x+e))^(-2+m)/f/(3+m)
```

Rubi [A] time = 0.33, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2860, 2689, 70, 69}

$$\frac{a^3 c^2 2^{m+\frac{1}{2}}(B(2-m) - A(m+3))\cos^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{5}{2}, \frac{1}{2}-m; \frac{7}{2}; \frac{1}{2}(1-\mathrm{si}\right.}{5f(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (2^(1/2 + m)*a^3*c^2*(B*(2 - m) - A*(3 + m))*Cos[e + f*x]^5*Hypergeometric2
F1[5/2, 1/2 - m, 7/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a
 + a*Sin[e + f*x])^(-3 + m))/(5*f*(3 + m)) - (a^2*B*c^2*Cos[e + f*x]^5*(a +
 a*Sin[e + f*x])^(-2 + m))/(f*(3 + m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(a+a\sin(e+fx))^{-2+}$$

$$= -\frac{a^2Bc^2\cos^5(e+fx)(a+a\sin(e+fx))^{-2+m}}{f(3+m)}$$

$$= -\frac{a^2Bc^2\cos^5(e+fx)(a+a\sin(e+fx))^{-2+m}}{f(3+m)}$$

$$= -\frac{a^2Bc^2\cos^5(e+fx)(a+a\sin(e+fx))^{-2+m}}{f(3+m)}$$

$$= -\frac{2^{\frac{1}{2}+m}a^3c^2\left(A-\frac{B(2-m)}{3+m}\right)\cos^5(e+fx)\,{}_2F_1\left(\frac{5}{2}\right.}{}$$

Mathematica [F] time = 180.08, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
2,x]
```

```
[Out] $Aborted
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\left((A-2B)c^2\cos\left(fx+e\right)^2-2(A-B)c^2+\left(Bc^2\cos\left(fx+e\right)^2+2(A-B)c^2\right)\sin\left(fx+e\right)\right)\left(a\sin\left(fx-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] integral(-((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x +
e)^2 + 2*(A - B)*c^2)*sin(f*x + e))*(a*sin(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(c\sin\left(fx+e\right)-c\right)^2\left(a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a)^
m, x)
```

maple [F] time = 8.31, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^m\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(c\sin\left(fx+e\right)-c\right)^{2}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a)^
m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(c-c\,\sin\left(e+f\,x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^2,x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int A\left(a\sin\left(e+fx\right)+a\right)^{m}\,dx+\int\left(-2A\left(a\sin\left(e+fx\right)+a\right)^{m}\sin\left(e+fx\right)\right)dx+\int A\left(a\sin\left(e+fx\right)+a\right)^{m}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)
```

```
[Out] c**2*(Integral(A*(a*sin(e + f*x) + a)**m, x) + Integral(-2*A*(a*sin(e + f*x
) + a)**m*sin(e + f*x), x) + Integral(A*(a*sin(e + f*x) + a)**m*sin(e + f*x
)**2, x) + Integral(B*(a*sin(e + f*x) + a)**m*sin(e + f*x), x) + Integral(-
2*B*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2, x) + Integral(B*(a*sin(e + f*x
) + a)**m*sin(e + f*x)**3, x))
```

3.198 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$

Optimal. Leaf size=139

$$\frac{a^2c2^{m+\frac{1}{2}}(B(1-m)-A(m+2))\cos^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sin\ldots\right.}{3f(m+2)}$$

```
[Out] 1/3*2^(1/2+m)*a^2*c*(B*(1-m)-A*(2+m))*cos(f*x+e)^3*hypergeom([3/2, 1/2-m],[
5/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(-2+m)/f/(
2+m)-a*B*c*cos(f*x+e)^3*(a+a*sin(f*x+e))^(-1+m)/f/(2+m)
```

Rubi [A] time = 0.29, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {2967, 2860, 2689, 70, 69}

$$\frac{a^2c2^{m+\frac{1}{2}}(B(1-m)-A(m+2))\cos^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sin\ldots\right.}{3f(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

```
[Out] (2^(1/2 + m)*a^2*c*(B*(1 - m) - A*(2 + m))*Cos[e + f*x]^3*Hypergeometric2F1
[3/2, 1/2 - m, 5/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a +
 a*Sin[e + f*x])^(-2 + m))/(3*f*(2 + m)) - (a*B*c*Cos[e + f*x]^3*(a + a*Sin
[e + f*x])^(-1 + m))/(f*(2 + m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx=(ac)\int \cos^2(e+fx)(a+a\sin(e+fx))^{-1+m}(A$$

$$=-\frac{aBc\cos^3(e+fx)(a+a\sin(e+fx))^{-1+m}}{f(2+m)}+$$

$$=-\frac{aBc\cos^3(e+fx)(a+a\sin(e+fx))^{-1+m}}{f(2+m)}+$$

$$=-\frac{aBc\cos^3(e+fx)(a+a\sin(e+fx))^{-1+m}}{f(2+m)}+$$

$$=-\frac{2^{\frac{1}{2}+m}a^2c\left(A-\frac{B(1-m)}{2+m}\right)\cos^3(e+fx)\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}\right.}{}$$

Mathematica [C] time = 4.22, size = 462, normalized size = 3.32

$$ic4^{-m-1}e^{ifmx}\left(1+ie^{-i(e+fx)}\right)^{-2m}\left(-(-1)^{3/4}e^{-\frac{1}{2}i(e+fx)}\left(e^{i(e+fx)}+i\right)\right)^{2m}(\sin(e+fx)-1)\sin^{-2m}\left(\frac{1}{4}(2e+2fx+\pi)\right)(a(s$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),
x]
```

```
[Out] (I*4^(-1 - m)*c*E^(I*f*m*x)*(-(((-1)^(3/4)*(I + E^(I*(e + f*x))))/E^((I/2)*
(e + f*x))))^(2*m)*(((-I)*B*Hypergeometric2F1[-2 - m, -2*m, -1 - m, (-I)/E^
(I*(e + f*x))])/(E^(I*(2*e + f*(2 + m)*x))*(2 + m)) + (2*((-I)*A + B)*Hyper
geometric2F1[-1 - m, -2*m, -m, (-I)/E^(I*(e + f*x))])/(E^(I*(e + f*(1 + m)*
x))*(1 + m)) + ((2*I)*A*E^(I*(e - f*(-1 + m)*x))*Hypergeometric2F1[1 - m, -
2*m, 2 - m, (-I)/E^(I*(e + f*x))])/(-1 + m) + (2*B*E^(I*(e - f*(-1 + m)*x))
*Hypergeometric2F1[1 - m, -2*m, 2 - m, (-I)/E^(I*(e + f*x))])/(-1 + m) + (I
*B*E^((2*I)*e - I*f*(-2 + m)*x)*Hypergeometric2F1[2 - m, -2*m, 3 - m, (-I)/
E^(I*(e + f*x))])/(-2 + m) + (4*A*Hypergeometric2F1[-2*m, -m, 1 - m, (-I)/E
^(I*(e + f*x))])/(E^(I*f*m*x)*m))*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x])
)^m)/((1 + I/E^(I*(e + f*x)))^(2*m)*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^2*Sin[(2*e + Pi + 2*f*x)/4]^(2*m))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bc\cos\left(fx+e\right)^{2}-(A-B)c\sin\left(fx+e\right)+(A-B)c\right)\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*(a*sin(f
*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int-\left(B\sin\left(fx+e\right)+A\right)\left(c\sin\left(fx+e\right)-c\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate(-(B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^m
, x)
```

maple [F] time = 4.98, size = 0, normalized size = 0.00

$$\int \left(a + a \sin\left(fx + e\right)\right)^{m} \left(A + B \sin\left(fx + e\right)\right) \left(c - c \sin\left(fx + e\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(B \sin\left(fx + e\right) + A\right)\left(c \sin\left(fx + e\right) - c\right)\left(a \sin\left(fx + e\right) + a\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^m
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + B \sin\left(e + f x\right)\right) \left(a + a \sin\left(e + f x\right)\right)^{m} \left(c - c \sin\left(e + f x\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x)),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int \left(-A\left(a \sin\left(e + fx\right) + a\right)^{m}\right) dx + \int A\left(a \sin\left(e + fx\right) + a\right)^{m} \sin\left(e + fx\right) dx + \int \left(-B\left(a \sin\left(e + fx\right) + a\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] -c*(Integral(-A*(a*sin(e + f*x) + a)**m, x) + Integral(A*(a*sin(e + f*x) +
a)**m*sin(e + f*x), x) + Integral(-B*(a*sin(e + f*x) + a)**m*sin(e + f*x),
x) + Integral(B*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2, x))
```

3.199 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=117

$$-\frac{2^{m+\frac{1}{2}}(Am+A+Bm)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(m+1)}$$

```
[Out] -B*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+m)-2^(1/2+m)*(A*m+B*m+A)*cos(f*x+e)*h
ypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+
a*sin(f*x+e))^m/f/(1+m)
```

Rubi [A] time = 0.08, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2751, 2652, 2651}

$$-\frac{2^{m+\frac{1}{2}}(Am+A+Bm)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m))) - (2^(1/2 + m)*(A +
A*m + B*m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f
*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m))
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)}+\frac{(A+Am+Bm)\int(a+a}{1+m}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)}+\frac{\big((A+Am+Bm)(1+\sin}{}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)}-\frac{2^{\frac{1}{2}+m}(A+Am+Bm)\cos}{}$$

Mathematica [C] time = 1.83, size = 275, normalized size = 2.35

$$-\frac{\sin^{-2m}\left(\frac{1}{4}(2e+2fx+\pi)\right)(a(\sin(e+fx)+1))^m\left(\frac{2\sqrt{2}A\sin\left(\frac{1}{4}(2e+2fx-\pi)\right)\cos^{2m+1}\left(\frac{1}{4}(2e+2fx-\pi)\right){}_2F_1\left(\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};\sin^2\left(\frac{1}{4}(2e+\right.\right.}{(2m+1)\sqrt{1-\sin(e+fx)}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -(((a*(1 + Sin[e + f*x]))^m*(((-1)^(1/4)*2^(-1 - 2*m)*B*(-(((-1)^(3/4)*(I +
 E^(I*(e + f*x))))/E^((I/2)*(e + f*x))))^(1 + 2*m)*(E^((2*I)*(e + f*x))*(-1
 + m)*Hypergeometric2F1[1, m, -m, (-I)/E^(I*(e + f*x))] - (1 + m)*Hypergeom
etric2F1[1, 2 + m, 2 - m, (-I)/E^(I*(e + f*x))]))/(E^(((3*I)/2)*(e + f*x))*
(-1 + m^2)) + (2*Sqrt[2]*A*Cos[(2*e - Pi + 2*f*x)/4]^(1 + 2*m)*Hypergeometr
ic2F1[1/2, 1/2 + m, 3/2 + m, Sin[(2*e + Pi + 2*f*x)/4]^2]*Sin[(2*e - Pi + 2
*f*x)/4])/((1 + 2*m)*Sqrt[1 - Sin[e + f*x]])))/(f*Sin[(2*e + Pi + 2*f*x)/4]
^(2*m)))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

maple [F] time = 2.53, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x)), x)
```

3.200 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=123

$$\frac{2^{m+\frac{1}{2}}(Am+Bm+B)\sec(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{cfm}$$

```
[Out] 2^(1/2+m)*(A*m+B*m+B)*hypergeom([-1/2, 1/2-m],[1/2],1/2-1/2*sin(f*x+e))*sec
(f*x+e)*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^m/c/f/m-B*sec(f*x+e)*(a+a*s
in(f*x+e))^(1+m)/a/c/f/m
```

Rubi [A] time = 0.30, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2860, 2689, 70, 69}

$$\frac{2^{m+\frac{1}{2}}(Am+Bm+B)\sec(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{cfm}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),x]
```

```
[Out] (2^(1/2 + m)*(B + A*m + B*m)*Hypergeometric2F1[-1/2, 1/2 - m, 1/2, (1 - Sin
[e + f*x])/2]*Sec[e + f*x]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x]
)^m)/(c*f*m) - (B*Sec[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*c*f*m)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \frac{\int \sec^2(e+fx)(a+a\sin(e+fx))^{1+m}(A+B\sin(e+fx))\,dx}{ac}$$

$$= -\frac{B\sec(e+fx)(a+a\sin(e+fx))^{1+m}}{acfm} + \frac{(B+Am+Bm)\int \sec^2(\ldots}{\ldots}$$

$$= -\frac{B\sec(e+fx)(a+a\sin(e+fx))^{1+m}}{acfm} + \frac{\left(a(B+Am+Bm)\sec(\ldots\right.}{\ldots}$$

$$= -\frac{B\sec(e+fx)(a+a\sin(e+fx))^{1+m}}{acfm} + \frac{\left(2^{-\frac{1}{2}+m}a(B+Am+Bm\ldots\right.}{\ldots}$$

$$= \frac{2^{\frac{1}{2}+m}(B+Am+Bm)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec(\ldots}{cfm}$$

Mathematica [C] time = 25.57, size = 7409, normalized size = 60.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^m}{c\sin\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{c\sin\left(fx+e\right)-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)
, x)
```

maple [F] time = 1.09, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)}{c-c\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{c\sin\left(fx+e\right)-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{m}}{c-c\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x)),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{A\left(a\sin\left(e+fx\right)+a\right)^{m}}{\sin\left(e+fx\right)-1}\,dx+\int \frac{B\left(a\sin\left(e+fx\right)+a\right)^{m}\sin\left(e+fx\right)}{\sin\left(e+fx\right)-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] -(Integral(A*(a*sin(e + f*x) + a)**m/(sin(e + f*x) - 1), x) + Integral(B*(a
*sin(e + f*x) + a)**m*sin(e + f*x)/(sin(e + f*x) - 1), x))/c
```

3.201 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=148

$$\frac{B\sec^3(e+fx)(a\sin(e+fx)+a)^{m+2}}{a^2c^2f(1-m)}+\frac{2^{m+\frac{1}{2}}(A(1-m)-B(m+2))\sec^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+}{3ac^2f(1-m)}$$

```
[Out] 1/3*2^(1/2+m)*(A*(1-m)-B*(2+m))*hypergeom([-3/2, 1/2-m],[-1/2],1/2-1/2*sin(
f*x+e))*sec(f*x+e)^3*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(1+m)/a/c^2/f/
(1-m)+B*sec(f*x+e)^3*(a+a*sin(f*x+e))^(2+m)/a^2/c^2/f/(1-m)
```

Rubi [A] time = 0.33, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2860, 2689, 70, 69}

$$\frac{B\sec^3(e+fx)(a\sin(e+fx)+a)^{m+2}}{a^2c^2f(1-m)}+\frac{2^{m+\frac{1}{2}}(A(1-m)-B(m+2))\sec^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+}{3ac^2f(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (2^(1/2 + m)*(A*(1 - m) - B*(2 + m))*Hypergeometric2F1[-3/2, 1/2 - m, -1/2,
 (1 - Sin[e + f*x])/2]*Sec[e + f*x]^3*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*S
in[e + f*x])^(1 + m))/(3*a*c^2*f*(1 - m)) + (B*Sec[e + f*x]^3*(a + a*Sin[e
+ f*x])^(2 + m))/(a^2*c^2*f*(1 - m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(a+a\sin(e+fx))^{2+m}(A+B\sin(e+fx))\,dx}{a^2c^2}$$

$$= \frac{B\sec^3(e+fx)(a+a\sin(e+fx))^{2+m}}{a^2c^2f(1-m)} + \frac{\left(A-\frac{B(2+m)}{1-m}\right)\int \sec^4(e$$

$$= \frac{B\sec^3(e+fx)(a+a\sin(e+fx))^{2+m}}{a^2c^2f(1-m)} + \frac{\left(\left(A-\frac{B(2+m)}{1-m}\right)\sec^3(e+\right.}{}$$

$$= \frac{B\sec^3(e+fx)(a+a\sin(e+fx))^{2+m}}{a^2c^2f(1-m)} + \frac{\left(2^{-\frac{1}{2}+m}\left(A-\frac{B(2+m)}{1-m}\right)\mathrm{s}\right.}{}$$

$$= \frac{2^{\frac{1}{2}+m}\left(A-\frac{B(2+m)}{1-m}\right) {}_2F_1\left(-\frac{3}{2},\frac{1}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec}{3ac^2f}$$

Mathematica [C] time = 6.93, size = 9240, normalized size = 62.43

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^2,x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^m}{c^2\cos\left(fx+e\right)^2+2\,c^2\sin\left(fx+e\right)-2\,c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c^2*cos(f*x + e)^2 +
 2*c^2*sin(f*x + e) - 2*c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(c\sin\left(fx+e\right)-c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)^
2, x)
```

maple [F] time = 8.47, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)}{\left(c-c\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(c\sin\left(fx+e\right)-c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)^
2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{m}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^2,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^2, x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\left(a\sin\left(e+fx\right)+a\right)^{m}}{\sin^{2}\left(e+fx\right)-2\sin\left(e+fx\right)+1}\,dx+\int \frac{B\left(a\sin\left(e+fx\right)+a\right)^{m}\sin\left(e+fx\right)}{\sin^{2}\left(e+fx\right)-2\sin\left(e+fx\right)+1}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

```
[Out] (Integral(A*(a*sin(e + f*x) + a)**m/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1),
 x) + Integral(B*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(sin(e + f*x)**2 - 2*
sin(e + f*x) + 1), x))/c**2
```

3.202 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=148

$$\frac{B\sec^5(e+fx)(a\sin(e+fx)+a)^{m+3}}{a^3c^3f(2-m)}+\frac{2^{m+\frac{1}{2}}(A(2-m)-B(m+3))\sec^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+f$$

```
[Out] 1/5*2^(1/2+m)*(A*(2-m)-B*(3+m))*hypergeom([-5/2, 1/2-m],[-3/2],1/2-1/2*sin(
f*x+e))*sec(f*x+e)^5*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(2+m)/a^2/c^3/
f/(2-m)+B*sec(f*x+e)^5*(a+a*sin(f*x+e))^(3+m)/a^3/c^3/f/(2-m)
```

Rubi [A] time = 0.33, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2860, 2689, 70, 69}

$$\frac{2^{m+\frac{1}{2}}(A(2-m)-B(m+3))\sec^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+2}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}-m;-\frac{3}{2};\frac{1}{2}(1-\sin\right.}{5a^2c^3f(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (2^(1/2 + m)*(A*(2 - m) - B*(3 + m))*Hypergeometric2F1[-5/2, 1/2 - m, -3/2,
 (1 - Sin[e + f*x])/2]*Sec[e + f*x]^5*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*S
in[e + f*x])^(2 + m))/(5*a^2*c^3*f*(2 - m)) + (B*Sec[e + f*x]^5*(a + a*Sin[
e + f*x])^(3 + m))/(a^3*c^3*f*(2 - m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(a+a\sin(e+fx))^{3+m}(A+B\sin(e+fx))\,dx}{a^3c^3}$$

$$= \frac{B\sec^5(e+fx)(a+a\sin(e+fx))^{3+m}}{a^3c^3f(2-m)} + \frac{\left(A-\frac{B(3+m)}{2-m}\right)\int \sec^6(e+\ldots}{\ldots}$$

$$= \frac{B\sec^5(e+fx)(a+a\sin(e+fx))^{3+m}}{a^3c^3f(2-m)} + \frac{\left(\left(A-\frac{B(3+m)}{2-m}\right)\sec^5(e+\ldots\right.}{\ldots}$$

$$= \frac{B\sec^5(e+fx)(a+a\sin(e+fx))^{3+m}}{a^3c^3f(2-m)} + \frac{\left(2^{-\frac{1}{2}+m}\left(A-\frac{B(3+m)}{2-m}\right)\sec\ldots\right.}{\ldots}$$

$$= \frac{2^{\frac{1}{2}+m}\left(A-\frac{B(3+m)}{2-m}\right){}_2F_1\left(-\frac{5}{2},\frac{1}{2}-m;-\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec^5(\ldots}{5a^2c^3f}$$

Mathematica [C] time = 7.18, size = 12580, normalized size = 85.00

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^3,x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^m}{3\,c^3\cos\left(fx+e\right)^2-4\,c^3-\left(c^3\cos\left(fx+e\right)^2-4\,c^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(3*c^3*cos(f*x + e)^2
 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(c\sin\left(fx+e\right)-c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] integrate(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)
^3, x)
```

maple [F] time = 9.30, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)}{\left(c-c\sin\left(fx+e\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(c\sin\left(fx+e\right)-c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] -integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)
^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{m}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^3,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^3, x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.203 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=118

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sin(e+fx)}}-\frac{2B\cos(e+fx)(a\sin(e+fx}{f(2m+1)\sqrt{c-c\sin(e+}}$$

```
[Out] -2*B*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)+(A+B)*c
os(f*x+e)*hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(a+a*sin(f*x+e))
^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2973, 2745, 2667, 68}

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sin(e+fx)}}-\frac{2B\cos(e+fx)(a\sin(e+fx}{f(2m+1)\sqrt{c-c\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] (-2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e +
f*x]]) + ((A + B)*Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 +
Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*
x]])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + (A+B)\int \frac{(a+a\sin(e}{\sqrt{c-c\sin(e}}$$

$$= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{((A+B)\cos(e+fx))\int}{\sqrt{a+a\sin(e}}$$

$$= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{(a(A+B)\cos(e+fx))\,S}{f\sqrt{a+a\sin(}}$$

$$= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\cos(e+fx)\,{}_2F_1}{}$$

Mathematica [A] time = 4.87, size = 200, normalized size = 1.69

$$\frac{2^{-2m-\frac{3}{2}}\sin\left(\frac{1}{4}(2e+2fx+\pi)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(\sin(e+fx)+1))^m\left(2^{2m+1}(A+B)\,{}_2F_1\left(1,2m\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e +
f*x]],x]
```

```
[Out] (2^(-3/2 - 2*m)*(-(2^(3 + 2*m)*B) + 2^(1 + 2*m)*(A + B)*Hypergeometric2F1[1
, 1 + 2*m, 2*(1 + m), Sin[(2*e + Pi + 2*f*x)/4]] + (A + B)*Hypergeometric2F
1[1 + 2*m, 1 + 2*m, 2*(1 + m), (1 - Tan[(2*e - Pi + 2*f*x)/8]^2)/2]*(Sec[(2
*e - Pi + 2*f*x)/8]^2)^(1 + 2*m))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*
(1 + Sin[e + f*x]))^m*Sin[(2*e + Pi + 2*f*x)/4])/((f + 2*f*m)*Sqrt[c - c*Si
n[e + f*x]])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}}{c\sin\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m/(c*sin(f*x + e) - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e)
+ c), x)
```

maple [F] time = 0.98, size = 0, normalized size = 0.00

$$\int\frac{\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)}{\sqrt{c-c\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e)
+ c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/2
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(A+B\sin\left(e+fx\right)\right)}{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(-c*(sin(e + f*
x) - 1)), x)
```

3.204 $\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx$

Optimal. Leaf size=118

$$\frac{(A+B)\cos(e+fx)(c\sin(e+fx)+c)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{a-a\sin(e+fx)}}-\frac{2B\cos(e+fx)(c\sin(e+fx)}{f(2m+1)\sqrt{a-a\sin(e+}}$$

```
[Out] -2*B*cos(f*x+e)*(c+c*sin(f*x+e))^m/f/(1+2*m)/(a-a*sin(f*x+e))^(1/2)+(A+B)*c
os(f*x+e)*hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(c+c*sin(f*x+e))
^m/f/(1+2*m)/(a-a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2973, 2745, 2667, 68}

$$\frac{(A+B)\cos(e+fx)(c\sin(e+fx)+c)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{a-a\sin(e+fx)}}-\frac{2B\cos(e+fx)(c\sin(e+fx)}{f(2m+1)\sqrt{a-a\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + c*Sin[e + f*x])^m)/Sqrt[a - a*Sin[e + f*x]],
x]
```

```
[Out] (-2*B*Cos[e + f*x]*(c + c*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[a - a*Sin[e +
f*x]]) + ((A + B)*Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 +
Sin[e + f*x])/2]*(c + c*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[a - a*Sin[e + f*
x]])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx = -\frac{2B\cos(e+fx)(c+c\sin(e+fx))^m}{f(1+2m)\sqrt{a-a\sin(e+fx)}} - (-A-B)\int \frac{(c+c\sin(e}{\sqrt{a-a\sin(e}}$$

$$= -\frac{2B\cos(e+fx)(c+c\sin(e+fx))^m}{f(1+2m)\sqrt{a-a\sin(e+fx)}} - \frac{((-A-B)\cos(e+fx))\int}{\sqrt{a-a\sin(e}}$$

$$= -\frac{2B\cos(e+fx)(c+c\sin(e+fx))^m}{f(1+2m)\sqrt{a-a\sin(e+fx)}} - \frac{((-A-B)c\cos(e+fx))}{f\sqrt{a-a\sin(}}$$

$$= -\frac{2B\cos(e+fx)(c+c\sin(e+fx))^m}{f(1+2m)\sqrt{a-a\sin(e+fx)}} + \frac{(A+B)\cos(e+fx)\,{}_2F_1\Big(}{}$$

Mathematica [A] time = 2.38, size = 200, normalized size = 1.69

$$\frac{2^{-2m-\frac{3}{2}}\sin\left(\frac{1}{4}(2e+2fx+\pi)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(c(\sin(e+fx)+1))^m\left(2^{2m+1}(A+B)\,{}_2F_1\left(1,2m\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + c*Sin[e + f*x])^m)/Sqrt[a - a*Sin[e +
f*x]],x]
```

```
[Out] (2^(-3/2 - 2*m)*(-(2^(3 + 2*m)*B) + 2^(1 + 2*m)*(A + B)*Hypergeometric2F1[1
, 1 + 2*m, 2*(1 + m), Sin[(2*e + Pi + 2*f*x)/4]] + (A + B)*Hypergeometric2F
1[1 + 2*m, 1 + 2*m, 2*(1 + m), (1 - Tan[(2*e - Pi + 2*f*x)/8]^2)/2]*(Sec[(2
*e - Pi + 2*f*x)/8]^2)^(1 + 2*m))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(c*
(1 + Sin[e + f*x]))^m*Sin[(2*e + Pi + 2*f*x)/4])/((f + 2*f*m)*Sqrt[a - a*Si
n[e + f*x]])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{-a\sin\left(fx+e\right)+a}\left(c\sin\left(fx+e\right)+c\right)^{m}}{a\sin\left(fx+e\right)-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) +
c)^m/(a*sin(f*x + e) - a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(c\sin\left(fx+e\right)+c\right)^{m}}{\sqrt{-a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) + c)^m/sqrt(-a*sin(f*x + e)
+ a), x)
```

maple [F] time = 1.12, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\sin\left(fx+e\right)\right)\left(c+c\sin\left(fx+e\right)\right)^{m}}{\sqrt{a-a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
```

```
[Out] int((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(c\sin\left(fx+e\right)+c\right)^{m}}{\sqrt{-a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) + c)^m/sqrt(-a*sin(f*x + e)
+ a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c+c\,\sin\left(e+f\,x\right)\right)}^m}{\sqrt{a-a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + c*sin(e + f*x))^m)/(a - a*sin(e + f*x))^(1/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + c*sin(e + f*x))^m)/(a - a*sin(e + f*x))^(1/2
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(A+B\sin\left(e+fx\right)\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))**m/(a-a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((c*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(-a*(sin(e + f*
x) - 1)), x)
```

3.205 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=275

$$-\frac{64c^3(B(5-2m)-A(2m+7))\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+5)(2m+7)\left(4m^2+8m+3\right)\sqrt{c-c\sin(e+fx)}}-\frac{16c^2(B(5-2m)-A(2m+7))\cos(e+fx)\sqrt{c-}}{f(2m+7)\left(4m^2+1\right.}$$

```
[Out] -2*c*(B*(5-2*m)-A*(7+2*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(
3/2)/f/(4*m^2+24*m+35)-2*B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(
5/2)/f/(7+2*m)-64*c^3*(B*(5-2*m)-A*(7+2*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m/f
/(5+2*m)/(7+2*m)/(4*m^2+8*m+3)/(c-c*sin(f*x+e))^(1/2)-16*c^2*(B*(5-2*m)-A*(
7+2*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2)/f/(7+2*m)/(4*m
^2+16*m+15)
```

Rubi [A] time = 0.50, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2973, 2740, 2738}

$$-\frac{16c^2(B(5-2m)-A(2m+7))\cos(e+fx)\sqrt{c-c\sin(e+fx)}\,(a\sin(e+fx)+a)^m}{f(2m+7)\left(4m^2+16m+15\right)}-\frac{64c^3(B(5-2m)-A(2m+}{f(2m+5)(2m+7)\left(4m\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (-64*c^3*(B*(5 - 2*m) - A*(7 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(
f*(5 + 2*m)*(7 + 2*m)*(3 + 8*m + 4*m^2)*Sqrt[c - c*Sin[e + f*x]]) - (16*c^2
*(B*(5 - 2*m) - A*(7 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[c - c
*Sin[e + f*x]])/(f*(7 + 2*m)*(15 + 16*m + 4*m^2)) - (2*c*(B*(5 - 2*m) - A*(
7 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3/2))/(
f*(5 + 2*m)*(7 + 2*m)) - (2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Si
n[e + f*x])^(5/2))/(f*(7 + 2*m))
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\,\mathrm{si}}{f(7+2m)}$$

$$= -\frac{2c(B(5-2m)-A(7+2m))\cos(e+fx)(a}{f(5+2m)(7}$$

$$= -\frac{16c^2(B(5-2m)-A(7+2m))\cos(e+fx)(}{f(3+2m)(5+2}$$

$$= -\frac{64c^3(B(5-2m)-A(7+2m))\cos(e+fx)(}{f(1+2m)(3+2m)(5+2m)(7+2m)\sqrt{c}}$$

Mathematica [C] time = 6.82, size = 667, normalized size = 2.43

$$\frac{(c-c\sin(e+fx))^{5/2}(a(\sin(e+fx)+1))^m\left(\frac{(24Am^2+184Am+350A-12Bm^2-104Bm-385B)\left(\left(\frac{1}{8}-\frac{i}{8}\right)\cos\left(\frac{3}{2}(e+fx)\right)-\left(\frac{1}{8}+\frac{i}{8}\right)\sin\left(\frac{3}{2}(e+fx\right.\right.}{(2m+3)(2m+5)(2m+7)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^(5/2)*(((2100*A - 1575*B + 1
272*A*m - 110*B*m + 304*A*m^2 - 68*B*m^2 + 32*A*m^3 - 8*B*m^3)*((1/8 + I/8)
*Cos[(e + f*x)/2] + (1/8 - I/8)*Sin[(e + f*x)/2]))/((1 + 2*m)*(3 + 2*m)*(5
+ 2*m)*(7 + 2*m)) + ((2100*A - 1575*B + 1272*A*m - 110*B*m + 304*A*m^2 - 68
*B*m^2 + 32*A*m^3 - 8*B*m^3)*((1/8 - I/8)*Cos[(e + f*x)/2] + (1/8 + I/8)*Si
n[(e + f*x)/2]))/((1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(7 + 2*m)) + ((350*A - 385*
B + 184*A*m - 104*B*m + 24*A*m^2 - 12*B*m^2)*((1/8 - I/8)*Cos[(3*(e + f*x))
/2] - (1/8 + I/8)*Sin[(3*(e + f*x))/2]))/((3 + 2*m)*(5 + 2*m)*(7 + 2*m)) +
((350*A - 385*B + 184*A*m - 104*B*m + 24*A*m^2 - 12*B*m^2)*((1/8 + I/8)*Cos
[(3*(e + f*x))/2] - (1/8 - I/8)*Sin[(3*(e + f*x))/2]))/((3 + 2*m)*(5 + 2*m)
*(7 + 2*m)) + ((14*A - 35*B + 4*A*m - 6*B*m)*((-1/8 + I/8)*Cos[(5*(e + f*x)
)/2] - (1/8 + I/8)*Sin[(5*(e + f*x))/2]))/((5 + 2*m)*(7 + 2*m)) + ((14*A -
35*B + 4*A*m - 6*B*m)*((-1/8 - I/8)*Cos[(5*(e + f*x))/2] - (1/8 - I/8)*Sin[
(5*(e + f*x))/2]))/((5 + 2*m)*(7 + 2*m)) + ((1/8 - I/8)*B*Cos[(7*(e + f*x))
/2] - (1/8 + I/8)*B*Sin[(7*(e + f*x))/2])/(7 + 2*m) + ((1/8 + I/8)*B*Cos[(7
*(e + f*x))/2] - (1/8 - I/8)*B*Sin[(7*(e + f*x))/2])/(7 + 2*m)))/(f*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^5)
```

fricas [B] time = 0.49, size = 562, normalized size = 2.04

$$\frac{2\left(\left(8\,Bc^2m^3+36\,Bc^2m^2+46\,Bc^2m+15\,Bc^2\right)\cos\left(fx+e\right)^4+64\,(A+B)c^2m-\left(8\,(A-2\,B)c^2m^3+4\,(11\,A-28\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] 2*((8*B*c^2*m^3 + 36*B*c^2*m^2 + 46*B*c^2*m + 15*B*c^2)*cos(f*x + e)^4 + 64
*(A + B)*c^2*m - (8*(A - 2*B)*c^2*m^3 + 4*(11*A - 28*B)*c^2*m^2 + 2*(31*A -
 86*B)*c^2*m + 3*(7*A - 20*B)*c^2)*cos(f*x + e)^3 + 32*(7*A - 5*B)*c^2 + (8
*(A - B)*c^2*m^3 + 4*(19*A - 11*B)*c^2*m^2 + 190*(A - B)*c^2*m + (77*A - 85
*B)*c^2)*cos(f*x + e)^2 + 2*(8*(A - B)*c^2*m^3 + 60*(A - B)*c^2*m^2 + 2*(79
*A - 63*B)*c^2*m + (161*A - 145*B)*c^2)*cos(f*x + e) + (64*(A + B)*c^2*m -
(8*B*c^2*m^3 + 36*B*c^2*m^2 + 46*B*c^2*m + 15*B*c^2)*cos(f*x + e)^3 + 32*(7
*A - 5*B)*c^2 - (8*(A - B)*c^2*m^3 + 4*(11*A - 19*B)*c^2*m^2 + 2*(31*A - 63
*B)*c^2*m + 3*(7*A - 15*B)*c^2)*cos(f*x + e)^2 - 2*(8*(A - B)*c^2*m^3 + 60*
(A - B)*c^2*m^2 + 2*(63*A - 79*B)*c^2*m + (49*A - 65*B)*c^2)*cos(f*x + e))*
sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(16*f*m^4 +
128*f*m^3 + 344*f*m^2 + 352*f*m + (16*f*m^4 + 128*f*m^3 + 344*f*m^2 + 352*f
*m + 105*f)*cos(f*x + e) - (16*f*m^4 + 128*f*m^3 + 344*f*m^2 + 352*f*m + 10
5*f)*sin(f*x + e) + 105*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(5/2)*(a*sin(f*x + e)
+ a)^m, x)
```

maple [F] time = 1.34, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

maxima [B] time = 0.55, size = 725, normalized size = 2.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] -2*(((4*m^2 + 24*m + 43)*a^m*c^(5/2) - (12*m^2 + 40*m - 15)*a^m*c^(5/2)*sin
(f*x + e)/(cos(f*x + e) + 1) + 2*(4*m^2 + 8*m + 35)*a^m*c^(5/2)*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 2*(4*m^2 + 8*m + 35)*a^m*c^(5/2)*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 - (12*m^2 + 40*m - 15)*a^m*c^(5/2)*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + (4*m^2 + 24*m + 43)*a^m*c^(5/2)*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5)*A*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((8*m^3 + 36*m^2 + 46*m + 15)*(sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2)) - 2*((4*m^2 + 40*m + 115)*a^m*c^(
5/2) - 2*(4*m^3 + 40*m^2 + 115*m)*a^m*c^(5/2)*sin(f*x + e)/(cos(f*x + e) +
1) + 2*(12*m^3 + 76*m^2 + 97*m + 175)*a^m*c^(5/2)*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - (16*m^3 + 76*m^2 + 260*m - 175)*a^m*c^(5/2)*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 - (16*m^3 + 76*m^2 + 260*m - 175)*a^m*c^(5/2)*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + 2*(12*m^3 + 76*m^2 + 97*m + 175)*a^m*c^(5/2)*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 - 2*(4*m^3 + 40*m^2 + 115*m)*a^m*c^(5/2)*s
```

```
in(f*x + e)^6/(cos(f*x + e) + 1)^6 + (4*m^2 + 40*m + 115)*a^m*c^(5/2)*sin(f
*x + e)^7/(cos(f*x + e) + 1)^7)*B*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1
) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((16*m^4 + 128*m^3
 + 344*m^2 + 352*m + (16*m^4 + 128*m^3 + 344*m^2 + 352*m + 105)*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 105)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(
5/2)))/f
```

mupad [B] time = 21.06, size = 749, normalized size = 2.72

$$-\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{B\,c^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(m^3\,8{}\mathrm{i}+m^2\,36{}\mathrm{i}+m\,46{}\mathrm{i}+15{}\mathrm{i}\right)}{4\,f\,\left(16\,m^4+128\,m^3+344\,m^2+352\,m+105\right)}-\frac{c^2\,{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(2100\,A-1575\,B+1272\,A\,m-\cdots\right)}{4\,f\,\left(16\,m^4+128\,m^3+344\,m^2\cdots\right)}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(5/2),
x)
```

```
[Out] -((c - c*sin(e + f*x))^(1/2)*((B*c^2*(a + a*sin(e + f*x))^m*(m*46i + m^2*36
i + m^3*8i + 15i))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^4 + 105)) - (c^2*
exp(e*3i + f*x*3i)*(a + a*sin(e + f*x))^m*(2100*A - 1575*B + 1272*A*m - 110
*B*m + 304*A*m^2 + 32*A*m^3 - 68*B*m^2 - 8*B*m^3))/(4*f*(352*m + 344*m^2 +
128*m^3 + 16*m^4 + 105)) + (c^2*exp(e*4i + f*x*4i)*(a + a*sin(e + f*x))^m*(
A*2100i - B*1575i + A*m*1272i - B*m*110i + A*m^2*304i + A*m^3*32i - B*m^2*6
8i - B*m^3*8i))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^4 + 105)) - (c^2*exp
(e*5i + f*x*5i)*(2*m + 1)*(a + a*sin(e + f*x))^m*(350*A - 385*B + 184*A*m -
 104*B*m + 24*A*m^2 - 12*B*m^2))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^4 +
 105)) + (c^2*exp(e*2i + f*x*2i)*(2*m + 1)*(a + a*sin(e + f*x))^m*(A*350i -
 B*385i + A*m*184i - B*m*104i + A*m^2*24i - B*m^2*12i))/(4*f*(352*m + 344*m
^2 + 128*m^3 + 16*m^4 + 105)) - (B*c^2*exp(e*7i + f*x*7i)*(a + a*sin(e + f*
x))^m*(46*m + 36*m^2 + 8*m^3 + 15))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^
4 + 105)) + (c^2*exp(e*1i + f*x*1i)*(a + a*sin(e + f*x))^m*(8*m + 4*m^2 + 3
)*(14*A - 35*B + 4*A*m - 6*B*m))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^4 +
 105)) - (c^2*exp(e*6i + f*x*6i)*(a + a*sin(e + f*x))^m*(8*m + 4*m^2 + 3)*(
A*14i - B*35i + A*m*4i - B*m*6i))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^4
+ 105))))/(exp(e*4i + f*x*4i) - (exp(e*3i + f*x*3i)*(m*352i + m^2*344i + m^
3*128i + m^4*16i + 105i))/(352*m + 344*m^2 + 128*m^3 + 16*m^4 + 105))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.206 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=166

$$-\frac{2Bc^2\cos(e+fx)(a\sin(e+fx)+a)^{m+2}}{a^2f(2m+5)\sqrt{c-c\sin(e+fx)}}+\frac{4c^2(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}-\frac{2c^2(A-3B)\cos(e+fx)(}{af(2m+3)\sqrt{c-}}$$

```
[Out] 4*(A-B)*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)-
2*(A-3*B)*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(3+2*m)/(c-c*sin(f*x+e)
)^(1/2)-2*B*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(2+m)/a^2/f/(5+2*m)/(c-c*sin(f*
x+e))^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 192, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2973, 2740, 2738}

$$-\frac{8c^2(B(3-2m)-A(2m+5))\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+5)\left(4m^2+8m+3\right)\sqrt{c-c\sin(e+fx)}}-\frac{2c(B(3-2m)-A(2m+5))\cos(e+fx)\sqrt{c-c\,\mathrm{si}}}{f(2m+3)(2m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (-8*c^2*(B*(3 - 2*m) - A*(5 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f
*(5 + 2*m)*(3 + 8*m + 4*m^2)*Sqrt[c - c*Sin[e + f*x]]) - (2*c*(B*(3 - 2*m)
- A*(5 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e + f*x]]
)/(f*(3 + 2*m)*(5 + 2*m)) - (2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c
*Sin[e + f*x])^(3/2))/(f*(5 + 2*m))
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
```

```
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=-\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\,\mathrm{s}}{f(5+2m)}$$

$$=-\frac{2c(B(3-2m)-A(5+2m))\cos(e+fx)(a}{f(3+2m)(5}$$

$$=-\frac{8c^2(B(3-2m)-A(5+2m))\cos(e+fx)(}{f(1+2m)(3+2m)(5+2m)\sqrt{c-c}}$$

Mathematica [A] time = 1.77, size = 174, normalized size = 1.05

$$\frac{c\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(a(\sin(e+fx)+1))^m\left(-2(2m+1)(2Am+5A-2Bm-\right.}{f(2m+1)(2m+3)(2m+5)\left(\cos\left(\frac{1}{2}(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^m*Sqrt[c -
c*Sin[e + f*x]]*(50*A - 39*B + 40*A*m - 16*B*m + 8*A*m^2 - 4*B*m^2 + B*(3 +
 8*m + 4*m^2)*Cos[2*(e + f*x)] - 2*(1 + 2*m)*(5*A - 9*B + 2*A*m - 2*B*m)*Si
n[e + f*x]))/(f*(1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2]))
```

fricas [A] time = 0.50, size = 313, normalized size = 1.89

$$\frac{2\left(\left(4\,Bcm^2+8\,Bcm+3\,Bc\right)\cos\left(fx+e\right)^3+8\,(A+B)cm+\left(4\,Acm^2+12\,(A-B)cm+(5\,A-6\,B)c\right)\cos\left(fx+e\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 2*((4*B*c*m^2 + 8*B*c*m + 3*B*c)*cos(f*x + e)^3 + 8*(A + B)*c*m + (4*A*c*m^
2 + 12*(A - B)*c*m + (5*A - 6*B)*c)*cos(f*x + e)^2 + 4*(5*A - 3*B)*c + (4*(
A - B)*c*m^2 + 4*(5*A - 3*B)*c*m + (25*A - 21*B)*c)*cos(f*x + e) + (8*(A +
B)*c*m + (4*B*c*m^2 + 8*B*c*m + 3*B*c)*cos(f*x + e)^2 + 4*(5*A - 3*B)*c - (
4*(A - B)*c*m^2 + 4*(3*A - 5*B)*c*m + (5*A - 9*B)*c)*cos(f*x + e))*sin(f*x
+ e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(8*f*m^3 + 36*f*m^2
+ 46*f*m + (8*f*m^3 + 36*f*m^2 + 46*f*m + 15*f)*cos(f*x + e) - (8*f*m^3 + 3
6*f*m^2 + 46*f*m + 15*f)*sin(f*x + e) + 15*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e)
+ a)^m, x)
```

maple [F] time = 1.28, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

maxima [B] time = 0.53, size = 498, normalized size = 3.00

$$-\frac{2\left(\frac{\left(a^m c^{\frac{3}{2}}(2m+5)-\frac{a^m c^{\frac{3}{2}}(2m-3)\sin(fx+e)}{\cos(fx+e)+1}-\frac{a^m c^{\frac{3}{2}}(2m-3)\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^m c^{\frac{3}{2}}(2m+5)\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)Ae^{\left(2m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-m\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)\right)}}{(4m^2+8m+3)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{3}{2}}}-\frac{2(\cdots}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] -2*((a^m*c^(3/2)*(2*m + 5) - a^m*c^(3/2)*(2*m - 3)*sin(f*x + e)/(cos(f*x +
e) + 1) - a^m*c^(3/2)*(2*m - 3)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^m*c
^(3/2)*(2*m + 5)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*A*e^(2*m*log(sin(f*x
+ e)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
1))/((4*m^2 + 8*m + 3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)) - 2
*(a^m*c^(3/2)*(2*m + 9) - 2*(2*m^2 + 9*m)*a^m*c^(3/2)*sin(f*x + e)/(cos(f*x
 + e) + 1) + (4*m^2 + 15)*a^m*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 (4*m^2 + 15)*a^m*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 2*(2*m^2 +
9*m)*a^m*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^m*c^(3/2)*(2*m + 9
)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*B*e^(2*m*log(sin(f*x + e)/(cos(f*x +
 e) + 1) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((8*m^3 + 3
6*m^2 + 46*m + (8*m^3 + 36*m^2 + 46*m + 15)*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 15)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)))/f
```

mupad [B] time = 19.20, size = 480, normalized size = 2.89

$$\frac{\sqrt{c-c\sin(e+fx)}\left(\frac{c\,e^{e\,3i+f\,x\,3i}\left(a+a\sin(e+fx)\right)^m\left(45A-30B+28Am+4Bm+4Am^2\right)}{f\left(m^3\,8i+m^2\,36i+m\,46i+15i\right)}+\frac{c\,e^{e\,2i+f\,x\,2i}\left(a+a\sin(e+fx)\right)^m\left(A\,45i-B\,30i+\cdots\right)}{f\left(m^3\,8i+m^2\,36i+m\,46i+1\cdots\right)}\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(3/2),
x)
```

```
[Out] ((c - c*sin(e + f*x))^(1/2)*((c*exp(e*3i + f*x*3i)*(a + a*sin(e + f*x))^m*(
45*A - 30*B + 28*A*m + 4*B*m + 4*A*m^2))/(f*(m*46i + m^2*36i + m^3*8i + 15i
)) + (c*exp(e*2i + f*x*2i)*(a + a*sin(e + f*x))^m*(A*45i - B*30i + A*m*28i
+ B*m*4i + A*m^2*4i))/(f*(m*46i + m^2*36i + m^3*8i + 15i)) + (B*c*(a + a*si
```

```
n(e + f*x))^m*(m*8i + m^2*4i + 3i))/(2*f*(m*46i + m^2*36i + m^3*8i + 15i))
+ (B*c*exp(e*5i + f*x*5i)*(a + a*sin(e + f*x))^m*(8*m + 4*m^2 + 3))/(2*f*(m
*46i + m^2*36i + m^3*8i + 15i)) + (c*exp(e*1i + f*x*1i)*(2*m + 1)*(a + a*si
n(e + f*x))^m*(10*A - 15*B + 4*A*m - 2*B*m))/(2*f*(m*46i + m^2*36i + m^3*8i
 + 15i)) + (c*exp(e*4i + f*x*4i)*(2*m + 1)*(a + a*sin(e + f*x))^m*(A*10i -
B*15i + A*m*4i - B*m*2i))/(2*f*(m*46i + m^2*36i + m^3*8i + 15i))))/(exp(e*3
i + f*x*3i) + (exp(e*2i + f*x*2i)*(46*m + 36*m^2 + 8*m^3 + 15))/(m*46i + m^
2*36i + m^3*8i + 15i))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.207 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}$

Optimal. Leaf size=104

$$\frac{2c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}+\frac{2Bc\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+3)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 2*(A-B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)+2*
B*c*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(3+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2971, 2738}

$$\frac{2c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}+\frac{2Bc\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+3)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*(A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*S
in[e + f*x]]) + (2*B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(3 +
 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{B\int(a+a\sin(e+fx))^{1+m}\sqrt{c-c\sin(e+fx)}\,d}{a}$$
$$= \frac{2(A-B)c\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \cdots$$

Mathematica [A] time = 0.44, size = 116, normalized size = 1.12

$$\frac{2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(a(\sin(e+fx)+1))^m(A(2m+3)+B(2m+1)\sin(e+f}{f(2m+1)(2m+3)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^m*Sqrt[c -
c*Sin[e + f*x]]*(-2*B + A*(3 + 2*m) + B*(1 + 2*m)*Sin[e + f*x]))/(f*(1 + 2*
m)*(3 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

fricas [A] time = 0.47, size = 165, normalized size = 1.59

$$-\frac{2\left((2\,Bm+B)\cos\left(fx+e\right)^2-2\,(A+B)m-(2\,Am+3\,A-2\,B)\cos\left(fx+e\right)-\left(2\,(A+B)m+(2\,Bm+B)\cos\left(\right.\right.\right.}{4\,fm^2+8\,fm+\left(4\,fm^2+8\,fm+3\,f\right)\cos\left(fx+e\right)-\left(4\,fm\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -2*((2*B*m + B)*cos(f*x + e)^2 - 2*(A + B)*m - (2*A*m + 3*A - 2*B)*cos(f*x
+ e) - (2*(A + B)*m + (2*B*m + B)*cos(f*x + e) + 3*A - B)*sin(f*x + e) - 3*
A + B)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(4*f*m^2 + 8*f*m +
(4*f*m^2 + 8*f*m + 3*f)*cos(f*x + e) - (4*f*m^2 + 8*f*m + 3*f)*sin(f*x + e)
 + 3*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m, x)
```

maple [F] time = 1.28, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\sqrt{c-c\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

maxima [B] time = 0.54, size = 323, normalized size = 3.11

$$-\frac{2\left(\frac{2\left(\frac{2a^{m}\sqrt{c}m\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{2a^{m}\sqrt{c}m\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-a^{m}\sqrt{c}-\frac{a^{m}\sqrt{c}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}\right)Be^{\left(2m\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)-m\log\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)\right)}}{\left(4m^{2}+8m+\frac{\left(4m^{2}+8m+3\right)\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+3\right)\sqrt{\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1}}+\frac{\left(a^{m}\sqrt{c}+\frac{a^{m}\sqrt{c}\sin\left(fx\right.}{\cos\left(fx+e\right)+}\right.}{}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] -2*(2*(2*a^m*sqrt(c)*m*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a^m*sqrt(c)*m*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 - a^m*sqrt(c) - a^m*sqrt(c)*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3)*B*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) -
 m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((4*m^2 + 8*m + (4*m^2 + 8
*m + 3)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3)*sqrt(sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 1)) + (a^m*sqrt(c) + a^m*sqrt(c)*sin(f*x + e)/(cos(f*x + e
) + 1))*A*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 1))/((2*m + 1)*sqrt(sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 1)))/f
```

mupad [B] time = 1.44, size = 105, normalized size = 1.01

$$-\frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\left(6A\cos\left(e+fx\right)-4B\cos\left(e+fx\right)+B\sin\left(2e+2fx\right)+\right.}{f\left(\sin\left(e+fx\right)-1\right)\left(4m^{2}+8m+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1/2),
x)
```

```
[Out] -((a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^(1/2)*(6*A*cos(e + f*x)
- 4*B*cos(e + f*x) + B*sin(2*e + 2*f*x) + 4*A*m*cos(e + f*x) + 2*B*m*sin(2*
e + 2*f*x)))/(f*(sin(e + f*x) - 1)*(8*m + 4*m^2 + 3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(e
 + f*x)), x)
```

3.208 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=118

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sin(e+fx)}}-\frac{2B\cos(e+fx)(a\sin(e+fx)}{f(2m+1)\sqrt{c-c\sin(e+}}$$

```
[Out] -2*B*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)+(A+B)*c
os(f*x+e)*hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(a+a*sin(f*x+e))
^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2973, 2745, 2667, 68}

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sin(e+fx)}}-\frac{2B\cos(e+fx)(a\sin(e+fx)}{f(2m+1)\sqrt{c-c\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] (-2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e +
f*x]]) + ((A + B)*Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 +
Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*
x]])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + (A+B)\int \frac{(a+a\sin(e}{\sqrt{c-c\sin(e}}$$

$$= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{((A+B)\cos(e+fx))\int}{\sqrt{a+a\sin(e}}$$

$$= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{(a(A+B)\cos(e+fx))\,\mathrm{S}}{f\sqrt{a+a\sin(}}$$

$$= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\cos(e+fx)\,{}_2F_1}{}$$

Mathematica [A] time = 2.23, size = 200, normalized size = 1.69

$$\frac{2^{-2m-\frac{3}{2}}\sin\left(\frac{1}{4}(2e+2fx+\pi)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(\sin(e+fx)+1))^m\left(2^{2m+1}(A+B)\,{}_2F_1\left(1,2m\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e +
f*x]],x]
```

```
[Out] (2^(-3/2 - 2*m)*(-(2^(3 + 2*m)*B) + 2^(1 + 2*m)*(A + B)*Hypergeometric2F1[1
, 1 + 2*m, 2*(1 + m), Sin[(2*e + Pi + 2*f*x)/4]] + (A + B)*Hypergeometric2F
1[1 + 2*m, 1 + 2*m, 2*(1 + m), (1 - Tan[(2*e - Pi + 2*f*x)/8]^2)/2]*(Sec[(2
*e - Pi + 2*f*x)/8]^2)^(1 + 2*m))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*
(1 + Sin[e + f*x]))^m*Sin[(2*e + Pi + 2*f*x)/4])/((f + 2*f*m)*Sqrt[c - c*Si
n[e + f*x]])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}}{c\sin\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m/(c*sin(f*x + e) - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e)
+ c), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)}{\sqrt{c-c\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e)
+ c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/2
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(e+fx)+1))^m\,(A+B\sin(e+fx))}{\sqrt{-c(\sin(e+fx)-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(-c*(sin(e + f*
x) - 1)), x)
```

3.209 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{(A(1-2m)-B(2m+3))\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{4cf(2m+1)\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos}{2f}$$

```
[Out] 1/2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(c-c*sin(f*x+e))^(3/2)+1/4*(A*(1-
2*m)-B*(3+2*m))*cos(f*x+e)*hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))
*(a+a*sin(f*x+e))^m/c/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2972, 2745, 2667, 68}

$$\frac{(A(1-2m)-B(2m+3))\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{4cf(2m+1)\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2
),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(2*f*(c - c*Sin[e + f*x])^(3/
2)) + ((A*(1 - 2*m) - B*(3 + 2*m))*Cos[e + f*x]*Hypergeometric2F1[1, 1/2 +
m, 3/2 + m, (1 + Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^m)/(4*c*f*(1 + 2*m)*
Sqrt[c - c*Sin[e + f*x]])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{2f(c-c\sin(e+fx))^{3/2}} + \frac{\left(Bc\left(-\frac{3}{2}-m\right)-Ac\left(\right.\right.}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{2f(c-c\sin(e+fx))^{3/2}} + \frac{\left(\left(Bc\left(-\frac{3}{2}-m\right)-Ac\right.\right.}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{2f(c-c\sin(e+fx))^{3/2}} + \frac{\left(a\left(Bc\left(-\frac{3}{2}-m\right)-A\right.\right.}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{2f(c-c\sin(e+fx))^{3/2}} + \frac{(A(1-2m)-B(3+}{}$$

Mathematica [C] time = 6.76, size = 3178, normalized size = 23.72

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (2^(-3/2 - 2*m)*B*(-(4^m*Hypergeometric2F1[1, 2*m, 1 + 2*m, Cos[(-e + Pi/2
- f*x)/2]]) + Hypergeometric2F1[2*m, 2*m, 1 + 2*m, (1 - Tan[(-e + Pi/2 - f*
x)/4]^2)/2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m))*(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2])^3*(a + a*Sin[e + f*x])^m)/(f*m*(c - c*Sin[e + f*x])^(3/2)) - ((A
 + B)*(Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^3*(a + a*Sin[e + f*x])^m*(AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*
x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Ta
n[(-e + Pi/2 - f*x)/4]^2 - (AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)
/4]^2, -Cot[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^2*(Csc[(-e + P
i/2 - f*x)/4]^2)^(2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e
 + Pi/2 - f*x)/4]^2)^(2*m) + (2^(1 - 2*m)*AppellF1[1 + 2*m, 2*m, 1, 2 + 2*m
, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2]*(-1 +
 Tan[(-e + Pi/2 - f*x)/4]^2)*(1 - Tan[(-e + Pi/2 - f*x)/4]^4)^(2*m))/(1 + 2
*m)))/(8*Sqrt[2]*f*(c - c*Sin[e + f*x])^(3/2)*(Cos[Pi/4 + (e - Pi/2 + f*x)/
2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^3*(-1/8*(m*Cos[(-e + Pi/2 - f*x)/4]*(C
os[(-e + Pi/2 - f*x)/4]^2)^(-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/4]*(AppellF1[1,
 -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Se
c[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2 - f*x)/4]^2 - (AppellF1[1, -
2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f*x)/4]^2]*Cot[(
-e + Pi/2 - f*x)/4]^2*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(1 - Tan[(-e + Pi/
2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(2*m) + (2^(1 - 2*m)
*AppellF1[1 + 2*m, 2*m, 1, 2 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 -
 Tan[(-e + Pi/2 - f*x)/4]^2]*(-1 + Tan[(-e + Pi/2 - f*x)/4]^2)*(1 - Tan[(-e
 + Pi/2 - f*x)/4]^4)^(2*m))/(1 + 2*m)))/Sqrt[2] + ((Cos[(-e + Pi/2 - f*x)/4
]^2)^(2*m)*((AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
 + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)*Tan[(-e + Pi/2
- f*x)/4])/2 + m*AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan
[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2
- f*x)/4]^3 + (Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2 - f*x)/4]^2
*(-1/2*(m*AppellF1[2, 1 - 2*m, 2*m, 3, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
 + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]) -
 (m*AppellF1[2, -2*m, 1 + 2*m, 3, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi
/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/2) + (
m*AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f
*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^3*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(1
- Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(2*m)
 + m*AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2
- f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^3*(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(-
1 - 2*m)*(Csc[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)*(1 - Tan[(-e + Pi/2 - f*x)/
4]^2)^(2*m) + (AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(
-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]*(Csc[(-e + Pi/2 - f*x)/4]^2
)^(1 + 2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(2*(1 - Cot[(-e + Pi/2
```

```
- f*x)/4]^2)^(2*m)) - (Cot[(-e + Pi/2 - f*x)/4]^2*(Csc[(-e + Pi/2 - f*x)/4]
^2)^(2*m)*((m*AppellF1[2, 1 - 2*m, 2*m, 3, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot
[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]*Csc[(-e + Pi/2 - f*x)/4]^
2)/2 + (m*AppellF1[2, -2*m, 1 + 2*m, 3, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-
e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]*Csc[(-e + Pi/2 - f*x)/4]^2)/
2)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e + Pi/2 - f*x)/4]^2)
^(2*m) + (m*AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e
+ Pi/2 - f*x)/4]^2]*Csc[(-e + Pi/2 - f*x)/4]*(Csc[(-e + Pi/2 - f*x)/4]^2)^(
2*m)*Sec[(-e + Pi/2 - f*x)/4]*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(-1 + 2*m))/
(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(2*m) + (AppellF1[1 + 2*m, 2*m, 1, 2 + 2*m
, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(
-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]*(1 - Tan[(-e + Pi/2 - f*x)/4
]^4)^(2*m))/(2^(2*m)*(1 + 2*m)) + (2^(1 - 2*m)*(-1/2*((1 + 2*m)*AppellF1[2
+ 2*m, 2*m, 2, 3 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + P
i/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(2 +
2*m) - (m*(1 + 2*m)*AppellF1[2 + 2*m, 1 + 2*m, 1, 3 + 2*m, (1 - Tan[(-e + P
i/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4
]^2*Tan[(-e + Pi/2 - f*x)/4])/(2*(2 + 2*m)))*(-1 + Tan[(-e + Pi/2 - f*x)/4]
^2)*(1 - Tan[(-e + Pi/2 - f*x)/4]^4)^(2*m))/(1 + 2*m) - (2^(2 - 2*m)*m*Appe
llF1[1 + 2*m, 2*m, 1, 2 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[
(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]
^3*(-1 + Tan[(-e + Pi/2 - f*x)/4]^2)*(1 - Tan[(-e + Pi/2 - f*x)/4]^4)^(-1 +
 2*m))/(1 + 2*m)))/(8*Sqrt[2])))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}}{c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\sin\left(fx+e\right)-2\,c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)
^(3/2), x)
```

maple [F] time = 1.01, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)}{\left(c-c\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)
^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{m}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(3/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(3/2
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(A+B\sin\left(e+fx\right)\right)}{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/(-c*(sin(e + f*x) -
 1))**(3/2), x)
```

3.210 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=134

$$\frac{(A(3-2m)-B(2m+5))\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(2,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{16c^2f(2m+1)\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos}{4f}$$

```
[Out] 1/4*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(c-c*sin(f*x+e))^(5/2)+1/16*(A*(3
-2*m)-B*(5+2*m))*cos(f*x+e)*hypergeom([2, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e)
)*(a+a*sin(f*x+e))^m/c^2/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2972, 2745, 2667, 68}

$$\frac{(A(3-2m)-B(2m+5))\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(2,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{16c^2f(2m+1)\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/2
),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(4*f*(c - c*Sin[e + f*x])^(5/
2)) + ((A*(3 - 2*m) - B*(5 + 2*m))*Cos[e + f*x]*Hypergeometric2F1[2, 1/2 +
m, 3/2 + m, (1 + Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^m)/(16*c^2*f*(1 + 2*
m)*Sqrt[c - c*Sin[e + f*x]])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{4f(c-c\sin(e+fx))^{5/2}} + \frac{\left(Bc\left(-\frac{5}{2}-m\right)-Ac\left(\right.\right.}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{4f(c-c\sin(e+fx))^{5/2}} + \frac{\left(\left(Bc\left(-\frac{5}{2}-m\right)-Ac\right.\right.}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{4f(c-c\sin(e+fx))^{5/2}} + \frac{\left(a^2\left(Bc\left(-\frac{5}{2}-m\right)-\right.\right.}{}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{4f(c-c\sin(e+fx))^{5/2}} + \frac{(A(3-2m)-B(5+}{}$$

Mathematica [C] time = 6.87, size = 8147, normalized size = 60.80

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}}{3\,c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}-\left(c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x +
 e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)
^(5/2), x)
```

maple [F] time = 1.04, size = 0, normalized size = 0.00

$$\int\frac{\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)}{\left(c-c\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)
^(5/2), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(5/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(5/2
), x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.211 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-m}\,dx$

Optimal. Leaf size=267

$$\frac{2(3A-2B(m+2))\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{c^3f(2m+5)(2m+7)\left(4m^2+8m+3\right)}+\frac{2(3A-2B(m+2))\cos(e+fx)(a\sin}{c^2f(2m+7)\left(}$$

```
[Out] (A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-4-m)/f/(7+2*m)+(3*A-
2*B*(2+m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m)/c/f/(4*m^2
+24*m+35)+2*(3*A-2*B*(2+m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^
(-2-m)/c^2/f/(8*m^3+60*m^2+142*m+105)+2*(3*A-2*B*(2+m))*cos(f*x+e)*(a+a*sin
(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/c^3/f/(16*m^4+128*m^3+344*m^2+352*m+105)
```

Rubi [A] time = 0.43, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2972, 2743, 2742}

$$\frac{2(3A-2B(m+2))\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{c^2f(2m+7)\left(4m^2+16m+15\right)}+\frac{2(3A-2B(m+2))\cos(e+fx)(a\sin}{c^3f(2m+5)(2m}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-4 -
m),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-4 - m))
/(f*(7 + 2*m)) + ((3*A - 2*B*(2 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(
c - c*Sin[e + f*x])^(-3 - m))/(c*f*(5 + 2*m)*(7 + 2*m)) + (2*(3*A - 2*B*(2
+ m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(c
^2*f*(7 + 2*m)*(15 + 16*m + 4*m^2)) + (2*(3*A - 2*B*(2 + m))*Cos[e + f*x]*(
a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(c^3*f*(5 + 2*m)*(7 +
2*m)*(3 + 8*m + 4*m^2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-m}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c}{f(7+2m)}$$
$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c}{f(7+2m)}$$
$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c}{f(7+2m)}$$
$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c}{f(7+2m)}$$

Mathematica [A] time = 14.13, size = 353, normalized size = 1.32

$$-\frac{2^{-m-18}\cos\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)\csc^{21}\left(\frac{1}{8}\left(-e-fx+\frac{\pi}{2}\right)\right)\sec^{7}\left(\frac{1}{8}\left(-e-fx+\frac{\pi}{2}\right)\right)\sin^{-2m}\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)(a\sin(e}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(-4 - m),x]
```

```
[Out] -((2^(-18 - m)*Cos[(-e + Pi/2 - f*x)/2]*Csc[(-e + Pi/2 - f*x)/8]^21*Sec[(-e
 + Pi/2 - f*x)/8]^7*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-4 - m)*(-
96*A + 58*B - 176*A*m + 64*B*m - 96*A*m^2 + 16*B*m^2 - 16*A*m^3 + 4*(2 + m)
*(-3*A + 2*B*(2 + m))*Cos[2*(-e + Pi/2 - f*x)] + 3*A*Cos[3*(-e + Pi/2 - f*x
)] - 4*B*Cos[3*(-e + Pi/2 - f*x)] - 2*B*m*Cos[3*(-e + Pi/2 - f*x)] + (29 +
32*m + 8*m^2)*(3*A - 2*B*(2 + m))*Sin[e + f*x]))/(f*(1 + 2*m)*(3 + 2*m)*(5
+ 2*m)*(7 + 2*m)*(-1 + Cot[(-e + Pi/2 - f*x)/8]^2)^7*Sin[(-e + Pi/2 - f*x)/
2]^(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-4 - m))))
```

fricas [A] time = 0.49, size = 205, normalized size = 0.77

$$\frac{\left(4\left(2\,Bm^2-(3\,A-8\,B)m-6\,A+8\,B\right)\cos\left(fx+e\right)^3+\left(8\,Am^3+12\,(4\,A-B)m^2+2\,(47\,A-24\,B)m+60\,A-4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x, al
gorithm="fricas")
```

```
[Out] (4*(2*B*m^2 - (3*A - 8*B)*m - 6*A + 8*B)*cos(f*x + e)^3 + (8*A*m^3 + 12*(4*
A - B)*m^2 + 2*(47*A - 24*B)*m + 60*A - 45*B)*cos(f*x + e) - (2*(2*B*m - 3*
A + 4*B)*cos(f*x + e)^3 - (8*B*m^3 - 12*(A - 4*B)*m^2 - 2*(24*A - 47*B)*m -
 45*A + 60*B)*cos(f*x + e))*sin(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*
x + e) + c)^(-m - 4)/(16*f*m^4 + 128*f*m^3 + 344*f*m^2 + 352*f*m + 105*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x, al
gorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 4), x)
```

maple [F] time = 5.94, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{-4-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x, al
gorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 4), x)
```

mupad [B] time = 22.23, size = 368, normalized size = 1.38

$$-\frac{\sin\left(4\,e+4\,f\,x\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(4\,B-3\,A+2\,B\,m\right)\,1{}\mathrm{i}}{4\,f\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{m+4}\,\left(m^4\,16{}\mathrm{i}+m^3\,128{}\mathrm{i}+m^2\,344{}\mathrm{i}+m\,352{}\mathrm{i}+105{}\mathrm{i}\right)}+\frac{\cos\left(e+f\,x\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^n}{4\,f\,\left(c-c\,\sin\left(e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m +
 4),x)
```

```
[Out] (cos(e + f*x)*(a + a*sin(e + f*x))^m*(A*168i - B*84i + A*m*340i - B*m*96i +
 A*m^2*192i + A*m^3*32i - B*m^2*24i))/(4*f*(c - c*sin(e + f*x))^(m + 4)*(m*
352i + m^2*344i + m^3*128i + m^4*16i + 105i)) - (sin(4*e + 4*f*x)*(a + a*si
n(e + f*x))^m*(4*B - 3*A + 2*B*m)*1i)/(4*f*(c - c*sin(e + f*x))^(m + 4)*(m*
352i + m^2*344i + m^3*128i + m^4*16i + 105i)) + (sin(2*e + 2*f*x)*(a + a*si
n(e + f*x))^m*(8*m + 2*m^2 + 7)*(4*B - 3*A + 2*B*m)*1i)/(f*(c - c*sin(e + f
*x))^(m + 4)*(m*352i + m^2*344i + m^3*128i + m^4*16i + 105i)) + (cos(3*e +
3*f*x)*(m + 2)*(a + a*sin(e + f*x))^m*(B*4i - A*3i + B*m*2i))/(f*(c - c*sin
(e + f*x))^(m + 4)*(m*352i + m^2*344i + m^3*128i + m^4*16i + 105i))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-4-m),x)
```

```
[Out] Timed out
```

3.212 $\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-3-m}\, dx$

Optimal. Leaf size=191

$$\frac{(2A - B(2m + 3))\cos(e + fx)(a\sin(e + fx) + a)^m (c - c\sin(e + fx))^{-m-1}}{c^2 f(2m + 5)\left(4m^2 + 8m + 3\right)} + \frac{(A + B)\cos(e + fx)(a\sin(e + fx) + \ldots}{f(2m + 5)\ldots}$$

```
[Out] (A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m)/f/(5+2*m)+(2*A-
B*(3+2*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m)/c/f/(4*m^2
+16*m+15)+(2*A-B*(3+2*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-
1-m)/c^2/f/(8*m^3+36*m^2+46*m+15)
```

Rubi [A] time = 0.31, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2972, 2743, 2742}

$$\frac{(2A - B(2m + 3))\cos(e + fx)(a\sin(e + fx) + a)^m (c - c\sin(e + fx))^{-m-1}}{c^2 f(2m + 5)\left(4m^2 + 8m + 3\right)} + \frac{(A + B)\cos(e + fx)(a\sin(e + fx) + \ldots}{f(2m + 5)\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-3 -
m),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 - m))
/(f*(5 + 2*m)) + ((2*A - B*(3 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(
c - c*Sin[e + f*x])^(-2 - m))/(c*f*(3 + 2*m)*(5 + 2*m)) + ((2*A - B*(3 + 2*
m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(c^2
*f*(5 + 2*m)*(3 + 8*m + 4*m^2))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
```

```
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-m}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c}{f(5+2m)}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c}{f(5+2m)}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c}{f(5+2m)}$$

Mathematica [A] time = 10.73, size = 269, normalized size = 1.41

$$\frac{2^{-m-13}\cos\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)\csc^{15}\left(\frac{1}{8}\left(-e-fx+\frac{\pi}{2}\right)\right)\sec^{5}\left(\frac{1}{8}\left(-e-fx+\frac{\pi}{2}\right)\right)\sin^{-2m}\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)(a\sin(e+}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(-3 - m),x]
```

```
[Out] (2^(-13 - m)*Cos[(-e + Pi/2 - f*x)/2]*Csc[(-e + Pi/2 - f*x)/8]^15*Sec[(-e +
 Pi/2 - f*x)/8]^5*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 - m)*(16*
A - 9*B + 24*A*m - 6*B*m + 8*A*m^2 + (2*A - 3*B - 2*B*m)*Cos[2*(-e + Pi/2 -
 f*x)] + 2*(3 + 2*m)*(-2*A + B*(3 + 2*m))*Sin[e + f*x]))/(f*(1 + 2*m)*(3 +
```

```
2*m)*(5 + 2*m)*(-1 + Cot[(-e + Pi/2 - f*x)/8]^2)^5*Sin[(-e + Pi/2 - f*x)/2]
^(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-3 - m)))
```

fricas [A] time = 0.48, size = 137, normalized size = 0.72

$$\frac{\left((2\,Bm-2\,A+3\,B)\cos\left(fx+e\right)^{3}+\left(4\,Bm^{2}-4\,(A-3\,B)m-6\,A+9\,B\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(4\,Am^{2}+4\right.\right.}{8\,fm^{3}+36\,fm^{2}+46\,fm+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x, al
gorithm="fricas")
```

```
[Out] ((2*B*m - 2*A + 3*B)*cos(f*x + e)^3 + (4*B*m^2 - 4*(A - 3*B)*m - 6*A + 9*B)
*cos(f*x + e)*sin(f*x + e) + (4*A*m^2 + 4*(3*A - B)*m + 9*A - 6*B)*cos(f*x
+ e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 3)/(8*f*m^3 + 36*f
*m^2 + 46*f*m + 15*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x, al
gorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 3), x)
```

maple [F] time = 6.18, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{-3-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x, al
gorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 3), x)
```

mupad [B] time = 15.48, size = 239, normalized size = 1.25

$$-\frac{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)^m\,\left(30\,A\,\cos\left(e+f\,x\right)-15\,B\,\cos\left(e+f\,x\right)-2\,A\,\cos\left(3\,e+3\,f\,x\right)+3\,B\,\cos\left(3\,e+3\,f\,x\right)\right.}{c^3\,f\,\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m +
 3),x)
```

```
[Out] -((a*(sin(e + f*x) + 1))^m*(30*A*cos(e + f*x) - 15*B*cos(e + f*x) - 2*A*cos
(3*e + 3*f*x) + 3*B*cos(3*e + 3*f*x) - 12*A*sin(2*e + 2*f*x) + 18*B*sin(2*e
 + 2*f*x) + 8*B*m^2*sin(2*e + 2*f*x) + 48*A*m*cos(e + f*x) - 10*B*m*cos(e +
 f*x) + 16*A*m^2*cos(e + f*x) + 2*B*m*cos(3*e + 3*f*x) - 8*A*m*sin(2*e + 2*
f*x) + 24*B*m*sin(2*e + 2*f*x)))/(c^3*f*(-c*(sin(e + f*x) - 1))^m*(46*m + 3
6*m^2 + 8*m^3 + 15)*(15*sin(e + f*x) + 6*cos(2*e + 2*f*x) - sin(3*e + 3*f*x
) - 10))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-3-m),x)
```

```
[Out] Timed out
```

3.213 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx$

Optimal. Leaf size=114

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}+\frac{(A-2B(m+1))\cos(e+fx)(a\sin(e+fx)+a}{cf(2m+1)(2m+3)}$$

```
[Out] (A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m)/f/(3+2*m)+(A-2*
B*(1+m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/c/f/(4*m^2+8
*m+3)
```

Rubi [**A**] time = 0.22, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2972, 2742}

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}+\frac{(A-2B(m+1))\cos(e+fx)(a\sin(e+fx)+a}{cf(2m+1)(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-2 -
m),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))
/(f*(3 + 2*m)) + ((A - 2*B*(1 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c
- c*Sin[e + f*x])^(-1 - m))/(c*f*(1 + 2*m)*(3 + 2*m))
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
```

```
+ 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c}{f(3+2m)}$$
$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c}{f(3+2m)}$$

Mathematica [A] time = 8.79, size = 211, normalized size = 1.85

$$-\frac{2^{-m-7}\cos\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)\csc^9\left(\frac{1}{8}\left(-e-fx+\frac{\pi}{2}\right)\right)\sec^3\left(\frac{1}{8}\left(-e-fx+\frac{\pi}{2}\right)\right)\sin^{-2m}\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)(a\sin(e+}{f\left(4m^2+8m+3\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(-2 - m),x]
```

```
[Out] -((2^(-7 - m)*Cos[(-e + Pi/2 - f*x)/2]*Csc[(-e + Pi/2 - f*x)/8]^9*Sec[(-e +
 Pi/2 - f*x)/8]^3*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m)*(B -
 2*A*(1 + m) + (A - 2*B*(1 + m))*Sin[e + f*x]))/(f*(3 + 8*m + 4*m^2)*(-1 +
Cot[(-e + Pi/2 - f*x)/8]^2)^3*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^(2*(-2 - m))))
```

fricas [A] time = 0.48, size = 89, normalized size = 0.78

$$\frac{\left((2\,Bm-A+2\,B)\cos\left(fx+e\right)\sin\left(fx+e\right)+(2\,Am+2\,A-B)\cos\left(fx+e\right)\right)\left(a\sin\left(fx+e\right)+a\right)^m\left(-c\sin\left(fx+\right.\right.}{4\,fm^2+8\,fm+3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x, al
gorithm="fricas")
```

```
[Out] ((2*B*m - A + 2*B)*cos(f*x + e)*sin(f*x + e) + (2*A*m + 2*A - B)*cos(f*x +
e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 2)/(4*f*m^2 + 8*f*m
+ 3*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x, al
gorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 2), x)
```

maple [F] time = 5.41, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{-2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x, al
gorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 2), x)
```

mupad [B] time = 14.16, size = 134, normalized size = 1.18

$$-\frac{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)^m\,\left(4\,A\,\cos\left(e+f\,x\right)-2\,B\,\cos\left(e+f\,x\right)-A\,\sin\left(2\,e+2\,f\,x\right)+2\,B\,\sin\left(2\,e+2\,f\,x\right)+4}{c^2\,f\,\left(-c\,\left(\sin\left(e+f\,x\right)-1\right)\right)^m\,\left(4\,m^2+8\,m+3\right)\,\left(4\,\sin\left(e+f\,x\right)+\cos\left(2\,e+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m +
 2),x)
```

```
[Out] -((a*(sin(e + f*x) + 1))^m*(4*A*cos(e + f*x) - 2*B*cos(e + f*x) - A*sin(2*e
 + 2*f*x) + 2*B*sin(2*e + 2*f*x) + 4*A*m*cos(e + f*x) + 2*B*m*sin(2*e + 2*f
*x)))/(c^2*f*(-c*(sin(e + f*x) - 1))^m*(8*m + 4*m^2 + 3)*(4*sin(e + f*x) +
cos(2*e + 2*f*x) - 3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-2-m),x)
```

```
[Out] Timed out
```

3.214 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=163

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{f(2m+1)} - \frac{B2^{\frac{1}{2}-m}\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e}{}$$

```
[Out] (A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/f/(1+2*m)-2^(1/
2-m)*B*cos(f*x+e)*hypergeom([1/2+m, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(1-s
in(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/f/(1+2*m)
```

Rubi [A] time = 0.31, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2972, 2745, 2689, 70, 69}

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{f(2m+1)} - \frac{B2^{\frac{1}{2}-m}\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-1 -
m),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))
/(f*(1 + 2*m)) - (2^(1/2 - m)*B*Cos[e + f*x]*Hypergeometric2F1[(1 + 2*m)/2,
 (1 + 2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 +
m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-m}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(}{f(1+2m)}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(}{f(1+2m)}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(}{f(1+2m)}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(}{f(1+2m)}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(}{f(1+2m)}$$

Mathematica [C] time = 11.62, size = 675, normalized size = 4.14

$$-\frac{2^{-m}(2m-3)\cos^2\left(\frac{1}{4}\left(-e-fx+\frac{\pi}{2}\right)\right)\cot\left(\frac{1}{4}\left(-e-fx+\frac{\pi}{2}\right)\right)\sin^{-2m}\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)(a\sin(e+fx)+a)^m(A+}{f\left(4m^2-1\right)\left((2m-3)\left((A+B)\left(\cos\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)+1\right)\left(1-\tan^2\left(\frac{1}{4}\left(-e-fx+\frac{\pi}{2}\right)\right.\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(-1 - m),x]
```

```
[Out] -(((-3 + 2*m)*Cos[(-e + Pi/2 - f*x)/4]^2*Cot[(-e + Pi/2 - f*x)/4]*(a + a*Si
n[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-1 - m)*(8*B*(1 +
2*m)*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(
-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2 - (A + B)*((-1 + 2*m)*Hyp
ergeometric2F1[-1/2 - m, -2*m, 1/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2] + (1 +
2*m)*Hypergeometric2F1[1/2 - m, -2*m, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2]*
Tan[(-e + Pi/2 - f*x)/4]^2)))/(2^m*f*(-1 + 4*m^2)*Sin[(-e + Pi/2 - f*x)/2]^
(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-1 - m))*(-64*B*m*AppellF1[
3/2 - m, 1 - 2*m, 1, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 -
 f*x)/4]^2]*Sin[(-e + Pi/2 - f*x)/4]^4 - 32*B*AppellF1[3/2 - m, -2*m, 2, 5/
2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-e + P
i/2 - f*x)/4]^4 + (-3 + 2*m)*(-4*B*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[
(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-e + Pi/2 - f*x)/
```

```
2]^2 + (A + B)*(1 + Cos[(-e + Pi/2 - f*x)/2])*(1 - Tan[(-e + Pi/2 - f*x)/4]
^2)^(2*m)))))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x, al
gorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
(-m - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x, al
gorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 1), x)
```

maple [F] time = 2.87, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{-1-m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x, al
gorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{m+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m +
 1),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m +
 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-1-m),x)
```

```
[Out] Timed out
```

3.215 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx$

Optimal. Leaf size=158

$$\frac{c2^{\frac{1}{2}-m}(A+2Bm)\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m+1),\frac{1}{2}(2\right.}{f(2m+1)}$$

```
[Out] 2^(1/2-m)*c*(2*B*m+A)*cos(f*x+e)*hypergeom([1/2+m, 1/2+m],[3/2+m],1/2+1/2*s
in(f*x+e))*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m
)/f/(1+2*m)-B*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/((c-c*sin(f*x+e))^m)
```

Rubi [A] time = 0.27, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2973, 2745, 2689, 70, 69}

$$\frac{c2^{\frac{1}{2}-m}(A+2Bm)\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m+1),\frac{1}{2}(2\right.}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^m,x]
```

```
[Out] (2^(1/2 - m)*c*(A + 2*B*m)*Cos[e + f*x]*Hypergeometric2F1[(1 + 2*m)/2, (1 +
 2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a
 + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m)) - (B*Cos[
e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(c - c*Sin[e + f*x])^m)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-m}\,dx = -\frac{B\cos(e + fx)(a + a\sin(e + fx))^m(c - c\sin}{f}$$

$$= -\frac{B\cos(e + fx)(a + a\sin(e + fx))^m(c - c\sin}{f}$$

$$= -\frac{B\cos(e + fx)(a + a\sin(e + fx))^m(c - c\sin}{f}$$

$$= -\frac{B\cos(e + fx)(a + a\sin(e + fx))^m(c - c\sin}{f}$$

$$= \frac{2^{\frac{1}{2}-m}c(A + 2Bm)\cos(e + fx)\,{}_2F_1\left(\frac{1}{2}(1 + 2m\right.}{}$$

Mathematica [C] time = 17.20, size = 2552, normalized size = 16.15

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^m,x]
```

```
[Out] (2^(2 - m)*((A + B)*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-e + Pi/2 - f*
x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*B*(-AppellF1[1/2 - m, -2*m, 2, 3/
2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[
1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*
x)/4]^2]))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*m)*(a + a*Sin[e + f*x])
^m*((A*Cos[(-e + Pi/2 - f*x)/2]^(2*m))/(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Si
n[Pi/4 + (e - Pi/2 + f*x)/2])^(2*m) + (B*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Sin
[e + f*x])/(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2]
)^(2*m))*Tan[(-e + Pi/2 - f*x)/4])/(f*(-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(
2*m)*(c - c*Sin[e + f*x])^m*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(-((2^(2
 - m)*m*((A + B)*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-e + Pi/2 - f*x)/
4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*B*(-AppellF1[1/2 - m, -2*m, 2, 3/2 -
 m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[1/2
 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/
4]^2]))*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e +
 Pi/2 - f*x)/4]^2*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(-1 - 2*m))/((-1 + 2*m)*
Sin[(-e + Pi/2 - f*x)/2]^(2*m))) - (((A + B)*AppellF1[1/2 - m, -2*m, 1, 3/2
 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*B*(-Appe
```

```
llF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2
 - f*x)/4]^2] + AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4
]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Sec[(-e
+ Pi/2 - f*x)/4]^2)/(2^m*(-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(1 - Tan
[(-e + Pi/2 - f*x)/4]^2)^(2*m)) + (2^(2 - m)*m*((A + B)*AppellF1[1/2 - m, -
2*m, 1, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] +
 8*B*(-AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan
[(-e + Pi/2 - f*x)/4]^2] + AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi
/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*Cos[(-e + Pi/2 - f*x)/2]^(1
+ 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(-1 - 2*m)*Tan[(-e + Pi/2 - f*x)/4])/((-1 +
 2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) + (2^(2 - m)*m*((A + B)*Appel
lF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2
- f*x)/4]^2] + 8*B*(-AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-e + Pi/2 - f
*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[1/2 - m, -2*m, 3, 3/2 - m
, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*Cos[(-e + Pi/2
 - f*x)/2]^(-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(1 - 2*m)*Tan[(-e + Pi/2 - f
*x)/4])/((-1 + 2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) - (2^(2 - m)*Co
s[(-e + Pi/2 - f*x)/2]^(2*m)*Tan[(-e + Pi/2 - f*x)/4]*((A + B)*(-(((1/2 - m
)*m*AppellF1[3/2 - m, 1 - 2*m, 1, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan
[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4
])/(3/2 - m)) - ((1/2 - m)*AppellF1[3/2 - m, -2*m, 2, 5/2 - m, Tan[(-e + Pi
/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan
[(-e + Pi/2 - f*x)/4])/(2*(3/2 - m))) + 8*B*(((1/2 - m)*m*AppellF1[3/2 - m,
 1 - 2*m, 2, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]
^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 - m) - ((1/2
- m)*m*AppellF1[3/2 - m, 1 - 2*m, 3, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -
Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x
)/4])/(3/2 - m) + ((1/2 - m)*AppellF1[3/2 - m, -2*m, 3, 5/2 - m, Tan[(-e +
Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*T
an[(-e + Pi/2 - f*x)/4])/(3/2 - m) - (3*(1/2 - m)*AppellF1[3/2 - m, -2*m, 4
, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e
 + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(2*(3/2 - m)))))/((-1 + 2*m)*
Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{m}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x, algor
ithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^
```

```
m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x, algor
ithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)
^m, x)
```

maple [F] time = 4.37, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)
^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{m}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^m,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^m, x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))**m),x)
```

```
[Out] Timed out
```

3.216 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx$

Optimal. Leaf size=170

$$\frac{c^2 2^{\frac{1}{2}-m}(2A-B(1-2m))\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m-\right.}{f(2m+1)}$$

```
[Out] 2^(1/2-m)*c^2*(2*A-B*(1-2*m))*cos(f*x+e)*hypergeom([1/2+m, -1/2+m],[3/2+m],
1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+
e))^(-1-m)/f/(1+2*m)-1/2*B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(
1-m)/f
```

Rubi [A] time = 0.33, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2973, 2745, 2689, 70, 69}

$$\frac{c^2 2^{\frac{1}{2}-m}(2A-B(1-2m))\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m-\right.}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(1 - m
),x]
```

```
[Out] (2^(1/2 - m)*c^2*(2*A - B*(1 - 2*m))*Cos[e + f*x]*Hypergeometric2F1[(-1 + 2
*m)/2, (1 + 2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(
1/2 + m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m)
) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(1 - m))/(2
*f)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
```

```
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\,\mathrm{si}}{2f}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\,\mathrm{si}}{2f}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\,\mathrm{si}}{2f}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\,\mathrm{si}}{2f}$$

$$= \frac{2^{\frac{1}{2}-m}c^2(2A-B(1-2m))\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}\right.}{}$$

Mathematica [C] time = 92.47, size = 3601, normalized size = 21.18

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(1 - m),x]
```

```
[Out] (2^(5 - m)*((A + B)*AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-e + Pi/2 - f*
x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - (A + 9*B)*AppellF1[1/2 - m, -2*m, 3
, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*B*(
2*AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
+ Pi/2 - f*x)/4]^2] - AppellF1[1/2 - m, -2*m, 5, 3/2 - m, Tan[(-e + Pi/2 -
f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*(a + a*Sin[e + f*x])^m*(c - c*Sin
[e + f*x])^(1 - m)*(Cos[Pi/4 + (e - Pi/2 + f*x)/2]^2*((A*Cos[(-e + Pi/2 - f
*x)/2]^(2*m))/(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)
/2])^(2*m) + (B*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Sin[e + f*x])/(Cos[Pi/4 + (e
 - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^(2*m)) + (A*Cos[(-e + P
i/2 - f*x)/2]^(2*m)*Sin[Pi/4 + (e - Pi/2 + f*x)/2]^2)/(Cos[Pi/4 + (e - Pi/2
 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^(2*m) + (B*Cos[(-e + Pi/2 - f*
x)/2]^(2*m)*Sin[e + f*x]*Sin[Pi/4 + (e - Pi/2 + f*x)/2]^2)/(Cos[Pi/4 + (e -
 Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^(2*m) + Cos[Pi/4 + (e - P
i/2 + f*x)/2]*((-2*A*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Sin[Pi/4 + (e - Pi/2 +
f*x)/2])/(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^
(2*m) - (2*B*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Sin[e + f*x]*Sin[Pi/4 + (e - Pi
```

/2 + f*x)/2])/(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)
/2])^(2*m)))*Tan[(-e + Pi/2 - f*x)/4])/(f*(-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/
2]^(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(1 - m))*(1 - Tan[(-e + P
i/2 - f*x)/4]^2)^(2*m)*(-((2^(5 - m)*m*((A + B)*AppellF1[1/2 - m, -2*m, 2,
3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - (A + 9*
B)*AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
 + Pi/2 - f*x)/4]^2] + 8*B*(2*AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-e +
 Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - AppellF1[1/2 - m, -2*m, 5
, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*Cos[(
-e + Pi/2 - f*x)/2]^(2*m)*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/
4]^2*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(-1 - 2*m))/((-1 + 2*m)*Sin[(-e + Pi/
2 - f*x)/2]^(2*m))) - (2^(3 - m)*((A + B)*AppellF1[1/2 - m, -2*m, 2, 3/2 -
m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - (A + 9*B)*App
ellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/
2 - f*x)/4]^2] + 8*B*(2*AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-e + Pi/2
- f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - AppellF1[1/2 - m, -2*m, 5, 3/2
- m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*Cos[(-e + P
i/2 - f*x)/2]^(2*m)*Sec[(-e + Pi/2 - f*x)/4]^2)/((-1 + 2*m)*Sin[(-e + Pi/2
- f*x)/2]^(2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) + (2^(5 - m)*m*((A
+ B)*AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(
-e + Pi/2 - f*x)/4]^2] - (A + 9*B)*AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[
(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*B*(2*AppellF1[1/2
- m, -2*m, 4, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4
]^2] - AppellF1[1/2 - m, -2*m, 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan
[(-e + Pi/2 - f*x)/4]^2]))*Cos[(-e + Pi/2 - f*x)/2]^(1 + 2*m)*Sin[(-e + Pi/
2 - f*x)/2]^(-1 - 2*m)*Tan[(-e + Pi/2 - f*x)/4])/((-1 + 2*m)*(1 - Tan[(-e +
 Pi/2 - f*x)/4]^2)^(2*m)) + (2^(5 - m)*m*((A + B)*AppellF1[1/2 - m, -2*m, 2
, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - (A +
9*B)*AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(
-e + Pi/2 - f*x)/4]^2] + 8*B*(2*AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-e
 + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - AppellF1[1/2 - m, -2*m,
 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*Cos
[(-e + Pi/2 - f*x)/2]^(-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(1 - 2*m)*Tan[(-e
 + Pi/2 - f*x)/4])/((-1 + 2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) - (2
^(5 - m)*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Tan[(-e + Pi/2 - f*x)/4]*((A + B)*(
-(((1/2 - m)*m*AppellF1[3/2 - m, 1 - 2*m, 2, 5/2 - m, Tan[(-e + Pi/2 - f*x)
/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi
/2 - f*x)/4])/(3/2 - m)) - ((1/2 - m)*AppellF1[3/2 - m, -2*m, 3, 5/2 - m, T
an[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*
x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 - m)) - (A + 9*B)*(-(((1/2 - m)*m*Ap
pellF1[3/2 - m, 1 - 2*m, 3, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
 Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/
2 - m)) - (3*(1/2 - m)*AppellF1[3/2 - m, -2*m, 4, 5/2 - m, Tan[(-e + Pi/2 -
 f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e
 + Pi/2 - f*x)/4])/(2*(3/2 - m))) + 8*B*(((1/2 - m)*m*AppellF1[3/2 - m, 1 -

```
 2*m, 5, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*
Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 - m) + (5*(1/2 -
m)*AppellF1[3/2 - m, -2*m, 6, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
 + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(
2*(3/2 - m)) + 2*(-(((1/2 - m)*m*AppellF1[3/2 - m, 1 - 2*m, 4, 5/2 - m, Tan
[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)
/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 - m)) - (2*(1/2 - m)*AppellF1[3/2 - m,
 -2*m, 5, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]
*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 - m)))))/((-1 +
2*m)*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))
))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x, alg
orithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
(-m + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m + 1), x)
```

maple [F] time = 3.81, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1 - m
),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1 - m
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1-m),x)
```

```
[Out] Timed out
```

3.217 $\int (a+a\sin(e+fx))^m (A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx$

Optimal. Leaf size=173

$$\frac{c^3 2^{\frac{5}{2}-m}(3A-2B(1-m))\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m-\right.}{3f(2m+1)}$$

```
[Out] 1/3*2^(5/2-m)*c^3*(3*A-2*B*(1-m))*cos(f*x+e)*hypergeom([1/2+m, -3/2+m],[3/2
+m],1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(
f*x+e))^(-1-m)/f/(1+2*m)-1/3*B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e
))^(2-m)/f
```

Rubi [A] time = 0.34, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2973, 2745, 2689, 70, 69}

$$\frac{c^3 2^{\frac{5}{2}-m}(3A-2B(1-m))\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m-\right.}{3f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(2 - m
),x]
```

```
[Out] (2^(5/2 - m)*c^3*(3*A - 2*B*(1 - m))*Cos[e + f*x]*Hypergeometric2F1[(-3 + 2
*m)/2, (1 + 2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(
1/2 + m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(3*f*(1 + 2*
m)) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(2 - m))/
(3*f)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
```

```
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\,\mathrm{si}}{3f}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\,\mathrm{si}}{3f}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\,\mathrm{si}}{3f}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\,\mathrm{si}}{3f}$$

$$= \frac{2^{\frac{5}{2}-m}c^3(3A-2B(1-m))\cos(e+fx)\,{}_2F_1\Big(\frac{1}{2}}{}$$

Mathematica [C] time = 49.22, size = 5163, normalized size = 29.84

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(2 - m),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x, alg
orithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
(-m + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m + 2), x)
```

maple **[F]** time = 6.07, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m + 2), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{m}\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(2 - m
),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(2 - m
), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(2-m),x)
[Out] Timed out
```

3.218 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^n(B(3-n)-B(4+n)\sin(e+fx))\,dx$

Optimal. Leaf size=34

$$\frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^{n-3}}{f}$$

```
[Out] a^3*B*c^3*cos(f*x+e)^7*(c-c*sin(f*x+e))^(-3+n)/f
```

Rubi [A] time = 0.27, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2967, 2854}

$$\frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^{n-3}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^n*(B*(3 - n) - B*(4 + n)*Si
n[e + f*x]),x]
```

```
[Out] (a^3*B*c^3*Cos[e + f*x]^7*(c - c*Sin[e + f*x])^(-3 + n))/f
```

Rule 2854

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] /; Fr
eeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b*c
*(m + p + 1), 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a + a\sin(e + fx))^3(c - c\sin(e + fx))^n(B(3 - n) - B(4 + n)\sin(e + fx))\,dx = \left(a^3c^3\right)\int \cos^6(e + fx)(c - c\sin(e$$

$$= \frac{a^3Bc^3\cos^7(e + fx)(c - c\sin(e +}{f}$$

Mathematica [A] time = 0.54, size = 63, normalized size = 1.85

$$\frac{a^3B(14\sin(2(e + fx)) - \sin(4(e + fx)) + 14\cos(e + fx) - 6\cos(3(e + fx)))(c - c\sin(e + fx))^n}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^n*(B*(3 - n) - B*(4 +
 n)*Sin[e + f*x]),x]
```

```
[Out] (a^3*B*(c - c*Sin[e + f*x])^n*(14*Cos[e + f*x] - 6*Cos[3*(e + f*x)] + 14*Si
n[2*(e + f*x)] - Sin[4*(e + f*x)]))/(8*f)
```

fricas [B] time = 0.48, size = 78, normalized size = 2.29

$$-\frac{\left(3\,Ba^3\cos\left(fx + e\right)^3 - 4\,Ba^3\cos\left(fx + e\right) + \left(Ba^3\cos\left(fx + e\right)^3 - 4\,Ba^3\cos\left(fx + e\right)\right)\sin\left(fx + e\right)\right)\left(-c\sin\left(fx +\right.\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e)
),x, algorithm="fricas")
```

```
[Out] -(3*B*a^3*cos(f*x + e)^3 - 4*B*a^3*cos(f*x + e) + (B*a^3*cos(f*x + e)^3 - 4
*B*a^3*cos(f*x + e))*sin(f*x + e))*(-c*sin(f*x + e) + c)^n/f
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(B(n + 4)\sin\left(fx + e\right) + B(n - 3)\right)\left(a\sin\left(fx + e\right) + a\right)^3\left(-c\sin\left(fx + e\right) + c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e)
),x, algorithm="giac")
```

```
[Out] integrate(-(B*(n + 4)*sin(f*x + e) + B*(n - 3))*(a*sin(f*x + e) + a)^3*(-c*
sin(f*x + e) + c)^n, x)
```

maple [F] time = 8.04, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{3}\left(c-c\sin\left(fx+e\right)\right)^{n}\left(B\left(3-n\right)-B\left(4+n\right)\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(B(n+4)\sin\left(fx+e\right)+B(n-3)\right)\left(a\sin\left(fx+e\right)+a\right)^{3}\left(-c\sin\left(fx+e\right)+c\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e)
),x, algorithm="maxima")
```

```
[Out] -integrate((B*(n + 4)*sin(f*x + e) + B*(n - 3))*(a*sin(f*x + e) + a)^3*(-c*
sin(f*x + e) + c)^n, x)
```

mupad [B] time = 14.45, size = 64, normalized size = 1.88

$$\frac{B\,a^3\,{\left(-c\,\left(\sin\left(e+f\,x\right)-1\right)\right)}^n\,\left(14\,\cos\left(e+f\,x\right)-6\,\cos\left(3\,e+3\,f\,x\right)+14\,\sin\left(2\,e+2\,f\,x\right)-\sin\left(4\,e+4\,f\,x\right)\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(B*(n - 3) + B*sin(e + f*x)*(n + 4))*(a + a*sin(e + f*x))^3*(c - c*sin
(e + f*x))^n,x)
```

```
[Out] (B*a^3*(-c*(sin(e + f*x) - 1))^n*(14*cos(e + f*x) - 6*cos(3*e + 3*f*x) + 14
*sin(2*e + 2*f*x) - sin(4*e + 4*f*x)))/(8*f)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**n*(B*(3-n)-B*(4+n)*sin(f*x+
e)),x)
```

```
[Out] Timed out
```

3.219 $\int (a-a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx$

Optimal. Leaf size=34

$$-\frac{a^3Bc^3\cos^7(e+fx)(c\sin(e+fx)+c)^{n-3}}{f}$$

```
[Out] -a^3*B*c^3*cos(f*x+e)^7*(c+c*sin(f*x+e))^(-3+n)/f
```

Rubi [A] time = 0.24, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.044, Rules used = {2967, 2854}

$$-\frac{a^3Bc^3\cos^7(e+fx)(c\sin(e+fx)+c)^{n-3}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[e + f*x])^3*(c + c*Sin[e + f*x])^n*(B*(3 - n) + B*(4 + n)*Si
n[e + f*x]),x]
```

```
[Out] -((a^3*B*c^3*Cos[e + f*x]^7*(c + c*Sin[e + f*x])^(-3 + n))/f)
```

Rule 2854

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] /; Fr
eeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b*c
*(m + p + 1), 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a - a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(c+c\sin$$
$$= -\frac{a^3Bc^3\cos^7(e+fx)(c+c\sin(}{f}$$

Mathematica [A] time = 1.14, size = 67, normalized size = 1.97

$$-\frac{a^3B\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(c(\sin(e+fx)+1))^n}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[e + f*x])^3*(c + c*Sin[e + f*x])^n*(B*(3 - n) + B*(4 +
 n)*Sin[e + f*x]),x]
```

```
[Out] -((a^3*B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])*(c*(1 + Sin[e + f*x]))^n)/f)
```

fricas [B] time = 0.47, size = 77, normalized size = 2.26

$$\frac{\left(3\,Ba^3\cos\left(fx+e\right)^3-4\,Ba^3\cos\left(fx+e\right)-\left(Ba^3\cos\left(fx+e\right)^3-4\,Ba^3\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\left(c\sin\left(fx+e\right.\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e)
),x, algorithm="fricas")
```

```
[Out] (3*B*a^3*cos(f*x + e)^3 - 4*B*a^3*cos(f*x + e) - (B*a^3*cos(f*x + e)^3 - 4*
B*a^3*cos(f*x + e))*sin(f*x + e))*(c*sin(f*x + e) + c)^n/f
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(B(n+4)\sin\left(fx+e\right)-B(n-3)\right)\left(a\sin\left(fx+e\right)-a\right)^3\left(c\sin\left(fx+e\right)+c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e)
),x, algorithm="giac")
```

```
[Out] integrate(-(B*(n + 4)*sin(f*x + e) - B*(n - 3))*(a*sin(f*x + e) - a)^3*(c*s
in(f*x + e) + c)^n, x)
```

maple [F] time = 8.06, size = 0, normalized size = 0.00

$$\int \left(a-a\sin\left(fx+e\right)\right)^{3}\left(c+c\sin\left(fx+e\right)\right)^{n}\left(B\left(3-n\right)+B\left(4+n\right)\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e)),x)
```

```
[Out] int((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(B(n+4)\sin\left(fx+e\right)-B(n-3)\right)\left(a\sin\left(fx+e\right)-a\right)^{3}\left(c\sin\left(fx+e\right)+c\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e)
),x, algorithm="maxima")
```

```
[Out] -integrate((B*(n + 4)*sin(f*x + e) - B*(n - 3))*(a*sin(f*x + e) - a)^3*(c*s
in(f*x + e) + c)^n, x)
```

mupad [B] time = 14.46, size = 61, normalized size = 1.79

$$-\frac{B\,a^3\,{\left(c\,\left(\sin\left(e+f\,x\right)+1\right)\right)}^n\,\left(14\,\cos\left(e+f\,x\right)-6\,\cos\left(3\,e+3\,f\,x\right)-14\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(4\,e+4\,f\,x\right)\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(B*(n - 3) - B*sin(e + f*x)*(n + 4))*(a - a*sin(e + f*x))^3*(c + c*sin
(e + f*x))^n,x)
```

```
[Out] -(B*a^3*(c*(sin(e + f*x) + 1))^n*(14*cos(e + f*x) - 6*cos(3*e + 3*f*x) - 14
*sin(2*e + 2*f*x) + sin(4*e + 4*f*x)))/(8*f)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))**3*(c+c*sin(f*x+e))**n*(B*(3-n)+B*(4+n)*sin(f*x+
e)),x)
```

```
[Out] Timed out
```

3.220 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3(B(-3+m)-B(4+m)\sin(e+fx))\,dx$

Optimal. Leaf size=33

$$\frac{a^3Bc^3\cos^7(e+fx)(a\sin(e+fx)+a)^{m-3}}{f}$$

```
[Out] a^3*B*c^3*cos(f*x+e)^7*(a+a*sin(f*x+e))^(-3+m)/f
```

Rubi [A] time = 0.24, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2967, 2854}

$$\frac{a^3Bc^3\cos^7(e+fx)(a\sin(e+fx)+a)^{m-3}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3*(B*(-3 + m) - B*(4 + m)*S
in[e + f*x]),x]
```

```
[Out] (a^3*B*c^3*Cos[e + f*x]^7*(a + a*Sin[e + f*x])^(-3 + m))/f
```

Rule 2854

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] /; Fr
eeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b*c
*(m + p + 1), 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3(B(-3+m)-B(4+m)\sin(e+fx))\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(a+a\sin$$
$$= \frac{a^3Bc^3\cos^7(e+fx)(a+a\sin(}{f}$$

Mathematica [A] time = 1.11, size = 66, normalized size = 2.00

$$\frac{Bc^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(a(\sin(e+fx)+1))^m}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3*(B*(-3 + m) - B*(4
+ m)*Sin[e + f*x]),x]
```

```
[Out] (B*c^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])*(a*(1 + Sin[e + f*x]))^m)/f
```

fricas [B] time = 0.49, size = 78, normalized size = 2.36

$$-\frac{\left(3\,Bc^3\cos\left(fx+e\right)^3-4\,Bc^3\cos\left(fx+e\right)-\left(Bc^3\cos\left(fx+e\right)^3-4\,Bc^3\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\left(a\sin\left(fx+e\right.\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+e
)),x, algorithm="fricas")
```

```
[Out] -(3*B*c^3*cos(f*x + e)^3 - 4*B*c^3*cos(f*x + e) - (B*c^3*cos(f*x + e)^3 - 4
*B*c^3*cos(f*x + e))*sin(f*x + e))*(a*sin(f*x + e) + a)^m/f
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B(m+4)\sin\left(fx+e\right)-B(m-3)\right)\left(c\sin\left(fx+e\right)-c\right)^3\left(a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+e
)),x, algorithm="giac")
```

```
[Out] integrate((B*(m + 4)*sin(f*x + e) - B*(m - 3))*(c*sin(f*x + e) - c)^3*(a*si
n(f*x + e) + a)^m, x)
```

maple [F] time = 7.86, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m} \left(c-c\sin\left(fx+e\right)\right)^{3} \left(B\left(m-3\right)-B\left(4+m\right)\sin\left(fx+e\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(m-3)-B*(4+m)*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(m-3)-B*(4+m)*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B(m+4)\sin\left(fx+e\right)-B(m-3)\right)\left(c\sin\left(fx+e\right)-c\right)^{3}\left(a\sin\left(fx+e\right)+a\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+e
)),x, algorithm="maxima")
```

```
[Out] integrate((B*(m + 4)*sin(f*x + e) - B*(m - 3))*(c*sin(f*x + e) - c)^3*(a*si
n(f*x + e) + a)^m, x)
```

mupad [B] time = 14.38, size = 61, normalized size = 1.85

$$\frac{B\,c^3\,{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)}^m\,\left(14\,\cos\left(e+f\,x\right)-6\,\cos\left(3\,e+3\,f\,x\right)-14\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(4\,e+4\,f\,x\right)\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*(m - 3) - B*sin(e + f*x)*(m + 4))*(a + a*sin(e + f*x))^m*(c - c*sin(
e + f*x))^3,x)
```

```
[Out] (B*c^3*(a*(sin(e + f*x) + 1))^m*(14*cos(e + f*x) - 6*cos(3*e + 3*f*x) - 14*
sin(2*e + 2*f*x) + sin(4*e + 4*f*x)))/(8*f)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**3*(B*(-3+m)-B*(4+m)*sin(f*x
+e)),x)
```

```
[Out] Timed out
```

3.221 $\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^3(B(-3+m)+B(4+m)\sin(e+fx))\,dx$

Optimal. Leaf size=35

$$-\frac{a^3Bc^3\cos^7(e+fx)(a-a\sin(e+fx))^{m-3}}{f}$$

```
[Out] -a^3*B*c^3*cos(f*x+e)^7*(a-a*sin(f*x+e))^(-3+m)/f
```

Rubi [A] time = 0.24, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {2967, 2854}

$$-\frac{a^3Bc^3\cos^7(e+fx)(a-a\sin(e+fx))^{m-3}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^3*(B*(-3 + m) + B*(4 + m)*S
in[e + f*x]),x]
```

```
[Out] -((a^3*B*c^3*Cos[e + f*x]^7*(a - a*Sin[e + f*x])^(-3 + m))/f)
```

Rule 2854

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] /; Fr
eeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b*c
*(m + p + 1), 0]
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^3(B(-3+m)+B(4+m)\sin(e+fx))\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(a-a$$
$$= -\frac{a^3Bc^3\cos^7(e+fx)(a-a\,\mathrm{s}}{f}$$

Mathematica [A] time = 0.54, size = 61, normalized size = 1.74

$$\frac{Bc^3(-14\sin(2(e+fx))+\sin(4(e+fx))-14\cos(e+fx)+6\cos(3(e+fx)))(a-a\sin(e+fx))^m}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^3*(B*(-3 + m) + B*(4
+ m)*Sin[e + f*x]),x]
```

```
[Out] (B*c^3*(a - a*Sin[e + f*x])^m*(-14*Cos[e + f*x] + 6*Cos[3*(e + f*x)] - 14*S
in[2*(e + f*x)] + Sin[4*(e + f*x)]))/(8*f)
```

fricas [B] time = 0.48, size = 77, normalized size = 2.20

$$\frac{\left(3\,Bc^3\cos\left(fx+e\right)^3-4\,Bc^3\cos\left(fx+e\right)+\left(Bc^3\cos\left(fx+e\right)^3-4\,Bc^3\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\left(-a\sin\left(fx+\right.\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+e
)),x, algorithm="fricas")
```

```
[Out] (3*B*c^3*cos(f*x + e)^3 - 4*B*c^3*cos(f*x + e) + (B*c^3*cos(f*x + e)^3 - 4*
B*c^3*cos(f*x + e))*sin(f*x + e))*(-a*sin(f*x + e) + a)^m/f
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B(m+4)\sin\left(fx+e\right)+B(m-3)\right)\left(c\sin\left(fx+e\right)+c\right)^3\left(-a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+e
)),x, algorithm="giac")
```

```
[Out] integrate((B*(m + 4)*sin(f*x + e) + B*(m - 3))*(c*sin(f*x + e) + c)^3*(-a*s
in(f*x + e) + a)^m, x)
```

maple [F] time = 8.12, size = 0, normalized size = 0.00

$$\int \left(a-a\sin\left(fx+e\right)\right)^{m}\left(c+c\sin\left(fx+e\right)\right)^{3}\left(B\left(m-3\right)+B\left(4+m\right)\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(m-3)+B*(4+m)*sin(f*x+e)),x)
```

```
[Out] int((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(m-3)+B*(4+m)*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B(m+4)\sin\left(fx+e\right)+B(m-3)\right)\left(c\sin\left(fx+e\right)+c\right)^{3}\left(-a\sin\left(fx+e\right)+a\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+e
)),x, algorithm="maxima")
```

```
[Out] integrate((B*(m + 4)*sin(f*x + e) + B*(m - 3))*(c*sin(f*x + e) + c)^3*(-a*s
in(f*x + e) + a)^m, x)
```

mupad [B] time = 14.36, size = 64, normalized size = 1.83

$$-\frac{B\,c^3\,{\left(-a\,\left(\sin\left(e+f\,x\right)-1\right)\right)}^m\,\left(14\,\cos\left(e+f\,x\right)-6\,\cos\left(3\,e+3\,f\,x\right)+14\,\sin\left(2\,e+2\,f\,x\right)-\sin\left(4\,e+4\,f\,x\right)\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*(m - 3) + B*sin(e + f*x)*(m + 4))*(a - a*sin(e + f*x))^m*(c + c*sin(
e + f*x))^3,x)
```

```
[Out] -(B*c^3*(-a*(sin(e + f*x) - 1))^m*(14*cos(e + f*x) - 6*cos(3*e + 3*f*x) + 1
4*sin(2*e + 2*f*x) - sin(4*e + 4*f*x)))/(8*f)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))**m*(c+c*sin(f*x+e))**3*(B*(-3+m)+B*(4+m)*sin(f*x
+e)),x)
```

```
[Out] Timed out
```

3.222 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx$

Optimal. Leaf size=36

$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n}{f}$$

```
[Out] B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n/f
```

Rubi [A] time = 0.13, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {2970}

$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n*(B*(m - n) - B*(1 + m + n
)*Sin[e + f*x]),x]
```

```
[Out] (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/f
```

Rule 2970

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0]
 && EqQ[a^2 - b^2, 0] && EqQ[A*b*(m + n + 1) + a*B*(m - n), 0] && NeQ[m, -2
^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx = \frac{B\cos(e+fx)(a+a\sin(e$$

Mathematica [A] time = 0.49, size = 36, normalized size = 1.00

$$\frac{B\cos(e+fx)(a(\sin(e+fx)+1))^m(c-c\sin(e+fx))^n}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n*(B*(m - n) - B*(1 +
 m + n)*Sin[e + f*x]),x]
```

```
[Out] (B*Cos[e + f*x]*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n)/f
```

fricas [A] time = 0.49, size = 36, normalized size = 1.00

$$\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}B\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x+
e)),x, algorithm="fricas")
```

```
[Out] (a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n*B*cos(f*x + e)/f
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x+
e)),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 7.66, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{n}\left(B\left(m-n\right)-B\left(n+m+1\right)\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(n+m+1)*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(n+m+1)*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(B(m+n+1)\sin\left(fx+e\right)-B(m-n)\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x+
e)),x, algorithm="maxima")
```

```
[Out] -integrate((B*(m + n + 1)*sin(f*x + e) - B*(m - n))*(a*sin(f*x + e) + a)^m*
(-c*sin(f*x + e) + c)^n, x)
```

mupad [**B**] time = 13.54, size = 36, normalized size = 1.00

$$\frac{B\cos\left(e+f\,x\right)\,{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)}^m\,{\left(-c\,\left(\sin\left(e+f\,x\right)-1\right)\right)}^n}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*(m - n) - B*sin(e + f*x)*(m + n + 1))*(a + a*sin(e + f*x))^m*(c - c*
sin(e + f*x))^n,x)
```

```
[Out] (B*cos(e + f*x)*(a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^n)/f
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n*(B*(m-n)-B*(1+m+n)*sin(f*
x+e)),x)
```

```
[Out] Timed out
```

3.223 $\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^n(B(m-n)+B(1+m+n)\sin(e+fx))\,dx$

Optimal. Leaf size=37

$$-\frac{B\cos(e+fx)(a-a\sin(e+fx))^m(c\sin(e+fx)+c)^n}{f}$$

```
[Out] -B*cos(f*x+e)*(a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n/f
```

Rubi [A] time = 0.12, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {2970}

$$-\frac{B\cos(e+fx)(a-a\sin(e+fx))^m(c\sin(e+fx)+c)^n}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^n*(B*(m - n) + B*(1 + m + n
)*Sin[e + f*x]),x]
```

```
[Out] -((B*Cos[e + f*x]*(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^n)/f)
```

Rule 2970

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0]
 && EqQ[a^2 - b^2, 0] && EqQ[A*b*(m + n + 1) + a*B*(m - n), 0] && NeQ[m, -2
^(-1)]
```

Rubi steps

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^n(B(m-n)+B(1+m+n)\sin(e+fx))\,dx=-\frac{B\cos(e+fx)(a-a\sin(e$$

Mathematica [A] time = 0.47, size = 37, normalized size = 1.00

$$-\frac{B\cos(e+fx)(a-a\sin(e+fx))^m(c(\sin(e+fx)+1))^n}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^n*(B*(m - n) + B*(1 +
 m + n)*Sin[e + f*x]),x]
```

```
[Out] -((B*Cos[e + f*x]*(c*(1 + Sin[e + f*x]))^n*(a - a*Sin[e + f*x])^m)/f)
```

fricas [A] time = 0.50, size = 37, normalized size = 1.00

$$-\frac{\left(-a\sin\left(fx+e\right)+a\right)^{m}\left(c\sin\left(fx+e\right)+c\right)^{n}B\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x+
e)),x, algorithm="fricas")
```

```
[Out] -(-a*sin(f*x + e) + a)^m*(c*sin(f*x + e) + c)^n*B*cos(f*x + e)/f
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x+
e)),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 8.40, size = 0, normalized size = 0.00

$$\int \left(a-a\sin\left(fx+e\right)\right)^{m}\left(c+c\sin\left(fx+e\right)\right)^{n}\left(B\left(m-n\right)+B\left(n+m+1\right)\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(n+m+1)*sin(f*x+e)),x)
```

```
[Out] int((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(n+m+1)*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B(m+n+1)\sin\left(fx+e\right)+B(m-n)\right)\left(-a\sin\left(fx+e\right)+a\right)^{m}\left(c\sin\left(fx+e\right)+c\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x+
e)),x, algorithm="maxima")
```

```
[Out] integrate((B*(m + n + 1)*sin(f*x + e) + B*(m - n))*(-a*sin(f*x + e) + a)^m*
(c*sin(f*x + e) + c)^n, x)
```

mupad [B] time = 13.50, size = 37, normalized size = 1.00

$$-\frac{B\,\cos\left(e+f\,x\right)\,{\left(-a\,\left(\sin\left(e+f\,x\right)-1\right)\right)}^m\,{\left(c\,\left(\sin\left(e+f\,x\right)+1\right)\right)}^n}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*(m - n) + B*sin(e + f*x)*(m + n + 1))*(a - a*sin(e + f*x))^m*(c + c*
sin(e + f*x))^n,x)
```

```
[Out] -(B*cos(e + f*x)*(-a*(sin(e + f*x) - 1))^m*(c*(sin(e + f*x) + 1))^n)/f
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))**m*(c+c*sin(f*x+e))**n*(B*(m-n)+B*(1+m+n)*sin(f*
x+e)),x)
```

```
[Out] Timed out
```

3.224 $\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=140

$$-\frac{a^3A\cos^7(c+dx)}{7d}+\frac{3a^3A\cos^5(c+dx)}{5d}-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\sin^5(c+dx)\cos(c+dx)}{3d}-\frac{a^3A\sin^3(c+dx)\mathrm{cc}}{12d}$$

```
[Out] 1/8*a^3*A*x-2/3*a^3*A*cos(d*x+c)^3/d+3/5*a^3*A*cos(d*x+c)^5/d-1/7*a^3*A*cos
(d*x+c)^7/d-1/8*a^3*A*cos(d*x+c)*sin(d*x+c)/d-1/12*a^3*A*cos(d*x+c)*sin(d*x
+c)^3/d+1/3*a^3*A*cos(d*x+c)*sin(d*x+c)^5/d
```

Rubi [A] time = 0.19, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2966, 2633, 2635, 8}

$$-\frac{a^3A\cos^7(c+dx)}{7d}+\frac{3a^3A\cos^5(c+dx)}{5d}-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\sin^5(c+dx)\cos(c+dx)}{3d}-\frac{a^3A\sin^3(c+dx)\mathrm{cc}}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*x)/8 - (2*a^3*A*Cos[c + d*x]^3)/(3*d) + (3*a^3*A*Cos[c + d*x]^5)/(5*
d) - (a^3*A*Cos[c + d*x]^7)/(7*d) - (a^3*A*Cos[c + d*x]*Sin[c + d*x])/(8*d)
 - (a^3*A*Cos[c + d*x]*Sin[c + d*x]^3)/(12*d) + (a^3*A*Cos[c + d*x]*Sin[c +
 d*x]^5)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \int \left(a^3A\sin^3(c+dx)+2a^3A\sin^4(c+dx)-2a^3A\sin\right.$$
$$= \left(a^3A\right)\int \sin^3(c+dx)\,dx-\left(a^3A\right)\int \sin^7(c+dx)\,dx$$
$$= -\frac{a^3A\cos(c+dx)\sin^3(c+dx)}{2d}+\frac{a^3A\cos(c+dx)\sin}{3d}$$
$$= -\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{3a^3A\cos^5(c+dx)}{5d}-\frac{a^3A\cos}{7}$$
$$= \frac{3}{4}a^3Ax-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{3a^3A\cos^5(c+dx)}{5d}-$$
$$= \frac{1}{8}a^3Ax-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{3a^3A\cos^5(c+dx)}{5d}-$$

Mathematica [A] time = 0.14, size = 87, normalized size = 0.62

$$\frac{a^3A(-210\sin(2(c+dx))-210\sin(4(c+dx))+70\sin(6(c+dx))-1365\cos(c+dx)-175\cos(3(c+dx))+147}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*(840*c + 840*d*x - 1365*Cos[c + d*x] - 175*Cos[3*(c + d*x)] + 147*Co
s[5*(c + d*x)] - 15*Cos[7*(c + d*x)] - 210*Sin[2*(c + d*x)] - 210*Sin[4*(c
+ d*x)] + 70*Sin[6*(c + d*x)]))/(6720*d)
```

fricas [A] time = 0.46, size = 105, normalized size = 0.75

$$-\frac{120\,Aa^3\cos(dx+c)^7-504\,Aa^3\cos(dx+c)^5+560\,Aa^3\cos(dx+c)^3-105\,Aa^3dx-35\left(8\,Aa^3\cos(dx+c)^5-\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/840*(120*A*a^3*cos(d*x + c)^7 - 504*A*a^3*cos(d*x + c)^5 + 560*A*a^3*cos
(d*x + c)^3 - 105*A*a^3*d*x - 35*(8*A*a^3*cos(d*x + c)^5 - 14*A*a^3*cos(d*x
 + c)^3 + 3*A*a^3*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.17, size = 131, normalized size = 0.94

$$\frac{1}{8}Aa^3x - \frac{Aa^3\cos(7dx+7c)}{448d} + \frac{7Aa^3\cos(5dx+5c)}{320d} - \frac{5Aa^3\cos(3dx+3c)}{192d} - \frac{13Aa^3\cos(dx+c)}{64d} + \frac{Aa^3\sin(6d}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/8*A*a^3*x - 1/448*A*a^3*cos(7*d*x + 7*c)/d + 7/320*A*a^3*cos(5*d*x + 5*c)
/d - 5/192*A*a^3*cos(3*d*x + 3*c)/d - 13/64*A*a^3*cos(d*x + c)/d + 1/96*A*a
^3*sin(6*d*x + 6*c)/d - 1/32*A*a^3*sin(4*d*x + 4*c)/d - 1/32*A*a^3*sin(2*d*
x + 2*c)/d
```

maple [A] time = 0.60, size = 158, normalized size = 1.13

$$\frac{\frac{a^3A\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(dx+c)\right)}{5}\right)\cos(dx+c)}{7} - 2a^3A\left(-\frac{\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6} + \frac{5dx}{16} + \frac{5c}{16}\right) + 2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/7*a^3*A*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*
x+c)-2*a^3*A*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+
c)+5/16*d*x+5/16*c)+2*a^3*A*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+
3/8*d*x+3/8*c)-1/3*a^3*A*(2+sin(d*x+c)^2)*cos(d*x+c))
```

maxima [A] time = 0.43, size = 157, normalized size = 1.12

$$-\frac{96\left(5\cos(dx+c)^7 - 21\cos(dx+c)^5 + 35\cos(dx+c)^3 - 35\cos(dx+c)\right)Aa^3 - 1120\left(\cos(dx+c)^3 - 3\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/3360*(96*(5*cos(d*x + c)^7 - 21*cos(d*x + c)^5 + 35*cos(d*x + c)^3 - 35*
cos(d*x + c))*A*a^3 - 1120*(cos(d*x + c)^3 - 3*cos(d*x + c))*A*a^3 + 35*(4*
sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*
c))*A*a^3 - 210*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*A*a
^3)/d
```

mupad [B] time = 15.30, size = 300, normalized size = 2.14

$$\frac{A\,a^3\left(105\,c - 210\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right) - 2464\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 - 1400\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 - 4032\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 6790\tan\left(\frac{c}{2}+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (A*a^3*(105*c - 210*tan(c/2 + (d*x)/2) - 2464*tan(c/2 + (d*x)/2)^2 - 1400*t
an(c/2 + (d*x)/2)^3 - 4032*tan(c/2 + (d*x)/2)^4 + 6790*tan(c/2 + (d*x)/2)^5
 + 2240*tan(c/2 + (d*x)/2)^6 - 14560*tan(c/2 + (d*x)/2)^8 - 6790*tan(c/2 +
(d*x)/2)^9 - 3360*tan(c/2 + (d*x)/2)^10 + 1400*tan(c/2 + (d*x)/2)^11 + 210*
tan(c/2 + (d*x)/2)^13 + 105*d*x + 735*tan(c/2 + (d*x)/2)^2*(c + d*x) + 2205
*tan(c/2 + (d*x)/2)^4*(c + d*x) + 3675*tan(c/2 + (d*x)/2)^6*(c + d*x) + 367
5*tan(c/2 + (d*x)/2)^8*(c + d*x) + 2205*tan(c/2 + (d*x)/2)^10*(c + d*x) + 7
35*tan(c/2 + (d*x)/2)^12*(c + d*x) + 105*tan(c/2 + (d*x)/2)^14*(c + d*x) -
352))/(840*d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

sympy [A] time = 8.49, size = 440, normalized size = 3.14

$$\begin{cases} -\frac{5Aa^3x\sin^6(c+dx)}{8} - \frac{15Aa^3x\sin^4(c+dx)\cos^2(c+dx)}{8} + \frac{3Aa^3x\sin^4(c+dx)}{4} - \frac{15Aa^3x\sin^2(c+dx)\cos^4(c+dx)}{8} + \frac{3Aa^3x\sin^2(c+dx)\cos^2(c+dx)}{2} \\ x\left(-A\sin(c)+A\right)\left(a\sin(c)+a\right)^3\sin^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Piecewise((-5*A*a**3*x*sin(c + d*x)**6/8 - 15*A*a**3*x*sin(c + d*x)**4*cos(
c + d*x)**2/8 + 3*A*a**3*x*sin(c + d*x)**4/4 - 15*A*a**3*x*sin(c + d*x)**2*
cos(c + d*x)**4/8 + 3*A*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/2 - 5*A*a**3
*x*cos(c + d*x)**6/8 + 3*A*a**3*x*cos(c + d*x)**4/4 + A*a**3*sin(c + d*x)**
6*cos(c + d*x)/d + 11*A*a**3*sin(c + d*x)**5*cos(c + d*x)/(8*d) + 2*A*a**3*
sin(c + d*x)**4*cos(c + d*x)**3/d + 5*A*a**3*sin(c + d*x)**3*cos(c + d*x)**
3/(3*d) - 5*A*a**3*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 8*A*a**3*sin(c + d*
x)**2*cos(c + d*x)**5/(5*d) - A*a**3*sin(c + d*x)**2*cos(c + d*x)/d + 5*A*a
```

```
**3*sin(c + d*x)*cos(c + d*x)**5/(8*d) - 3*A*a**3*sin(c + d*x)*cos(c + d*x)
**3/(4*d) + 16*A*a**3*cos(c + d*x)**7/(35*d) - 2*A*a**3*cos(c + d*x)**3/(3*
d), Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**3*sin(c)**3, True))
```

3.225 $\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=121

$$\frac{2a^3A\cos^5(c+dx)}{5d}-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\sin^5(c+dx)\cos(c+dx)}{6d}+\frac{5a^3A\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{3a^3A\sin}{}$$

```
[Out] 3/16*a^3*A*x-2/3*a^3*A*cos(d*x+c)^3/d+2/5*a^3*A*cos(d*x+c)^5/d-3/16*a^3*A*c
os(d*x+c)*sin(d*x+c)/d+5/24*a^3*A*cos(d*x+c)*sin(d*x+c)^3/d+1/6*a^3*A*cos(d
*x+c)*sin(d*x+c)^5/d
```

Rubi [A] time = 0.17, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2966, 2635, 8, 2633}

$$\frac{2a^3A\cos^5(c+dx)}{5d}-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\sin^5(c+dx)\cos(c+dx)}{6d}+\frac{5a^3A\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{3a^3A\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (3*a^3*A*x)/16 - (2*a^3*A*Cos[c + d*x]^3)/(3*d) + (2*a^3*A*Cos[c + d*x]^5)/
(5*d) - (3*a^3*A*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (5*a^3*A*Cos[c + d*x]*
Sin[c + d*x]^3)/(24*d) + (a^3*A*Cos[c + d*x]*Sin[c + d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \int \left(a^3A\sin^2(c+dx)+2a^3A\sin^3(c+dx)-2a^3A\sin\right.$$
$$= \left(a^3A\right)\int \sin^2(c+dx)\,dx-\left(a^3A\right)\int \sin^6(c+dx)\,dx$$
$$= -\frac{a^3A\cos(c+dx)\sin(c+dx)}{2d}+\frac{a^3A\cos(c+dx)\sin}{6d}$$
$$= \frac{1}{2}a^3Ax-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{2a^3A\cos^5(c+dx)}{5d}-$$
$$= \frac{1}{2}a^3Ax-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{2a^3A\cos^5(c+dx)}{5d}-$$
$$= \frac{3}{16}a^3Ax-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{2a^3A\cos^5(c+dx)}{5d}$$

Mathematica [A] time = 0.10, size = 77, normalized size = 0.64

$$\frac{a^3A(-15\sin(2(c+dx))-45\sin(4(c+dx))+5\sin(6(c+dx))-240\cos(c+dx)-40\cos(3(c+dx))+24\cos(5}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*(180*c + 180*d*x - 240*Cos[c + d*x] - 40*Cos[3*(c + d*x)] + 24*Cos[5
*(c + d*x)] - 15*Sin[2*(c + d*x)] - 45*Sin[4*(c + d*x)] + 5*Sin[6*(c + d*x)
]))/(960*d)
```

fricas [A] time = 0.46, size = 91, normalized size = 0.75

$$\frac{96\,Aa^3\cos\left(dx+c\right)^5-160\,Aa^3\cos\left(dx+c\right)^3+45\,Aa^3dx+5\left(8\,Aa^3\cos\left(dx+c\right)^5-26\,Aa^3\cos\left(dx+c\right)^3+9\,Aa\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(96*A*a^3*cos(d*x + c)^5 - 160*A*a^3*cos(d*x + c)^3 + 45*A*a^3*d*x +
5*(8*A*a^3*cos(d*x + c)^5 - 26*A*a^3*cos(d*x + c)^3 + 9*A*a^3*cos(d*x + c))
*sin(d*x + c))/d
```

giac [A] time = 0.17, size = 113, normalized size = 0.93

$$\frac{3}{16}\,Aa^3x+\frac{Aa^3\cos\left(5\,dx+5\,c\right)}{40\,d}-\frac{Aa^3\cos\left(3\,dx+3\,c\right)}{24\,d}-\frac{Aa^3\cos\left(dx+c\right)}{4\,d}+\frac{Aa^3\sin\left(6\,dx+6\,c\right)}{192\,d}-\frac{3\,Aa^3\sin\left(4\,dx+4\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 3/16*A*a^3*x + 1/40*A*a^3*cos(5*d*x + 5*c)/d - 1/24*A*a^3*cos(3*d*x + 3*c)/
d - 1/4*A*a^3*cos(d*x + c)/d + 1/192*A*a^3*sin(6*d*x + 6*c)/d - 3/64*A*a^3*
sin(4*d*x + 4*c)/d - 1/64*A*a^3*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.50, size = 136, normalized size = 1.12

$$\frac{-a^3A\left(-\frac{\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{2a^3A\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}-\frac{2a^3A\left(2+\sin^2(dx\right.}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

```
[Out] 1/d*(-a^3*A*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c
)+5/16*d*x+5/16*c)+2/5*a^3*A*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)
-2/3*a^3*A*(2+sin(d*x+c)^2)*cos(d*x+c)+a^3*A*(-1/2*cos(d*x+c)*sin(d*x+c)+1/
2*d*x+1/2*c))
```

maxima [A] time = 0.40, size = 138, normalized size = 1.14

$$\frac{128\left(3\,\cos\left(dx+c\right)^5-10\,\cos\left(dx+c\right)^3+15\,\cos\left(dx+c\right)\right)Aa^3+640\left(\cos\left(dx+c\right)^3-3\,\cos\left(dx+c\right)\right)Aa^3-5\left(4\,\mathrm{s}\right.}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/960*(128*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*A*a^3 +
 640*(cos(d*x + c)^3 - 3*cos(d*x + c))*A*a^3 - 5*(4*sin(2*d*x + 2*c)^3 + 60
```

```
*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*A*a^3 + 240*(2*d*x
+ 2*c - sin(2*d*x + 2*c))*A*a^3)/d
```

mupad [B] time = 15.29, size = 256, normalized size = 2.12

$$\frac{A\,a^3\,\left(45\,c-90\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-768\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+130\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+1500\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-1280\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (A*a^3*(45*c - 90*tan(c/2 + (d*x)/2) - 768*tan(c/2 + (d*x)/2)^2 + 130*tan(c
/2 + (d*x)/2)^3 + 1500*tan(c/2 + (d*x)/2)^5 - 1280*tan(c/2 + (d*x)/2)^6 - 1
500*tan(c/2 + (d*x)/2)^7 - 1920*tan(c/2 + (d*x)/2)^8 - 130*tan(c/2 + (d*x)/
2)^9 + 90*tan(c/2 + (d*x)/2)^11 + 45*d*x + 270*tan(c/2 + (d*x)/2)^2*(c + d*
x) + 675*tan(c/2 + (d*x)/2)^4*(c + d*x) + 900*tan(c/2 + (d*x)/2)^6*(c + d*x
) + 675*tan(c/2 + (d*x)/2)^8*(c + d*x) + 270*tan(c/2 + (d*x)/2)^10*(c + d*x
) + 45*tan(c/2 + (d*x)/2)^12*(c + d*x) - 128))/(240*d*(tan(c/2 + (d*x)/2)^2
 + 1)^6)
```

sympy [A] time = 5.69, size = 359, normalized size = 2.97

$$\begin{cases} -\frac{5Aa^3x\sin^6(c+dx)}{16}-\frac{15Aa^3x\sin^4(c+dx)\cos^2(c+dx)}{16}-\frac{15Aa^3x\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{Aa^3x\sin^2(c+dx)}{2}-\frac{5Aa^3x\cos^6(c+dx)}{16}+\frac{Aa^3x\,c}{} \\ x\left(-A\sin(c)+A\right)\left(a\sin(c)+a\right)^3\sin^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Piecewise((-5*A*a**3*x*sin(c + d*x)**6/16 - 15*A*a**3*x*sin(c + d*x)**4*cos
(c + d*x)**2/16 - 15*A*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + A*a**3*x
*sin(c + d*x)**2/2 - 5*A*a**3*x*cos(c + d*x)**6/16 + A*a**3*x*cos(c + d*x)*
*2/2 + 11*A*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 2*A*a**3*sin(c + d*x
)**4*cos(c + d*x)/d + 5*A*a**3*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 8*A*
a**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 2*A*a**3*sin(c + d*x)**2*cos(c
 + d*x)/d + 5*A*a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) - A*a**3*sin(c + d
*x)*cos(c + d*x)/(2*d) + 16*A*a**3*cos(c + d*x)**5/(15*d) - 4*A*a**3*cos(c
+ d*x)**3/(3*d), Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**3*sin(c)**2,
 True))
```

3.226 $\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=96

$$\frac{a^3A\cos^5(c+dx)}{5d}-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\sin^3(c+dx)\cos(c+dx)}{2d}-\frac{a^3A\sin(c+dx)\cos(c+dx)}{4d}+\frac{1}{4}a^3Ax$$

```
[Out] 1/4*a^3*A*x-2/3*a^3*A*cos(d*x+c)^3/d+1/5*a^3*A*cos(d*x+c)^5/d-1/4*a^3*A*cos
(d*x+c)*sin(d*x+c)/d+1/2*a^3*A*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.12, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2966, 2638, 2635, 8, 2633}

$$\frac{a^3A\cos^5(c+dx)}{5d}-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\sin^3(c+dx)\cos(c+dx)}{2d}-\frac{a^3A\sin(c+dx)\cos(c+dx)}{4d}+\frac{1}{4}a^3Ax$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*x)/4 - (2*a^3*A*Cos[c + d*x]^3)/(3*d) + (a^3*A*Cos[c + d*x]^5)/(5*d)
 - (a^3*A*Cos[c + d*x]*Sin[c + d*x])/(4*d) + (a^3*A*Cos[c + d*x]*Sin[c + d*
x]^3)/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \int \left(a^3A\sin(c+dx)+2a^3A\sin^2(c+dx)-2a^3A\sin\right.$$
$$= \left(a^3A\right)\int \sin(c+dx)\,dx-\left(a^3A\right)\int \sin^5(c+dx)\,dx+$$
$$= -\frac{a^3A\cos(c+dx)}{d}-\frac{a^3A\cos(c+dx)\sin(c+dx)}{d}+$$
$$= a^3Ax-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\cos^5(c+dx)}{5d}-$$
$$= \frac{1}{4}a^3Ax-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\cos^5(c+dx)}{5d}-$$

Mathematica [A] time = 0.48, size = 55, normalized size = 0.57

$$\frac{a^3A(-90\cos(c+dx)-25\cos(3(c+dx))+3(-5\sin(4(c+dx))+\cos(5(c+dx))+20dx))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*(-90*Cos[c + d*x] - 25*Cos[3*(c + d*x)] + 3*(20*d*x + Cos[5*(c + d*x
)] - 5*Sin[4*(c + d*x)])))/(240*d)
```

fricas [A] time = 0.44, size = 77, normalized size = 0.80

$$\frac{12\,Aa^3\cos(dx+c)^5-40\,Aa^3\cos(dx+c)^3+15\,Aa^3dx-15\left(2\,Aa^3\cos(dx+c)^3-Aa^3\cos(dx+c)\right)\sin(dx+}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/60*(12*A*a^3*cos(d*x + c)^5 - 40*A*a^3*cos(d*x + c)^3 + 15*A*a^3*d*x - 15
*(2*A*a^3*cos(d*x + c)^3 - A*a^3*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.16, size = 77, normalized size = 0.80

$$\frac{1}{4}\,Aa^3x+\frac{Aa^3\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{5\,Aa^3\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{3\,Aa^3\cos\left(dx+c\right)}{8\,d}-\frac{Aa^3\sin\left(4\,dx+4\,c\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/4*A*a^3*x + 1/80*A*a^3*cos(5*d*x + 5*c)/d - 5/48*A*a^3*cos(3*d*x + 3*c)/d
 - 3/8*A*a^3*cos(d*x + c)/d - 1/16*A*a^3*sin(4*d*x + 4*c)/d
```

maple [A] time = 0.40, size = 117, normalized size = 1.22

$$\frac{\frac{a^3A\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}-2a^3A\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2a^3A\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\cdots\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/5*a^3*A*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)-2*a^3*A*(-1/4
*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+2*a^3*A*(-1/2*cos(
d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-a^3*A*cos(d*x+c))
```

maxima [A] time = 0.37, size = 112, normalized size = 1.17

$$\frac{16\left(3\,\cos\left(dx+c\right)^5-10\,\cos\left(dx+c\right)^3+15\,\cos\left(dx+c\right)\right)Aa^3-15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)-8\,\sin\left(2\,dx+\cdots\right.\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/240*(16*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*A*a^3 -
15*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*A*a^3 + 120*(2*d
*x + 2*c - sin(2*d*x + 2*c))*A*a^3 - 240*A*a^3*cos(d*x + c))/d
```

mupad [B] time = 15.03, size = 292, normalized size = 3.04

$$\frac{A\,a^3\,x}{4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(\frac{A\,a^3\,(15\,c+15\,d\,x)}{12}-\frac{A\,a^3\,(75\,c+75\,d\,x-120)}{60}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{A\,a^3\,(15\,c+15\,d\,x)}{12}-\frac{A\,a^3\,(75\,c+75\,d\,x-160)}{60}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (A*a^3*x)/4 - (tan(c/2 + (d*x)/2)^8*((A*a^3*(15*c + 15*d*x))/12 - (A*a^3*(7
5*c + 75*d*x - 120))/60) + tan(c/2 + (d*x)/2)^2*((A*a^3*(15*c + 15*d*x))/12
 - (A*a^3*(75*c + 75*d*x - 160))/60) + tan(c/2 + (d*x)/2)^4*((A*a^3*(15*c +
 15*d*x))/6 - (A*a^3*(150*c + 150*d*x - 80))/60) + tan(c/2 + (d*x)/2)^6*((A
*a^3*(15*c + 15*d*x))/6 - (A*a^3*(150*c + 150*d*x - 480))/60) + (A*a^3*tan(
c/2 + (d*x)/2))/2 - 3*A*a^3*tan(c/2 + (d*x)/2)^3 + 3*A*a^3*tan(c/2 + (d*x)/
2)^7 - (A*a^3*tan(c/2 + (d*x)/2)^9)/2 + (A*a^3*(15*c + 15*d*x))/60 - (A*a^3
*(15*c + 15*d*x - 56))/60)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

sympy [A] time = 3.65, size = 267, normalized size = 2.78

$$\begin{cases} -\frac{3Aa^3x\sin^4(c+dx)}{4}-\frac{3Aa^3x\sin^2(c+dx)\cos^2(c+dx)}{2}+Aa^3x\sin^2(c+dx)-\frac{3Aa^3x\cos^4(c+dx)}{4}+Aa^3x\cos^2(c+dx)+\frac{Aa^3\sin}{} \\ x\left(-A\sin(c)+A\right)\left(a\sin(c)+a\right)^3\sin(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Piecewise((-3*A*a**3*x*sin(c + d*x)**4/4 - 3*A*a**3*x*sin(c + d*x)**2*cos(c
 + d*x)**2/2 + A*a**3*x*sin(c + d*x)**2 - 3*A*a**3*x*cos(c + d*x)**4/4 + A*
a**3*x*cos(c + d*x)**2 + A*a**3*sin(c + d*x)**4*cos(c + d*x)/d + 5*A*a**3*s
in(c + d*x)**3*cos(c + d*x)/(4*d) + 4*A*a**3*sin(c + d*x)**2*cos(c + d*x)**
3/(3*d) + 3*A*a**3*sin(c + d*x)*cos(c + d*x)**3/(4*d) - A*a**3*sin(c + d*x)
*cos(c + d*x)/d + 8*A*a**3*cos(c + d*x)**5/(15*d) - A*a**3*cos(c + d*x)/d,
Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**3*sin(c), True))
```

3.227 $\int (a + a\sin(c + dx))^3 (A - A\sin(c + dx))\, dx$

Optimal. Leaf size=82

$$-\frac{5a^3A\cos^3(c+dx)}{12d}-\frac{A\cos^3(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{4d}+\frac{5a^3A\sin(c+dx)\cos(c+dx)}{8d}+\frac{5}{8}a^3Ax$$

```
[Out] 5/8*a^3*A*x-5/12*a^3*A*cos(d*x+c)^3/d+5/8*a^3*A*cos(d*x+c)*sin(d*x+c)/d-1/4
*A*cos(d*x+c)^3*(a^3+a^3*sin(d*x+c))/d
```

Rubi [A] time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2736, 2678, 2669, 2635, 8}

$$-\frac{5a^3A\cos^3(c+dx)}{12d}-\frac{A\cos^3(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{4d}+\frac{5a^3A\sin(c+dx)\cos(c+dx)}{8d}+\frac{5}{8}a^3Ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (5*a^3*A*x)/8 - (5*a^3*A*Cos[c + d*x]^3)/(12*d) + (5*a^3*A*Cos[c + d*x]*Sin
[c + d*x])/(8*d) - (A*Cos[c + d*x]^3*(a^3 + a^3*Sin[c + d*x]))/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
```

```
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = (aA)\int \cos^2(c+dx)(a+a\sin(c+dx))^2\,dx$$
$$= -\frac{A\cos^3(c+dx)\left(a^3+a^3\sin(c+dx)\right)}{4d}+\frac{1}{4}\left(5a^2A\right)\int \cos^2(c+d$$
$$= -\frac{5a^3A\cos^3(c+dx)}{12d}-\frac{A\cos^3(c+dx)\left(a^3+a^3\sin(c+dx)\right)}{4d}+\frac{1}{4}$$
$$= -\frac{5a^3A\cos^3(c+dx)}{12d}+\frac{5a^3A\cos(c+dx)\sin(c+dx)}{8d}-\frac{A\cos^3(c$$
$$= \frac{5}{8}a^3Ax-\frac{5a^3A\cos^3(c+dx)}{12d}+\frac{5a^3A\cos(c+dx)\sin(c+dx)}{8d}-$$

Mathematica [A] time = 0.35, size = 54, normalized size = 0.66

$$\frac{a^3A(24\sin(2(c+dx))-3\sin(4(c+dx))-48\cos(c+dx)-16\cos(3(c+dx))+60dx)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*(60*d*x - 48*Cos[c + d*x] - 16*Cos[3*(c + d*x)] + 24*Sin[2*(c + d*x)
] - 3*Sin[4*(c + d*x)]))/(96*d)
```

fricas [A] time = 0.45, size = 63, normalized size = 0.77

$$-\frac{16\,Aa^3\cos(dx+c)^3-15\,Aa^3dx+3\left(2\,Aa^3\cos(dx+c)^3-5\,Aa^3\cos(dx+c)\right)\sin(dx+c)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/24*(16*A*a^3*cos(d*x + c)^3 - 15*A*a^3*d*x + 3*(2*A*a^3*cos(d*x + c)^3 -
 5*A*a^3*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.17, size = 77, normalized size = 0.94

$$\frac{5}{8}\,Aa^3x - \frac{Aa^3\cos\left(3\,dx + 3\,c\right)}{6\,d} - \frac{Aa^3\cos\left(dx + c\right)}{2\,d} - \frac{Aa^3\sin\left(4\,dx + 4\,c\right)}{32\,d} + \frac{Aa^3\sin\left(2\,dx + 2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 5/8*A*a^3*x - 1/6*A*a^3*cos(3*d*x + 3*c)/d - 1/2*A*a^3*cos(d*x + c)/d - 1/3
2*A*a^3*sin(4*d*x + 4*c)/d + 1/4*A*a^3*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.41, size = 89, normalized size = 1.09

$$\frac{-a^3A\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8}\right) + \frac{2a^3A\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3} - 2a^3A\cos\left(dx + c\right) + a^3A\left(dx + c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

```
[Out] 1/d*(-a^3*A*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+2
/3*a^3*A*(2+sin(d*x+c)^2)*cos(d*x+c)-2*a^3*A*cos(d*x+c)+a^3*A*(d*x+c))
```

maxima [A] time = 0.39, size = 86, normalized size = 1.05

$$-\frac{64\left(\cos\left(dx + c\right)^3 - 3\,\cos\left(dx + c\right)\right)Aa^3 + 3\left(12\,dx + 12\,c + \sin\left(4\,dx + 4\,c\right) - 8\,\sin\left(2\,dx + 2\,c\right)\right)Aa^3 - 96\left(dx + c\right)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(64*(cos(d*x + c)^3 - 3*cos(d*x + c))*A*a^3 + 3*(12*d*x + 12*c + sin(
4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*A*a^3 - 96*(d*x + c)*A*a^3 + 192*A*a^3*c
os(d*x + c))/d
```

mupad [B] time = 15.19, size = 250, normalized size = 3.05

$$\frac{5\,A\,a^3\,x}{8}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{A\,a^3\,(15\,c+15\,d\,x)}{6}-\frac{A\,a^3\,(60\,c+60\,d\,x-32)}{24}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(\frac{A\,a^3\,(15\,c+15\,d\,x)}{6}-\frac{A\,a^3\,(60\,c+60\,d\,x-96)}{24}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (5*A*a^3*x)/8 - (tan(c/2 + (d*x)/2)^2*((A*a^3*(15*c + 15*d*x))/6 - (A*a^3*(
60*c + 60*d*x - 32))/24) + tan(c/2 + (d*x)/2)^6*((A*a^3*(15*c + 15*d*x))/6
- (A*a^3*(60*c + 60*d*x - 96))/24) + tan(c/2 + (d*x)/2)^4*((A*a^3*(15*c + 1
5*d*x))/4 - (A*a^3*(90*c + 90*d*x - 96))/24) - (3*A*a^3*tan(c/2 + (d*x)/2))
/4 - (11*A*a^3*tan(c/2 + (d*x)/2)^3)/4 + (11*A*a^3*tan(c/2 + (d*x)/2)^5)/4
+ (3*A*a^3*tan(c/2 + (d*x)/2)^7)/4 + (A*a^3*(15*c + 15*d*x))/24 - (A*a^3*(1
5*c + 15*d*x - 32))/24)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

sympy [A] time = 1.37, size = 196, normalized size = 2.39

$$\begin{cases} -\frac{3Aa^3x\sin^4(c+dx)}{8}-\frac{3Aa^3x\sin^2(c+dx)\cos^2(c+dx)}{4}-\frac{3Aa^3x\cos^4(c+dx)}{8}+Aa^3x+\frac{5Aa^3\sin^3(c+dx)\cos(c+dx)}{8d}+\frac{2Aa^3\sin^2(c+dx)\cos}{d} \\ x\left(-A\sin(c)+A\right)\left(a\sin(c)+a\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Piecewise((-3*A*a**3*x*sin(c + d*x)**4/8 - 3*A*a**3*x*sin(c + d*x)**2*cos(c
 + d*x)**2/4 - 3*A*a**3*x*cos(c + d*x)**4/8 + A*a**3*x + 5*A*a**3*sin(c + d
*x)**3*cos(c + d*x)/(8*d) + 2*A*a**3*sin(c + d*x)**2*cos(c + d*x)/d + 3*A*a
**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 4*A*a**3*cos(c + d*x)**3/(3*d) - 2
*A*a**3*cos(c + d*x)/d, Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**3, Tr
ue))
```

3.228 $\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=76

$$-\frac{a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\cos(c+dx)}{d}+\frac{a^3A\sin(c+dx)\cos(c+dx)}{d}-\frac{a^3A\tanh^{-1}(\cos(c+dx))}{d}+a^3Ax$$

```
[Out] a^3*A*x-a^3*A*arctanh(cos(d*x+c))/d+a^3*A*cos(d*x+c)/d-1/3*a^3*A*cos(d*x+c)
^3/d+a^3*A*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.10, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2966, 3770, 2635, 8, 2633}

$$-\frac{a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\cos(c+dx)}{d}+\frac{a^3A\sin(c+dx)\cos(c+dx)}{d}-\frac{a^3A\tanh^{-1}(\cos(c+dx))}{d}+a^3Ax$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] a^3*A*x - (a^3*A*ArcTanh[Cos[c + d*x]])/d + (a^3*A*Cos[c + d*x])/d - (a^3*A
*Cos[c + d*x]^3)/(3*d) + (a^3*A*Cos[c + d*x]*Sin[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
```

```
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \int \left(2a^3A + a^3A\csc(c+dx) - 2a^3A\sin^2(c+dx) - a\right.$$
$$= 2a^3Ax + \left(a^3A\right)\int \csc(c+dx)\,dx - \left(a^3A\right)\int \sin^3(c$$
$$= 2a^3Ax - \frac{a^3A\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^3A\cos(c+dx}{d}$$
$$= a^3Ax - \frac{a^3A\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^3A\cos(c+dx)}{d}$$

Mathematica [A] time = 0.15, size = 74, normalized size = 0.97

$$\frac{a^3A\left(9\cos(c+dx) - \cos(3(c+dx)) + 6\left(\sin(2(c+dx)) + 2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 2c\right.\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*(9*Cos[c + d*x] - Cos[3*(c + d*x)] + 6*(-2*c + 2*d*x - 2*Log[Cos[(c
+ d*x)/2]] + 2*Log[Sin[(c + d*x)/2]] + Sin[2*(c + d*x)])))/(12*d)
```

fricas [A] time = 0.48, size = 92, normalized size = 1.21

$$-\frac{2\,Aa^3\cos\left(dx+c\right)^3 - 6\,Aa^3dx - 6\,Aa^3\cos\left(dx+c\right)\sin\left(dx+c\right) - 6\,Aa^3\cos\left(dx+c\right) + 3\,Aa^3\log\left(\frac{1}{2}\cos\left(dx+\right.\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fric
as")
```

[Out] -1/6*(2*A*a^3*cos(d*x + c)^3 - 6*A*a^3*d*x - 6*A*a^3*cos(d*x + c)*sin(d*x + c) - 6*A*a^3*cos(d*x + c) + 3*A*a^3*log(1/2*cos(d*x + c) + 1/2) - 3*A*a^3*log(-1/2*cos(d*x + c) + 1/2))/d

giac [A] time = 0.16, size = 107, normalized size = 1.41

$$\frac{3\,(dx+c)Aa^3 + 3\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) - \frac{2\left(3\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 6\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 3\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 2\,Aa^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="giac")

[Out] 1/3*(3*(d*x + c)*A*a^3 + 3*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(3*A*a^3*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 3*A*a^3*tan(1/2*d*x + 1/2*c) - 2*A*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d

maple [A] time = 0.43, size = 99, normalized size = 1.30

$$\frac{A\cos(dx+c)\left(\sin^2(dx+c)\right)a^3}{3d}+\frac{2a^3A\cos(dx+c)}{3d}+\frac{a^3A\cos(dx+c)\sin(dx+c)}{d}+a^3Ax+\frac{a^3Ac}{d}+\frac{a^3A\ln(\csc(d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)

[Out] 1/3/d*A*cos(d*x+c)*sin(d*x+c)^2*a^3+2/3*a^3*A*cos(d*x+c)/d+a^3*A*cos(d*x+c)*sin(d*x+c)/d+a^3*A*x+1/d*a^3*A*c+1/d*a^3*A*ln(csc(d*x+c)-cot(d*x+c))

maxima [A] time = 0.39, size = 85, normalized size = 1.12

$$-\frac{2\left(\cos(dx+c)^3-3\,\cos(dx+c)\right)Aa^3+3\,(2\,dx+2\,c-\sin(2\,dx+2\,c))Aa^3-12\,(dx+c)Aa^3+6\,Aa^3\log(\cot(d}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="maxima")

[Out] -1/6*(2*(cos(d*x + c)^3 - 3*cos(d*x + c))*A*a^3 + 3*(2*d*x + 2*c - sin(2*d*x + 2*c))*A*a^3 - 12*(d*x + c)*A*a^3 + 6*A*a^3*log(cot(d*x + c) + csc(d*x + c)))/d

mupad [B] time = 13.23, size = 212, normalized size = 2.79

$$\frac{-2\,A\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\,A\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,A\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{4\,A\,a^3}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{2\,A\,a^3\,\mathrm{atan}\left(\frac{4\,A^2\,a^6}{4\,A^2\,a^6-4\,A^2\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x),x)
```

```
[Out] ((4*A*a^3)/3 + 2*A*a^3*tan(c/2 + (d*x)/2) + 4*A*a^3*tan(c/2 + (d*x)/2)^2 -
2*A*a^3*tan(c/2 + (d*x)/2)^5)/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x
)/2)^4 + tan(c/2 + (d*x)/2)^6 + 1)) + (2*A*a^3*atan((4*A^2*a^6)/(4*A^2*a^6
- 4*A^2*a^6*tan(c/2 + (d*x)/2)) + (4*A^2*a^6*tan(c/2 + (d*x)/2))/(4*A^2*a^6
 - 4*A^2*a^6*tan(c/2 + (d*x)/2))))/d + (A*a^3*log(tan(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-Aa^3\left(\int(-2\sin(c+dx)\csc(c+dx))\,dx+\int 2\sin^3(c+dx)\csc(c+dx)\,dx+\int\sin^4(c+dx)\csc(c+dx)\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] -A*a**3*(Integral(-2*sin(c + d*x)*csc(c + d*x), x) + Integral(2*sin(c + d*x
)**3*csc(c + d*x), x) + Integral(sin(c + d*x)**4*csc(c + d*x), x) + Integra
l(-csc(c + d*x), x))
```

3.229 $\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=79

$$\frac{2a^3A\cos(c+dx)}{d}-\frac{a^3A\cot(c+dx)}{d}+\frac{a^3A\sin(c+dx)\cos(c+dx)}{2d}-\frac{2a^3A\tanh^{-1}(\cos(c+dx))}{d}-\frac{1}{2}a^3Ax$$

```
[Out] -1/2*a^3*A*x-2*a^3*A*arctanh(cos(d*x+c))/d+2*a^3*A*cos(d*x+c)/d-a^3*A*cot(d
*x+c)/d+1/2*a^3*A*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.18, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2950, 2709, 3770, 3767, 8, 2638, 2635}

$$\frac{2a^3A\cos(c+dx)}{d}-\frac{a^3A\cot(c+dx)}{d}+\frac{a^3A\sin(c+dx)\cos(c+dx)}{2d}-\frac{2a^3A\tanh^{-1}(\cos(c+dx))}{d}-\frac{1}{2}a^3Ax$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] -(a^3*A*x)/2 - (2*a^3*A*ArcTanh[Cos[c + d*x]])/d + (2*a^3*A*Cos[c + d*x])/d
 - (a^3*A*Cot[c + d*x])/d + (a^3*A*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
```

```
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2950

```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^n*c^n,
 Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
0] && IntegerQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = (aA)\int \cot^2(c+dx)(a+a\sin(c+dx))^2\,dx$$
$$= \frac{A\int \left(2a^4\csc(c+dx)+a^4\csc^2(c+dx)-2a^4\sin(c\right.}{a}$$
$$= \left(a^3A\right)\int \csc^2(c+dx)\,dx-\left(a^3A\right)\int \sin^2(c+dx)\,dx$$
$$= -\frac{2a^3A\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^3A\cos(c+dx)}{d}+$$
$$= -\frac{1}{2}a^3Ax-\frac{2a^3A\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^3A\cos(}{d}$$

Mathematica [A] time = 0.19, size = 77, normalized size = 0.97

$$\frac{a^3A\left(-8\sin(c)\sin(dx)+\sin(2(c+dx))+8\cos(c)\cos(dx)-4\cot(c+dx)+8\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-8\log\left(\cos\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*(-2*c - 2*d*x + 8*Cos[c]*Cos[d*x] - 4*Cot[c + d*x] - 8*Log[Cos[(c +
d*x)/2]] + 8*Log[Sin[(c + d*x)/2]] - 8*Sin[c]*Sin[d*x] + Sin[2*(c + d*x)]))
/(4*d)
```

fricas [A] time = 0.49, size = 111, normalized size = 1.41

$$-\frac{Aa^3\cos(dx+c)^3+2\,Aa^3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-2\,Aa^3\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+}{2\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/2*(A*a^3*cos(d*x + c)^3 + 2*A*a^3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x +
c) - 2*A*a^3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + A*a^3*cos(d*x + c)
 + (A*a^3*d*x - 4*A*a^3*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

giac [B] time = 0.19, size = 153, normalized size = 1.94

$$-\frac{(dx+c)Aa^3-4\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{4\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+Aa^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\left(Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/2*((d*x + c)*A*a^3 - 4*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - A*a^3*tan(
1/2*d*x + 1/2*c) + (4*A*a^3*tan(1/2*d*x + 1/2*c) + A*a^3)/tan(1/2*d*x + 1/2
*c) + 2*(A*a^3*tan(1/2*d*x + 1/2*c)^3 - 4*A*a^3*tan(1/2*d*x + 1/2*c)^2 - A*
a^3*tan(1/2*d*x + 1/2*c) - 4*A*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.36, size = 95, normalized size = 1.20

$$\frac{a^3A\cos(dx+c)\sin(dx+c)}{2d}-\frac{a^3Ax}{2}-\frac{a^3Ac}{2d}+\frac{2a^3A\cos(dx+c)}{d}+\frac{2a^3A\ln(\csc(dx+c)-\cot(dx+c))}{d}-\frac{a^3A\cot(}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

```
[Out] 1/2*a^3*A*cos(d*x+c)*sin(d*x+c)/d-1/2*a^3*A*x-1/2/d*a^3*A*c+2*a^3*A*cos(d*x
+c)/d+2/d*a^3*A*ln(csc(d*x+c)-cot(d*x+c))-a^3*A*cot(d*x+c)/d
```

maxima [A] time = 0.55, size = 83, normalized size = 1.05

$$-\frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))Aa^3+4\,Aa^3\big(\log(\cos(dx+c)+1)-\log(\cos(dx+c)-1)\big)-8\,Aa^3\cos(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*A*a^3 + 4*A*a^3*(log(cos(d*x + c) +
1) - log(cos(d*x + c) - 1)) - 8*A*a^3*cos(d*x + c) + 4*A*a^3/tan(d*x + c))/
d
```

mupad [B] time = 13.19, size = 226, normalized size = 2.86

$$\frac{A\,a^3\,\mathrm{atan}\left(\frac{A^2\,a^6}{4\,A^2\,a^6+A^2\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{4\,A^2\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,A^2\,a^6+A^2\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{3\,A\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-8\,A\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-8\,A\,a^3\,t}{d\,\left(2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^2,x)
```

```
[Out] (A*a^3*atan((A^2*a^6)/(4*A^2*a^6 + A^2*a^6*tan(c/2 + (d*x)/2)) - (4*A^2*a^6
*tan(c/2 + (d*x)/2))/(4*A^2*a^6 + A^2*a^6*tan(c/2 + (d*x)/2))))/d - (A*a^3
- 8*A*a^3*tan(c/2 + (d*x)/2) - 8*A*a^3*tan(c/2 + (d*x)/2)^3 + 3*A*a^3*tan(c
/2 + (d*x)/2)^4)/(d*(2*tan(c/2 + (d*x)/2) + 4*tan(c/2 + (d*x)/2)^3 + 2*tan(
c/2 + (d*x)/2)^5)) + (2*A*a^3*log(tan(c/2 + (d*x)/2)))/d + (A*a^3*tan(c/2 +
 (d*x)/2))/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-Aa^3\left(\int\left(-2\sin(c+dx)\csc^2(c+dx)\right)dx+\int 2\sin^3(c+dx)\csc^2(c+dx)\,dx+\int\sin^4(c+dx)\csc^2(c+dx)\,d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] -A*a**3*(Integral(-2*sin(c + d*x)*csc(c + d*x)**2, x) + Integral(2*sin(c +
d*x)**3*csc(c + d*x)**2, x) + Integral(sin(c + d*x)**4*csc(c + d*x)**2, x)
+ Integral(-csc(c + d*x)**2, x))
```

3.230 $\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=78

$$\frac{a^3A\cos(c+dx)}{d}-\frac{2a^3A\cot(c+dx)}{d}-\frac{a^3A\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3A\cot(c+dx)\csc(c+dx)}{2d}-2a^3Ax$$

```
[Out] -2*a^3*A*x-1/2*a^3*A*arctanh(cos(d*x+c))/d+a^3*A*cos(d*x+c)/d-2*a^3*A*cot(d
*x+c)/d-1/2*a^3*A*cot(d*x+c)*csc(d*x+c)/d
```

Rubi [A] time = 0.12, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2966, 3767, 8, 3768, 3770, 2638}

$$\frac{a^3A\cos(c+dx)}{d}-\frac{2a^3A\cot(c+dx)}{d}-\frac{a^3A\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3A\cot(c+dx)\csc(c+dx)}{2d}-2a^3Ax$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] -2*a^3*A*x - (a^3*A*ArcTanh[Cos[c + d*x]])/(2*d) + (a^3*A*Cos[c + d*x])/d -
 (2*a^3*A*Cot[c + d*x])/d - (a^3*A*Cot[c + d*x]*Csc[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \int \left(-2a^3A+2a^3A\csc^2(c+dx)+a^3A\csc^3(c+dx)\right.$$

$$= -2a^3Ax+\left(a^3A\right)\int \csc^3(c+dx)\,dx-\left(a^3A\right)\int \sin($$

$$= -2a^3Ax+\frac{a^3A\cos(c+dx)}{d}-\frac{a^3A\cot(c+dx)\csc(c}{2d}$$

$$= -2a^3Ax-\frac{a^3A\tanh^{-1}(\cos(c+dx))}{2d}+\frac{a^3A\cos(c+}{d}$$

Mathematica [A] time = 0.04, size = 142, normalized size = 1.82

$$-\frac{a^3A\sin(c)\sin(dx)}{d}+\frac{a^3A\cos(c)\cos(dx)}{d}-\frac{2a^3A\cot(c+dx)}{d}-\frac{a^3A\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a^3A\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a^3A}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] -2*a^3*A*x + (a^3*A*Cos[c]*Cos[d*x])/d - (2*a^3*A*Cot[c + d*x])/d - (a^3*A*
Csc[(c + d*x)/2]^2)/(8*d) - (a^3*A*Log[Cos[(c + d*x)/2]])/(2*d) + (a^3*A*Lo
g[Sin[(c + d*x)/2]])/(2*d) + (a^3*A*Sec[(c + d*x)/2]^2)/(8*d) - (a^3*A*Sin[
c]*Sin[d*x])/d
```

fricas [B] time = 0.48, size = 152, normalized size = 1.95

$$-\frac{8\,Aa^3dx\cos(dx+c)^2-4\,Aa^3\cos(dx+c)^3-8\,Aa^3dx-8\,Aa^3\cos(dx+c)\sin(dx+c)+2\,Aa^3\cos(dx+c)+}{4\left(d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/4*(8*A*a^3*d*x*cos(d*x + c)^2 - 4*A*a^3*cos(d*x + c)^3 - 8*A*a^3*d*x - 8
*A*a^3*cos(d*x + c)*sin(d*x + c) + 2*A*a^3*cos(d*x + c) + (A*a^3*cos(d*x +
c)^2 - A*a^3)*log(1/2*cos(d*x + c) + 1/2) - (A*a^3*cos(d*x + c)^2 - A*a^3)*
log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.20, size = 137, normalized size = 1.76

$$\frac{Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\,(dx+c)Aa^3+4\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+8\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{16\,Aa^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/8*(A*a^3*tan(1/2*d*x + 1/2*c)^2 - 16*(d*x + c)*A*a^3 + 4*A*a^3*log(abs(ta
n(1/2*d*x + 1/2*c))) + 8*A*a^3*tan(1/2*d*x + 1/2*c) + 16*A*a^3/(tan(1/2*d*x
 + 1/2*c)^2 + 1) - (6*A*a^3*tan(1/2*d*x + 1/2*c)^2 + 8*A*a^3*tan(1/2*d*x +
1/2*c) + A*a^3)/tan(1/2*d*x + 1/2*c)^2)/d
```

maple [A] time = 0.56, size = 94, normalized size = 1.21

$$\frac{a^3A\cos(dx+c)}{d}-2a^3Ax-\frac{2a^3Ac}{d}-\frac{2a^3A\cot(dx+c)}{d}-\frac{a^3A\cot(dx+c)\csc(dx+c)}{2d}+\frac{a^3A\ln(\csc(dx+c)-\cot(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

```
[Out] a^3*A*cos(d*x+c)/d-2*a^3*A*x-2/d*a^3*A*c-2*a^3*A*cot(d*x+c)/d-1/2*a^3*A*cot
(d*x+c)*csc(d*x+c)/d+1/2/d*a^3*A*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.40, size = 90, normalized size = 1.15

$$-\frac{8\,(dx+c)Aa^3-Aa^3\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)-4\,Aa^3\cos(dx+c)+\frac{8\,Aa^3}{\tan(dx+c)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/4*(8*(d*x + c)*A*a^3 - A*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(
cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 4*A*a^3*cos(d*x + c) + 8*A*a^3
/tan(d*x + c))/d
```

mupad [**B**] time = 13.50, size = 220, normalized size = 2.82

$$\frac{A\,a^3\left(\frac{\cos(c+d\,x)}{2}-4\,\mathrm{atan}\left(\frac{\sqrt{17}\,\left(4\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{17\,\cos\left(\frac{c}{2}-\mathrm{atan}(4)+\frac{d\,x}{2}\right)}\right)-\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}+\cos(2\,c+2\,d\,x)+\frac{\cos(3\,c+3\,d\,x)}{2}+2\,\sin(2\,c\right.}{2\,d\,\left(\cos(c+d\,x)^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^3,x)
```

```
[Out] (A*a^3*(cos(c + d*x)/2 - 4*atan((17^(1/2)*(4*cos(c/2 + (d*x)/2) - sin(c/2 +
 (d*x)/2)))/(17*cos(c/2 - atan(4) + (d*x)/2))) - log(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2))/2 + cos(2*c + 2*d*x) + cos(3*c + 3*d*x)/2 + 2*sin(2*c + 2*
d*x) + 4*atan((17^(1/2)*(4*cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2)))/(17*co
s(c/2 - atan(4) + (d*x)/2)))*cos(2*c + 2*d*x) + (log(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/2 - 1))/(2*d*(cos(c + d*x)^2 - 1))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.231 $\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=78

$$-\frac{a^3A\cot^3(c+dx)}{3d}-\frac{a^3A\cot(c+dx)}{d}+\frac{a^3A\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3A\cot(c+dx)\csc(c+dx)}{d}-a^3Ax$$

```
[Out] -a^3*A*x+a^3*A*arctanh(cos(d*x+c))/d-a^3*A*cot(d*x+c)/d-1/3*a^3*A*cot(d*x+c
)^3/d-a^3*A*cot(d*x+c)*csc(d*x+c)/d
```

Rubi [A] time = 0.13, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2966, 3770, 3768, 3767}

$$-\frac{a^3A\cot^3(c+dx)}{3d}-\frac{a^3A\cot(c+dx)}{d}+\frac{a^3A\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3A\cot(c+dx)\csc(c+dx)}{d}-a^3Ax$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] -(a^3*A*x) + (a^3*A*ArcTanh[Cos[c + d*x]])/d - (a^3*A*Cot[c + d*x])/d - (a^
3*A*Cot[c + d*x]^3)/(3*d) - (a^3*A*Cot[c + d*x]*Csc[c + d*x])/d
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \int \left(-a^3A-2a^3A\csc(c+dx)+2a^3A\csc^3(c+dx)\right.$$
$$= -a^3Ax+\left(a^3A\right)\int \csc^4(c+dx)\,dx-\left(2a^3A\right)\int \csc($$
$$= -a^3Ax+\frac{2a^3A\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3A\cot(c+}{}$$
$$= -a^3Ax+\frac{a^3A\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3A\cot(c+d}{d}$$

Mathematica [A] time = 0.49, size = 141, normalized size = 1.81

$$-\frac{a^3A\left(-8\tan\left(\frac{1}{2}(c+dx)\right)+8\cot\left(\frac{1}{2}(c+dx)\right)+6\csc^2\left(\frac{1}{2}(c+dx)\right)-6\sec^2\left(\frac{1}{2}(c+dx)\right)+24\log\left(\sin\left(\frac{1}{2}(c+dx\right)\right.\right.}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] -1/24*(a^3*A*(24*c + 24*d*x + 8*Cot[(c + d*x)/2] + 6*Csc[(c + d*x)/2]^2 - 2
4*Log[Cos[(c + d*x)/2]] + 24*Log[Sin[(c + d*x)/2]] - 6*Sec[(c + d*x)/2]^2 -
 8*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + (Csc[(c + d*x)/2]^4*Sin[c + d*x])/2
- 8*Tan[(c + d*x)/2]))/d
```

fricas [B] time = 0.46, size = 175, normalized size = 2.24

$$-\frac{4\,Aa^3\cos(dx+c)^3-6\,Aa^3\cos(dx+c)-3\left(Aa^3\cos(dx+c)^2-Aa^3\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+}{6\left(d\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/6*(4*A*a^3*cos(d*x + c)^3 - 6*A*a^3*cos(d*x + c) - 3*(A*a^3*cos(d*x + c)
^2 - A*a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(A*a^3*cos(d*x + c
)^2 - A*a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 6*(A*a^3*d*x*cos(d
*x + c)^2 - A*a^3*d*x - A*a^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^
2 - d)*sin(d*x + c))
```

giac [A] time = 0.18, size = 150, normalized size = 1.92

$$\frac{Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\,(dx+c)Aa^3-24\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+9\,Aa^3\tan\left(\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/24*(A*a^3*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 24*(d
*x + c)*A*a^3 - 24*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 9*A*a^3*tan(1/2*d
*x + 1/2*c) + (44*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 9*A*a^3*tan(1/2*d*x + 1/2*
c)^2 - 6*A*a^3*tan(1/2*d*x + 1/2*c) - A*a^3)/tan(1/2*d*x + 1/2*c)^3)/d
```

maple [A] time = 0.53, size = 103, normalized size = 1.32

$$-a^3Ax-\frac{a^3Ac}{d}-\frac{a^3A\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{d}-\frac{a^3A\cot\left(dx+c\right)\csc\left(dx+c\right)}{d}-\frac{2a^3A\cot\left(dx+c\right)}{3d}-\frac{a^3A\cot(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

```
[Out] -a^3*A*x-1/d*a^3*A*c-1/d*a^3*A*ln(csc(d*x+c)-cot(d*x+c))-a^3*A*cot(d*x+c)*c
sc(d*x+c)/d-2/3*a^3*A*cot(d*x+c)/d-1/3/d*a^3*A*cot(d*x+c)*csc(d*x+c)^2
```

maxima [A] time = 0.45, size = 117, normalized size = 1.50

$$-\frac{6\,(dx+c)Aa^3-3\,Aa^3\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-6\,Aa^3\left(\log\left(\cos\left(dx+c\right)+\right.\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/6*(6*(d*x + c)*A*a^3 - 3*A*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - lo
g(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 6*A*a^3*(log(cos(d*x + c) +
```

```
1) - log(cos(d*x + c) - 1)) + 2*(3*tan(d*x + c)^2 + 1)*A*a^3/tan(d*x + c)^3
)/d
```

mupad [B] time = 13.32, size = 245, normalized size = 3.14

$$-\frac{\frac{A\,a^3\,\sin(2\,c+2\,d\,x)}{2}-\frac{A\,a^3\,\cos(3\,c+3\,d\,x)}{6}+\frac{A\,a^3\,\cos(c+d\,x)}{2}-\frac{A\,a^3\,\sin(3\,c+3\,d\,x)\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{d\,x}{2}\right)}\right)}{2}+\frac{3\,A\,a^3\,\sin(c+d\,x)\,\ln\left(\right.}{4}}{\frac{3\,d\,\sin(c+d\,x)}{4}-\frac{d\,\sin(3\,c+3\,d\,x)}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^4,x)
```

```
[Out] -((A*a^3*sin(2*c + 2*d*x))/2 - (A*a^3*cos(3*c + 3*d*x))/6 + (A*a^3*cos(c +
d*x))/2 - (A*a^3*sin(3*c + 3*d*x)*atan((2^(1/2)*(cos(c/2 + (d*x)/2) + sin(c
/2 + (d*x)/2)))/(2*cos(c/2 + pi/4 + (d*x)/2))))/2 + (3*A*a^3*sin(c + d*x)*l
og(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 + (3*A*a^3*sin(c + d*x)*atan((
2^(1/2)*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2)))/(2*cos(c/2 + pi/4 + (d*x
)/2))))/2 - (A*a^3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d
*x))/4)/((3*d*sin(c + d*x))/4 - (d*sin(3*c + 3*d*x))/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.232 $\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=86

$$-\frac{2a^3A\cot^3(c+dx)}{3d}+\frac{5a^3A\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^3A\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a^3A\cot(c+dx)\csc(c+dx)}{8d}$$

```
[Out] 5/8*a^3*A*arctanh(cos(d*x+c))/d-2/3*a^3*A*cot(d*x+c)^3/d-3/8*a^3*A*cot(d*x+
c)*csc(d*x+c)/d-1/4*a^3*A*cot(d*x+c)*csc(d*x+c)^3/d
```

Rubi [A] time = 0.15, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2966, 3770, 3767, 8, 3768}

$$-\frac{2a^3A\cot^3(c+dx)}{3d}+\frac{5a^3A\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^3A\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a^3A\cot(c+dx)\csc(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (5*a^3*A*ArcTanh[Cos[c + d*x]])/(8*d) - (2*a^3*A*Cot[c + d*x]^3)/(3*d) - (3
*a^3*A*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a^3*A*Cot[c + d*x]*Csc[c + d*x]^
3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx &= \int \left(-a^3A\csc(c+dx)-2a^3A\csc^2(c+dx)+2a^3A\,c\right.\\ &= -\left(\left(a^3A\right)\int \csc(c+dx)\,dx\right)+\left(a^3A\right)\int \csc^5(c+dx\\ &= \frac{a^3A\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3A\cot(c+dx)\csc^3(c}{4d}\\ &= \frac{a^3A\tanh^{-1}(\cos(c+dx))}{d}-\frac{2a^3A\cot^3(c+dx)}{3d}-\frac{3a}{}\\ &= \frac{5a^3A\tanh^{-1}(\cos(c+dx))}{8d}-\frac{2a^3A\cot^3(c+dx)}{3d}-\frac{3}{}\end{aligned}$$

Mathematica [B] time = 0.07, size = 210, normalized size = 2.44

$$a^3A\left(-\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{3d}+\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{3d}-\frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{3\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{3\sec^2\left(\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] a^3*A*(Cot[(c + d*x)/2]/(3*d) - (3*Csc[(c + d*x)/2]^2)/(32*d) - (Cot[(c + d
*x)/2]*Csc[(c + d*x)/2]^2)/(12*d) - Csc[(c + d*x)/2]^4/(64*d) + (5*Log[Cos[
(c + d*x)/2]])/(8*d) - (5*Log[Sin[(c + d*x)/2]])/(8*d) + (3*Sec[(c + d*x)/2
]^2)/(32*d) + Sec[(c + d*x)/2]^4/(64*d) - Tan[(c + d*x)/2]/(3*d) + (Sec[(c
+ d*x)/2]^2*Tan[(c + d*x)/2])/(12*d))
```

fricas [B] time = 0.45, size = 166, normalized size = 1.93

$$-\frac{32\,Aa^3\cos\left(dx+c\right)^3\sin\left(dx+c\right)-18\,Aa^3\cos\left(dx+c\right)^3+30\,Aa^3\cos\left(dx+c\right)-15\left(Aa^3\cos\left(dx+c\right)^4-2\,Aa^3\mathrm{c}\right.}{48\left(d\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/48*(32*A*a^3*cos(d*x + c)^3*sin(d*x + c) - 18*A*a^3*cos(d*x + c)^3 + 30*
A*a^3*cos(d*x + c) - 15*(A*a^3*cos(d*x + c)^4 - 2*A*a^3*cos(d*x + c)^2 + A*
a^3)*log(1/2*cos(d*x + c) + 1/2) + 15*(A*a^3*cos(d*x + c)^4 - 2*A*a^3*cos(d
*x + c)^2 + A*a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^4 - 2*d*co
s(d*x + c)^2 + d)
```

giac [B] time = 0.20, size = 174, normalized size = 2.02

$$\frac{3\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+16\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-120\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/192*(3*A*a^3*tan(1/2*d*x + 1/2*c)^4 + 16*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 2
4*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 120*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) -
 48*A*a^3*tan(1/2*d*x + 1/2*c) + (250*A*a^3*tan(1/2*d*x + 1/2*c)^4 + 48*A*a
^3*tan(1/2*d*x + 1/2*c)^3 - 24*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 16*A*a^3*tan(
1/2*d*x + 1/2*c) - 3*A*a^3)/tan(1/2*d*x + 1/2*c)^4)/d
```

maple [A] time = 0.62, size = 109, normalized size = 1.27

$$-\frac{5a^3A\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{8d}+\frac{2a^3A\cot\left(dx+c\right)}{3d}-\frac{2a^3A\cot\left(dx+c\right)\left(\csc^2\left(dx+c\right)\right)}{3d}-\frac{a^3A\cot\left(dx+c\right)\left(\mathrm{cs}\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

```
[Out] -5/8/d*a^3*A*ln(csc(d*x+c)-cot(d*x+c))+2/3*a^3*A*cot(d*x+c)/d-2/3/d*a^3*A*c
ot(d*x+c)*csc(d*x+c)^2-1/4*a^3*A*cot(d*x+c)*csc(d*x+c)^3/d-3/8*a^3*A*cot(d*
x+c)*csc(d*x+c)/d
```

maxima [A] time = 0.43, size = 145, normalized size = 1.69

$$\frac{3\,Aa^3\left(\frac{2\left(3\,\cos(dx+c)^3-5\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+24\,Aa^3\left(\log\left(\cos\left(dx+c\right)\right.\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/48*(3*A*a^3*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*co
s(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 24
*A*a^3*(log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)) + 96*A*a^3/tan(d*x +
 c) - 32*(3*tan(d*x + c)^2 + 1)*A*a^3/tan(d*x + c)^3)/d
```

mupad [B] time = 13.12, size = 244, normalized size = 2.84

$$-\frac{A\,a^3\left(3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-16\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+16\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^5,x)
```

```
[Out] -(A*a^3*(3*cos(c/2 + (d*x)/2)^8 - 3*sin(c/2 + (d*x)/2)^8 - 16*cos(c/2 + (d*
x)/2)*sin(c/2 + (d*x)/2)^7 + 16*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2) - 2
4*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^6 + 48*cos(c/2 + (d*x)/2)^3*sin(c
/2 + (d*x)/2)^5 - 48*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^3 + 24*cos(c/2
 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2 + 120*log(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2))*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^4))/(192*d*cos(c/2 + (d*x)
/2)^4*sin(c/2 + (d*x)/2)^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.233 $\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=105

$$-\frac{a^3A\cot^5(c+dx)}{5d}-\frac{2a^3A\cot^3(c+dx)}{3d}+\frac{a^3A\tanh^{-1}(\cos(c+dx))}{4d}-\frac{a^3A\cot(c+dx)\csc^3(c+dx)}{2d}+\frac{a^3A\cot(c+dx)}{4d}$$

```
[Out] 1/4*a^3*A*arctanh(cos(d*x+c))/d-2/3*a^3*A*cot(d*x+c)^3/d-1/5*a^3*A*cot(d*x+
c)^5/d+1/4*a^3*A*cot(d*x+c)*csc(d*x+c)/d-1/2*a^3*A*cot(d*x+c)*csc(d*x+c)^3/
d
```

Rubi [A] time = 0.23, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2950, 2709, 3767, 8, 3768, 3770}

$$-\frac{a^3A\cot^5(c+dx)}{5d}-\frac{2a^3A\cot^3(c+dx)}{3d}+\frac{a^3A\tanh^{-1}(\cos(c+dx))}{4d}-\frac{a^3A\cot(c+dx)\csc^3(c+dx)}{2d}+\frac{a^3A\cot(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*ArcTanh[Cos[c + d*x]])/(4*d) - (2*a^3*A*Cot[c + d*x]^3)/(3*d) - (a^3
*A*Cot[c + d*x]^5)/(5*d) + (a^3*A*Cot[c + d*x]*Csc[c + d*x])/(4*d) - (a^3*A
*Cot[c + d*x]*Csc[c + d*x]^3)/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2950

```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^n*c^n,
 Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
```

```
0] && IntegerQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \left(a^3A^3\right)\int \frac{\cot^6(c+dx)}{(A-A\sin(c+dx))^2}\,dx$$

$$= \frac{a^3\int\left(-A^4\csc^2(c+dx)-2A^4\csc^3(c+dx)+2A^4 \text{c}\right.}{A^3}$$

$$= -\left(\left(a^3A\right)\int\csc^2(c+dx)\,dx\right)+\left(a^3A\right)\int\csc^6(c+d$$

$$= \frac{a^3A\cot(c+dx)\csc(c+dx)}{d}-\frac{a^3A\cot(c+dx)\csc^3}{2d}$$

$$= \frac{a^3A\tanh^{-1}(\cos(c+dx))}{d}-\frac{2a^3A\cot^3(c+dx)}{3d}-\frac{a^3}{}$$

$$= \frac{a^3A\tanh^{-1}(\cos(c+dx))}{4d}-\frac{2a^3A\cot^3(c+dx)}{3d}-\frac{a^3}{}$$

Mathematica [B] time = 0.07, size = 268, normalized size = 2.55

$$a^3A\left(-\frac{7\tan\left(\frac{1}{2}(c+dx)\right)}{30d}+\frac{7\cot\left(\frac{1}{2}(c+dx)\right)}{30d}-\frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{16d}+\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{\sec^2\left(\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] a^3*A*((7*Cot[(c + d*x)/2])/(30*d) + Csc[(c + d*x)/2]^2/(16*d) - (19*Cot[(c
 + d*x)/2]*Csc[(c + d*x)/2]^2)/(480*d) - Csc[(c + d*x)/2]^4/(32*d) - (Cot[(
c + d*x)/2]*Csc[(c + d*x)/2]^4)/(160*d) + Log[Cos[(c + d*x)/2]]/(4*d) - Log
[Sin[(c + d*x)/2]]/(4*d) - Sec[(c + d*x)/2]^2/(16*d) + Sec[(c + d*x)/2]^4/(
32*d) - (7*Tan[(c + d*x)/2])/(30*d) + (19*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2])/(480*d) + (Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(160*d))
```

fricas [B] time = 0.43, size = 201, normalized size = 1.91

$$56\,Aa^3\cos(dx+c)^5 - 80\,Aa^3\cos(dx+c)^3 + 15\left(Aa^3\cos(dx+c)^4 - 2\,Aa^3\cos(dx+c)^2 + Aa^3\right)\log\left(\frac{1}{2}\,\cos(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(56*A*a^3*cos(d*x + c)^5 - 80*A*a^3*cos(d*x + c)^3 + 15*(A*a^3*cos(d*
x + c)^4 - 2*A*a^3*cos(d*x + c)^2 + A*a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(
d*x + c) - 15*(A*a^3*cos(d*x + c)^4 - 2*A*a^3*cos(d*x + c)^2 + A*a^3)*log(-
1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 30*(A*a^3*cos(d*x + c)^3 + A*a^3*cos
(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d
*x + c))
```

giac [A] time = 0.19, size = 174, normalized size = 1.66

$$\frac{3\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 15\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 25\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 120\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/480*(3*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 15*A*a^3*tan(1/2*d*x + 1/2*c)^4 + 2
5*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 120*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) -
 90*A*a^3*tan(1/2*d*x + 1/2*c) + (274*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 90*A*a
^3*tan(1/2*d*x + 1/2*c)^4 - 25*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 15*A*a^3*tan(
1/2*d*x + 1/2*c) - 3*A*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

maple [A] time = 0.72, size = 132, normalized size = 1.26

$$\frac{7a^3A\cot(dx+c)}{15d}+\frac{a^3A\cot(dx+c)\csc(dx+c)}{4d}-\frac{a^3A\ln(\csc(dx+c)-\cot(dx+c))}{4d}-\frac{a^3A\cot(dx+c)\left(\csc^3(dx\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

```
[Out] 7/15*a^3*A*cot(d*x+c)/d+1/4*a^3*A*cot(d*x+c)*csc(d*x+c)/d-1/4/d*a^3*A*ln(cs
c(d*x+c)-cot(d*x+c))-1/2*a^3*A*cot(d*x+c)*csc(d*x+c)^3/d-1/5/d*a^3*A*cot(d*
x+c)*csc(d*x+c)^4-4/15/d*a^3*A*cot(d*x+c)*csc(d*x+c)^2
```

maxima [A] time = 0.44, size = 175, normalized size = 1.67

$$\frac{15\,Aa^3\left(\frac{2\left(3\,\cos(dx+c)^3-5\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-60\,Aa^3\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-\log\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/120*(15*A*a^3*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*
cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) -
60*A*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) + log
(cos(d*x + c) - 1)) + 120*A*a^3/tan(d*x + c) - 8*(15*tan(d*x + c)^4 + 10*ta
n(d*x + c)^2 + 3)*A*a^3/tan(d*x + c)^5)/d
```

mupad [B] time = 13.16, size = 244, normalized size = 2.32

$$-\frac{A\,a^3\,\left(3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-15\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+15\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^6,x)
```

```
[Out] -(A*a^3*(3*cos(c/2 + (d*x)/2)^10 - 3*sin(c/2 + (d*x)/2)^10 - 15*cos(c/2 + (
d*x)/2)*sin(c/2 + (d*x)/2)^9 + 15*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2) -
 25*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^8 + 90*cos(c/2 + (d*x)/2)^4*sin
(c/2 + (d*x)/2)^6 - 90*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 + 25*cos(c
/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^2 + 120*log(sin(c/2 + (d*x)/2)/cos(c/2 +
```

```
 (d*x)/2))*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5))/(480*d*cos(c/2 + (d*
x)/2)^5*sin(c/2 + (d*x)/2)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.234 $\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=130

$$-\frac{2a^3A\cot^5(c+dx)}{5d}-\frac{2a^3A\cot^3(c+dx)}{3d}+\frac{3a^3A\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^3A\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{5a^3A\cot(c}{}$$

```
[Out] 3/16*a^3*A*arctanh(cos(d*x+c))/d-2/3*a^3*A*cot(d*x+c)^3/d-2/5*a^3*A*cot(d*x
+c)^5/d+3/16*a^3*A*cot(d*x+c)*csc(d*x+c)/d-5/24*a^3*A*cot(d*x+c)*csc(d*x+c)
^3/d-1/6*a^3*A*cot(d*x+c)*csc(d*x+c)^5/d
```

Rubi [A] time = 0.20, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2966, 3768, 3770, 3767}

$$-\frac{2a^3A\cot^5(c+dx)}{5d}-\frac{2a^3A\cot^3(c+dx)}{3d}+\frac{3a^3A\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^3A\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{5a^3A\cot(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (3*a^3*A*ArcTanh[Cos[c + d*x]])/(16*d) - (2*a^3*A*Cot[c + d*x]^3)/(3*d) - (
2*a^3*A*Cot[c + d*x]^5)/(5*d) + (3*a^3*A*Cot[c + d*x]*Csc[c + d*x])/(16*d)
- (5*a^3*A*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a^3*A*Cot[c + d*x]*Csc[c
+ d*x]^5)/(6*d)
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \int \left(-a^3A\csc^3(c+dx) - 2a^3A\csc^4(c+dx) + 2a^3A\,\mathrm{cs}\right.$$

$$= -\left(\left(a^3A\right)\int \csc^3(c+dx)\,dx\right) + \left(a^3A\right)\int \csc^7(c+dx)$$

$$= \frac{a^3A\cot(c+dx)\csc(c+dx)}{2d} - \frac{a^3A\cot(c+dx)\csc^5(c}{6d}$$

$$= \frac{a^3A\tanh^{-1}(\cos(c+dx))}{2d} - \frac{2a^3A\cot^3(c+dx)}{3d} - \frac{2a^3}{}$$

$$= \frac{a^3A\tanh^{-1}(\cos(c+dx))}{2d} - \frac{2a^3A\cot^3(c+dx)}{3d} - \frac{2a^3}{}$$

$$= \frac{3a^3A\tanh^{-1}(\cos(c+dx))}{16d} - \frac{2a^3A\cot^3(c+dx)}{3d} - \frac{2a}{}$$

Mathematica [B] time = 0.08, size = 306, normalized size = 2.35

$$a^3A\left(-\frac{2\tan\left(\frac{1}{2}(c+dx)\right)}{15d} + \frac{2\cot\left(\frac{1}{2}(c+dx)\right)}{15d} - \frac{\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d} - \frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{3\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{\sec^6\left(\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] a^3*A*((2*Cot[(c + d*x)/2])/(15*d) + (3*Csc[(c + d*x)/2]^2)/(64*d) + (Cot[(
c + d*x)/2]*Csc[(c + d*x)/2]^2)/(240*d) - Csc[(c + d*x)/2]^4/(64*d) - (Cot[
(c + d*x)/2]*Csc[(c + d*x)/2]^4)/(80*d) - Csc[(c + d*x)/2]^6/(384*d) + (3*L
og[Cos[(c + d*x)/2]])/(16*d) - (3*Log[Sin[(c + d*x)/2]])/(16*d) - (3*Sec[(c
 + d*x)/2]^2)/(64*d) + Sec[(c + d*x)/2]^4/(64*d) + Sec[(c + d*x)/2]^6/(384*
d) - (2*Tan[(c + d*x)/2])/(15*d) - (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(2
40*d) + (Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(80*d))
```

fricas [B] time = 0.47, size = 240, normalized size = 1.85

$$-\frac{90\,Aa^3\cos\left(dx+c\right)^5-80\,Aa^3\cos\left(dx+c\right)^3-90\,Aa^3\cos\left(dx+c\right)-45\left(Aa^3\cos\left(dx+c\right)^6-3\,Aa^3\cos\left(dx+c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/480*(90*A*a^3*cos(d*x + c)^5 - 80*A*a^3*cos(d*x + c)^3 - 90*A*a^3*cos(d*
x + c) - 45*(A*a^3*cos(d*x + c)^6 - 3*A*a^3*cos(d*x + c)^4 + 3*A*a^3*cos(d*
x + c)^2 - A*a^3)*log(1/2*cos(d*x + c) + 1/2) + 45*(A*a^3*cos(d*x + c)^6 -
3*A*a^3*cos(d*x + c)^4 + 3*A*a^3*cos(d*x + c)^2 - A*a^3)*log(-1/2*cos(d*x +
 c) + 1/2) + 64*(2*A*a^3*cos(d*x + c)^5 - 5*A*a^3*cos(d*x + c)^3)*sin(d*x +
 c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

giac [B] time = 0.20, size = 242, normalized size = 1.86

$$\frac{5\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+24\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+45\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+40\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/1920*(5*A*a^3*tan(1/2*d*x + 1/2*c)^6 + 24*A*a^3*tan(1/2*d*x + 1/2*c)^5 +
45*A*a^3*tan(1/2*d*x + 1/2*c)^4 + 40*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^
3*tan(1/2*d*x + 1/2*c)^2 - 360*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 240*A
*a^3*tan(1/2*d*x + 1/2*c) + (882*A*a^3*tan(1/2*d*x + 1/2*c)^6 + 240*A*a^3*t
an(1/2*d*x + 1/2*c)^5 + 15*A*a^3*tan(1/2*d*x + 1/2*c)^4 - 40*A*a^3*tan(1/2*
d*x + 1/2*c)^3 - 45*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 24*A*a^3*tan(1/2*d*x + 1
/2*c) - 5*A*a^3)/tan(1/2*d*x + 1/2*c)^6)/d
```

maple [A] time = 0.73, size = 155, normalized size = 1.19

$$\frac{3a^3A\cot\left(dx+c\right)\csc\left(dx+c\right)}{16d}-\frac{3a^3A\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{16d}+\frac{4a^3A\cot\left(dx+c\right)}{15d}+\frac{2a^3A\cot\left(dx+c\right)\left(\csc\right.}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

```
[Out] 3/16*a^3*A*cot(d*x+c)*csc(d*x+c)/d-3/16/d*a^3*A*ln(csc(d*x+c)-cot(d*x+c))+4
/15*a^3*A*cot(d*x+c)/d+2/15/d*a^3*A*cot(d*x+c)*csc(d*x+c)^2-2/5/d*a^3*A*cot
(d*x+c)*csc(d*x+c)^4-1/6*a^3*A*cot(d*x+c)*csc(d*x+c)^5/d-5/24*a^3*A*cot(d*x
+c)*csc(d*x+c)^3/d
```

maxima [A] time = 0.60, size = 207, normalized size = 1.59

$$\frac{5\,Aa^3\left(\frac{2\left(15\,\cos(dx+c)^5-40\,\cos(dx+c)^3+33\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)-120\,Aa^3\left(\ldots\right.}{4\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/480*(5*A*a^3*(2*(15*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 33*cos(d*x + c))
/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 15*log(cos(d*
x + c) + 1) + 15*log(cos(d*x + c) - 1)) - 120*A*a^3*(2*cos(d*x + c)/(cos(d*
x + c)^2 - 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 320*(3*tan
(d*x + c)^2 + 1)*A*a^3/tan(d*x + c)^3 - 64*(15*tan(d*x + c)^4 + 10*tan(d*x
+ c)^2 + 3)*A*a^3/tan(d*x + c)^5)/d
```

mupad [B] time = 13.37, size = 340, normalized size = 2.62

$$-\frac{A\,a^3\left(5\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-24\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+24\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^7,x)
```

```
[Out] -(A*a^3*(5*cos(c/2 + (d*x)/2)^12 - 5*sin(c/2 + (d*x)/2)^12 - 24*cos(c/2 + (
d*x)/2)*sin(c/2 + (d*x)/2)^11 + 24*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)
 - 45*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 - 40*cos(c/2 + (d*x)/2)^3*
sin(c/2 + (d*x)/2)^9 + 15*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 + 240*c
os(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 - 240*cos(c/2 + (d*x)/2)^7*sin(c/2
 + (d*x)/2)^5 - 15*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 + 40*cos(c/2 +
 (d*x)/2)^9*sin(c/2 + (d*x)/2)^3 + 45*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)
/2)^2 + 360*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^6
*sin(c/2 + (d*x)/2)^6))/(1920*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**7*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Timed out
```

3.235 $\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=129

$$-\frac{4A\cos(c+dx)}{a^3d}+\frac{A\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{199A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}+\frac{41A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}-\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)$$

```
[Out] -19/2*A*x/a^3-4*A*cos(d*x+c)/a^3/d+1/2*A*cos(d*x+c)*sin(d*x+c)/a^3/d-2/5*A*
cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3+41/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^2
-199/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A] time = 0.21, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2966, 2638, 2635, 8, 2650, 2648}

$$-\frac{4A\cos(c+dx)}{a^3d}+\frac{A\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{199A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}+\frac{41A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}-\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^4*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-19*A*x)/(2*a^3) - (4*A*Cos[c + d*x])/(a^3*d) + (A*Cos[c + d*x]*Sin[c + d*
x])/(2*a^3*d) - (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) + (41*A*C
os[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x])^2) - (199*A*Cos[c + d*x])/(15*a^3
*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \int \left(-\frac{9A}{a^3}+\frac{4A\sin(c+dx)}{a^3}-\frac{A\sin^2(c+dx)}{a^3}+\frac{2A}{a^3(1+\sin(c+dx))^3}-\right.$$

$$= -\frac{9Ax}{a^3}-\frac{A\int\sin^2(c+dx)\,dx}{a^3}+\frac{(2A)\int\frac{1}{(1+\sin(c+dx))^3}\,dx}{a^3}+\frac{(4A)\int\sin(c}{a^3}$$

$$= -\frac{9Ax}{a^3}-\frac{4A\cos(c+dx)}{a^3d}+\frac{A\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{2A\cos(c+}{5a^3d(1+\sin(c}$$

$$= -\frac{19Ax}{2a^3}-\frac{4A\cos(c+dx)}{a^3d}+\frac{A\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{2A\cos(c}{5a^3d(1+\sin(}$$

$$= -\frac{19Ax}{2a^3}-\frac{4A\cos(c+dx)}{a^3d}+\frac{A\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{2A\cos(c}{5a^3d(1+\sin(}$$

Mathematica [A] time = 0.94, size = 254, normalized size = 1.97

$$\frac{A\left(-11400dx\sin\left(c+\frac{dx}{2}\right)-5700dx\sin\left(c+\frac{3dx}{2}\right)+1830\sin\left(2c+\frac{3dx}{2}\right)-4234\sin\left(2c+\frac{5dx}{2}\right)+1140dx\sin\left(3c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^4*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (A*(-11400*d*x*Cos[(d*x)/2] + 12060*Cos[c + (d*x)/2] - 14090*Cos[c + (3*d*x
)/2] + 5700*d*x*Cos[2*c + (3*d*x)/2] + 1140*d*x*Cos[2*c + (5*d*x)/2] + 1050
*Cos[3*c + (5*d*x)/2] + 165*Cos[3*c + (7*d*x)/2] + 15*Cos[5*c + (9*d*x)/2]
+ 19780*Sin[(d*x)/2] - 11400*d*x*Sin[c + (d*x)/2] - 5700*d*x*Sin[c + (3*d*x
)/2] + 1830*Sin[2*c + (3*d*x)/2] - 4234*Sin[2*c + (5*d*x)/2] + 1140*d*x*Sin
[3*c + (5*d*x)/2] + 165*Sin[4*c + (7*d*x)/2] - 15*Sin[4*c + (9*d*x)/2]))/(4
80*a^3*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

fricas [B] time = 0.46, size = 248, normalized size = 1.92

$$-\frac{15\,A\cos\left(dx+c\right)^{5}+90\,A\cos\left(dx+c\right)^{4}+\left(285\,Adx+683\,A\right)\cos\left(dx+c\right)^{3}-1140\,Adx+\left(855\,Adx-526\,A\right)\cos}{30\left(a^{3}d\cos\left(dx+c\right)^{3}+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] -1/30*(15*A*cos(d*x + c)^5 + 90*A*cos(d*x + c)^4 + (285*A*d*x + 683*A)*cos(
d*x + c)^3 - 1140*A*d*x + (855*A*d*x - 526*A)*cos(d*x + c)^2 - 6*(95*A*d*x
+ 191*A)*cos(d*x + c) - (15*A*cos(d*x + c)^4 - 75*A*cos(d*x + c)^3 + 1140*A
*d*x - 19*(15*A*d*x - 32*A)*cos(d*x + c)^2 + 6*(95*A*d*x + 189*A)*cos(d*x +
 c) - 12*A)*sin(d*x + c) - 12*A)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x +
c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos
(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

giac [A] time = 0.21, size = 156, normalized size = 1.21

$$-\frac{\frac{285\,(dx+c)A}{a^{3}}+\frac{30\left(A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+8\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,A\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}a^{3}}+\frac{4\left(135\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+615\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+1025\,A\tan\right.}{a^{3}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/30*(285*(d*x + c)*A/a^3 + 30*(A*tan(1/2*d*x + 1/2*c)^3 + 8*A*tan(1/2*d*x
 + 1/2*c)^2 - A*tan(1/2*d*x + 1/2*c) + 8*A)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2
*a^3) + 4*(135*A*tan(1/2*d*x + 1/2*c)^4 + 615*A*tan(1/2*d*x + 1/2*c)^3 + 10
25*A*tan(1/2*d*x + 1/2*c)^2 + 685*A*tan(1/2*d*x + 1/2*c) + 164*A)/(a^3*(tan
(1/2*d*x + 1/2*c) + 1)^5))/d
```

maple [B] time = 0.45, size = 257, normalized size = 1.99

$$-\frac{A\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{3}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{8A\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{3}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^{3}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{8A}{d\,a^{3}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-19$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/d*A/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-8/d*A/a^3/(1+tan
(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2+1/d*A/a^3/(1+tan(1/2*d*x+1/2*c)^2
)^2*tan(1/2*d*x+1/2*c)-8/d*A/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2-19/d*A/a^3*arct
an(tan(1/2*d*x+1/2*c))-16/5/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^5+8/d*A/a^3/(tan
(1/2*d*x+1/2*c)+1)^4+4/3/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^3-10/d*A/a^3/(tan(1
/2*d*x+1/2*c)+1)^2-18/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.59, size = 715, normalized size = 5.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/15*(A*((1325*sin(d*x + c)/(cos(d*x + c) + 1) + 2673*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 3805*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4329*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 + 3575*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 227
5*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 975*sin(d*x + c)^7/(cos(d*x + c) +
1)^7 + 195*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 304)/(a^3 + 5*a^3*sin(d*x
+ c)/(cos(d*x + c) + 1) + 12*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 20*a
^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 26*a^3*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 + 26*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 20*a^3*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 + 12*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 5*a^
3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + a^3*sin(d*x + c)^9/(cos(d*x + c) +
1)^9) + 195*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) + 6*A*((105*sin(d*
x + c)/(cos(d*x + c) + 1) + 189*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 200*s
in(d*x + c)^3/(cos(d*x + c) + 1)^3 + 160*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + 75*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^6/(cos(d*x + c
) + 1)^6 + 24)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 11*a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
+ 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 11*a^3*sin(d*x + c)^5/(cos(d
*x + c) + 1)^5 + 5*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^3*sin(d*x +
c)^7/(cos(d*x + c) + 1)^7) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3
))/d
```

mupad [B] time = 17.08, size = 326, normalized size = 2.53

$$\frac{\left(\frac{95\,A\,(c+d\,x)}{2}-\frac{A\,(1425\,c+1425\,d\,x+570)}{30}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\left(114\,A\,(c+d\,x)-\frac{A\,(3420\,c+3420\,d\,x+2850)}{30}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(190\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^4*(A - A*sin(c + d*x)))/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((95*A*(c + d*x))/2 - (A*(1425*c + 1425*d*x + 3910))/30
) + tan(c/2 + (d*x)/2)^8*((95*A*(c + d*x))/2 - (A*(1425*c + 1425*d*x + 570)
)/30) + tan(c/2 + (d*x)/2)^7*(114*A*(c + d*x) - (A*(3420*c + 3420*d*x + 285
0))/30) + tan(c/2 + (d*x)/2)^2*(114*A*(c + d*x) - (A*(3420*c + 3420*d*x + 7
902))/30) + tan(c/2 + (d*x)/2)^6*(190*A*(c + d*x) - (A*(5700*c + 5700*d*x +
 6650))/30) + tan(c/2 + (d*x)/2)^3*(190*A*(c + d*x) - (A*(5700*c + 5700*d*x
 + 11270))/30) + tan(c/2 + (d*x)/2)^5*(247*A*(c + d*x) - (A*(7410*c + 7410*
d*x + 10450))/30) + tan(c/2 + (d*x)/2)^4*(247*A*(c + d*x) - (A*(7410*c + 74
10*d*x + 12846))/30) + (19*A*(c + d*x))/2 - (A*(285*c + 285*d*x + 896))/30)
/(a^3*d*(tan(c/2 + (d*x)/2) + 1)^5*(tan(c/2 + (d*x)/2)^2 + 1)^2) - (19*A*x)
/(2*a^3)
```

sympy [A] time = 78.44, size = 3614, normalized size = 28.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-285*A*d*x*tan(c/2 + d*x/2)**9/(30*a**3*d*tan(c/2 + d*x/2)**9 +
150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*
d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2
 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)
**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 1425*A*d*x*tan(c/2 + d*x/2
)**8/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*
a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*ta
n(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d
*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 3
0*a**3*d) - 3420*A*d*x*tan(c/2 + d*x/2)**7/(30*a**3*d*tan(c/2 + d*x/2)**9 +
 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3
*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/
2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2
)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 5700*A*d*x*tan(c/2 + d*x/
```

2)**6/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360
*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*t
an(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 +
d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) +
30*a**3*d) - 7410*A*d*x*tan(c/2 + d*x/2)**5/(30*a**3*d*tan(c/2 + d*x/2)**9
+ 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**
3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c
/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/
2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 7410*A*d*x*tan(c/2 + d*x
/2)**4/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 36
0*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*
tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 +
d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) +
30*a**3*d) - 5700*A*d*x*tan(c/2 + d*x/2)**3/(30*a**3*d*tan(c/2 + d*x/2)**9
+ 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a*
*3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(
c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x
/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 3420*A*d*x*tan(c/2 + d*
x/2)**2/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 3
60*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d
*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2
+ d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2)
+ 30*a**3*d) - 1425*A*d*x*tan(c/2 + d*x/2)/(30*a**3*d*tan(c/2 + d*x/2)**9 +
150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3
*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/
2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2
)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 285*A*d*x/(30*a**3*d*tan(
c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x
/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 +
780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*
d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 570*A*ta
n(c/2 + d*x/2)**8/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x
/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 +
780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*
d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2
+ d*x/2) + 30*a**3*d) - 2850*A*tan(c/2 + d*x/2)**7/(30*a**3*d*tan(c/2 + d*
x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 +
600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3
*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/
2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 6650*A*tan(c/2 +
d*x/2)**6/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8
+ 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**
3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c
/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/
2) + 30*a**3*d) - 10450*A*tan(c/2 + d*x/2)**5/(30*a**3*d*tan(c/2 + d*x/2)**

```
9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a
**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan
(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*
x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 12846*A*tan(c/2 + d*x/
2)**4/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360
*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*t
an(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 +
d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) +
30*a**3*d) - 11270*A*tan(c/2 + d*x/2)**3/(30*a**3*d*tan(c/2 + d*x/2)**9 + 1
50*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d
*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2
+ d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)*
*2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 7902*A*tan(c/2 + d*x/2)**2/
(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*
d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2
 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)
**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**
3*d) - 3910*A*tan(c/2 + d*x/2)/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*
tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 +
 d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**
4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a
**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 896*A/(30*a**3*d*tan(c/2 + d*x/2)**9
+ 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**
3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c
/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/
2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d), Ne(d, 0)), (x*(-A*sin(c)
+ A)*sin(c)**4/(a*sin(c) + a)**3, True))
```

3.236 $\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=103

$$\frac{A\cos(c+dx)}{a^3d}+\frac{104A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}-\frac{31A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}+\frac{4Ax}{a^3}$$

```
[Out] 4*A*x/a^3+A*cos(d*x+c)/a^3/d+2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3-31/15*
A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^2+104/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c)
)
```

Rubi [A] time = 0.19, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2966, 2638, 2650, 2648}

$$\frac{A\cos(c+dx)}{a^3d}+\frac{104A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}-\frac{31A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}+\frac{4Ax}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (4*A*x)/a^3 + (A*Cos[c + d*x])/(a^3*d) + (2*A*Cos[c + d*x])/(5*a^3*d*(1 + S
in[c + d*x])^3) - (31*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x])^2) + (10
4*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \int \left(\frac{4A}{a^3}-\frac{A\sin(c+dx)}{a^3}-\frac{2A}{a^3(1+\sin(c+dx))^3}+\frac{7A}{a^3(1+\sin(c+dx))^2}\right.$$

$$=\frac{4Ax}{a^3}-\frac{A\int\sin(c+dx)\,dx}{a^3}-\frac{(2A)\int\frac{1}{(1+\sin(c+dx))^3}\,dx}{a^3}+\frac{(7A)\int\frac{1}{(1+\sin(c+dx}}{a^3}$$

$$=\frac{4Ax}{a^3}+\frac{A\cos(c+dx)}{a^3d}+\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}-\frac{7A\cos(c+dx)}{3a^3d(1+\sin(c+dx))}$$

$$=\frac{4Ax}{a^3}+\frac{A\cos(c+dx)}{a^3d}+\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}-\frac{31A\cos(c+dx)}{15a^3d(1+\sin(c+dx)}$$

$$=\frac{4Ax}{a^3}+\frac{A\cos(c+dx)}{a^3d}+\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}-\frac{31A\cos(c+dx)}{15a^3d(1+\sin(c+dx)}$$

Mathematica [B] time = 0.80, size = 228, normalized size = 2.21

$$-\frac{A\left(-1200dx\sin\left(c+\frac{dx}{2}\right)-600dx\sin\left(c+\frac{3dx}{2}\right)+405\sin\left(2c+\frac{3dx}{2}\right)-491\sin\left(2c+\frac{5dx}{2}\right)+120dx\sin\left(3c+\frac{5dx}{2}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/120*(A*(-1200*d*x*Cos[(d*x)/2] + 1665*Cos[c + (d*x)/2] - 1675*Cos[c + (3
*d*x)/2] + 600*d*x*Cos[2*c + (3*d*x)/2] + 120*d*x*Cos[2*c + (5*d*x)/2] + 75
*Cos[3*c + (5*d*x)/2] + 15*Cos[3*c + (7*d*x)/2] + 2495*Sin[(d*x)/2] - 1200*
d*x*Sin[c + (d*x)/2] - 600*d*x*Sin[c + (3*d*x)/2] + 405*Sin[2*c + (3*d*x)/2
] - 491*Sin[2*c + (5*d*x)/2] + 120*d*x*Sin[3*c + (5*d*x)/2] + 15*Sin[4*c +
(7*d*x)/2]))/(a^3*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^5)
```

fricas [B] time = 0.45, size = 225, normalized size = 2.18

$$\frac{15\,A\cos\left(dx+c\right)^{4}+\left(60\,Adx+149\,A\right)\cos\left(dx+c\right)^{3}-240\,Adx+\left(180\,Adx-103\,A\right)\cos\left(dx+c\right)^{2}-3\,(40\,Adx\cdots}{15\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}-2\,a^{3}a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(15*A*cos(d*x + c)^4 + (60*A*d*x + 149*A)*cos(d*x + c)^3 - 240*A*d*x +
 (180*A*d*x - 103*A)*cos(d*x + c)^2 - 3*(40*A*d*x + 81*A)*cos(d*x + c) + (1
5*A*cos(d*x + c)^3 - 240*A*d*x + 2*(30*A*d*x - 67*A)*cos(d*x + c)^2 - 3*(40
*A*d*x + 79*A)*cos(d*x + c) + 6*A)*sin(d*x + c) - 6*A)/(a^3*d*cos(d*x + c)^
3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*
x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

giac [A] time = 0.18, size = 113, normalized size = 1.10

$$\frac{2\left(\frac{30\,(dx+c)A}{a^{3}}+\frac{15\,A}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)a^{3}}+\frac{60\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+285\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+505\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+335\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+79\,A}{a^{3}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{5}}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 2/15*(30*(d*x + c)*A/a^3 + 15*A/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^3) + (60*A*
tan(1/2*d*x + 1/2*c)^4 + 285*A*tan(1/2*d*x + 1/2*c)^3 + 505*A*tan(1/2*d*x +
 1/2*c)^2 + 335*A*tan(1/2*d*x + 1/2*c) + 79*A)/(a^3*(tan(1/2*d*x + 1/2*c) +
 1)^5))/d
```

maple [A] time = 0.43, size = 155, normalized size = 1.50

$$\frac{2A}{d\,a^{3}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{8A\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{3}}+\frac{16A}{5d\,a^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{8A}{d\,a^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{\cdots}{3d\,a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/d*A/a^3/(1+tan(1/2*d*x+1/2*c)^2)+8/d*A/a^3*arctan(tan(1/2*d*x+1/2*c))+16/
5/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^5-8/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^4+4/3/d
```

```
*A/a^3/(tan(1/2*d*x+1/2*c)+1)^3+6/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^2+8/d*A/a^
3/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.56, size = 543, normalized size = 5.27

$$\frac{2\left(3A\left(\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{189\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{200\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{160\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{75\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+24}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{11a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{11a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}+\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right.\right.}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] 2/15*(3*A*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 189*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 200*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 160*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 + 75*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 + 24)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c
) + 1) + 11*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^3*sin(d*x + c)^3
/(cos(d*x + c) + 1)^3 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 11*a^3
*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*a^3*sin(d*x + c)^6/(cos(d*x + c) +
 1)^6 + a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7) + 15*arctan(sin(d*x + c)/(
cos(d*x + c) + 1))/a^3) + A*((95*sin(d*x + c)/(cos(d*x + c) + 1) + 145*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + 75*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 +
15*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 22)/(a^3 + 5*a^3*sin(d*x + c)/(cos
(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
 a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5) + 15*arctan(sin(d*x + c)/(cos(d*x
 + c) + 1))/a^3))/d
```

mupad [B] time = 16.93, size = 261, normalized size = 2.53

$$\frac{4Ax}{a^3}-\frac{\left(20A\,(c+d\,x)-\frac{4A\,(75c+75d\,x+30)}{15}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(44A\,(c+d\,x)-\frac{4A\,(165c+165d\,x+150)}{15}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(A - A*sin(c + d*x)))/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (4*A*x)/a^3 - (tan(c/2 + (d*x)/2)*(20*A*(c + d*x) - (4*A*(75*c + 75*d*x + 2
05))/15) + tan(c/2 + (d*x)/2)^6*(20*A*(c + d*x) - (4*A*(75*c + 75*d*x + 30)
)/15) + tan(c/2 + (d*x)/2)^5*(44*A*(c + d*x) - (4*A*(165*c + 165*d*x + 150)
)/15) + tan(c/2 + (d*x)/2)^2*(44*A*(c + d*x) - (4*A*(165*c + 165*d*x + 367)
```

```
)/15) + tan(c/2 + (d*x)/2)^4*(60*A*(c + d*x) - (4*A*(225*c + 225*d*x + 320)
)/15) + tan(c/2 + (d*x)/2)^3*(60*A*(c + d*x) - (4*A*(225*c + 225*d*x + 385)
)/15) + 4*A*(c + d*x) - (4*A*(15*c + 15*d*x + 47))/15)/(a^3*d*(tan(c/2 + (d
*x)/2) + 1)^5*(tan(c/2 + (d*x)/2)^2 + 1))
```

sympy [A] time = 45.41, size = 2290, normalized size = 22.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((60*A*d*x*tan(c/2 + d*x/2)**7/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75
*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*t
an(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 +
d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 300*A*d*x*tan(c/2 + d
*x/2)**6/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 1
65*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d
*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 +
 d*x/2) + 15*a**3*d) + 660*A*d*x*tan(c/2 + d*x/2)**5/(15*a**3*d*tan(c/2 + d
*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 +
 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3
*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 900*A*d*
x*tan(c/2 + d*x/2)**4/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 +
d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4
 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**
3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 900*A*d*x*tan(c/2 + d*x/2)**3/(15*a**3*
d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2
+ d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)*
*3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*
d) + 660*A*d*x*tan(c/2 + d*x/2)**2/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3
*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/
2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2
)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 300*A*d*x*tan(c/2 + d*x/2)
/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*
d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2
 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2)
+ 15*a**3*d) + 60*A*d*x/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2
+ d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)*
*4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a
**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 120*A*tan(c/2 + d*x/2)**6/(15*a**3*d*
tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 +
d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3
 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d)
 + 600*A*tan(c/2 + d*x/2)**5/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan
```

```
(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*
x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 +
 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 1280*A*tan(c/2 + d*x/2)**4/(15*a
**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(
c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x
/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a
**3*d) + 1540*A*tan(c/2 + d*x/2)**3/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**
3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c
/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/
2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 1468*A*tan(c/2 + d*x/2)**
2/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3
*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/
2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2)
 + 15*a**3*d) + 820*A*tan(c/2 + d*x/2)/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*
a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*ta
n(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d
*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 188*A/(15*a**3*d*tan(c
/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2
)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 16
5*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d), Ne(
d, 0)), (x*(-A*sin(c) + A)*sin(c)**3/(a*sin(c) + a)**3, True))
```

3.237 $\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=89

$$-\frac{13A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)}+\frac{7A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^2}-\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}-\frac{Ax}{a^3}$$

```
[Out] -A*x/a^3-2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3+7/5*A*cos(d*x+c)/a^3/d/(1+
sin(d*x+c))^2-13/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A] time = 0.17, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2966, 2650, 2648}

$$-\frac{13A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)}+\frac{7A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^2}-\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}-\frac{Ax}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((A*x)/a^3) - (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) + (7*A*Cos
[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^2) - (13*A*Cos[c + d*x])/(5*a^3*d*(1
 + Sin[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \int \left(-\frac{A}{a^3} + \frac{2A}{a^3(1+\sin(c+dx))^3} - \frac{5A}{a^3(1+\sin(c+dx))^2} + \frac{4A}{a^3(1+\sin(c+\right.$$

$$= -\frac{Ax}{a^3} + \frac{(2A)\int \frac{1}{(1+\sin(c+dx))^3}\,dx}{a^3} + \frac{(4A)\int \frac{1}{1+\sin(c+dx)}\,dx}{a^3} - \frac{(5A)\int \frac{1}{(1+\sin(c-}}{a^3}$$

$$= -\frac{Ax}{a^3} - \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} + \frac{5A\cos(c+dx)}{3a^3d(1+\sin(c+dx))^2} - \frac{4A\cos(c+}{a^3d(1+\sin(}$$

$$= -\frac{Ax}{a^3} - \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} + \frac{7A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^2} - \frac{7A\cos(c}{3a^3d(1+\sin}$$

$$= -\frac{Ax}{a^3} - \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} + \frac{7A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^2} - \frac{13A\cos(c}{5a^3d(1+\sin}$$

Mathematica [B] time = 0.76, size = 189, normalized size = 2.12

$$\frac{A\left(-50dx\sin\left(c+\frac{dx}{2}\right)-25dx\sin\left(c+\frac{3dx}{2}\right)+40\sin\left(2c+\frac{3dx}{2}\right)-26\sin\left(2c+\frac{5dx}{2}\right)+5dx\sin\left(3c+\frac{5dx}{2}\right)+110\cos\right.}{20a^3d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (A*(-50*d*x*Cos[(d*x)/2] + 110*Cos[c + (d*x)/2] - 90*Cos[c + (3*d*x)/2] + 2
5*d*x*Cos[2*c + (3*d*x)/2] + 5*d*x*Cos[2*c + (5*d*x)/2] + 150*Sin[(d*x)/2]
- 50*d*x*Sin[c + (d*x)/2] - 25*d*x*Sin[c + (3*d*x)/2] + 40*Sin[2*c + (3*d*x
)/2] - 26*Sin[2*c + (5*d*x)/2] + 5*d*x*Sin[3*c + (5*d*x)/2]))/(20*a^3*d*(Co
s[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

fricas [B] time = 0.45, size = 204, normalized size = 2.29

$$-\frac{(5\,Adx+13\,A)\cos(dx+c)^3-20\,Adx+3\,(5\,Adx-2\,A)\cos(dx+c)^2-(10\,Adx+21\,A)\cos(dx+c)-\left(20\,Ad\right.}{5\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2-2\,a^3d\cos(dx+c)-4\,a^3d+\left(a^3d\,c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] -1/5*((5*A*d*x + 13*A)*cos(d*x + c)^3 - 20*A*d*x + 3*(5*A*d*x - 2*A)*cos(d*
x + c)^2 - (10*A*d*x + 21*A)*cos(d*x + c) - (20*A*d*x - (5*A*d*x - 13*A)*co
s(d*x + c)^2 + (10*A*d*x + 19*A)*cos(d*x + c) - 2*A)*sin(d*x + c) - 2*A)/(a
^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3
*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

giac [A] time = 0.19, size = 93, normalized size = 1.04

$$-\frac{\frac{5\,(dx+c)A}{a^3}+\frac{2\left(5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+25\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+55\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+35\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,A\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/5*(5*(d*x + c)*A/a^3 + 2*(5*A*tan(1/2*d*x + 1/2*c)^4 + 25*A*tan(1/2*d*x
+ 1/2*c)^3 + 55*A*tan(1/2*d*x + 1/2*c)^2 + 35*A*tan(1/2*d*x + 1/2*c) + 8*A)
/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

maple [A] time = 0.43, size = 131, normalized size = 1.47

$$-\frac{2A\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}-\frac{16A}{5d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{8A}{d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{4A}{d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{}{d\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -2/d*A/a^3*arctan(tan(1/2*d*x+1/2*c))-16/5/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^5
+8/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^4-4/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^3-2/d*
A/a^3/(tan(1/2*d*x+1/2*c)+1)^2-2/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.64, size = 392, normalized size = 4.40

$$-\frac{2\left(A\left(\frac{\frac{95\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{145\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{75\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+22}{a^3+\frac{5\,a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10\,a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{15\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)+\frac{2\,A}{a^3+\frac{5\,a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,a^3\sin}{(\cos(dx+}}\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -2/15*(A*((95*sin(d*x + c)/(cos(d*x + c) + 1) + 145*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 75*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 + 22)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10
*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) +
2*A*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 1)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a
^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) +
 1)^5))/d
```

mupad [B] time = 14.92, size = 178, normalized size = 2.00

$$\frac{\left(5\,A\,\left(c+d\,x\right)-\frac{A\,\left(25\,c+25\,d\,x+10\right)}{5}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(10\,A\,\left(c+d\,x\right)-\frac{A\,\left(50\,c+50\,d\,x+50\right)}{5}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(10\,A\,\left(c+d\right.\right.}{a^3\,d\,\left(\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(A - A*sin(c + d*x)))/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(5*A*(c + d*x) - (A*(25*c + 25*d*x + 70))/5) + tan(c/2
+ (d*x)/2)^4*(5*A*(c + d*x) - (A*(25*c + 25*d*x + 10))/5) + tan(c/2 + (d*x)
/2)^3*(10*A*(c + d*x) - (A*(50*c + 50*d*x + 50))/5) + tan(c/2 + (d*x)/2)^2*
(10*A*(c + d*x) - (A*(50*c + 50*d*x + 110))/5) + A*(c + d*x) - (A*(5*c + 5*
d*x + 16))/5)/(a^3*d*(tan(c/2 + (d*x)/2) + 1)^5) - (A*x)/a^3
```

sympy [A] time = 25.57, size = 1268, normalized size = 14.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-5*A*d*x*tan(c/2 + d*x/2)**5/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*
a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(
c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 25*A*d*x*tan(c/2
 + d*x/2)**4/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4
+ 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d
*tan(c/2 + d*x/2) + 5*a**3*d) - 50*A*d*x*tan(c/2 + d*x/2)**3/(5*a**3*d*tan(
c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2
```

```
)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*
d) - 50*A*d*x*tan(c/2 + d*x/2)**2/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d
*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 +
d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 25*A*d*x*tan(c/2 + d*x
/2)/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3
*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2
+ d*x/2) + 5*a**3*d) - 5*A*d*x/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*ta
n(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x
/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 10*A*tan(c/2 + d*x/2)**4/
(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*t
an(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*
x/2) + 5*a**3*d) - 50*A*tan(c/2 + d*x/2)**3/(5*a**3*d*tan(c/2 + d*x/2)**5 +
 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*
tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 110*A*tan(c/
2 + d*x/2)**2/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4
 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*
d*tan(c/2 + d*x/2) + 5*a**3*d) - 70*A*tan(c/2 + d*x/2)/(5*a**3*d*tan(c/2 +
d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 +
 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 1
6*A/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3
*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2
+ d*x/2) + 5*a**3*d), Ne(d, 0)), (x*(-A*sin(c) + A)*sin(c)**2/(a*sin(c) + a
)**3, True))
```

3.238 $\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=82

$$\frac{4A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}-\frac{11A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}$$

```
[Out] 2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3-11/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x
+c))^2+4/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A] time = 0.14, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2966, 2650, 2648}

$$\frac{4A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}-\frac{11A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) - (11*A*Cos[c + d*x])/(15
*a^3*d*(1 + Sin[c + d*x])^2) + (4*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*
x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \int \left(-\frac{2A}{a^3(1+\sin(c+dx))^3}+\frac{3A}{a^3(1+\sin(c+dx))^2}-\frac{A}{a^3(1+\sin(c+dx))}\right.$$
$$= -\frac{A\int \frac{1}{1+\sin(c+dx)}\,dx}{a^3}-\frac{(2A)\int \frac{1}{(1+\sin(c+dx))^3}\,dx}{a^3}+\frac{(3A)\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{a^3}$$
$$= \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}-\frac{A\cos(c+dx)}{a^3d(1+\sin(c+dx))^2}+\frac{A\cos(c+dx)}{a^3d(1+\sin(c+dx))}$$
$$= \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}-\frac{11A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2}-\frac{(4A)\int \frac{1}{1+\sin(c+dx)}\,d}{15a^3}$$
$$= \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}-\frac{11A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2}+\frac{4A\cos(c+dx)}{15a^3d(1+\sin(c+}$$

Mathematica [A] time = 0.46, size = 107, normalized size = 1.30

$$-\frac{A\left(15\sin\left(2c+\frac{3dx}{2}\right)-4\sin\left(2c+\frac{5dx}{2}\right)+15\cos\left(c+\frac{dx}{2}\right)-5\cos\left(c+\frac{3dx}{2}\right)+25\sin\left(\frac{dx}{2}\right)\right)}{30a^3d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/30*(A*(15*Cos[c + (d*x)/2] - 5*Cos[c + (3*d*x)/2] + 25*Sin[(d*x)/2] + 15
*Sin[2*c + (3*d*x)/2] - 4*Sin[2*c + (5*d*x)/2]))/(a^3*d*(Cos[c/2] + Sin[c/2
])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

fricas [B] time = 0.43, size = 156, normalized size = 1.90

$$\frac{4\,A\cos\left(dx+c\right)^3+7\,A\cos\left(dx+c\right)^2-3\,A\cos\left(dx+c\right)-\left(4\,A\cos\left(dx+c\right)^2-3\,A\cos\left(dx+c\right)-6\,A\right)\sin}{15\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2-2\,a^3d\cos\left(dx+c\right)-4\,a^3d+\left(a^3d\cos\left(dx+c\right)^2-2\,a^3d\cos\left(dx+c\right)\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] 1/15*(4*A*cos(d*x + c)^3 + 7*A*cos(d*x + c)^2 - 3*A*cos(d*x + c) - (4*A*cos
(d*x + c)^2 - 3*A*cos(d*x + c) - 6*A)*sin(d*x + c) - 6*A)/(a^3*d*cos(d*x +
```

```
c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos
(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

giac [A] time = 0.19, size = 63, normalized size = 0.77

$$-\frac{2\left(15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+A\right)}{15\,a^{3}d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] -2/15*(15*A*tan(1/2*d*x + 1/2*c)^3 - 5*A*tan(1/2*d*x + 1/2*c)^2 + 5*A*tan(1
/2*d*x + 1/2*c) + A)/(a^3*d*(tan(1/2*d*x + 1/2*c) + 1)^5)
```

maple [A] time = 0.42, size = 71, normalized size = 0.87

$$\frac{4A\left(-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{5}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}\right)}{d\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 4/d*A/a^3*(-1/2/(tan(1/2*d*x+1/2*c)+1)^2-2/(tan(1/2*d*x+1/2*c)+1)^4+5/3/(ta
n(1/2*d*x+1/2*c)+1)^3+4/5/(tan(1/2*d*x+1/2*c)+1)^5)
```

maxima [B] time = 0.62, size = 348, normalized size = 4.24

$$\frac{2\left(\frac{2\,A\left(\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)}{a^{3}+\frac{5\,a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{10\,a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{5\,a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{a^{3}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}}-\frac{3\,A\left(\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{5\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\cdots\right)}{a^{3}+\frac{5\,a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{10\,a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{5\,a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}\cdots}\cdots\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] 2/15*(2*A*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 1)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d
```

```
*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5) - 3*A*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1)/(a^3 + 5*a^
3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5))/d
```

mupad [B] time = 13.45, size = 110, normalized size = 1.34

$$-\frac{2\,A\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+5\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-5\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+15\sin\left(\frac{c}{2}\right.\right.}{15\,a^3\,d\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(A - A*sin(c + d*x)))/(a + a*sin(c + d*x))^3,x)
```

```
[Out] -(2*A*cos(c/2 + (d*x)/2)^2*(cos(c/2 + (d*x)/2)^3 + 15*sin(c/2 + (d*x)/2)^3
- 5*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^2 + 5*cos(c/2 + (d*x)/2)^2*sin(c/
2 + (d*x)/2)))/(15*a^3*d*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^5)
```

sympy [A] time = 14.11, size = 461, normalized size = 5.62

$$\begin{cases} -\frac{30A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^3d\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^3d}+\frac{}{15a^3d\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\,\mathrm{t}} \\ \frac{x(-A\sin(c)+A)\sin(c)}{(a\sin(c)+a)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-30*A*tan(c/2 + d*x/2)**3/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a*
*3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(
c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 10*A*tan(c/2 +
d*x/2)**2/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 +
150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d
*tan(c/2 + d*x/2) + 15*a**3*d) - 10*A*tan(c/2 + d*x/2)/(15*a**3*d*tan(c/2 +
 d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3
 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d)
 - 2*A/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 + 150
*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*ta
n(c/2 + d*x/2) + 15*a**3*d), Ne(d, 0)), (x*(-A*sin(c) + A)*sin(c)/(a*sin(c)
 + a)**3, True))
```

3.239 $\int \frac{A-A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=58

$$-\frac{A\cos^3(c+dx)}{15d(a\sin(c+dx)+a)^3}-\frac{aA\cos^3(c+dx)}{5d(a\sin(c+dx)+a)^4}$$

```
[Out] -1/5*a*A*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^4-1/15*A*cos(d*x+c)^3/d/(a+a*sin(d
*x+c))^3
```

Rubi [A] time = 0.11, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2736, 2672, 2671}

$$-\frac{A\cos^3(c+dx)}{15d(a\sin(c+dx)+a)^3}-\frac{aA\cos^3(c+dx)}{5d(a\sin(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A - A*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a*A*Cos[c + d*x]^3)/(5*d*(a + a*Sin[c + d*x])^4) - (A*Cos[c + d*x]^3)/(15
*d*(a + a*Sin[c + d*x])^3)
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
```

```
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{A-A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= (aA)\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx\\ &= -\frac{aA\cos^3(c+dx)}{5d(a+a\sin(c+dx))^4}+\frac{1}{5}A\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx\\ &= -\frac{aA\cos^3(c+dx)}{5d(a+a\sin(c+dx))^4}-\frac{A\cos^3(c+dx)}{15d(a+a\sin(c+dx))^3}\end{aligned}$$

Mathematica [A] time = 0.24, size = 92, normalized size = 1.59

$$\frac{A\left(\sin\left(2c+\frac{5dx}{2}\right)-15\cos\left(c+\frac{dx}{2}\right)+5\cos\left(c+\frac{3dx}{2}\right)+5\sin\left(\frac{dx}{2}\right)\right)}{30a^3d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A - A*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (A*(-15*Cos[c + (d*x)/2] + 5*Cos[c + (3*d*x)/2] + 5*Sin[(d*x)/2] + Sin[2*c
+ (5*d*x)/2]))/(30*a^3*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])^5)
```

fricas [B] time = 0.41, size = 154, normalized size = 2.66

$$\frac{A\cos\left(dx+c\right)^3-2\,A\cos\left(dx+c\right)^2+3\,A\cos\left(dx+c\right)-\left(A\cos\left(dx+c\right)^2+3\,A\cos\left(dx+c\right)+6\,A\right)\sin\left(\right.}{15\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2-2\,a^3d\cos\left(dx+c\right)-4\,a^3d+\left(a^3d\cos\left(dx+c\right)^2-2\,a^3d\cos\left(dx+c\right)\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(A*cos(d*x + c)^3 - 2*A*cos(d*x + c)^2 + 3*A*cos(d*x + c) - (A*cos(d*x
 + c)^2 + 3*A*cos(d*x + c) + 6*A)*sin(d*x + c) + 6*A)/(a^3*d*cos(d*x + c)^3
 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x
 + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

giac [A] time = 0.17, size = 79, normalized size = 1.36

$$-\frac{2\left(15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+25\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,A\right)}{15\,a^{3}d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*A*tan(1/2*d*x + 1/2*c)^4 + 15*A*tan(1/2*d*x + 1/2*c)^3 + 25*A*tan
(1/2*d*x + 1/2*c)^2 + 5*A*tan(1/2*d*x + 1/2*c) + 4*A)/(a^3*d*(tan(1/2*d*x +
 1/2*c) + 1)^5)
```

maple [A] time = 0.41, size = 86, normalized size = 1.48

$$\frac{2A\left(-\frac{8}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{3}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}-\frac{14}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}\right)}{d\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/d*A/a^3*(-8/5/(tan(1/2*d*x+1/2*c)+1)^5+3/(tan(1/2*d*x+1/2*c)+1)^2-1/(tan(
1/2*d*x+1/2*c)+1)+4/(tan(1/2*d*x+1/2*c)+1)^4-14/3/(tan(1/2*d*x+1/2*c)+1)^3)
```

maxima [B] time = 0.53, size = 387, normalized size = 6.67

$$-\frac{2\left(\frac{A\left(\frac{20\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{30\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{15\,\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+7\right)}{a^{3}+\frac{5\,a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{10\,a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{5\,a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{a^{3}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}}-\frac{3\,A\left(\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{5\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right.}{a^{3}+\frac{5\,a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{10\,a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{5\,a^{3}\sin(dx+}{(\cos(dx+c)+}}\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(A*(20*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 30*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 7)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^
3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(c
os(d*x + c) + 1)^5) - 3*A*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x +
```

```
c)^2/(cos(d*x + c) + 1)^2 + 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1)/(a^3
 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5))/d
```

mupad [B] time = 13.31, size = 134, normalized size = 2.31

$$-\frac{2\,A\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+25\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+15\,\cos\right.}{15\,a^3\,d\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A - A*sin(c + d*x))/(a + a*sin(c + d*x))^3,x)
```

```
[Out] -(2*A*cos(c/2 + (d*x)/2)*(4*cos(c/2 + (d*x)/2)^4 + 15*sin(c/2 + (d*x)/2)^4
+ 15*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^3 + 5*cos(c/2 + (d*x)/2)^3*sin(c
/2 + (d*x)/2) + 25*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2))/(15*a^3*d*(c
os(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^5)
```

sympy [A] time = 9.11, size = 573, normalized size = 9.88

$$\begin{cases} -\frac{30A\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^3d\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^3d} - \frac{}{15a^3d\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan\ldots} \\ \frac{x(-A\sin(c)+A)}{(a\sin(c)+a)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-30*A*tan(c/2 + d*x/2)**4/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a*
*3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(
c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) - 30*A*tan(c/2 +
d*x/2)**3/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 +
150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d
*tan(c/2 + d*x/2) + 15*a**3*d) - 50*A*tan(c/2 + d*x/2)**2/(15*a**3*d*tan(c/
2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)
**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3
*d) - 10*A*tan(c/2 + d*x/2)/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(
c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x
/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) - 8*A/(15*a**3*d*tan(c/2 +
 d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3
 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d)
, Ne(d, 0)), (x*(-A*sin(c) + A)/(a*sin(c) + a)**3, True))
```

3.240 $\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=98

$$\frac{8A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)}+\frac{3A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}-\frac{A\tanh^{-1}(\cos(c+dx))}{a^3d}$$

```
[Out] -A*arctanh(cos(d*x+c))/a^3/d+2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3+3/5*A*
cos(d*x+c)/a^3/d/(1+sin(d*x+c))^2+8/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A] time = 0.16, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2966, 3770, 2650, 2648}

$$\frac{8A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)}+\frac{3A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}-\frac{A\tanh^{-1}(\cos(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((A*ArcTanh[Cos[c + d*x]])/(a^3*d)) + (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin
[c + d*x])^3) + (3*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^2) + (8*A*Co
s[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
```

```
[m] && IntegerQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \int \left(\frac{A\csc(c+dx)}{a^3} - \frac{2A}{a^3(1+\sin(c+dx))^3} - \frac{A}{a^3(1+\sin(c+dx))^2} - \frac{}{a^3(1}\right.$$
$$= \frac{A\int \csc(c+dx)\,dx}{a^3} - \frac{A\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{a^3} - \frac{A\int \frac{1}{1+\sin(c+dx)}\,dx}{a^3} - \frac{(2A)\int}{}$$
$$= -\frac{A\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} + \frac{A\cos(c+dx)}{3a^3d(1+\sin(c+}$$
$$= -\frac{A\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} + \frac{3A\cos(c+dx}{5a^3d(1+\sin(c+}$$
$$= -\frac{A\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} + \frac{3A\cos(c+dx}{5a^3d(1+\sin(c+}$$

Mathematica [B] time = 1.01, size = 313, normalized size = 3.19

$$\frac{(A-A\sin(c+dx))\left(2\sin\left(\frac{dx}{2}\right)(-19\sin(c+dx)+4\cos(2(c+dx))-17)+\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(2*Cos[c/2] - 2*Sin[c/2] + 3*Cos[c/
2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 3*Sin[c/2]*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^2 - 5*Log[Cos[(c + d*x)/2]]*(Cos[c/2] + Sin[c/2])*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^4 + 5*Log[Sin[(c + d*x)/2]]*(Cos[c/2] + Sin
[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4) + 2*Sin[(d*x)/2]*(-17 + 4*C
os[2*(c + d*x)] - 19*Sin[c + d*x]))*(A - A*Sin[c + d*x]))/(5*a^3*d*(Cos[c/2
] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^5)
```

fricas [B] time = 0.44, size = 310, normalized size = 3.16

$$\frac{16\,A\cos\left(dx+c\right)^3-22\,A\cos\left(dx+c\right)^2-42\,A\cos\left(dx+c\right)-5\left(A\cos\left(dx+c\right)^3+3\,A\cos\left(dx+c\right)^2-2\,A\cos\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] 1/10*(16*A*cos(d*x + c)^3 - 22*A*cos(d*x + c)^2 - 42*A*cos(d*x + c) - 5*(A*
cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + (A*cos(d*x + c)^2
- 2*A*cos(d*x + c) - 4*A)*sin(d*x + c) - 4*A)*log(1/2*cos(d*x + c) + 1/2) +
 5*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + (A*cos(d*x +
 c)^2 - 2*A*cos(d*x + c) - 4*A)*sin(d*x + c) - 4*A)*log(-1/2*cos(d*x + c) +
 1/2) - 2*(8*A*cos(d*x + c)^2 + 19*A*cos(d*x + c) - 2*A)*sin(d*x + c) - 4*A
)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4
*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x +
c))
```

giac [A] time = 0.19, size = 99, normalized size = 1.01

$$\frac{\frac{5\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}+\frac{2\left(20\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+55\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+75\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+45\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+13\,A\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] 1/5*(5*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 2*(20*A*tan(1/2*d*x + 1/2*c)^
4 + 55*A*tan(1/2*d*x + 1/2*c)^3 + 75*A*tan(1/2*d*x + 1/2*c)^2 + 45*A*tan(1/
2*d*x + 1/2*c) + 13*A)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

maple [A] time = 0.73, size = 130, normalized size = 1.33

$$\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}+\frac{16A}{5d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{8A}{d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{12A}{d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{10}{d\,a^3\left(\tan\left(\frac{d}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*A/a^3*ln(tan(1/2*d*x+1/2*c))+16/5/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^5-8/d*
A/a^3/(tan(1/2*d*x+1/2*c)+1)^4+12/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^3-10/d*A/a
^3/(tan(1/2*d*x+1/2*c)+1)^2+8/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.59, size = 433, normalized size = 4.42

$$\frac{A\left(\frac{2\left(\frac{115\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{185\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{135\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{45\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+32\right)}{a^3+\frac{5\,a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10\,a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{15\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)+\frac{2\,A\left(\frac{20\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\,\sin(dx+c}{(\cos(dx+c)+}\right.}{a^3+\frac{5\,a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/15*(A*(2*(115*sin(d*x + c)/(cos(d*x + c) + 1) + 185*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 135*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 45*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 32)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) +
 10*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c
)^5/(cos(d*x + c) + 1)^5) + 15*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) +
2*A*(20*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 30*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 7)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*
x + c) + 1)^5))/d
```

mupad [B] time = 14.77, size = 199, normalized size = 2.03

$$\frac{A\left(5\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+90\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+150\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+110\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+40\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+25\,\ln\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A - A*sin(c + d*x))/(sin(c + d*x)*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (A*(5*log(tan(c/2 + (d*x)/2)) + 90*tan(c/2 + (d*x)/2) + 150*tan(c/2 + (d*x)
/2)^2 + 110*tan(c/2 + (d*x)/2)^3 + 40*tan(c/2 + (d*x)/2)^4 + 25*log(tan(c/2
 + (d*x)/2))*tan(c/2 + (d*x)/2) + 50*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x
)/2)^2 + 50*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^3 + 25*log(tan(c/2 +
```

```
 (d*x)/2))*tan(c/2 + (d*x)/2)^4 + 5*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)
/2)^5 + 26))/(5*a^3*d*(tan(c/2 + (d*x)/2) + 1)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{A\left(\int\left(-\frac{\csc\left(c+dx\right)}{\sin^{3}\left(c+dx\right)+3\sin^{2}\left(c+dx\right)+3\sin\left(c+dx\right)+1}\right)dx+\int\frac{\sin\left(c+dx\right)\csc\left(c+dx\right)}{\sin^{3}\left(c+dx\right)+3\sin^{2}\left(c+dx\right)+3\sin\left(c+dx\right)+1}\,dx\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] -A*(Integral(-csc(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c +
 d*x) + 1), x) + Integral(sin(c + d*x)*csc(c + d*x)/(sin(c + d*x)**3 + 3*si
n(c + d*x)**2 + 3*sin(c + d*x) + 1), x))/a**3
```

3.241 $\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=113

$$-\frac{A\cot(c+dx)}{a^3d}+\frac{4A\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{104A\cot(c+dx)}{15a^3d(\csc(c+dx)+1)}+\frac{31A\cot(c+dx)}{15a^3d(\csc(c+dx)+1)^2}-\frac{2A\cot(c+dx}{5a^3d(\csc(c+dx)}$$

```
[Out] 4*A*arctanh(cos(d*x+c))/a^3/d-94/15*A*cot(d*x+c)/a^3/d+2/5*A*cot(d*x+c)/a^3
/d/(1+sin(d*x+c))^3+13/15*A*cot(d*x+c)/a^3/d/(1+sin(d*x+c))^2+4*A*cot(d*x+c
)/a^3/d/(1+sin(d*x+c))
```

Rubi [A] time = 0.40, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2950, 2709, 3770, 3767, 8, 3777, 3922, 3919, 3794}

$$-\frac{A\cot(c+dx)}{a^3d}+\frac{4A\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{104A\cot(c+dx)}{15a^3d(\csc(c+dx)+1)}+\frac{31A\cot(c+dx)}{15a^3d(\csc(c+dx)+1)^2}-\frac{2A\cot(c+dx}{5a^3d(\csc(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (4*A*ArcTanh[Cos[c + d*x]])/(a^3*d) - (A*Cot[c + d*x])/(a^3*d) - (2*A*Cot[c
 + d*x])/(5*a^3*d*(1 + Csc[c + d*x])^3) + (31*A*Cot[c + d*x])/(15*a^3*d*(1
+ Csc[c + d*x])^2) - (104*A*Cot[c + d*x])/(15*a^3*d*(1 + Csc[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2950

```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^n*c^n,
 Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
```

```
0] && IntegerQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = (aA)\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx$$

$$= \frac{A\int\left(\frac{9}{a^2}-\frac{4\csc(c+dx)}{a^2}+\frac{\csc^2(c+dx)}{a^2}-\frac{2}{a^2(1+\csc(c+dx))^3}+\frac{9}{a^2(1+\csc(c+dx))^2}-\frac{}{a^2(1}\right.}{a}$$

$$= \frac{9Ax}{a^3}+\frac{A\int\csc^2(c+dx)\,dx}{a^3}-\frac{(2A)\int\frac{1}{(1+\csc(c+dx))^3}\,dx}{a^3}-\frac{(4A)\int\csc(c+}{a^3}$$

$$= \frac{9Ax}{a^3}+\frac{4A\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{2A\cot(c+dx)}{5a^3d(1+\csc(c+dx))^3}+\frac{3A\cot}{a^3d(1+\csc}$$

$$= \frac{2Ax}{a^3}+\frac{4A\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{A\cot(c+dx)}{a^3d}-\frac{2A\cot(c+dx)}{5a^3d(1+\csc(c+d}$$

$$= \frac{4A\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{A\cot(c+dx)}{a^3d}-\frac{2A\cot(c+dx)}{5a^3d(1+\csc(c+dx))^3}+\frac{}{1}$$

$$= \frac{4A\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{A\cot(c+dx)}{a^3d}-\frac{2A\cot(c+dx)}{5a^3d(1+\csc(c+dx))^3}+\frac{}{1}$$

Mathematica [A] time = 2.97, size = 167, normalized size = 1.48

$$-\frac{A\left(-15\tan\left(\frac{1}{2}(c+dx)\right)+15\cot\left(\frac{1}{2}(c+dx)\right)+120\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-120\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\frac{2\sin\left(\frac{1}{2}(c\right.}{}\right.}{30a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/30*(A*(15*Cot[(c + d*x)/2] - 120*Log[Cos[(c + d*x)/2]] + 120*Log[Sin[(c
+ d*x)/2]] + 12/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + 38/(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^2 + (2*Sin[(c + d*x)/2]*(-287 + 79*Cos[2*(c + d*x)]
 - 354*Sin[c + d*x]))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5 - 15*Tan[(c +
 d*x)/2]))/(a^3*d)
```

fricas [B] time = 0.45, size = 406, normalized size = 3.59

$$\frac{94\,A\cos\left(dx+c\right)^4+222\,A\cos\left(dx+c\right)^3-115\,A\cos\left(dx+c\right)^2-237\,A\cos\left(dx+c\right)+30\left(A\cos\left(dx+c\right)^4-2\,A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(94*A*cos(d*x + c)^4 + 222*A*cos(d*x + c)^3 - 115*A*cos(d*x + c)^2 - 2
37*A*cos(d*x + c) + 30*(A*cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*cos(d*x
 + c)^2 + 2*A*cos(d*x + c) - (A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*A*c
os(d*x + c) - 4*A)*sin(d*x + c) + 4*A)*log(1/2*cos(d*x + c) + 1/2) - 30*(A*
cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*cos(d*x + c)^2 + 2*A*cos(d*x + c)
 - (A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) - 4*A)*sin(d*x
 + c) + 4*A)*log(-1/2*cos(d*x + c) + 1/2) + (94*A*cos(d*x + c)^3 - 128*A*co
s(d*x + c)^2 - 243*A*cos(d*x + c) - 6*A)*sin(d*x + c) + 6*A)/(a^3*d*cos(d*x
 + c)^4 - 2*a^3*d*cos(d*x + c)^3 - 5*a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x
 + c) + 4*a^3*d - (a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*
cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

giac [A] time = 0.22, size = 146, normalized size = 1.29

$$-\frac{\frac{120\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}-\frac{15\left(8\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-A\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{4\,(135\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+435\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+605\,A\tan(}{a^3(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/30*(120*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 15*A*tan(1/2*d*x + 1/2*c)
/a^3 - 15*(8*A*tan(1/2*d*x + 1/2*c) - A)/(a^3*tan(1/2*d*x + 1/2*c)) + 4*(13
5*A*tan(1/2*d*x + 1/2*c)^4 + 435*A*tan(1/2*d*x + 1/2*c)^3 + 605*A*tan(1/2*d
*x + 1/2*c)^2 + 385*A*tan(1/2*d*x + 1/2*c) + 104*A)/(a^3*(tan(1/2*d*x + 1/2
*c) + 1)^5))/d
```

maple [A] time = 0.73, size = 169, normalized size = 1.50

$$\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3d}-\frac{A}{2a^3d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{4A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}-\frac{16A}{5d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{8A}{d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/2*A/a^3/d*tan(1/2*d*x+1/2*c)-1/2*A/a^3/d/tan(1/2*d*x+1/2*c)-4/d*A/a^3*ln(
tan(1/2*d*x+1/2*c))-16/5/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^5+8/d*A/a^3/(tan(1/
2*d*x+1/2*c)+1)^4-44/3/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^3+14/d*A/a^3/(tan(1/2
*d*x+1/2*c)+1)^2-18/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.69, size = 519, normalized size = 4.59

$$-\frac{3\,A\left(\frac{\frac{121\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{410\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{610\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{425\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{125\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+5}{\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10\,a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{10\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{5\,a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{30\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{5\,\sin(dx+c)}{a^3(\cos(dx+c)+1)}\right)+}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/30*(3*A*((121*sin(d*x + c)/(cos(d*x + c) + 1) + 410*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 610*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 425*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + 125*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5)/(a^
3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^3*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 5*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^3*sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6) + 30*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3
 - 5*sin(d*x + c)/(a^3*(cos(d*x + c) + 1))) + 2*A*(2*(115*sin(d*x + c)/(cos
(d*x + c) + 1) + 185*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 135*sin(d*x + c)
^3/(cos(d*x + c) + 1)^3 + 45*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 32)/(a^3
 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5) + 15*lo
g(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

mupad [B] time = 15.77, size = 210, normalized size = 1.86

$$\frac{A\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^3\,d}-\frac{4\,A\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{37\,A\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+121\,A\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{514\,A\tan\left(\frac{c}{2}\right.}{3}}{d\left(2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+10\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+20\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+20\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A - A*sin(c + d*x))/(sin(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (A*tan(c/2 + (d*x)/2))/(2*a^3*d) - (4*A*log(tan(c/2 + (d*x)/2)))/(a^3*d) -
(A + (491*A*tan(c/2 + (d*x)/2))/15 + (338*A*tan(c/2 + (d*x)/2)^2)/3 + (514*
```

```
A*tan(c/2 + (d*x)/2)^3)/3 + 121*A*tan(c/2 + (d*x)/2)^4 + 37*A*tan(c/2 + (d*
x)/2)^5)/(d*(10*a^3*tan(c/2 + (d*x)/2)^2 + 20*a^3*tan(c/2 + (d*x)/2)^3 + 20
*a^3*tan(c/2 + (d*x)/2)^4 + 10*a^3*tan(c/2 + (d*x)/2)^5 + 2*a^3*tan(c/2 + (
d*x)/2)^6 + 2*a^3*tan(c/2 + (d*x)/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{A\left(\int\left(-\frac{\csc^2(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\right)dx+\int\frac{\sin(c+dx)\csc^2(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] -A*(Integral(-csc(c + d*x)**2/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(
c + d*x) + 1), x) + Integral(sin(c + d*x)*csc(c + d*x)**2/(sin(c + d*x)**3
+ 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x))/a**3
```

3.242 $\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=138

$$\frac{4A\cot(c+dx)}{a^3d}+\frac{164A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}+\frac{29A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}-\frac{19A\tanh^{-1}(\cos(c+$$

```
[Out] -19/2*A*arctanh(cos(d*x+c))/a^3/d+4*A*cot(d*x+c)/a^3/d-1/2*A*cot(d*x+c)*csc
(d*x+c)/a^3/d+2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3+29/15*A*cos(d*x+c)/a^
3/d/(1+sin(d*x+c))^2+164/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A] time = 0.22, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2966, 3770, 3767, 8, 3768, 2650, 2648}

$$\frac{4A\cot(c+dx)}{a^3d}+\frac{164A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}+\frac{29A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}-\frac{19A\tanh^{-1}(\cos(c+$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-19*A*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (4*A*Cot[c + d*x])/(a^3*d) - (A*C
ot[c + d*x]*Csc[c + d*x])/(2*a^3*d) + (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[
c + d*x])^3) + (29*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x])^2) + (164*A
*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx &= \int \left(\frac{9A\csc(c+dx)}{a^3}-\frac{4A\csc^2(c+dx)}{a^3}+\frac{A\csc^3(c+dx)}{a^3}-\frac{2A}{a^3(1+\sin(c\right.\\
&= \frac{A\int \csc^3(c+dx)\,dx}{a^3}-\frac{(2A)\int \frac{1}{(1+\sin(c+dx))^3}\,dx}{a^3}-\frac{(4A)\int \csc^2(c+dx)\,dx}{a^3}\\
&= -\frac{9A\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{A\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{2A\cos(c+}{5a^3d(1+\sin(c}\\
&= -\frac{19A\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{4A\cot(c+dx)}{a^3d}-\frac{A\cot(c+dx)\csc(c+dx}{2a^3d}\\
&= -\frac{19A\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{4A\cot(c+dx)}{a^3d}-\frac{A\cot(c+dx)\csc(c+dx}{2a^3d}
\end{aligned}$$

Mathematica [A] time = 3.89, size = 245, normalized size = 1.78

$$\frac{A\left(-240\tan\left(\frac{1}{2}(c+dx)\right)+240\cot\left(\frac{1}{2}(c+dx)\right)-15\csc^2\left(\frac{1}{2}(c+dx)\right)+15\sec^2\left(\frac{1}{2}(c+dx)\right)+1140\log\left(\sin\left(\frac{1}{2}(c\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (A*(240*Cot[(c + d*x)/2] - 15*Csc[(c + d*x)/2]^2 - 1140*Log[Cos[(c + d*x)/2
]] + 1140*Log[Sin[(c + d*x)/2]] + 15*Sec[(c + d*x)/2]^2 - (96*Sin[(c + d*x)
/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5 + 48/(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])^4 - (464*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
])^3 + 232/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - (2624*Sin[(c + d*x)/2]
)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 240*Tan[(c + d*x)/2]))/(120*a^3*d
)
```

fricas [B] time = 0.45, size = 498, normalized size = 3.61

$$\frac{896\,A\cos(dx+c)^5-1222\,A\cos(dx+c)^4-3218\,A\cos(dx+c)^3+1168\,A\cos(dx+c)^2+2292\,A\cos(dx+c)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/60*(896*A*cos(d*x + c)^5 - 1222*A*cos(d*x + c)^4 - 3218*A*cos(d*x + c)^3
+ 1168*A*cos(d*x + c)^2 + 2292*A*cos(d*x + c) - 285*(A*cos(d*x + c)^5 + 3*A
*cos(d*x + c)^4 - 3*A*cos(d*x + c)^3 - 7*A*cos(d*x + c)^2 + 2*A*cos(d*x + c
) + (A*cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*cos(d*x + c)^2 + 2*A*cos(d
*x + c) + 4*A)*sin(d*x + c) + 4*A)*log(1/2*cos(d*x + c) + 1/2) + 285*(A*cos
(d*x + c)^5 + 3*A*cos(d*x + c)^4 - 3*A*cos(d*x + c)^3 - 7*A*cos(d*x + c)^2
+ 2*A*cos(d*x + c) + (A*cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*cos(d*x +
 c)^2 + 2*A*cos(d*x + c) + 4*A)*sin(d*x + c) + 4*A)*log(-1/2*cos(d*x + c) +
 1/2) - 2*(448*A*cos(d*x + c)^4 + 1059*A*cos(d*x + c)^3 - 550*A*cos(d*x + c
)^2 - 1134*A*cos(d*x + c) + 12*A)*sin(d*x + c) + 24*A)/(a^3*d*cos(d*x + c)^
5 + 3*a^3*d*cos(d*x + c)^4 - 3*a^3*d*cos(d*x + c)^3 - 7*a^3*d*cos(d*x + c)^
2 + 2*a^3*d*cos(d*x + c) + 4*a^3*d + (a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*
x + c)^3 - 5*a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + 4*a^3*d)*sin(d*x
 + c))
```

giac [A] time = 0.30, size = 180, normalized size = 1.30

$$\frac{\frac{1140\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{15\left(114\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+A\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}+\frac{15\left(Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^6}+\frac{16\left(240\right.}{}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/120*(1140*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 15*(114*A*tan(1/2*d*x +
1/2*c)^2 - 16*A*tan(1/2*d*x + 1/2*c) + A)/(a^3*tan(1/2*d*x + 1/2*c)^2) + 15
*(A*a^3*tan(1/2*d*x + 1/2*c)^2 - 16*A*a^3*tan(1/2*d*x + 1/2*c))/a^6 + 16*(2
40*A*tan(1/2*d*x + 1/2*c)^4 + 825*A*tan(1/2*d*x + 1/2*c)^3 + 1165*A*tan(1/2
*d*x + 1/2*c)^2 + 755*A*tan(1/2*d*x + 1/2*c) + 199*A)/(a^3*(tan(1/2*d*x + 1
/2*c) + 1)^5))/d
```

maple [A] time = 0.87, size = 209, normalized size = 1.51

$$\frac{A\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^3}-\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a^3d}-\frac{A}{8d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{2A}{a^3d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{19A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^3}+\frac{}{5d\,a^3\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/8/d*A/a^3*tan(1/2*d*x+1/2*c)^2-2*A/a^3/d*tan(1/2*d*x+1/2*c)-1/8/d*A/a^3/t
an(1/2*d*x+1/2*c)^2+2*A/a^3/d/tan(1/2*d*x+1/2*c)+19/2/d*A/a^3*ln(tan(1/2*d*
x+1/2*c))+16/5/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^5-8/d*A/a^3/(tan(1/2*d*x+1/2*
c)+1)^4+52/3/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^3-18/d*A/a^3/(tan(1/2*d*x+1/2*c
)+1)^2+32/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.41, size = 622, normalized size = 4.51

$$\frac{12\,A\left(\frac{\frac{121\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{410\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{610\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{425\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{125\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+5}{\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10\,a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{10\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{5\,a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{5\,\sin(dx+c)}{a^3(\cos(dx+c)+1)}\right)+A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] 1/120*(12*A*((121*sin(d*x + c)/(cos(d*x + c) + 1) + 410*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + 610*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 425*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 + 125*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5)/(a
^3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^3*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 + 5*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^3*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6) + 30*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^
3 - 5*sin(d*x + c)/(a^3*(cos(d*x + c) + 1))) + A*((105*sin(d*x + c)/(cos(d*
x + c) + 1) + 2782*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 9410*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3 + 13645*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 9285*s
in(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2580*sin(d*x + c)^6/(cos(d*x + c) + 1)
^6 - 15)/(a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 5*a^3*sin(d*x + c)^3/(c
os(d*x + c) + 1)^3 + 10*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*a^3*si
n(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)
^6 + a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7) - 15*(12*sin(d*x + c)/(cos(d*
x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/a^3 + 780*log(sin(d*x +
c)/(cos(d*x + c) + 1))/a^3))/d
```

mupad [B] time = 15.74, size = 288, normalized size = 2.09

$$\frac{A\left(165\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4234\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+14090\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+19780\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+12060\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A - A*sin(c + d*x))/(sin(c + d*x)^3*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (A*(165*tan(c/2 + (d*x)/2) + 4234*tan(c/2 + (d*x)/2)^2 + 14090*tan(c/2 + (d
*x)/2)^3 + 19780*tan(c/2 + (d*x)/2)^4 + 12060*tan(c/2 + (d*x)/2)^5 + 1830*t
an(c/2 + (d*x)/2)^6 - 1050*tan(c/2 + (d*x)/2)^7 - 165*tan(c/2 + (d*x)/2)^8
+ 15*tan(c/2 + (d*x)/2)^9 + 1140*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)
^2 + 5700*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^3 + 11400*log(tan(c/2
+ (d*x)/2))*tan(c/2 + (d*x)/2)^4 + 11400*log(tan(c/2 + (d*x)/2))*tan(c/2 +
(d*x)/2)^5 + 5700*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^6 + 1140*log(t
an(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^7 - 15))/(120*a^3*d*tan(c/2 + (d*x)/2
)^2*(tan(c/2 + (d*x)/2) + 1)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{A\left(\int\left(-\frac{\csc^3(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\right)dx+\int\frac{\sin(c+dx)\csc^3(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] -A*(Integral(-csc(c + d*x)**3/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(
c + d*x) + 1), x) + Integral(sin(c + d*x)*csc(c + d*x)**3/(sin(c + d*x)**3
+ 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x))/a**3
```

3.243 $\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=153

$$-\frac{A\cot^3(c+dx)}{3a^3d}-\frac{10A\cot(c+dx)}{a^3d}-\frac{93A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)}-\frac{13A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^2}-\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}+18A$$

```
[Out] 18*A*arctanh(cos(d*x+c))/a^3/d-10*A*cot(d*x+c)/a^3/d-1/3*A*cot(d*x+c)^3/a^3
/d+2*A*cot(d*x+c)*csc(d*x+c)/a^3/d-2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3-
13/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^2-93/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x
+c))
```

Rubi [A] time = 0.25, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2966, 3770, 3767, 8, 3768, 2650, 2648}

$$-\frac{A\cot^3(c+dx)}{3a^3d}-\frac{10A\cot(c+dx)}{a^3d}-\frac{93A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)}-\frac{13A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^2}-\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}+18A$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^4*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (18*A*ArcTanh[Cos[c + d*x]])/(a^3*d) - (10*A*Cot[c + d*x])/(a^3*d) - (A*Cot
[c + d*x]^3)/(3*a^3*d) + (2*A*Cot[c + d*x]*Csc[c + d*x])/(a^3*d) - (2*A*Cos
[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) - (13*A*Cos[c + d*x])/(5*a^3*d*(1
 + Sin[c + d*x])^2) - (93*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \int \left(-\frac{16A\csc(c+dx)}{a^3}+\frac{9A\csc^2(c+dx)}{a^3}-\frac{4A\csc^3(c+dx)}{a^3}+\frac{A\csc^4(c\right.$$

$$= \frac{A\int \csc^4(c+dx)\,dx}{a^3}+\frac{(2A)\int \frac{1}{(1+\sin(c+dx))^3}\,dx}{a^3}-\frac{(4A)\int \csc^3(c+dx)\,dx}{a^3}$$

$$= \frac{16A\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{2A\cot(c+dx)\csc(c+dx)}{a^3d}-\frac{2A\cos(c}{5a^3d(1+\sin(}$$

$$= \frac{18A\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{10A\cot(c+dx)}{a^3d}-\frac{A\cot^3(c+dx)}{3a^3d}+\frac{2A\cot}{}$$

$$= \frac{18A\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{10A\cot(c+dx)}{a^3d}-\frac{A\cot^3(c+dx)}{3a^3d}+\frac{2A\cot}{}$$

Mathematica [B] time = 6.23, size = 348, normalized size = 2.27

$$A\left(\frac{29\tan\left(\frac{1}{2}(c+dx)\right)}{6d}-\frac{29\cot\left(\frac{1}{2}(c+dx)\right)}{6d}+\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{2d}-\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{2d}-\frac{18\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{18\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{186}{5d\left(\sin\left(\frac{1}{2}(c\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Csc[c + d*x]^4*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (A*((-29*Cot[(c + d*x)/2])/(6*d) + Csc[(c + d*x)/2]^2/(2*d) - (Cot[(c + d*x
)/2]*Csc[(c + d*x)/2]^2)/(24*d) + (18*Log[Cos[(c + d*x)/2]])/d - (18*Log[Si
n[(c + d*x)/2]])/d - Sec[(c + d*x)/2]^2/(2*d) + (4*Sin[(c + d*x)/2])/(5*d*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5) - 2/(5*d*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])^4) + (26*Sin[(c + d*x)/2])/(5*d*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^3) - 13/(5*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (186*Sin[(c
+ d*x)/2])/(5*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (29*Tan[(c + d*x)/
2])/(6*d) + (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*d)))/a^3
```

fricas [B] time = 0.46, size = 594, normalized size = 3.88

$$424\,A\cos\left(dx+c\right)^6+1002\,A\cos\left(dx+c\right)^5-944\,A\cos\left(dx+c\right)^4-2074\,A\cos\left(dx+c\right)^3+531\,A\cos\left(dx+c\right)^2-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(424*A*cos(d*x + c)^6 + 1002*A*cos(d*x + c)^5 - 944*A*cos(d*x + c)^4 -
 2074*A*cos(d*x + c)^3 + 531*A*cos(d*x + c)^2 + 1077*A*cos(d*x + c) + 135*(
A*cos(d*x + c)^6 - 2*A*cos(d*x + c)^5 - 6*A*cos(d*x + c)^4 + 4*A*cos(d*x +
c)^3 + 9*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) - (A*cos(d*x + c)^5 + 3*A*cos(
d*x + c)^4 - 3*A*cos(d*x + c)^3 - 7*A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + 4
*A)*sin(d*x + c) - 4*A)*log(1/2*cos(d*x + c) + 1/2) - 135*(A*cos(d*x + c)^6
 - 2*A*cos(d*x + c)^5 - 6*A*cos(d*x + c)^4 + 4*A*cos(d*x + c)^3 + 9*A*cos(d
*x + c)^2 - 2*A*cos(d*x + c) - (A*cos(d*x + c)^5 + 3*A*cos(d*x + c)^4 - 3*A
*cos(d*x + c)^3 - 7*A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + 4*A)*sin(d*x + c)
 - 4*A)*log(-1/2*cos(d*x + c) + 1/2) + (424*A*cos(d*x + c)^5 - 578*A*cos(d*
x + c)^4 - 1522*A*cos(d*x + c)^3 + 552*A*cos(d*x + c)^2 + 1083*A*cos(d*x +
c) + 6*A)*sin(d*x + c) - 6*A)/(a^3*d*cos(d*x + c)^6 - 2*a^3*d*cos(d*x + c)^
5 - 6*a^3*d*cos(d*x + c)^4 + 4*a^3*d*cos(d*x + c)^3 + 9*a^3*d*cos(d*x + c)^
2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d - (a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*
x + c)^4 - 3*a^3*d*cos(d*x + c)^3 - 7*a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*
x + c) + 4*a^3*d)*sin(d*x + c))
```

giac [A] time = 0.30, size = 213, normalized size = 1.39

$$-\frac{\frac{2160\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{5\left(792\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-117\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-A\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}+\frac{48\left(125\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+445\,A\tan\cdots\right.}{\cdots}}{120\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/120*(2160*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 5*(792*A*tan(1/2*d*x +
1/2*c)^3 - 117*A*tan(1/2*d*x + 1/2*c)^2 + 12*A*tan(1/2*d*x + 1/2*c) - A)/(a
^3*tan(1/2*d*x + 1/2*c)^3) + 48*(125*A*tan(1/2*d*x + 1/2*c)^4 + 445*A*tan(1
/2*d*x + 1/2*c)^3 + 635*A*tan(1/2*d*x + 1/2*c)^2 + 415*A*tan(1/2*d*x + 1/2*
c) + 108*A)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^5) - 5*(A*a^6*tan(1/2*d*x + 1/2
*c)^3 - 12*A*a^6*tan(1/2*d*x + 1/2*c)^2 + 117*A*a^6*tan(1/2*d*x + 1/2*c))/a
^9)/d
```

maple [A] time = 0.83, size = 249, normalized size = 1.63

$$\frac{A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d\,a^3}-\frac{A\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^3}+\frac{39A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3d}-\frac{A}{24d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{A}{2d\,a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{39\cdots}{8a^3d\tan\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/24/d*A/a^3*tan(1/2*d*x+1/2*c)^3-1/2/d*A/a^3*tan(1/2*d*x+1/2*c)^2+39/8*A/a
^3/d*tan(1/2*d*x+1/2*c)-1/24/d*A/a^3/tan(1/2*d*x+1/2*c)^3+1/2/d*A/a^3/tan(1
/2*d*x+1/2*c)^2-39/8*A/a^3/d/tan(1/2*d*x+1/2*c)-18/d*A/a^3*ln(tan(1/2*d*x+1
/2*c))-16/5/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^5+8/d*A/a^3/(tan(1/2*d*x+1/2*c)+
1)^4-20/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^3+22/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^
2-50/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.46, size = 706, normalized size = 4.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/120*(A*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 2782*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 9410*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 13645*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 + 9285*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 25
80*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 15)/(a^3*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 5*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^3*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + 10*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*a
^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^3*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7) - 15*(12*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2)/a^3 + 780*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - A*((20*si
n(d*x + c)/(cos(d*x + c) + 1) - 230*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4
777*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 15785*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 - 22390*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14940*sin(d*x + c)^6
/(cos(d*x + c) + 1)^6 - 4005*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 5)/(a^3*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) +
1)^4 + 10*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 10*a^3*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + 5*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + a^3*sin(d
*x + c)^8/(cos(d*x + c) + 1)^8) + 5*(81*sin(d*x + c)/(cos(d*x + c) + 1) - 9
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)
/a^3 - 1380*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

mupad [B] time = 15.28, size = 314, normalized size = 2.05

$$-\frac{A\left(335\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+7559\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+24610\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+33170\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A - A*sin(c + d*x))/(sin(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] -(A*(335*tan(c/2 + (d*x)/2)^2 - 35*tan(c/2 + (d*x)/2) + 7559*tan(c/2 + (d*x
)/2)^3 + 24610*tan(c/2 + (d*x)/2)^4 + 33170*tan(c/2 + (d*x)/2)^5 + 18670*ta
n(c/2 + (d*x)/2)^6 + 1310*tan(c/2 + (d*x)/2)^7 - 2375*tan(c/2 + (d*x)/2)^8
- 335*tan(c/2 + (d*x)/2)^9 + 35*tan(c/2 + (d*x)/2)^10 - 5*tan(c/2 + (d*x)/2
)^11 + 2160*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^3 + 10800*log(tan(c/
2 + (d*x)/2))*tan(c/2 + (d*x)/2)^4 + 21600*log(tan(c/2 + (d*x)/2))*tan(c/2
+ (d*x)/2)^5 + 21600*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^6 + 10800*l
og(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^7 + 2160*log(tan(c/2 + (d*x)/2))*
tan(c/2 + (d*x)/2)^8 + 5))/(120*a^3*d*tan(c/2 + (d*x)/2)^3*(tan(c/2 + (d*x)
/2) + 1)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{A\left(\int\left(-\frac{\csc^4(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\right)dx+\int\frac{\sin(c+dx)\csc^4(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] -A*(Integral(-csc(c + d*x)**4/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(
c + d*x) + 1), x) + Integral(sin(c + d*x)*csc(c + d*x)**4/(sin(c + d*x)**3
+ 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x))/a**3
```

3.244 $\int (a + a \sin(e + fx))(A + B \sin(e + fx))(c + d \sin(e + fx))^3 \, dx$

Optimal. Leaf size=327

$$-\frac{a\left(5Ad\left(6c^2+20cd+9d^2\right)-B\left(6c^3-30c^2d-71cd^2-45d^3\right)\right)\sin(e+fx)\cos(e+fx)}{120f}+\frac{1}{8}ax\left(A\left(8c^3+12c^2d+\right.\right.$$

```
[Out] 1/8*a*(B*(4*c^3+12*c^2*d+9*c*d^2+3*d^3)+A*(8*c^3+12*c^2*d+12*c*d^2+3*d^3))*
x-1/30*a*(5*A*d*(3*c^3+16*c^2*d+12*c*d^2+4*d^3)-B*(3*c^4-15*c^3*d-52*c^2*d^
2-60*c*d^3-16*d^4))*cos(f*x+e)/d/f-1/120*a*(5*A*d*(6*c^2+20*c*d+9*d^2)-B*(6
*c^3-30*c^2*d-71*c*d^2-45*d^3))*cos(f*x+e)*sin(f*x+e)/f-1/60*a*(4*(5*A+4*B)
*d^2-3*c*(B*c-5*(A+B)*d))*cos(f*x+e)*(c+d*sin(f*x+e))^2/d/f+1/20*a*(B*c-5*(
A+B)*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3/d/f-1/5*a*B*cos(f*x+e)*(c+d*sin(f*x+e
))^4/d/f
```

Rubi [A] time = 0.58, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2968, 3023, 2753, 2734}

$$-\frac{a\left(5Ad\left(16c^2d+3c^3+12cd^2+4d^3\right)-B\left(-52c^2d^2-15c^3d+3c^4-60cd^3-16d^4\right)\right)\cos(e+fx)}{30df}-\frac{a\left(5Ad\left(6c^2+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a*(B*(4*c^3 + 12*c^2*d + 9*c*d^2 + 3*d^3) + A*(8*c^3 + 12*c^2*d + 12*c*d^2
 + 3*d^3))*x)/8 - (a*(5*A*d*(3*c^3 + 16*c^2*d + 12*c*d^2 + 4*d^3) - B*(3*c^
4 - 15*c^3*d - 52*c^2*d^2 - 60*c*d^3 - 16*d^4))*Cos[e + f*x])/(30*d*f) - (a
*(5*A*d*(6*c^2 + 20*c*d + 9*d^2) - B*(6*c^3 - 30*c^2*d - 71*c*d^2 - 45*d^3)
)*Cos[e + f*x]*Sin[e + f*x])/(120*f) - (a*(4*(5*A + 4*B)*d^2 - 3*c*(B*c - 5
*(A + B)*d))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(60*d*f) + (a*(B*c - 5*(A
 + B)*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(20*d*f) - (a*B*Cos[e + f*x]*
(c + d*Sin[e + f*x])^4)/(5*d*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \int (c+d\sin(e+fx))^3\left(aA+(aA+aB)\sin(e+\right.$$

$$= -\frac{aB\cos(e+fx)(c+d\sin(e+fx))^4}{5df} + \frac{\int (c+d}{}$$

$$= \frac{a(Bc-5(A+B)d)\cos(e+fx)(c+d\sin(e+fx}{20df}$$

$$= -\frac{a\left(4(5A+4B)d^2-3c(Bc-5(A+B)d)\right)\cos(e}{60df}$$

$$= \frac{1}{8}a\left(B\left(4c^3+12c^2d+9cd^2+3d^3\right)+A\left(8c^3+1\right.\right.$$

Mathematica [A] time = 2.00, size = 267, normalized size = 0.82

$$\frac{a(\sin(e+fx)+1)\left(10d\left(4Ad(3c+d)+B\left(12c^2+12cd+5d^2\right)\right)\cos(3(e+fx))+15\left(-8\left(Ad\left(3c^2+3cd+d^2\right)+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,
x]
```

```
[Out] (a*(1 + Sin[e + f*x])*(-60*(2*A*(4*c^3 + 12*c^2*d + 9*c*d^2 + 3*d^3) + B*(8
*c^3 + 18*c^2*d + 18*c*d^2 + 5*d^3))*Cos[e + f*x] + 10*d*(4*A*d*(3*c + d) +
 B*(12*c^2 + 12*c*d + 5*d^2))*Cos[3*(e + f*x)] - 6*B*d^3*Cos[5*(e + f*x)] +
 15*(4*(B*(4*c^3 + 12*c^2*d + 9*c*d^2 + 3*d^3) + A*(8*c^3 + 12*c^2*d + 12*c
*d^2 + 3*d^3))*f*x - 8*(B*(c + d)^3 + A*d*(3*c^2 + 3*c*d + d^2))*Sin[2*(e +
 f*x)] + d^2*(A*d + B*(3*c + d))*Sin[4*(e + f*x)])))/(480*f*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^2)
```

fricas [A] time = 0.47, size = 239, normalized size = 0.73

$$-\frac{24\,Bad^3\cos\left(fx+e\right)^5-40\left(3\,Bac^2d+3\,(A+B)acd^2+(A+2\,B)ad^3\right)\cos\left(fx+e\right)^3-15\left(4\,(2\,A+B)ac^3+12\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -1/120*(24*B*a*d^3*cos(f*x + e)^5 - 40*(3*B*a*c^2*d + 3*(A + B)*a*c*d^2 + (
A + 2*B)*a*d^3)*cos(f*x + e)^3 - 15*(4*(2*A + B)*a*c^3 + 12*(A + B)*a*c^2*d
 + 3*(4*A + 3*B)*a*c*d^2 + 3*(A + B)*a*d^3)*f*x + 120*((A + B)*a*c^3 + 3*(A
 + B)*a*c^2*d + 3*(A + B)*a*c*d^2 + (A + B)*a*d^3)*cos(f*x + e) - 15*(2*(3*
B*a*c*d^2 + (A + B)*a*d^3)*cos(f*x + e)^3 - (4*B*a*c^3 + 12*(A + B)*a*c^2*d
 + 3*(4*A + 5*B)*a*c*d^2 + 5*(A + B)*a*d^3)*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.18, size = 314, normalized size = 0.96

$$-\frac{Bad^3\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{1}{8}\left(8\,Aac^3+4\,Bac^3+12\,Aac^2d+12\,Bac^2d+12\,Aacd^2+9\,Bacd^2+3\,Aad^3+3\,Bad^3\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -1/80*B*a*d^3*cos(5*f*x + 5*e)/f + 1/8*(8*A*a*c^3 + 4*B*a*c^3 + 12*A*a*c^2*
d + 12*B*a*c^2*d + 12*A*a*c*d^2 + 9*B*a*c*d^2 + 3*A*a*d^3 + 3*B*a*d^3)*x +
1/48*(12*B*a*c^2*d + 12*A*a*c*d^2 + 12*B*a*c*d^2 + 4*A*a*d^3 + 5*B*a*d^3)*c
os(3*f*x + 3*e)/f - 1/8*(8*A*a*c^3 + 8*B*a*c^3 + 24*A*a*c^2*d + 18*B*a*c^2*
d + 18*A*a*c*d^2 + 18*B*a*c*d^2 + 6*A*a*d^3 + 5*B*a*d^3)*cos(f*x + e)/f + 1
/32*(3*B*a*c*d^2 + A*a*d^3 + B*a*d^3)*sin(4*f*x + 4*e)/f - 1/4*(B*a*c^3 + 3
*A*a*c^2*d + 3*B*a*c^2*d + 3*A*a*c*d^2 + 3*B*a*c*d^2 + A*a*d^3 + B*a*d^3)*s
in(2*f*x + 2*e)/f
```

maple [A] time = 0.52, size = 422, normalized size = 1.29

$$-A c^3 a \cos\left(fx+e\right) + 3A c^2 da \left(-\frac{\sin(fx+e)\cos(fx+e)}{2} + \frac{fx}{2} + \frac{e}{2}\right) - Ac\, d^2 a \left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right) + A d^3 a \Bigg(\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(-A*c^3*a*cos(f*x+e)+3*A*c^2*d*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/
2*e)-A*c*d^2*a*(2+sin(f*x+e)^2)*cos(f*x+e)+A*d^3*a*(-1/4*(sin(f*x+e)^3+3/2*
sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+B*c^3*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1
/2*f*x+1/2*e)-B*c^2*d*a*(2+sin(f*x+e)^2)*cos(f*x+e)+3*B*c*d^2*a*(-1/4*(sin(
f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*B*d^3*a*(8/3+sin(f*x
+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+A*c^3*a*(f*x+e)-3*A*c^2*d*a*cos(f*x+e)+3
*A*c*d^2*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/3*A*d^3*a*(2+sin(f*
x+e)^2)*cos(f*x+e)-B*c^3*a*cos(f*x+e)+3*B*c^2*d*a*(-1/2*sin(f*x+e)*cos(f*x+
e)+1/2*f*x+1/2*e)-B*c*d^2*a*(2+sin(f*x+e)^2)*cos(f*x+e)+B*d^3*a*(-1/4*(sin(
f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e))
```

maxima [A] time = 0.36, size = 406, normalized size = 1.24

$$480\left(fx+e\right)Aac^3 + 120\left(2fx+2e-\sin\left(2fx+2e\right)\right)Bac^3 + 360\left(2fx+2e-\sin\left(2fx+2e\right)\right)Aac^2d + 480\Big(\mathrm{co}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] 1/480*(480*(f*x + e)*A*a*c^3 + 120*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^3
 + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a*c^2*d + 480*(cos(f*x + e)^3 - 3
*cos(f*x + e))*B*a*c^2*d + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^2*d +
 480*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*c*d^2 + 360*(2*f*x + 2*e - sin(2
```

```
*f*x + 2*e))*A*a*c*d^2 + 480*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a*c*d^2 +
45*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a*c*d^2 + 160*
(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*d^3 + 15*(12*f*x + 12*e + sin(4*f*x +
 4*e) - 8*sin(2*f*x + 2*e))*A*a*d^3 - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e
)^3 + 15*cos(f*x + e))*B*a*d^3 + 15*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*s
in(2*f*x + 2*e))*B*a*d^3 - 480*A*a*c^3*cos(f*x + e) - 480*B*a*c^3*cos(f*x +
 e) - 1440*A*a*c^2*d*cos(f*x + e))/f
```

mupad [B] time = 15.58, size = 830, normalized size = 2.54

$$\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(8\,A\,c^3+3\,A\,d^3+4\,B\,c^3+3\,B\,d^3+12\,A\,c\,d^2+12\,A\,c^2\,d+9\,B\,c\,d^2+12\,B\,c^2\,d\right)}{4\left(2\,A\,a\,c^3+\frac{3\,A\,a\,d^3}{4}+B\,a\,c^3+\frac{3\,B\,a\,d^3}{4}+3\,A\,a\,c\,d^2+3\,A\,a\,c^2\,d+\frac{9\,B\,a\,c\,d^2}{4}+3\,B\,a\,c^2\,d\right)}\right)\left(8\,A\,c^3+3\,A\,d^3+4\,B\,c^3+3\,B\,d^3+12\right.}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^3,x)
```

```
[Out] (a*atan((a*tan(e/2 + (f*x)/2)*(8*A*c^3 + 3*A*d^3 + 4*B*c^3 + 3*B*d^3 + 12*A
*c*d^2 + 12*A*c^2*d + 9*B*c*d^2 + 12*B*c^2*d))/(4*(2*A*a*c^3 + (3*A*a*d^3)/
4 + B*a*c^3 + (3*B*a*d^3)/4 + 3*A*a*c*d^2 + 3*A*a*c^2*d + (9*B*a*c*d^2)/4 +
 3*B*a*c^2*d)))*(8*A*c^3 + 3*A*d^3 + 4*B*c^3 + 3*B*d^3 + 12*A*c*d^2 + 12*A*
c^2*d + 9*B*c*d^2 + 12*B*c^2*d))/(4*f) - (tan(e/2 + (f*x)/2)*((3*A*a*d^3)/4
 + B*a*c^3 + (3*B*a*d^3)/4 + 3*A*a*c*d^2 + 3*A*a*c^2*d + (9*B*a*c*d^2)/4 +
3*B*a*c^2*d) + tan(e/2 + (f*x)/2)^8*(2*A*a*c^3 + 2*B*a*c^3 + 6*A*a*c^2*d) +
 tan(e/2 + (f*x)/2)^2*(8*A*a*c^3 + (20*A*a*d^3)/3 + 8*B*a*c^3 + (16*B*a*d^3
)/3 + 20*A*a*c*d^2 + 24*A*a*c^2*d + 20*B*a*c*d^2 + 20*B*a*c^2*d) + tan(e/2
+ (f*x)/2)^4*(12*A*a*c^3 + (28*A*a*d^3)/3 + 12*B*a*c^3 + (32*B*a*d^3)/3 + 2
8*A*a*c*d^2 + 36*A*a*c^2*d + 28*B*a*c*d^2 + 28*B*a*c^2*d) + tan(e/2 + (f*x)
/2)^6*(8*A*a*c^3 + 4*A*a*d^3 + 8*B*a*c^3 + 12*A*a*c*d^2 + 24*A*a*c^2*d + 12
*B*a*c*d^2 + 12*B*a*c^2*d) - tan(e/2 + (f*x)/2)^9*((3*A*a*d^3)/4 + B*a*c^3
+ (3*B*a*d^3)/4 + 3*A*a*c*d^2 + 3*A*a*c^2*d + (9*B*a*c*d^2)/4 + 3*B*a*c^2*d
) + tan(e/2 + (f*x)/2)^3*((7*A*a*d^3)/2 + 2*B*a*c^3 + (7*B*a*d^3)/2 + 6*A*a
*c*d^2 + 6*A*a*c^2*d + (21*B*a*c*d^2)/2 + 6*B*a*c^2*d) - tan(e/2 + (f*x)/2)
^7*((7*A*a*d^3)/2 + 2*B*a*c^3 + (7*B*a*d^3)/2 + 6*A*a*c*d^2 + 6*A*a*c^2*d +
 (21*B*a*c*d^2)/2 + 6*B*a*c^2*d) + 2*A*a*c^3 + (4*A*a*d^3)/3 + 2*B*a*c^3 +
(16*B*a*d^3)/15 + 4*A*a*c*d^2 + 6*A*a*c^2*d + 4*B*a*c*d^2 + 4*B*a*c^2*d)/(f
*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^
6 + 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 + 1))
```

sympy [A] time = 5.37, size = 996, normalized size = 3.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((A*a*c**3*x - A*a*c**3*cos(e + f*x)/f + 3*A*a*c**2*d*x*sin(e + f*
x)**2/2 + 3*A*a*c**2*d*x*cos(e + f*x)**2/2 - 3*A*a*c**2*d*sin(e + f*x)*cos(
e + f*x)/(2*f) - 3*A*a*c**2*d*cos(e + f*x)/f + 3*A*a*c*d**2*x*sin(e + f*x)*
*2/2 + 3*A*a*c*d**2*x*cos(e + f*x)**2/2 - 3*A*a*c*d**2*sin(e + f*x)**2*cos(
e + f*x)/f - 3*A*a*c*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a*c*d**2*co
s(e + f*x)**3/f + 3*A*a*d**3*x*sin(e + f*x)**4/8 + 3*A*a*d**3*x*sin(e + f*x
)**2*cos(e + f*x)**2/4 + 3*A*a*d**3*x*cos(e + f*x)**4/8 - 5*A*a*d**3*sin(e
+ f*x)**3*cos(e + f*x)/(8*f) - A*a*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*
A*a*d**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 2*A*a*d**3*cos(e + f*x)**3/(3
*f) + B*a*c**3*x*sin(e + f*x)**2/2 + B*a*c**3*x*cos(e + f*x)**2/2 - B*a*c**
3*sin(e + f*x)*cos(e + f*x)/(2*f) - B*a*c**3*cos(e + f*x)/f + 3*B*a*c**2*d*
x*sin(e + f*x)**2/2 + 3*B*a*c**2*d*x*cos(e + f*x)**2/2 - 3*B*a*c**2*d*sin(e
 + f*x)**2*cos(e + f*x)/f - 3*B*a*c**2*d*sin(e + f*x)*cos(e + f*x)/(2*f) -
2*B*a*c**2*d*cos(e + f*x)**3/f + 9*B*a*c*d**2*x*sin(e + f*x)**4/8 + 9*B*a*c
*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 9*B*a*c*d**2*x*cos(e + f*x)**4/
8 - 15*B*a*c*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*B*a*c*d**2*sin(e +
 f*x)**2*cos(e + f*x)/f - 9*B*a*c*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) -
 2*B*a*c*d**2*cos(e + f*x)**3/f + 3*B*a*d**3*x*sin(e + f*x)**4/8 + 3*B*a*d*
*3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a*d**3*x*cos(e + f*x)**4/8 - B
*a*d**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a*d**3*sin(e + f*x)**3*cos(e +
 f*x)/(8*f) - 4*B*a*d**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 3*B*a*d**3
*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 8*B*a*d**3*cos(e + f*x)**5/(15*f), Ne
(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))**3*(a*sin(e) + a), True))
```

3.245 $\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))^2\,dx$

Optimal. Leaf size=213

$$\frac{1}{8}ax\left(4A\left(2c^2+2cd+d^2\right)+B\left(4c^2+8cd+3d^2\right)\right)-\frac{a\left(4Ad\left(c^2+3cd+d^2\right)-B\left(c^3-4c^2d-8cd^2-4d^3\right)\right)\cos(e+$$

```
[Out] 1/8*a*(4*A*(2*c^2+2*c*d+d^2)+B*(4*c^2+8*c*d+3*d^2))*x-1/6*a*(4*A*d*(c^2+3*c
*d+d^2)-B*(c^3-4*c^2*d-8*c*d^2-4*d^3))*cos(f*x+e)/d/f-1/24*a*(3*(4*A+3*B)*d
^2-2*c*(B*c-4*(A+B)*d))*cos(f*x+e)*sin(f*x+e)/f+1/12*a*(B*c-4*(A+B)*d)*cos(
f*x+e)*(c+d*sin(f*x+e))^2/d/f-1/4*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^3/d/f
```

Rubi [A] time = 0.36, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2968, 3023, 2753, 2734}

$$-\frac{a\left(4Ad\left(c^2+3cd+d^2\right)-B\left(-4c^2d+c^3-8cd^2-4d^3\right)\right)\cos(e+fx)}{6df}+\frac{a\left(-8cd(A+B)-3d^2(4A+3B)+2Bc^2\right)}{24f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a*(4*A*(2*c^2 + 2*c*d + d^2) + B*(4*c^2 + 8*c*d + 3*d^2))*x)/8 - (a*(4*A*d
*(c^2 + 3*c*d + d^2) - B*(c^3 - 4*c^2*d - 8*c*d^2 - 4*d^3))*Cos[e + f*x])/(
6*d*f) + (a*(2*B*c^2 - 8*(A + B)*c*d - 3*(4*A + 3*B)*d^2)*Cos[e + f*x]*Sin[
e + f*x])/(24*f) + (a*(B*c - 4*(A + B)*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])
^2)/(12*d*f) - (a*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(4*d*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

```
&& IntegerQ[2*m]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int (a + a\sin(e+fx))(A + B\sin(e+fx))(c + d\sin(e+fx))^2\,dx = \int (c + d\sin(e+fx))^2 \left(aA + (aA + aB)\sin(e +\right.$$

$$= -\frac{aB\cos(e+fx)(c+d\sin(e+fx))^3}{4df} + \frac{\int (c + d}{}$$

$$= \frac{a(Bc - 4(A+B)d)\cos(e+fx)(c + d\sin(e + f}{12df}$$

$$= \frac{1}{8}a\left(4A\left(2c^2 + 2cd + d^2\right) + B\left(4c^2 + 8cd + 3d^2\right)\right.$$

Mathematica [A] time = 1.12, size = 185, normalized size = 0.87

$$\frac{a(\sin(e+fx)+1)\left(3\left(4fx\left(4A\left(2c^2+2cd+d^2\right)+B\left(4c^2+8cd+3d^2\right)\right)-8\left(Ad(2c+d)+B(c+d)^2\right)\sin(2(e+f\right.\right.}{96f\left(\sin\left(\frac{1}{2}(e\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,
x]
```

```
[Out] (a*(1 + Sin[e + f*x])*(-24*(B*(4*c^2 + 6*c*d + 3*d^2) + A*(4*c^2 + 8*c*d +
3*d^2))*Cos[e + f*x] + 8*d*(A*d + B*(2*c + d))*Cos[3*(e + f*x)] + 3*(4*(4*A
*(2*c^2 + 2*c*d + d^2) + B*(4*c^2 + 8*c*d + 3*d^2))*f*x - 8*(B*(c + d)^2 +
A*d*(2*c + d))*Sin[2*(e + f*x)] + B*d^2*Sin[4*(e + f*x)])))/(96*f*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^2)
```

fricas [A] time = 0.47, size = 160, normalized size = 0.75

$$8\left(2\,Bacd + (A+B)ad^2\right)\cos\left(fx+e\right)^3 + 3\left(4\,(2\,A+B)ac^2 + 8\,(A+B)acd + (4\,A+3\,B)ad^2\right)fx - 24\left((A+B)ac\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] 1/24*(8*(2*B*a*c*d + (A + B)*a*d^2)*cos(f*x + e)^3 + 3*(4*(2*A + B)*a*c^2 +
 8*(A + B)*a*c*d + (4*A + 3*B)*a*d^2)*f*x - 24*((A + B)*a*c^2 + 2*(A + B)*a
*c*d + (A + B)*a*d^2)*cos(f*x + e) + 3*(2*B*a*d^2*cos(f*x + e)^3 - (4*B*a*c
^2 + 8*(A + B)*a*c*d + (4*A + 5*B)*a*d^2)*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.15, size = 198, normalized size = 0.93

$$\frac{Bad^2\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{1}{8}\left(8\,Aac^2+4\,Bac^2+8\,Aacd+8\,Bacd+4\,Aad^2+3\,Bad^2\right)x+\frac{\left(2\,Bacd+Aad^2+Bad^2\right)\text{co}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] 1/32*B*a*d^2*sin(4*f*x + 4*e)/f + 1/8*(8*A*a*c^2 + 4*B*a*c^2 + 8*A*a*c*d +
8*B*a*c*d + 4*A*a*d^2 + 3*B*a*d^2)*x + 1/12*(2*B*a*c*d + A*a*d^2 + B*a*d^2)
*cos(3*f*x + 3*e)/f - 1/4*(4*A*a*c^2 + 4*B*a*c^2 + 8*A*a*c*d + 6*B*a*c*d +
3*A*a*d^2 + 3*B*a*d^2)*cos(f*x + e)/f - 1/4*(B*a*c^2 + 2*A*a*c*d + 2*B*a*c*
d + A*a*d^2 + B*a*d^2)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.41, size = 274, normalized size = 1.29

$$-A\,c^2a\cos\left(fx+e\right)+2Acda\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{A\,d^2a\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}+B\,c^2a\left(-\frac{\sin\left(fx+e\right)\cos\left(fx\right.}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(-A*c^2*a*cos(f*x+e)+2*A*c*d*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*
e)-1/3*A*d^2*a*(2+sin(f*x+e)^2)*cos(f*x+e)+B*c^2*a*(-1/2*sin(f*x+e)*cos(f*x
+e)+1/2*f*x+1/2*e)-2/3*B*c*d*a*(2+sin(f*x+e)^2)*cos(f*x+e)+B*d^2*a*(-1/4*(s
in(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+A*c^2*a*(f*x+e)-2*A*c
*d*a*cos(f*x+e)+A*d^2*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-B*c^2*a*
cos(f*x+e)+2*B*c*d*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/3*B*d^2*a
*(2+sin(f*x+e)^2)*cos(f*x+e))
```

maxima [A] time = 0.37, size = 264, normalized size = 1.24

$$\frac{96\left(fx+e\right)Aac^2+24\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)Bac^2+48\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)Aacd+64\left(\cos\left(fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] 1/96*(96*(f*x + e)*A*a*c^2 + 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^2 +
48*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a*c*d + 64*(cos(f*x + e)^3 - 3*cos(f*
x + e))*B*a*c*d + 48*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c*d + 32*(cos(f*x
 + e)^3 - 3*cos(f*x + e))*A*a*d^2 + 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a
*d^2 + 32*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a*d^2 + 3*(12*f*x + 12*e + si
n(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a*d^2 - 96*A*a*c^2*cos(f*x + e) - 96
*B*a*c^2*cos(f*x + e) - 192*A*a*c*d*cos(f*x + e))/f
```

mupad [B] time = 15.41, size = 547, normalized size = 2.57

$$\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(8\,A\,c^2+4\,A\,d^2+4\,B\,c^2+3\,B\,d^2+8\,A\,c\,d+8\,B\,c\,d\right)}{4\left(2\,A\,a\,c^2+A\,a\,d^2+B\,a\,c^2+\frac{3\,B\,a\,d^2}{4}+2\,A\,a\,c\,d+2\,B\,a\,c\,d\right)}\right)\left(8\,A\,c^2+4\,A\,d^2+4\,B\,c^2+3\,B\,d^2+8\,A\,c\,d+8\,B\,c\,d\right)}{4\,f}-\frac{\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^2,x)
```

```
[Out] (a*atan((a*tan(e/2 + (f*x)/2)*(8*A*c^2 + 4*A*d^2 + 4*B*c^2 + 3*B*d^2 + 8*A*
c*d + 8*B*c*d))/(4*(2*A*a*c^2 + A*a*d^2 + B*a*c^2 + (3*B*a*d^2)/4 + 2*A*a*c
*d + 2*B*a*c*d)))*(8*A*c^2 + 4*A*d^2 + 4*B*c^2 + 3*B*d^2 + 8*A*c*d + 8*B*c*
d))/(4*f) - (tan(e/2 + (f*x)/2)^6*(2*A*a*c^2 + 2*B*a*c^2 + 4*A*a*c*d) + tan
(e/2 + (f*x)/2)*(A*a*d^2 + B*a*c^2 + (3*B*a*d^2)/4 + 2*A*a*c*d + 2*B*a*c*d)
```

```
 + tan(e/2 + (f*x)/2)^4*(6*A*a*c^2 + 4*A*a*d^2 + 6*B*a*c^2 + 4*B*a*d^2 + 12
*A*a*c*d + 8*B*a*c*d) + tan(e/2 + (f*x)/2)^2*(6*A*a*c^2 + (16*A*a*d^2)/3 +
6*B*a*c^2 + (16*B*a*d^2)/3 + 12*A*a*c*d + (32*B*a*c*d)/3) - tan(e/2 + (f*x)
/2)^7*(A*a*d^2 + B*a*c^2 + (3*B*a*d^2)/4 + 2*A*a*c*d + 2*B*a*c*d) + tan(e/2
 + (f*x)/2)^3*(A*a*d^2 + B*a*c^2 + (11*B*a*d^2)/4 + 2*A*a*c*d + 2*B*a*c*d)
- tan(e/2 + (f*x)/2)^5*(A*a*d^2 + B*a*c^2 + (11*B*a*d^2)/4 + 2*A*a*c*d + 2*
B*a*c*d) + 2*A*a*c^2 + (4*A*a*d^2)/3 + 2*B*a*c^2 + (4*B*a*d^2)/3 + 4*A*a*c*
d + (8*B*a*c*d)/3)/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*
tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))
```

sympy [A] time = 2.31, size = 571, normalized size = 2.68

$$\begin{cases} Aac^2x - \frac{Aac^2\cos\left(e+fx\right)}{f} + Aacdx\sin^2\left(e+fx\right) + Aacdx\cos^2\left(e+fx\right) - \frac{Aacd\sin\left(e+fx\right)\cos\left(e+fx\right)}{f} - \frac{2Aacd\cos\left(e+fx\right)}{f} + \\ x\left(A+B\sin\left(e\right)\right)\left(c+d\sin\left(e\right)\right)^2\left(a\sin\left(e\right)+a\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((A*a*c**2*x - A*a*c**2*cos(e + f*x)/f + A*a*c*d*x*sin(e + f*x)**2
 + A*a*c*d*x*cos(e + f*x)**2 - A*a*c*d*sin(e + f*x)*cos(e + f*x)/f - 2*A*a*
c*d*cos(e + f*x)/f + A*a*d**2*x*sin(e + f*x)**2/2 + A*a*d**2*x*cos(e + f*x)
**2/2 - A*a*d**2*sin(e + f*x)**2*cos(e + f*x)/f - A*a*d**2*sin(e + f*x)*cos
(e + f*x)/(2*f) - 2*A*a*d**2*cos(e + f*x)**3/(3*f) + B*a*c**2*x*sin(e + f*x
)**2/2 + B*a*c**2*x*cos(e + f*x)**2/2 - B*a*c**2*sin(e + f*x)*cos(e + f*x)/
(2*f) - B*a*c**2*cos(e + f*x)/f + B*a*c*d*x*sin(e + f*x)**2 + B*a*c*d*x*cos
(e + f*x)**2 - 2*B*a*c*d*sin(e + f*x)**2*cos(e + f*x)/f - B*a*c*d*sin(e + f
*x)*cos(e + f*x)/f - 4*B*a*c*d*cos(e + f*x)**3/(3*f) + 3*B*a*d**2*x*sin(e +
 f*x)**4/8 + 3*B*a*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a*d**2*x*
cos(e + f*x)**4/8 - 5*B*a*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - B*a*d**
2*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a*d**2*sin(e + f*x)*cos(e + f*x)**3/
(8*f) - 2*B*a*d**2*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(A + B*sin(e))*(c +
 d*sin(e))**2*(a*sin(e) + a), True))
```

3.246 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=111

$$-\frac{a(3A(c+d)+B(3c+d))\cos(e+fx)}{3f}-\frac{a(3Ad+3Bc-Bd)\sin(e+fx)\cos(e+fx)}{6f}+\frac{1}{2}ax(A(2c+d)+B(c+d))-\frac{Ba$$

```
[Out] 1/2*a*(B*(c+d)+A*(2*c+d))*x-1/3*a*(3*A*(c+d)+B*(3*c+d))*cos(f*x+e)/f-1/6*a*
(3*A*d+3*B*c-B*d)*cos(f*x+e)*sin(f*x+e)/f-1/3*B*d*cos(f*x+e)*(a+a*sin(f*x+e
))^2/a/f
```

Rubi [A] time = 0.16, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2968, 3023, 2734}

$$-\frac{a(3A(c+d)+B(3c+d))\cos(e+fx)}{3f}-\frac{a(3Ad+3Bc-Bd)\sin(e+fx)\cos(e+fx)}{6f}+\frac{1}{2}ax(A(2c+d)+B(c+d))-\frac{Ba$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (a*(B*(c + d) + A*(2*c + d))*x)/2 - (a*(3*A*(c + d) + B*(3*c + d))*Cos[e +
f*x])/(3*f) - (a*(3*B*c + 3*A*d - B*d)*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (
B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/(3*a*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int (a+a\sin(e+fx))\left(Ac+(Bc+Ad)\sin(e+\right.$$

$$= -\frac{Bd\cos(e+fx)(a+a\sin(e+fx))^2}{3af} + \frac{\int (a+}{}$$

$$= \frac{1}{2}a(B(c+d)+A(2c+d))x - \frac{a(3A(c+d)+B(}{}$$

Mathematica [A] time = 0.44, size = 104, normalized size = 0.94

$$\frac{a(-3(4A(c+d)+B(4c+3d))\cos(e+fx)+12Acfx-3Ad\sin(2(e+fx))+6Adfx-3Bc\sin(2(e+fx))+6Bc}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (a*(12*A*c*f*x + 6*B*c*f*x + 6*A*d*f*x + 6*B*d*f*x - 3*(4*A*(c + d) + B*(4*
c + 3*d))*Cos[e + f*x] + B*d*Cos[3*(e + f*x)] - 3*B*c*Sin[2*(e + f*x)] - 3*
A*d*Sin[2*(e + f*x)] - 3*B*d*Sin[2*(e + f*x)]))/(12*f)
```

fricas [A] time = 0.47, size = 84, normalized size = 0.76

$$\frac{2\,Bad\cos\left(fx+e\right)^3+3\,((2\,A+B)ac+(A+B)ad)fx-3\,(Bac+(A+B)ad)\cos\left(fx+e\right)\sin\left(fx+e\right)-6\,((A+}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] 1/6*(2*B*a*d*cos(f*x + e)^3 + 3*((2*A + B)*a*c + (A + B)*a*d)*f*x - 3*(B*a*
c + (A + B)*a*d)*cos(f*x + e)*sin(f*x + e) - 6*((A + B)*a*c + (A + B)*a*d)*
cos(f*x + e))/f
```

giac [A] time = 0.15, size = 101, normalized size = 0.91

$$\frac{Bad\cos\left(3\,fx+3\,e\right)}{12\,f}+\frac{1}{2}\,(2\,Aac+Bac+Aad+Bad)x-\frac{(4\,Aac+4\,Bac+4\,Aad+3\,Bad)\cos\left(fx+e\right)}{4\,f}-\frac{(Bac+Aa}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] 1/12*B*a*d*cos(3*f*x + 3*e)/f + 1/2*(2*A*a*c + B*a*c + A*a*d + B*a*d)*x - 1
/4*(4*A*a*c + 4*B*a*c + 4*A*a*d + 3*B*a*d)*cos(f*x + e)/f - 1/4*(B*a*c + A*
a*d + B*a*d)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.30, size = 147, normalized size = 1.32

$$\frac{-\frac{Bad\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}+Aad\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+Bac\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+Bad\left(-\frac{\sin(f}{}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] 1/f*(-1/3*B*a*d*(2+sin(f*x+e)^2)*cos(f*x+e)+A*a*d*(-1/2*sin(f*x+e)*cos(f*x+
e)+1/2*f*x+1/2*e)+B*a*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+B*a*d*(-
1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-A*a*c*cos(f*x+e)-A*a*d*cos(f*x+e)-
B*a*c*cos(f*x+e)+A*a*c*(f*x+e))
```

maxima [A] time = 0.37, size = 143, normalized size = 1.29

$$\frac{12\left(fx+e\right)Aac+3\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)Bac+3\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)Aad+4\left(\cos\left(fx+e\right)^3-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] 1/12*(12*(f*x + e)*A*a*c + 3*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c + 3*(2*
f*x + 2*e - sin(2*f*x + 2*e))*A*a*d + 4*(cos(f*x + e)^3 - 3*cos(f*x + e))*B
*a*d + 3*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*d - 12*A*a*c*cos(f*x + e) - 1
2*B*a*c*cos(f*x + e) - 12*A*a*d*cos(f*x + e))/f
```

mupad [B] time = 13.31, size = 134, normalized size = 1.21

$$-\frac{\frac{3\,A\,a\,d\,\sin\left(2\,e+2\,f\,x\right)}{2}-\frac{B\,a\,d\,\cos\left(3\,e+3\,f\,x\right)}{2}+\frac{3\,B\,a\,c\,\sin\left(2\,e+2\,f\,x\right)}{2}+\frac{3\,B\,a\,d\,\sin\left(2\,e+2\,f\,x\right)}{2}+6\,A\,a\,c\,\cos\left(e+f\,x\right)+6\,A\,a\,d\,\cos}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x)),x)
```

```
[Out] -((3*A*a*d*sin(2*e + 2*f*x))/2 - (B*a*d*cos(3*e + 3*f*x))/2 + (3*B*a*c*sin(
2*e + 2*f*x))/2 + (3*B*a*d*sin(2*e + 2*f*x))/2 + 6*A*a*c*cos(e + f*x) + 6*A
*a*d*cos(e + f*x) + 6*B*a*c*cos(e + f*x) + (9*B*a*d*cos(e + f*x))/2 - 6*A*a
*c*f*x - 3*A*a*d*f*x - 3*B*a*c*f*x - 3*B*a*d*f*x)/(6*f)
```

sympy [A] time = 1.00, size = 277, normalized size = 2.50

$$\begin{cases} Aacx - \frac{Aac\cos\left(e+fx\right)}{f} + \frac{Aadx\sin^2\left(e+fx\right)}{2} + \frac{Aadx\cos^2\left(e+fx\right)}{2} - \frac{Aad\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f} - \frac{Aad\cos\left(e+fx\right)}{f} + \frac{Bacx\sin^2\left(e+fx\right)}{2} + \\ x\left(A+B\sin\left(e\right)\right)\left(c+d\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((A*a*c*x - A*a*c*cos(e + f*x)/f + A*a*d*x*sin(e + f*x)**2/2 + A*a
*d*x*cos(e + f*x)**2/2 - A*a*d*sin(e + f*x)*cos(e + f*x)/(2*f) - A*a*d*cos(
e + f*x)/f + B*a*c*x*sin(e + f*x)**2/2 + B*a*c*x*cos(e + f*x)**2/2 - B*a*c*
sin(e + f*x)*cos(e + f*x)/(2*f) - B*a*c*cos(e + f*x)/f + B*a*d*x*sin(e + f*
x)**2/2 + B*a*d*x*cos(e + f*x)**2/2 - B*a*d*sin(e + f*x)**2*cos(e + f*x)/f
- B*a*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*B*a*d*cos(e + f*x)**3/(3*f), Ne
(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))*(a*sin(e) + a), True))
```

3.247 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=48

$$-\frac{a(A+B)\cos(e+fx)}{f}+\frac{1}{2}ax(2A+B)-\frac{aB\sin(e+fx)\cos(e+fx)}{2f}$$

```
[Out] 1/2*a*(2*A+B)*x-a*(A+B)*cos(f*x+e)/f-1/2*a*B*cos(f*x+e)*sin(f*x+e)/f
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2734}

$$-\frac{a(A+B)\cos(e+fx)}{f}+\frac{1}{2}ax(2A+B)-\frac{aB\sin(e+fx)\cos(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]),x]
```

```
[Out] (a*(2*A + B)*x)/2 - (a*(A + B)*Cos[e + f*x])/f - (a*B*Cos[e + f*x]*Sin[e +
f*x])/(2*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\,dx=\frac{1}{2}a(2A+B)x-\frac{a(A+B)\cos(e+fx)}{f}-\frac{aB\cos(e+fx)\sin(e+fx)}{2f}$$

Mathematica [A] time = 0.10, size = 45, normalized size = 0.94

$$\frac{a(-4(A+B)\cos(e+fx)+4Afx-B\sin(2(e+fx))+2Be+2Bfx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]),x]
```

[Out] (a*(2*B*e + 4*A*f*x + 2*B*f*x - 4*(A + B)*Cos[e + f*x] - B*Sin[2*(e + f*x)]
))/(4*f)

fricas [A] time = 0.43, size = 43, normalized size = 0.90

$$\frac{(2A+B)afx - Ba\cos\left(fx+e\right)\sin\left(fx+e\right) - 2(A+B)a\cos\left(fx+e\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="fricas")

[Out] 1/2*((2*A + B)*a*f*x - B*a*cos(f*x + e)*sin(f*x + e) - 2*(A + B)*a*cos(f*x
+ e))/f

giac [A] time = 0.13, size = 48, normalized size = 1.00

$$\frac{1}{2}(2Aa+Ba)x - \frac{Ba\sin\left(2fx+2e\right)}{4f} - \frac{(Aa+Ba)\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="giac")

[Out] 1/2*(2*A*a + B*a)*x - 1/4*B*a*sin(2*f*x + 2*e)/f - (A*a + B*a)*cos(f*x + e)
/f

maple [A] time = 0.16, size = 59, normalized size = 1.23

$$\frac{aB\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right) - aA\cos\left(fx+e\right) - aB\cos\left(fx+e\right) + aA\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)

[Out] 1/f*(a*B*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-a*A*cos(f*x+e)-a*B*cos(
f*x+e)+a*A*(f*x+e))

maxima [A] time = 0.45, size = 57, normalized size = 1.19

$$\frac{4\left(fx+e\right)Aa + \left(2fx+2e-\sin\left(2fx+2e\right)\right)Ba - 4Aa\cos\left(fx+e\right) - 4Ba\cos\left(fx+e\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*A*a + (2*f*x + 2*e - sin(2*f*x + 2*e))*B*a - 4*A*a*cos(f*x
 + e) - 4*B*a*cos(f*x + e))/f
```

mupad [B] time = 13.26, size = 100, normalized size = 2.08

$$A\,a\,x-\frac{-B\,a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+\left(2\,A\,a+2\,B\,a\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+B\,a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+2\,A\,a+2\,B\,a}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}+\frac{B\,a\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x)),x)
```

```
[Out] A*a*x - (2*A*a + 2*B*a + tan(e/2 + (f*x)/2)^2*(2*A*a + 2*B*a) - B*a*tan(e/2
 + (f*x)/2)^3 + B*a*tan(e/2 + (f*x)/2))/(f*(2*tan(e/2 + (f*x)/2)^2 + tan(e/
2 + (f*x)/2)^4 + 1)) + (B*a*x)/2
```

sympy [A] time = 0.35, size = 94, normalized size = 1.96

$$\begin{cases} Aax - \frac{Aa\cos\left(e+fx\right)}{f} + \frac{Bax\sin^2\left(e+fx\right)}{2} + \frac{Bax\cos^2\left(e+fx\right)}{2} - \frac{Ba\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f} - \frac{Ba\cos\left(e+fx\right)}{f} & \text{for}\: f \neq 0 \\ x\left(A+B\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)
```

```
[Out] Piecewise((A*a*x - A*a*cos(e + f*x)/f + B*a*x*sin(e + f*x)**2/2 + B*a*x*cos
(e + f*x)**2/2 - B*a*sin(e + f*x)*cos(e + f*x)/(2*f) - B*a*cos(e + f*x)/f,
Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a), True))
```

3.248 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=98

$$\frac{2a(c-d)(Bc-Ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\sqrt{c^2-d^2}}-\frac{ax(Bc-d(A+B))}{d^2}-\frac{aB\cos(e+fx)}{df}$$

```
[Out] -a*(B*c-(A+B)*d)*x/d^2-a*B*cos(f*x+e)/d/f+2*a*(c-d)*(-A*d+B*c)*arctan((d+c*
tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^2/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2968, 3023, 2735, 2660, 618, 204}

$$\frac{2a(c-d)(Bc-Ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\sqrt{c^2-d^2}}-\frac{ax(Bc-d(A+B))}{d^2}-\frac{aB\cos(e+fx)}{df}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```

```
[Out] -((a*(B*c - (A + B)*d)*x)/d^2) + (2*a*(c - d)*(B*c - A*d)*ArcTan[(d + c*Tan
[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^2*Sqrt[c^2 - d^2]*f) - (a*B*Cos[e + f*x
])/(d*f)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \int \frac{aA+(aA+aB)\sin(e+fx)+aB\sin^2(e+fx)}{c+d\sin(e+fx)}\,dx$$
$$= -\frac{aB\cos(e+fx)}{df} + \frac{\int \frac{aAd-a(Bc-(A+B)d)\sin(e+fx)}{c+d\sin(e+fx)}\,dx}{d}$$
$$= -\frac{a(Bc-(A+B)d)x}{d^2} - \frac{aB\cos(e+fx)}{df} + \frac{(a(c-d)(Bc-Ad))\int \frac{}{c}$$
$$= -\frac{a(Bc-(A+B)d)x}{d^2} - \frac{aB\cos(e+fx)}{df} + \frac{(2a(c-d)(Bc-Ad))\,\mathrm{S}}{}$$
$$= -\frac{a(Bc-(A+B)d)x}{d^2} - \frac{aB\cos(e+fx)}{df} - \frac{(4a(c-d)(Bc-Ad))\,\mathrm{S}}{}$$
$$= -\frac{a(Bc-(A+B)d)x}{d^2} + \frac{2a(c-d)(Bc-Ad)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right.}{d^2\sqrt{c^2-d^2}\,f}$$

Mathematica [C] time = 0.65, size = 196, normalized size = 2.00

$$\frac{a(\sin(e+fx)+1)\left(\frac{2(c-d)(\cos(e)-i\sin(e))(Bc-Ad)\tan^{-1}\left(\frac{(\cos(e)-i\sin(e))\sec\left(\frac{fx}{2}\right)\left(c\sin\left(\frac{fx}{2}\right)+d\cos\left(e+\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)}{f\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}} + Adx + Bx(d-c) + \frac{Bd\sin(e)}{f}\right.}{d^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),
x]
```

```
[Out] (a*(A*d*x + B*(-c + d)*x - (B*d*Cos[e]*Cos[f*x])/f + (2*(c - d)*(B*c - A*d)
*ArcTan[(Sec[(f*x)/2]*(Cos[e] - I*Sin[e])*(d*Cos[e + (f*x)/2] + c*Sin[(f*x)
/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(Cos[e] - I*Sin[e]))/(
Sqrt[c^2 - d^2]*f*Sqrt[(Cos[e] - I*Sin[e])^2]) + (B*d*Sin[e]*Sin[f*x])/f)*(
1 + Sin[e + f*x]))/(d^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)
```

fricas [A] time = 0.49, size = 292, normalized size = 2.98

$$\left[-\frac{2\,Bad\cos\left(fx+e\right)+2\,(Bac-(A+B)ad)fx-(Bac-Aad)\sqrt{-\frac{c-d}{c+d}}\,\log\left(-\frac{\left(2\,c^2-d^2\right)\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2-2\,(}{d^2\cos\left(fx+e\right)^2}\right.}{2\,d^2f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] [-1/2*(2*B*a*d*cos(f*x + e) + 2*(B*a*c - (A + B)*a*d)*f*x - (B*a*c - A*a*d)
*sqrt(-(c - d)/(c + d))*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x
+ e) - c^2 - d^2 - 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*c
os(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x +
e) - c^2 - d^2)))/(d^2*f), -(B*a*d*cos(f*x + e) + (B*a*c - (A + B)*a*d)*f*x
 + (B*a*c - A*a*d)*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt(
(c - d)/(c + d))/((c - d)*cos(f*x + e))))/(d^2*f)]
```

giac [A] time = 0.15, size = 141, normalized size = 1.44

$$-\frac{\frac{(Bac-Aad-Bad)\left(fx+e\right)}{d^2}+\frac{2\,Ba}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)d}-\frac{2\,\left(Bac^2-Aacd-Bacd+Aad^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}\,d^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] -((B*a*c - A*a*d - B*a*d)*(f*x + e)/d^2 + 2*B*a/((tan(1/2*f*x + 1/2*e)^2 +
1)*d) - 2*(B*a*c^2 - A*a*c*d - B*a*c*d + A*a*d^2)*(pi*floor(1/2*(f*x + e)/p
i + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sq
rt(c^2 - d^2)*d^2))/f
```

maple [B] time = 0.40, size = 294, normalized size = 3.00

$$-\frac{2a\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)Ac}{fd\sqrt{c^2-d^2}}+\frac{2a\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)A}{f\sqrt{c^2-d^2}}+\frac{2a\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)B\,c^2}{f\,d^2\sqrt{c^2-d^2}}-\frac{2a\arctan\left(\frac{2c\tan}{2}\right.}{fd\sqrt{c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] -2/f*a/d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^
(1/2))*A*c+2/f*a/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c
^2-d^2)^(1/2))*A+2/f*a/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*
e)+2*d)/(c^2-d^2)^(1/2))*B*c^2-2/f*a/d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(
1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c-2/f*a/d*B/(1+tan(1/2*f*x+1/2*e)^2)
+2/f*a/d*A*arctan(tan(1/2*f*x+1/2*e))-2/f*a/d^2*B*arctan(tan(1/2*f*x+1/2*e)
)*c+2/f*a/d*B*arctan(tan(1/2*f*x+1/2*e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 16.58, size = 3074, normalized size = 31.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c + d*sin(e + f*x)),x)
```

```
[Out] (2*A*a*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c + d)) + (2*B*a*at
an(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c + d)) - (B*a*cos(e + f*x))
/(f*(c + d)) + (2*A*a*c*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(d*f*(
c + d)) - (A*a*atan((A^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*3i + A^2*
d^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i - B^2*c^4*sin(e/2 + (f*x)/2)*(d
^2 - c^2)^(3/2)*2i - B^2*c^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + B^2*
d^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + A*B*d^6*sin(e/2 + (f*x)/2)*(d
^2 - c^2)^(1/2)*4i + A^2*c*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*1i + A^
2*c*d^5*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + B^2*c*d^5*cos(e/2 + (f*x)
/2)*(d^2 - c^2)^(1/2)*1i + B^2*c^3*d*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*1
i + B^2*c^5*d*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + A^2*c*d^5*sin(e/2 +
 (f*x)/2)*(d^2 - c^2)^(1/2)*4i + A^2*c^2*d^4*cos(e/2 + (f*x)/2)*(d^2 - c^2)
^(1/2)*2i + A^2*c^3*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i - B^2*c^3*d
```

^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A^2*c^2*d^2*sin(e/2 + (f*x)/2)
*(d^2 - c^2)^(3/2)*2i + A^2*c^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*3i
 - A^2*c^3*d^3*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A^2*c^4*d^2*sin(e/
2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + B^2*c^2*d^2*sin(e/2 + (f*x)/2)*(d^2 - c
^2)^(3/2)*3i - B^2*c^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i + B^2*c^
4*d^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i + A*B*c*d^5*cos(e/2 + (f*x)/2
)*(d^2 - c^2)^(1/2)*2i - A*B*c*d^3*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*6i
+ A*B*c*d^5*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i + A*B*c^3*d*sin(e/2 + (
f*x)/2)*(d^2 - c^2)^(3/2)*4i + A*B*c^5*d*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/
2)*4i - A*B*c^2*d^2*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*2i + A*B*c^2*d^4*c
os(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A*B*c^3*d^3*cos(e/2 + (f*x)/2)*(d^
2 - c^2)^(1/2)*2i - A*B*c^4*d^2*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A
*B*c^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i - A*B*c^3*d^3*sin(e/2 +
(f*x)/2)*(d^2 - c^2)^(1/2)*10i + A*B*c^4*d^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)
^(1/2)*2i)/(4*A^2*d^7*sin(e/2 + (f*x)/2) + 2*B^2*d^7*sin(e/2 + (f*x)/2) + 2
*A^2*c^2*d^5*cos(e/2 + (f*x)/2) - 2*A^2*c^3*d^4*cos(e/2 + (f*x)/2) - 2*A^2*
c^4*d^3*cos(e/2 + (f*x)/2) - 2*B^2*c^3*d^4*cos(e/2 + (f*x)/2) + B^2*c^5*d^2
*cos(e/2 + (f*x)/2) - 4*A^2*c^2*d^5*sin(e/2 + (f*x)/2) - 4*A^2*c^3*d^4*sin(
e/2 + (f*x)/2) - 4*B^2*c^2*d^5*sin(e/2 + (f*x)/2) + 2*B^2*c^4*d^3*sin(e/2 +
 (f*x)/2) + 4*A*B*d^7*sin(e/2 + (f*x)/2) + 2*A^2*c*d^6*cos(e/2 + (f*x)/2) +
 B^2*c*d^6*cos(e/2 + (f*x)/2) + 4*A^2*c*d^6*sin(e/2 + (f*x)/2) - 4*A*B*c^3*
d^4*cos(e/2 + (f*x)/2) + 2*A*B*c^5*d^2*cos(e/2 + (f*x)/2) - 8*A*B*c^2*d^5*s
in(e/2 + (f*x)/2) + 4*A*B*c^4*d^3*sin(e/2 + (f*x)/2) + 2*A*B*c*d^6*cos(e/2
+ (f*x)/2)))*(d^2 - c^2)^(1/2)*2i)/(d*f*(c + d)) - (2*B*a*c^2*atan(sin(e/2
+ (f*x)/2)/cos(e/2 + (f*x)/2)))/(d^2*f*(c + d)) - (B*a*c*cos(e + f*x))/(d*f
*(c + d)) + (B*a*c*atan((A^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*3i +
A^2*d^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i - B^2*c^4*sin(e/2 + (f*x)/2
)*(d^2 - c^2)^(3/2)*2i - B^2*c^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i +
B^2*d^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + A*B*d^6*sin(e/2 + (f*x)/2
)*(d^2 - c^2)^(1/2)*4i + A^2*c*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*1i
+ A^2*c*d^5*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + B^2*c*d^5*cos(e/2 + (
f*x)/2)*(d^2 - c^2)^(1/2)*1i + B^2*c^3*d*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(3/
2)*1i + B^2*c^5*d*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + A^2*c*d^5*sin(e
/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*4i + A^2*c^2*d^4*cos(e/2 + (f*x)/2)*(d^2 -
c^2)^(1/2)*2i + A^2*c^3*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i - B^2*c
^3*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A^2*c^2*d^2*sin(e/2 + (f*x
)/2)*(d^2 - c^2)^(3/2)*2i + A^2*c^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2
)*3i - A^2*c^3*d^3*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A^2*c^4*d^2*si
n(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + B^2*c^2*d^2*sin(e/2 + (f*x)/2)*(d^2
 - c^2)^(3/2)*3i - B^2*c^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i + B^
2*c^4*d^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i + A*B*c*d^5*cos(e/2 + (f*
x)/2)*(d^2 - c^2)^(1/2)*2i - A*B*c*d^3*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)
*6i + A*B*c*d^5*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i + A*B*c^3*d*sin(e/2
 + (f*x)/2)*(d^2 - c^2)^(3/2)*4i + A*B*c^5*d*sin(e/2 + (f*x)/2)*(d^2 - c^2)
^(1/2)*4i - A*B*c^2*d^2*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*2i + A*B*c^2*d

```
^4*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A*B*c^3*d^3*cos(e/2 + (f*x)/2)
*(d^2 - c^2)^(1/2)*2i - A*B*c^4*d^2*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i
 - A*B*c^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i - A*B*c^3*d^3*sin(e/
2 + (f*x)/2)*(d^2 - c^2)^(1/2)*10i + A*B*c^4*d^2*sin(e/2 + (f*x)/2)*(d^2 -
c^2)^(1/2)*2i)/(4*A^2*d^7*sin(e/2 + (f*x)/2) + 2*B^2*d^7*sin(e/2 + (f*x)/2)
 + 2*A^2*c^2*d^5*cos(e/2 + (f*x)/2) - 2*A^2*c^3*d^4*cos(e/2 + (f*x)/2) - 2*
A^2*c^4*d^3*cos(e/2 + (f*x)/2) - 2*B^2*c^3*d^4*cos(e/2 + (f*x)/2) + B^2*c^5
*d^2*cos(e/2 + (f*x)/2) - 4*A^2*c^2*d^5*sin(e/2 + (f*x)/2) - 4*A^2*c^3*d^4*
sin(e/2 + (f*x)/2) - 4*B^2*c^2*d^5*sin(e/2 + (f*x)/2) + 2*B^2*c^4*d^3*sin(e
/2 + (f*x)/2) + 4*A*B*d^7*sin(e/2 + (f*x)/2) + 2*A^2*c*d^6*cos(e/2 + (f*x)/
2) + B^2*c*d^6*cos(e/2 + (f*x)/2) + 4*A^2*c*d^6*sin(e/2 + (f*x)/2) - 4*A*B*
c^3*d^4*cos(e/2 + (f*x)/2) + 2*A*B*c^5*d^2*cos(e/2 + (f*x)/2) - 8*A*B*c^2*d
^5*sin(e/2 + (f*x)/2) + 4*A*B*c^4*d^3*sin(e/2 + (f*x)/2) + 2*A*B*c*d^6*cos(
e/2 + (f*x)/2)))*(d^2 - c^2)^(1/2)*2i)/(d^2*f*(c + d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.249 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=124

$$\frac{2a\left(d^2(A+B)(c-d)-Bc\left(c^2-d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\left(c^2-d^2\right)^{3/2}}+\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}+\frac{aBx}{d^2}$$

```
[Out] a*B*x/d^2+2*a*((A+B)*(c-d)*d^2-B*c*(c^2-d^2))*arctan((d+c*tan(1/2*f*x+1/2*e
))/(c^2-d^2)^(1/2))/d^2/(c^2-d^2)^(3/2)/f+a*(-A*d+B*c)*cos(f*x+e)/d/(c+d)/f
/(c+d*sin(f*x+e))
```

Rubi [A] time = 0.33, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2968, 3021, 2735, 2660, 618, 204}

$$\frac{2a\left(d^2(A+B)(c-d)-Bc\left(c^2-d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\left(c^2-d^2\right)^{3/2}}+\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}+\frac{aBx}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a*B*x)/d^2 + (2*a*((A + B)*(c - d)*d^2 - B*c*(c^2 - d^2))*ArcTan[(d + c*Ta
n[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^2*(c^2 - d^2)^(3/2)*f) + (a*(B*c - A*d
)*Cos[e + f*x])/(d*(c + d)*f*(c + d*Sin[e + f*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \int \frac{aA+(aA+aB)\sin(e+fx)+aB\sin^2(e+fx)}{(c+d\sin(e+fx))^2}\,dx$$

$$= \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))} - \frac{\int \frac{-a(A+B)(c-d)d-aB\left(c^2-d^2\right)\sin(e+fx)}{c+d\sin(e+fx)}\,dx}{d\left(c^2-d^2\right)}$$

$$= \frac{aBx}{d^2} + \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))} + \frac{\left(a\left(Ad^2-B\left(c^2+cd-d^2\right)\right)\right.}{d^2(c+d)}$$

$$= \frac{aBx}{d^2} + \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))} + \frac{\left(2a\left(Ad^2-B\left(c^2+cd-d^2\right.\right.\right.}{}$$

$$= \frac{aBx}{d^2} + \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))} - \frac{\left(4a\left(Ad^2-B\left(c^2+cd-d^2\right.\right.\right.}{}$$

$$= \frac{aBx}{d^2} + \frac{2a\left(Ad^2-B\left(c^2+cd-d^2\right)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2(c+d)\sqrt{c^2-d^2}\,f} + \frac{a(}{d(c-}$$

Mathematica [C] time = 1.27, size = 217, normalized size = 1.75

$$\frac{a(\sin(e+fx)+1)\left(\frac{2(\cos(e)-i\sin(e))\left(Ad^2-B\left(c^2+cd-d^2\right)\right)\tan^{-1}\left(\frac{(\cos(e)-i\sin(e))\sec\left(\frac{fx}{2}\right)\left(c\sin\left(\frac{fx}{2}\right)+d\cos\left(e+\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)}{f(c+d)\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}} + \frac{\csc(e)(Ad-Bc)(c\cos(e)+d\sin(}{f(c+d)(c+d\sin(e+fx))}\right.}{d^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^
2,x]
```

```
[Out] (a*(1 + Sin[e + f*x])*(B*x + (2*(A*d^2 - B*(c^2 + c*d - d^2))*ArcTan[(Sec[(
f*x)/2]*(Cos[e] - I*Sin[e])*(d*Cos[e + (f*x)/2] + c*Sin[(f*x)/2]))/(Sqrt[c^
2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(Cos[e] - I*Sin[e]))/((c + d)*Sqrt[c
^2 - d^2]*f*Sqrt[(Cos[e] - I*Sin[e])^2]) + ((-(B*c) + A*d)*Csc[e]*(c*Cos[e]
 + d*Sin[f*x]))/((c + d)*f*(c + d*Sin[e + f*x]))))/(d^2*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^2)
```

fricas [B] time = 0.49, size = 655, normalized size = 5.28

$$\left[\frac{2\left(Bac^3d+Bac^2d^2-Bacd^3-Bad^4\right)fx\sin\left(fx+e\right)+2\left(Bac^4+Bac^3d-Bac^2d^2-Bacd^3\right)fx+\left(Bac^3+Bac^2d-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] [1/2*(2*(B*a*c^3*d + B*a*c^2*d^2 - B*a*c*d^3 - B*a*d^4)*f*x*sin(f*x + e) +
2*(B*a*c^4 + B*a*c^3*d - B*a*c^2*d^2 - B*a*c*d^3)*f*x + (B*a*c^3 + B*a*c^2*
d - (A + B)*a*c*d^2 + (B*a*c^2*d + B*a*c*d^2 - (A + B)*a*d^3)*sin(f*x + e))
*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) -
c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^
2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(B*a*c^3*d -
 A*a*c^2*d^2 - B*a*c*d^3 + A*a*d^4)*cos(f*x + e))/((c^3*d^3 + c^2*d^4 - c*d
^5 - d^6)*f*sin(f*x + e) + (c^4*d^2 + c^3*d^3 - c^2*d^4 - c*d^5)*f), ((B*a*
c^3*d + B*a*c^2*d^2 - B*a*c*d^3 - B*a*d^4)*f*x*sin(f*x + e) + (B*a*c^4 + B*
a*c^3*d - B*a*c^2*d^2 - B*a*c*d^3)*f*x + (B*a*c^3 + B*a*c^2*d - (A + B)*a*c
*d^2 + (B*a*c^2*d + B*a*c*d^2 - (A + B)*a*d^3)*sin(f*x + e))*sqrt(c^2 - d^2
)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (B*a*c^3*d
 - A*a*c^2*d^2 - B*a*c*d^3 + A*a*d^4)*cos(f*x + e))/((c^3*d^3 + c^2*d^4 - c
*d^5 - d^6)*f*sin(f*x + e) + (c^4*d^2 + c^3*d^3 - c^2*d^4 - c*d^5)*f)]
```

giac [A] time = 0.20, size = 204, normalized size = 1.65

$$\frac{\frac{(fx+e)Ba}{d^2}-\frac{2\left(Bac^2+Bacd-Aad^2-Bad^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(cd^2+d^3\right)\sqrt{c^2-d^2}}+\frac{2\left(Bacd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-Aad^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+Bac^2-A\right.}{\left(c^2d+cd^2\right)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c\right.}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] ((f*x + e)*B*a/d^2 - 2*(B*a*c^2 + B*a*c*d - A*a*d^2 - B*a*d^2)*(pi*floor(1/
2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2
 - d^2)))/((c*d^2 + d^3)*sqrt(c^2 - d^2)) + 2*(B*a*c*d*tan(1/2*f*x + 1/2*e)
 - A*a*d^2*tan(1/2*f*x + 1/2*e) + B*a*c^2 - A*a*c*d)/((c^2*d + c*d^2)*(c*ta
n(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)))/f
```

maple [B] time = 0.49, size = 434, normalized size = 3.50

$$-\frac{2ad\tan\left(\frac{fx}{2}+\frac{e}{2}\right)A}{f\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d+c\right)(c+d)\,c}+\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)B}{f\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d+c\right)(c+d)}-\frac{}{f\left(\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -2*a/f*d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)/c*tan(1/2*
f*x+1/2*e)*A+2*a/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*
tan(1/2*f*x+1/2*e)*B-2*a/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c
)/(c+d)*A+2*a/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*B
*c+2*a/f/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2
-d^2)^(1/2))*A-2*a/f/d^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+
1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^2-2*a/f/d/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2
*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c+2*a/f/(c+d)/(c^2-d^2)^(1
/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B+2*a/f*B/d^2*
arctan(tan(1/2*f*x+1/2*e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 20.32, size = 5102, normalized size = 41.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c + d*sin(e + f*x))^2,x)
```

```
[Out] (2*B*a*atan(((B*a*((32*(B^2*a^2*c^2*d^3 + 2*B^2*a^2*c^3*d^2 + B^2*a^2*c^4*d
))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(6*B^2*a^2*c^2*d^4 +
```

2*B^2*a^2*c^3*d^3 - 4*B^2*a^2*c^4*d^2 - A^2*a^2*c*d^5 + B^2*a^2*c*d^5 - 2*B
^2*a^2*c^5*d + 2*A*B*a^2*c^2*d^4 + 2*A*B*a^2*c^3*d^3 - 2*A*B*a^2*c*d^5))/(2
*c*d^4 + d^5 + c^2*d^3) + (B*a*((32*tan(e/2 + (f*x)/2)*(2*A*a*c*d^7 + 2*B*a
*c*d^7 + 2*A*a*c^2*d^6 - 4*B*a*c^3*d^5 - 2*B*a*c^4*d^4))/(2*c*d^4 + d^5 + c
^2*d^3) - (32*(B*a*c*d^6 - A*a*c^2*d^5 - A*a*c^3*d^4 + B*a*c^2*d^5))/(2*c*d
^3 + d^4 + c^2*d^2) + (B*a*((32*(c^2*d^7 + 2*c^3*d^6 + c^4*d^5))/(2*c*d^3 +
d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^9 + 6*c^2*d^8 + c^3*d^7 - 4
*c^4*d^6 - 2*c^5*d^5))/(2*c*d^4 + d^5 + c^2*d^3))*1i)/d^2)*1i)/d^2))/d^2 +
(B*a*((32*(B^2*a^2*c^2*d^3 + 2*B^2*a^2*c^3*d^2 + B^2*a^2*c^4*d))/(2*c*d^3 +
d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(6*B^2*a^2*c^2*d^4 + 2*B^2*a^2*c^3
*d^3 - 4*B^2*a^2*c^4*d^2 - A^2*a^2*c*d^5 + B^2*a^2*c*d^5 - 2*B^2*a^2*c^5*d
+ 2*A*B*a^2*c^2*d^4 + 2*A*B*a^2*c^3*d^3 - 2*A*B*a^2*c*d^5))/(2*c*d^4 + d^5
+ c^2*d^3) + (B*a*((32*(B*a*c*d^6 - A*a*c^2*d^5 - A*a*c^3*d^4 + B*a*c^2*d^5
))/(2*c*d^3 + d^4 + c^2*d^2) - (32*tan(e/2 + (f*x)/2)*(2*A*a*c*d^7 + 2*B*a*
c*d^7 + 2*A*a*c^2*d^6 - 4*B*a*c^3*d^5 - 2*B*a*c^4*d^4))/(2*c*d^4 + d^5 + c^
2*d^3) + (B*a*((32*(c^2*d^7 + 2*c^3*d^6 + c^4*d^5))/(2*c*d^3 + d^4 + c^2*d^
2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^9 + 6*c^2*d^8 + c^3*d^7 - 4*c^4*d^6 - 2*
c^5*d^5))/(2*c*d^4 + d^5 + c^2*d^3))*1i)/d^2)*1i)/d^2))/d^2)/((64*(B^3*a^3*
c^3 + A*B^2*a^3*c^3 - B^3*a^3*c*d^2 + B^3*a^3*c^2*d - 2*A*B^2*a^3*c*d^2 + A
*B^2*a^3*c^2*d - A^2*B*a^3*c*d^2))/(2*c*d^3 + d^4 + c^2*d^2) - (64*tan(e/2
+ (f*x)/2)*(2*B^3*a^3*c*d^3 - 2*B^3*a^3*c^4 - 4*B^3*a^3*c^3*d + 2*A*B^2*a^3
*c*d^3 + 2*A*B^2*a^3*c^2*d^2))/(2*c*d^4 + d^5 + c^2*d^3) - (B*a*((32*(B^2*a
^2*c^2*d^3 + 2*B^2*a^2*c^3*d^2 + B^2*a^2*c^4*d))/(2*c*d^3 + d^4 + c^2*d^2)
+ (32*tan(e/2 + (f*x)/2)*(6*B^2*a^2*c^2*d^4 + 2*B^2*a^2*c^3*d^3 - 4*B^2*a^2
*c^4*d^2 - A^2*a^2*c*d^5 + B^2*a^2*c*d^5 - 2*B^2*a^2*c^5*d + 2*A*B*a^2*c^2*
d^4 + 2*A*B*a^2*c^3*d^3 - 2*A*B*a^2*c*d^5))/(2*c*d^4 + d^5 + c^2*d^3) + (B*
a*((32*tan(e/2 + (f*x)/2)*(2*A*a*c*d^7 + 2*B*a*c*d^7 + 2*A*a*c^2*d^6 - 4*B*
a*c^3*d^5 - 2*B*a*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^3) - (32*(B*a*c*d^6 - A*
a*c^2*d^5 - A*a*c^3*d^4 + B*a*c^2*d^5))/(2*c*d^3 + d^4 + c^2*d^2) + (B*a*((
32*(c^2*d^7 + 2*c^3*d^6 + c^4*d^5))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2
+ (f*x)/2)*(3*c*d^9 + 6*c^2*d^8 + c^3*d^7 - 4*c^4*d^6 - 2*c^5*d^5))/(2*c*d
^4 + d^5 + c^2*d^3))*1i)/d^2)*1i)/d^2)*1i)/d^2 + (B*a*((32*(B^2*a^2*c^2*d^3
+ 2*B^2*a^2*c^3*d^2 + B^2*a^2*c^4*d))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(
e/2 + (f*x)/2)*(6*B^2*a^2*c^2*d^4 + 2*B^2*a^2*c^3*d^3 - 4*B^2*a^2*c^4*d^2 -
A^2*a^2*c*d^5 + B^2*a^2*c*d^5 - 2*B^2*a^2*c^5*d + 2*A*B*a^2*c^2*d^4 + 2*A*
B*a^2*c^3*d^3 - 2*A*B*a^2*c*d^5))/(2*c*d^4 + d^5 + c^2*d^3) + (B*a*((32*(B*
a*c*d^6 - A*a*c^2*d^5 - A*a*c^3*d^4 + B*a*c^2*d^5))/(2*c*d^3 + d^4 + c^2*d^
2) - (32*tan(e/2 + (f*x)/2)*(2*A*a*c*d^7 + 2*B*a*c*d^7 + 2*A*a*c^2*d^6 - 4*
B*a*c^3*d^5 - 2*B*a*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^3) + (B*a*((32*(c^2*d^
7 + 2*c^3*d^6 + c^4*d^5))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2
)*(3*c*d^9 + 6*c^2*d^8 + c^3*d^7 - 4*c^4*d^6 - 2*c^5*d^5))/(2*c*d^4 + d^5 +
c^2*d^3))*1i)/d^2)*1i)/d^2)*1i)/d^2)))/(d^2*f) - ((2*(A*a*d - B*a*c))/(d*(
c + d)) + (2*a*tan(e/2 + (f*x)/2)*(A*d - B*c))/(c*(c + d)))/(f*(c + 2*d*tan
(e/2 + (f*x)/2) + c*tan(e/2 + (f*x)/2)^2)) + (a*atan(((a*(-(c + d)^3*(c - d
))^(1/2)*((32*(B^2*a^2*c^2*d^3 + 2*B^2*a^2*c^3*d^2 + B^2*a^2*c^4*d))/(2*c*d

```
^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(6*B^2*a^2*c^2*d^4 + 2*B^2*a^2
*c^3*d^3 - 4*B^2*a^2*c^4*d^2 - A^2*a^2*c*d^5 + B^2*a^2*c*d^5 - 2*B^2*a^2*c^
5*d + 2*A*B*a^2*c^2*d^4 + 2*A*B*a^2*c^3*d^3 - 2*A*B*a^2*c*d^5))/(2*c*d^4 +
d^5 + c^2*d^3) + (a*(-(c + d)^3*(c - d))^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*A
*a*c*d^7 + 2*B*a*c*d^7 + 2*A*a*c^2*d^6 - 4*B*a*c^3*d^5 - 2*B*a*c^4*d^4))/(2
*c*d^4 + d^5 + c^2*d^3) - (32*(B*a*c*d^6 - A*a*c^2*d^5 - A*a*c^3*d^4 + B*a*
c^2*d^5))/(2*c*d^3 + d^4 + c^2*d^2) + (a*((32*(c^2*d^7 + 2*c^3*d^6 + c^4*d^
5))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^9 + 6*c^2*d^8
 + c^3*d^7 - 4*c^4*d^6 - 2*c^5*d^5))/(2*c*d^4 + d^5 + c^2*d^3))*(-(c + d)^3
*(c - d))^(1/2)*(A*d^2 - B*c^2 + B*d^2 - B*c*d))/(2*c*d^5 + d^6 - 2*c^3*d^3
 - c^4*d^2))*(A*d^2 - B*c^2 + B*d^2 - B*c*d))/(2*c*d^5 + d^6 - 2*c^3*d^3 -
c^4*d^2))*(A*d^2 - B*c^2 + B*d^2 - B*c*d)*1i)/(2*c*d^5 + d^6 - 2*c^3*d^3 -
c^4*d^2) + (a*(-(c + d)^3*(c - d))^(1/2)*((32*(B^2*a^2*c^2*d^3 + 2*B^2*a^2*
c^3*d^2 + B^2*a^2*c^4*d))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2
)*(6*B^2*a^2*c^2*d^4 + 2*B^2*a^2*c^3*d^3 - 4*B^2*a^2*c^4*d^2 - A^2*a^2*c*d^
5 + B^2*a^2*c*d^5 - 2*B^2*a^2*c^5*d + 2*A*B*a^2*c^2*d^4 + 2*A*B*a^2*c^3*d^3
 - 2*A*B*a^2*c*d^5))/(2*c*d^4 + d^5 + c^2*d^3) + (a*(-(c + d)^3*(c - d))^(1
/2)*((32*(B*a*c*d^6 - A*a*c^2*d^5 - A*a*c^3*d^4 + B*a*c^2*d^5))/(2*c*d^3 +
d^4 + c^2*d^2) - (32*tan(e/2 + (f*x)/2)*(2*A*a*c*d^7 + 2*B*a*c*d^7 + 2*A*a*
c^2*d^6 - 4*B*a*c^3*d^5 - 2*B*a*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^3) + (a*((
32*(c^2*d^7 + 2*c^3*d^6 + c^4*d^5))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2
 + (f*x)/2)*(3*c*d^9 + 6*c^2*d^8 + c^3*d^7 - 4*c^4*d^6 - 2*c^5*d^5))/(2*c*d
^4 + d^5 + c^2*d^3))*(-(c + d)^3*(c - d))^(1/2)*(A*d^2 - B*c^2 + B*d^2 - B*
c*d))/(2*c*d^5 + d^6 - 2*c^3*d^3 - c^4*d^2))*(A*d^2 - B*c^2 + B*d^2 - B*c*d
))/(2*c*d^5 + d^6 - 2*c^3*d^3 - c^4*d^2))*(A*d^2 - B*c^2 + B*d^2 - B*c*d)*1
i)/(2*c*d^5 + d^6 - 2*c^3*d^3 - c^4*d^2))/((64*(B^3*a^3*c^3 + A*B^2*a^3*c^3
 - B^3*a^3*c*d^2 + B^3*a^3*c^2*d - 2*A*B^2*a^3*c*d^2 + A*B^2*a^3*c^2*d - A^
2*B*a^3*c*d^2))/(2*c*d^3 + d^4 + c^2*d^2) - (64*tan(e/2 + (f*x)/2)*(2*B^3*a
^3*c*d^3 - 2*B^3*a^3*c^4 - 4*B^3*a^3*c^3*d + 2*A*B^2*a^3*c*d^3 + 2*A*B^2*a^
3*c^2*d^2))/(2*c*d^4 + d^5 + c^2*d^3) - (a*(-(c + d)^3*(c - d))^(1/2)*((32*
(B^2*a^2*c^2*d^3 + 2*B^2*a^2*c^3*d^2 + B^2*a^2*c^4*d))/(2*c*d^3 + d^4 + c^2
*d^2) + (32*tan(e/2 + (f*x)/2)*(6*B^2*a^2*c^2*d^4 + 2*B^2*a^2*c^3*d^3 - 4*B
^2*a^2*c^4*d^2 - A^2*a^2*c*d^5 + B^2*a^2*c*d^5 - 2*B^2*a^2*c^5*d + 2*A*B*a^
2*c^2*d^4 + 2*A*B*a^2*c^3*d^3 - 2*A*B*a^2*c*d^5))/(2*c*d^4 + d^5 + c^2*d^3)
 + (a*(-(c + d)^3*(c - d))^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*A*a*c*d^7 + 2*B
*a*c*d^7 + 2*A*a*c^2*d^6 - 4*B*a*c^3*d^5 - 2*B*a*c^4*d^4))/(2*c*d^4 + d^5 +
 c^2*d^3) - (32*(B*a*c*d^6 - A*a*c^2*d^5 - A*a*c^3*d^4 + B*a*c^2*d^5))/(2*c
*d^3 + d^4 + c^2*d^2) + (a*((32*(c^2*d^7 + 2*c^3*d^6 + c^4*d^5))/(2*c*d^3 +
 d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^9 + 6*c^2*d^8 + c^3*d^7 - 4
*c^4*d^6 - 2*c^5*d^5))/(2*c*d^4 + d^5 + c^2*d^3))*(-(c + d)^3*(c - d))^(1/2
)*(A*d^2 - B*c^2 + B*d^2 - B*c*d))/(2*c*d^5 + d^6 - 2*c^3*d^3 - c^4*d^2))*(
A*d^2 - B*c^2 + B*d^2 - B*c*d))/(2*c*d^5 + d^6 - 2*c^3*d^3 - c^4*d^2))*(A*d
^2 - B*c^2 + B*d^2 - B*c*d))/(2*c*d^5 + d^6 - 2*c^3*d^3 - c^4*d^2) + (a*(-(
c + d)^3*(c - d))^(1/2)*((32*(B^2*a^2*c^2*d^3 + 2*B^2*a^2*c^3*d^2 + B^2*a^2
*c^4*d))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(6*B^2*a^2*c^2*
```

```
d^4 + 2*B^2*a^2*c^3*d^3 - 4*B^2*a^2*c^4*d^2 - A^2*a^2*c*d^5 + B^2*a^2*c*d^5
 - 2*B^2*a^2*c^5*d + 2*A*B*a^2*c^2*d^4 + 2*A*B*a^2*c^3*d^3 - 2*A*B*a^2*c*d^
5))/(2*c*d^4 + d^5 + c^2*d^3) + (a*(-(c + d)^3*(c - d))^(1/2)*((32*(B*a*c*d
^6 - A*a*c^2*d^5 - A*a*c^3*d^4 + B*a*c^2*d^5))/(2*c*d^3 + d^4 + c^2*d^2) -
(32*tan(e/2 + (f*x)/2)*(2*A*a*c*d^7 + 2*B*a*c*d^7 + 2*A*a*c^2*d^6 - 4*B*a*c
^3*d^5 - 2*B*a*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^3) + (a*((32*(c^2*d^7 + 2*c
^3*d^6 + c^4*d^5))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*
d^9 + 6*c^2*d^8 + c^3*d^7 - 4*c^4*d^6 - 2*c^5*d^5))/(2*c*d^4 + d^5 + c^2*d^
3))*(-(c + d)^3*(c - d))^(1/2)*(A*d^2 - B*c^2 + B*d^2 - B*c*d))/(2*c*d^5 +
d^6 - 2*c^3*d^3 - c^4*d^2))*(A*d^2 - B*c^2 + B*d^2 - B*c*d))/(2*c*d^5 + d^6
 - 2*c^3*d^3 - c^4*d^2))*(A*d^2 - B*c^2 + B*d^2 - B*c*d))/(2*c*d^5 + d^6 -
2*c^3*d^3 - c^4*d^2)))*(-(c + d)^3*(c - d))^(1/2)*(A*d^2 - B*c^2 + B*d^2 -
B*c*d)*2i)/(f*(2*c*d^5 + d^6 - 2*c^3*d^3 - c^4*d^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
[Out] Timed out
```

3.250 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=176

$$\frac{a(2Ac-Ad+Bc-2Bd)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)\left(c^2-d^2\right)^{3/2}}-\frac{a\left(Ad(c-2d)+B\left(c^2+2cd-2d^2\right)\right)\cos(e+fx)}{2df(c-d)(c+d)^2(c+d\sin(e+fx))}+\frac{a(Bc-Ad}{2df(c+d)(c}$$

```
[Out] a*(2*A*c-A*d+B*c-2*B*d)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(c
+d)/(c^2-d^2)^(3/2)/f+1/2*a*(-A*d+B*c)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(f*x+e)
)^2-1/2*a*(A*(c-2*d)*d+B*(c^2+2*c*d-2*d^2))*cos(f*x+e)/(c-d)/d/(c+d)^2/f/(c
+d*sin(f*x+e))
```

Rubi [A] time = 0.42, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2968, 3021, 2754, 12, 2660, 618, 204}

$$\frac{a(2Ac-Ad+Bc-2Bd)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)\left(c^2-d^2\right)^{3/2}}-\frac{a\left(Ad(c-2d)+B\left(c^2+2cd-2d^2\right)\right)\cos(e+fx)}{2df(c-d)(c+d)^2(c+d\sin(e+fx))}+\frac{a(Bc-Ad}{2df(c+d)(c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a*(2*A*c + B*c - A*d - 2*B*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d
^2]])/((c + d)*(c^2 - d^2)^(3/2)*f) + (a*(B*c - A*d)*Cos[e + f*x])/(2*d*(c
+ d)*f*(c + d*Sin[e + f*x])^2) - (a*(A*(c - 2*d)*d + B*(c^2 + 2*c*d - 2*d^2
))*Cos[e + f*x])/(2*(c - d)*d*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a + a\sin(e+fx))(A + B\sin(e+fx))}{(c + d\sin(e+fx))^3}\,dx = \int \frac{aA + (aA + aB)\sin(e+fx) + aB\sin^2(e+fx)}{(c+d\sin(e+fx))^3}\,dx$$

$$= \frac{a(Bc - Ad)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{\int \frac{-2a(A+B)(c-d)d - a(c-d)(Ad+B(c+2d))}{(c+d\sin(e+fx))^2}}{2d\left(c^2-d^2\right)}$$

$$= \frac{a(Bc - Ad)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a\left(A(c-2d)d + B\left(c^2 + 2cd - 2d\right.\right.}{2(c-d)d(c+d)^2 f(c+d\sin}$$

$$= \frac{a(Bc - Ad)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a\left(A(c-2d)d + B\left(c^2 + 2cd - 2d\right.\right.}{2(c-d)d(c+d)^2 f(c+d\sin}$$

$$= \frac{a(Bc - Ad)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a\left(A(c-2d)d + B\left(c^2 + 2cd - 2d\right.\right.}{2(c-d)d(c+d)^2 f(c+d\sin}$$

$$= \frac{a(Bc - Ad)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a\left(A(c-2d)d + B\left(c^2 + 2cd - 2d\right.\right.}{2(c-d)d(c+d)^2 f(c+d\sin}$$

$$= \frac{a(2Ac + Bc - Ad - 2Bd)\tan^{-1}\left(\frac{d + c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c-d)(c+d)^2\sqrt{c^2-d^2}\,f} + \frac{a(Bc - Ad)}{2d(c+d)f(c+}$$

Mathematica [C] time = 2.71, size = 345, normalized size = 1.96

$$\frac{a(\sin(e+fx)+1)\left(\frac{d\csc(e)\left(\left(Ad^2(d-2c)+Bc\left(2c^2+2cd-3d^2\right)\right)\sin(2e+fx) - d\left(Ad(c-2d)+B\left(c^2+2cd-2d^2\right)\right)\cos(e+2fx) + \sin(fx)\left(Bc\left(2c^2+6cd-5d^2\right)-\right.\right.}{d^2(c+d\sin(e+fx))^2}\right.}{4f(c-d)(c+d)^2\left(\sin\left(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^
3,x]
```

```
[Out] (a*(1 + Sin[e + f*x])*((4*(2*A*c + B*c - A*d - 2*B*d)*ArcTan[(Sec[(f*x)/2]*
(Cos[e] - I*Sin[e])*(d*Cos[e + (f*x)/2] + c*Sin[(f*x)/2]))/(Sqrt[c^2 - d^2]
*Sqrt[(Cos[e] - I*Sin[e])^2])]*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(
```

```
Cos[e] - I*Sin[e])^2]) + ((2*c^2 + d^2)*(A*(c - 2*d)*d + B*(c^2 + 2*c*d - 2
*d^2))*Cot[e] + d*Csc[e]*(-(d*(A*(c - 2*d)*d + B*(c^2 + 2*c*d - 2*d^2))*Cos
[e + 2*f*x]) + (B*c*(2*c^2 + 6*c*d - 5*d^2) - A*d*(-4*c^2 + 6*c*d + d^2))*S
in[f*x] + (A*d^2*(-2*c + d) + B*c*(2*c^2 + 2*c*d - 3*d^2))*Sin[2*e + f*x]))
/(d^2*(c + d*Sin[e + f*x])^2)))/(4*(c - d)*(c + d)^2*f*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^2)
```

fricas [B] time = 0.53, size = 967, normalized size = 5.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] [1/4*(2*(B*a*c^4 + (A + 2*B)*a*c^3*d - (2*A + 3*B)*a*c^2*d^2 - (A + 2*B)*a*
c*d^3 + 2*(A + B)*a*d^4)*cos(f*x + e)*sin(f*x + e) + ((2*A + B)*a*c^3 - (A
+ 2*B)*a*c^2*d + (2*A + B)*a*c*d^2 - (A + 2*B)*a*d^3 - ((2*A + B)*a*c*d^2 -
 (A + 2*B)*a*d^3)*cos(f*x + e)^2 + 2*((2*A + B)*a*c^2*d - (A + 2*B)*a*c*d^2
)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*
sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))
*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) +
 2*(2*(A + B)*a*c^4 - (2*A + B)*a*c^3*d - (3*A + 2*B)*a*c^2*d^2 + (2*A + B)
*a*c*d^3 + A*a*d^4)*cos(f*x + e))/((c^5*d^2 + c^4*d^3 - 2*c^3*d^4 - 2*c^2*d
^5 + c*d^6 + d^7)*f*cos(f*x + e)^2 - 2*(c^6*d + c^5*d^2 - 2*c^4*d^3 - 2*c^3
*d^4 + c^2*d^5 + c*d^6)*f*sin(f*x + e) - (c^7 + c^6*d - c^5*d^2 - c^4*d^3 -
 c^3*d^4 - c^2*d^5 + c*d^6 + d^7)*f), 1/2*((B*a*c^4 + (A + 2*B)*a*c^3*d - (
2*A + 3*B)*a*c^2*d^2 - (A + 2*B)*a*c*d^3 + 2*(A + B)*a*d^4)*cos(f*x + e)*si
n(f*x + e) + ((2*A + B)*a*c^3 - (A + 2*B)*a*c^2*d + (2*A + B)*a*c*d^2 - (A
+ 2*B)*a*d^3 - ((2*A + B)*a*c*d^2 - (A + 2*B)*a*d^3)*cos(f*x + e)^2 + 2*((2
*A + B)*a*c^2*d - (A + 2*B)*a*c*d^2)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-
(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (2*(A + B)*a*c^4 - (
2*A + B)*a*c^3*d - (3*A + 2*B)*a*c^2*d^2 + (2*A + B)*a*c*d^3 + A*a*d^4)*cos
(f*x + e))/((c^5*d^2 + c^4*d^3 - 2*c^3*d^4 - 2*c^2*d^5 + c*d^6 + d^7)*f*cos
(f*x + e)^2 - 2*(c^6*d + c^5*d^2 - 2*c^4*d^3 - 2*c^3*d^4 + c^2*d^5 + c*d^6)
*f*sin(f*x + e) - (c^7 + c^6*d - c^5*d^2 - c^4*d^3 - c^3*d^4 - c^2*d^5 + c*
d^6 + d^7)*f)]
```

giac [B] time = 0.23, size = 594, normalized size = 3.38

$$\frac{(2\,Aac+Bac-Aad-2\,Bad)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(c^3+c^2d-cd^2-d^3\right)\sqrt{c^2-d^2}}+\frac{Bac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,Aac^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,Bac^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] ((2*A*a*c + B*a*c - A*a*d - 2*B*a*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(
c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^3 + c^2*d -
c*d^2 - d^3)*sqrt(c^2 - d^2)) + (B*a*c^4*tan(1/2*f*x + 1/2*e)^3 - 3*A*a*c^3
*d*tan(1/2*f*x + 1/2*e)^3 - 2*B*a*c^3*d*tan(1/2*f*x + 1/2*e)^3 + 2*A*a*c^2*
d^2*tan(1/2*f*x + 1/2*e)^3 + 2*A*a*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 2*A*a*c^4
*tan(1/2*f*x + 1/2*e)^2 - 2*B*a*c^4*tan(1/2*f*x + 1/2*e)^2 + 2*A*a*c^3*d*ta
n(1/2*f*x + 1/2*e)^2 + B*a*c^3*d*tan(1/2*f*x + 1/2*e)^2 - 3*A*a*c^2*d^2*tan
(1/2*f*x + 1/2*e)^2 - 4*B*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 + 4*A*a*c*d^3*ta
n(1/2*f*x + 1/2*e)^2 + 2*B*a*c*d^3*tan(1/2*f*x + 1/2*e)^2 + 2*A*a*d^4*tan(1
/2*f*x + 1/2*e)^2 - B*a*c^4*tan(1/2*f*x + 1/2*e) - 5*A*a*c^3*d*tan(1/2*f*x
+ 1/2*e) - 6*B*a*c^3*d*tan(1/2*f*x + 1/2*e) + 6*A*a*c^2*d^2*tan(1/2*f*x + 1
/2*e) + 4*B*a*c^2*d^2*tan(1/2*f*x + 1/2*e) + 2*A*a*c*d^3*tan(1/2*f*x + 1/2*
e) - 2*A*a*c^4 - 2*B*a*c^4 + 2*A*a*c^3*d + B*a*c^3*d + A*a*c^2*d^2)/((c^5 +
 c^4*d - c^3*d^2 - c^2*d^3)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1
/2*e) + c)^2))/f
```

maple [B] time = 0.52, size = 2021, normalized size = 11.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -4*a/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2
-d^3)*tan(1/2*f*x+1/2*e)^2*B*d^2+6*a/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*
x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*tan(1/2*f*x+1/2*e)*A*d^2+4*a/f/(tan(1
/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*tan(1/2
*f*x+1/2*e)*B*d^2-a/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c
^2/(c^3+c^2*d-c*d^2-d^3)*tan(1/2*f*x+1/2*e)*B+2*a/f/(tan(1/2*f*x+1/2*e)^2*c
+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*A*c*d+a/f/(tan(1/2*f*x+1
/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*B*c*d+a/f/(tan(
1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^3+c^2*d-c*d^2-d^3)*ta
n(1/2*f*x+1/2*e)^3*B+2*a/f/(c^3+c^2*d-c*d^2-d^3)/(c^2-d^2)^(1/2)*arctan(1/2
*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c-a/f/(c^3+c^2*d-c*d^2-d^3
)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*
A*d-6*a/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^3+c^2*d-
c*d^2-d^3)*tan(1/2*f*x+1/2*e)*B*d+2*a/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f
*x+1/2*e)*d+c)^2/c/(c^3+c^2*d-c*d^2-d^3)*tan(1/2*f*x+1/2*e)*A*d^3-3*a/f/(ta
n(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^3+c^2*d-c*d^2-d^3)*ta
n(1/2*f*x+1/2*e)^3*A*d+2*a/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d
```

```
+c)^2/c/(c^3+c^2*d-c*d^2-d^3)*tan(1/2*f*x+1/2*e)^3*A*d^3-5*a/f/(tan(1/2*f*x
+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^3+c^2*d-c*d^2-d^3)*tan(1/2*f*x
+1/2*e)*A*d-2*a/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^
3+c^2*d-c*d^2-d^3)*tan(1/2*f*x+1/2*e)^3*B*d+2*a/f/(tan(1/2*f*x+1/2*e)^2*c+2
*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*c*tan(1/2*f*x+1/2*e)^2*A*d
+2*a/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2
-d^3)/c*tan(1/2*f*x+1/2*e)^2*B*d^3+4*a/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*
f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)/c*tan(1/2*f*x+1/2*e)^2*A*d^3+2*a/f/
(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)/c
^2*tan(1/2*f*x+1/2*e)^2*A*d^4+a/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2
*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*c*tan(1/2*f*x+1/2*e)^2*B*d-2*a/f/(tan(1/2*
f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*A*c^2+a/f/
(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*A
*d^2-2*a/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c
*d^2-d^3)*B*c^2+a/f/(c^3+c^2*d-c*d^2-d^3)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*t
an(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c-2*a/f/(c^3+c^2*d-c*d^2-d^3)/(c^
2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*d-2
*a/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d
^3)*c^2*tan(1/2*f*x+1/2*e)^2*A-2*a/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+
1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*c^2*tan(1/2*f*x+1/2*e)^2*B+2*a/f/(tan(1
/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*tan(1/2
*f*x+1/2*e)^3*A*d^2-3*a/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)
^2/(c^3+c^2*d-c*d^2-d^3)*tan(1/2*f*x+1/2*e)^2*A*d^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 15.59, size = 554, normalized size = 3.15

$$-\frac{\frac{A\,a\,d^2-2\,A\,a\,c^2-2\,B\,a\,c^2+2\,A\,a\,c\,d+B\,a\,c\,d}{-c^3-c^2\,d+c\,d^2+d^3}+\frac{a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,A\,d^3-B\,c^3+6\,A\,c\,d^2-5\,A\,c^2\,d+4\,B\,c\,d^2-6\,B\,c^2\,d\right)}{c\,\left(-c^3-c^2\,d+c\,d^2+d^3\right)}+\frac{a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(2\,A\,d^3+B\,c^3+2\,A\right.}{c\,\left(-c^3-c^2\,d+c\,d^2\right.}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(2\,c^2+4\,d^2\right)+c^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+c^2+4\,c\,d\,\tan\left(\frac{e}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c + d*sin(e + f*x))^3,x)
```

```
[Out] - ((A*a*d^2 - 2*A*a*c^2 - 2*B*a*c^2 + 2*A*a*c*d + B*a*c*d)/(c*d^2 - c^2*d -
 c^3 + d^3) + (a*tan(e/2 + (f*x)/2)*(2*A*d^3 - B*c^3 + 6*A*c*d^2 - 5*A*c^2*
d + 4*B*c*d^2 - 6*B*c^2*d))/(c*(c*d^2 - c^2*d - c^3 + d^3)) + (a*tan(e/2 +
(f*x)/2)^3*(2*A*d^3 + B*c^3 + 2*A*c*d^2 - 3*A*c^2*d - 2*B*c^2*d))/(c*(c*d^2
 - c^2*d - c^3 + d^3)) + (a*tan(e/2 + (f*x)/2)^2*(c^2 + 2*d^2)*(A*d^2 - 2*A
*c^2 - 2*B*c^2 + 2*A*c*d + B*c*d))/(c^2*(c*d^2 - c^2*d - c^3 + d^3)))/(f*(t
an(e/2 + (f*x)/2)^2*(2*c^2 + 4*d^2) + c^2*tan(e/2 + (f*x)/2)^4 + c^2 + 4*c*
d*tan(e/2 + (f*x)/2)^3 + 4*c*d*tan(e/2 + (f*x)/2))) - (a*atan((((a*(2*A*c -
 A*d + B*c - 2*B*d)*(2*c*d^3 - 2*c^3*d + 2*d^4 - 2*c^2*d^2))/(2*(c + d)^(5/
2)*(c - d)^(3/2)*(c*d^2 - c^2*d - c^3 + d^3)) + (a*c*tan(e/2 + (f*x)/2)*(2*
A*c - A*d + B*c - 2*B*d))/((c + d)^(5/2)*(c - d)^(3/2)))*(c*d^2 - c^2*d - c
^3 + d^3))/(2*A*a*c - A*a*d + B*a*c - 2*B*a*d))*(2*A*c - A*d + B*c - 2*B*d)
)/(f*(c + d)^(5/2)*(c - d)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.251 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=464

$$\frac{a^2\left(6Ad(c-10d)-B\left(2c^2-12cd+55d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{120d^2f}+\frac{a^2\left(6Ad\left(c^2-10cd-12d^2\right)-B\left(2c\right.\right.}{}$$

```
[Out] 1/16*a^2*(6*A*(4*c^3+8*c^2*d+7*c*d^2+2*d^3)+B*(16*c^3+42*c^2*d+36*c*d^2+11*
d^3))*x+1/60*a^2*(6*A*d*(c^4-10*c^3*d-44*c^2*d^2-40*c*d^3-12*d^4)-B*(2*c^5-
12*c^4*d+47*c^3*d^2+208*c^2*d^3+216*c*d^4+64*d^5))*cos(f*x+e)/d^2/f+1/240*a
^2*(6*A*d*(2*c^3-20*c^2*d-57*c*d^2-30*d^3)-B*(4*c^4-24*c^3*d+96*c^2*d^2+284
*c*d^3+165*d^4))*cos(f*x+e)*sin(f*x+e)/d/f+1/120*a^2*(6*A*d*(c^2-10*c*d-12*
d^2)-B*(2*c^3-12*c^2*d+51*c*d^2+64*d^3))*cos(f*x+e)*(c+d*sin(f*x+e))^2/d^2/
f+1/120*a^2*(6*A*(c-10*d)*d-B*(2*c^2-12*c*d+55*d^2))*cos(f*x+e)*(c+d*sin(f*
x+e))^3/d^2/f+1/30*a^2*(-6*A*d+2*B*c-7*B*d)*cos(f*x+e)*(c+d*sin(f*x+e))^4/d
^2/f-1/6*B*cos(f*x+e)*(a^2+a^2*sin(f*x+e))*(c+d*sin(f*x+e))^4/d/f
```

Rubi [A] time = 0.95, antiderivative size = 464, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2976, 2968, 3023, 2753, 2734}

$$\frac{a^2\left(6Ad\left(-44c^2d^2-10c^3d+c^4-40cd^3-12d^4\right)-B\left(47c^3d^2+208c^2d^3-12c^4d+2c^5+216cd^4+64d^5\right)\right)\cos(e+}{60d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a^2*(6*A*(4*c^3 + 8*c^2*d + 7*c*d^2 + 2*d^3) + B*(16*c^3 + 42*c^2*d + 36*c
*d^2 + 11*d^3))*x)/16 + (a^2*(6*A*d*(c^4 - 10*c^3*d - 44*c^2*d^2 - 40*c*d^3
 - 12*d^4) - B*(2*c^5 - 12*c^4*d + 47*c^3*d^2 + 208*c^2*d^3 + 216*c*d^4 + 6
4*d^5))*Cos[e + f*x])/(60*d^2*f) + (a^2*(6*A*d*(2*c^3 - 20*c^2*d - 57*c*d^2
 - 30*d^3) - B*(4*c^4 - 24*c^3*d + 96*c^2*d^2 + 284*c*d^3 + 165*d^4))*Cos[e
 + f*x]*Sin[e + f*x])/(240*d*f) + (a^2*(6*A*d*(c^2 - 10*c*d - 12*d^2) - B*(
2*c^3 - 12*c^2*d + 51*c*d^2 + 64*d^3))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)
/(120*d^2*f) + (a^2*(6*A*(c - 10*d)*d - B*(2*c^2 - 12*c*d + 55*d^2))*Cos[e
+ f*x]*(c + d*Sin[e + f*x])^3)/(120*d^2*f) + (a^2*(2*B*c - 6*A*d - 7*B*d)*C
os[e + f*x]*(c + d*Sin[e + f*x])^4)/(30*d^2*f) - (B*Cos[e + f*x]*(a^2 + a^2
*Sin[e + f*x])*(c + d*Sin[e + f*x])^4)/(6*d*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
```

```
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(c+d\,\mathrm{s}}{6df}$$

$$= -\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(c+d\,\mathrm{s}}{6df}$$

$$= \frac{a^2(2Bc-6Ad-7Bd)\cos(e+fx)(c+d\sin(}{30d^2f}$$

$$= \frac{a^2\left(6A(c-10d)d-B\left(2c^2-12cd+55d^2\right)\right)\mathrm{c}}{120d^2f}$$

$$= \frac{a^2\left(6Ad\left(c^2-10cd-12d^2\right)-B\left(2c^3-12c^2d\right.\right.}{1}$$

$$= \frac{1}{16}a^2\left(6A\left(4c^3+8c^2d+7cd^2+2d^3\right)+B\left(16\right.\right.$$

Mathematica [A] time = 3.13, size = 437, normalized size = 0.94

$$-\frac{a^2\cos(e+fx)\left(60\left(6A\left(4c^3+8c^2d+7cd^2+2d^3\right)+B\left(16c^3+42c^2d+36cd^2+11d^3\right)\right)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
3,x]
```

```
[Out] -1/480*(a^2*Cos[e + f*x]*(60*(6*A*(4*c^3 + 8*c^2*d + 7*c*d^2 + 2*d^3) + B*(
16*c^3 + 42*c^2*d + 36*c*d^2 + 11*d^3))*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[
2]] + Sqrt[Cos[e + f*x]^2]*(960*A*c^3 + 880*B*c^3 + 2640*A*c^2*d + 2400*B*c
^2*d + 2400*A*c*d^2 + 2268*B*c*d^2 + 756*A*d^3 + 712*B*d^3 - 16*(3*A*d*(5*c
^2 + 10*c*d + 4*d^2) + B*(5*c^3 + 30*c^2*d + 36*c*d^2 + 14*d^3))*Cos[2*(e +
 f*x)] + 12*d^2*(3*B*c + A*d + 2*B*d)*Cos[4*(e + f*x)] + 240*A*c^3*Sin[e +
f*x] + 480*B*c^3*Sin[e + f*x] + 1440*A*c^2*d*Sin[e + f*x] + 1530*B*c^2*d*Si
n[e + f*x] + 1530*A*c*d^2*Sin[e + f*x] + 1620*B*c*d^2*Sin[e + f*x] + 540*A*
d^3*Sin[e + f*x] + 545*B*d^3*Sin[e + f*x] - 90*B*c^2*d*Sin[3*(e + f*x)] - 9
0*A*c*d^2*Sin[3*(e + f*x)] - 180*B*c*d^2*Sin[3*(e + f*x)] - 60*A*d^3*Sin[3*
(e + f*x)] - 80*B*d^3*Sin[3*(e + f*x)] + 5*B*d^3*Sin[5*(e + f*x)])))/(f*Sqr
t[Cos[e + f*x]^2])
```

fricas [A] time = 0.50, size = 364, normalized size = 0.78

$$-\frac{48\left(3\,Ba^2cd^2+(A+2\,B)a^2d^3\right)\cos\left(fx+e\right)^5-80\left(Ba^2c^3+3\,(A+2\,B)a^2c^2d+3\,(2\,A+3\,B)a^2cd^2+(3\,A+4\,B)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/240*(48*(3*B*a^2*c*d^2 + (A + 2*B)*a^2*d^3)*cos(f*x + e)^5 - 80*(B*a^2*c
^3 + 3*(A + 2*B)*a^2*c^2*d + 3*(2*A + 3*B)*a^2*c*d^2 + (3*A + 4*B)*a^2*d^3)
*cos(f*x + e)^3 - 15*(8*(3*A + 2*B)*a^2*c^3 + 6*(8*A + 7*B)*a^2*c^2*d + 6*(
7*A + 6*B)*a^2*c*d^2 + (12*A + 11*B)*a^2*d^3)*f*x + 480*((A + B)*a^2*c^3 +
3*(A + B)*a^2*c^2*d + 3*(A + B)*a^2*c*d^2 + (A + B)*a^2*d^3)*cos(f*x + e) +
 5*(8*B*a^2*d^3*cos(f*x + e)^5 - 2*(18*B*a^2*c^2*d + 18*(A + 2*B)*a^2*c*d^2
 + (12*A + 19*B)*a^2*d^3)*cos(f*x + e)^3 + 3*(8*(A + 2*B)*a^2*c^3 + 6*(8*A
+ 9*B)*a^2*c^2*d + 6*(9*A + 10*B)*a^2*c*d^2 + (20*A + 21*B)*a^2*d^3)*cos(f*
x + e))*sin(f*x + e))/f
```

giac [A] time = 0.22, size = 474, normalized size = 1.02

$$-\frac{Ba^2d^3\sin\left(6\,fx+6\,e\right)}{192\,f}+\frac{1}{16}\left(24\,Aa^2c^3+16\,Ba^2c^3+48\,Aa^2c^2d+42\,Ba^2c^2d+42\,Aa^2cd^2+36\,Ba^2cd^2+12\,Aa^2d^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -1/192*B*a^2*d^3*sin(6*f*x + 6*e)/f + 1/16*(24*A*a^2*c^3 + 16*B*a^2*c^3 + 4
8*A*a^2*c^2*d + 42*B*a^2*c^2*d + 42*A*a^2*c*d^2 + 36*B*a^2*c*d^2 + 12*A*a^2
*d^3 + 11*B*a^2*d^3)*x - 1/80*(3*B*a^2*c*d^2 + A*a^2*d^3 + 2*B*a^2*d^3)*cos
(5*f*x + 5*e)/f + 1/48*(4*B*a^2*c^3 + 12*A*a^2*c^2*d + 24*B*a^2*c^2*d + 24*
A*a^2*c*d^2 + 27*B*a^2*c*d^2 + 9*A*a^2*d^3 + 10*B*a^2*d^3)*cos(3*f*x + 3*e)
/f - 1/8*(16*A*a^2*c^3 + 14*B*a^2*c^3 + 42*A*a^2*c^2*d + 36*B*a^2*c^2*d + 3
6*A*a^2*c*d^2 + 33*B*a^2*c*d^2 + 11*A*a^2*d^3 + 10*B*a^2*d^3)*cos(f*x + e)/
f + 1/64*(6*B*a^2*c^2*d + 6*A*a^2*c*d^2 + 12*B*a^2*c*d^2 + 4*A*a^2*d^3 + 5*
B*a^2*d^3)*sin(4*f*x + 4*e)/f - 1/64*(16*A*a^2*c^3 + 32*B*a^2*c^3 + 96*A*a^
2*c^2*d + 96*B*a^2*c^2*d + 96*A*a^2*c*d^2 + 96*B*a^2*c*d^2 + 32*A*a^2*d^3 +
 31*B*a^2*d^3)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.64, size = 745, normalized size = 1.61

$$a^2Ac^3\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-a^2Ac^2d\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)+3a^2Acd^2\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+\right.}{2}\right.}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(a^2*A*c^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-a^2*A*c^2*d*(2+si
n(f*x+e)^2)*cos(f*x+e)+3*a^2*A*c*d^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*co
s(f*x+e)+3/8*f*x+3/8*e)-1/5*a^2*A*d^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*c
os(f*x+e)-1/3*B*a^2*c^3*(2+sin(f*x+e)^2)*cos(f*x+e)+3*B*a^2*c^2*d*(-1/4*(si
n(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3/5*B*a^2*c*d^2*(8/3+s
in(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+B*a^2*d^3*(-1/6*(sin(f*x+e)^5+5/4*
sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)-2*a^2*A*c^3*cos(f
*x+e)+6*a^2*A*c^2*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2*a^2*A*c*d^
2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a^2*A*d^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e
))*cos(f*x+e)+3/8*f*x+3/8*e)+2*B*a^2*c^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*
x+1/2*e)-2*B*a^2*c^2*d*(2+sin(f*x+e)^2)*cos(f*x+e)+6*B*a^2*c*d^2*(-1/4*(sin
(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/5*B*a^2*d^3*(8/3+sin(
f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+a^2*A*c^3*(f*x+e)-3*a^2*A*c^2*d*cos(f
*x+e)+3*a^2*A*c*d^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/3*a^2*A*d^
3*(2+sin(f*x+e)^2)*cos(f*x+e)-B*a^2*c^3*cos(f*x+e)+3*B*a^2*c^2*d*(-1/2*sin(
f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-B*a^2*c*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+B*
a^2*d^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e))
```

maxima [A] time = 0.46, size = 724, normalized size = 1.56

$$240\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)Aa^2c^3+960\left(fx+e\right)Aa^2c^3+320\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)Ba^2c^3+48$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 1/960*(240*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^3 + 960*(f*x + e)*A*a^2
*c^3 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^3 + 480*(2*f*x + 2*e -
 sin(2*f*x + 2*e))*B*a^2*c^3 + 960*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*
c^2*d + 1440*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^2*d + 1920*(cos(f*x +
 e)^3 - 3*cos(f*x + e))*B*a^2*c^2*d + 90*(12*f*x + 12*e + sin(4*f*x + 4*e)
```

```
- 8*sin(2*f*x + 2*e))*B*a^2*c^2*d + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*
a^2*c^2*d + 1920*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c*d^2 + 90*(12*f*x
 + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*c*d^2 + 720*(2*f*x +
 2*e - sin(2*f*x + 2*e))*A*a^2*c*d^2 - 192*(3*cos(f*x + e)^5 - 10*cos(f*x +
 e)^3 + 15*cos(f*x + e))*B*a^2*c*d^2 + 960*(cos(f*x + e)^3 - 3*cos(f*x + e)
)*B*a^2*c*d^2 + 180*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))
*B*a^2*c*d^2 - 64*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*
A*a^2*d^3 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*d^3 + 60*(12*f*x +
12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*d^3 - 128*(3*cos(f*x +
e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*d^3 + 5*(4*sin(2*f*x + 2*
e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^2*d^3
+ 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^2*d^3 - 19
20*A*a^2*c^3*cos(f*x + e) - 960*B*a^2*c^3*cos(f*x + e) - 2880*A*a^2*c^2*d*c
os(f*x + e))/f
```

mupad [B] time = 15.99, size = 1291, normalized size = 2.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^3,x)
```

```
[Out] (a^2*atan((a^2*tan(e/2 + (f*x)/2)*(24*A*c^3 + 12*A*d^3 + 16*B*c^3 + 11*B*d^
3 + 42*A*c*d^2 + 48*A*c^2*d + 36*B*c*d^2 + 42*B*c^2*d))/(8*(3*A*a^2*c^3 + (
3*A*a^2*d^3)/2 + 2*B*a^2*c^3 + (11*B*a^2*d^3)/8 + (21*A*a^2*c*d^2)/4 + 6*A*
a^2*c^2*d + (9*B*a^2*c*d^2)/2 + (21*B*a^2*c^2*d)/4)))*(24*A*c^3 + 12*A*d^3
+ 16*B*c^3 + 11*B*d^3 + 42*A*c*d^2 + 48*A*c^2*d + 36*B*c*d^2 + 42*B*c^2*d))
/(8*f) - (tan(e/2 + (f*x)/2)*(A*a^2*c^3 + (3*A*a^2*d^3)/2 + 2*B*a^2*c^3 + (
11*B*a^2*d^3)/8 + (21*A*a^2*c*d^2)/4 + 6*A*a^2*c^2*d + (9*B*a^2*c*d^2)/2 +
(21*B*a^2*c^2*d)/4) + tan(e/2 + (f*x)/2)^8*(20*A*a^2*c^3 + 4*A*a^2*d^3 + 14
*B*a^2*c^3 + 24*A*a^2*c*d^2 + 42*A*a^2*c^2*d + 12*B*a^2*c*d^2 + 24*B*a^2*c^
2*d) - tan(e/2 + (f*x)/2)^11*(A*a^2*c^3 + (3*A*a^2*d^3)/2 + 2*B*a^2*c^3 + (
11*B*a^2*d^3)/8 + (21*A*a^2*c*d^2)/4 + 6*A*a^2*c^2*d + (9*B*a^2*c*d^2)/2 +
(21*B*a^2*c^2*d)/4) + tan(e/2 + (f*x)/2)^5*(2*A*a^2*c^3 + 7*A*a^2*d^3 + 4*B
*a^2*c^3 + (47*B*a^2*d^3)/4 + (33*A*a^2*c*d^2)/2 + 12*A*a^2*c^2*d + 21*B*a^
2*c*d^2 + (33*B*a^2*c^2*d)/2) - tan(e/2 + (f*x)/2)^7*(2*A*a^2*c^3 + 7*A*a^2
*d^3 + 4*B*a^2*c^3 + (47*B*a^2*d^3)/4 + (33*A*a^2*c*d^2)/2 + 12*A*a^2*c^2*d
 + 21*B*a^2*c*d^2 + (33*B*a^2*c^2*d)/2) + tan(e/2 + (f*x)/2)^3*(3*A*a^2*c^3
 + (17*A*a^2*d^3)/2 + 6*B*a^2*c^3 + (187*B*a^2*d^3)/24 + (87*A*a^2*c*d^2)/4
 + 18*A*a^2*c^2*d + (51*B*a^2*c*d^2)/2 + (87*B*a^2*c^2*d)/4) - tan(e/2 + (f
*x)/2)^9*(3*A*a^2*c^3 + (17*A*a^2*d^3)/2 + 6*B*a^2*c^3 + (187*B*a^2*d^3)/24
 + (87*A*a^2*c*d^2)/4 + 18*A*a^2*c^2*d + (51*B*a^2*c*d^2)/2 + (87*B*a^2*c^2
*d)/4) + tan(e/2 + (f*x)/2)^4*(40*A*a^2*c^3 + 32*A*a^2*d^3 + 36*B*a^2*c^3 +
 32*B*a^2*d^3 + 96*A*a^2*c*d^2 + 108*A*a^2*c^2*d + 96*B*a^2*c*d^2 + 96*B*a^
2*c^2*d) + tan(e/2 + (f*x)/2)^2*(20*A*a^2*c^3 + (72*A*a^2*d^3)/5 + 18*B*a^2
```

```
*c^3 + (64*B*a^2*d^3)/5 + 48*A*a^2*c*d^2 + 54*A*a^2*c^2*d + (216*B*a^2*c*d^
2)/5 + 48*B*a^2*c^2*d) + tan(e/2 + (f*x)/2)^6*(40*A*a^2*c^3 + 24*A*a^2*d^3
+ (100*B*a^2*c^3)/3 + (64*B*a^2*d^3)/3 + 80*A*a^2*c*d^2 + 100*A*a^2*c^2*d +
 72*B*a^2*c*d^2 + 80*B*a^2*c^2*d) + tan(e/2 + (f*x)/2)^10*(4*A*a^2*c^3 + 2*
B*a^2*c^3 + 6*A*a^2*c^2*d) + 4*A*a^2*c^3 + (12*A*a^2*d^3)/5 + (10*B*a^2*c^3
)/3 + (32*B*a^2*d^3)/15 + 8*A*a^2*c*d^2 + 10*A*a^2*c^2*d + (36*B*a^2*c*d^2)
/5 + 8*B*a^2*c^2*d)/(f*(6*tan(e/2 + (f*x)/2)^2 + 15*tan(e/2 + (f*x)/2)^4 +
20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 + 6*tan(e/2 + (f*x)/2)^10
 + tan(e/2 + (f*x)/2)^12 + 1))
```

sympy [A] time = 11.95, size = 1865, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((A*a**2*c**3*x*sin(e + f*x)**2/2 + A*a**2*c**3*x*cos(e + f*x)**2/
2 + A*a**2*c**3*x - A*a**2*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**2*
c**3*cos(e + f*x)/f + 3*A*a**2*c**2*d*x*sin(e + f*x)**2 + 3*A*a**2*c**2*d*x
*cos(e + f*x)**2 - 3*A*a**2*c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**
2*c**2*d*sin(e + f*x)*cos(e + f*x)/f - 2*A*a**2*c**2*d*cos(e + f*x)**3/f -
3*A*a**2*c**2*d*cos(e + f*x)/f + 9*A*a**2*c*d**2*x*sin(e + f*x)**4/8 + 9*A*
a**2*c*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*A*a**2*c*d**2*x*sin(e +
 f*x)**2/2 + 9*A*a**2*c*d**2*x*cos(e + f*x)**4/8 + 3*A*a**2*c*d**2*x*cos(e
+ f*x)**2/2 - 15*A*a**2*c*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 6*A*a**
2*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 9*A*a**2*c*d**2*sin(e + f*x)*cos(
e + f*x)**3/(8*f) - 3*A*a**2*c*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 4*A*a
**2*c*d**2*cos(e + f*x)**3/f + 3*A*a**2*d**3*x*sin(e + f*x)**4/4 + 3*A*a**2
*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + 3*A*a**2*d**3*x*cos(e + f*x)**4
/4 - A*a**2*d**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**2*d**3*sin(e + f*x
)**3*cos(e + f*x)/(4*f) - 4*A*a**2*d**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*
f) - A*a**2*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**2*d**3*sin(e + f*x
)*cos(e + f*x)**3/(4*f) - 8*A*a**2*d**3*cos(e + f*x)**5/(15*f) - 2*A*a**2*d
**3*cos(e + f*x)**3/(3*f) + B*a**2*c**3*x*sin(e + f*x)**2 + B*a**2*c**3*x*c
os(e + f*x)**2 - B*a**2*c**3*sin(e + f*x)**2*cos(e + f*x)/f - B*a**2*c**3*s
in(e + f*x)*cos(e + f*x)/f - 2*B*a**2*c**3*cos(e + f*x)**3/(3*f) - B*a**2*c
**3*cos(e + f*x)/f + 9*B*a**2*c**2*d*x*sin(e + f*x)**4/8 + 9*B*a**2*c**2*d*
x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a**2*c**2*d*x*sin(e + f*x)**2/2 +
 9*B*a**2*c**2*d*x*cos(e + f*x)**4/8 + 3*B*a**2*c**2*d*x*cos(e + f*x)**2/2
- 15*B*a**2*c**2*d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 6*B*a**2*c**2*d*sin
(e + f*x)**2*cos(e + f*x)/f - 9*B*a**2*c**2*d*sin(e + f*x)*cos(e + f*x)**3/
(8*f) - 3*B*a**2*c**2*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 4*B*a**2*c**2*d*c
os(e + f*x)**3/f + 9*B*a**2*c*d**2*x*sin(e + f*x)**4/4 + 9*B*a**2*c*d**2*x*
sin(e + f*x)**2*cos(e + f*x)**2/2 + 9*B*a**2*c*d**2*x*cos(e + f*x)**4/4 - 3
```

```
*B*a**2*c*d**2*sin(e + f*x)**4*cos(e + f*x)/f - 15*B*a**2*c*d**2*sin(e + f*
x)**3*cos(e + f*x)/(4*f) - 4*B*a**2*c*d**2*sin(e + f*x)**2*cos(e + f*x)**3/
f - 3*B*a**2*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 9*B*a**2*c*d**2*sin(e
+ f*x)*cos(e + f*x)**3/(4*f) - 8*B*a**2*c*d**2*cos(e + f*x)**5/(5*f) - 2*B*
a**2*c*d**2*cos(e + f*x)**3/f + 5*B*a**2*d**3*x*sin(e + f*x)**6/16 + 15*B*a
**2*d**3*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 3*B*a**2*d**3*x*sin(e + f*x
)**4/8 + 15*B*a**2*d**3*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 3*B*a**2*d**
3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 5*B*a**2*d**3*x*cos(e + f*x)**6/16
+ 3*B*a**2*d**3*x*cos(e + f*x)**4/8 - 11*B*a**2*d**3*sin(e + f*x)**5*cos(e
+ f*x)/(16*f) - 2*B*a**2*d**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a**2*d**
3*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 5*B*a**2*d**3*sin(e + f*x)**3*cos
(e + f*x)/(8*f) - 8*B*a**2*d**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 5*B
*a**2*d**3*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 3*B*a**2*d**3*sin(e + f*x)
*cos(e + f*x)**3/(8*f) - 16*B*a**2*d**3*cos(e + f*x)**5/(15*f), Ne(f, 0)),
(x*(A + B*sin(e))*(c + d*sin(e))**3*(a*sin(e) + a)**2, True))
```

3.252 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=336

$$\frac{a^2\left(5Ad(c-8d)-2B\left(c^2-5cd+18d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^2}{60d^2f}+\frac{1}{8}a^2x\left(12Ac^2+16Acd+7Ad^2+8Bc^2\right.$$

```
[Out] 1/8*a^2*(12*A*c^2+16*A*c*d+7*A*d^2+8*B*c^2+14*B*c*d+6*B*d^2)*x+1/30*a^2*(5*
A*d*(c^3-8*c^2*d-20*c*d^2-8*d^3)-2*B*(c^4-5*c^3*d+16*c^2*d^2+40*c*d^3+18*d^
4))*cos(f*x+e)/d^2/f+1/120*a^2*(5*A*d*(2*c^2-16*c*d-21*d^2)-B*(4*c^3-20*c^2
*d+66*c*d^2+90*d^3))*cos(f*x+e)*sin(f*x+e)/d/f+1/60*a^2*(5*A*(c-8*d)*d-2*B*
(c^2-5*c*d+18*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^2/d^2/f+1/20*a^2*(2*B*(c-3*
d)-5*A*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3/d^2/f-1/5*B*cos(f*x+e)*(a^2+a^2*sin
(f*x+e))*(c+d*sin(f*x+e))^3/d/f
```

Rubi [A] time = 0.70, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2976, 2968, 3023, 2753, 2734}

$$\frac{a^2\left(5Ad\left(-8c^2d+c^3-20cd^2-8d^3\right)-2B\left(16c^2d^2-5c^3d+c^4+40cd^3+18d^4\right)\right)\cos(e+fx)}{30d^2f}+\frac{a^2\left(5Ad(c-8d)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*(12*A*c^2 + 8*B*c^2 + 16*A*c*d + 14*B*c*d + 7*A*d^2 + 6*B*d^2)*x)/8 +
(a^2*(5*A*d*(c^3 - 8*c^2*d - 20*c*d^2 - 8*d^3) - 2*B*(c^4 - 5*c^3*d + 16*c^
2*d^2 + 40*c*d^3 + 18*d^4))*Cos[e + f*x])/(30*d^2*f) + (a^2*(5*A*d*(2*c^2 -
 16*c*d - 21*d^2) - B*(4*c^3 - 20*c^2*d + 66*c*d^2 + 90*d^3))*Cos[e + f*x]*
Sin[e + f*x])/(120*d*f) + (a^2*(5*A*(c - 8*d)*d - 2*B*(c^2 - 5*c*d + 18*d^2
))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(60*d^2*f) + (a^2*(2*B*(c - 3*d) -
5*A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(20*d^2*f) - (B*Cos[e + f*x]*(a
^2 + a^2*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(5*d*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = -\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(c+d\,\mathrm{s}}{5df}$$
$$= -\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(c+d\,\mathrm{s}}{5df}$$
$$= \frac{a^2(2B(c-3d)-5Ad)\cos(e+fx)(c+d\sin(}{20d^2f}$$
$$= \frac{a^2\left(5A(c-8d)d-2B\left(c^2-5cd+18d^2\right)\right)\cos}{60d^2f}$$
$$= \frac{1}{8}a^2\left(12Ac^2+8Bc^2+16Acd+14Bcd+7Ad\right.$$

Mathematica [A] time = 1.57, size = 296, normalized size = 0.88

$$-\frac{a^2\cos(e+fx)\left(60\left(A\left(12c^2+16cd+7d^2\right)+2B\left(4c^2+7cd+3d^2\right)\right)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\,\left(-8\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
2,x]
```

```
[Out] -1/240*(a^2*Cos[e + f*x]*(60*(2*B*(4*c^2 + 7*c*d + 3*d^2) + A*(12*c^2 + 16*
c*d + 7*d^2))*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]
*(480*A*c^2 + 440*B*c^2 + 880*A*c*d + 800*B*c*d + 400*A*d^2 + 378*B*d^2 - 8
*(10*A*d*(c + d) + B*(5*c^2 + 20*c*d + 12*d^2))*Cos[2*(e + f*x)] + 6*B*d^2*
Cos[4*(e + f*x)] + 120*A*c^2*Sin[e + f*x] + 240*B*c^2*Sin[e + f*x] + 480*A*
c*d*Sin[e + f*x] + 510*B*c*d*Sin[e + f*x] + 255*A*d^2*Sin[e + f*x] + 270*B*
d^2*Sin[e + f*x] - 30*B*c*d*Sin[3*(e + f*x)] - 15*A*d^2*Sin[3*(e + f*x)] -
30*B*d^2*Sin[3*(e + f*x)])))/(f*Sqrt[Cos[e + f*x]^2])
```

fricas [A] time = 0.46, size = 245, normalized size = 0.73

$$-\frac{24\,Ba^2d^2\cos\left(fx+e\right)^5-40\left(Ba^2c^2+2\,(A+2\,B)a^2cd+(2\,A+3\,B)a^2d^2\right)\cos\left(fx+e\right)^3-15\left(4\,(3\,A+2\,B)a^2c^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/120*(24*B*a^2*d^2*cos(f*x + e)^5 - 40*(B*a^2*c^2 + 2*(A + 2*B)*a^2*c*d +
 (2*A + 3*B)*a^2*d^2)*cos(f*x + e)^3 - 15*(4*(3*A + 2*B)*a^2*c^2 + 2*(8*A +
 7*B)*a^2*c*d + (7*A + 6*B)*a^2*d^2)*f*x + 240*((A + B)*a^2*c^2 + 2*(A + B)
*a^2*c*d + (A + B)*a^2*d^2)*cos(f*x + e) - 15*(2*(2*B*a^2*c*d + (A + 2*B)*a
^2*d^2)*cos(f*x + e)^3 - (4*(A + 2*B)*a^2*c^2 + 2*(8*A + 9*B)*a^2*c*d + (9*
A + 10*B)*a^2*d^2)*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.18, size = 311, normalized size = 0.93

$$-\frac{Ba^2d^2\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{1}{8}\left(12\,Aa^2c^2+8\,Ba^2c^2+16\,Aa^2cd+14\,Ba^2cd+7\,Aa^2d^2+6\,Ba^2d^2\right)x+\frac{\left(4\,Ba^2c^2+8\,Aa\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] -1/80*B*a^2*d^2*cos(5*f*x + 5*e)/f + 1/8*(12*A*a^2*c^2 + 8*B*a^2*c^2 + 16*A
*a^2*c*d + 14*B*a^2*c*d + 7*A*a^2*d^2 + 6*B*a^2*d^2)*x + 1/48*(4*B*a^2*c^2
+ 8*A*a^2*c*d + 16*B*a^2*c*d + 8*A*a^2*d^2 + 9*B*a^2*d^2)*cos(3*f*x + 3*e)/
f - 1/8*(16*A*a^2*c^2 + 14*B*a^2*c^2 + 28*A*a^2*c*d + 24*B*a^2*c*d + 12*A*a
^2*d^2 + 11*B*a^2*d^2)*cos(f*x + e)/f + 1/32*(2*B*a^2*c*d + A*a^2*d^2 + 2*B
*a^2*d^2)*sin(4*f*x + 4*e)/f - 1/4*(A*a^2*c^2 + 2*B*a^2*c^2 + 4*A*a^2*c*d +
 4*B*a^2*c*d + 2*A*a^2*d^2 + 2*B*a^2*d^2)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.52, size = 496, normalized size = 1.48

$$\frac{a^2A\,c^2\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{2a^2Acd\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}+a^2A\,d^2\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(a^2*A*c^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2/3*a^2*A*c*d*(2+
sin(f*x+e)^2)*cos(f*x+e)+a^2*A*d^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(
f*x+e)+3/8*f*x+3/8*e)-1/3*B*a^2*c^2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*B*a^2*c*d
*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*B*a^2*d^
2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-2*a^2*A*c^2*cos(f*x+e)+4*a
^2*A*c*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2/3*a^2*A*d^2*(2+sin(f*
x+e)^2)*cos(f*x+e)+2*B*a^2*c^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-4
/3*B*a^2*c*d*(2+sin(f*x+e)^2)*cos(f*x+e)+2*B*a^2*d^2*(-1/4*(sin(f*x+e)^3+3/
2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+a^2*A*c^2*(f*x+e)-2*a^2*A*c*d*cos(f
```

```
*x+e)+a^2*A*d^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-B*a^2*c^2*cos(f*
x+e)+2*B*a^2*c*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/3*B*a^2*d^2*(
2+sin(f*x+e)^2)*cos(f*x+e))
```

maxima [A] time = 0.44, size = 478, normalized size = 1.42

$$\frac{120\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)Aa^2c^2+480\left(fx+e\right)Aa^2c^2+160\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)Ba^2c^2+24}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/480*(120*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^2 + 480*(f*x + e)*A*a^2
*c^2 + 160*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^2 + 240*(2*f*x + 2*e -
 sin(2*f*x + 2*e))*B*a^2*c^2 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*
c*d + 480*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c*d + 640*(cos(f*x + e)^3
- 3*cos(f*x + e))*B*a^2*c*d + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(
2*f*x + 2*e))*B*a^2*c*d + 240*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*c*d +
320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*d^2 + 15*(12*f*x + 12*e + sin(4
*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*d^2 + 120*(2*f*x + 2*e - sin(2*f*x
+ 2*e))*A*a^2*d^2 - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x +
 e))*B*a^2*d^2 + 160*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*d^2 + 30*(12*f
*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^2*d^2 - 960*A*a^2*c^
2*cos(f*x + e) - 480*B*a^2*c^2*cos(f*x + e) - 960*A*a^2*c*d*cos(f*x + e))/f
```

mupad [B] time = 15.71, size = 765, normalized size = 2.28

$$\frac{a^2\,\mathrm{atan}\left(\frac{a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(12\,A\,c^2+7\,A\,d^2+8\,B\,c^2+6\,B\,d^2+16\,A\,c\,d+14\,B\,c\,d\right)}{4\left(3\,A\,a^2\,c^2+\frac{7\,A\,a^2\,d^2}{4}+2\,B\,a^2\,c^2+\frac{3\,B\,a^2\,d^2}{2}+4\,A\,a^2\,c\,d+\frac{7\,B\,a^2\,c\,d}{2}\right)}\right)\left(12\,A\,c^2+7\,A\,d^2+8\,B\,c^2+6\,B\,d^2+16\,A\,c\,d+14\,B\right.}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^2,x)
```

```
[Out] (a^2*atan((a^2*tan(e/2 + (f*x)/2)*(12*A*c^2 + 7*A*d^2 + 8*B*c^2 + 6*B*d^2 +
 16*A*c*d + 14*B*c*d))/(4*(3*A*a^2*c^2 + (7*A*a^2*d^2)/4 + 2*B*a^2*c^2 + (3
*B*a^2*d^2)/2 + 4*A*a^2*c*d + (7*B*a^2*c*d)/2)))*(12*A*c^2 + 7*A*d^2 + 8*B*
c^2 + 6*B*d^2 + 16*A*c*d + 14*B*c*d))/(4*f) - (tan(e/2 + (f*x)/2)^8*(4*A*a^
2*c^2 + 2*B*a^2*c^2 + 4*A*a^2*c*d) + tan(e/2 + (f*x)/2)*(A*a^2*c^2 + (7*A*a
^2*d^2)/4 + 2*B*a^2*c^2 + (3*B*a^2*d^2)/2 + 4*A*a^2*c*d + (7*B*a^2*c*d)/2)
```

```
- tan(e/2 + (f*x)/2)^9*(A*a^2*c^2 + (7*A*a^2*d^2)/4 + 2*B*a^2*c^2 + (3*B*a^
2*d^2)/2 + 4*A*a^2*c*d + (7*B*a^2*c*d)/2) + tan(e/2 + (f*x)/2)^3*(2*A*a^2*c
^2 + (11*A*a^2*d^2)/2 + 4*B*a^2*c^2 + 7*B*a^2*d^2 + 8*A*a^2*c*d + 11*B*a^2*
c*d) - tan(e/2 + (f*x)/2)^7*(2*A*a^2*c^2 + (11*A*a^2*d^2)/2 + 4*B*a^2*c^2 +
 7*B*a^2*d^2 + 8*A*a^2*c*d + 11*B*a^2*c*d) + tan(e/2 + (f*x)/2)^6*(16*A*a^2
*c^2 + 8*A*a^2*d^2 + 12*B*a^2*c^2 + 4*B*a^2*d^2 + 24*A*a^2*c*d + 16*B*a^2*c
*d) + tan(e/2 + (f*x)/2)^2*(16*A*a^2*c^2 + (40*A*a^2*d^2)/3 + (44*B*a^2*c^2
)/3 + 12*B*a^2*d^2 + (88*A*a^2*c*d)/3 + (80*B*a^2*c*d)/3) + tan(e/2 + (f*x)
/2)^4*(24*A*a^2*c^2 + (56*A*a^2*d^2)/3 + (64*B*a^2*c^2)/3 + 20*B*a^2*d^2 +
(128*A*a^2*c*d)/3 + (112*B*a^2*c*d)/3) + 4*A*a^2*c^2 + (8*A*a^2*d^2)/3 + (1
0*B*a^2*c^2)/3 + (12*B*a^2*d^2)/5 + (20*A*a^2*c*d)/3 + (16*B*a^2*c*d)/3)/(f
*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^
6 + 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 + 1))
```

sympy [A] time = 6.00, size = 1129, normalized size = 3.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((A*a**2*c**2*x*sin(e + f*x)**2/2 + A*a**2*c**2*x*cos(e + f*x)**2/
2 + A*a**2*c**2*x - A*a**2*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**2*
c**2*cos(e + f*x)/f + 2*A*a**2*c*d*x*sin(e + f*x)**2 + 2*A*a**2*c*d*x*cos(e
 + f*x)**2 - 2*A*a**2*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 2*A*a**2*c*d*sin
(e + f*x)*cos(e + f*x)/f - 4*A*a**2*c*d*cos(e + f*x)**3/(3*f) - 2*A*a**2*c*
d*cos(e + f*x)/f + 3*A*a**2*d**2*x*sin(e + f*x)**4/8 + 3*A*a**2*d**2*x*sin(
e + f*x)**2*cos(e + f*x)**2/4 + A*a**2*d**2*x*sin(e + f*x)**2/2 + 3*A*a**2*
d**2*x*cos(e + f*x)**4/8 + A*a**2*d**2*x*cos(e + f*x)**2/2 - 5*A*a**2*d**2*
sin(e + f*x)**3*cos(e + f*x)/(8*f) - 2*A*a**2*d**2*sin(e + f*x)**2*cos(e +
f*x)/f - 3*A*a**2*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - A*a**2*d**2*sin
(e + f*x)*cos(e + f*x)/(2*f) - 4*A*a**2*d**2*cos(e + f*x)**3/(3*f) + B*a**2
*c**2*x*sin(e + f*x)**2 + B*a**2*c**2*x*cos(e + f*x)**2 - B*a**2*c**2*sin(e
 + f*x)**2*cos(e + f*x)/f - B*a**2*c**2*sin(e + f*x)*cos(e + f*x)/f - 2*B*a
**2*c**2*cos(e + f*x)**3/(3*f) - B*a**2*c**2*cos(e + f*x)/f + 3*B*a**2*c*d*
x*sin(e + f*x)**4/4 + 3*B*a**2*c*d*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + B*
a**2*c*d*x*sin(e + f*x)**2 + 3*B*a**2*c*d*x*cos(e + f*x)**4/4 + B*a**2*c*d*
x*cos(e + f*x)**2 - 5*B*a**2*c*d*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 4*B*a
**2*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**2*c*d*sin(e + f*x)*cos(e +
f*x)**3/(4*f) - B*a**2*c*d*sin(e + f*x)*cos(e + f*x)/f - 8*B*a**2*c*d*cos(e
 + f*x)**3/(3*f) + 3*B*a**2*d**2*x*sin(e + f*x)**4/4 + 3*B*a**2*d**2*x*sin(
e + f*x)**2*cos(e + f*x)**2/2 + 3*B*a**2*d**2*x*cos(e + f*x)**4/4 - B*a**2*
d**2*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a**2*d**2*sin(e + f*x)**3*cos(e +
 f*x)/(4*f) - 4*B*a**2*d**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - B*a**2*
d**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**2*d**2*sin(e + f*x)*cos(e + f*
```

```
x)**3/(4*f) - 8*B*a**2*d**2*cos(e + f*x)**5/(15*f) - 2*B*a**2*d**2*cos(e +
f*x)**3/(3*f), Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))**2*(a*sin(e) + a
)**2, True))
```

3.253 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=166

$$-\frac{a^2(12Ac+8Ad+8Bc+7Bd)\cos(e+fx)}{6f}-\frac{a^2(12Ac+8Ad+8Bc+7Bd)\sin(e+fx)\cos(e+fx)}{24f}+\frac{1}{8}a^2x(12Ac+$$

```
[Out] 1/8*a^2*(12*A*c+8*A*d+8*B*c+7*B*d)*x-1/6*a^2*(12*A*c+8*A*d+8*B*c+7*B*d)*cos
(f*x+e)/f-1/24*a^2*(12*A*c+8*A*d+8*B*c+7*B*d)*cos(f*x+e)*sin(f*x+e)/f-1/12*
(4*A*d+4*B*c-B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^2/f-1/4*B*d*cos(f*x+e)*(a+a*s
in(f*x+e))^3/a/f
```

Rubi [A] time = 0.27, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2968, 3023, 2751, 2644}

$$-\frac{a^2(12Ac+8Ad+8Bc+7Bd)\cos(e+fx)}{6f}-\frac{a^2(12Ac+8Ad+8Bc+7Bd)\sin(e+fx)\cos(e+fx)}{24f}+\frac{1}{8}a^2x(12Ac+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (a^2*(12*A*c + 8*B*c + 8*A*d + 7*B*d)*x)/8 - (a^2*(12*A*c + 8*B*c + 8*A*d +
 7*B*d)*Cos[e + f*x])/(6*f) - (a^2*(12*A*c + 8*B*c + 8*A*d + 7*B*d)*Cos[e +
 f*x]*Sin[e + f*x])/(24*f) - ((4*B*c + 4*A*d - B*d)*Cos[e + f*x]*(a + a*Sin
[e + f*x])^2)/(12*f) - (B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^3)/(4*a*f)
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int (a+a\sin(e+fx))^2\left(Ac+(Bc+Ad)\sin(e\right.$$

$$= -\frac{Bd\cos(e+fx)(a+a\sin(e+fx))^3}{4af} + \frac{\int (a+}{}$$

$$= -\frac{(4Bc+4Ad-Bd)\cos(e+fx)(a+a\sin(e+}{12f}$$

$$= \frac{1}{8}a^2(12Ac+8Bc+8Ad+7Bd)x - \frac{a^2(12Ac+}{}$$

Mathematica [A] time = 0.76, size = 160, normalized size = 0.96

$$-\frac{a^2\cos(e+fx)\left(6(12Ac+8Ad+8Bc+7Bd)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\,\left(8(Ad+B(c+2d))\sin^2(e\right.\right.}{24f\sqrt{\cos^2(e+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),
x]
```

```
[Out] -1/24*(a^2*Cos[e + f*x]*(6*(12*A*c + 8*B*c + 8*A*d + 7*B*d)*ArcSin[Sqrt[1 -
 Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(8*(6*A*c + 5*B*c + 5*A*d +
4*B*d) + 3*(4*A*c + 8*B*c + 8*A*d + 7*B*d)*Sin[e + f*x] + 8*(A*d + B*(c + 2
*d))*Sin[e + f*x]^2 + 6*B*d*Sin[e + f*x]^3)))/(f*Sqrt[Cos[e + f*x]^2])
```

fricas [A] time = 0.45, size = 144, normalized size = 0.87

$$\frac{8\left(Ba^2c+(A+2B)a^2d\right)\cos\left(fx+e\right)^3+3\left(4\left(3A+2B\right)a^2c+(8A+7B)a^2d\right)fx-48\left(\left(A+B\right)a^2c+(A+B)a^2d\right)}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/24*(8*(B*a^2*c + (A + 2*B)*a^2*d)*cos(f*x + e)^3 + 3*(4*(3*A + 2*B)*a^2*c
 + (8*A + 7*B)*a^2*d)*f*x - 48*((A + B)*a^2*c + (A + B)*a^2*d)*cos(f*x + e)
 + 3*(2*B*a^2*d*cos(f*x + e)^3 - (4*(A + 2*B)*a^2*c + (8*A + 9*B)*a^2*d)*co
s(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.17, size = 172, normalized size = 1.04

$$\frac{Ba^2d\sin\left(4fx+4e\right)}{32f}+\frac{1}{8}\left(12Aa^2c+8Ba^2c+8Aa^2d+7Ba^2d\right)x+\frac{\left(Ba^2c+Aa^2d+2Ba^2d\right)\cos\left(3fx+3e\right)}{12f}-\frac{(8A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] 1/32*B*a^2*d*sin(4*f*x + 4*e)/f + 1/8*(12*A*a^2*c + 8*B*a^2*c + 8*A*a^2*d +
 7*B*a^2*d)*x + 1/12*(B*a^2*c + A*a^2*d + 2*B*a^2*d)*cos(3*f*x + 3*e)/f - 1
/4*(8*A*a^2*c + 7*B*a^2*c + 7*A*a^2*d + 6*B*a^2*d)*cos(f*x + e)/f - 1/4*(A*
a^2*c + 2*B*a^2*c + 2*A*a^2*d + 2*B*a^2*d)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.41, size = 278, normalized size = 1.67

$$\frac{a^2Ac\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{a^2Ad\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}-\frac{Ba^2c\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}+Ba^2d\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3}{}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] 1/f*(a^2*A*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/3*a^2*A*d*(2+sin(
f*x+e)^2)*cos(f*x+e)-1/3*B*a^2*c*(2+sin(f*x+e)^2)*cos(f*x+e)+B*a^2*d*(-1/4*
(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2*a^2*A*c*cos(f*x+e
)+2*a^2*A*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2*B*a^2*c*(-1/2*sin(
```

```
f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2/3*B*a^2*d*(2+sin(f*x+e)^2)*cos(f*x+e)+a^
2*A*c*(f*x+e)-a^2*A*d*cos(f*x+e)-B*a^2*c*cos(f*x+e)+B*a^2*d*(-1/2*sin(f*x+e
)*cos(f*x+e)+1/2*f*x+1/2*e))
```

maxima [A] time = 0.42, size = 268, normalized size = 1.61

$$\frac{24\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)Aa^2c+96\left(fx+e\right)Aa^2c+32\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)Ba^2c+48\left(2\,fx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] 1/96*(24*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c + 96*(f*x + e)*A*a^2*c +
32*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c + 48*(2*f*x + 2*e - sin(2*f*x
+ 2*e))*B*a^2*c + 32*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*d + 48*(2*f*x
+ 2*e - sin(2*f*x + 2*e))*A*a^2*d + 64*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*
a^2*d + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^2*d +
 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*d - 192*A*a^2*c*cos(f*x + e) - 9
6*B*a^2*c*cos(f*x + e) - 96*A*a^2*d*cos(f*x + e))/f
```

mupad [B] time = 14.60, size = 492, normalized size = 2.96

$$\frac{a^2\,\mathrm{atan}\left(\frac{a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(12\,A\,c+8\,A\,d+8\,B\,c+7\,B\,d\right)}{4\,\left(3\,A\,a^2\,c+2\,A\,a^2\,d+2\,B\,a^2\,c+\frac{7\,B\,a^2\,d}{4}\right)}\right)\,\left(12\,A\,c+8\,A\,d+8\,B\,c+7\,B\,d\right)}{4\,f}-\frac{a^2\,\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{f\,x}{2}\right)\,\left(12\,A\right.}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x)),x)
```

```
[Out] (a^2*atan((a^2*tan(e/2 + (f*x)/2)*(12*A*c + 8*A*d + 8*B*c + 7*B*d))/(4*(3*A
*a^2*c + 2*A*a^2*d + 2*B*a^2*c + (7*B*a^2*d)/4)))*(12*A*c + 8*A*d + 8*B*c +
 7*B*d))/(4*f) - (a^2*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2)*(12*A*c + 8*A*d
+ 8*B*c + 7*B*d))/(4*f) - (tan(e/2 + (f*x)/2)^3*(A*a^2*c + 2*A*a^2*d + 2*B*
a^2*c + (15*B*a^2*d)/4) - tan(e/2 + (f*x)/2)^7*(A*a^2*c + 2*A*a^2*d + 2*B*a
^2*c + (7*B*a^2*d)/4) - tan(e/2 + (f*x)/2)^5*(A*a^2*c + 2*A*a^2*d + 2*B*a^2
*c + (15*B*a^2*d)/4) + tan(e/2 + (f*x)/2)^4*(12*A*a^2*c + 10*A*a^2*d + 10*B
*a^2*c + 8*B*a^2*d) + tan(e/2 + (f*x)/2)^2*(12*A*a^2*c + (34*A*a^2*d)/3 + (
34*B*a^2*c)/3 + (32*B*a^2*d)/3) + tan(e/2 + (f*x)/2)^6*(4*A*a^2*c + 2*A*a^2
*d + 2*B*a^2*c) + tan(e/2 + (f*x)/2)*(A*a^2*c + 2*A*a^2*d + 2*B*a^2*c + (7*
B*a^2*d)/4) + 4*A*a^2*c + (10*A*a^2*d)/3 + (10*B*a^2*c)/3 + (8*B*a^2*d)/3)/
```

```
(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/2 + (f*x)/2)^
6 + tan(e/2 + (f*x)/2)^8 + 1))
```

sympy [A] time = 2.76, size = 571, normalized size = 3.44

$$\begin{cases} \frac{Aa^2cx\sin^2\left(e+fx\right)}{2} + \frac{Aa^2cx\cos^2\left(e+fx\right)}{2} + Aa^2cx - \frac{Aa^2c\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f} - \frac{2Aa^2c\cos\left(e+fx\right)}{f} + Aa^2dx\sin^2\left(e+fx\right) + Aa^2d \\ x\left(A+B\sin\left(e\right)\right)\left(c+d\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((A*a**2*c*x*sin(e + f*x)**2/2 + A*a**2*c*x*cos(e + f*x)**2/2 + A*
a**2*c*x - A*a**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**2*c*cos(e + f*
x)/f + A*a**2*d*x*sin(e + f*x)**2 + A*a**2*d*x*cos(e + f*x)**2 - A*a**2*d*s
in(e + f*x)**2*cos(e + f*x)/f - A*a**2*d*sin(e + f*x)*cos(e + f*x)/f - 2*A*
a**2*d*cos(e + f*x)**3/(3*f) - A*a**2*d*cos(e + f*x)/f + B*a**2*c*x*sin(e +
 f*x)**2 + B*a**2*c*x*cos(e + f*x)**2 - B*a**2*c*sin(e + f*x)**2*cos(e + f*
x)/f - B*a**2*c*sin(e + f*x)*cos(e + f*x)/f - 2*B*a**2*c*cos(e + f*x)**3/(3
*f) - B*a**2*c*cos(e + f*x)/f + 3*B*a**2*d*x*sin(e + f*x)**4/8 + 3*B*a**2*d
*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + B*a**2*d*x*sin(e + f*x)**2/2 + 3*B*a
**2*d*x*cos(e + f*x)**4/8 + B*a**2*d*x*cos(e + f*x)**2/2 - 5*B*a**2*d*sin(e
 + f*x)**3*cos(e + f*x)/(8*f) - 2*B*a**2*d*sin(e + f*x)**2*cos(e + f*x)/f -
 3*B*a**2*d*sin(e + f*x)*cos(e + f*x)**3/(8*f) - B*a**2*d*sin(e + f*x)*cos(
e + f*x)/(2*f) - 4*B*a**2*d*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(A + B*sin
(e))*(c + d*sin(e))*(a*sin(e) + a)**2, True))
```

3.254 $\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))\,dx$

Optimal. Leaf size=94

$$-\frac{2a^2(3A+2B)\cos(e+fx)}{3f}-\frac{a^2(3A+2B)\sin(e+fx)\cos(e+fx)}{6f}+\frac{1}{2}a^2x(3A+2B)-\frac{B\cos(e+fx)(a\sin(e+fx)}{3f}$$

```
[Out] 1/2*a^2*(3*A+2*B)*x-2/3*a^2*(3*A+2*B)*cos(f*x+e)/f-1/6*a^2*(3*A+2*B)*cos(f*
x+e)*sin(f*x+e)/f-1/3*B*cos(f*x+e)*(a+a*sin(f*x+e))^2/f
```

Rubi [A] time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2751, 2644}

$$-\frac{2a^2(3A+2B)\cos(e+fx)}{3f}-\frac{a^2(3A+2B)\sin(e+fx)\cos(e+fx)}{6f}+\frac{1}{2}a^2x(3A+2B)-\frac{B\cos(e+fx)(a\sin(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x]
```

```
[Out] (a^2*(3*A + 2*B)*x)/2 - (2*a^2*(3*A + 2*B)*Cos[e + f*x])/(3*f) - (a^2*(3*A
+ 2*B)*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*
x])^2)/(3*f)
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^2}{3f}+\frac{1}{3}(3A+2B)\int (a+a\sin(e$$
$$=\frac{1}{2}a^2(3A+2B)x-\frac{2a^2(3A+2B)\cos(e+fx)}{3f}-\frac{a^2(3A+2B)\cos(e}{6}$$

Mathematica [A] time = 0.32, size = 106, normalized size = 1.13

$$-\frac{a^2\cos(e+fx)\left(6(3A+2B)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\left(3(A+2B)\sin(e+fx)+2(6A+5B)+2B\sin\right.\right.}{6f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x]
```

```
[Out] -1/6*(a^2*Cos[e + f*x]*(6*(3*A + 2*B)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]
] + Sqrt[Cos[e + f*x]^2]*(2*(6*A + 5*B) + 3*(A + 2*B)*Sin[e + f*x] + 2*B*Si
n[e + f*x]^2)))/(f*Sqrt[Cos[e + f*x]^2])
```

fricas [A] time = 0.46, size = 70, normalized size = 0.74

$$\frac{2\,Ba^2\cos\left(fx+e\right)^3+3\,(3\,A+2\,B)a^2fx-3\,(A+2\,B)a^2\cos\left(fx+e\right)\sin\left(fx+e\right)-12\,(A+B)a^2\cos\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*B*a^2*cos(f*x + e)^3 + 3*(3*A + 2*B)*a^2*f*x - 3*(A + 2*B)*a^2*cos(f
*x + e)*sin(f*x + e) - 12*(A + B)*a^2*cos(f*x + e))/f
```

giac [A] time = 0.16, size = 88, normalized size = 0.94

$$\frac{Ba^2\cos\left(3\,fx+3\,e\right)}{12\,f}+\frac{1}{2}\left(3\,Aa^2+2\,Ba^2\right)x-\frac{\left(8\,Aa^2+7\,Ba^2\right)\cos\left(fx+e\right)}{4\,f}-\frac{\left(Aa^2+2\,Ba^2\right)\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/12*B*a^2*cos(3*f*x + 3*e)/f + 1/2*(3*A*a^2 + 2*B*a^2)*x - 1/4*(8*A*a^2 +
7*B*a^2)*cos(f*x + e)/f - 1/4*(A*a^2 + 2*B*a^2)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.30, size = 117, normalized size = 1.24

$$\frac{a^2A\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{B\,a^2\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}-2a^2A\cos\left(fx+e\right)+2B\,a^2\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x)
```

```
[Out] 1/f*(a^2*A*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/3*B*a^2*(2+sin(f*x+
e)^2)*cos(f*x+e)-2*a^2*A*cos(f*x+e)+2*B*a^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2
*f*x+1/2*e)+a^2*A*(f*x+e)-B*a^2*cos(f*x+e))
```

maxima [A] time = 0.44, size = 114, normalized size = 1.21

$$\frac{3\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)Aa^2+12\left(fx+e\right)Aa^2+4\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)Ba^2+6\left(2\,fx+2\,e-\right.}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2 + 12*(f*x + e)*A*a^2 + 4*(co
s(f*x + e)^3 - 3*cos(f*x + e))*B*a^2 + 6*(2*f*x + 2*e - sin(2*f*x + 2*e))*B
*a^2 - 24*A*a^2*cos(f*x + e) - 12*B*a^2*cos(f*x + e))/f
```

mupad [B] time = 13.16, size = 91, normalized size = 0.97

$$-\frac{\frac{3\,A\,a^2\,\sin\left(2\,e+2\,f\,x\right)}{2}-\frac{B\,a^2\,\cos\left(3\,e+3\,f\,x\right)}{2}+3\,B\,a^2\,\sin\left(2\,e+2\,f\,x\right)+12\,A\,a^2\,\cos\left(e+f\,x\right)+\frac{21\,B\,a^2\,\cos\left(e+f\,x\right)}{2}-9\,A\,a^2\,f}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2,x)
```

```
[Out] -((3*A*a^2*sin(2*e + 2*f*x))/2 - (B*a^2*cos(3*e + 3*f*x))/2 + 3*B*a^2*sin(2
*e + 2*f*x) + 12*A*a^2*cos(e + f*x) + (21*B*a^2*cos(e + f*x))/2 - 9*A*a^2*f
*x - 6*B*a^2*f*x)/(6*f)
```

sympy [A] time = 0.95, size = 199, normalized size = 2.12

$$\begin{cases}\frac{Aa^2x\sin^2\left(e+fx\right)}{2}+\frac{Aa^2x\cos^2\left(e+fx\right)}{2}+Aa^2x-\frac{Aa^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{2Aa^2\cos\left(e+fx\right)}{f}+Ba^2x\sin^2\left(e+fx\right)+Ba^2x\cos \\ x\left(A+B\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right)^2\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e)),x)
```

```
[Out] Piecewise((A*a**2*x*sin(e + f*x)**2/2 + A*a**2*x*cos(e + f*x)**2/2 + A*a**2
*x - A*a**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**2*cos(e + f*x)/f + B*a
**2*x*sin(e + f*x)**2 + B*a**2*x*cos(e + f*x)**2 - B*a**2*sin(e + f*x)**2*c
os(e + f*x)/f - B*a**2*sin(e + f*x)*cos(e + f*x)/f - 2*B*a**2*cos(e + f*x)*
*3/(3*f) - B*a**2*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) +
a)**2, True))
```

3.255 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=171

$$-\frac{2a^2(c-d)^2(Bc-Ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f\sqrt{c^2-d^2}}-\frac{a^2x\left(2Ad(c-2d)-B\left(2c^2-4cd+3d^2\right)\right)}{2d^3}+\frac{a^2(-2Ad+2Bc-3Bc}{2d^2f}$$

```
[Out] -1/2*a^2*(2*A*(c-2*d)*d-B*(2*c^2-4*c*d+3*d^2))*x/d^3+1/2*a^2*(-2*A*d+2*B*c-
3*B*d)*cos(f*x+e)/d^2/f-1/2*B*cos(f*x+e)*(a^2+a^2*sin(f*x+e))/d/f-2*a^2*(c-
d)^2*(-A*d+B*c)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^3/f/(c^2
-d^2)^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2976, 2968, 3023, 2735, 2660, 618, 204}

$$-\frac{2a^2(c-d)^2(Bc-Ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f\sqrt{c^2-d^2}}-\frac{a^2x\left(2Ad(c-2d)-B\left(2c^2-4cd+3d^2\right)\right)}{2d^3}+\frac{a^2(-2Ad+2Bc-3Bc}{2d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```

```
[Out] -(a^2*(2*A*(c - 2*d)*d - B*(2*c^2 - 4*c*d + 3*d^2))*x)/(2*d^3) - (2*a^2*(c
- d)^2*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3*S
qrt[c^2 - d^2]*f) + (a^2*(2*B*c - 2*A*d - 3*B*d)*Cos[e + f*x])/(2*d^2*f) -
(B*Cos[e + f*x]*(a^2 + a^2*Sin[e + f*x]))/(2*d*f)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = -\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2df} + \frac{\int \frac{(a+a\sin(e+fx))(a(Bc+2Ad)}{c+d\sin}}{}$$

$$= -\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2df} + \frac{\int \frac{a^2(Bc+2Ad)+\left(a^2(Bc+2Ad)\right.}{}}{}$$

$$= \frac{a^2(2Bc-2Ad-3Bd)\cos(e+fx)}{2d^2f} - \frac{B\cos(e+fx)\left(a^2+a^2\sin\right.}{2df}$$

$$= -\frac{a^2\left(2A(c-2d)d-B\left(2c^2-4cd+3d^2\right)\right)x}{2d^3} + \frac{a^2(2Bc-2Ad-3}{2d^2}$$

$$= -\frac{a^2\left(2A(c-2d)d-B\left(2c^2-4cd+3d^2\right)\right)x}{2d^3} + \frac{a^2(2Bc-2Ad-3}{2d^2}$$

$$= -\frac{a^2\left(2A(c-2d)d-B\left(2c^2-4cd+3d^2\right)\right)x}{2d^3} + \frac{a^2(2Bc-2Ad-3}{2d^2}$$

$$= -\frac{a^2\left(2A(c-2d)d-B\left(2c^2-4cd+3d^2\right)\right)x}{2d^3} - \frac{2a^2(c-d)^2(Bc-}{d^3}$$

Mathematica [A] time = 0.62, size = 177, normalized size = 1.04

$$\frac{a^2(\sin(e+fx)+1)^2\left(2(e+fx)\left(2Ad(2d-c)+B\left(2c^2-4cd+3d^2\right)\right)-\frac{8(c-d)^2(Bc-Ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}-4d(Ad\right.}{4d^3f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
),x]
```

```
[Out] (a^2*(1 + Sin[e + f*x])^2*(2*(2*A*d*(-c + 2*d) + B*(2*c^2 - 4*c*d + 3*d^2))
*(e + f*x) - (8*(c - d)^2*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[
c^2 - d^2]])/Sqrt[c^2 - d^2] - 4*d*(-(B*c) + A*d + 2*B*d)*Cos[e + f*x] - B*
d^2*Sin[2*(e + f*x)]))/(4*d^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)
```

fricas [A] time = 0.51, size = 452, normalized size = 2.64

$$\left[-\frac{Ba^2d^2\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(2\,Ba^2c^2-2\,(A+2\,B)a^2cd+(4\,A+3\,B)a^2d^2\right)fx+\left(Ba^2c^2-(A+B)a^2cd+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] [-1/2*(B*a^2*d^2*cos(f*x + e)*sin(f*x + e) - (2*B*a^2*c^2 - 2*(A + 2*B)*a^2
*c*d + (4*A + 3*B)*a^2*d^2)*f*x + (B*a^2*c^2 - (A + B)*a^2*c*d + A*a^2*d^2)
*sqrt(-(c - d)/(c + d))*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x
+ e) - c^2 - d^2 - 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*c
os(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x +
e) - c^2 - d^2)) - 2*(B*a^2*c*d - (A + 2*B)*a^2*d^2)*cos(f*x + e))/(d^3*f),
 -1/2*(B*a^2*d^2*cos(f*x + e)*sin(f*x + e) - (2*B*a^2*c^2 - 2*(A + 2*B)*a^2
*c*d + (4*A + 3*B)*a^2*d^2)*f*x - 2*(B*a^2*c^2 - (A + B)*a^2*c*d + A*a^2*d^
2)*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/(c + d))
/((c - d)*cos(f*x + e))) - 2*(B*a^2*c*d - (A + 2*B)*a^2*d^2)*cos(f*x + e))/
(d^3*f)]
```

giac [A] time = 0.19, size = 314, normalized size = 1.84

$$\frac{\frac{\left(2\,Ba^2c^2-2\,Aa^2cd-4\,Ba^2cd+4\,Aa^2d^2+3\,Ba^2d^2\right)\left(fx+e\right)}{d^3}-\frac{4\left(Ba^2c^3-Aa^2c^2d-2\,Ba^2c^2d+2\,Aa^2cd^2+Ba^2cd^2-Aa^2d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\right.}{\sqrt{c^2}}\right.\right.}{\sqrt{c^2-d^2}\,d^3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] 1/2*((2*B*a^2*c^2 - 2*A*a^2*c*d - 4*B*a^2*c*d + 4*A*a^2*d^2 + 3*B*a^2*d^2)*
(f*x + e)/d^3 - 4*(B*a^2*c^3 - A*a^2*c^2*d - 2*B*a^2*c^2*d + 2*A*a^2*c*d^2
+ B*a^2*c*d^2 - A*a^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arcta
n((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*d^3) + 2*
(B*a^2*d*tan(1/2*f*x + 1/2*e)^3 + 2*B*a^2*c*tan(1/2*f*x + 1/2*e)^2 - 2*A*a^
2*d*tan(1/2*f*x + 1/2*e)^2 - 4*B*a^2*d*tan(1/2*f*x + 1/2*e)^2 - B*a^2*d*tan
(1/2*f*x + 1/2*e) + 2*B*a^2*c - 2*A*a^2*d - 4*B*a^2*d)/((tan(1/2*f*x + 1/2*
e)^2 + 1)^2*d^2))/f
```

maple [B] time = 0.43, size = 713, normalized size = 4.17

$$\frac{2a^2\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)Ac^2}{f\,d^2\sqrt{c^2-d^2}}-\frac{4a^2\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)Ac}{f\,d\sqrt{c^2-d^2}}+\frac{2a^2\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)A}{f\sqrt{c^2-d^2}}-\frac{2a^2\arctan\left(\right.}{f\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/f*a^2/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^
2)^(1/2))*A*c^2-4/f*a^2/d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e
)+2*d)/(c^2-d^2)^(1/2))*A*c+2/f*a^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2
*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A-2/f*a^2/d^3/(c^2-d^2)^(1/2)*arctan(1/2*
(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^3+4/f*a^2/d^2/(c^2-d^2)^(
1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^2-2/f*a^2
/d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))
*B*c+1/f*a^2/d/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*x+1/2*e)^3-2/f*a^2/d/
(1+tan(1/2*f*x+1/2*e)^2)^2*A*tan(1/2*f*x+1/2*e)^2+2/f*a^2/d^2/(1+tan(1/2*f*
x+1/2*e)^2)^2*B*tan(1/2*f*x+1/2*e)^2*c-4/f*a^2/d/(1+tan(1/2*f*x+1/2*e)^2)^2
*B*tan(1/2*f*x+1/2*e)^2-1/f*a^2/d/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*x+
1/2*e)-2/f*a^2/d/(1+tan(1/2*f*x+1/2*e)^2)^2*A+2/f*a^2/d^2/(1+tan(1/2*f*x+1/
2*e)^2)^2*B*c-4/f*a^2/d/(1+tan(1/2*f*x+1/2*e)^2)^2*B-2/f*a^2/d^2*arctan(tan
(1/2*f*x+1/2*e))*A*c+4/f*a^2/d*arctan(tan(1/2*f*x+1/2*e))*A+2/f*a^2/d^3*arc
tan(tan(1/2*f*x+1/2*e))*B*c^2-4/f*a^2/d^2*arctan(tan(1/2*f*x+1/2*e))*B*c+3/
f*a^2/d*arctan(tan(1/2*f*x+1/2*e))*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 20.07, size = 7371, normalized size = 43.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c + d*sin(e + f*x)),x)
```

```
[Out] (atan(((((8*(16*A^2*a^4*c^2*d^6 - 16*A^2*a^4*c^3*d^5 + 4*A^2*a^4*c^4*d^4 +
9*B^2*a^4*c^2*d^6 - 24*B^2*a^4*c^3*d^5 + 28*B^2*a^4*c^4*d^4 - 16*B^2*a^4*c^
5*d^3 + 4*B^2*a^4*c^6*d^2 + 24*A*B*a^4*c^2*d^6 - 44*A*B*a^4*c^3*d^5 + 32*A*
B*a^4*c^4*d^4 - 8*A*B*a^4*c^5*d^3))/d^5 + ((((32*c^2*d^3 + (8*tan(e/2 + (f*
x)/2)*(12*c*d^10 - 8*c^3*d^8))/d^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*B)*1i
)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 - (8*(8*A*a^2*c*d^8 + 6*B*a^2*c*d^
8 - 8*A*a^2*c^2*d^7 - 8*B*a^2*c^2*d^7 + 2*B*a^2*c^3*d^6))/d^5 + (8*tan(e/2
+ (f*x)/2)*(8*A*a^2*c*d^9 - 16*A*a^2*c^2*d^8 + 8*A*a^2*c^3*d^7 - 8*B*a^2*c^
2*d^8 + 16*B*a^2*c^3*d^7 - 8*B*a^2*c^4*d^6))/d^6)*(B*a^2*c^2*1i + (a^2*d^2*
(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 + (8*tan(e/2 + (f*x)
/2)*(32*A^2*a^4*c^4*d^5 - 32*A^2*a^4*c^3*d^6 - 16*A^2*a^4*c^2*d^7 - 8*A^2*a
^4*c^5*d^4 - 48*B^2*a^4*c^2*d^7 + 43*B^2*a^4*c^3*d^6 + 8*B^2*a^4*c^4*d^5 -
44*B^2*a^4*c^5*d^4 + 32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 + 28*A^2*a^4*c*
d^8 + 18*B^2*a^4*c*d^8 - 80*A*B*a^4*c^2*d^7 + 8*A*B*a^4*c^3*d^6 + 76*A*B*a^
4*c^4*d^5 - 64*A*B*a^4*c^5*d^4 + 16*A*B*a^4*c^6*d^3 + 48*A*B*a^4*c*d^8))/d^
6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/
2)*1i)/d^3 + (((8*(16*A^2*a^4*c^2*d^6 - 16*A^2*a^4*c^3*d^5 + 4*A^2*a^4*c^4*
d^4 + 9*B^2*a^4*c^2*d^6 - 24*B^2*a^4*c^3*d^5 + 28*B^2*a^4*c^4*d^4 - 16*B^2*
a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2 + 24*A*B*a^4*c^2*d^6 - 44*A*B*a^4*c^3*d^5 +
 32*A*B*a^4*c^4*d^4 - 8*A*B*a^4*c^5*d^3))/d^5 + (((8*(8*A*a^2*c*d^8 + 6*B*a
^2*c*d^8 - 8*A*a^2*c^2*d^7 - 8*B*a^2*c^2*d^7 + 2*B*a^2*c^3*d^6))/d^5 + ((32
*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*c^3*d^8))/d^6)*(B*a^2*c^2*1
i + (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 - (8*ta
n(e/2 + (f*x)/2)*(8*A*a^2*c*d^9 - 16*A*a^2*c^2*d^8 + 8*A*a^2*c^3*d^7 - 8*B*
a^2*c^2*d^8 + 16*B*a^2*c^3*d^7 - 8*B*a^2*c^4*d^6))/d^6)*(B*a^2*c^2*1i + (a^
2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 + (8*tan(e/2 +
 (f*x)/2)*(32*A^2*a^4*c^4*d^5 - 32*A^2*a^4*c^3*d^6 - 16*A^2*a^4*c^2*d^7 - 8
*A^2*a^4*c^5*d^4 - 48*B^2*a^4*c^2*d^7 + 43*B^2*a^4*c^3*d^6 + 8*B^2*a^4*c^4*
d^5 - 44*B^2*a^4*c^5*d^4 + 32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 + 28*A^2*
a^4*c*d^8 + 18*B^2*a^4*c*d^8 - 80*A*B*a^4*c^2*d^7 + 8*A*B*a^4*c^3*d^6 + 76*
A*B*a^4*c^4*d^5 - 64*A*B*a^4*c^5*d^4 + 16*A*B*a^4*c^6*d^3 + 48*A*B*a^4*c*d^
8))/d^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c
)*1i)/2)*1i)/d^3)/((16*(2*B^3*a^6*c^7 + 20*A^3*a^6*c^2*d^5 - 16*A^3*a^6*c^3
*d^4 + 4*A^3*a^6*c^4*d^3 + 3*B^3*a^6*c^3*d^4 - 10*B^3*a^6*c^4*d^3 + 13*B^3*
a^6*c^5*d^2 - 8*A^3*a^6*c*d^6 - 8*B^3*a^6*c^6*d - 6*A^2*B*a^6*c*d^6 + 3*A*B
^2*a^6*c^2*d^5 - 6*A*B^2*a^6*c^3*d^4 + 3*A*B^2*a^6*c^4*d^3 + 24*A^2*B*a^6*c
^2*d^5 - 36*A^2*B*a^6*c^3*d^4 + 24*A^2*B*a^6*c^4*d^3 - 6*A^2*B*a^6*c^5*d^2)
)/d^5 - (((8*(16*A^2*a^4*c^2*d^6 - 16*A^2*a^4*c^3*d^5 + 4*A^2*a^4*c^4*d^4 +
 9*B^2*a^4*c^2*d^6 - 24*B^2*a^4*c^3*d^5 + 28*B^2*a^4*c^4*d^4 - 16*B^2*a^4*c
^5*d^3 + 4*B^2*a^4*c^6*d^2 + 24*A*B*a^4*c^2*d^6 - 44*A*B*a^4*c^3*d^5 + 32*A
*B*a^4*c^4*d^4 - 8*A*B*a^4*c^5*d^3))/d^5 + ((((32*c^2*d^3 + (8*tan(e/2 + (f
```

```
*x)/2)*(12*c*d^10 - 8*c^3*d^8))/d^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*B)*1
i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 - (8*(8*A*a^2*c*d^8 + 6*B*a^2*c*d
^8 - 8*A*a^2*c^2*d^7 - 8*B*a^2*c^2*d^7 + 2*B*a^2*c^3*d^6))/d^5 + (8*tan(e/2
 + (f*x)/2)*(8*A*a^2*c*d^9 - 16*A*a^2*c^2*d^8 + 8*A*a^2*c^3*d^7 - 8*B*a^2*c
^2*d^8 + 16*B*a^2*c^3*d^7 - 8*B*a^2*c^4*d^6))/d^6)*(B*a^2*c^2*1i + (a^2*d^2
*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 + (8*tan(e/2 + (f*x
)/2)*(32*A^2*a^4*c^4*d^5 - 32*A^2*a^4*c^3*d^6 - 16*A^2*a^4*c^2*d^7 - 8*A^2*
a^4*c^5*d^4 - 48*B^2*a^4*c^2*d^7 + 43*B^2*a^4*c^3*d^6 + 8*B^2*a^4*c^4*d^5 -
 44*B^2*a^4*c^5*d^4 + 32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 + 28*A^2*a^4*c
*d^8 + 18*B^2*a^4*c*d^8 - 80*A*B*a^4*c^2*d^7 + 8*A*B*a^4*c^3*d^6 + 76*A*B*a
^4*c^4*d^5 - 64*A*B*a^4*c^5*d^4 + 16*A*B*a^4*c^6*d^3 + 48*A*B*a^4*c*d^8))/d
^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)
/2))/d^3 + (((8*(16*A^2*a^4*c^2*d^6 - 16*A^2*a^4*c^3*d^5 + 4*A^2*a^4*c^4*d^
4 + 9*B^2*a^4*c^2*d^6 - 24*B^2*a^4*c^3*d^5 + 28*B^2*a^4*c^4*d^4 - 16*B^2*a^
4*c^5*d^3 + 4*B^2*a^4*c^6*d^2 + 24*A*B*a^4*c^2*d^6 - 44*A*B*a^4*c^3*d^5 + 3
2*A*B*a^4*c^4*d^4 - 8*A*B*a^4*c^5*d^3))/d^5 + (((8*(8*A*a^2*c*d^8 + 6*B*a^2
*c*d^8 - 8*A*a^2*c^2*d^7 - 8*B*a^2*c^2*d^7 + 2*B*a^2*c^3*d^6))/d^5 + ((32*c
^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*c^3*d^8))/d^6)*(B*a^2*c^2*1i
+ (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 - (8*tan(
e/2 + (f*x)/2)*(8*A*a^2*c*d^9 - 16*A*a^2*c^2*d^8 + 8*A*a^2*c^3*d^7 - 8*B*a^
2*c^2*d^8 + 16*B*a^2*c^3*d^7 - 8*B*a^2*c^4*d^6))/d^6)*(B*a^2*c^2*1i + (a^2*
d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 + (8*tan(e/2 + (
f*x)/2)*(32*A^2*a^4*c^4*d^5 - 32*A^2*a^4*c^3*d^6 - 16*A^2*a^4*c^2*d^7 - 8*A
^2*a^4*c^5*d^4 - 48*B^2*a^4*c^2*d^7 + 43*B^2*a^4*c^3*d^6 + 8*B^2*a^4*c^4*d^
5 - 44*B^2*a^4*c^5*d^4 + 32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 + 28*A^2*a^
4*c*d^8 + 18*B^2*a^4*c*d^8 - 80*A*B*a^4*c^2*d^7 + 8*A*B*a^4*c^3*d^6 + 76*A*
B*a^4*c^4*d^5 - 64*A*B*a^4*c^5*d^4 + 16*A*B*a^4*c^6*d^3 + 48*A*B*a^4*c*d^8)
)/d^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*
1i)/2))/d^3 - (16*tan(e/2 + (f*x)/2)*(104*A^3*a^6*c^3*d^5 - 96*A^3*a^6*c^2*
d^6 - 8*B^3*a^6*c^8 - 48*A^3*a^6*c^4*d^4 + 8*A^3*a^6*c^5*d^3 - 18*B^3*a^6*c
^2*d^6 + 84*B^3*a^6*c^3*d^5 - 170*B^3*a^6*c^4*d^4 + 192*B^3*a^6*c^5*d^3 - 1
28*B^3*a^6*c^6*d^2 + 32*A^3*a^6*c*d^7 + 48*B^3*a^6*c^7*d + 18*A*B^2*a^6*c*d
^7 + 24*A*B^2*a^6*c^7*d + 48*A^2*B*a^6*c*d^7 - 132*A*B^2*a^6*c^2*d^6 + 354*
A*B^2*a^6*c^3*d^5 - 480*A*B^2*a^6*c^4*d^4 + 360*A*B^2*a^6*c^5*d^3 - 144*A*B
^2*a^6*c^6*d^2 - 216*A^2*B*a^6*c^2*d^6 + 384*A^2*B*a^6*c^3*d^5 - 336*A^2*B*
a^6*c^4*d^4 + 144*A^2*B*a^6*c^5*d^3 - 24*A^2*B*a^6*c^6*d^2))/d^6))*(B*a^2*c
^2*1i + (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2)*2i)/(d^3
*f) - ((2*(A*a^2*d - B*a^2*c + 2*B*a^2*d))/d^2 + (2*tan(e/2 + (f*x)/2)^2*(A
*a^2*d - B*a^2*c + 2*B*a^2*d))/d^2 - (B*a^2*tan(e/2 + (f*x)/2)^3)/d + (B*a^
2*tan(e/2 + (f*x)/2))/d)/(f*(2*tan(e/2 + (f*x)/2)^2 + tan(e/2 + (f*x)/2)^4
+ 1)) + (a^2*atan(((a^2*(A*d - B*c)*(-(c + d)*(c - d)^3)^(1/2)*((8*(16*A^2*
a^4*c^2*d^6 - 16*A^2*a^4*c^3*d^5 + 4*A^2*a^4*c^4*d^4 + 9*B^2*a^4*c^2*d^6 -
24*B^2*a^4*c^3*d^5 + 28*B^2*a^4*c^4*d^4 - 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^
6*d^2 + 24*A*B*a^4*c^2*d^6 - 44*A*B*a^4*c^3*d^5 + 32*A*B*a^4*c^4*d^4 - 8*A*
B*a^4*c^5*d^3))/d^5 + (8*tan(e/2 + (f*x)/2)*(32*A^2*a^4*c^4*d^5 - 32*A^2*a^
```

4*c^3*d^6 - 16*A^2*a^4*c^2*d^7 - 8*A^2*a^4*c^5*d^4 - 48*B^2*a^4*c^2*d^7 + 4
3*B^2*a^4*c^3*d^6 + 8*B^2*a^4*c^4*d^5 - 44*B^2*a^4*c^5*d^4 + 32*B^2*a^4*c^6
*d^3 - 8*B^2*a^4*c^7*d^2 + 28*A^2*a^4*c*d^8 + 18*B^2*a^4*c*d^8 - 80*A*B*a^4
*c^2*d^7 + 8*A*B*a^4*c^3*d^6 + 76*A*B*a^4*c^4*d^5 - 64*A*B*a^4*c^5*d^4 + 16
*A*B*a^4*c^6*d^3 + 48*A*B*a^4*c*d^8))/d^6 + (a^2*(A*d - B*c)*(-(c + d)*(c -
d)^3)^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*A*a^2*c*d^9 - 16*A*a^2*c^2*d^8 + 8*A
*a^2*c^3*d^7 - 8*B*a^2*c^2*d^8 + 16*B*a^2*c^3*d^7 - 8*B*a^2*c^4*d^6))/d^6 -
(8*(8*A*a^2*c*d^8 + 6*B*a^2*c*d^8 - 8*A*a^2*c^2*d^7 - 8*B*a^2*c^2*d^7 + 2*
B*a^2*c^3*d^6))/d^5 + (a^2*(32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 -
8*c^3*d^8))/d^6)*(A*d - B*c)*(-(c + d)*(c - d)^3)^(1/2))/(c*d^3 + d^4)))/(
c*d^3 + d^4))*1i)/(c*d^3 + d^4) + (a^2*(A*d - B*c)*(-(c + d)*(c - d)^3)^(1/
2)*((8*(16*A^2*a^4*c^2*d^6 - 16*A^2*a^4*c^3*d^5 + 4*A^2*a^4*c^4*d^4 + 9*B^2
*a^4*c^2*d^6 - 24*B^2*a^4*c^3*d^5 + 28*B^2*a^4*c^4*d^4 - 16*B^2*a^4*c^5*d^3
+ 4*B^2*a^4*c^6*d^2 + 24*A*B*a^4*c^2*d^6 - 44*A*B*a^4*c^3*d^5 + 32*A*B*a^4
*c^4*d^4 - 8*A*B*a^4*c^5*d^3))/d^5 + (8*tan(e/2 + (f*x)/2)*(32*A^2*a^4*c^4*
d^5 - 32*A^2*a^4*c^3*d^6 - 16*A^2*a^4*c^2*d^7 - 8*A^2*a^4*c^5*d^4 - 48*B^2*
a^4*c^2*d^7 + 43*B^2*a^4*c^3*d^6 + 8*B^2*a^4*c^4*d^5 - 44*B^2*a^4*c^5*d^4 +
32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 + 28*A^2*a^4*c*d^8 + 18*B^2*a^4*c*d
^8 - 80*A*B*a^4*c^2*d^7 + 8*A*B*a^4*c^3*d^6 + 76*A*B*a^4*c^4*d^5 - 64*A*B*a
^4*c^5*d^4 + 16*A*B*a^4*c^6*d^3 + 48*A*B*a^4*c*d^8))/d^6 + (a^2*(A*d - B*c)
(-(c + d)(c - d)^3)^(1/2)*((8*(8*A*a^2*c*d^8 + 6*B*a^2*c*d^8 - 8*A*a^2*c^
2*d^7 - 8*B*a^2*c^2*d^7 + 2*B*a^2*c^3*d^6))/d^5 - (8*tan(e/2 + (f*x)/2)*(8*
A*a^2*c*d^9 - 16*A*a^2*c^2*d^8 + 8*A*a^2*c^3*d^7 - 8*B*a^2*c^2*d^8 + 16*B*a
^2*c^3*d^7 - 8*B*a^2*c^4*d^6))/d^6 + (a^2*(32*c^2*d^3 + (8*tan(e/2 + (f*x)/
2)*(12*c*d^10 - 8*c^3*d^8))/d^6)*(A*d - B*c)*(-(c + d)*(c - d)^3)^(1/2))/(c
*d^3 + d^4)))/(c*d^3 + d^4))*1i)/(c*d^3 + d^4))/((16*(2*B^3*a^6*c^7 + 20*A^
3*a^6*c^2*d^5 - 16*A^3*a^6*c^3*d^4 + 4*A^3*a^6*c^4*d^3 + 3*B^3*a^6*c^3*d^4
- 10*B^3*a^6*c^4*d^3 + 13*B^3*a^6*c^5*d^2 - 8*A^3*a^6*c*d^6 - 8*B^3*a^6*c^6
*d - 6*A^2*B*a^6*c*d^6 + 3*A*B^2*a^6*c^2*d^5 - 6*A*B^2*a^6*c^3*d^4 + 3*A*B^
2*a^6*c^4*d^3 + 24*A^2*B*a^6*c^2*d^5 - 36*A^2*B*a^6*c^3*d^4 + 24*A^2*B*a^6*
c^4*d^3 - 6*A^2*B*a^6*c^5*d^2))/d^5 - (16*tan(e/2 + (f*x)/2)*(104*A^3*a^6*c
^3*d^5 - 96*A^3*a^6*c^2*d^6 - 8*B^3*a^6*c^8 - 48*A^3*a^6*c^4*d^4 + 8*A^3*a^
6*c^5*d^3 - 18*B^3*a^6*c^2*d^6 + 84*B^3*a^6*c^3*d^5 - 170*B^3*a^6*c^4*d^4 +
192*B^3*a^6*c^5*d^3 - 128*B^3*a^6*c^6*d^2 + 32*A^3*a^6*c*d^7 + 48*B^3*a^6*
c^7*d + 18*A*B^2*a^6*c*d^7 + 24*A*B^2*a^6*c^7*d + 48*A^2*B*a^6*c*d^7 - 132*
A*B^2*a^6*c^2*d^6 + 354*A*B^2*a^6*c^3*d^5 - 480*A*B^2*a^6*c^4*d^4 + 360*A*B
^2*a^6*c^5*d^3 - 144*A*B^2*a^6*c^6*d^2 - 216*A^2*B*a^6*c^2*d^6 + 384*A^2*B*
a^6*c^3*d^5 - 336*A^2*B*a^6*c^4*d^4 + 144*A^2*B*a^6*c^5*d^3 - 24*A^2*B*a^6*
c^6*d^2))/d^6 - (a^2*(A*d - B*c)*(-(c + d)*(c - d)^3)^(1/2)*((8*(16*A^2*a^4
*c^2*d^6 - 16*A^2*a^4*c^3*d^5 + 4*A^2*a^4*c^4*d^4 + 9*B^2*a^4*c^2*d^6 - 24*
B^2*a^4*c^3*d^5 + 28*B^2*a^4*c^4*d^4 - 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d
^2 + 24*A*B*a^4*c^2*d^6 - 44*A*B*a^4*c^3*d^5 + 32*A*B*a^4*c^4*d^4 - 8*A*B*a
^4*c^5*d^3))/d^5 + (8*tan(e/2 + (f*x)/2)*(32*A^2*a^4*c^4*d^5 - 32*A^2*a^4*c
^3*d^6 - 16*A^2*a^4*c^2*d^7 - 8*A^2*a^4*c^5*d^4 - 48*B^2*a^4*c^2*d^7 + 43*B
^2*a^4*c^3*d^6 + 8*B^2*a^4*c^4*d^5 - 44*B^2*a^4*c^5*d^4 + 32*B^2*a^4*c^6*d^

```
3 - 8*B^2*a^4*c^7*d^2 + 28*A^2*a^4*c*d^8 + 18*B^2*a^4*c*d^8 - 80*A*B*a^4*c^
2*d^7 + 8*A*B*a^4*c^3*d^6 + 76*A*B*a^4*c^4*d^5 - 64*A*B*a^4*c^5*d^4 + 16*A*
B*a^4*c^6*d^3 + 48*A*B*a^4*c*d^8))/d^6 + (a^2*(A*d - B*c)*(-(c + d)*(c - d)
^3)^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*A*a^2*c*d^9 - 16*A*a^2*c^2*d^8 + 8*A*a^
2*c^3*d^7 - 8*B*a^2*c^2*d^8 + 16*B*a^2*c^3*d^7 - 8*B*a^2*c^4*d^6))/d^6 - (8
*(8*A*a^2*c*d^8 + 6*B*a^2*c*d^8 - 8*A*a^2*c^2*d^7 - 8*B*a^2*c^2*d^7 + 2*B*a
^2*c^3*d^6))/d^5 + (a^2*(32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*
c^3*d^8))/d^6)*(A*d - B*c)*(-(c + d)*(c - d)^3)^(1/2))/(c*d^3 + d^4)))/(c*d
^3 + d^4)))/(c*d^3 + d^4) + (a^2*(A*d - B*c)*(-(c + d)*(c - d)^3)^(1/2)*((8
*(16*A^2*a^4*c^2*d^6 - 16*A^2*a^4*c^3*d^5 + 4*A^2*a^4*c^4*d^4 + 9*B^2*a^4*c
^2*d^6 - 24*B^2*a^4*c^3*d^5 + 28*B^2*a^4*c^4*d^4 - 16*B^2*a^4*c^5*d^3 + 4*B
^2*a^4*c^6*d^2 + 24*A*B*a^4*c^2*d^6 - 44*A*B*a^4*c^3*d^5 + 32*A*B*a^4*c^4*d
^4 - 8*A*B*a^4*c^5*d^3))/d^5 + (8*tan(e/2 + (f*x)/2)*(32*A^2*a^4*c^4*d^5 -
32*A^2*a^4*c^3*d^6 - 16*A^2*a^4*c^2*d^7 - 8*A^2*a^4*c^5*d^4 - 48*B^2*a^4*c^
2*d^7 + 43*B^2*a^4*c^3*d^6 + 8*B^2*a^4*c^4*d^5 - 44*B^2*a^4*c^5*d^4 + 32*B^
2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 + 28*A^2*a^4*c*d^8 + 18*B^2*a^4*c*d^8 - 8
0*A*B*a^4*c^2*d^7 + 8*A*B*a^4*c^3*d^6 + 76*A*B*a^4*c^4*d^5 - 64*A*B*a^4*c^5
*d^4 + 16*A*B*a^4*c^6*d^3 + 48*A*B*a^4*c*d^8))/d^6 + (a^2*(A*d - B*c)*(-(c
+ d)*(c - d)^3)^(1/2)*((8*(8*A*a^2*c*d^8 + 6*B*a^2*c*d^8 - 8*A*a^2*c^2*d^7
- 8*B*a^2*c^2*d^7 + 2*B*a^2*c^3*d^6))/d^5 - (8*tan(e/2 + (f*x)/2)*(8*A*a^2*
c*d^9 - 16*A*a^2*c^2*d^8 + 8*A*a^2*c^3*d^7 - 8*B*a^2*c^2*d^8 + 16*B*a^2*c^3
*d^7 - 8*B*a^2*c^4*d^6))/d^6 + (a^2*(32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12
*c*d^10 - 8*c^3*d^8))/d^6)*(A*d - B*c)*(-(c + d)*(c - d)^3)^(1/2))/(c*d^3 +
 d^4)))/(c*d^3 + d^4)))/(c*d^3 + d^4)))*(A*d - B*c)*(-(c + d)*(c - d)^3)^(1
/2)*2i)/(f*(c*d^3 + d^4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.256 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=198

$$-\frac{2a^2(c-d)\left(Ad(c+2d)-B\left(2c^2+2cd-d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f(c+d)\sqrt{c^2-d^2}}-\frac{a^2x(-Ad+2Bc-2Bd)}{d^3}+\frac{a^2(Ad-B(2c+ \cdots}{d^2f(c \cdots}$$

```
[Out] -a^2*(-A*d+2*B*c-2*B*d)*x/d^3+a^2*(A*d-B*(2*c+d))*cos(f*x+e)/d^2/(c+d)/f+(-
A*d+B*c)*cos(f*x+e)*(a^2+a^2*sin(f*x+e))/d/(c+d)/f/(c+d*sin(f*x+e))-2*a^2*(
c-d)*(A*d*(c+2*d)-B*(2*c^2+2*c*d-d^2))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2
-d^2)^(1/2))/d^3/(c+d)/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2975, 2968, 3023, 2735, 2660, 618, 204}

$$-\frac{2a^2(c-d)\left(Ad(c+2d)-B\left(2c^2+2cd-d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f(c+d)\sqrt{c^2-d^2}}+\frac{a^2(Ad-B(2c+d))\cos(e+fx)}{d^2f(c+d)}-\frac{a^2x(-Ad \cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -((a^2*(2*B*c - A*d - 2*B*d)*x)/d^3) - (2*a^2*(c - d)*(A*d*(c + 2*d) - B*(2
*c^2 + 2*c*d - d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3
*(c + d)*Sqrt[c^2 - d^2]*f) + (a^2*(A*d - B*(2*c + d))*Cos[e + f*x])/(d^2*(
c + d)*f) + ((B*c - A*d)*Cos[e + f*x]*(a^2 + a^2*Sin[e + f*x]))/(d*(c + d)*
f*(c + d*Sin[e + f*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))} + \int \frac{(a+a\sin(e+fx))(-a(\cdots}{\cdots}$$

$$= \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))} + \int \frac{-a^2(B(c-d)-2Ad)+(\cdots}{\cdots}$$

$$= \frac{a^2(Ad-B(2c+d))\cos(e+fx)}{d^2(c+d)f} + \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a\cdots\right.}{d(c+d)f(c+d\sin(e\cdots}$$

$$= -\frac{a^2(2Bc-Ad-2Bd)x}{d^3} + \frac{a^2(Ad-B(2c+d))\cos(e+fx)}{d^2(c+d)f} + \frac{(Bc\cdots}{\cdots}$$

$$= -\frac{a^2(2Bc-Ad-2Bd)x}{d^3} + \frac{a^2(Ad-B(2c+d))\cos(e+fx)}{d^2(c+d)f} + \frac{(Bc\cdots}{\cdots}$$

$$= -\frac{a^2(2Bc-Ad-2Bd)x}{d^3} + \frac{a^2(Ad-B(2c+d))\cos(e+fx)}{d^2(c+d)f} + \frac{(Bc\cdots}{\cdots}$$

$$= -\frac{a^2(2Bc-Ad-2Bd)x}{d^3} - \frac{2a^2(c-d)\left(Ad(c+2d)-B\left(2c^2+2cd\cdots\right.\right.}{d^3(c+d)\sqrt{c^2-\cdots}}$$

<u>Mathematica</u> [A] time = 1.00, size = 192, normalized size = 0.97

$$\frac{a^2(\sin(e+fx)+1)^2\left(\frac{2(c-d)\left(B\left(2c^2+2cd-d^2\right)-Ad(c+2d)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}} + (e+fx)(Ad-2Bc+2Bd) - \frac{d(d-c)(Ad-Bc)\cos\cdots}{(c+d)(c+d\sin(e+\cdots}\right.}{d^3f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^2,x]
```

```
[Out] (a^2*(1 + Sin[e + f*x])^2*((-2*B*c + A*d + 2*B*d)*(e + f*x) + (2*(c - d)*(-
(A*d*(c + 2*d)) + B*(2*c^2 + 2*c*d - d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/
Sqrt[c^2 - d^2]])/((c + d)*Sqrt[c^2 - d^2]) - B*d*Cos[e + f*x] - (d*(-c + d
```

```
)*(-(B*c) + A*d)*Cos[e + f*x])/((c + d)*(c + d*Sin[e + f*x]))))/(d^3*f*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^4)
```

fricas [A] time = 0.57, size = 731, normalized size = 3.69

$$\left[-\frac{2\left(2\,Ba^2c^3 - Aa^2c^2d - (A+2\,B)a^2cd^2\right)fx + \left(2\,Ba^2c^3 - (A-2\,B)a^2c^2d - (2\,A+B)a^2cd^2 + \left(2\,Ba^2c^2d - (A-2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] [-1/2*(2*(2*B*a^2*c^3 - A*a^2*c^2*d - (A + 2*B)*a^2*c*d^2)*f*x + (2*B*a^2*c
^3 - (A - 2*B)*a^2*c^2*d - (2*A + B)*a^2*c*d^2 + (2*B*a^2*c^2*d - (A - 2*B)
*a^2*c*d^2 - (2*A + B)*a^2*d^3)*sin(f*x + e))*sqrt(-(c - d)/(c + d))*log(((
2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*((c^2 + c*
d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c +
 d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(2*B*a^2*c
^2*d - A*a^2*c*d^2 + A*a^2*d^3)*cos(f*x + e) + 2*((2*B*a^2*c^2*d - A*a^2*c*
d^2 - (A + 2*B)*a^2*d^3)*f*x + (B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x + e))*sin(
f*x + e))/((c*d^4 + d^5)*f*sin(f*x + e) + (c^2*d^3 + c*d^4)*f), -((2*B*a^2*
c^3 - A*a^2*c^2*d - (A + 2*B)*a^2*c*d^2)*f*x + (2*B*a^2*c^3 - (A - 2*B)*a^2
*c^2*d - (2*A + B)*a^2*c*d^2 + (2*B*a^2*c^2*d - (A - 2*B)*a^2*c*d^2 - (2*A
+ B)*a^2*d^3)*sin(f*x + e))*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) +
 d)*sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e))) + (2*B*a^2*c^2*d - A*a^2*
c*d^2 + A*a^2*d^3)*cos(f*x + e) + ((2*B*a^2*c^2*d - A*a^2*c*d^2 - (A + 2*B)
*a^2*d^3)*f*x + (B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x + e))*sin(f*x + e))/((c*d
^4 + d^5)*f*sin(f*x + e) + (c^2*d^3 + c*d^4)*f)]
```

giac [B] time = 0.20, size = 498, normalized size = 2.52

$$\frac{\frac{2\left(2\,Ba^2c^3 - Aa^2c^2d - Aa^2cd^2 - 3\,Ba^2cd^2 + 2\,Aa^2d^3 + Ba^2d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi} + \frac{1}{2}\right\rfloor \mathrm{sgn}(c) + \arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) + d}{\sqrt{c^2-d^2}}\right)\right)}{\left(cd^3+d^4\right)\sqrt{c^2-d^2}} - \frac{2\left(Ba^2c^2d\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^3 - Aa^2cd^2\tan\left(\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] (2*(2*B*a^2*c^3 - A*a^2*c^2*d - A*a^2*c*d^2 - 3*B*a^2*c*d^2 + 2*A*a^2*d^3 +
 B*a^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*
x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c*d^3 + d^4)*sqrt(c^2 - d^2)) - 2*(B*a^
2*c^2*d*tan(1/2*f*x + 1/2*e)^3 - A*a^2*c*d^2*tan(1/2*f*x + 1/2*e)^3 - B*a^2
*c*d^2*tan(1/2*f*x + 1/2*e)^3 + A*a^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 2*B*a^2*
c^3*tan(1/2*f*x + 1/2*e)^2 - A*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^2 + A*a^2*c*d
^2*tan(1/2*f*x + 1/2*e)^2 + 3*B*a^2*c^2*d*tan(1/2*f*x + 1/2*e) - A*a^2*c*d^
2*tan(1/2*f*x + 1/2*e) + B*a^2*c*d^2*tan(1/2*f*x + 1/2*e) + A*a^2*d^3*tan(1
/2*f*x + 1/2*e) + 2*B*a^2*c^3 - A*a^2*c^2*d + A*a^2*c*d^2)/((c*tan(1/2*f*x
+ 1/2*e)^4 + 2*d*tan(1/2*f*x + 1/2*e)^3 + 2*c*tan(1/2*f*x + 1/2*e)^2 + 2*d*
tan(1/2*f*x + 1/2*e) + c)*(c^2*d^2 + c*d^3)) - (2*B*a^2*c - A*a^2*d - 2*B*a
^2*d)*(f*x + e)/d^3)/f
```

maple [B] time = 0.51, size = 848, normalized size = 4.28

$$\frac{2a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)A}{f\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d+c\right)(c+d)}-\frac{2a^2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)A}{f\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d+c\right)(c+d)c}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*tan(1/2*f*x
+1/2*e)*A-2*a^2/f*d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)
/c*tan(1/2*f*x+1/2*e)*A-2*a^2/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2
*e)*d+c)/(c+d)*c*tan(1/2*f*x+1/2*e)*B+2*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan
(1/2*f*x+1/2*e)*d+c)/(c+d)*tan(1/2*f*x+1/2*e)*B+2*a^2/f/d/(tan(1/2*f*x+1/2*
e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*A*c-2*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+
2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*A-2*a^2/f/d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan
(1/2*f*x+1/2*e)*d+c)/(c+d)*B*c^2+2*a^2/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/
2*f*x+1/2*e)*d+c)/(c+d)*B*c-2*a^2/f/d^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2
*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c^2-2*a^2/f/d/(c+d)/(c^2-d^2)
^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c+4*a^2/f
/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1
/2))*A+4*a^2/f/d^3/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)
+2*d)/(c^2-d^2)^(1/2))*B*c^3-6*a^2/f/d/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*
c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c+2*a^2/f/(c+d)/(c^2-d^2)^(1/2
)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B-2*a^2/f/d^2*B/
(1+tan(1/2*f*x+1/2*e)^2)+2*a^2/f/d^2*A*arctan(tan(1/2*f*x+1/2*e))-4*a^2/f/d
^3*B*arctan(tan(1/2*f*x+1/2*e))*c+4*a^2/f/d^2*B*arctan(tan(1/2*f*x+1/2*e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 21.58, size = 8706, normalized size = 43.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c + d*sin(e + f*x))^2,x)
```

```
[Out] - ((2*(A*a^2*d^2 + 2*B*a^2*c^2 - A*a^2*c*d))/(d^2*(c + d)) + (2*tan(e/2 + (
f*x)/2)^2*(A*a^2*d^2 + 2*B*a^2*c^2 - A*a^2*c*d))/(d^2*(c + d)) + (2*tan(e/2
 + (f*x)/2)*(A*a^2*d^2 + 3*B*a^2*c^2 - A*a^2*c*d + B*a^2*c*d))/(c*d*(c + d)
) + (2*tan(e/2 + (f*x)/2)^3*(A*a^2*d^2 + B*a^2*c^2 - A*a^2*c*d - B*a^2*c*d)
)/(c*d*(c + d)))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + 2*c*tan(e/2 + (f*x)/2)^2
+ c*tan(e/2 + (f*x)/2)^4 + 2*d*tan(e/2 + (f*x)/2)^3)) - (atan((((B*a^2*c*2i
 - a^2*d*(A + 2*B)*1i)*((32*(A^2*a^4*c^2*d^6 + 2*A^2*a^4*c^3*d^5 + A^2*a^4*
c^4*d^4 + 4*B^2*a^4*c^2*d^6 - 8*B^2*a^4*c^4*d^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B
*a^4*c^2*d^6 + 4*A*B*a^4*c^3*d^5 - 4*A*B*a^4*c^4*d^4 - 4*A*B*a^4*c^5*d^3))/
(2*c*d^6 + d^7 + c^2*d^5) + ((B*a^2*c*2i - a^2*d*(A + 2*B)*1i)*((((32*(c^2*
d^10 + 2*c^3*d^9 + c^4*d^8))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x
)/2)*(3*c*d^12 + 6*c^2*d^11 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 +
 d^8 + c^2*d^6))*(B*a^2*c*2i - a^2*d*(A + 2*B)*1i))/d^3 - (32*(A*a^2*c*d^9
+ 2*B*a^2*c*d^9 - A*a^2*c^3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3*d^7 - B*a^2*c
^4*d^6))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(4*A*a^2*c*d^10
 + 2*B*a^2*c*d^10 + 2*A*a^2*c^2*d^9 - 4*A*a^2*c^3*d^8 - 2*A*a^2*c^4*d^7 - 4
*B*a^2*c^2*d^9 - 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 4*B*a^2*c^5*d^6))/(2*c
*d^7 + d^8 + c^2*d^6)))/d^3 + (32*tan(e/2 + (f*x)/2)*(8*A^2*a^4*c^2*d^7 + 4
*A^2*a^4*c^3*d^6 - 4*A^2*a^4*c^4*d^5 - 2*A^2*a^4*c^5*d^4 + 6*B^2*a^4*c^2*d^
7 - 29*B^2*a^4*c^3*d^6 - 4*B^2*a^4*c^4*d^5 + 28*B^2*a^4*c^5*d^4 - 8*B^2*a^4
*c^7*d^2 - 2*A^2*a^4*c*d^8 + 7*B^2*a^4*c*d^8 + 22*A*B*a^4*c^2*d^7 - 16*A*B*
a^4*c^3*d^6 - 26*A*B*a^4*c^4*d^5 + 8*A*B*a^4*c^5*d^4 + 8*A*B*a^4*c^6*d^3 +
4*A*B*a^4*c*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*1i)/d^3 + ((B*a^2*c*2i - a^2*d
*(A + 2*B)*1i)*((32*(A^2*a^4*c^2*d^6 + 2*A^2*a^4*c^3*d^5 + A^2*a^4*c^4*d^4
+ 4*B^2*a^4*c^2*d^6 - 8*B^2*a^4*c^4*d^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B*a^4*c^2
*d^6 + 4*A*B*a^4*c^3*d^5 - 4*A*B*a^4*c^4*d^4 - 4*A*B*a^4*c^5*d^3))/(2*c*d^6
 + d^7 + c^2*d^5) + ((B*a^2*c*2i - a^2*d*(A + 2*B)*1i)*((32*(A*a^2*c*d^9 +
2*B*a^2*c*d^9 - A*a^2*c^3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3*d^7 - B*a^2*c^4
```

```
*d^6))/(2*c*d^6 + d^7 + c^2*d^5) + (((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8))/
(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^11 +
 c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*(B*a^2*c*2i
- a^2*d*(A + 2*B)*1i))/d^3 - (32*tan(e/2 + (f*x)/2)*(4*A*a^2*c*d^10 + 2*B*a
^2*c*d^10 + 2*A*a^2*c^2*d^9 - 4*A*a^2*c^3*d^8 - 2*A*a^2*c^4*d^7 - 4*B*a^2*c
^2*d^9 - 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 4*B*a^2*c^5*d^6))/(2*c*d^7 + d
^8 + c^2*d^6)))/d^3 + (32*tan(e/2 + (f*x)/2)*(8*A^2*a^4*c^2*d^7 + 4*A^2*a^4
*c^3*d^6 - 4*A^2*a^4*c^4*d^5 - 2*A^2*a^4*c^5*d^4 + 6*B^2*a^4*c^2*d^7 - 29*B
^2*a^4*c^3*d^6 - 4*B^2*a^4*c^4*d^5 + 28*B^2*a^4*c^5*d^4 - 8*B^2*a^4*c^7*d^2
 - 2*A^2*a^4*c*d^8 + 7*B^2*a^4*c*d^8 + 22*A*B*a^4*c^2*d^7 - 16*A*B*a^4*c^3*
d^6 - 26*A*B*a^4*c^4*d^5 + 8*A*B*a^4*c^5*d^4 + 8*A*B*a^4*c^6*d^3 + 4*A*B*a^
4*c*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*1i)/d^3)/((64*(4*B^3*a^6*c^6 - 2*A^3*a
^6*c^2*d^4 - 2*A^3*a^6*c^3*d^3 - 10*B^3*a^6*c^2*d^4 + 14*B^3*a^6*c^3*d^3 -
2*B^3*a^6*c^4*d^2 + 4*A^3*a^6*c*d^5 + 2*B^3*a^6*c*d^5 - 8*B^3*a^6*c^5*d + 9
*A*B^2*a^6*c*d^5 - 12*A*B^2*a^6*c^5*d + 12*A^2*B*a^6*c*d^5 - 30*A*B^2*a^6*c
^2*d^4 + 21*A*B^2*a^6*c^3*d^3 + 12*A*B^2*a^6*c^4*d^2 - 21*A^2*B*a^6*c^2*d^4
 + 9*A^2*B*a^6*c^4*d^2))/(2*c*d^6 + d^7 + c^2*d^5) + ((B*a^2*c*2i - a^2*d*(
A + 2*B)*1i)*((32*(A^2*a^4*c^2*d^6 + 2*A^2*a^4*c^3*d^5 + A^2*a^4*c^4*d^4 +
4*B^2*a^4*c^2*d^6 - 8*B^2*a^4*c^4*d^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B*a^4*c^2*d
^6 + 4*A*B*a^4*c^3*d^5 - 4*A*B*a^4*c^4*d^4 - 4*A*B*a^4*c^5*d^3))/(2*c*d^6 +
 d^7 + c^2*d^5) + ((B*a^2*c*2i - a^2*d*(A + 2*B)*1i)*((((32*(c^2*d^10 + 2*c
^3*d^9 + c^4*d^8))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*
d^12 + 6*c^2*d^11 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 + d^8 + c^2
*d^6))*(B*a^2*c*2i - a^2*d*(A + 2*B)*1i))/d^3 - (32*(A*a^2*c*d^9 + 2*B*a^2*
c*d^9 - A*a^2*c^3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3*d^7 - B*a^2*c^4*d^6))/(
2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(4*A*a^2*c*d^10 + 2*B*a^2
*c*d^10 + 2*A*a^2*c^2*d^9 - 4*A*a^2*c^3*d^8 - 2*A*a^2*c^4*d^7 - 4*B*a^2*c^2
*d^9 - 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 4*B*a^2*c^5*d^6))/(2*c*d^7 + d^8
 + c^2*d^6)))/d^3 + (32*tan(e/2 + (f*x)/2)*(8*A^2*a^4*c^2*d^7 + 4*A^2*a^4*c
^3*d^6 - 4*A^2*a^4*c^4*d^5 - 2*A^2*a^4*c^5*d^4 + 6*B^2*a^4*c^2*d^7 - 29*B^2
*a^4*c^3*d^6 - 4*B^2*a^4*c^4*d^5 + 28*B^2*a^4*c^5*d^4 - 8*B^2*a^4*c^7*d^2 -
 2*A^2*a^4*c*d^8 + 7*B^2*a^4*c*d^8 + 22*A*B*a^4*c^2*d^7 - 16*A*B*a^4*c^3*d^
6 - 26*A*B*a^4*c^4*d^5 + 8*A*B*a^4*c^5*d^4 + 8*A*B*a^4*c^6*d^3 + 4*A*B*a^4*
c*d^8))/(2*c*d^7 + d^8 + c^2*d^6)))/d^3 - ((B*a^2*c*2i - a^2*d*(A + 2*B)*1i
)*((32*(A^2*a^4*c^2*d^6 + 2*A^2*a^4*c^3*d^5 + A^2*a^4*c^4*d^4 + 4*B^2*a^4*c
^2*d^6 - 8*B^2*a^4*c^4*d^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B*a^4*c^2*d^6 + 4*A*B*
a^4*c^3*d^5 - 4*A*B*a^4*c^4*d^4 - 4*A*B*a^4*c^5*d^3))/(2*c*d^6 + d^7 + c^2*
d^5) + ((B*a^2*c*2i - a^2*d*(A + 2*B)*1i)*((32*(A*a^2*c*d^9 + 2*B*a^2*c*d^9
 - A*a^2*c^3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3*d^7 - B*a^2*c^4*d^6))/(2*c*d
^6 + d^7 + c^2*d^5) + (((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8))/(2*c*d^6 + d^
7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^11 + c^3*d^10 - 4
*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*(B*a^2*c*2i - a^2*d*(A +
2*B)*1i))/d^3 - (32*tan(e/2 + (f*x)/2)*(4*A*a^2*c*d^10 + 2*B*a^2*c*d^10 + 2
*A*a^2*c^2*d^9 - 4*A*a^2*c^3*d^8 - 2*A*a^2*c^4*d^7 - 4*B*a^2*c^2*d^9 - 6*B*
a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 4*B*a^2*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6)
```

))/d^3 + (32*tan(e/2 + (f*x)/2)*(8*A^2*a^4*c^2*d^7 + 4*A^2*a^4*c^3*d^6 - 4*A^2*a^4*c^4*d^5 - 2*A^2*a^4*c^5*d^4 + 6*B^2*a^4*c^2*d^7 - 29*B^2*a^4*c^3*d^6 - 4*B^2*a^4*c^4*d^5 + 28*B^2*a^4*c^5*d^4 - 8*B^2*a^4*c^7*d^2 - 2*A^2*a^4*c*d^8 + 7*B^2*a^4*c*d^8 + 22*A*B*a^4*c^2*d^7 - 16*A*B*a^4*c^3*d^6 - 26*A*B*a^4*c^4*d^5 + 8*A*B*a^4*c^5*d^4 + 8*A*B*a^4*c^6*d^3 + 4*A*B*a^4*c*d^8))/(2*c*d^7 + d^8 + c^2*d^6)))/d^3 + (64*tan(e/2 + (f*x)/2)*(16*B^3*a^6*c^7 + 2*A^3*a^6*c^2*d^5 - 4*A^3*a^6*c^3*d^4 - 2*A^3*a^6*c^4*d^3 - 32*B^3*a^6*c^2*d^5 + 16*B^3*a^6*c^3*d^4 + 48*B^3*a^6*c^4*d^3 - 40*B^3*a^6*c^5*d^2 + 4*A^3*a^6*c*d^6 + 8*B^3*a^6*c*d^6 - 16*B^3*a^6*c^6*d + 24*A*B^2*a^6*c*d^6 - 24*A*B^2*a^6*c^6*d + 18*A^2*B*a^6*c*d^6 - 48*A*B^2*a^6*c^2*d^5 - 24*A*B^2*a^6*c^3*d^4 + 72*A*B^2*a^6*c^4*d^3 - 12*A^2*B*a^6*c^2*d^5 - 30*A^2*B*a^6*c^3*d^4 + 12*A^2*B*a^6*c^4*d^3 + 12*A^2*B*a^6*c^5*d^2))/(2*c*d^7 + d^8 + c^2*d^6)))*(B*a^2*c*2i - a^2*d*(A + 2*B)*1i)*2i)/(d^3*f) - (a^2*atan(((a^2*(-(c + d)^3*(c - d))^(1/2)*((32*(A^2*a^4*c^2*d^6 + 2*A^2*a^4*c^3*d^5 + A^2*a^4*c^4*d^4 + 4*B^2*a^4*c^2*d^6 - 8*B^2*a^4*c^4*d^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B*a^4*c^2*d^6 + 4*A*B*a^4*c^3*d^5 - 4*A*B*a^4*c^4*d^4 - 4*A*B*a^4*c^5*d^3))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(8*A^2*a^4*c^2*d^7 + 4*A^2*a^4*c^3*d^6 - 4*A^2*a^4*c^4*d^5 - 2*A^2*a^4*c^5*d^4 + 6*B^2*a^4*c^2*d^7 - 29*B^2*a^4*c^3*d^6 - 4*B^2*a^4*c^4*d^5 + 28*B^2*a^4*c^5*d^4 - 8*B^2*a^4*c^7*d^2 - 2*A^2*a^4*c*d^8 + 7*B^2*a^4*c*d^8 + 22*A*B*a^4*c^2*d^7 - 16*A*B*a^4*c^3*d^6 - 26*A*B*a^4*c^4*d^5 + 8*A*B*a^4*c^5*d^4 + 8*A*B*a^4*c^6*d^3 + 4*A*B*a^4*c*d^8))/(2*c*d^7 + d^8 + c^2*d^6) + (a^2*(-(c + d)^3*(c - d))^(1/2)*((32*tan(e/2 + (f*x)/2)*(4*A*a^2*c*d^10 + 2*B*a^2*c*d^10 + 2*A*a^2*c^2*d^9 - 4*A*a^2*c^3*d^8 - 2*A*a^2*c^4*d^7 - 4*B*a^2*c^2*d^9 - 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 4*B*a^2*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6) - (32*(A*a^2*c*d^9 + 2*B*a^2*c*d^9 - A*a^2*c^3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3*d^7 - B*a^2*c^4*d^6))/(2*c*d^6 + d^7 + c^2*d^5) + (a^2*((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^11 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*(-(c + d)^3*(c - d))^(1/2)*(2*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))*(2*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))*(2*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d)*1i)/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3) + (a^2*(-(c + d)^3*(c - d))^(1/2)*((32*(A^2*a^4*c^2*d^6 + 2*A^2*a^4*c^3*d^5 + A^2*a^4*c^4*d^4 + 4*B^2*a^4*c^2*d^6 - 8*B^2*a^4*c^4*d^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B*a^4*c^2*d^6 + 4*A*B*a^4*c^3*d^5 - 4*A*B*a^4*c^4*d^4 - 4*A*B*a^4*c^5*d^3))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(8*A^2*a^4*c^2*d^7 + 4*A^2*a^4*c^3*d^6 - 4*A^2*a^4*c^4*d^5 - 2*A^2*a^4*c^5*d^4 + 6*B^2*a^4*c^2*d^7 - 29*B^2*a^4*c^3*d^6 - 4*B^2*a^4*c^4*d^5 + 28*B^2*a^4*c^5*d^4 - 8*B^2*a^4*c^7*d^2 - 2*A^2*a^4*c*d^8 + 7*B^2*a^4*c*d^8 + 22*A*B*a^4*c^2*d^7 - 16*A*B*a^4*c^3*d^6 - 26*A*B*a^4*c^4*d^5 + 8*A*B*a^4*c^5*d^4 + 8*A*B*a^4*c^6*d^3 + 4*A*B*a^4*c*d^8))/(2*c*d^7 + d^8 + c^2*d^6) + (a^2*(-(c + d)^3*(c - d))^(1/2)*((32*(A*a^2*c*d^9 + 2*B*a^2*c*d^9 - A*a^2*c^3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3*d^7 - B*a^2*c^4*d^6))/(2*c*d^6 + d^7 + c^2*d^5) - (32*tan(e/2 + (f*x)/2)*(4*A*a^2*c*d^10 + 2*B*a^2*c*d^10 + 2*A*a^2*c^2*d^9 - 4*A*a^2*c^3*d^8 - 2*A*a^2*c^4 *d^7 - 4*B*a^2*c^2*d^9 - 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 4*B*a^2*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6) + (a^2*((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^11 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*(-(c + d)^3*(c - d))^(1/2)*(2*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))*(2*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))*(2*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d)*1i)/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))/((64*(4*B^3*a^6*c^6 - 2*A^3*a^6*c^2*d^4 - 2*A^3*a^6*c^3*d^3 - 10*B^3*a^6*c^2*d^4 + 14*B^3*a^6*c^3*d^3 - 2*B^3*a^6*c^4*d^2 + 4*A^3*a^6*c*d^5 + 2*B^3*a^6*c*d^5 - 8*B^3*a^6*c^5*d + 9*A*B^2*a^6*c*d^5 - 12*A*B^2*a^6*c^5*d + 12*A^2*B*a^6*c*d^5 - 30*A*B^2*a^6*c^2*d^4 + 21*A*B^2*a^6*c^3*d^3 + 12*A*B^2*a^6*c^4*d^2 - 21*A^2*B*a^6*c^2*d^4 + 9*A^2*B*a^6*c^4*d^2))/(2*c*d^6 + d^7 + c^2*d^5) + (64*tan(e/2 + (f*x)/2)*(16*B^3*a^6*c^7 + 2*A^3*a^6*c^2*d^5 - 4*A^3*a^6*c^3*d^4 - 2*A^3*a^6*c^4*d^3 - 32*B^3*a^6*c^2*d^5 + 16*B^3*a^6*c^3*d^4 + 48*B^3*a^6*c^4*d^3 - 40*B^3*a^6*c^5*d^2 + 4*A^3*a^6*c*d^6 + 8*B^3*a^6*c*d^6 - 16*B^3*a^6*c^6*d + 24*A*B^2*a^6*c*d^6 - 24*A*B^2*a^6*c^6*d + 18*A^2*B*a^6*c*d^6 - 48*A*B^2*a^6*c^2*d^5 - 24*A*B^2*a^6*c^3*d^4 + 72*A*B^2*a^6*c^4*d^3 - 12*A^2*B*a^6*c^2*d^5 - 30*A^2*B*a^6*c^3*d^4 + 12*A^2*B*a^6*c^4*d^3 + 12*A^2*B*a^6*c^5*d^2))/(2*c*d^7 + d^8 + c^2*d^6) + (a^2*(-(c + d)^3*(c - d))^(1/2)*((32*(A^2*a^4*c^2*d^6 + 2*A^2*a^4*c^3*d^5 + A^2*a^4*c^4*d^4 + 4*B^2*a^4*c^2*d^6 - 8*B^2*a^4*c^4*d^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B*a^4*c^2*d^6 + 4*A*B*a^4*c^3*d^5 - 4*A*B*a^4*c^4*d^4 - 4*A*B*a^4*c^5*d^3))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(8*A^2*a^4*c^2*d^7 + 4*A^2*a^4*c^3*d^6 - 4*A^2*a^4*c^4*d^5 - 2*A^2*a^4*c^5*d^4 + 6*B^2*a^4*c^2*d^7 - 29*B^2*a^4*c^3*d^6 - 4*B^2*a^4*c^4*d^5 + 28*B^2*a^4*c^5*d^4 - 8*B^2*a^4*c^7*d^2 - 2*A^2*a^4*c*d^8 + 7*B^2*a^4*c*d^8 + 22*A*B*a^4*c^2*d^7 - 16*A*B*a^4*c^3*d^6 - 26*A*B*a^4*c^4*d^5 + 8*A*B*a^4*c^5*d^4 + 8*A*B*a^4*c^6*d^3 + 4*A*B*a^4*c*d^8))/(2*c*d^7 + d^8 + c^2*d^6) + (a^2*(-(c + d)^3*(c - d))^(1/2)*((32*tan(e/2 + (f*x)/2)*(4*A*a^2*c*d^10 + 2*B*a^2*c*d^10 + 2*A*a^2*c^2*d^9 - 4*A*a^2*c^3*d^8 - 2*A*a^2*c^4*d^7 - 4*B*a^2*c^2*d^9 - 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 4*B*a^2*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6) - (32*(A*a^2*c*d^9 + 2*B*a^2*c*d^9 - A*a^2*c^3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3*d^7 - B*a^2*c^4*d^6))/(2*c*d^6 + d^7 + c^2*d^5) + (a^2*((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^11 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*(-(c + d)^3*(c - d))^(1/2)*(2*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))*(2*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))*(2*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3) - (a^2*(-(c + d)^3*(c - d))^(1/2)*((32*(A^2*a^4*c^2*d^6 + 2*A^2*a^4*c^3*d^5 + A^2*a^4*c^4*d^4 + 4*B^2*a^4*c^2*d^6 - 8*B^2*a^4*c^4*d^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B*a^4*c^2*d^6 + 4*A*B*a^4*c^3*d^5 - 4*A*B*a^4*c^4*d^4 - 4*A*B*a^4*c^5*d^3))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(8*A^2*a^4*c^2*d^7 + 4*A^2*a^4*c^3*d^6 - 4*A^2*a^4*c^4*d^5 - 2*A^2*a^4*c^5*d^4 + 6*B^2*a^4*c^2*d^7 - 29*B^2*a^4*c^3*d^6 - 4*B^2*a^4*c^4*d^5 + 28

```
*B^2*a^4*c^5*d^4 - 8*B^2*a^4*c^7*d^2 - 2*A^2*a^4*c*d^8 + 7*B^2*a^4*c*d^8 +
22*A*B*a^4*c^2*d^7 - 16*A*B*a^4*c^3*d^6 - 26*A*B*a^4*c^4*d^5 + 8*A*B*a^4*c^
5*d^4 + 8*A*B*a^4*c^6*d^3 + 4*A*B*a^4*c*d^8))/(2*c*d^7 + d^8 + c^2*d^6) + (
a^2*(-(c + d)^3*(c - d))^(1/2)*((32*(A*a^2*c*d^9 + 2*B*a^2*c*d^9 - A*a^2*c^
3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3*d^7 - B*a^2*c^4*d^6))/(2*c*d^6 + d^7 +
c^2*d^5) - (32*tan(e/2 + (f*x)/2)*(4*A*a^2*c*d^10 + 2*B*a^2*c*d^10 + 2*A*a^
2*c^2*d^9 - 4*A*a^2*c^3*d^8 - 2*A*a^2*c^4*d^7 - 4*B*a^2*c^2*d^9 - 6*B*a^2*c
^3*d^8 + 4*B*a^2*c^4*d^7 + 4*B*a^2*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6) + (a
^2*((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8))/(2*c*d^6 + d^7 + c^2*d^5) + (32*t
an(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^11 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8
))/(2*c*d^7 + d^8 + c^2*d^6))*(-(c + d)^3*(c - d))^(1/2)*(2*A*d^2 - 2*B*c^2
 + B*d^2 + A*c*d - 2*B*c*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))*(2*A*d^
2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^
3))*(2*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d))/(3*c*d^5 + d^6 + 3*c^2*d
^4 + c^3*d^3)))*(-(c + d)^3*(c - d))^(1/2)*(2*A*d^2 - 2*B*c^2 + B*d^2 + A*c
*d - 2*B*c*d)*2i)/(f*(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.257 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=215

$$-\frac{a^2\left(3Ad^2-B\left(2c^2+3cd-2d^2\right)\right)\cos(e+fx)}{2d^2f(c+d)^2(c+d\sin(e+fx))}+\frac{a^2\left(3Ad^3-B\left(2c^3+4c^2d+cd^2-4d^3\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f(c+d)^2\sqrt{c^2-d^2}}+\cdots$$

```
[Out] a^2*B*x/d^3+1/2*(-A*d+B*c)*cos(f*x+e)*(a^2+a^2*sin(f*x+e))/d/(c+d)/f/(c+d*s
in(f*x+e))^2-1/2*a^2*(3*A*d^2-B*(2*c^2+3*c*d-2*d^2))*cos(f*x+e)/d^2/(c+d)^2
/f/(c+d*sin(f*x+e))+a^2*(3*A*d^3-B*(2*c^3+4*c^2*d+c*d^2-4*d^3))*arctan((d+c
*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^3/(c+d)^2/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2975, 2968, 3021, 2735, 2660, 618, 204}

$$\frac{a^2\left(3Ad^3-B\left(4c^2d+2c^3+cd^2-4d^3\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f(c+d)^2\sqrt{c^2-d^2}}-\frac{a^2\left(3Ad^2-B\left(2c^2+3cd-2d^2\right)\right)\cos(e+fx)}{2d^2f(c+d)^2(c+d\sin(e+fx))}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a^2*B*x)/d^3 + (a^2*(3*A*d^3 - B*(2*c^3 + 4*c^2*d + c*d^2 - 4*d^3))*ArcTan
[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3*(c + d)^2*Sqrt[c^2 - d^2]*
f) + ((B*c - A*d)*Cos[e + f*x]*(a^2 + a^2*Sin[e + f*x]))/(2*d*(c + d)*f*(c
+ d*Sin[e + f*x])^2) - (a^2*(3*A*d^2 - B*(2*c^2 + 3*c*d - 2*d^2))*Cos[e + f
*x])/(2*d^2*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{\int \frac{(a+a\sin(e+fx))(-a(}{(}}{}$$

$$= \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{\int \frac{-a^2(Bc-3Ad-2Bd)+}{}}{}$$

$$= \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2\left(3Ad^2-B\left(2c^2\right.\right.}{2d^2(c+d)^2}$$

$$= \frac{a^2Bx}{d^3} + \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2\left(3Ad^2\right.}{2d^2}$$

$$= \frac{a^2Bx}{d^3} + \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2\left(3Ad^2\right.}{2d^2}$$

$$= \frac{a^2Bx}{d^3} + \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2\left(3Ad^2\right.}{2d^2}$$

$$= \frac{a^2Bx}{d^3} - \frac{a^2\left(2Bc(c+d)^2-d^2(3Ad+B(c+4d))\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e\right.}{\sqrt{c^2-d^2}}\right.}{d^3(c+d)^2\sqrt{c^2-d^2}\,f}$$

Mathematica [A] time = 1.37, size = 226, normalized size = 1.05

$$\frac{a^2(\sin(e+fx)+1)^2\left(-\frac{d\left(Ad(c+4d)+B\left(-3c^2-4cd+2d^2\right)\right)\cos(e+fx)}{(c+d)^2(c+d\sin(e+fx))} - \frac{2\left(B\left(2c^3+4c^2d+cd^2-4d^3\right)-3Ad^3\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{(c+d)^2\sqrt{c^2-d^2}} - \frac{d(d-c)(Ad}{(c+d)(c+}\right.}{2d^3f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^3,x]
```

```
[Out] (a^2*(1 + Sin[e + f*x])^2*(2*B*(e + f*x) - (2*(-3*A*d^3 + B*(2*c^3 + 4*c^2*
d + c*d^2 - 4*d^3))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c +
 d)^2*Sqrt[c^2 - d^2]) - (d*(-c + d)*(-(B*c) + A*d)*Cos[e + f*x])/((c + d)*
(c + d*Sin[e + f*x])^2) - (d*(A*d*(c + 4*d) + B*(-3*c^2 - 4*c*d + 2*d^2))*C
```

```
os[e + f*x])/((c + d)^2*(c + d*Sin[e + f*x]))))/(2*d^3*f*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^4)
```

fricas [B] time = 0.55, size = 1483, normalized size = 6.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")

[Out] [1/4*(4*(B*a^2*c^4*d^2 + 2*B*a^2*c^3*d^3 - 2*B*a^2*c*d^5 - B*a^2*d^6)*f*x*c
os(f*x + e)^2 - 4*(B*a^2*c^6 + 2*B*a^2*c^5*d + B*a^2*c^4*d^2 - B*a^2*c^2*d^
4 - 2*B*a^2*c*d^5 - B*a^2*d^6)*f*x - (2*B*a^2*c^5 + 4*B*a^2*c^4*d + 3*B*a^2
*c^3*d^2 - 3*A*a^2*c^2*d^3 + B*a^2*c*d^4 - (3*A + 4*B)*a^2*d^5 - (2*B*a^2*c
^3*d^2 + 4*B*a^2*c^2*d^3 + B*a^2*c*d^4 - (3*A + 4*B)*a^2*d^5)*cos(f*x + e)^
2 + 2*(2*B*a^2*c^4*d + 4*B*a^2*c^3*d^2 + B*a^2*c^2*d^3 - (3*A + 4*B)*a^2*c*
d^4)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c
*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x +
e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)
) - 2*(2*B*a^2*c^5*d + 4*B*a^2*c^4*d^2 - (4*A + 3*B)*a^2*c^3*d^3 - (A + 4*B
)*a^2*c^2*d^4 + (4*A + B)*a^2*c*d^5 + A*a^2*d^6)*cos(f*x + e) - 2*(4*(B*a^2
*c^5*d + 2*B*a^2*c^4*d^2 - 2*B*a^2*c^2*d^4 - B*a^2*c*d^5)*f*x + (3*B*a^2*c^
4*d^2 - (A - 4*B)*a^2*c^3*d^3 - (4*A + 5*B)*a^2*c^2*d^4 + (A - 4*B)*a^2*c*d
^5 + 2*(2*A + B)*a^2*d^6)*cos(f*x + e))*sin(f*x + e))/((c^4*d^5 + 2*c^3*d^6
 - 2*c*d^8 - d^9)*f*cos(f*x + e)^2 - 2*(c^5*d^4 + 2*c^4*d^5 - 2*c^2*d^7 - c
*d^8)*f*sin(f*x + e) - (c^6*d^3 + 2*c^5*d^4 + c^4*d^5 - c^2*d^7 - 2*c*d^8 -
 d^9)*f), 1/2*(2*(B*a^2*c^4*d^2 + 2*B*a^2*c^3*d^3 - 2*B*a^2*c*d^5 - B*a^2*d
^6)*f*x*cos(f*x + e)^2 - 2*(B*a^2*c^6 + 2*B*a^2*c^5*d + B*a^2*c^4*d^2 - B*a
^2*c^2*d^4 - 2*B*a^2*c*d^5 - B*a^2*d^6)*f*x - (2*B*a^2*c^5 + 4*B*a^2*c^4*d
+ 3*B*a^2*c^3*d^2 - 3*A*a^2*c^2*d^3 + B*a^2*c*d^4 - (3*A + 4*B)*a^2*d^5 - (
2*B*a^2*c^3*d^2 + 4*B*a^2*c^2*d^3 + B*a^2*c*d^4 - (3*A + 4*B)*a^2*d^5)*cos(
f*x + e)^2 + 2*(2*B*a^2*c^4*d + 4*B*a^2*c^3*d^2 + B*a^2*c^2*d^3 - (3*A + 4*
B)*a^2*c*d^4)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(s
qrt(c^2 - d^2)*cos(f*x + e))) - (2*B*a^2*c^5*d + 4*B*a^2*c^4*d^2 - (4*A + 3
*B)*a^2*c^3*d^3 - (A + 4*B)*a^2*c^2*d^4 + (4*A + B)*a^2*c*d^5 + A*a^2*d^6)*
cos(f*x + e) - (4*(B*a^2*c^5*d + 2*B*a^2*c^4*d^2 - 2*B*a^2*c^2*d^4 - B*a^2*
c*d^5)*f*x + (3*B*a^2*c^4*d^2 - (A - 4*B)*a^2*c^3*d^3 - (4*A + 5*B)*a^2*c^2
*d^4 + (A - 4*B)*a^2*c*d^5 + 2*(2*A + B)*a^2*d^6)*cos(f*x + e))*sin(f*x + e
))/((c^4*d^5 + 2*c^3*d^6 - 2*c*d^8 - d^9)*f*cos(f*x + e)^2 - 2*(c^5*d^4 + 2
*c^4*d^5 - 2*c^2*d^7 - c*d^8)*f*sin(f*x + e) - (c^6*d^3 + 2*c^5*d^4 + c^4*d
^5 - c^2*d^7 - 2*c*d^8 - d^9)*f)]
```

giac [B] time = 0.24, size = 703, normalized size = 3.27

$$\frac{\frac{(fx+e)Ba^2}{d^3} - \frac{\left(2\,Ba^2c^3+4\,Ba^2c^2d+Ba^2cd^2-3\,Aa^2d^3-4\,Ba^2d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(c^2d^3+2\,cd^4+d^5\right)\sqrt{c^2-d^2}} + \frac{Ba^2c^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+Aa^2c^3d^2\tan \ldots}{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] ((f*x + e)*B*a^2/d^3 - (2*B*a^2*c^3 + 4*B*a^2*c^2*d + B*a^2*c*d^2 - 3*A*a^2
*d^3 - 4*B*a^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*ta
n(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^2*d^3 + 2*c*d^4 + d^5)*sqrt(c
^2 - d^2)) + (B*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^3 + A*a^2*c^3*d^2*tan(1/2*f*
x + 1/2*e)^3 + 4*B*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 4*A*a^2*c^2*d^3*tan
(1/2*f*x + 1/2*e)^3 - 2*A*a^2*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*B*a^2*c^5*ta
n(1/2*f*x + 1/2*e)^2 + 4*B*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^2 - 4*A*a^2*c^3*d
^2*tan(1/2*f*x + 1/2*e)^2 + 3*B*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^2 - A*a^2*
c^2*d^3*tan(1/2*f*x + 1/2*e)^2 + 8*B*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^2 - 8
*A*a^2*c*d^4*tan(1/2*f*x + 1/2*e)^2 - 2*B*a^2*c*d^4*tan(1/2*f*x + 1/2*e)^2
- 2*A*a^2*d^5*tan(1/2*f*x + 1/2*e)^2 + 7*B*a^2*c^4*d*tan(1/2*f*x + 1/2*e) -
 A*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e) + 12*B*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)
 - 12*A*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e) - 4*B*a^2*c^2*d^3*tan(1/2*f*x + 1/
2*e) - 2*A*a^2*c*d^4*tan(1/2*f*x + 1/2*e) + 2*B*a^2*c^5 + 4*B*a^2*c^4*d - 4
*A*a^2*c^3*d^2 - B*a^2*c^3*d^2 - A*a^2*c^2*d^3)/((c^4*d^2 + 2*c^3*d^3 + c^2
*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2))/f
```

maple [B] time = 0.54, size = 1916, normalized size = 8.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -a^2/f/d/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)
+2*d)/(c^2-d^2)^(1/2))*B*c-2*a^2/f/d^3/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arct
an(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^3-2*a^2/f*d^2/(tan
(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*tan(1/2*f
*x+1/2*e)^3*A-4*a^2/f/d^2/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*t
an(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^2+a^2/f/d/(tan(1/2*f*x+1/2*e)^2
*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^2*tan(1/2*f*x+1/2*e)^3*B-8
*a^2/f*d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d
```

```
^2)/c*tan(1/2*f*x+1/2*e)^2*A-2*a^2/f*d^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*
f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/2*e)^2*A+2*a^2/f/d^2/(t
an(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^3*tan(1
/2*f*x+1/2*e)^2*B+4*a^2/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+
c)^2/(c^2+2*c*d+d^2)*c^2*tan(1/2*f*x+1/2*e)^2*B-2*a^2/f*d^2/(tan(1/2*f*x+1/
2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^2
*B-2*a^2/f*d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^2+2
*c*d+d^2)*tan(1/2*f*x+1/2*e)*A+7*a^2/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*
f*x+1/2*e)*d+c)^2*c^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*B-a^2/f/(tan(1/2*f
*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*B*c-4*a^2/f/(tan(
1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*A*c+3*a^2/f/
(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^
2-d^2)^(1/2))*A-a^2/f*d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2
/(c^2+2*c*d+d^2)*A+a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^
2/(c^2+2*c*d+d^2)*c*tan(1/2*f*x+1/2*e)^3*A+4*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+
2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*tan(1/2*f*x+1/2*e)^3*B-4*a^2/
f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*tan
(1/2*f*x+1/2*e)^2*A+3*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+
c)^2/(c^2+2*c*d+d^2)*c*tan(1/2*f*x+1/2*e)^2*B-a^2/f/(tan(1/2*f*x+1/2*e)^2*c
+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*A+12*a^2/
f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*tan
(1/2*f*x+1/2*e)*B-12*a^2/f*d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d
+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*A-4*a^2/f*d/(tan(1/2*f*x+1/2*e)^2*
c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*B+4*a^2/f/
d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*B*c^2
+2*a^2/f/d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d
+d^2)*B*c^3-4*a^2/f*d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(
c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*A-a^2/f*d/(tan(1/2*f*x+1/2*e)^2*c+2*tan
(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2*A+8*a^2/f*d/(ta
n(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*
x+1/2*e)^2*B+4*a^2/f/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/
2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B+2*a^2/f*B/d^3*arctan(tan(1/2*f*x+1/2*e
))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 22.50, size = 8632, normalized size = 40.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c + d*sin(e + f*x))^3,x)
```

```
[Out] (2*B*a^2*atan(((B*a^2*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d^5 + 24*B^2*
a^4*c^4*d^4 + 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2))/(4*c*d^8 + d^9 + 6*c
^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(40*B^2*a^4*c^2*d^7 +
 75*B^2*a^4*c^3*d^6 + 24*B^2*a^4*c^4*d^5 - 36*B^2*a^4*c^5*d^4 - 32*B^2*a^4*
c^6*d^3 - 8*B^2*a^4*c^7*d^2 - 9*A^2*a^4*c*d^8 - 8*B^2*a^4*c*d^8 + 6*A*B*a^4
*c^2*d^7 + 24*A*B*a^4*c^3*d^6 + 12*A*B*a^4*c^4*d^5 - 24*A*B*a^4*c*d^8))/(4*
c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (B*a^2*((8*tan(e/2 + (f*x
)/2)*(12*A*a^2*c*d^11 + 16*B*a^2*c*d^11 + 24*A*a^2*c^2*d^10 + 12*A*a^2*c^3*
d^9 + 28*B*a^2*c^2*d^10 - 8*B*a^2*c^3*d^9 - 44*B*a^2*c^4*d^8 - 32*B*a^2*c^5
*d^7 - 8*B*a^2*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6)
 - (8*(4*B*a^2*c*d^10 - 6*A*a^2*c^2*d^9 - 12*A*a^2*c^3*d^8 - 6*A*a^2*c^4*d^
7 + 8*B*a^2*c^2*d^9 + 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 2*B*a^2*c^5*d^6))
/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (B*a^2*((8*(4*c^2*d^12
 + 16*c^3*d^11 + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*
c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*
d^13 + 64*c^3*d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(
4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3)*1i)/d^3))/d^3 +
 (B*a^2*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d^5 + 24*B^2*a^4*c^4*d^4 +
16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3
*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(40*B^2*a^4*c^2*d^7 + 75*B^2*a^4*c^
3*d^6 + 24*B^2*a^4*c^4*d^5 - 36*B^2*a^4*c^5*d^4 - 32*B^2*a^4*c^6*d^3 - 8*B^
2*a^4*c^7*d^2 - 9*A^2*a^4*c*d^8 - 8*B^2*a^4*c*d^8 + 6*A*B*a^4*c^2*d^7 + 24*
A*B*a^4*c^3*d^6 + 12*A*B*a^4*c^4*d^5 - 24*A*B*a^4*c*d^8))/(4*c*d^9 + d^10 +
 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (B*a^2*((8*(4*B*a^2*c*d^10 - 6*A*a^2*c^
2*d^9 - 12*A*a^2*c^3*d^8 - 6*A*a^2*c^4*d^7 + 8*B*a^2*c^2*d^9 + 6*B*a^2*c^3*
d^8 + 4*B*a^2*c^4*d^7 + 2*B*a^2*c^5*d^6))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^
3*d^6 + c^4*d^5) - (8*tan(e/2 + (f*x)/2)*(12*A*a^2*c*d^11 + 16*B*a^2*c*d^11
 + 24*A*a^2*c^2*d^10 + 12*A*a^2*c^3*d^9 + 28*B*a^2*c^2*d^10 - 8*B*a^2*c^3*d
^9 - 44*B*a^2*c^4*d^8 - 32*B*a^2*c^5*d^7 - 8*B*a^2*c^6*d^6))/(4*c*d^9 + d^1
0 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (B*a^2*((8*(4*c^2*d^12 + 16*c^3*d^11
 + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^
3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^13 + 64*c^3*
d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4*c*d^9 + d^10
 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3)*1i)/d^3))/d^3)/((16*(2*B^3*a^
6*c^5 + 17*B^3*a^6*c^3*d^2 - 16*B^3*a^6*c*d^4 + 12*B^3*a^6*c^4*d - 24*A*B^2
```

```
*a^6*c*d^4 + 6*A*B^2*a^6*c^4*d - 9*A^2*B*a^6*c*d^4 + 12*A*B^2*a^6*c^3*d^2))
/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) - (16*tan(e/2 + (f*x)/2)
*(28*B^3*a^6*c^2*d^4 - 8*B^3*a^6*c^6 - 8*B^3*a^6*c^3*d^3 - 44*B^3*a^6*c^4*d
^2 + 16*B^3*a^6*c*d^5 - 32*B^3*a^6*c^5*d + 12*A*B^2*a^6*c*d^5 + 24*A*B^2*a^
6*c^2*d^4 + 12*A*B^2*a^6*c^3*d^3))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7
+ c^4*d^6) - (B*a^2*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d^5 + 24*B^2*a^
4*c^4*d^4 + 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2))/(4*c*d^8 + d^9 + 6*c^2
*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(40*B^2*a^4*c^2*d^7 + 7
5*B^2*a^4*c^3*d^6 + 24*B^2*a^4*c^4*d^5 - 36*B^2*a^4*c^5*d^4 - 32*B^2*a^4*c^
6*d^3 - 8*B^2*a^4*c^7*d^2 - 9*A^2*a^4*c*d^8 - 8*B^2*a^4*c*d^8 + 6*A*B*a^4*c
^2*d^7 + 24*A*B*a^4*c^3*d^6 + 12*A*B*a^4*c^4*d^5 - 24*A*B*a^4*c*d^8))/(4*c*
d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (B*a^2*((8*tan(e/2 + (f*x)/
2)*(12*A*a^2*c*d^11 + 16*B*a^2*c*d^11 + 24*A*a^2*c^2*d^10 + 12*A*a^2*c^3*d^
9 + 28*B*a^2*c^2*d^10 - 8*B*a^2*c^3*d^9 - 44*B*a^2*c^4*d^8 - 32*B*a^2*c^5*d
^7 - 8*B*a^2*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) -
 (8*(4*B*a^2*c*d^10 - 6*A*a^2*c^2*d^9 - 12*A*a^2*c^3*d^8 - 6*A*a^2*c^4*d^7
+ 8*B*a^2*c^2*d^9 + 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 2*B*a^2*c^5*d^6))/(
4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (B*a^2*((8*(4*c^2*d^12 +
 16*c^3*d^11 + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*c^
2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^
13 + 64*c^3*d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4*
c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3)*1i)/d^3)*1i)/d^3
+ (B*a^2*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d^5 + 24*B^2*a^4*c^4*d^4 +
 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^
3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(40*B^2*a^4*c^2*d^7 + 75*B^2*a^4*c
^3*d^6 + 24*B^2*a^4*c^4*d^5 - 36*B^2*a^4*c^5*d^4 - 32*B^2*a^4*c^6*d^3 - 8*B
^2*a^4*c^7*d^2 - 9*A^2*a^4*c*d^8 - 8*B^2*a^4*c*d^8 + 6*A*B*a^4*c^2*d^7 + 24
*A*B*a^4*c^3*d^6 + 12*A*B*a^4*c^4*d^5 - 24*A*B*a^4*c*d^8))/(4*c*d^9 + d^10
+ 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (B*a^2*((8*(4*B*a^2*c*d^10 - 6*A*a^2*c
^2*d^9 - 12*A*a^2*c^3*d^8 - 6*A*a^2*c^4*d^7 + 8*B*a^2*c^2*d^9 + 6*B*a^2*c^3
*d^8 + 4*B*a^2*c^4*d^7 + 2*B*a^2*c^5*d^6))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c
^3*d^6 + c^4*d^5) - (8*tan(e/2 + (f*x)/2)*(12*A*a^2*c*d^11 + 16*B*a^2*c*d^1
1 + 24*A*a^2*c^2*d^10 + 12*A*a^2*c^3*d^9 + 28*B*a^2*c^2*d^10 - 8*B*a^2*c^3*
d^9 - 44*B*a^2*c^4*d^8 - 32*B*a^2*c^5*d^7 - 8*B*a^2*c^6*d^6))/(4*c*d^9 + d^
10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (B*a^2*((8*(4*c^2*d^12 + 16*c^3*d^1
1 + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c
^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^13 + 64*c^3
*d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4*c*d^9 + d^1
0 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3)*1i)/d^3)*1i)/d^3)))/(d^3*f)
- ((A*a^2*d^3 - 2*B*a^2*c^3 + 4*A*a^2*c*d^2 + B*a^2*c*d^2 - 4*B*a^2*c^2*d)/
(d^2*(2*c*d + c^2 + d^2)) - (tan(e/2 + (f*x)/2)^3*(B*a^2*c^3 - 2*A*a^2*d^3
- 4*A*a^2*c*d^2 + A*a^2*c^2*d + 4*B*a^2*c^2*d))/(c*d*(2*c*d + c^2 + d^2)) +
 (tan(e/2 + (f*x)/2)*(2*A*a^2*d^3 - 7*B*a^2*c^3 + 12*A*a^2*c*d^2 + A*a^2*c^
2*d + 4*B*a^2*c*d^2 - 12*B*a^2*c^2*d))/(c*d*(2*c*d + c^2 + d^2)) + (tan(e/2
 + (f*x)/2)^2*(c^2 + 2*d^2)*(A*a^2*d^3 - 2*B*a^2*c^3 + 4*A*a^2*c*d^2 + B*a^
```

2*c*d^2 - 4*B*a^2*c^2*d))/(c^2*d^2*(2*c*d + c^2 + d^2)))/(f*(tan(e/2 + (f*x)/2)^2*(2*c^2 + 4*d^2) + c^2*tan(e/2 + (f*x)/2)^4 + c^2 + 4*c*d*tan(e/2 + (f*x)/2)^3 + 4*c*d*tan(e/2 + (f*x)/2))) + (a^2*atan(((a^2*(-(c + d)^5*(c - d))^(1/2)*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d^5 + 24*B^2*a^4*c^4*d^4 + 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(40*B^2*a^4*c^2*d^7 + 75*B^2*a^4*c^3*d^6 + 24*B^2*a^4*c^4*d^5 - 36*B^2*a^4*c^5*d^4 - 32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 - 9*A^2*a^4*c*d^8 - 8*B^2*a^4*c*d^8 + 6*A*B*a^4*c^2*d^7 + 24*A*B*a^4*c^3*d^6 + 12*A*B*a^4*c^4*d^5 - 24*A*B*a^4*c*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (a^2*(-(c + d)^5*(c - d))^(1/2)*((8*tan(e/2 + (f*x)/2)*(12*A*a^2*c*d^11 + 16*B*a^2*c*d^11 + 24*A*a^2*c^2*d^10 + 12*A*a^2*c^3*d^9 + 28*B*a^2*c^2*d^10 - 8*B*a^2*c^3*d^9 - 44*B*a^2*c^4*d^8 - 32*B*a^2*c^5*d^7 - 8*B*a^2*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (8*(4*B*a^2*c*d^10 - 6*A*a^2*c^2*d^9 - 12*A*a^2*c^3*d^8 - 6*A*a^2*c^4*d^7 + 8*B*a^2*c^2*d^9 + 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 2*B*a^2*c^5*d^6))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (a^2*(-(c + d)^5*(c - d))^(1/2)*((8*(4*c^2*d^12 + 16*c^3*d^11 + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^13 + 64*c^3*d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*(2*B*c^3 - 3*A*d^3 - 4*B*d^3 + B*c*d^2 + 4*B*c^2*d))/(2*(4*c*d^8 + d^9 + 5*c^2*d^7 - 5*c^4*d^5 - 4*c^5*d^4 - c^6*d^3)))*(2*B*c^3 - 3*A*d^3 - 4*B*d^3 + B*c*d^2 + 4*B*c^2*d))/(2*(4*c*d^8 + d^9 + 5*c^2*d^7 - 5*c^4*d^5 - 4*c^5*d^4 - c^6*d^3)))*(2*B*c^3 - 3*A*d^3 - 4*B*d^3 + B*c*d^2 + 4*B*c^2*d)*1i)/(2*(4*c*d^8 + d^9 + 5*c^2*d^7 - 5*c^4*d^5 - 4*c^5*d^4 - c^6*d^3)) + (a^2*(-(c + d)^5*(c - d))^(1/2)*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d^5 + 24*B^2*a^4*c^4*d^4 + 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(40*B^2*a^4*c^2*d^7 + 75*B^2*a^4*c^3*d^6 + 24*B^2*a^4*c^4*d^5 - 36*B^2*a^4*c^5*d^4 - 32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 - 9*A^2*a^4*c*d^8 - 8*B^2*a^4*c*d^8 + 6*A*B*a^4*c^2*d^7 + 24*A*B*a^4*c^3*d^6 + 12*A*B*a^4*c^4*d^5 - 24*A*B*a^4*c*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (a^2*(-(c + d)^5*(c - d))^(1/2)*((8*(4*B*a^2*c*d^10 - 6*A*a^2*c^2*d^9 - 12*A*a^2*c^3*d^8 - 6*A*a^2*c^4*d^7 + 8*B*a^2*c^2*d^9 + 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 2*B*a^2*c^5*d^6))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) - (8*tan(e/2 + (f*x)/2)*(12*A*a^2*c*d^11 + 16*B*a^2*c*d^11 + 24*A*a^2*c^2*d^10 + 12*A*a^2*c^3*d^9 + 28*B*a^2*c^2*d^10 - 8*B*a^2*c^3*d^9 - 44*B*a^2*c^4*d^8 - 32*B*a^2*c^5*d^7 - 8*B*a^2*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (a^2*(-(c + d)^5*(c - d))^(1/2)*((8*(4*c^2*d^12 + 16*c^3*d^11 + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^13 + 64*c^3*d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*(2*B*c^3 - 3*A*d^3 - 4*B*d^3 + B*c*d^2 + 4*B*c^2*d))/(2*(4*c*d^8 + d^9 + 5*c^2*d^7 - 5*c^4*d^5 - 4*c^5*d^4 - c^6*d^3)))*(2*B*c^3 - 3*A*d^3 - 4*B*d^3 + B*c*d^2 + 4*B*c^2*d))/(2*(4*c*d^8 +

d^9 + 5*c^2*d^7 - 5*c^4*d^5 - 4*c^5*d^4 - c^6*d^3)))*(2*B*c^3 - 3*A*d^3 - 4*B*d^3 + B*c*d^2 + 4*B*c^2*d)*1i)/(2*(4*c*d^8 + d^9 + 5*c^2*d^7 - 5*c^4*d^5 - 4*c^5*d^4 - c^6*d^3)))/((16*(2*B^3*a^6*c^5 + 17*B^3*a^6*c^3*d^2 - 16*B^3*a^6*c*d^4 + 12*B^3*a^6*c^4*d - 24*A*B^2*a^6*c*d^4 + 6*A*B^2*a^6*c^4*d - 9*A^2*B*a^6*c*d^4 + 12*A*B^2*a^6*c^3*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) - (16*tan(e/2 + (f*x)/2)*(28*B^3*a^6*c^2*d^4 - 8*B^3*a^6*c^6 - 8*B^3*a^6*c^3*d^3 - 44*B^3*a^6*c^4*d^2 + 16*B^3*a^6*c*d^5 - 32*B^3*a^6*c^5*d + 12*A*B^2*a^6*c*d^5 + 24*A*B^2*a^6*c^2*d^4 + 12*A*B^2*a^6*c^3*d^3))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (a^2*(-(c + d)^5*(c - d))^(1/2)*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d^5 + 24*B^2*a^4*c^4*d^4 + 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(40*B^2*a^4*c^2*d^7 + 75*B^2*a^4*c^3*d^6 + 24*B^2*a^4*c^4*d^5 - 36*B^2*a^4*c^5*d^4 - 32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 - 9*A^2*a^4*c*d^8 - 8*B^2*a^4*c*d^8 + 6*A*B*a^4*c^2*d^7 + 24*A*B*a^4*c^3*d^6 + 12*A*B*a^4*c^4*d^5 - 24*A*B*a^4*c*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (a^2*(-(c + d)^5*(c - d))^(1/2)*((8*tan(e/2 + (f*x)/2)*(12*A*a^2*c*d^11 + 16*B*a^2*c*d^11 + 24*A*a^2*c^2*d^10 + 12*A*a^2*c^3*d^9 + 28*B*a^2*c^2*d^10 - 8*B*a^2*c^3*d^9 - 44*B*a^2*c^4*d^8 - 32*B*a^2*c^5*d^7 - 8*B*a^2*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (8*(4*B*a^2*c*d^10 - 6*A*a^2*c^2*d^9 - 12*A*a^2*c^3*d^8 - 6*A*a^2*c^4*d^7 + 8*B*a^2*c^2*d^9 + 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 2*B*a^2*c^5*d^6))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (a^2*(-(c + d)^5*(c - d))^(1/2)*((8*(4*c^2*d^12 + 16*c^3*d^11 + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^13 + 64*c^3*d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*(2*B*c^3 - 3*A*d^3 - 4*B*d^3 + B*c*d^2 + 4*B*c^2*d))/(2*(4*c*d^8 + d^9 + 5*c^2*d^7 - 5*c^4*d^5 - 4*c^5*d^4 - c^6*d^3)))*(2*B*c^3 - 3*A*d^3 - 4*B*d^3 + B*c*d^2 + 4*B*c^2*d))/(2*(4*c*d^8 + d^9 + 5*c^2*d^7 - 5*c^4*d^5 - 4*c^5*d^4 - c^6*d^3)))*(2*B*c^3 - 3*A*d^3 - 4*B*d^3 + B*c*d^2 + 4*B*c^2*d))/(2*(4*c*d^8 + d^9 + 5*c^2*d^7 - 5*c^4*d^5 - 4*c^5*d^4 - c^6*d^3)) + (a^2*(-(c + d)^5*(c - d))^(1/2)*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d^5 + 24*B^2*a^4*c^4*d^4 + 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(40*B^2*a^4*c^2*d^7 + 75*B^2*a^4*c^3*d^6 + 24*B^2*a^4*c^4*d^5 - 36*B^2*a^4*c^5*d^4 - 32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 - 9*A^2*a^4*c*d^8 - 8*B^2*a^4*c*d^8 + 6*A*B*a^4*c^2*d^7 + 24*A*B*a^4*c^3*d^6 + 12*A*B*a^4*c^4*d^5 - 24*A*B*a^4*c*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (a^2*(-(c + d)^5*(c - d))^(1/2)*((8*(4*B*a^2*c*d^10 - 6*A*a^2*c^2*d^9 - 12*A*a^2*c^3*d^8 - 6*A*a^2*c^4*d^7 + 8*B*a^2*c^2*d^9 + 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 2*B*a^2*c^5*d^6))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) - (8*tan(e/2 + (f*x)/2)*(12*A*a^2*c*d^11 + 16*B*a^2*c*d^11 + 24*A*a^2*c^2*d^10 + 12*A*a^2*c^3*d^9 + 28*B*a^2*c^2*d^10 - 8*B*a^2*c^3*d^9 - 44*B*a^2*c^4*d^8 - 32*B*a^2*c^5*d^7 - 8*B*a^2*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (a^2*(-(c + d)^5*(c - d))^(1/2)*((8*(4*c^2*d^12 + 16*c^3*d^11

```
+ 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3
*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^13 + 64*c^3*d
^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4*c*d^9 + d^10
+ 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*(2*B*c^3 - 3*A*d^3 - 4*B*d^3 + B*c*d^2
+ 4*B*c^2*d))/(2*(4*c*d^8 + d^9 + 5*c^2*d^7 - 5*c^4*d^5 - 4*c^5*d^4 - c^6*d
^3)))*(2*B*c^3 - 3*A*d^3 - 4*B*d^3 + B*c*d^2 + 4*B*c^2*d))/(2*(4*c*d^8 + d^
9 + 5*c^2*d^7 - 5*c^4*d^5 - 4*c^5*d^4 - c^6*d^3)))*(2*B*c^3 - 3*A*d^3 - 4*B
*d^3 + B*c*d^2 + 4*B*c^2*d))/(2*(4*c*d^8 + d^9 + 5*c^2*d^7 - 5*c^4*d^5 - 4*
c^5*d^4 - c^6*d^3))))*(-(c + d)^5*(c - d))^(1/2)*(2*B*c^3 - 3*A*d^3 - 4*B*d
^3 + B*c*d^2 + 4*B*c^2*d)*1i)/(f*(4*c*d^8 + d^9 + 5*c^2*d^7 - 5*c^4*d^5 - 4
*c^5*d^4 - c^6*d^3))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.258 $\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c + d\sin(e + fx))^3 \, dx$

Optimal. Leaf size=604

$$-\frac{a^3\left(-14Acd+91Ad^2+6Bc^2-27Bcd+87Bd^2\right)\cos(e+fx)(c+d\sin(e+fx))^4}{210d^3f}-\frac{a^3\left(7Ad\left(2c^2-18cd+115d^2\right.\right.}{}$$

```
[Out] 1/16*a^3*(3*B*(10*c^3+26*c^2*d+23*c*d^2+7*d^3)+A*(40*c^3+90*c^2*d+78*c*d^2+
23*d^3))*x-1/420*a^3*(7*A*d*(2*c^5-18*c^4*d+107*c^3*d^2+472*c^2*d^3+456*c*d
^4+136*d^5)-3*B*(2*c^6-14*c^5*d+51*c^4*d^2-189*c^3*d^3-920*c^2*d^4-952*c*d^
5-288*d^6))*cos(f*x+e)/d^3/f-1/1680*a^3*(7*A*d*(4*c^4-36*c^3*d+216*c^2*d^2+
626*c*d^3+345*d^4)-3*B*(4*c^5-28*c^4*d+104*c^3*d^2-392*c^2*d^3-1263*c*d^4-7
35*d^5))*cos(f*x+e)*sin(f*x+e)/d^2/f-1/840*a^3*(7*A*d*(2*c^3-18*c^2*d+111*c
*d^2+136*d^3)-B*(6*c^4-42*c^3*d+165*c^2*d^2-651*c*d^3-864*d^4))*cos(f*x+e)*
(c+d*sin(f*x+e))^2/d^3/f-1/840*a^3*(7*A*d*(2*c^2-18*c*d+115*d^2)-B*(6*c^3-4
2*c^2*d+177*c*d^2-735*d^3))*cos(f*x+e)*(c+d*sin(f*x+e))^3/d^3/f-1/210*a^3*(
-14*A*c*d+91*A*d^2+6*B*c^2-27*B*c*d+87*B*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^4
/d^3/f-1/7*a*B*cos(f*x+e)*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^4/d/f+1/42*(3
*B*(c-3*d)-7*A*d)*cos(f*x+e)*(a^3+a^3*sin(f*x+e))*(c+d*sin(f*x+e))^4/d^2/f
```

Rubi [A] time = 1.49, antiderivative size = 604, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2976, 2968, 3023, 2753, 2734}

$$-\frac{a^3\left(7Ad\left(107c^3d^2+472c^2d^3-18c^4d+2c^5+456cd^4+136d^5\right)-3B\left(51c^4d^2-189c^3d^3-920c^2d^4-14c^5d+2c\right.\right.}{420d^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a^3*(3*B*(10*c^3 + 26*c^2*d + 23*c*d^2 + 7*d^3) + A*(40*c^3 + 90*c^2*d + 7
8*c*d^2 + 23*d^3))*x)/16 - (a^3*(7*A*d*(2*c^5 - 18*c^4*d + 107*c^3*d^2 + 47
2*c^2*d^3 + 456*c*d^4 + 136*d^5) - 3*B*(2*c^6 - 14*c^5*d + 51*c^4*d^2 - 189
*c^3*d^3 - 920*c^2*d^4 - 952*c*d^5 - 288*d^6))*Cos[e + f*x])/(420*d^3*f) -
(a^3*(7*A*d*(4*c^4 - 36*c^3*d + 216*c^2*d^2 + 626*c*d^3 + 345*d^4) - 3*B*(4
*c^5 - 28*c^4*d + 104*c^3*d^2 - 392*c^2*d^3 - 1263*c*d^4 - 735*d^5))*Cos[e
+ f*x]*Sin[e + f*x])/(1680*d^2*f) - (a^3*(7*A*d*(2*c^3 - 18*c^2*d + 111*c*d
^2 + 136*d^3) - B*(6*c^4 - 42*c^3*d + 165*c^2*d^2 - 651*c*d^3 - 864*d^4))*C
os[e + f*x]*(c + d*Sin[e + f*x])^2)/(840*d^3*f) - (a^3*(7*A*d*(2*c^2 - 18*c
*d + 115*d^2) - B*(6*c^3 - 42*c^2*d + 177*c*d^2 - 735*d^3))*Cos[e + f*x]*(c
 + d*Sin[e + f*x])^3)/(840*d^3*f) - (a^3*(6*B*c^2 - 14*A*c*d - 27*B*c*d + 9
```

```
1*A*d^2 + 87*B*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(210*d^3*f) - (a*B
*Cos[e + f*x]*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^4)/(7*d*f) + ((3*
B*(c - 3*d) - 7*A*d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x])*(c + d*Sin[e + f
*x])^4)/(42*d^2*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
```

```
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin}{7df}$$

$$= -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin}{7df}$$

$$= -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin}{7df}$$

$$= -\frac{a^3\left(6Bc^2-14Acd-27Bcd+91Ad^2+87B}{210d^3f}$$

$$= -\frac{a^3\left(7Ad\left(2c^2-18cd+115d^2\right)-B\left(6c^3-42}{}$$

$$= -\frac{a^3\left(7Ad\left(2c^3-18c^2d+111cd^2+136d^3\right)-}{}$$

$$= \frac{1}{16}a^3\left(3B\left(10c^3+26c^2d+23cd^2+7d^3\right)+A$$

Mathematica [A] time = 4.76, size = 528, normalized size = 0.87

$$-\frac{a^3\cos(e+fx)\left(420\left(A\left(40c^3+90c^2d+78cd^2+23d^3\right)+3B\left(10c^3+26c^2d+23cd^2+7d^3\right)\right)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
3,x]
```

```
[Out] -1/3360*(a^3*Cos[e + f*x]*(420*(3*B*(10*c^3 + 26*c^2*d + 23*c*d^2 + 7*d^3)
+ A*(40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3))*ArcSin[Sqrt[1 - Sin[e + f*x]]/
Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(12880*A*c^3 + 11760*B*c^3 + 35280*A*c^2*d
+ 32676*B*c^2*d + 32676*A*c*d^2 + 30828*B*c*d^2 + 10276*A*d^3 + 9762*B*d^3
- (112*A*(5*c^3 + 45*c^2*d + 66*c*d^2 + 26*d^3) + 3*B*(560*c^3 + 2464*c^2*d
 + 2912*c*d^2 + 1083*d^3))*Cos[2*(e + f*x)] + 18*d*(14*A*d*(c + d) + B*(14*
c^2 + 42*c*d + 23*d^2))*Cos[4*(e + f*x)] - 15*B*d^3*Cos[6*(e + f*x)] + 5040
```

```
*A*c^3*Sin[e + f*x] + 6930*B*c^3*Sin[e + f*x] + 20790*A*c^2*d*Sin[e + f*x]
+ 22050*B*c^2*d*Sin[e + f*x] + 22050*A*c*d^2*Sin[e + f*x] + 22785*B*c*d^2*S
in[e + f*x] + 7595*A*d^3*Sin[e + f*x] + 7665*B*d^3*Sin[e + f*x] - 210*B*c^3
*Sin[3*(e + f*x)] - 630*A*c^2*d*Sin[3*(e + f*x)] - 1890*B*c^2*d*Sin[3*(e +
f*x)] - 1890*A*c*d^2*Sin[3*(e + f*x)] - 2940*B*c*d^2*Sin[3*(e + f*x)] - 980
*A*d^3*Sin[3*(e + f*x)] - 1260*B*d^3*Sin[3*(e + f*x)] + 105*B*c*d^2*Sin[5*(
e + f*x)] + 35*A*d^3*Sin[5*(e + f*x)] + 105*B*d^3*Sin[5*(e + f*x)]))/(f*Sq
rt[Cos[e + f*x]^2])
```

fricas [A] time = 0.51, size = 432, normalized size = 0.72

$$\frac{240\,Ba^3d^3\cos\left(fx+e\right)^7-1008\left(Ba^3c^2d+(A+3\,B)a^3cd^2+(A+2\,B)a^3d^3\right)\cos\left(fx+e\right)^5+560\left((A+3\,B)a^3c^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/1680*(240*B*a^3*d^3*cos(f*x + e)^7 - 1008*(B*a^3*c^2*d + (A + 3*B)*a^3*c*
d^2 + (A + 2*B)*a^3*d^3)*cos(f*x + e)^5 + 560*((A + 3*B)*a^3*c^3 + 3*(3*A +
 5*B)*a^3*c^2*d + 3*(5*A + 7*B)*a^3*c*d^2 + (7*A + 9*B)*a^3*d^3)*cos(f*x +
e)^3 + 105*(10*(4*A + 3*B)*a^3*c^3 + 6*(15*A + 13*B)*a^3*c^2*d + 3*(26*A +
23*B)*a^3*c*d^2 + (23*A + 21*B)*a^3*d^3)*f*x - 6720*((A + B)*a^3*c^3 + 3*(A
 + B)*a^3*c^2*d + 3*(A + B)*a^3*c*d^2 + (A + B)*a^3*d^3)*cos(f*x + e) - 35*
(8*(3*B*a^3*c*d^2 + (A + 3*B)*a^3*d^3)*cos(f*x + e)^5 - 2*(6*B*a^3*c^3 + 18
*(A + 3*B)*a^3*c^2*d + 3*(18*A + 31*B)*a^3*c*d^2 + (31*A + 45*B)*a^3*d^3)*c
os(f*x + e)^3 + 3*(2*(12*A + 17*B)*a^3*c^3 + 6*(17*A + 19*B)*a^3*c^2*d + 3*
(38*A + 41*B)*a^3*c*d^2 + (41*A + 43*B)*a^3*d^3)*cos(f*x + e))*sin(f*x + e)
)/f
```

giac [A] time = 0.25, size = 566, normalized size = 0.94

$$\frac{Ba^3d^3\cos\left(7\,fx+7\,e\right)}{448\,f}+\frac{1}{16}\left(40\,Aa^3c^3+30\,Ba^3c^3+90\,Aa^3c^2d+78\,Ba^3c^2d+78\,Aa^3cd^2+69\,Ba^3cd^2+23\,Aa^3d^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/448*B*a^3*d^3*cos(7*f*x + 7*e)/f + 1/16*(40*A*a^3*c^3 + 30*B*a^3*c^3 + 90
*A*a^3*c^2*d + 78*B*a^3*c^2*d + 78*A*a^3*c*d^2 + 69*B*a^3*c*d^2 + 23*A*a^3*
d^3 + 21*B*a^3*d^3)*x - 1/320*(12*B*a^3*c^2*d + 12*A*a^3*c*d^2 + 36*B*a^3*c
*d^2 + 12*A*a^3*d^3 + 19*B*a^3*d^3)*cos(5*f*x + 5*e)/f + 1/192*(16*A*a^3*c^
```

```
3 + 48*B*a^3*c^3 + 144*A*a^3*c^2*d + 204*B*a^3*c^2*d + 204*A*a^3*c*d^2 + 22
8*B*a^3*c*d^2 + 76*A*a^3*d^3 + 81*B*a^3*d^3)*cos(3*f*x + 3*e)/f - 1/64*(240
*A*a^3*c^3 + 208*B*a^3*c^3 + 624*A*a^3*c^2*d + 552*B*a^3*c^2*d + 552*A*a^3*
c*d^2 + 504*B*a^3*c*d^2 + 168*A*a^3*d^3 + 155*B*a^3*d^3)*cos(f*x + e)/f - 1
/192*(3*B*a^3*c*d^2 + A*a^3*d^3 + 3*B*a^3*d^3)*sin(6*f*x + 6*e)/f + 1/64*(2
*B*a^3*c^3 + 6*A*a^3*c^2*d + 18*B*a^3*c^2*d + 18*A*a^3*c*d^2 + 27*B*a^3*c*d
^2 + 9*A*a^3*d^3 + 11*B*a^3*d^3)*sin(4*f*x + 4*e)/f - 1/64*(48*A*a^3*c^3 +
64*B*a^3*c^3 + 192*A*a^3*c^2*d + 192*B*a^3*c^2*d + 192*A*a^3*c*d^2 + 189*B*
a^3*c*d^2 + 63*A*a^3*d^3 + 61*B*a^3*d^3)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.65, size = 1077, normalized size = 1.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(-B*a^3*c*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+3*B*a^3*c^3*(-1/2*sin(f*x+e)*
cos(f*x+e)+1/2*f*x+1/2*e)+a^3*A*c^3*(f*x+e)+3*B*a^3*d^3*(-1/6*(sin(f*x+e)^5
+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)-3*a^3*A*c^3*
cos(f*x+e)+3*a^3*A*d^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f
*x+3/8*e)+B*a^3*c^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+
3/8*e)+3*a^3*A*c^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+a^3*A*d^3*(-1
/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16
*e)+B*a^3*d^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)
-B*a^3*c^3*cos(f*x+e)-3/5*a^3*A*d^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos
(f*x+e)-B*a^3*c^3*(2+sin(f*x+e)^2)*cos(f*x+e)-1/3*a^3*A*c^3*(2+sin(f*x+e)^2
)*cos(f*x+e)-1/3*a^3*A*d^3*(2+sin(f*x+e)^2)*cos(f*x+e)-1/7*B*a^3*d^3*(16/5+
sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)+9*B*a^3*c*d^2*(-
1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3/5*B*a^3*d^3*(
8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+9*a^3*A*c^2*d*(-1/2*sin(f*x+e
)*cos(f*x+e)+1/2*f*x+1/2*e)+3*B*a^3*c^2*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f
*x+1/2*e)+9*a^3*A*c*d^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*
f*x+3/8*e)+3*a^3*A*c^2*d*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8
*f*x+3/8*e)+9*B*a^3*c^2*d*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/
8*f*x+3/8*e)+3*B*a^3*c*d^2*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*
x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+3*a^3*A*c*d^2*(-1/2*sin(f*x+e)*cos(f*x+e)
+1/2*f*x+1/2*e)-3*a^3*A*c^2*d*cos(f*x+e)-3*a^3*A*c^2*d*(2+sin(f*x+e)^2)*cos
(f*x+e)-9/5*B*a^3*c*d^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-3*a^
3*A*c*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)-3*B*a^3*c^2*d*(2+sin(f*x+e)^2)*cos(f*
x+e)-3/5*a^3*A*c*d^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-3/5*B*a
^3*c^2*d*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e))
```

maxima [A] time = 0.63, size = 1056, normalized size = 1.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 1/6720*(2240*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c^3 + 5040*(2*f*x + 2*
e - sin(2*f*x + 2*e))*A*a^3*c^3 + 6720*(f*x + e)*A*a^3*c^3 + 6720*(cos(f*x
+ e)^3 - 3*cos(f*x + e))*B*a^3*c^3 + 210*(12*f*x + 12*e + sin(4*f*x + 4*e)
- 8*sin(2*f*x + 2*e))*B*a^3*c^3 + 5040*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a
^3*c^3 + 20160*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c^2*d + 630*(12*f*x
+ 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c^2*d + 15120*(2*f*x
+ 2*e - sin(2*f*x + 2*e))*A*a^3*c^2*d - 1344*(3*cos(f*x + e)^5 - 10*cos(f*x
 + e)^3 + 15*cos(f*x + e))*B*a^3*c^2*d + 20160*(cos(f*x + e)^3 - 3*cos(f*x
+ e))*B*a^3*c^2*d + 1890*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x +
2*e))*B*a^3*c^2*d + 5040*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c^2*d - 134
4*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^3*c*d^2 + 20
160*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c*d^2 + 1890*(12*f*x + 12*e + s
in(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c*d^2 + 5040*(2*f*x + 2*e - sin
(2*f*x + 2*e))*A*a^3*c*d^2 - 4032*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 1
5*cos(f*x + e))*B*a^3*c*d^2 + 6720*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*
c*d^2 + 105*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48
*sin(2*f*x + 2*e))*B*a^3*c*d^2 + 1890*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8
*sin(2*f*x + 2*e))*B*a^3*c*d^2 - 1344*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3
 + 15*cos(f*x + e))*A*a^3*d^3 + 2240*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^
3*d^3 + 35*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*
sin(2*f*x + 2*e))*A*a^3*d^3 + 630*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin
(2*f*x + 2*e))*A*a^3*d^3 + 192*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*c
os(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*d^3 - 1344*(3*cos(f*x + e)^5 - 10*co
s(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*d^3 + 105*(4*sin(2*f*x + 2*e)^3 + 60*
f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^3*d^3 + 210*(12*
f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*d^3 - 20160*A*a^3
*c^3*cos(f*x + e) - 6720*B*a^3*c^3*cos(f*x + e) - 20160*A*a^3*c^2*d*cos(f*x
 + e))/f
```

mupad [B] time = 16.18, size = 1395, normalized size = 2.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^3,x)
```

```
[Out] (a^3*atan((a^3*tan(e/2 + (f*x)/2)*(40*A*c^3 + 23*A*d^3 + 30*B*c^3 + 21*B*d^
3 + 78*A*c*d^2 + 90*A*c^2*d + 69*B*c*d^2 + 78*B*c^2*d))/(8*(5*A*a^3*c^3 + (
23*A*a^3*d^3)/8 + (15*B*a^3*c^3)/4 + (21*B*a^3*d^3)/8 + (39*A*a^3*c*d^2)/4
+ (45*A*a^3*c^2*d)/4 + (69*B*a^3*c*d^2)/8 + (39*B*a^3*c^2*d)/4)))*(40*A*c^3
```

```
 + 23*A*d^3 + 30*B*c^3 + 21*B*d^3 + 78*A*c*d^2 + 90*A*c^2*d + 69*B*c*d^2 +
78*B*c^2*d))/(8*f) - (tan(e/2 + (f*x)/2)*(3*A*a^3*c^3 + (23*A*a^3*d^3)/8 +
(15*B*a^3*c^3)/4 + (21*B*a^3*d^3)/8 + (39*A*a^3*c*d^2)/4 + (45*A*a^3*c^2*d)
/4 + (69*B*a^3*c*d^2)/8 + (39*B*a^3*c^2*d)/4) + tan(e/2 + (f*x)/2)^10*(40*A
*a^3*c^3 + 4*A*a^3*d^3 + 24*B*a^3*c^3 + 36*A*a^3*c*d^2 + 72*A*a^3*c^2*d + 1
2*B*a^3*c*d^2 + 36*B*a^3*c^2*d) - tan(e/2 + (f*x)/2)^13*(3*A*a^3*c^3 + (23*
A*a^3*d^3)/8 + (15*B*a^3*c^3)/4 + (21*B*a^3*d^3)/8 + (39*A*a^3*c*d^2)/4 + (
45*A*a^3*c^2*d)/4 + (69*B*a^3*c*d^2)/8 + (39*B*a^3*c^2*d)/4) + tan(e/2 + (f
*x)/2)^3*(12*A*a^3*c^3 + (115*A*a^3*d^3)/6 + 17*B*a^3*c^3 + (35*B*a^3*d^3)/
2 + 57*A*a^3*c*d^2 + 51*A*a^3*c^2*d + (115*B*a^3*c*d^2)/2 + 57*B*a^3*c^2*d)
 - tan(e/2 + (f*x)/2)^11*(12*A*a^3*c^3 + (115*A*a^3*d^3)/6 + 17*B*a^3*c^3 +
 (35*B*a^3*d^3)/2 + 57*A*a^3*c*d^2 + 51*A*a^3*c^2*d + (115*B*a^3*c*d^2)/2 +
 57*B*a^3*c^2*d) + tan(e/2 + (f*x)/2)^8*((322*A*a^3*c^3)/3 + (148*A*a^3*d^3
)/3 + 82*B*a^3*c^3 + 32*B*a^3*d^3 + 188*A*a^3*c*d^2 + 246*A*a^3*c^2*d + 148
*B*a^3*c*d^2 + 188*B*a^3*c^2*d) + tan(e/2 + (f*x)/2)^6*((448*A*a^3*c^3)/3 +
 (328*A*a^3*d^3)/3 + 128*B*a^3*c^3 + 112*B*a^3*d^3 + 344*A*a^3*c*d^2 + 384*
A*a^3*c^2*d + 328*B*a^3*c*d^2 + 344*B*a^3*c^2*d) + tan(e/2 + (f*x)/2)^2*((1
36*A*a^3*c^3)/3 + (476*A*a^3*d^3)/15 + 40*B*a^3*c^3 + (144*B*a^3*d^3)/5 + (
532*A*a^3*c*d^2)/5 + 120*A*a^3*c^2*d + (476*B*a^3*c*d^2)/5 + (532*B*a^3*c^2
*d)/5) + tan(e/2 + (f*x)/2)^5*(15*A*a^3*c^3 + (841*A*a^3*d^3)/24 + (91*B*a^
3*c^3)/4 + (345*B*a^3*d^3)/8 + (339*A*a^3*c*d^2)/4 + (273*A*a^3*c^2*d)/4 +
(841*B*a^3*c*d^2)/8 + (339*B*a^3*c^2*d)/4) - tan(e/2 + (f*x)/2)^9*(15*A*a^3
*c^3 + (841*A*a^3*d^3)/24 + (91*B*a^3*c^3)/4 + (345*B*a^3*d^3)/8 + (339*A*a
^3*c*d^2)/4 + (273*A*a^3*c^2*d)/4 + (841*B*a^3*c*d^2)/8 + (339*B*a^3*c^2*d)
/4) + tan(e/2 + (f*x)/2)^4*(114*A*a^3*c^3 + (456*A*a^3*d^3)/5 + 102*B*a^3*c
^3 + (432*B*a^3*d^3)/5 + (1416*A*a^3*c*d^2)/5 + 306*A*a^3*c^2*d + (1368*B*a
^3*c*d^2)/5 + (1416*B*a^3*c^2*d)/5) + tan(e/2 + (f*x)/2)^12*(6*A*a^3*c^3 +
2*B*a^3*c^3 + 6*A*a^3*c^2*d) + (22*A*a^3*c^3)/3 + (68*A*a^3*d^3)/15 + 6*B*a
^3*c^3 + (144*B*a^3*d^3)/35 + (76*A*a^3*c*d^2)/5 + 18*A*a^3*c^2*d + (68*B*a
^3*c*d^2)/5 + (76*B*a^3*c^2*d)/5)/(f*(7*tan(e/2 + (f*x)/2)^2 + 21*tan(e/2 +
 (f*x)/2)^4 + 35*tan(e/2 + (f*x)/2)^6 + 35*tan(e/2 + (f*x)/2)^8 + 21*tan(e/
2 + (f*x)/2)^10 + 7*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (f*x)/2)^14 + 1))
```

sympy [A] time = 22.09, size = 2878, normalized size = 4.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((3*A*a**3*c**3*x*sin(e + f*x)**2/2 + 3*A*a**3*c**3*x*cos(e + f*x)
**2/2 + A*a**3*c**3*x - A*a**3*c**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a*
*3*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**3*c**3*cos(e + f*x)**3/(3*
f) - 3*A*a**3*c**3*cos(e + f*x)/f + 9*A*a**3*c**2*d*x*sin(e + f*x)**4/8 + 9
*A*a**3*c**2*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 9*A*a**3*c**2*d*x*sin(
```

e + f*x)**2/2 + 9*A*a**3*c**2*d*x*cos(e + f*x)**4/8 + 9*A*a**3*c**2*d*x*cos
(e + f*x)**2/2 - 15*A*a**3*c**2*d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 9*A*
a**3*c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 9*A*a**3*c**2*d*sin(e + f*x)*c
os(e + f*x)**3/(8*f) - 9*A*a**3*c**2*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 6*
A*a**3*c**2*d*cos(e + f*x)**3/f - 3*A*a**3*c**2*d*cos(e + f*x)/f + 27*A*a**
3*c*d**2*x*sin(e + f*x)**4/8 + 27*A*a**3*c*d**2*x*sin(e + f*x)**2*cos(e + f
*x)**2/4 + 3*A*a**3*c*d**2*x*sin(e + f*x)**2/2 + 27*A*a**3*c*d**2*x*cos(e +
 f*x)**4/8 + 3*A*a**3*c*d**2*x*cos(e + f*x)**2/2 - 3*A*a**3*c*d**2*sin(e +
f*x)**4*cos(e + f*x)/f - 45*A*a**3*c*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f
) - 4*A*a**3*c*d**2*sin(e + f*x)**2*cos(e + f*x)**3/f - 9*A*a**3*c*d**2*sin
(e + f*x)**2*cos(e + f*x)/f - 27*A*a**3*c*d**2*sin(e + f*x)*cos(e + f*x)**3
/(8*f) - 3*A*a**3*c*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 8*A*a**3*c*d**2*
cos(e + f*x)**5/(5*f) - 6*A*a**3*c*d**2*cos(e + f*x)**3/f + 5*A*a**3*d**3*x
*sin(e + f*x)**6/16 + 15*A*a**3*d**3*x*sin(e + f*x)**4*cos(e + f*x)**2/16 +
 9*A*a**3*d**3*x*sin(e + f*x)**4/8 + 15*A*a**3*d**3*x*sin(e + f*x)**2*cos(e
 + f*x)**4/16 + 9*A*a**3*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 5*A*a**
3*d**3*x*cos(e + f*x)**6/16 + 9*A*a**3*d**3*x*cos(e + f*x)**4/8 - 11*A*a**3
*d**3*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 3*A*a**3*d**3*sin(e + f*x)**4*c
os(e + f*x)/f - 5*A*a**3*d**3*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 15*A*
a**3*d**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*A*a**3*d**3*sin(e + f*x)**
2*cos(e + f*x)**3/f - A*a**3*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 5*A*a**3
*d**3*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 9*A*a**3*d**3*sin(e + f*x)*cos(
e + f*x)**3/(8*f) - 8*A*a**3*d**3*cos(e + f*x)**5/(5*f) - 2*A*a**3*d**3*cos
(e + f*x)**3/(3*f) + 3*B*a**3*c**3*x*sin(e + f*x)**4/8 + 3*B*a**3*c**3*x*si
n(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a**3*c**3*x*sin(e + f*x)**2/2 + 3*B*a
3*c3*x*cos(e + f*x)**4/8 + 3*B*a**3*c**3*x*cos(e + f*x)**2/2 - 5*B*a**3
*c**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*B*a**3*c**3*sin(e + f*x)**2*co
s(e + f*x)/f - 3*B*a**3*c**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 3*B*a**3*
c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*B*a**3*c**3*cos(e + f*x)**3/f - B*
a**3*c**3*cos(e + f*x)/f + 27*B*a**3*c**2*d*x*sin(e + f*x)**4/8 + 27*B*a**3
*c**2*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a**3*c**2*d*x*sin(e + f*x
)**2/2 + 27*B*a**3*c**2*d*x*cos(e + f*x)**4/8 + 3*B*a**3*c**2*d*x*cos(e + f
*x)**2/2 - 3*B*a**3*c**2*d*sin(e + f*x)**4*cos(e + f*x)/f - 45*B*a**3*c**2*
d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*B*a**3*c**2*d*sin(e + f*x)**2*cos(
e + f*x)**3/f - 9*B*a**3*c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 27*B*a**3*
c**2*d*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 3*B*a**3*c**2*d*sin(e + f*x)*co
s(e + f*x)/(2*f) - 8*B*a**3*c**2*d*cos(e + f*x)**5/(5*f) - 6*B*a**3*c**2*d*
cos(e + f*x)**3/f + 15*B*a**3*c*d**2*x*sin(e + f*x)**6/16 + 45*B*a**3*c*d**
2*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 27*B*a**3*c*d**2*x*sin(e + f*x)**4
/8 + 45*B*a**3*c*d**2*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 27*B*a**3*c*d*
*2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 15*B*a**3*c*d**2*x*cos(e + f*x)**6
/16 + 27*B*a**3*c*d**2*x*cos(e + f*x)**4/8 - 33*B*a**3*c*d**2*sin(e + f*x)*
*5*cos(e + f*x)/(16*f) - 9*B*a**3*c*d**2*sin(e + f*x)**4*cos(e + f*x)/f - 5
*B*a**3*c*d**2*sin(e + f*x)**3*cos(e + f*x)**3/(2*f) - 45*B*a**3*c*d**2*sin
(e + f*x)**3*cos(e + f*x)/(8*f) - 12*B*a**3*c*d**2*sin(e + f*x)**2*cos(e +

```
f*x)**3/f - 3*B*a**3*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 15*B*a**3*c*d*
*2*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 27*B*a**3*c*d**2*sin(e + f*x)*cos(
e + f*x)**3/(8*f) - 24*B*a**3*c*d**2*cos(e + f*x)**5/(5*f) - 2*B*a**3*c*d**
2*cos(e + f*x)**3/f + 15*B*a**3*d**3*x*sin(e + f*x)**6/16 + 45*B*a**3*d**3*
x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 3*B*a**3*d**3*x*sin(e + f*x)**4/8 +
45*B*a**3*d**3*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 3*B*a**3*d**3*x*sin(e
 + f*x)**2*cos(e + f*x)**2/4 + 15*B*a**3*d**3*x*cos(e + f*x)**6/16 + 3*B*a*
*3*d**3*x*cos(e + f*x)**4/8 - B*a**3*d**3*sin(e + f*x)**6*cos(e + f*x)/f -
33*B*a**3*d**3*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 2*B*a**3*d**3*sin(e +
f*x)**4*cos(e + f*x)**3/f - 3*B*a**3*d**3*sin(e + f*x)**4*cos(e + f*x)/f -
5*B*a**3*d**3*sin(e + f*x)**3*cos(e + f*x)**3/(2*f) - 5*B*a**3*d**3*sin(e +
 f*x)**3*cos(e + f*x)/(8*f) - 8*B*a**3*d**3*sin(e + f*x)**2*cos(e + f*x)**5
/(5*f) - 4*B*a**3*d**3*sin(e + f*x)**2*cos(e + f*x)**3/f - 15*B*a**3*d**3*s
in(e + f*x)*cos(e + f*x)**5/(16*f) - 3*B*a**3*d**3*sin(e + f*x)*cos(e + f*x
)**3/(8*f) - 16*B*a**3*d**3*cos(e + f*x)**7/(35*f) - 8*B*a**3*d**3*cos(e +
f*x)**5/(5*f), Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))**3*(a*sin(e) + a
)**3, True))
```

3.259 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=463

$$\frac{1}{16}a^3x\left(A\left(40c^2+60cd+26d^2\right)+B\left(30c^2+52cd+23d^2\right)\right)+\frac{a^3\left(2Ad(2c-11d)-B\left(2c^2-8cd+21d^2\right)\right)\cos(e+f$$
$$40d^3f$$

```
[Out] 1/16*a^3*(B*(30*c^2+52*c*d+23*d^2)+A*(40*c^2+60*c*d+26*d^2))*x-1/60*a^3*(2*
A*d*(2*c^4-15*c^3*d+72*c^2*d^2+180*c*d^3+76*d^4)-B*(2*c^5-12*c^4*d+37*c^3*d
^2-112*c^2*d^3-304*c*d^4-136*d^5))*cos(f*x+e)/d^3/f-1/240*a^3*(2*A*d*(4*c^3
-30*c^2*d+146*c*d^2+195*d^3)-B*(4*c^4-24*c^3*d+76*c^2*d^2-236*c*d^3-345*d^4
))*cos(f*x+e)*sin(f*x+e)/d^2/f-1/120*a^3*(2*A*d*(2*c^2-15*c*d+76*d^2)-B*(2*
c^3-12*c^2*d+41*c*d^2-136*d^3))*cos(f*x+e)*(c+d*sin(f*x+e))^2/d^3/f+1/40*a^
3*(2*A*(2*c-11*d)*d-B*(2*c^2-8*c*d+21*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^3/d
^3/f-1/6*a*B*cos(f*x+e)*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^3/d/f+1/30*(-6*
A*d+3*B*c-8*B*d)*cos(f*x+e)*(a^3+a^3*sin(f*x+e))*(c+d*sin(f*x+e))^3/d^2/f
```

Rubi [A] time = 1.13, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2976, 2968, 3023, 2753, 2734}

$$-\frac{a^3\left(2Ad\left(72c^2d^2-15c^3d+2c^4+180cd^3+76d^4\right)-B\left(37c^3d^2-112c^2d^3-12c^4d+2c^5-304cd^4-136d^5\right)\right)\cos(e}{60d^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a^3*(B*(30*c^2 + 52*c*d + 23*d^2) + A*(40*c^2 + 60*c*d + 26*d^2))*x)/16 -
(a^3*(2*A*d*(2*c^4 - 15*c^3*d + 72*c^2*d^2 + 180*c*d^3 + 76*d^4) - B*(2*c^5
 - 12*c^4*d + 37*c^3*d^2 - 112*c^2*d^3 - 304*c*d^4 - 136*d^5))*Cos[e + f*x]
)/(60*d^3*f) - (a^3*(2*A*d*(4*c^3 - 30*c^2*d + 146*c*d^2 + 195*d^3) - B*(4*
c^4 - 24*c^3*d + 76*c^2*d^2 - 236*c*d^3 - 345*d^4))*Cos[e + f*x]*Sin[e + f*
x])/(240*d^2*f) - (a^3*(2*A*d*(2*c^2 - 15*c*d + 76*d^2) - B*(2*c^3 - 12*c^2
*d + 41*c*d^2 - 136*d^3))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(120*d^3*f)
+ (a^3*(2*A*(2*c - 11*d)*d - B*(2*c^2 - 8*c*d + 21*d^2))*Cos[e + f*x]*(c +
d*Sin[e + f*x])^3)/(40*d^3*f) - (a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^2*(c
 + d*Sin[e + f*x])^3)/(6*d*f) + ((3*B*c - 6*A*d - 8*B*d)*Cos[e + f*x]*(a^3
+ a^3*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(30*d^2*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
```

```
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin}{6df}$$

$$= -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin}{6df}$$

$$= -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin}{6df}$$

$$= \frac{a^3\left(2A(2c-11d)d-B\left(2c^2-8cd+21d^2\right)\right)\cos}{40d^3f}$$

$$= -\frac{a^3\left(2Ad\left(2c^2-15cd+76d^2\right)-B\left(2c^3-12c^2\right.\right.}{}$$

$$= \frac{1}{16}a^3\left(B\left(30c^2+52cd+23d^2\right)+A\left(40c^2+60\right.\right.$$

Mathematica [A] time = 2.42, size = 355, normalized size = 0.77

$$-\frac{a^3\cos(e+fx)\left(60\left(A\left(40c^2+60cd+26d^2\right)+B\left(30c^2+52cd+23d^2\right)\right)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\,\left(-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
2,x]
```

```
[Out] -1/480*(a^3*Cos[e + f*x]*(60*(B*(30*c^2 + 52*c*d + 23*d^2) + A*(40*c^2 + 60
*c*d + 26*d^2))*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^
2]*(1840*A*c^2 + 1680*B*c^2 + 3360*A*c*d + 3112*B*c*d + 1556*A*d^2 + 1468*B
*d^2 - 16*(A*(5*c^2 + 30*c*d + 22*d^2) + B*(15*c^2 + 44*c*d + 26*d^2))*Cos[
2*(e + f*x)] + 12*d*(2*B*c + A*d + 3*B*d)*Cos[4*(e + f*x)] + 720*A*c^2*Sin[
e + f*x] + 990*B*c^2*Sin[e + f*x] + 1980*A*c*d*Sin[e + f*x] + 2100*B*c*d*Si
n[e + f*x] + 1050*A*d^2*Sin[e + f*x] + 1085*B*d^2*Sin[e + f*x] - 30*B*c^2*S
in[3*(e + f*x)] - 60*A*c*d*Sin[3*(e + f*x)] - 180*B*c*d*Sin[3*(e + f*x)] -
90*A*d^2*Sin[3*(e + f*x)] - 140*B*d^2*Sin[3*(e + f*x)] + 5*B*d^2*Sin[5*(e +
 f*x)])))/(f*Sqrt[Cos[e + f*x]^2])
```

fricas [A] time = 0.49, size = 299, normalized size = 0.65

$$-\frac{48\left(2\,Ba^3cd+(A+3\,B)a^3d^2\right)\cos\left(fx+e\right)^5-80\left((A+3\,B)a^3c^2+2\,(3\,A+5\,B)a^3cd+(5\,A+7\,B)a^3d^2\right)\cos\left(fx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/240*(48*(2*B*a^3*c*d + (A + 3*B)*a^3*d^2)*cos(f*x + e)^5 - 80*((A + 3*B)
*a^3*c^2 + 2*(3*A + 5*B)*a^3*c*d + (5*A + 7*B)*a^3*d^2)*cos(f*x + e)^3 - 15
*(10*(4*A + 3*B)*a^3*c^2 + 4*(15*A + 13*B)*a^3*c*d + (26*A + 23*B)*a^3*d^2)
*f*x + 960*((A + B)*a^3*c^2 + 2*(A + B)*a^3*c*d + (A + B)*a^3*d^2)*cos(f*x
+ e) + 5*(8*B*a^3*d^2*cos(f*x + e)^5 - 2*(6*B*a^3*c^2 + 12*(A + 3*B)*a^3*c*
d + (18*A + 31*B)*a^3*d^2)*cos(f*x + e)^3 + 3*(2*(12*A + 17*B)*a^3*c^2 + 4*
(17*A + 19*B)*a^3*c*d + (38*A + 41*B)*a^3*d^2)*cos(f*x + e))*sin(f*x + e))/
f
```

giac [A] time = 0.21, size = 380, normalized size = 0.82

$$-\frac{Ba^3d^2\sin\left(6\,fx+6\,e\right)}{192\,f}+\frac{1}{16}\left(40\,Aa^3c^2+30\,Ba^3c^2+60\,Aa^3cd+52\,Ba^3cd+26\,Aa^3d^2+23\,Ba^3d^2\right)x-\frac{\left(2\,Ba^3cd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] -1/192*B*a^3*d^2*sin(6*f*x + 6*e)/f + 1/16*(40*A*a^3*c^2 + 30*B*a^3*c^2 + 6
0*A*a^3*c*d + 52*B*a^3*c*d + 26*A*a^3*d^2 + 23*B*a^3*d^2)*x - 1/80*(2*B*a^3
*c*d + A*a^3*d^2 + 3*B*a^3*d^2)*cos(5*f*x + 5*e)/f + 1/48*(4*A*a^3*c^2 + 12
*B*a^3*c^2 + 24*A*a^3*c*d + 34*B*a^3*c*d + 17*A*a^3*d^2 + 19*B*a^3*d^2)*cos
(3*f*x + 3*e)/f - 1/8*(30*A*a^3*c^2 + 26*B*a^3*c^2 + 52*A*a^3*c*d + 46*B*a^
3*c*d + 23*A*a^3*d^2 + 21*B*a^3*d^2)*cos(f*x + e)/f + 1/64*(2*B*a^3*c^2 + 4
*A*a^3*c*d + 12*B*a^3*c*d + 6*A*a^3*d^2 + 9*B*a^3*d^2)*sin(4*f*x + 4*e)/f -
 1/64*(48*A*a^3*c^2 + 64*B*a^3*c^2 + 128*A*a^3*c*d + 128*B*a^3*c*d + 64*A*a
^3*d^2 + 63*B*a^3*d^2)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.62, size = 725, normalized size = 1.57

$$\frac{-\frac{a^3Ac^2\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}+2a^3Acd\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{a^3Ad^2\left(\frac{8}{3}+\sin^4\left(fx+e\right)+\frac{4\left(\sin^2\left(fx+e\right)\right)}{3}\right.}{5}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(-1/3*a^3*A*c^2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a^3*A*c*d*(-1/4*(sin(f*x+
e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*a^3*A*d^2*(8/3+sin(f*x+e
)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+B*a^3*c^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+
e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/5*B*a^3*c*d*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e
)^2)*cos(f*x+e)+B*a^3*d^2*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x
+e))*cos(f*x+e)+5/16*f*x+5/16*e)+3*a^3*A*c^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/
2*f*x+1/2*e)-2*a^3*A*c*d*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a^3*A*d^2*(-1/4*(sin
(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-B*a^3*c^2*(2+sin(f*x+e)
^2)*cos(f*x+e)+6*B*a^3*c*d*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3
/8*f*x+3/8*e)-3/5*B*a^3*d^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-
3*a^3*A*c^2*cos(f*x+e)+6*a^3*A*c*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*
e)-a^3*A*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+3*B*a^3*c^2*(-1/2*sin(f*x+e)*cos(f
*x+e)+1/2*f*x+1/2*e)-2*B*a^3*c*d*(2+sin(f*x+e)^2)*cos(f*x+e)+3*B*a^3*d^2*(-
1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+a^3*A*c^2*(f*x+
e)-2*a^3*A*c*d*cos(f*x+e)+a^3*A*d^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2
*e)-B*a^3*c^2*cos(f*x+e)+2*B*a^3*c*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/
2*e)-1/3*B*a^3*d^2*(2+sin(f*x+e)^2)*cos(f*x+e))
```

maxima [A] time = 0.46, size = 704, normalized size = 1.52

$$\frac{320\left(\cos\left(fx+e\right)^{3}-3\cos\left(fx+e\right)\right)Aa^{3}c^{2}+720\left(2fx+2e-\sin\left(2fx+2e\right)\right)Aa^{3}c^{2}+960\left(fx+e\right)Aa^{3}c^{2}+96}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/960*(320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c^2 + 720*(2*f*x + 2*e -
 sin(2*f*x + 2*e))*A*a^3*c^2 + 960*(f*x + e)*A*a^3*c^2 + 960*(cos(f*x + e)^
3 - 3*cos(f*x + e))*B*a^3*c^2 + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*si
n(2*f*x + 2*e))*B*a^3*c^2 + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c^2
+ 1920*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c*d + 60*(12*f*x + 12*e + si
n(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c*d + 1440*(2*f*x + 2*e - sin(2*
f*x + 2*e))*A*a^3*c*d - 128*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(
f*x + e))*B*a^3*c*d + 1920*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*c*d + 18
0*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c*d + 480*(
2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c*d - 64*(3*cos(f*x + e)^5 - 10*cos(f
*x + e)^3 + 15*cos(f*x + e))*A*a^3*d^2 + 960*(cos(f*x + e)^3 - 3*cos(f*x +
e))*A*a^3*d^2 + 90*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*
A*a^3*d^2 + 240*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*d^2 - 192*(3*cos(f*x
 + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*d^2 + 320*(cos(f*x + e
)^3 - 3*cos(f*x + e))*B*a^3*d^2 + 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e +
 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^3*d^2 + 90*(12*f*x + 12*e +
```

```
sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*d^2 - 2880*A*a^3*c^2*cos(f*x +
 e) - 960*B*a^3*c^2*cos(f*x + e) - 1920*A*a^3*c*d*cos(f*x + e))/f
```

mupad [B] time = 15.63, size = 976, normalized size = 2.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^2,x)
```

```
[Out] (a^3*atan((a^3*tan(e/2 + (f*x)/2)*(40*A*c^2 + 26*A*d^2 + 30*B*c^2 + 23*B*d^
2 + 60*A*c*d + 52*B*c*d))/(8*(5*A*a^3*c^2 + (13*A*a^3*d^2)/4 + (15*B*a^3*c^
2)/4 + (23*B*a^3*d^2)/8 + (15*A*a^3*c*d)/2 + (13*B*a^3*c*d)/2)))*(40*A*c^2
+ 26*A*d^2 + 30*B*c^2 + 23*B*d^2 + 60*A*c*d + 52*B*c*d))/(8*f) - (tan(e/2 +
 (f*x)/2)^10*(6*A*a^3*c^2 + 2*B*a^3*c^2 + 4*A*a^3*c*d) + tan(e/2 + (f*x)/2)
*(3*A*a^3*c^2 + (13*A*a^3*d^2)/4 + (15*B*a^3*c^2)/4 + (23*B*a^3*d^2)/8 + (1
5*A*a^3*c*d)/2 + (13*B*a^3*c*d)/2) - tan(e/2 + (f*x)/2)^11*(3*A*a^3*c^2 + (
13*A*a^3*d^2)/4 + (15*B*a^3*c^2)/4 + (23*B*a^3*d^2)/8 + (15*A*a^3*c*d)/2 +
(13*B*a^3*c*d)/2) + tan(e/2 + (f*x)/2)^8*(34*A*a^3*c^2 + 12*A*a^3*d^2 + 22*
B*a^3*c^2 + 4*B*a^3*d^2 + 44*A*a^3*c*d + 24*B*a^3*c*d) + tan(e/2 + (f*x)/2)
^5*(6*A*a^3*c^2 + (25*A*a^3*d^2)/2 + (19*B*a^3*c^2)/2 + (75*B*a^3*d^2)/4 +
19*A*a^3*c*d + 25*B*a^3*c*d) - tan(e/2 + (f*x)/2)^7*(6*A*a^3*c^2 + (25*A*a^
3*d^2)/2 + (19*B*a^3*c^2)/2 + (75*B*a^3*d^2)/4 + 19*A*a^3*c*d + 25*B*a^3*c*
d) + tan(e/2 + (f*x)/2)^4*(76*A*a^3*c^2 + 64*A*a^3*d^2 + 68*B*a^3*c^2 + 64*
B*a^3*d^2 + 136*A*a^3*c*d + 128*B*a^3*c*d) + tan(e/2 + (f*x)/2)^3*(9*A*a^3*
c^2 + (63*A*a^3*d^2)/4 + (53*B*a^3*c^2)/4 + (391*B*a^3*d^2)/24 + (53*A*a^3*
c*d)/2 + (63*B*a^3*c*d)/2) - tan(e/2 + (f*x)/2)^9*(9*A*a^3*c^2 + (63*A*a^3*
d^2)/4 + (53*B*a^3*c^2)/4 + (391*B*a^3*d^2)/24 + (53*A*a^3*c*d)/2 + (63*B*a
^3*c*d)/2) + tan(e/2 + (f*x)/2)^2*(38*A*a^3*c^2 + (152*A*a^3*d^2)/5 + 34*B*
a^3*c^2 + (136*B*a^3*d^2)/5 + 68*A*a^3*c*d + (304*B*a^3*c*d)/5) + tan(e/2 +
 (f*x)/2)^6*((220*A*a^3*c^2)/3 + (152*A*a^3*d^2)/3 + 60*B*a^3*c^2 + (136*B*
a^3*d^2)/3 + 120*A*a^3*c*d + (304*B*a^3*c*d)/3) + (22*A*a^3*c^2)/3 + (76*A*
a^3*d^2)/15 + 6*B*a^3*c^2 + (68*B*a^3*d^2)/15 + 12*A*a^3*c*d + (152*B*a^3*c
*d)/15)/(f*(6*tan(e/2 + (f*x)/2)^2 + 15*tan(e/2 + (f*x)/2)^4 + 20*tan(e/2 +
 (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 + 6*tan(e/2 + (f*x)/2)^10 + tan(e/2 +
 (f*x)/2)^12 + 1))
```

sympy [A] time = 10.85, size = 1804, normalized size = 3.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*A*a**3*c**2*x*sin(e + f*x)**2/2 + 3*A*a**3*c**2*x*cos(e + f*x)
**2/2 + A*a**3*c**2*x - A*a**3*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a*
*3*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**3*c**2*cos(e + f*x)**3/(3*
f) - 3*A*a**3*c**2*cos(e + f*x)/f + 3*A*a**3*c*d*x*sin(e + f*x)**4/4 + 3*A*
a**3*c*d*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + 3*A*a**3*c*d*x*sin(e + f*x)*
*2 + 3*A*a**3*c*d*x*cos(e + f*x)**4/4 + 3*A*a**3*c*d*x*cos(e + f*x)**2 - 5*
A*a**3*c*d*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 6*A*a**3*c*d*sin(e + f*x)**
2*cos(e + f*x)/f - 3*A*a**3*c*d*sin(e + f*x)*cos(e + f*x)**3/(4*f) - 3*A*a*
*3*c*d*sin(e + f*x)*cos(e + f*x)/f - 4*A*a**3*c*d*cos(e + f*x)**3/f - 2*A*a
**3*c*d*cos(e + f*x)/f + 9*A*a**3*d**2*x*sin(e + f*x)**4/8 + 9*A*a**3*d**2*
x*sin(e + f*x)**2*cos(e + f*x)**2/4 + A*a**3*d**2*x*sin(e + f*x)**2/2 + 9*A
*a**3*d**2*x*cos(e + f*x)**4/8 + A*a**3*d**2*x*cos(e + f*x)**2/2 - A*a**3*d
**2*sin(e + f*x)**4*cos(e + f*x)/f - 15*A*a**3*d**2*sin(e + f*x)**3*cos(e +
 f*x)/(8*f) - 4*A*a**3*d**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 3*A*a**
3*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 9*A*a**3*d**2*sin(e + f*x)*cos(e +
f*x)**3/(8*f) - A*a**3*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 8*A*a**3*d**2
*cos(e + f*x)**5/(15*f) - 2*A*a**3*d**2*cos(e + f*x)**3/f + 3*B*a**3*c**2*x
*sin(e + f*x)**4/8 + 3*B*a**3*c**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*
B*a**3*c**2*x*sin(e + f*x)**2/2 + 3*B*a**3*c**2*x*cos(e + f*x)**4/8 + 3*B*a
**3*c**2*x*cos(e + f*x)**2/2 - 5*B*a**3*c**2*sin(e + f*x)**3*cos(e + f*x)/(
8*f) - 3*B*a**3*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**3*c**2*sin(e +
 f*x)*cos(e + f*x)**3/(8*f) - 3*B*a**3*c**2*sin(e + f*x)*cos(e + f*x)/(2*f)
 - 2*B*a**3*c**2*cos(e + f*x)**3/f - B*a**3*c**2*cos(e + f*x)/f + 9*B*a**3*
c*d*x*sin(e + f*x)**4/4 + 9*B*a**3*c*d*x*sin(e + f*x)**2*cos(e + f*x)**2/2
+ B*a**3*c*d*x*sin(e + f*x)**2 + 9*B*a**3*c*d*x*cos(e + f*x)**4/4 + B*a**3*
c*d*x*cos(e + f*x)**2 - 2*B*a**3*c*d*sin(e + f*x)**4*cos(e + f*x)/f - 15*B*
a**3*c*d*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 8*B*a**3*c*d*sin(e + f*x)**2*
cos(e + f*x)**3/(3*f) - 6*B*a**3*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 9*B*a
**3*c*d*sin(e + f*x)*cos(e + f*x)**3/(4*f) - B*a**3*c*d*sin(e + f*x)*cos(e
+ f*x)/f - 16*B*a**3*c*d*cos(e + f*x)**5/(15*f) - 4*B*a**3*c*d*cos(e + f*x)
**3/f + 5*B*a**3*d**2*x*sin(e + f*x)**6/16 + 15*B*a**3*d**2*x*sin(e + f*x)*
*4*cos(e + f*x)**2/16 + 9*B*a**3*d**2*x*sin(e + f*x)**4/8 + 15*B*a**3*d**2*
x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 9*B*a**3*d**2*x*sin(e + f*x)**2*cos(
e + f*x)**2/4 + 5*B*a**3*d**2*x*cos(e + f*x)**6/16 + 9*B*a**3*d**2*x*cos(e
+ f*x)**4/8 - 11*B*a**3*d**2*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 3*B*a**3
*d**2*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a**3*d**2*sin(e + f*x)**3*cos(e
+ f*x)**3/(6*f) - 15*B*a**3*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*B*a
**3*d**2*sin(e + f*x)**2*cos(e + f*x)**3/f - B*a**3*d**2*sin(e + f*x)**2*co
s(e + f*x)/f - 5*B*a**3*d**2*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 9*B*a**3
*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 8*B*a**3*d**2*cos(e + f*x)**5/(5
*f) - 2*B*a**3*d**2*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(A + B*sin(e))*(c
+ d*sin(e))**2*(a*sin(e) + a)**3, True))
```

3.260 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=201

$$\frac{a^3(20Ac+15Ad+15Bc+13Bd)\cos^3(e+fx)}{60f}-\frac{a^3(20Ac+15Ad+15Bc+13Bd)\cos(e+fx)}{5f}-\frac{3a^3(20Ac+15A}{}$$

```
[Out] 1/8*a^3*(20*A*c+15*A*d+15*B*c+13*B*d)*x-1/5*a^3*(20*A*c+15*A*d+15*B*c+13*B*
d)*cos(f*x+e)/f+1/60*a^3*(20*A*c+15*A*d+15*B*c+13*B*d)*cos(f*x+e)^3/f-3/40*
a^3*(20*A*c+15*A*d+15*B*c+13*B*d)*cos(f*x+e)*sin(f*x+e)/f-1/20*(5*A*d+5*B*c
-B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^3/f-1/5*B*d*cos(f*x+e)*(a+a*sin(f*x+e))^4
/a/f
```

Rubi [A] time = 0.33, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2968, 3023, 2751, 2645, 2638, 2635, 8, 2633}

$$\frac{a^3(20Ac+15Ad+15Bc+13Bd)\cos^3(e+fx)}{60f}-\frac{a^3(20Ac+15Ad+15Bc+13Bd)\cos(e+fx)}{5f}-\frac{3a^3(20Ac+15A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (a^3*(20*A*c + 15*B*c + 15*A*d + 13*B*d)*x)/8 - (a^3*(20*A*c + 15*B*c + 15*
A*d + 13*B*d)*Cos[e + f*x])/(5*f) + (a^3*(20*A*c + 15*B*c + 15*A*d + 13*B*d
)*Cos[e + f*x]^3)/(60*f) - (3*a^3*(20*A*c + 15*B*c + 15*A*d + 13*B*d)*Cos[e
 + f*x]*Sin[e + f*x])/(40*f) - ((5*B*c + 5*A*d - B*d)*Cos[e + f*x]*(a + a*S
in[e + f*x])^3)/(20*f) - (B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^4)/(5*a*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int (a+a\sin(e+fx))^3\left(Ac+(Bc+Ad)\sin(e\right.$$

$$= -\frac{Bd\cos(e+fx)(a+a\sin(e+fx))^4}{5af} + \frac{\int(a+}{}$$

$$= -\frac{(5Bc+5Ad-Bd)\cos(e+fx)(a+a\sin(e+}{20f}$$

$$= -\frac{(5Bc+5Ad-Bd)\cos(e+fx)(a+a\sin(e+}{20f}$$

$$= \frac{1}{20}a^3(20Ac+15Bc+15Ad+13Bd)x - \frac{(5Bc}{}$$

$$= \frac{1}{20}a^3(20Ac+15Bc+15Ad+13Bd)x - \frac{3a^3(2}{}$$

$$= \frac{1}{8}a^3(20Ac+15Bc+15Ad+13Bd)x - \frac{a^3(20A}{}$$

Mathematica [A] time = 0.91, size = 156, normalized size = 0.78

$$\frac{\cos(e+fx)\left(-\frac{1}{4}a^4(5Ad+5Bc-Bd)(\sin(e+fx)+1)^3 - \frac{a^4(20Ac+15Ad+15Bc+13Bd)\left(30\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\left(2\sin^2(e+fx)+9\right.\right.}{24\sqrt{\cos^2(e+fx)}}\right.}{5af}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),
x]
```

```
[Out] (Cos[e + f*x]*(-1/4*(a^4*(5*B*c + 5*A*d - B*d)*(1 + Sin[e + f*x])^3) - B*d*
(a + a*Sin[e + f*x])^4 - (a^4*(20*A*c + 15*B*c + 15*A*d + 13*B*d)*(30*ArcSi
n[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(22 + 9*Sin[e + f*
x] + 2*Sin[e + f*x]^2)))/(24*Sqrt[Cos[e + f*x]^2])))/(5*a*f)
```

fricas [A] time = 0.47, size = 178, normalized size = 0.89

$$-\frac{24\,Ba^3d\cos\left(fx+e\right)^5 - 40\left((A+3\,B)a^3c+(3\,A+5\,B)a^3d\right)\cos\left(fx+e\right)^3 - 15\left(5\,(4\,A+3\,B)a^3c+(15\,A+13\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] -1/120*(24*B*a^3*d*cos(f*x + e)^5 - 40*((A + 3*B)*a^3*c + (3*A + 5*B)*a^3*d
)*cos(f*x + e)^3 - 15*(5*(4*A + 3*B)*a^3*c + (15*A + 13*B)*a^3*d)*f*x + 480
*((A + B)*a^3*c + (A + B)*a^3*d)*cos(f*x + e) - 15*(2*(B*a^3*c + (A + 3*B)*
a^3*d)*cos(f*x + e)^3 - ((12*A + 17*B)*a^3*c + (17*A + 19*B)*a^3*d)*cos(f*x
 + e))*sin(f*x + e))/f
```

giac [A] time = 0.18, size = 217, normalized size = 1.08

$$-\frac{Ba^3d\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{1}{8}\left(20\,Aa^3c+15\,Ba^3c+15\,Aa^3d+13\,Ba^3d\right)x+\frac{\left(4\,Aa^3c+12\,Ba^3c+12\,Aa^3d+17\,Ba^3d\right)}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/80*B*a^3*d*cos(5*f*x + 5*e)/f + 1/8*(20*A*a^3*c + 15*B*a^3*c + 15*A*a^3*
d + 13*B*a^3*d)*x + 1/48*(4*A*a^3*c + 12*B*a^3*c + 12*A*a^3*d + 17*B*a^3*d)
*cos(3*f*x + 3*e)/f - 1/8*(30*A*a^3*c + 26*B*a^3*c + 26*A*a^3*d + 23*B*a^3*
d)*cos(f*x + e)/f + 1/32*(B*a^3*c + A*a^3*d + 3*B*a^3*d)*sin(4*f*x + 4*e)/f
 - 1/4*(3*A*a^3*c + 4*B*a^3*c + 4*A*a^3*d + 4*B*a^3*d)*sin(2*f*x + 2*e)/f
```

maple [B] time = 0.51, size = 414, normalized size = 2.06

$$\frac{-\frac{a^3Ac\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}+a^3Ad\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+B\,a^3c\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] 1/f*(-1/3*a^3*A*c*(2+sin(f*x+e)^2)*cos(f*x+e)+a^3*A*d*(-1/4*(sin(f*x+e)^3+3
/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+B*a^3*c*(-1/4*(sin(f*x+e)^3+3/2*si
n(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*B*a^3*d*(8/3+sin(f*x+e)^4+4/3*sin(f
*x+e)^2)*cos(f*x+e)+3*a^3*A*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-a^
3*A*d*(2+sin(f*x+e)^2)*cos(f*x+e)-B*a^3*c*(2+sin(f*x+e)^2)*cos(f*x+e)+3*B*a
^3*d*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3*a^3*A*
c*cos(f*x+e)+3*a^3*A*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+3*B*a^3*c
*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-B*a^3*d*(2+sin(f*x+e)^2)*cos(f*
x+e)+a^3*A*c*(f*x+e)-a^3*A*d*cos(f*x+e)-B*a^3*c*cos(f*x+e)+B*a^3*d*(-1/2*si
n(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e))
```

maxima [B] time = 0.43, size = 398, normalized size = 1.98

$$160\left(\cos\left(fx+e\right)^{3}-3\cos\left(fx+e\right)\right)Aa^{3}c+360\left(2fx+2e-\sin\left(2fx+2e\right)\right)Aa^{3}c+480\left(fx+e\right)Aa^{3}c+480$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] 1/480*(160*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c + 360*(2*f*x + 2*e - s
in(2*f*x + 2*e))*A*a^3*c + 480*(f*x + e)*A*a^3*c + 480*(cos(f*x + e)^3 - 3*
cos(f*x + e))*B*a^3*c + 15*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x
+ 2*e))*B*a^3*c + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c + 480*(cos(f
*x + e)^3 - 3*cos(f*x + e))*A*a^3*d + 15*(12*f*x + 12*e + sin(4*f*x + 4*e)
- 8*sin(2*f*x + 2*e))*A*a^3*d + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*
d - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*d + 4
80*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*d + 45*(12*f*x + 12*e + sin(4*f*
x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*d + 120*(2*f*x + 2*e - sin(2*f*x + 2*e
))*B*a^3*d - 1440*A*a^3*c*cos(f*x + e) - 480*B*a^3*c*cos(f*x + e) - 480*A*a
^3*d*cos(f*x + e))/f
```

mupad [B] time = 14.62, size = 550, normalized size = 2.74

$$\frac{a^{3}\,\mathrm{atan}\left(\frac{a^{3}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(20Ac+15Ad+15Bc+13Bd)}{4\left(5Aa^{3}c+\frac{15Aa^{3}d}{4}+\frac{15Ba^{3}c}{4}+\frac{13Ba^{3}d}{4}\right)}\right)(20Ac+15Ad+15Bc+13Bd)}{4f}-\frac{a^{3}\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\frac{fx}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c + d*sin(e + f*x)),x)
```

```
[Out] (a^3*atan((a^3*tan(e/2 + (f*x)/2)*(20*A*c + 15*A*d + 15*B*c + 13*B*d))/(4*(
5*A*a^3*c + (15*A*a^3*d)/4 + (15*B*a^3*c)/4 + (13*B*a^3*d)/4)))*(20*A*c + 1
5*A*d + 15*B*c + 13*B*d))/(4*f) - (a^3*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2)
*(20*A*c + 15*A*d + 15*B*c + 13*B*d))/(4*f) - (tan(e/2 + (f*x)/2)^3*(6*A*a^
3*c + (19*A*a^3*d)/2 + (19*B*a^3*c)/2 + (25*B*a^3*d)/2) - tan(e/2 + (f*x)/2
)^9*(3*A*a^3*c + (15*A*a^3*d)/4 + (15*B*a^3*c)/4 + (13*B*a^3*d)/4) - tan(e/
2 + (f*x)/2)^7*(6*A*a^3*c + (19*A*a^3*d)/2 + (19*B*a^3*c)/2 + (25*B*a^3*d)/
2) + tan(e/2 + (f*x)/2)^6*(28*A*a^3*c + 20*A*a^3*d + 20*B*a^3*c + 12*B*a^3*
d) + tan(e/2 + (f*x)/2)^2*((92*A*a^3*c)/3 + 28*A*a^3*d + 28*B*a^3*c + (76*B
*a^3*d)/3) + tan(e/2 + (f*x)/2)^4*((136*A*a^3*c)/3 + 40*A*a^3*d + 40*B*a^3*
c + (116*B*a^3*d)/3) + tan(e/2 + (f*x)/2)^8*(6*A*a^3*c + 2*A*a^3*d + 2*B*a^
```

```
3*c) + tan(e/2 + (f*x)/2)*(3*A*a^3*c + (15*A*a^3*d)/4 + (15*B*a^3*c)/4 + (1
3*B*a^3*d)/4) + (22*A*a^3*c)/3 + 6*A*a^3*d + 6*B*a^3*c + (76*B*a^3*d)/15)/(
f*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)
^6 + 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 + 1))
```

sympy [A] time = 5.28, size = 960, normalized size = 4.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((3*A*a**3*c*x*sin(e + f*x)**2/2 + 3*A*a**3*c*x*cos(e + f*x)**2/2
+ A*a**3*c*x - A*a**3*c*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**3*c*sin(e +
 f*x)*cos(e + f*x)/(2*f) - 2*A*a**3*c*cos(e + f*x)**3/(3*f) - 3*A*a**3*c*co
s(e + f*x)/f + 3*A*a**3*d*x*sin(e + f*x)**4/8 + 3*A*a**3*d*x*sin(e + f*x)**
2*cos(e + f*x)**2/4 + 3*A*a**3*d*x*sin(e + f*x)**2/2 + 3*A*a**3*d*x*cos(e +
 f*x)**4/8 + 3*A*a**3*d*x*cos(e + f*x)**2/2 - 5*A*a**3*d*sin(e + f*x)**3*co
s(e + f*x)/(8*f) - 3*A*a**3*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**3*d*s
in(e + f*x)*cos(e + f*x)**3/(8*f) - 3*A*a**3*d*sin(e + f*x)*cos(e + f*x)/(2
*f) - 2*A*a**3*d*cos(e + f*x)**3/f - A*a**3*d*cos(e + f*x)/f + 3*B*a**3*c*x
*sin(e + f*x)**4/8 + 3*B*a**3*c*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a
**3*c*x*sin(e + f*x)**2/2 + 3*B*a**3*c*x*cos(e + f*x)**4/8 + 3*B*a**3*c*x*c
os(e + f*x)**2/2 - 5*B*a**3*c*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*B*a**3
*c*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**3*c*sin(e + f*x)*cos(e + f*x)**3
/(8*f) - 3*B*a**3*c*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*B*a**3*c*cos(e + f*
x)**3/f - B*a**3*c*cos(e + f*x)/f + 9*B*a**3*d*x*sin(e + f*x)**4/8 + 9*B*a*
*3*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + B*a**3*d*x*sin(e + f*x)**2/2 + 9
*B*a**3*d*x*cos(e + f*x)**4/8 + B*a**3*d*x*cos(e + f*x)**2/2 - B*a**3*d*sin
(e + f*x)**4*cos(e + f*x)/f - 15*B*a**3*d*sin(e + f*x)**3*cos(e + f*x)/(8*f
) - 4*B*a**3*d*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 3*B*a**3*d*sin(e + f
*x)**2*cos(e + f*x)/f - 9*B*a**3*d*sin(e + f*x)*cos(e + f*x)**3/(8*f) - B*a
**3*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 8*B*a**3*d*cos(e + f*x)**5/(15*f) -
 2*B*a**3*d*cos(e + f*x)**3/f, Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))*
(a*sin(e) + a)**3, True))
```

3.261 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=127

$$-\frac{5a^3(4A+3B)\cos(e+fx)}{6f}-\frac{5a^3(4A+3B)\sin(e+fx)\cos(e+fx)}{24f}+\frac{5}{8}a^3x(4A+3B)-\frac{a(4A+3B)\cos(e+fx)(a}{12f}$$

```
[Out] 5/8*a^3*(4*A+3*B)*x-5/6*a^3*(4*A+3*B)*cos(f*x+e)/f-5/24*a^3*(4*A+3*B)*cos(f
*x+e)*sin(f*x+e)/f-1/12*a*(4*A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^2/f-1/4*B*c
os(f*x+e)*(a+a*sin(f*x+e))^3/f
```

Rubi [A] time = 0.10, antiderivative size = 117, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2751, 2645, 2638, 2635, 8, 2633}

$$\frac{a^3(4A+3B)\cos^3(e+fx)}{12f}-\frac{a^3(4A+3B)\cos(e+fx)}{f}-\frac{3a^3(4A+3B)\sin(e+fx)\cos(e+fx)}{8f}+\frac{5}{8}a^3x(4A+3B)-\frac{B}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]),x]
```

```
[Out] (5*a^3*(4*A + 3*B)*x)/8 - (a^3*(4*A + 3*B)*Cos[e + f*x])/f + (a^3*(4*A + 3*
B)*Cos[e + f*x]^3)/(12*f) - (3*a^3*(4*A + 3*B)*Cos[e + f*x]*Sin[e + f*x])/(
8*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^3)/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx &= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^3}{4f}+\frac{1}{4}(4A+3B)\int (a+a\sin(e\\ &= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^3}{4f}+\frac{1}{4}(4A+3B)\int \left(a^3+3a^3\sin\right.\\ &= \frac{1}{4}a^3(4A+3B)x-\frac{B\cos(e+fx)(a+a\sin(e+fx))^3}{4f}+\frac{1}{4}\left(a^3(4A\right.\\ &= \frac{1}{4}a^3(4A+3B)x-\frac{3a^3(4A+3B)\cos(e+fx)}{4f}-\frac{3a^3(4A+3B)\cos}{8}\\ &= \frac{5}{8}a^3(4A+3B)x-\frac{a^3(4A+3B)\cos(e+fx)}{f}+\frac{a^3(4A+3B)\cos^3(e}{12f}\end{aligned}$$

Mathematica [A] time = 0.50, size = 120, normalized size = 0.94

$$-\frac{a^3\cos(e+fx)\left(30(4A+3B)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\,\left(8(A+3B)\sin^2(e+fx)+9(4A+5B)\sin(e\right.\right.}{24f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]),x]
```

```
[Out] -1/24*(a^3*Cos[e + f*x]*(30*(4*A + 3*B)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[
2]] + Sqrt[Cos[e + f*x]^2]*(88*A + 72*B + 9*(4*A + 5*B)*Sin[e + f*x] + 8*(A
 + 3*B)*Sin[e + f*x]^2 + 6*B*Sin[e + f*x]^3)))/(f*Sqrt[Cos[e + f*x]^2])
```

fricas [A] time = 0.45, size = 93, normalized size = 0.73

$$\frac{8\,(A+3\,B)a^3\cos\left(fx+e\right)^3+15\,(4\,A+3\,B)a^3fx-96\,(A+B)a^3\cos\left(fx+e\right)+3\left(2\,Ba^3\cos\left(fx+e\right)^3-(12\,A\right.}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/24*(8*(A + 3*B)*a^3*cos(f*x + e)^3 + 15*(4*A + 3*B)*a^3*f*x - 96*(A + B)*
a^3*cos(f*x + e) + 3*(2*B*a^3*cos(f*x + e)^3 - (12*A + 17*B)*a^3*cos(f*x +
e))*sin(f*x + e))/f
```

giac [A] time = 0.17, size = 116, normalized size = 0.91

$$\frac{Ba^3\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{5}{8}\left(4\,Aa^3+3\,Ba^3\right)x+\frac{\left(Aa^3+3\,Ba^3\right)\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{\left(15\,Aa^3+13\,Ba^3\right)\cos\left(fx+e\right)}{4\,f}-\frac{(3\,A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/32*B*a^3*sin(4*f*x + 4*e)/f + 5/8*(4*A*a^3 + 3*B*a^3)*x + 1/12*(A*a^3 + 3
*B*a^3)*cos(3*f*x + 3*e)/f - 1/4*(15*A*a^3 + 13*B*a^3)*cos(f*x + e)/f - 1/4
*(3*A*a^3 + 4*B*a^3)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.41, size = 178, normalized size = 1.40

$$\frac{-\frac{a^3A\left(2+\sin^2\left(fx+e\right)\right)\cos\left(fx+e\right)}{3}+B\,a^3\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+3a^3A\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x)
```

```
[Out] 1/f*(-1/3*a^3*A*(2+sin(f*x+e)^2)*cos(f*x+e)+B*a^3*(-1/4*(sin(f*x+e)^3+3/2*s
in(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+3*a^3*A*(-1/2*sin(f*x+e)*cos(f*x+e)+1/
2*f*x+1/2*e)-B*a^3*(2+sin(f*x+e)^2)*cos(f*x+e)-3*a^3*A*cos(f*x+e)+3*B*a^3*(
-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+a^3*A*(f*x+e)-B*a^3*cos(f*x+e))
```

maxima [A] time = 0.38, size = 171, normalized size = 1.35

$$\frac{32\left(\cos\left(fx+e\right)^{3}-3\,\cos\left(fx+e\right)\right)Aa^{3}+72\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)Aa^{3}+96\left(fx+e\right)Aa^{3}+96\left(\cos\left(fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/96*(32*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3 + 72*(2*f*x + 2*e - sin(2*
f*x + 2*e))*A*a^3 + 96*(f*x + e)*A*a^3 + 96*(cos(f*x + e)^3 - 3*cos(f*x + e
))*B*a^3 + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3
+ 72*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3 - 288*A*a^3*cos(f*x + e) - 96*B
*a^3*cos(f*x + e))/f
```

mupad [B] time = 14.26, size = 330, normalized size = 2.60

$$\frac{5\,a^3\,\mathrm{atan}\left(\frac{5\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(4\,A+3\,B\right)}{4\,\left(5\,A\,a^3+\frac{15\,B\,a^3}{4}\right)}\right)\,\left(4\,A+3\,B\right)}{4\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(3\,A\,a^3+\frac{15\,B\,a^3}{4}\right)+\frac{22\,A\,a^3}{3}+6\,B\,a^3+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3,x)
```

```
[Out] (5*a^3*atan((5*a^3*tan(e/2 + (f*x)/2)*(4*A + 3*B))/(4*(5*A*a^3 + (15*B*a^3)
/4)))*(4*A + 3*B))/(4*f) - (tan(e/2 + (f*x)/2)*(3*A*a^3 + (15*B*a^3)/4) + (
22*A*a^3)/3 + 6*B*a^3 + tan(e/2 + (f*x)/2)^6*(6*A*a^3 + 2*B*a^3) - tan(e/2
+ (f*x)/2)^7*(3*A*a^3 + (15*B*a^3)/4) + tan(e/2 + (f*x)/2)^3*(3*A*a^3 + (23
*B*a^3)/4) - tan(e/2 + (f*x)/2)^5*(3*A*a^3 + (23*B*a^3)/4) + tan(e/2 + (f*x
)/2)^4*(22*A*a^3 + 18*B*a^3) + tan(e/2 + (f*x)/2)^2*((70*A*a^3)/3 + 22*B*a^
3))/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/2 + (f*x)
/2)^6 + tan(e/2 + (f*x)/2)^8 + 1)) - (5*a^3*(4*A + 3*B)*(atan(tan(e/2 + (f*
x)/2)) - (f*x)/2))/(4*f)
```

sympy [A] time = 2.08, size = 371, normalized size = 2.92

$$\begin{cases} \frac{3Aa^3x\sin^2\left(e+fx\right)}{2}+\frac{3Aa^3x\cos^2\left(e+fx\right)}{2}+Aa^3x-\frac{Aa^3\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{3Aa^3\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{2Aa^3\cos^3\left(e+fx\right)}{3f}-\frac{3Aa^3}{} \\ x\left(A+B\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e)),x)
```

```
[Out] Piecewise((3*A*a**3*x*sin(e + f*x)**2/2 + 3*A*a**3*x*cos(e + f*x)**2/2 + A*
a**3*x - A*a**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**3*sin(e + f*x)*cos(
e + f*x)/(2*f) - 2*A*a**3*cos(e + f*x)**3/(3*f) - 3*A*a**3*cos(e + f*x)/f +
 3*B*a**3*x*sin(e + f*x)**4/8 + 3*B*a**3*x*sin(e + f*x)**2*cos(e + f*x)**2/
4 + 3*B*a**3*x*sin(e + f*x)**2/2 + 3*B*a**3*x*cos(e + f*x)**4/8 + 3*B*a**3*
x*cos(e + f*x)**2/2 - 5*B*a**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*B*a**
3*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**3*sin(e + f*x)*cos(e + f*x)**3/(8
*f) - 3*B*a**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*B*a**3*cos(e + f*x)**3/f
 - B*a**3*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**3, T
rue))
```

3.262 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=246

$$\frac{2a^3(c-d)^3(Bc-Ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^4f\sqrt{c^2-d^2}}+\frac{a^3\left(Ad(2c-5d)-B\left(2c^2-5cd+5d^2\right)\right)\cos(e+fx)}{2d^3f}+\frac{a^3x\left(Ad\left(2c^2\right.\right.}{}$$

```
[Out] 1/2*a^3*(A*d*(2*c^2-6*c*d+7*d^2)-B*(2*c^3-6*c^2*d+7*c*d^2-5*d^3))*x/d^4+1/2
*a^3*(A*(2*c-5*d)*d-B*(2*c^2-5*c*d+5*d^2))*cos(f*x+e)/d^3/f-1/3*a*B*cos(f*x
+e)*(a+a*sin(f*x+e))^2/d/f+1/6*(-3*A*d+3*B*c-5*B*d)*cos(f*x+e)*(a^3+a^3*sin
(f*x+e))/d^2/f+2*a^3*(c-d)^3*(-A*d+B*c)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^
2-d^2)^(1/2))/d^4/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 0.90, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2976, 2968, 3023, 2735, 2660, 618, 204}

$$\frac{a^3\left(Ad(2c-5d)-B\left(2c^2-5cd+5d^2\right)\right)\cos(e+fx)}{2d^3f}+\frac{2a^3(c-d)^3(Bc-Ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^4f\sqrt{c^2-d^2}}+\frac{a^3x\left(Ad\left(2c^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```

```
[Out] (a^3*(A*d*(2*c^2 - 6*c*d + 7*d^2) - B*(2*c^3 - 6*c^2*d + 7*c*d^2 - 5*d^3))*
x)/(2*d^4) + (2*a^3*(c - d)^3*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/S
qrt[c^2 - d^2]])/(d^4*Sqrt[c^2 - d^2]*f) + (a^3*(A*(2*c - 5*d)*d - B*(2*c^2
 - 5*c*d + 5*d^2))*Cos[e + f*x])/(2*d^3*f) - (a*B*Cos[e + f*x]*(a + a*Sin[e
 + f*x])^2)/(3*d*f) + ((3*B*c - 3*A*d - 5*B*d)*Cos[e + f*x]*(a^3 + a^3*Sin[
e + f*x]))/(6*d^2*f)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2}{3df} + \frac{\int \frac{(a+a\sin(e+fx))^2(a(2Bc+3Ad)}{c+d\sin}}{3}$$

$$= -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2}{3df} + \frac{(3Bc-3Ad-5Bd)\cos(e}{6}$$

$$= -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2}{3df} + \frac{(3Bc-3Ad-5Bd)\cos(e}{6}$$

$$= \frac{a^3\left(A(2c-5d)d-B\left(2c^2-5cd+5d^2\right)\right)\cos(e+fx)}{2d^3f} - \frac{aB\cos(e+}{}$$

$$= \frac{a^3\left(Ad\left(2c^2-6cd+7d^2\right)-B\left(2c^3-6c^2d+7cd^2-5d^3\right)\right)x}{2d^4} + \frac{a^3}{}$$

$$= \frac{a^3\left(Ad\left(2c^2-6cd+7d^2\right)-B\left(2c^3-6c^2d+7cd^2-5d^3\right)\right)x}{2d^4} + \frac{a^3}{}$$

$$= \frac{a^3\left(Ad\left(2c^2-6cd+7d^2\right)-B\left(2c^3-6c^2d+7cd^2-5d^3\right)\right)x}{2d^4} + \frac{a^3}{}$$

$$= \frac{a^3\left(Ad\left(2c^2-6cd+7d^2\right)-B\left(2c^3-6c^2d+7cd^2-5d^3\right)\right)x}{2d^4} + \frac{2a^3}{}$$

Mathematica [A] time = 0.98, size = 233, normalized size = 0.95

$$\frac{a^3(\sin(e+fx)+1)^3\left(-3d\left(4Ad(3d-c)+B\left(4c^2-12cd+15d^2\right)\right)\cos(e+fx)+\frac{24(c-d)^3(Bc-Ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}+\right.}{12d^4f\left(\sin\left(\frac{1}{2}(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
),x]
```

```
[Out] (a^3*(1 + Sin[e + f*x])^3*(6*(A*d*(2*c^2 - 6*c*d + 7*d^2) + B*(-2*c^3 + 6*c
^2*d - 7*c*d^2 + 5*d^3))*(e + f*x) + (24*(c - d)^3*(B*c - A*d)*ArcTan[(d +
c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2] - 3*d*(4*A*d*(-c + 3*
d) + B*(4*c^2 - 12*c*d + 15*d^2))*Cos[e + f*x] + B*d^3*Cos[3*(e + f*x)] - 3
*d^2*(-(B*c) + A*d + 3*B*d)*Sin[2*(e + f*x)]))/(12*d^4*f*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^6)
```

fricas [A] time = 0.55, size = 627, normalized size = 2.55

$$\left[\frac{2\,Ba^3d^3\cos\left(fx+e\right)^3-3\left(2\,Ba^3c^3-2\,(A+3\,B)a^3c^2d+(6\,A+7\,B)a^3cd^2-(7\,A+5\,B)a^3d^3\right)fx+3\left(Ba^3cd^2-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] [1/6*(2*B*a^3*d^3*cos(f*x + e)^3 - 3*(2*B*a^3*c^3 - 2*(A + 3*B)*a^3*c^2*d +
 (6*A + 7*B)*a^3*c*d^2 - (7*A + 5*B)*a^3*d^3)*f*x + 3*(B*a^3*c*d^2 - (A + 3
*B)*a^3*d^3)*cos(f*x + e)*sin(f*x + e) + 3*(B*a^3*c^3 - (A + 2*B)*a^3*c^2*d
 + (2*A + B)*a^3*c*d^2 - A*a^3*d^3)*sqrt(-(c - d)/(c + d))*log(-((2*c^2 - d
^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 - 2*((c^2 + c*d)*cos(f*
x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^
2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 6*(B*a^3*c^2*d - (A +
 3*B)*a^3*c*d^2 + (3*A + 4*B)*a^3*d^3)*cos(f*x + e))/(d^4*f), 1/6*(2*B*a^3*
d^3*cos(f*x + e)^3 - 3*(2*B*a^3*c^3 - 2*(A + 3*B)*a^3*c^2*d + (6*A + 7*B)*a
^3*c*d^2 - (7*A + 5*B)*a^3*d^3)*f*x + 3*(B*a^3*c*d^2 - (A + 3*B)*a^3*d^3)*c
os(f*x + e)*sin(f*x + e) - 6*(B*a^3*c^3 - (A + 2*B)*a^3*c^2*d + (2*A + B)*a
^3*c*d^2 - A*a^3*d^3)*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sq
rt((c - d)/(c + d))/((c - d)*cos(f*x + e))) - 6*(B*a^3*c^2*d - (A + 3*B)*a^
3*c*d^2 + (3*A + 4*B)*a^3*d^3)*cos(f*x + e))/(d^4*f)]
```

giac [B] time = 0.21, size = 617, normalized size = 2.51

$$-\frac{\frac{3\left(2\,Ba^3c^3-2\,Aa^3c^2d-6\,Ba^3c^2d+6\,Aa^3cd^2+7\,Ba^3cd^2-7\,Aa^3d^3-5\,Ba^3d^3\right)\left(fx+e\right)}{d^4}-\frac{12\left(Ba^3c^4-Aa^3c^3d-3\,Ba^3c^3d+3\,Aa^3c^2d^2+3\,Ba^3c^2d^2-3\,Aa^3cd^3-\right.}{\sqrt{c^2-}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/6*(3*(2*B*a^3*c^3 - 2*A*a^3*c^2*d - 6*B*a^3*c^2*d + 6*A*a^3*c*d^2 + 7*B*
a^3*c*d^2 - 7*A*a^3*d^3 - 5*B*a^3*d^3)*(f*x + e)/d^4 - 12*(B*a^3*c^4 - A*a^
3*c^3*d - 3*B*a^3*c^3*d + 3*A*a^3*c^2*d^2 + 3*B*a^3*c^2*d^2 - 3*A*a^3*c*d^3
 - B*a^3*c*d^3 + A*a^3*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arct
an((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*d^4) + 2
*(3*B*a^3*c*d*tan(1/2*f*x + 1/2*e)^5 - 3*A*a^3*d^2*tan(1/2*f*x + 1/2*e)^5 -
 9*B*a^3*d^2*tan(1/2*f*x + 1/2*e)^5 + 6*B*a^3*c^2*tan(1/2*f*x + 1/2*e)^4 -
6*A*a^3*c*d*tan(1/2*f*x + 1/2*e)^4 - 18*B*a^3*c*d*tan(1/2*f*x + 1/2*e)^4 +
18*A*a^3*d^2*tan(1/2*f*x + 1/2*e)^4 + 18*B*a^3*d^2*tan(1/2*f*x + 1/2*e)^4 +
 12*B*a^3*c^2*tan(1/2*f*x + 1/2*e)^2 - 12*A*a^3*c*d*tan(1/2*f*x + 1/2*e)^2
- 36*B*a^3*c*d*tan(1/2*f*x + 1/2*e)^2 + 36*A*a^3*d^2*tan(1/2*f*x + 1/2*e)^2
 + 48*B*a^3*d^2*tan(1/2*f*x + 1/2*e)^2 - 3*B*a^3*c*d*tan(1/2*f*x + 1/2*e) +
 3*A*a^3*d^2*tan(1/2*f*x + 1/2*e) + 9*B*a^3*d^2*tan(1/2*f*x + 1/2*e) + 6*B*
a^3*c^2 - 6*A*a^3*c*d - 18*B*a^3*c*d + 18*A*a^3*d^2 + 22*B*a^3*d^2)/((tan(1
/2*f*x + 1/2*e)^2 + 1)^3*d^3))/f
```

maple [B] time = 0.48, size = 1357, normalized size = 5.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/f*a^3/d^2/(1+tan(1/2*f*x+1/2*e)^2)^3*A*tan(1/2*f*x+1/2*e)^4*c-2/f*a^3/d^3
/(1+tan(1/2*f*x+1/2*e)^2)^3*B*tan(1/2*f*x+1/2*e)^4*c^2+6/f*a^3/d^2/(1+tan(1
/2*f*x+1/2*e)^2)^3*B*tan(1/2*f*x+1/2*e)^4*c+4/f*a^3/d^2/(1+tan(1/2*f*x+1/2*
e)^2)^3*A*tan(1/2*f*x+1/2*e)^2*c-4/f*a^3/d^3/(1+tan(1/2*f*x+1/2*e)^2)^3*B*t
an(1/2*f*x+1/2*e)^2*c^2+12/f*a^3/d^2/(1+tan(1/2*f*x+1/2*e)^2)^3*B*tan(1/2*f
*x+1/2*e)^2*c+1/f*a^3/d^2/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*B*c
-16/f*a^3/d/(1+tan(1/2*f*x+1/2*e)^2)^3*B*tan(1/2*f*x+1/2*e)^2-1/f*a^3/d/(1+
tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*A+6/f*a^3/d^3*arctan(tan(1/2*f*x
+1/2*e))*B*c^2-7/f*a^3/d^2*arctan(tan(1/2*f*x+1/2*e))*B*c-3/f*a^3/d/(1+tan(
1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*B+2/f*a^3/d^2/(1+tan(1/2*f*x+1/2*e)^
2)^3*A*c-2/f*a^3/d^3/(1+tan(1/2*f*x+1/2*e)^2)^3*B*c^2+2/f*a^3/d^3*arctan(ta
n(1/2*f*x+1/2*e))*A*c^2-6/f*a^3/d^2*arctan(tan(1/2*f*x+1/2*e))*A*c+6/f*a^3/
d^2/(1+tan(1/2*f*x+1/2*e)^2)^3*B*c-2/f*a^3/d^4*arctan(tan(1/2*f*x+1/2*e))*B
*c^3+1/f*a^3/d/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*A+3/f*a^3/d/
(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*B-6/f*a^3/d/(1+tan(1/2*f*x+
1/2*e)^2)^3*A*tan(1/2*f*x+1/2*e)^4-6/f*a^3/d/(1+tan(1/2*f*x+1/2*e)^2)^3*B*t
an(1/2*f*x+1/2*e)^4-12/f*a^3/d/(1+tan(1/2*f*x+1/2*e)^2)^3*A*tan(1/2*f*x+1/2
*e)^2-2/f*a^3/d^3/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(
c^2-d^2)^(1/2))*A*c^3+6/f*a^3/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f
*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c^2-6/f*a^3/d/(c^2-d^2)^(1/2)*arctan(1/2*
(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c+2/f*a^3/d^4/(c^2-d^2)^(1/
2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^4-6/f*a^3/d
```

```
^3/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))
*B*c^3+6/f*a^3/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/
(c^2-d^2)^(1/2))*B*c^2-2/f*a^3/d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*
x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c-1/f*a^3/d^2/(1+tan(1/2*f*x+1/2*e)^2)^3*t
an(1/2*f*x+1/2*e)^5*B*c+7/f*a^3/d*arctan(tan(1/2*f*x+1/2*e))*A+5/f*a^3/d*ar
ctan(tan(1/2*f*x+1/2*e))*B-6/f*a^3/d/(1+tan(1/2*f*x+1/2*e)^2)^3*A-22/3/f*a^
3/d/(1+tan(1/2*f*x+1/2*e)^2)^3*B+2/f*a^3/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*ta
n(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 21.76, size = 10256, normalized size = 41.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c + d*sin(e + f*x)),x)
```

```
[Out] - ((2*(9*A*a^3*d^2 + 3*B*a^3*c^2 + 11*B*a^3*d^2 - 3*A*a^3*c*d - 9*B*a^3*c*d
))/(3*d^3) - (tan(e/2 + (f*x)/2)^5*(A*a^3*d - B*a^3*c + 3*B*a^3*d))/d^2 + (
4*tan(e/2 + (f*x)/2)^2*(3*A*a^3*d^2 + B*a^3*c^2 + 4*B*a^3*d^2 - A*a^3*c*d -
 3*B*a^3*c*d))/d^3 + (2*tan(e/2 + (f*x)/2)^4*(3*A*a^3*d^2 + B*a^3*c^2 + 3*B
*a^3*d^2 - A*a^3*c*d - 3*B*a^3*c*d))/d^3 + (tan(e/2 + (f*x)/2)*(A*a^3*d - B
*a^3*c + 3*B*a^3*d))/d^2)/(f*(3*tan(e/2 + (f*x)/2)^2 + 3*tan(e/2 + (f*x)/2)
^4 + tan(e/2 + (f*x)/2)^6 + 1)) - (atan(((((8*(49*A^2*a^6*c^2*d^9 - 84*A^2*
a^6*c^3*d^8 + 64*A^2*a^6*c^4*d^7 - 24*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 +
 25*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 + 109*B^2*a^6*c^4*d^7 - 104*B^2*a^
6*c^5*d^6 + 64*B^2*a^6*c^6*d^5 - 24*B^2*a^6*c^7*d^4 + 4*B^2*a^6*c^8*d^3 + 7
0*A*B*a^6*c^2*d^9 - 158*A*B*a^6*c^3*d^8 + 188*A*B*a^6*c^4*d^7 - 128*A*B*a^6
*c^5*d^6 + 48*A*B*a^6*c^6*d^5 - 8*A*B*a^6*c^7*d^4))/d^8 + (8*tan(e/2 + (f*x
)/2)*(19*A^2*a^6*c^3*d^9 - 144*A^2*a^6*c^2*d^10 + 116*A^2*a^6*c^4*d^8 - 116
*A^2*a^6*c^5*d^7 + 48*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 - 140*B^2*a^6*c^2
*d^10 + 189*B^2*a^6*c^3*d^9 - 114*B^2*a^6*c^4*d^8 - 41*B^2*a^6*c^5*d^7 + 13
6*B^2*a^6*c^6*d^6 - 116*B^2*a^6*c^7*d^5 + 48*B^2*a^6*c^8*d^4 - 8*B^2*a^6*c^
```

```
9*d^3 + 94*A^2*a^6*c*d^11 + 50*B^2*a^6*c*d^11 - 308*A*B*a^6*c^2*d^10 + 258*
A*B*a^6*c^3*d^9 + 22*A*B*a^6*c^4*d^8 - 252*A*B*a^6*c^5*d^7 + 232*A*B*a^6*c^
6*d^6 - 96*A*B*a^6*c^7*d^5 + 16*A*B*a^6*c^8*d^4 + 140*A*B*a^6*c*d^11))/d^9
+ ((((32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^13 - 8*c^3*d^11))/d^9)*(B*
a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^3*(7*A + 5*B)*1i)/2 -
(a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2))/d^4 - (8*(14*A*a^3*c*d^11 + 10*B*a^3*c*d
^11 - 16*A*a^3*c^2*d^10 + 2*A*a^3*c^3*d^9 - 14*B*a^3*c^2*d^10 + 6*B*a^3*c^3
*d^9 - 2*B*a^3*c^4*d^8))/d^8 + (8*tan(e/2 + (f*x)/2)*(8*A*a^3*c*d^12 - 24*A
*a^3*c^2*d^11 + 24*A*a^3*c^3*d^10 - 8*A*a^3*c^4*d^9 - 8*B*a^3*c^2*d^11 + 24
*B*a^3*c^3*d^10 - 24*B*a^3*c^4*d^9 + 8*B*a^3*c^5*d^8))/d^9)*(B*a^3*c^3*1i +
 (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^3*(7*A + 5*B)*1i)/2 - (a^3*d*(2*A*
c^2 + 6*B*c^2)*1i)/2))/d^4)*(B*a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2
- (a^3*d^3*(7*A + 5*B)*1i)/2 - (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2)*1i)/d^4 +
(((8*(49*A^2*a^6*c^2*d^9 - 84*A^2*a^6*c^3*d^8 + 64*A^2*a^6*c^4*d^7 - 24*A^2
*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 25*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8
+ 109*B^2*a^6*c^4*d^7 - 104*B^2*a^6*c^5*d^6 + 64*B^2*a^6*c^6*d^5 - 24*B^2*a
^6*c^7*d^4 + 4*B^2*a^6*c^8*d^3 + 70*A*B*a^6*c^2*d^9 - 158*A*B*a^6*c^3*d^8 +
 188*A*B*a^6*c^4*d^7 - 128*A*B*a^6*c^5*d^6 + 48*A*B*a^6*c^6*d^5 - 8*A*B*a^6
*c^7*d^4))/d^8 + (8*tan(e/2 + (f*x)/2)*(19*A^2*a^6*c^3*d^9 - 144*A^2*a^6*c^
2*d^10 + 116*A^2*a^6*c^4*d^8 - 116*A^2*a^6*c^5*d^7 + 48*A^2*a^6*c^6*d^6 - 8
*A^2*a^6*c^7*d^5 - 140*B^2*a^6*c^2*d^10 + 189*B^2*a^6*c^3*d^9 - 114*B^2*a^6
*c^4*d^8 - 41*B^2*a^6*c^5*d^7 + 136*B^2*a^6*c^6*d^6 - 116*B^2*a^6*c^7*d^5 +
 48*B^2*a^6*c^8*d^4 - 8*B^2*a^6*c^9*d^3 + 94*A^2*a^6*c*d^11 + 50*B^2*a^6*c*
d^11 - 308*A*B*a^6*c^2*d^10 + 258*A*B*a^6*c^3*d^9 + 22*A*B*a^6*c^4*d^8 - 25
2*A*B*a^6*c^5*d^7 + 232*A*B*a^6*c^6*d^6 - 96*A*B*a^6*c^7*d^5 + 16*A*B*a^6*c
^8*d^4 + 140*A*B*a^6*c*d^11))/d^9 + (((8*(14*A*a^3*c*d^11 + 10*B*a^3*c*d^11
 - 16*A*a^3*c^2*d^10 + 2*A*a^3*c^3*d^9 - 14*B*a^3*c^2*d^10 + 6*B*a^3*c^3*d^
9 - 2*B*a^3*c^4*d^8))/d^8 + ((32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^13
 - 8*c^3*d^11))/d^9)*(B*a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*
d^3*(7*A + 5*B)*1i)/2 - (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2))/d^4 - (8*tan(e/2
 + (f*x)/2)*(8*A*a^3*c*d^12 - 24*A*a^3*c^2*d^11 + 24*A*a^3*c^3*d^10 - 8*A*a
^3*c^4*d^9 - 8*B*a^3*c^2*d^11 + 24*B*a^3*c^3*d^10 - 24*B*a^3*c^4*d^9 + 8*B*
a^3*c^5*d^8))/d^9)*(B*a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^
3*(7*A + 5*B)*1i)/2 - (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2))/d^4)*(B*a^3*c^3*1i
 + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^3*(7*A + 5*B)*1i)/2 - (a^3*d*(2*
A*c^2 + 6*B*c^2)*1i)/2)*1i)/d^4)/((16*(2*B^3*a^9*c^10 - 47*A^3*a^9*c^2*d^8
+ 55*A^3*a^9*c^3*d^7 - 21*A^3*a^9*c^4*d^6 - 7*A^3*a^9*c^5*d^5 + 8*A^3*a^9*c
^6*d^4 - 2*A^3*a^9*c^7*d^3 - 15*B^3*a^9*c^3*d^7 + 71*B^3*a^9*c^4*d^6 - 148*
B^3*a^9*c^5*d^5 + 180*B^3*a^9*c^6*d^4 - 139*B^3*a^9*c^7*d^3 + 67*B^3*a^9*c^
8*d^2 + 14*A^3*a^9*c*d^9 - 18*B^3*a^9*c^9*d - 6*A*B^2*a^9*c^9*d + 10*A^2*B*
a^9*c*d^9 + 5*A*B^2*a^9*c^2*d^8 - 53*A*B^2*a^9*c^3*d^7 + 174*A*B^2*a^9*c^4*
d^6 - 280*A*B^2*a^9*c^5*d^5 + 257*A*B^2*a^9*c^6*d^4 - 141*A*B^2*a^9*c^7*d^3
 + 44*A*B^2*a^9*c^8*d^2 - 32*A^2*B*a^9*c^2*d^8 + 21*A^2*B*a^9*c^3*d^7 + 45*
A^2*B*a^9*c^4*d^6 - 97*A^2*B*a^9*c^5*d^5 + 81*A^2*B*a^9*c^6*d^4 - 34*A^2*B*
a^9*c^7*d^3 + 6*A^2*B*a^9*c^8*d^2))/d^8 + (((8*(49*A^2*a^6*c^2*d^9 - 84*A^2
```

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*a^6*c^3*d^8 + 64*A^2*a^6*c^4*d^7 - 24*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5
+ 25*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 + 109*B^2*a^6*c^4*d^7 - 104*B^2*a
^6*c^5*d^6 + 64*B^2*a^6*c^6*d^5 - 24*B^2*a^6*c^7*d^4 + 4*B^2*a^6*c^8*d^3 +
70*A*B*a^6*c^2*d^9 - 158*A*B*a^6*c^3*d^8 + 188*A*B*a^6*c^4*d^7 - 128*A*B*a^
6*c^5*d^6 + 48*A*B*a^6*c^6*d^5 - 8*A*B*a^6*c^7*d^4))/d^8 + (8*tan(e/2 + (f*
x)/2)*(19*A^2*a^6*c^3*d^9 - 144*A^2*a^6*c^2*d^10 + 116*A^2*a^6*c^4*d^8 - 11
6*A^2*a^6*c^5*d^7 + 48*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 - 140*B^2*a^6*c^
2*d^10 + 189*B^2*a^6*c^3*d^9 - 114*B^2*a^6*c^4*d^8 - 41*B^2*a^6*c^5*d^7 + 1
36*B^2*a^6*c^6*d^6 - 116*B^2*a^6*c^7*d^5 + 48*B^2*a^6*c^8*d^4 - 8*B^2*a^6*c
^9*d^3 + 94*A^2*a^6*c*d^11 + 50*B^2*a^6*c*d^11 - 308*A*B*a^6*c^2*d^10 + 258
*A*B*a^6*c^3*d^9 + 22*A*B*a^6*c^4*d^8 - 252*A*B*a^6*c^5*d^7 + 232*A*B*a^6*c
^6*d^6 - 96*A*B*a^6*c^7*d^5 + 16*A*B*a^6*c^8*d^4 + 140*A*B*a^6*c*d^11))/d^9
 + ((((32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^13 - 8*c^3*d^11))/d^9)*(B
*a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^3*(7*A + 5*B)*1i)/2 -
 (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2))/d^4 - (8*(14*A*a^3*c*d^11 + 10*B*a^3*c*
d^11 - 16*A*a^3*c^2*d^10 + 2*A*a^3*c^3*d^9 - 14*B*a^3*c^2*d^10 + 6*B*a^3*c^
3*d^9 - 2*B*a^3*c^4*d^8))/d^8 + (8*tan(e/2 + (f*x)/2)*(8*A*a^3*c*d^12 - 24*
A*a^3*c^2*d^11 + 24*A*a^3*c^3*d^10 - 8*A*a^3*c^4*d^9 - 8*B*a^3*c^2*d^11 + 2
4*B*a^3*c^3*d^10 - 24*B*a^3*c^4*d^9 + 8*B*a^3*c^5*d^8))/d^9)*(B*a^3*c^3*1i
+ (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^3*(7*A + 5*B)*1i)/2 - (a^3*d*(2*A
*c^2 + 6*B*c^2)*1i)/2))/d^4)*(B*a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2
 - (a^3*d^3*(7*A + 5*B)*1i)/2 - (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2))/d^4 - ((
(8*(49*A^2*a^6*c^2*d^9 - 84*A^2*a^6*c^3*d^8 + 64*A^2*a^6*c^4*d^7 - 24*A^2*a
^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 25*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 +
109*B^2*a^6*c^4*d^7 - 104*B^2*a^6*c^5*d^6 + 64*B^2*a^6*c^6*d^5 - 24*B^2*a^6
*c^7*d^4 + 4*B^2*a^6*c^8*d^3 + 70*A*B*a^6*c^2*d^9 - 158*A*B*a^6*c^3*d^8 + 1
88*A*B*a^6*c^4*d^7 - 128*A*B*a^6*c^5*d^6 + 48*A*B*a^6*c^6*d^5 - 8*A*B*a^6*c
^7*d^4))/d^8 + (8*tan(e/2 + (f*x)/2)*(19*A^2*a^6*c^3*d^9 - 144*A^2*a^6*c^2*
d^10 + 116*A^2*a^6*c^4*d^8 - 116*A^2*a^6*c^5*d^7 + 48*A^2*a^6*c^6*d^6 - 8*A
^2*a^6*c^7*d^5 - 140*B^2*a^6*c^2*d^10 + 189*B^2*a^6*c^3*d^9 - 114*B^2*a^6*c
^4*d^8 - 41*B^2*a^6*c^5*d^7 + 136*B^2*a^6*c^6*d^6 - 116*B^2*a^6*c^7*d^5 + 4
8*B^2*a^6*c^8*d^4 - 8*B^2*a^6*c^9*d^3 + 94*A^2*a^6*c*d^11 + 50*B^2*a^6*c*d^
11 - 308*A*B*a^6*c^2*d^10 + 258*A*B*a^6*c^3*d^9 + 22*A*B*a^6*c^4*d^8 - 252*
A*B*a^6*c^5*d^7 + 232*A*B*a^6*c^6*d^6 - 96*A*B*a^6*c^7*d^5 + 16*A*B*a^6*c^8
*d^4 + 140*A*B*a^6*c*d^11))/d^9 + (((8*(14*A*a^3*c*d^11 + 10*B*a^3*c*d^11 -
 16*A*a^3*c^2*d^10 + 2*A*a^3*c^3*d^9 - 14*B*a^3*c^2*d^10 + 6*B*a^3*c^3*d^9
- 2*B*a^3*c^4*d^8))/d^8 + ((32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^13 -
 8*c^3*d^11))/d^9)*(B*a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^
3*(7*A + 5*B)*1i)/2 - (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2))/d^4 - (8*tan(e/2 +
 (f*x)/2)*(8*A*a^3*c*d^12 - 24*A*a^3*c^2*d^11 + 24*A*a^3*c^3*d^10 - 8*A*a^3
*c^4*d^9 - 8*B*a^3*c^2*d^11 + 24*B*a^3*c^3*d^10 - 24*B*a^3*c^4*d^9 + 8*B*a^
3*c^5*d^8))/d^9)*(B*a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^3*
(7*A + 5*B)*1i)/2 - (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2))/d^4)*(B*a^3*c^3*1i +
 (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^3*(7*A + 5*B)*1i)/2 - (a^3*d*(2*A*
c^2 + 6*B*c^2)*1i)/2))/d^4 + (16*tan(e/2 + (f*x)/2)*(8*B^3*a^9*c^11 - 462*A
```

^3*a^9*c^2*d^9 + 926*A^3*a^9*c^3*d^8 - 1034*A^3*a^9*c^4*d^7 + 704*A^3*a^9*c
^5*d^6 - 296*A^3*a^9*c^6*d^5 + 72*A^3*a^9*c^7*d^4 - 8*A^3*a^9*c^8*d^3 - 50*
B^3*a^9*c^2*d^9 + 290*B^3*a^9*c^3*d^8 - 788*B^3*a^9*c^4*d^7 + 1332*B^3*a^9*
c^5*d^6 - 1546*B^3*a^9*c^6*d^5 + 1274*B^3*a^9*c^7*d^4 - 744*B^3*a^9*c^8*d^3
 + 296*B^3*a^9*c^9*d^2 + 98*A^3*a^9*c*d^10 - 72*B^3*a^9*c^10*d + 50*A*B^2*a
^9*c*d^10 - 24*A*B^2*a^9*c^10*d + 140*A^2*B*a^9*c*d^10 - 430*A*B^2*a^9*c^2*
d^9 + 1524*A*B^2*a^9*c^3*d^8 - 3076*A*B^2*a^9*c^4*d^7 + 4018*A*B^2*a^9*c^5*
d^6 - 3582*A*B^2*a^9*c^6*d^5 + 2192*A*B^2*a^9*c^7*d^4 - 888*A*B^2*a^9*c^8*d
^3 + 216*A*B^2*a^9*c^9*d^2 - 834*A^2*B*a^9*c^2*d^9 + 2206*A^2*B*a^9*c^3*d^8
 - 3398*A^2*B*a^9*c^4*d^7 + 3342*A^2*B*a^9*c^5*d^6 - 2152*A^2*B*a^9*c^6*d^5
 + 888*A^2*B*a^9*c^7*d^4 - 216*A^2*B*a^9*c^8*d^3 + 24*A^2*B*a^9*c^9*d^2))/d
^9))*(B*a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^3*(7*A + 5*B)*
1i)/2 - (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2)*2i)/(d^4*f) - (a^3*atan(((a^3*(A*
d - B*c)*(-(c + d)*(c - d)^5)^(1/2)*((8*(49*A^2*a^6*c^2*d^9 - 84*A^2*a^6*c^
3*d^8 + 64*A^2*a^6*c^4*d^7 - 24*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 25*B^
2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 + 109*B^2*a^6*c^4*d^7 - 104*B^2*a^6*c^5*
d^6 + 64*B^2*a^6*c^6*d^5 - 24*B^2*a^6*c^7*d^4 + 4*B^2*a^6*c^8*d^3 + 70*A*B*
a^6*c^2*d^9 - 158*A*B*a^6*c^3*d^8 + 188*A*B*a^6*c^4*d^7 - 128*A*B*a^6*c^5*d
^6 + 48*A*B*a^6*c^6*d^5 - 8*A*B*a^6*c^7*d^4))/d^8 + (8*tan(e/2 + (f*x)/2)*(
19*A^2*a^6*c^3*d^9 - 144*A^2*a^6*c^2*d^10 + 116*A^2*a^6*c^4*d^8 - 116*A^2*a
^6*c^5*d^7 + 48*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 - 140*B^2*a^6*c^2*d^10
 + 189*B^2*a^6*c^3*d^9 - 114*B^2*a^6*c^4*d^8 - 41*B^2*a^6*c^5*d^7 + 136*B^2*
a^6*c^6*d^6 - 116*B^2*a^6*c^7*d^5 + 48*B^2*a^6*c^8*d^4 - 8*B^2*a^6*c^9*d^3
 + 94*A^2*a^6*c*d^11 + 50*B^2*a^6*c*d^11 - 308*A*B*a^6*c^2*d^10 + 258*A*B*a^
6*c^3*d^9 + 22*A*B*a^6*c^4*d^8 - 252*A*B*a^6*c^5*d^7 + 232*A*B*a^6*c^6*d^6
 - 96*A*B*a^6*c^7*d^5 + 16*A*B*a^6*c^8*d^4 + 140*A*B*a^6*c*d^11))/d^9 + (a^3
*(A*d - B*c)*(-(c + d)*(c - d)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*A*a^3*c*d
^12 - 24*A*a^3*c^2*d^11 + 24*A*a^3*c^3*d^10 - 8*A*a^3*c^4*d^9 - 8*B*a^3*c^2
*d^11 + 24*B*a^3*c^3*d^10 - 24*B*a^3*c^4*d^9 + 8*B*a^3*c^5*d^8))/d^9 - (8*(
14*A*a^3*c*d^11 + 10*B*a^3*c*d^11 - 16*A*a^3*c^2*d^10 + 2*A*a^3*c^3*d^9 - 1
4*B*a^3*c^2*d^10 + 6*B*a^3*c^3*d^9 - 2*B*a^3*c^4*d^8))/d^8 + (a^3*(32*c^2*d
^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^13 - 8*c^3*d^11))/d^9)*(A*d - B*c)*(-(c
 + d)*(c - d)^5)^(1/2))/(c*d^4 + d^5)))/(c*d^4 + d^5))*1i)/(c*d^4 + d^5) + (
a^3*(A*d - B*c)*(-(c + d)*(c - d)^5)^(1/2)*((8*(49*A^2*a^6*c^2*d^9 - 84*A^2
*a^6*c^3*d^8 + 64*A^2*a^6*c^4*d^7 - 24*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5
 + 25*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 + 109*B^2*a^6*c^4*d^7 - 104*B^2*a
^6*c^5*d^6 + 64*B^2*a^6*c^6*d^5 - 24*B^2*a^6*c^7*d^4 + 4*B^2*a^6*c^8*d^3 +
70*A*B*a^6*c^2*d^9 - 158*A*B*a^6*c^3*d^8 + 188*A*B*a^6*c^4*d^7 - 128*A*B*a^
6*c^5*d^6 + 48*A*B*a^6*c^6*d^5 - 8*A*B*a^6*c^7*d^4))/d^8 + (8*tan(e/2 + (f*
x)/2)*(19*A^2*a^6*c^3*d^9 - 144*A^2*a^6*c^2*d^10 + 116*A^2*a^6*c^4*d^8 - 11
6*A^2*a^6*c^5*d^7 + 48*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 - 140*B^2*a^6*c^
2*d^10 + 189*B^2*a^6*c^3*d^9 - 114*B^2*a^6*c^4*d^8 - 41*B^2*a^6*c^5*d^7 + 1
36*B^2*a^6*c^6*d^6 - 116*B^2*a^6*c^7*d^5 + 48*B^2*a^6*c^8*d^4 - 8*B^2*a^6*c
^9*d^3 + 94*A^2*a^6*c*d^11 + 50*B^2*a^6*c*d^11 - 308*A*B*a^6*c^2*d^10 + 258
*A*B*a^6*c^3*d^9 + 22*A*B*a^6*c^4*d^8 - 252*A*B*a^6*c^5*d^7 + 232*A*B*a^6*c

^6*d^6 - 96*A*B*a^6*c^7*d^5 + 16*A*B*a^6*c^8*d^4 + 140*A*B*a^6*c*d^11))/d^9
+ (a^3*(A*d - B*c)*(-(c + d)*(c - d)^5)^(1/2)*((8*(14*A*a^3*c*d^11 + 10*B*
a^3*c*d^11 - 16*A*a^3*c^2*d^10 + 2*A*a^3*c^3*d^9 - 14*B*a^3*c^2*d^10 + 6*B*
a^3*c^3*d^9 - 2*B*a^3*c^4*d^8))/d^8 - (8*tan(e/2 + (f*x)/2)*(8*A*a^3*c*d^12
- 24*A*a^3*c^2*d^11 + 24*A*a^3*c^3*d^10 - 8*A*a^3*c^4*d^9 - 8*B*a^3*c^2*d^
11 + 24*B*a^3*c^3*d^10 - 24*B*a^3*c^4*d^9 + 8*B*a^3*c^5*d^8))/d^9 + (a^3*(3
2*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^13 - 8*c^3*d^11))/d^9)*(A*d - B*c
)*(-(c + d)*(c - d)^5)^(1/2))/(c*d^4 + d^5)))/(c*d^4 + d^5))*1i)/(c*d^4 + d
^5))/((16*(2*B^3*a^9*c^10 - 47*A^3*a^9*c^2*d^8 + 55*A^3*a^9*c^3*d^7 - 21*A^
3*a^9*c^4*d^6 - 7*A^3*a^9*c^5*d^5 + 8*A^3*a^9*c^6*d^4 - 2*A^3*a^9*c^7*d^3 -
15*B^3*a^9*c^3*d^7 + 71*B^3*a^9*c^4*d^6 - 148*B^3*a^9*c^5*d^5 + 180*B^3*a^
9*c^6*d^4 - 139*B^3*a^9*c^7*d^3 + 67*B^3*a^9*c^8*d^2 + 14*A^3*a^9*c*d^9 - 1
8*B^3*a^9*c^9*d - 6*A*B^2*a^9*c^9*d + 10*A^2*B*a^9*c*d^9 + 5*A*B^2*a^9*c^2*
d^8 - 53*A*B^2*a^9*c^3*d^7 + 174*A*B^2*a^9*c^4*d^6 - 280*A*B^2*a^9*c^5*d^5
+ 257*A*B^2*a^9*c^6*d^4 - 141*A*B^2*a^9*c^7*d^3 + 44*A*B^2*a^9*c^8*d^2 - 32
*A^2*B*a^9*c^2*d^8 + 21*A^2*B*a^9*c^3*d^7 + 45*A^2*B*a^9*c^4*d^6 - 97*A^2*B
*a^9*c^5*d^5 + 81*A^2*B*a^9*c^6*d^4 - 34*A^2*B*a^9*c^7*d^3 + 6*A^2*B*a^9*c^
8*d^2))/d^8 + (16*tan(e/2 + (f*x)/2)*(8*B^3*a^9*c^11 - 462*A^3*a^9*c^2*d^9
+ 926*A^3*a^9*c^3*d^8 - 1034*A^3*a^9*c^4*d^7 + 704*A^3*a^9*c^5*d^6 - 296*A^
3*a^9*c^6*d^5 + 72*A^3*a^9*c^7*d^4 - 8*A^3*a^9*c^8*d^3 - 50*B^3*a^9*c^2*d^9
+ 290*B^3*a^9*c^3*d^8 - 788*B^3*a^9*c^4*d^7 + 1332*B^3*a^9*c^5*d^6 - 1546*
B^3*a^9*c^6*d^5 + 1274*B^3*a^9*c^7*d^4 - 744*B^3*a^9*c^8*d^3 + 296*B^3*a^9*
c^9*d^2 + 98*A^3*a^9*c*d^10 - 72*B^3*a^9*c^10*d + 50*A*B^2*a^9*c*d^10 - 24*
A*B^2*a^9*c^10*d + 140*A^2*B*a^9*c*d^10 - 430*A*B^2*a^9*c^2*d^9 + 1524*A*B^
2*a^9*c^3*d^8 - 3076*A*B^2*a^9*c^4*d^7 + 4018*A*B^2*a^9*c^5*d^6 - 3582*A*B^
2*a^9*c^6*d^5 + 2192*A*B^2*a^9*c^7*d^4 - 888*A*B^2*a^9*c^8*d^3 + 216*A*B^2*
a^9*c^9*d^2 - 834*A^2*B*a^9*c^2*d^9 + 2206*A^2*B*a^9*c^3*d^8 - 3398*A^2*B*a
^9*c^4*d^7 + 3342*A^2*B*a^9*c^5*d^6 - 2152*A^2*B*a^9*c^6*d^5 + 888*A^2*B*a^
9*c^7*d^4 - 216*A^2*B*a^9*c^8*d^3 + 24*A^2*B*a^9*c^9*d^2))/d^9 + (a^3*(A*d
- B*c)*(-(c + d)*(c - d)^5)^(1/2)*((8*(49*A^2*a^6*c^2*d^9 - 84*A^2*a^6*c^3*
d^8 + 64*A^2*a^6*c^4*d^7 - 24*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 25*B^2*
a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 + 109*B^2*a^6*c^4*d^7 - 104*B^2*a^6*c^5*d^
6 + 64*B^2*a^6*c^6*d^5 - 24*B^2*a^6*c^7*d^4 + 4*B^2*a^6*c^8*d^3 + 70*A*B*a^
6*c^2*d^9 - 158*A*B*a^6*c^3*d^8 + 188*A*B*a^6*c^4*d^7 - 128*A*B*a^6*c^5*d^6
+ 48*A*B*a^6*c^6*d^5 - 8*A*B*a^6*c^7*d^4))/d^8 + (8*tan(e/2 + (f*x)/2)*(19
*A^2*a^6*c^3*d^9 - 144*A^2*a^6*c^2*d^10 + 116*A^2*a^6*c^4*d^8 - 116*A^2*a^6
*c^5*d^7 + 48*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 - 140*B^2*a^6*c^2*d^10 +
189*B^2*a^6*c^3*d^9 - 114*B^2*a^6*c^4*d^8 - 41*B^2*a^6*c^5*d^7 + 136*B^2*a^
6*c^6*d^6 - 116*B^2*a^6*c^7*d^5 + 48*B^2*a^6*c^8*d^4 - 8*B^2*a^6*c^9*d^3 +
94*A^2*a^6*c*d^11 + 50*B^2*a^6*c*d^11 - 308*A*B*a^6*c^2*d^10 + 258*A*B*a^6*
c^3*d^9 + 22*A*B*a^6*c^4*d^8 - 252*A*B*a^6*c^5*d^7 + 232*A*B*a^6*c^6*d^6 -
96*A*B*a^6*c^7*d^5 + 16*A*B*a^6*c^8*d^4 + 140*A*B*a^6*c*d^11))/d^9 + (a^3*(
A*d - B*c)*(-(c + d)*(c - d)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*A*a^3*c*d^1
2 - 24*A*a^3*c^2*d^11 + 24*A*a^3*c^3*d^10 - 8*A*a^3*c^4*d^9 - 8*B*a^3*c^2*d
^11 + 24*B*a^3*c^3*d^10 - 24*B*a^3*c^4*d^9 + 8*B*a^3*c^5*d^8))/d^9 - (8*(14

```
*A*a^3*c*d^11 + 10*B*a^3*c*d^11 - 16*A*a^3*c^2*d^10 + 2*A*a^3*c^3*d^9 - 14*
B*a^3*c^2*d^10 + 6*B*a^3*c^3*d^9 - 2*B*a^3*c^4*d^8))/d^8 + (a^3*(32*c^2*d^3
 + (8*tan(e/2 + (f*x)/2)*(12*c*d^13 - 8*c^3*d^11))/d^9)*(A*d - B*c)*(-(c +
d)*(c - d)^5)^(1/2))/(c*d^4 + d^5)))/(c*d^4 + d^5)))/(c*d^4 + d^5) - (a^3*(
A*d - B*c)*(-(c + d)*(c - d)^5)^(1/2)*((8*(49*A^2*a^6*c^2*d^9 - 84*A^2*a^6*
c^3*d^8 + 64*A^2*a^6*c^4*d^7 - 24*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 25*
B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 + 109*B^2*a^6*c^4*d^7 - 104*B^2*a^6*c^
5*d^6 + 64*B^2*a^6*c^6*d^5 - 24*B^2*a^6*c^7*d^4 + 4*B^2*a^6*c^8*d^3 + 70*A*
B*a^6*c^2*d^9 - 158*A*B*a^6*c^3*d^8 + 188*A*B*a^6*c^4*d^7 - 128*A*B*a^6*c^5
*d^6 + 48*A*B*a^6*c^6*d^5 - 8*A*B*a^6*c^7*d^4))/d^8 + (8*tan(e/2 + (f*x)/2)
*(19*A^2*a^6*c^3*d^9 - 144*A^2*a^6*c^2*d^10 + 116*A^2*a^6*c^4*d^8 - 116*A^2
*a^6*c^5*d^7 + 48*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 - 140*B^2*a^6*c^2*d^1
0 + 189*B^2*a^6*c^3*d^9 - 114*B^2*a^6*c^4*d^8 - 41*B^2*a^6*c^5*d^7 + 136*B^
2*a^6*c^6*d^6 - 116*B^2*a^6*c^7*d^5 + 48*B^2*a^6*c^8*d^4 - 8*B^2*a^6*c^9*d^
3 + 94*A^2*a^6*c*d^11 + 50*B^2*a^6*c*d^11 - 308*A*B*a^6*c^2*d^10 + 258*A*B*
a^6*c^3*d^9 + 22*A*B*a^6*c^4*d^8 - 252*A*B*a^6*c^5*d^7 + 232*A*B*a^6*c^6*d^
6 - 96*A*B*a^6*c^7*d^5 + 16*A*B*a^6*c^8*d^4 + 140*A*B*a^6*c*d^11))/d^9 + (a
^3*(A*d - B*c)*(-(c + d)*(c - d)^5)^(1/2)*((8*(14*A*a^3*c*d^11 + 10*B*a^3*c
*d^11 - 16*A*a^3*c^2*d^10 + 2*A*a^3*c^3*d^9 - 14*B*a^3*c^2*d^10 + 6*B*a^3*c
^3*d^9 - 2*B*a^3*c^4*d^8))/d^8 - (8*tan(e/2 + (f*x)/2)*(8*A*a^3*c*d^12 - 24
*A*a^3*c^2*d^11 + 24*A*a^3*c^3*d^10 - 8*A*a^3*c^4*d^9 - 8*B*a^3*c^2*d^11 +
24*B*a^3*c^3*d^10 - 24*B*a^3*c^4*d^9 + 8*B*a^3*c^5*d^8))/d^9 + (a^3*(32*c^2
*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^13 - 8*c^3*d^11))/d^9)*(A*d - B*c)*(-(
c + d)*(c - d)^5)^(1/2))/(c*d^4 + d^5)))/(c*d^4 + d^5)))/(c*d^4 + d^5)))*(A
*d - B*c)*(-(c + d)*(c - d)^5)^(1/2)*2i)/(f*(c*d^4 + d^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.263 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=283

$$\frac{2a^3(c-d)^2\left(Ad(2c+3d)-B\left(3c^2+3cd-d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^4f(c+d)\sqrt{c^2-d^2}}-\frac{a^3x\left(2Ad(2c-3d)-B\left(6c^2-12cd+7d\right.\right.}{2d^4}$$

```
[Out] -1/2*a^3*(2*A*(2*c-3*d)*d-B*(6*c^2-12*c*d+7*d^2))*x/d^4-1/2*a^3*(4*A*c*d-B*
(6*c^2-3*c*d-5*d^2))*cos(f*x+e)/d^3/(c+d)/f+1/2*(2*A*d-B*(3*c+d))*cos(f*x+e
)*(a^3+a^3*sin(f*x+e))/d^2/(c+d)/f+a*(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))
^2/d/(c+d)/f/(c+d*sin(f*x+e))+2*a^3*(c-d)^2*(A*d*(2*c+3*d)-B*(3*c^2+3*c*d-d
^2))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^4/(c+d)/f/(c^2-d^2)
^(1/2)
```

Rubi [A] time = 0.94, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2975, 2976, 2968, 3023, 2735, 2660, 618, 204}

$$-\frac{a^3\left(4Acd-B\left(6c^2-3cd-5d^2\right)\right)\cos(e+fx)}{2d^3f(c+d)}+\frac{2a^3(c-d)^2\left(Ad(2c+3d)-B\left(3c^2+3cd-d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+f\right.}{\sqrt{c^2-d}}\right.}{d^4f(c+d)\sqrt{c^2-d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -(a^3*(2*A*(2*c - 3*d)*d - B*(6*c^2 - 12*c*d + 7*d^2))*x)/(2*d^4) + (2*a^3*
(c - d)^2*(A*d*(2*c + 3*d) - B*(3*c^2 + 3*c*d - d^2))*ArcTan[(d + c*Tan[(e
+ f*x)/2])/Sqrt[c^2 - d^2]])/(d^4*(c + d)*Sqrt[c^2 - d^2]*f) - (a^3*(4*A*c*
d - B*(6*c^2 - 3*c*d - 5*d^2))*Cos[e + f*x])/(2*d^3*(c + d)*f) + ((2*A*d -
B*(3*c + d))*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(2*d^2*(c + d)*f) + (a*
(B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/(d*(c + d)*f*(c + d*Sin[e
+ f*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
```

```
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{d(c+d)f(c+d\sin(e+fx))} + \frac{\int \frac{(a+a\sin(e+fx))^2(-}{}}{}$$

$$= \frac{(2Ad-B(3c+d))\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d^2(c+d)f} + \frac{a(Bc-}{a}$$

$$= \frac{(2Ad-B(3c+d))\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d^2(c+d)f} + \frac{a(Bc-}{a}$$

$$= -\frac{a^3\left(4Acd-B\left(6c^2-3cd-5d^2\right)\right)\cos(e+fx)}{2d^3(c+d)f} + \frac{(2Ad-B(3c}{}$$

$$= -\frac{a^3\left(2A(2c-3d)d-B\left(6c^2-12cd+7d^2\right)\right)x}{2d^4} - \frac{a^3\left(4Acd-B\right(}{}$$

$$= -\frac{a^3\left(2A(2c-3d)d-B\left(6c^2-12cd+7d^2\right)\right)x}{2d^4} - \frac{a^3\left(4Acd-B\right(}{}$$

$$= -\frac{a^3\left(2A(2c-3d)d-B\left(6c^2-12cd+7d^2\right)\right)x}{2d^4} - \frac{a^3\left(4Acd-B\right(}{}$$

$$= -\frac{a^3\left(2A(2c-3d)d-B\left(6c^2-12cd+7d^2\right)\right)x}{2d^4} + \frac{2a^3(c-d)^2\left(Ad\right.}{}$$

Mathematica [A] time = 1.50, size = 244, normalized size = 0.86

$$\frac{a^3(\sin(e+fx)+1)^3\left(2(e+fx)\left(2Ad(3d-2c)+B\left(6c^2-12cd+7d^2\right)\right)-\frac{8(c-d)^2\left(B\left(3c^2+3cd-d^2\right)-Ad(2c+3d)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e\right.}{\sqrt{c^2}}\right.}{(c+d)\sqrt{c^2-d^2}}\right.}{4d^4f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^2,x]
```

```
[Out] (a^3*(1 + Sin[e + f*x])^3*(2*(2*A*d*(-2*c + 3*d) + B*(6*c^2 - 12*c*d + 7*d^
2))*(e + f*x) - (8*(c - d)^2*(-(A*d*(2*c + 3*d)) + B*(3*c^2 + 3*c*d - d^2))
*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)*Sqrt[c^2 - d^2]
) - 4*d*(-2*B*c + A*d + 3*B*d)*Cos[e + f*x] + (4*(c - d)^2*d*(B*c - A*d)*Co
s[e + f*x])/((c + d)*(c + d*Sin[e + f*x])) - B*d^2*Sin[2*(e + f*x)]))/(4*d^
4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

fricas [A] time = 0.59, size = 1027, normalized size = 3.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] [1/2*((B*a^3*c*d^3 + B*a^3*d^4)*cos(f*x + e)^3 + (6*B*a^3*c^4 - 2*(2*A + 3*
B)*a^3*c^3*d + (2*A - 5*B)*a^3*c^2*d^2 + (6*A + 7*B)*a^3*c*d^3)*f*x + (3*B*
a^3*c^4 - 2*A*a^3*c^3*d - (A + 4*B)*a^3*c^2*d^2 + (3*A + B)*a^3*c*d^3 + (3*
B*a^3*c^3*d - 2*A*a^3*c^2*d^2 - (A + 4*B)*a^3*c*d^3 + (3*A + B)*a^3*d^4)*si
n(f*x + e))*sqrt(-(c - d)/(c + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*
d*sin(f*x + e) - c^2 - d^2 + 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*
d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*
sin(f*x + e) - c^2 - d^2)) + (6*B*a^3*c^3*d - 2*(2*A + 3*B)*a^3*c^2*d^2 + (
2*A - 5*B)*a^3*c*d^3 - (2*A + B)*a^3*d^4)*cos(f*x + e) + ((6*B*a^3*c^3*d -
2*(2*A + 3*B)*a^3*c^2*d^2 + (2*A - 5*B)*a^3*c*d^3 + (6*A + 7*B)*a^3*d^4)*f*
x + (3*B*a^3*c^2*d^2 - (2*A + 3*B)*a^3*c*d^3 - 2*(A + 3*B)*a^3*d^4)*cos(f*x
 + e))*sin(f*x + e))/((c*d^5 + d^6)*f*sin(f*x + e) + (c^2*d^4 + c*d^5)*f),
1/2*((B*a^3*c*d^3 + B*a^3*d^4)*cos(f*x + e)^3 + (6*B*a^3*c^4 - 2*(2*A + 3*B
)*a^3*c^3*d + (2*A - 5*B)*a^3*c^2*d^2 + (6*A + 7*B)*a^3*c*d^3)*f*x + 2*(3*B
*a^3*c^4 - 2*A*a^3*c^3*d - (A + 4*B)*a^3*c^2*d^2 + (3*A + B)*a^3*c*d^3 + (3
*B*a^3*c^3*d - 2*A*a^3*c^2*d^2 - (A + 4*B)*a^3*c*d^3 + (3*A + B)*a^3*d^4)*s
```

in(f*x + e))*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt((c - d
)/(c + d))/((c - d)*cos(f*x + e))) + (6*B*a^3*c^3*d - 2*(2*A + 3*B)*a^3*c^2
*d^2 + (2*A - 5*B)*a^3*c*d^3 - (2*A + B)*a^3*d^4)*cos(f*x + e) + ((6*B*a^3*
c^3*d - 2*(2*A + 3*B)*a^3*c^2*d^2 + (2*A - 5*B)*a^3*c*d^3 + (6*A + 7*B)*a^3
*d^4)*f*x + (3*B*a^3*c^2*d^2 - (2*A + 3*B)*a^3*c*d^3 - 2*(A + 3*B)*a^3*d^4)
*cos(f*x + e))*sin(f*x + e))/((c*d^5 + d^6)*f*sin(f*x + e) + (c^2*d^4 + c*d
^5)*f)]

giac [B] time = 0.21, size = 588, normalized size = 2.08

$$-\frac{\frac{4\left(3\,Ba^3c^4-2\,Aa^3c^3d-3\,Ba^3c^3d+Aa^3c^2d^2-4\,Ba^3c^2d^2+4\,Aa^3cd^3+5\,Ba^3cd^3-3\,Aa^3d^4-Ba^3d^4\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(cd^4+d^5\right)\sqrt{c^2-d^2}}-\frac{4\left(B\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="giac")

[Out] -1/2*(4*(3*B*a^3*c^4 - 2*A*a^3*c^3*d - 3*B*a^3*c^3*d + A*a^3*c^2*d^2 - 4*B*
a^3*c^2*d^2 + 4*A*a^3*c*d^3 + 5*B*a^3*c*d^3 - 3*A*a^3*d^4 - B*a^3*d^4)*(pi*
floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/
sqrt(c^2 - d^2)))/((c*d^4 + d^5)*sqrt(c^2 - d^2)) - 4*(B*a^3*c^3*d*tan(1/2*
f*x + 1/2*e) - A*a^3*c^2*d^2*tan(1/2*f*x + 1/2*e) - 2*B*a^3*c^2*d^2*tan(1/2
*f*x + 1/2*e) + 2*A*a^3*c*d^3*tan(1/2*f*x + 1/2*e) + B*a^3*c*d^3*tan(1/2*f*
x + 1/2*e) - A*a^3*d^4*tan(1/2*f*x + 1/2*e) + B*a^3*c^4 - A*a^3*c^3*d - 2*B
*a^3*c^3*d + 2*A*a^3*c^2*d^2 + B*a^3*c^2*d^2 - A*a^3*c*d^3)/((c^2*d^3 + c*d
^4)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) - (6*B*a^3*c
^2 - 4*A*a^3*c*d - 12*B*a^3*c*d + 6*A*a^3*d^2 + 7*B*a^3*d^2)*(f*x + e)/d^4
- 2*(B*a^3*d*tan(1/2*f*x + 1/2*e)^3 + 4*B*a^3*c*tan(1/2*f*x + 1/2*e)^2 - 2*
A*a^3*d*tan(1/2*f*x + 1/2*e)^2 - 6*B*a^3*d*tan(1/2*f*x + 1/2*e)^2 - B*a^3*d
*tan(1/2*f*x + 1/2*e) + 4*B*a^3*c - 2*A*a^3*d - 6*B*a^3*d)/((tan(1/2*f*x +
1/2*e)^2 + 1)^2*d^3))/f

maple [B] time = 0.56, size = 1534, normalized size = 5.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)

[Out] -2*a^3/f*d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)/c*tan(1/
2*f*x+1/2*e)*A+2*a^3/f/d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c

```
)/(c+d)*c^2*tan(1/2*f*x+1/2*e)*B-4*a^3/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/
2*f*x+1/2*e)*d+c)/(c+d)*c*tan(1/2*f*x+1/2*e)*B+4*a^3/f/d^3/(c+d)/(c^2-d^2)^
(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c^3-2*a^3/
f/d^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^
2)^(1/2))*A*c^2-8*a^3/f/d/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x
+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c-6*a^3/f/d^4/(c+d)/(c^2-d^2)^(1/2)*arctan(
1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c^4*B+6*a^3/f/d^3/(c+d)/(
c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c
^3+8*a^3/f/d^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d
)/(c^2-d^2)^(1/2))*B*c^2-10*a^3/f/d/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*t
an(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c-2*a^3/f/d/(tan(1/2*f*x+1/2*e)^2
*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*c*tan(1/2*f*x+1/2*e)*A+4*a^3/f/d/(tan(1/
2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*A*c+2*a^3/f/d^3/(tan(1/2*f
*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*B*c^3-4*a^3/f/d^2/(tan(1/2*f*
x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*B*c^2+2*a^3/f/d/(tan(1/2*f*x+1
/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*B*c+4*a^3/f/d^3/(1+tan(1/2*f*x+1/
2*e)^2)^2*B*tan(1/2*f*x+1/2*e)^2*c-2*a^3/f/d^2/(tan(1/2*f*x+1/2*e)^2*c+2*ta
n(1/2*f*x+1/2*e)*d+c)/(c+d)*A*c^2-a^3/f/d^2/(1+tan(1/2*f*x+1/2*e)^2)^2*B*ta
n(1/2*f*x+1/2*e)+4*a^3/f/d^3/(1+tan(1/2*f*x+1/2*e)^2)^2*B*c-4*a^3/f/d^3*arc
tan(tan(1/2*f*x+1/2*e))*A*c+6*a^3/f/d^4*arctan(tan(1/2*f*x+1/2*e))*B*c^2-12
*a^3/f/d^3*arctan(tan(1/2*f*x+1/2*e))*B*c+6*a^3/f/(c+d)/(c^2-d^2)^(1/2)*arc
tan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A+2*a^3/f/(c+d)/(c^2-
d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B+4*a^3
/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*tan(1/2*f*x+1/2*
e)*A+2*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*tan(1/
2*f*x+1/2*e)*B+a^3/f/d^2/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*x+1/2*e)^3-
2*a^3/f/d^2/(1+tan(1/2*f*x+1/2*e)^2)^2*A*tan(1/2*f*x+1/2*e)^2-6*a^3/f/d^2/(
1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*x+1/2*e)^2-2*a^3/f/(tan(1/2*f*x+1/2*e
)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*A-2*a^3/f/d^2/(1+tan(1/2*f*x+1/2*e)^2
)^2*A-6*a^3/f/d^2/(1+tan(1/2*f*x+1/2*e)^2)^2*B+6*a^3/f/d^2*arctan(tan(1/2*f
*x+1/2*e))*A+7*a^3/f/d^2*arctan(tan(1/2*f*x+1/2*e))*B
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 23.88, size = 11993, normalized size = 42.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c + d*sin(e + f*x))^2,x)
```

```
[Out] - ((2*(A*a^3*d^3 - 3*B*a^3*c^3 - A*a^3*c*d^2 + 2*A*a^3*c^2*d + 2*B*a^3*c*d^
2 + 3*B*a^3*c^2*d))/(d^3*(c + d)) + (2*tan(e/2 + (f*x)/2)^4*(A*a^3*d^3 - 3*
B*a^3*c^3 - B*a^3*d^3 - A*a^3*c*d^2 + 2*A*a^3*c^2*d + B*a^3*c*d^2 + 3*B*a^3
*c^2*d))/(d^3*(c + d)) + (2*tan(e/2 + (f*x)/2)^2*(2*A*a^3*d^3 - 6*B*a^3*c^3
 + B*a^3*d^3 - 2*A*a^3*c*d^2 + 4*A*a^3*c^2*d + 5*B*a^3*c*d^2 + 6*B*a^3*c^2*
d))/(d^3*(c + d)) + (4*tan(e/2 + (f*x)/2)^3*(A*a^3*d^3 - 3*B*a^3*c^3 - A*a^
3*c*d^2 + 2*A*a^3*c^2*d + 2*B*a^3*c*d^2 + 3*B*a^3*c^2*d))/(c*d^2*(c + d)) +
 (tan(e/2 + (f*x)/2)^5*(2*A*a^3*d^3 - 3*B*a^3*c^3 - 4*A*a^3*c*d^2 + 2*A*a^3
*c^2*d - 2*B*a^3*c*d^2 + 3*B*a^3*c^2*d))/(c*d^2*(c + d)) + (tan(e/2 + (f*x)
/2)*(2*A*a^3*d^3 - 9*B*a^3*c^3 + 6*A*a^3*c^2*d + 10*B*a^3*c*d^2 + 9*B*a^3*c
^2*d))/(c*d^2*(c + d)))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + 3*c*tan(e/2 + (f*x
)/2)^2 + 3*c*tan(e/2 + (f*x)/2)^4 + c*tan(e/2 + (f*x)/2)^6 + 4*d*tan(e/2 +
(f*x)/2)^3 + 2*d*tan(e/2 + (f*x)/2)^5)) - (atan(((((8*(36*A^2*a^6*c^2*d^9 +
 24*A^2*a^6*c^3*d^8 - 44*A^2*a^6*c^4*d^7 - 16*A^2*a^6*c^5*d^6 + 16*A^2*a^6*
c^6*d^5 + 49*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 - 59*B^2*a^6*c^4*d^7 + 14
4*B^2*a^6*c^5*d^6 - 24*B^2*a^6*c^6*d^5 - 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^
8*d^3 + 84*A*B*a^6*c^2*d^9 - 32*A*B*a^6*c^3*d^8 - 148*A*B*a^6*c^4*d^7 + 88*
A*B*a^6*c^5*d^6 + 72*A*B*a^6*c^6*d^5 - 48*A*B*a^6*c^7*d^4))/(2*c*d^9 + d^10
 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)*(144*A^2*a^6*c^2*d^10 - 164*A^2*a^6*c^3
*d^9 - 136*A^2*a^6*c^4*d^8 + 136*A^2*a^6*c^5*d^7 + 32*A^2*a^6*c^6*d^6 - 32*
A^2*a^6*c^7*d^5 - 100*B^2*a^6*c^2*d^10 - 299*B^2*a^6*c^3*d^9 + 494*B^2*a^6*
c^4*d^8 + 91*B^2*a^6*c^5*d^7 - 504*B^2*a^6*c^6*d^6 + 156*B^2*a^6*c^7*d^5 +
144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 + 36*A^2*a^6*c*d^11 + 94*B^2*a^6*c
*d^11 + 88*A*B*a^6*c^2*d^10 - 628*A*B*a^6*c^3*d^9 + 208*A*B*a^6*c^4*d^8 + 5
72*A*B*a^6*c^5*d^7 - 320*A*B*a^6*c^6*d^6 - 144*A*B*a^6*c^7*d^5 + 96*A*B*a^6
*c^8*d^4 + 144*A*B*a^6*c*d^11))/(2*c*d^10 + d^11 + c^2*d^9) + (((((8*(4*c^2
*d^13 + 8*c^3*d^12 + 4*c^4*d^11))/(2*c*d^9 + d^10 + c^2*d^8) + (8*tan(e/2 +
 (f*x)/2)*(12*c*d^15 + 24*c^2*d^14 + 4*c^3*d^13 - 16*c^4*d^12 - 8*c^5*d^11)
)/(2*c*d^10 + d^11 + c^2*d^9))*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7*B)/2)*1i -
 (a^3*d*(4*A*c + 12*B*c)*1i)/2))/d^4 - (8*(12*A*a^3*c*d^12 + 14*B*a^3*c*d^1
2 + 4*A*a^3*c^2*d^11 - 12*A*a^3*c^3*d^10 - 4*A*a^3*c^4*d^9 - 20*B*a^3*c^3*d
^10 + 6*B*a^3*c^5*d^8))/(2*c*d^9 + d^10 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)*
(24*A*a^3*c*d^13 + 8*B*a^3*c*d^13 - 8*A*a^3*c^2*d^12 - 40*A*a^3*c^3*d^11 +
8*A*a^3*c^4*d^10 + 16*A*a^3*c^5*d^9 - 32*B*a^3*c^2*d^12 - 8*B*a^3*c^3*d^11
+ 56*B*a^3*c^4*d^10 - 24*B*a^3*c^6*d^8))/(2*c*d^10 + d^11 + c^2*d^9))*(B*a^
3*c^2*3i + a^3*d^2*(3*A + (7*B)/2)*1i - (a^3*d*(4*A*c + 12*B*c)*1i)/2))/d^4
)*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7*B)/2)*1i - (a^3*d*(4*A*c + 12*B*c)*1i)/
```

2)*1i)/d^4 + (((8*(36*A^2*a^6*c^2*d^9 + 24*A^2*a^6*c^3*d^8 - 44*A^2*a^6*c^4
*d^7 - 16*A^2*a^6*c^5*d^6 + 16*A^2*a^6*c^6*d^5 + 49*B^2*a^6*c^2*d^9 - 70*B^
2*a^6*c^3*d^8 - 59*B^2*a^6*c^4*d^7 + 144*B^2*a^6*c^5*d^6 - 24*B^2*a^6*c^6*d
^5 - 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8*d^3 + 84*A*B*a^6*c^2*d^9 - 32*A*B*
a^6*c^3*d^8 - 148*A*B*a^6*c^4*d^7 + 88*A*B*a^6*c^5*d^6 + 72*A*B*a^6*c^6*d^5
 - 48*A*B*a^6*c^7*d^4))/(2*c*d^9 + d^10 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)*
(144*A^2*a^6*c^2*d^10 - 164*A^2*a^6*c^3*d^9 - 136*A^2*a^6*c^4*d^8 + 136*A^2
*a^6*c^5*d^7 + 32*A^2*a^6*c^6*d^6 - 32*A^2*a^6*c^7*d^5 - 100*B^2*a^6*c^2*d^
10 - 299*B^2*a^6*c^3*d^9 + 494*B^2*a^6*c^4*d^8 + 91*B^2*a^6*c^5*d^7 - 504*B
^2*a^6*c^6*d^6 + 156*B^2*a^6*c^7*d^5 + 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9
*d^3 + 36*A^2*a^6*c*d^11 + 94*B^2*a^6*c*d^11 + 88*A*B*a^6*c^2*d^10 - 628*A*
B*a^6*c^3*d^9 + 208*A*B*a^6*c^4*d^8 + 572*A*B*a^6*c^5*d^7 - 320*A*B*a^6*c^6
*d^6 - 144*A*B*a^6*c^7*d^5 + 96*A*B*a^6*c^8*d^4 + 144*A*B*a^6*c*d^11))/(2*c
*d^10 + d^11 + c^2*d^9) + (((8*(12*A*a^3*c*d^12 + 14*B*a^3*c*d^12 + 4*A*a^3
*c^2*d^11 - 12*A*a^3*c^3*d^10 - 4*A*a^3*c^4*d^9 - 20*B*a^3*c^3*d^10 + 6*B*a
^3*c^5*d^8))/(2*c*d^9 + d^10 + c^2*d^8) + (((8*(4*c^2*d^13 + 8*c^3*d^12 + 4
*c^4*d^11))/(2*c*d^9 + d^10 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)*(12*c*d^15 +
 24*c^2*d^14 + 4*c^3*d^13 - 16*c^4*d^12 - 8*c^5*d^11))/(2*c*d^10 + d^11 + c
^2*d^9))*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7*B)/2)*1i - (a^3*d*(4*A*c + 12*B*
c)*1i)/2))/d^4 - (8*tan(e/2 + (f*x)/2)*(24*A*a^3*c*d^13 + 8*B*a^3*c*d^13 -
8*A*a^3*c^2*d^12 - 40*A*a^3*c^3*d^11 + 8*A*a^3*c^4*d^10 + 16*A*a^3*c^5*d^9
- 32*B*a^3*c^2*d^12 - 8*B*a^3*c^3*d^11 + 56*B*a^3*c^4*d^10 - 24*B*a^3*c^6*d
^8))/(2*c*d^10 + d^11 + c^2*d^9))*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7*B)/2)*1
i - (a^3*d*(4*A*c + 12*B*c)*1i)/2))/d^4)*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7*
B)/2)*1i - (a^3*d*(4*A*c + 12*B*c)*1i)/2)*1i)/d^4)/((16*(132*A^3*a^9*c^3*d^
6 - 252*A^3*a^9*c^2*d^7 - 54*B^3*a^9*c^9 + 76*A^3*a^9*c^4*d^5 - 80*A^3*a^9*
c^5*d^4 + 16*A^3*a^9*c^6*d^3 - 115*B^3*a^9*c^2*d^7 + 350*B^3*a^9*c^3*d^6 -
537*B^3*a^9*c^4*d^5 + 387*B^3*a^9*c^5*d^4 + 36*B^3*a^9*c^6*d^3 - 297*B^3*a^
9*c^7*d^2 + 108*A^3*a^9*c*d^8 + 14*B^3*a^9*c*d^8 + 216*B^3*a^9*c^8*d + 96*A
*B^2*a^9*c*d^8 + 108*A*B^2*a^9*c^8*d + 198*A^2*B*a^9*c*d^8 - 573*A*B^2*a^9*
c^2*d^7 + 1239*A*B^2*a^9*c^3*d^6 - 1125*A*B^2*a^9*c^4*d^5 + 93*A*B^2*a^9*c^
5*d^4 + 630*A*B^2*a^9*c^6*d^3 - 468*A*B^2*a^9*c^7*d^2 - 768*A^2*B*a^9*c^2*d
^7 + 996*A^2*B*a^9*c^3*d^6 - 288*A^2*B*a^9*c^4*d^5 - 402*A^2*B*a^9*c^5*d^4
+ 336*A^2*B*a^9*c^6*d^3 - 72*A^2*B*a^9*c^7*d^2))/(2*c*d^9 + d^10 + c^2*d^8)
 + (16*tan(e/2 + (f*x)/2)*(520*A^3*a^9*c^4*d^6 - 360*A^3*a^9*c^2*d^8 - 168*
A^3*a^9*c^3*d^7 - 216*B^3*a^9*c^10 - 112*A^3*a^9*c^5*d^5 - 160*A^3*a^9*c^6*
d^4 + 64*A^3*a^9*c^7*d^3 - 728*B^3*a^9*c^2*d^8 + 1702*B^3*a^9*c^3*d^7 - 109
0*B^3*a^9*c^4*d^6 - 1584*B^3*a^9*c^5*d^5 + 2898*B^3*a^9*c^6*d^4 - 1080*B^3*
a^9*c^7*d^3 - 864*B^3*a^9*c^8*d^2 + 216*A^3*a^9*c*d^9 + 98*B^3*a^9*c*d^9 +
864*B^3*a^9*c^9*d + 462*A*B^2*a^9*c*d^9 + 432*A*B^2*a^9*c^9*d + 576*A^2*B*a
^9*c*d^9 - 2178*A*B^2*a^9*c^2*d^8 + 2982*A*B^2*a^9*c^3*d^7 + 594*A*B^2*a^9*
c^4*d^6 - 4668*A*B^2*a^9*c^5*d^5 + 3096*A*B^2*a^9*c^6*d^4 + 792*A*B^2*a^9*c
^7*d^3 - 1512*A*B^2*a^9*c^8*d^2 - 1752*A^2*B*a^9*c^2*d^8 + 912*A^2*B*a^9*c^
3*d^7 + 2016*A^2*B*a^9*c^4*d^6 - 2352*A^2*B*a^9*c^5*d^5 + 24*A^2*B*a^9*c^6*
d^4 + 864*A^2*B*a^9*c^7*d^3 - 288*A^2*B*a^9*c^8*d^2))/(2*c*d^10 + d^11 + c^

2*d^9) + (((8*(36*A^2*a^6*c^2*d^9 + 24*A^2*a^6*c^3*d^8 - 44*A^2*a^6*c^4*d^7
 - 16*A^2*a^6*c^5*d^6 + 16*A^2*a^6*c^6*d^5 + 49*B^2*a^6*c^2*d^9 - 70*B^2*a^
6*c^3*d^8 - 59*B^2*a^6*c^4*d^7 + 144*B^2*a^6*c^5*d^6 - 24*B^2*a^6*c^6*d^5 -
 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8*d^3 + 84*A*B*a^6*c^2*d^9 - 32*A*B*a^6*
c^3*d^8 - 148*A*B*a^6*c^4*d^7 + 88*A*B*a^6*c^5*d^6 + 72*A*B*a^6*c^6*d^5 - 4
8*A*B*a^6*c^7*d^4))/(2*c*d^9 + d^10 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)*(144
*A^2*a^6*c^2*d^10 - 164*A^2*a^6*c^3*d^9 - 136*A^2*a^6*c^4*d^8 + 136*A^2*a^6
*c^5*d^7 + 32*A^2*a^6*c^6*d^6 - 32*A^2*a^6*c^7*d^5 - 100*B^2*a^6*c^2*d^10 -
 299*B^2*a^6*c^3*d^9 + 494*B^2*a^6*c^4*d^8 + 91*B^2*a^6*c^5*d^7 - 504*B^2*a
^6*c^6*d^6 + 156*B^2*a^6*c^7*d^5 + 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3
 + 36*A^2*a^6*c*d^11 + 94*B^2*a^6*c*d^11 + 88*A*B*a^6*c^2*d^10 - 628*A*B*a^
6*c^3*d^9 + 208*A*B*a^6*c^4*d^8 + 572*A*B*a^6*c^5*d^7 - 320*A*B*a^6*c^6*d^6
 - 144*A*B*a^6*c^7*d^5 + 96*A*B*a^6*c^8*d^4 + 144*A*B*a^6*c*d^11))/(2*c*d^1
0 + d^11 + c^2*d^9) + (((((8*(4*c^2*d^13 + 8*c^3*d^12 + 4*c^4*d^11))/(2*c*d
^9 + d^10 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)*(12*c*d^15 + 24*c^2*d^14 + 4*c
^3*d^13 - 16*c^4*d^12 - 8*c^5*d^11))/(2*c*d^10 + d^11 + c^2*d^9))*(B*a^3*c^
2*3i + a^3*d^2*(3*A + (7*B)/2)*1i - (a^3*d*(4*A*c + 12*B*c)*1i)/2))/d^4 - (
8*(12*A*a^3*c*d^12 + 14*B*a^3*c*d^12 + 4*A*a^3*c^2*d^11 - 12*A*a^3*c^3*d^10
 - 4*A*a^3*c^4*d^9 - 20*B*a^3*c^3*d^10 + 6*B*a^3*c^5*d^8))/(2*c*d^9 + d^10
 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)*(24*A*a^3*c*d^13 + 8*B*a^3*c*d^13 - 8*A*
a^3*c^2*d^12 - 40*A*a^3*c^3*d^11 + 8*A*a^3*c^4*d^10 + 16*A*a^3*c^5*d^9 - 32
*B*a^3*c^2*d^12 - 8*B*a^3*c^3*d^11 + 56*B*a^3*c^4*d^10 - 24*B*a^3*c^6*d^8))
/(2*c*d^10 + d^11 + c^2*d^9))*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7*B)/2)*1i -
(a^3*d*(4*A*c + 12*B*c)*1i)/2))/d^4)*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7*B)/2
)*1i - (a^3*d*(4*A*c + 12*B*c)*1i)/2))/d^4 - (((8*(36*A^2*a^6*c^2*d^9 + 24*
A^2*a^6*c^3*d^8 - 44*A^2*a^6*c^4*d^7 - 16*A^2*a^6*c^5*d^6 + 16*A^2*a^6*c^6*
d^5 + 49*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 - 59*B^2*a^6*c^4*d^7 + 144*B^
2*a^6*c^5*d^6 - 24*B^2*a^6*c^6*d^5 - 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8*d^
3 + 84*A*B*a^6*c^2*d^9 - 32*A*B*a^6*c^3*d^8 - 148*A*B*a^6*c^4*d^7 + 88*A*B*
a^6*c^5*d^6 + 72*A*B*a^6*c^6*d^5 - 48*A*B*a^6*c^7*d^4))/(2*c*d^9 + d^10 + c
^2*d^8) + (8*tan(e/2 + (f*x)/2)*(144*A^2*a^6*c^2*d^10 - 164*A^2*a^6*c^3*d^9
 - 136*A^2*a^6*c^4*d^8 + 136*A^2*a^6*c^5*d^7 + 32*A^2*a^6*c^6*d^6 - 32*A^2*
a^6*c^7*d^5 - 100*B^2*a^6*c^2*d^10 - 299*B^2*a^6*c^3*d^9 + 494*B^2*a^6*c^4*
d^8 + 91*B^2*a^6*c^5*d^7 - 504*B^2*a^6*c^6*d^6 + 156*B^2*a^6*c^7*d^5 + 144*
B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 + 36*A^2*a^6*c*d^11 + 94*B^2*a^6*c*d^1
1 + 88*A*B*a^6*c^2*d^10 - 628*A*B*a^6*c^3*d^9 + 208*A*B*a^6*c^4*d^8 + 572*A
*B*a^6*c^5*d^7 - 320*A*B*a^6*c^6*d^6 - 144*A*B*a^6*c^7*d^5 + 96*A*B*a^6*c^8
*d^4 + 144*A*B*a^6*c*d^11))/(2*c*d^10 + d^11 + c^2*d^9) + (((8*(12*A*a^3*c*
d^12 + 14*B*a^3*c*d^12 + 4*A*a^3*c^2*d^11 - 12*A*a^3*c^3*d^10 - 4*A*a^3*c^4
*d^9 - 20*B*a^3*c^3*d^10 + 6*B*a^3*c^5*d^8))/(2*c*d^9 + d^10 + c^2*d^8) + (
((8*(4*c^2*d^13 + 8*c^3*d^12 + 4*c^4*d^11))/(2*c*d^9 + d^10 + c^2*d^8) + (8
*tan(e/2 + (f*x)/2)*(12*c*d^15 + 24*c^2*d^14 + 4*c^3*d^13 - 16*c^4*d^12 - 8
*c^5*d^11))/(2*c*d^10 + d^11 + c^2*d^9))*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7*
B)/2)*1i - (a^3*d*(4*A*c + 12*B*c)*1i)/2))/d^4 - (8*tan(e/2 + (f*x)/2)*(24*
A*a^3*c*d^13 + 8*B*a^3*c*d^13 - 8*A*a^3*c^2*d^12 - 40*A*a^3*c^3*d^11 + 8*A*

a^3*c^4*d^10 + 16*A*a^3*c^5*d^9 - 32*B*a^3*c^2*d^12 - 8*B*a^3*c^3*d^11 + 56
*B*a^3*c^4*d^10 - 24*B*a^3*c^6*d^8))/(2*c*d^10 + d^11 + c^2*d^9))*(B*a^3*c^
2*3i + a^3*d^2*(3*A + (7*B)/2)*1i - (a^3*d*(4*A*c + 12*B*c)*1i)/2))/d^4)*(B
*a^3*c^2*3i + a^3*d^2*(3*A + (7*B)/2)*1i - (a^3*d*(4*A*c + 12*B*c)*1i)/2))/
d^4))*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7*B)/2)*1i - (a^3*d*(4*A*c + 12*B*c)*
1i)/2)*2i)/(d^4*f) - (a^3*atan(((a^3*(-(c + d)^3*(c - d)^3)^(1/2)*((8*(36*A
^2*a^6*c^2*d^9 + 24*A^2*a^6*c^3*d^8 - 44*A^2*a^6*c^4*d^7 - 16*A^2*a^6*c^5*d
^6 + 16*A^2*a^6*c^6*d^5 + 49*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 - 59*B^2*
a^6*c^4*d^7 + 144*B^2*a^6*c^5*d^6 - 24*B^2*a^6*c^6*d^5 - 72*B^2*a^6*c^7*d^4
 + 36*B^2*a^6*c^8*d^3 + 84*A*B*a^6*c^2*d^9 - 32*A*B*a^6*c^3*d^8 - 148*A*B*a
^6*c^4*d^7 + 88*A*B*a^6*c^5*d^6 + 72*A*B*a^6*c^6*d^5 - 48*A*B*a^6*c^7*d^4))
/(2*c*d^9 + d^10 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)*(144*A^2*a^6*c^2*d^10 -
 164*A^2*a^6*c^3*d^9 - 136*A^2*a^6*c^4*d^8 + 136*A^2*a^6*c^5*d^7 + 32*A^2*a
^6*c^6*d^6 - 32*A^2*a^6*c^7*d^5 - 100*B^2*a^6*c^2*d^10 - 299*B^2*a^6*c^3*d^
9 + 494*B^2*a^6*c^4*d^8 + 91*B^2*a^6*c^5*d^7 - 504*B^2*a^6*c^6*d^6 + 156*B^
2*a^6*c^7*d^5 + 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 + 36*A^2*a^6*c*d^1
1 + 94*B^2*a^6*c*d^11 + 88*A*B*a^6*c^2*d^10 - 628*A*B*a^6*c^3*d^9 + 208*A*B
*a^6*c^4*d^8 + 572*A*B*a^6*c^5*d^7 - 320*A*B*a^6*c^6*d^6 - 144*A*B*a^6*c^7*
d^5 + 96*A*B*a^6*c^8*d^4 + 144*A*B*a^6*c*d^11))/(2*c*d^10 + d^11 + c^2*d^9)
 + (a^3*(-(c + d)^3*(c - d)^3)^(1/2)*((8*tan(e/2 + (f*x)/2)*(24*A*a^3*c*d^1
3 + 8*B*a^3*c*d^13 - 8*A*a^3*c^2*d^12 - 40*A*a^3*c^3*d^11 + 8*A*a^3*c^4*d^1
0 + 16*A*a^3*c^5*d^9 - 32*B*a^3*c^2*d^12 - 8*B*a^3*c^3*d^11 + 56*B*a^3*c^4*
d^10 - 24*B*a^3*c^6*d^8))/(2*c*d^10 + d^11 + c^2*d^9) - (8*(12*A*a^3*c*d^12
 + 14*B*a^3*c*d^12 + 4*A*a^3*c^2*d^11 - 12*A*a^3*c^3*d^10 - 4*A*a^3*c^4*d^9
 - 20*B*a^3*c^3*d^10 + 6*B*a^3*c^5*d^8))/(2*c*d^9 + d^10 + c^2*d^8) + (a^3*
((8*(4*c^2*d^13 + 8*c^3*d^12 + 4*c^4*d^11))/(2*c*d^9 + d^10 + c^2*d^8) + (8
*tan(e/2 + (f*x)/2)*(12*c*d^15 + 24*c^2*d^14 + 4*c^3*d^13 - 16*c^4*d^12 - 8
*c^5*d^11))/(2*c*d^10 + d^11 + c^2*d^9))*(-(c + d)^3*(c - d)^3)^(1/2)*(3*A*
d^2 - 3*B*c^2 + B*d^2 + 2*A*c*d - 3*B*c*d))/(3*c*d^6 + d^7 + 3*c^2*d^5 + c^
3*d^4))*(3*A*d^2 - 3*B*c^2 + B*d^2 + 2*A*c*d - 3*B*c*d))/(3*c*d^6 + d^7 + 3
*c^2*d^5 + c^3*d^4))*(3*A*d^2 - 3*B*c^2 + B*d^2 + 2*A*c*d - 3*B*c*d)*1i)/(3
*c*d^6 + d^7 + 3*c^2*d^5 + c^3*d^4) + (a^3*(-(c + d)^3*(c - d)^3)^(1/2)*((8
*(36*A^2*a^6*c^2*d^9 + 24*A^2*a^6*c^3*d^8 - 44*A^2*a^6*c^4*d^7 - 16*A^2*a^6
*c^5*d^6 + 16*A^2*a^6*c^6*d^5 + 49*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 - 5
9*B^2*a^6*c^4*d^7 + 144*B^2*a^6*c^5*d^6 - 24*B^2*a^6*c^6*d^5 - 72*B^2*a^6*c
^7*d^4 + 36*B^2*a^6*c^8*d^3 + 84*A*B*a^6*c^2*d^9 - 32*A*B*a^6*c^3*d^8 - 148
*A*B*a^6*c^4*d^7 + 88*A*B*a^6*c^5*d^6 + 72*A*B*a^6*c^6*d^5 - 48*A*B*a^6*c^7
*d^4))/(2*c*d^9 + d^10 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)*(144*A^2*a^6*c^2*
d^10 - 164*A^2*a^6*c^3*d^9 - 136*A^2*a^6*c^4*d^8 + 136*A^2*a^6*c^5*d^7 + 32
*A^2*a^6*c^6*d^6 - 32*A^2*a^6*c^7*d^5 - 100*B^2*a^6*c^2*d^10 - 299*B^2*a^6*
c^3*d^9 + 494*B^2*a^6*c^4*d^8 + 91*B^2*a^6*c^5*d^7 - 504*B^2*a^6*c^6*d^6 +
156*B^2*a^6*c^7*d^5 + 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 + 36*A^2*a^6
*c*d^11 + 94*B^2*a^6*c*d^11 + 88*A*B*a^6*c^2*d^10 - 628*A*B*a^6*c^3*d^9 + 2
08*A*B*a^6*c^4*d^8 + 572*A*B*a^6*c^5*d^7 - 320*A*B*a^6*c^6*d^6 - 144*A*B*a^
6*c^7*d^5 + 96*A*B*a^6*c^8*d^4 + 144*A*B*a^6*c*d^11))/(2*c*d^10 + d^11 + c^

2*d^9) + (a^3*(-(c + d)^3*(c - d)^3)^(1/2)*((8*(12*A*a^3*c*d^12 + 14*B*a^3*
c*d^12 + 4*A*a^3*c^2*d^11 - 12*A*a^3*c^3*d^10 - 4*A*a^3*c^4*d^9 - 20*B*a^3*
c^3*d^10 + 6*B*a^3*c^5*d^8))/(2*c*d^9 + d^10 + c^2*d^8) - (8*tan(e/2 + (f*x
)/2)*(24*A*a^3*c*d^13 + 8*B*a^3*c*d^13 - 8*A*a^3*c^2*d^12 - 40*A*a^3*c^3*d^
11 + 8*A*a^3*c^4*d^10 + 16*A*a^3*c^5*d^9 - 32*B*a^3*c^2*d^12 - 8*B*a^3*c^3*
d^11 + 56*B*a^3*c^4*d^10 - 24*B*a^3*c^6*d^8))/(2*c*d^10 + d^11 + c^2*d^9) +
 (a^3*((8*(4*c^2*d^13 + 8*c^3*d^12 + 4*c^4*d^11))/(2*c*d^9 + d^10 + c^2*d^8
) + (8*tan(e/2 + (f*x)/2)*(12*c*d^15 + 24*c^2*d^14 + 4*c^3*d^13 - 16*c^4*d^
12 - 8*c^5*d^11))/(2*c*d^10 + d^11 + c^2*d^9))*(-(c + d)^3*(c - d)^3)^(1/2)
*(3*A*d^2 - 3*B*c^2 + B*d^2 + 2*A*c*d - 3*B*c*d))/(3*c*d^6 + d^7 + 3*c^2*d^
5 + c^3*d^4))*(3*A*d^2 - 3*B*c^2 + B*d^2 + 2*A*c*d - 3*B*c*d))/(3*c*d^6 + d
^7 + 3*c^2*d^5 + c^3*d^4))*(3*A*d^2 - 3*B*c^2 + B*d^2 + 2*A*c*d - 3*B*c*d)*
1i)/(3*c*d^6 + d^7 + 3*c^2*d^5 + c^3*d^4))/((16*(132*A^3*a^9*c^3*d^6 - 252*
A^3*a^9*c^2*d^7 - 54*B^3*a^9*c^9 + 76*A^3*a^9*c^4*d^5 - 80*A^3*a^9*c^5*d^4
+ 16*A^3*a^9*c^6*d^3 - 115*B^3*a^9*c^2*d^7 + 350*B^3*a^9*c^3*d^6 - 537*B^3*
a^9*c^4*d^5 + 387*B^3*a^9*c^5*d^4 + 36*B^3*a^9*c^6*d^3 - 297*B^3*a^9*c^7*d^
2 + 108*A^3*a^9*c*d^8 + 14*B^3*a^9*c*d^8 + 216*B^3*a^9*c^8*d + 96*A*B^2*a^9
*c*d^8 + 108*A*B^2*a^9*c^8*d + 198*A^2*B*a^9*c*d^8 - 573*A*B^2*a^9*c^2*d^7
+ 1239*A*B^2*a^9*c^3*d^6 - 1125*A*B^2*a^9*c^4*d^5 + 93*A*B^2*a^9*c^5*d^4 +
630*A*B^2*a^9*c^6*d^3 - 468*A*B^2*a^9*c^7*d^2 - 768*A^2*B*a^9*c^2*d^7 + 996
*A^2*B*a^9*c^3*d^6 - 288*A^2*B*a^9*c^4*d^5 - 402*A^2*B*a^9*c^5*d^4 + 336*A^
2*B*a^9*c^6*d^3 - 72*A^2*B*a^9*c^7*d^2))/(2*c*d^9 + d^10 + c^2*d^8) + (16*t
an(e/2 + (f*x)/2)*(520*A^3*a^9*c^4*d^6 - 360*A^3*a^9*c^2*d^8 - 168*A^3*a^9*
c^3*d^7 - 216*B^3*a^9*c^10 - 112*A^3*a^9*c^5*d^5 - 160*A^3*a^9*c^6*d^4 + 64
*A^3*a^9*c^7*d^3 - 728*B^3*a^9*c^2*d^8 + 1702*B^3*a^9*c^3*d^7 - 1090*B^3*a^
9*c^4*d^6 - 1584*B^3*a^9*c^5*d^5 + 2898*B^3*a^9*c^6*d^4 - 1080*B^3*a^9*c^7*
d^3 - 864*B^3*a^9*c^8*d^2 + 216*A^3*a^9*c*d^9 + 98*B^3*a^9*c*d^9 + 864*B^3*
a^9*c^9*d + 462*A*B^2*a^9*c*d^9 + 432*A*B^2*a^9*c^9*d + 576*A^2*B*a^9*c*d^9
 - 2178*A*B^2*a^9*c^2*d^8 + 2982*A*B^2*a^9*c^3*d^7 + 594*A*B^2*a^9*c^4*d^6
- 4668*A*B^2*a^9*c^5*d^5 + 3096*A*B^2*a^9*c^6*d^4 + 792*A*B^2*a^9*c^7*d^3 -
 1512*A*B^2*a^9*c^8*d^2 - 1752*A^2*B*a^9*c^2*d^8 + 912*A^2*B*a^9*c^3*d^7 +
2016*A^2*B*a^9*c^4*d^6 - 2352*A^2*B*a^9*c^5*d^5 + 24*A^2*B*a^9*c^6*d^4 + 86
4*A^2*B*a^9*c^7*d^3 - 288*A^2*B*a^9*c^8*d^2))/(2*c*d^10 + d^11 + c^2*d^9) +
 (a^3*(-(c + d)^3*(c - d)^3)^(1/2)*((8*(36*A^2*a^6*c^2*d^9 + 24*A^2*a^6*c^3
*d^8 - 44*A^2*a^6*c^4*d^7 - 16*A^2*a^6*c^5*d^6 + 16*A^2*a^6*c^6*d^5 + 49*B^
2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 - 59*B^2*a^6*c^4*d^7 + 144*B^2*a^6*c^5*d
^6 - 24*B^2*a^6*c^6*d^5 - 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8*d^3 + 84*A*B*
a^6*c^2*d^9 - 32*A*B*a^6*c^3*d^8 - 148*A*B*a^6*c^4*d^7 + 88*A*B*a^6*c^5*d^6
 + 72*A*B*a^6*c^6*d^5 - 48*A*B*a^6*c^7*d^4))/(2*c*d^9 + d^10 + c^2*d^8) + (
8*tan(e/2 + (f*x)/2)*(144*A^2*a^6*c^2*d^10 - 164*A^2*a^6*c^3*d^9 - 136*A^2*
a^6*c^4*d^8 + 136*A^2*a^6*c^5*d^7 + 32*A^2*a^6*c^6*d^6 - 32*A^2*a^6*c^7*d^5
 - 100*B^2*a^6*c^2*d^10 - 299*B^2*a^6*c^3*d^9 + 494*B^2*a^6*c^4*d^8 + 91*B^
2*a^6*c^5*d^7 - 504*B^2*a^6*c^6*d^6 + 156*B^2*a^6*c^7*d^5 + 144*B^2*a^6*c^8
*d^4 - 72*B^2*a^6*c^9*d^3 + 36*A^2*a^6*c*d^11 + 94*B^2*a^6*c*d^11 + 88*A*B*
a^6*c^2*d^10 - 628*A*B*a^6*c^3*d^9 + 208*A*B*a^6*c^4*d^8 + 572*A*B*a^6*c^5*

```
d^7 - 320*A*B*a^6*c^6*d^6 - 144*A*B*a^6*c^7*d^5 + 96*A*B*a^6*c^8*d^4 + 144*
A*B*a^6*c*d^11))/(2*c*d^10 + d^11 + c^2*d^9) + (a^3*(-(c + d)^3*(c - d)^3)^
(1/2)*((8*tan(e/2 + (f*x)/2)*(24*A*a^3*c*d^13 + 8*B*a^3*c*d^13 - 8*A*a^3*c^
2*d^12 - 40*A*a^3*c^3*d^11 + 8*A*a^3*c^4*d^10 + 16*A*a^3*c^5*d^9 - 32*B*a^3
*c^2*d^12 - 8*B*a^3*c^3*d^11 + 56*B*a^3*c^4*d^10 - 24*B*a^3*c^6*d^8))/(2*c*
d^10 + d^11 + c^2*d^9) - (8*(12*A*a^3*c*d^12 + 14*B*a^3*c*d^12 + 4*A*a^3*c^
2*d^11 - 12*A*a^3*c^3*d^10 - 4*A*a^3*c^4*d^9 - 20*B*a^3*c^3*d^10 + 6*B*a^3*
c^5*d^8))/(2*c*d^9 + d^10 + c^2*d^8) + (a^3*((8*(4*c^2*d^13 + 8*c^3*d^12 +
4*c^4*d^11))/(2*c*d^9 + d^10 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)*(12*c*d^15
+ 24*c^2*d^14 + 4*c^3*d^13 - 16*c^4*d^12 - 8*c^5*d^11))/(2*c*d^10 + d^11 +
c^2*d^9))*(-(c + d)^3*(c - d)^3)^(1/2)*(3*A*d^2 - 3*B*c^2 + B*d^2 + 2*A*c*d
 - 3*B*c*d))/(3*c*d^6 + d^7 + 3*c^2*d^5 + c^3*d^4))*(3*A*d^2 - 3*B*c^2 + B*
d^2 + 2*A*c*d - 3*B*c*d))/(3*c*d^6 + d^7 + 3*c^2*d^5 + c^3*d^4))*(3*A*d^2 -
 3*B*c^2 + B*d^2 + 2*A*c*d - 3*B*c*d))/(3*c*d^6 + d^7 + 3*c^2*d^5 + c^3*d^4
) - (a^3*(-(c + d)^3*(c - d)^3)^(1/2)*((8*(36*A^2*a^6*c^2*d^9 + 24*A^2*a^6*
c^3*d^8 - 44*A^2*a^6*c^4*d^7 - 16*A^2*a^6*c^5*d^6 + 16*A^2*a^6*c^6*d^5 + 49
*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 - 59*B^2*a^6*c^4*d^7 + 144*B^2*a^6*c^
5*d^6 - 24*B^2*a^6*c^6*d^5 - 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8*d^3 + 84*A
*B*a^6*c^2*d^9 - 32*A*B*a^6*c^3*d^8 - 148*A*B*a^6*c^4*d^7 + 88*A*B*a^6*c^5*
d^6 + 72*A*B*a^6*c^6*d^5 - 48*A*B*a^6*c^7*d^4))/(2*c*d^9 + d^10 + c^2*d^8)
+ (8*tan(e/2 + (f*x)/2)*(144*A^2*a^6*c^2*d^10 - 164*A^2*a^6*c^3*d^9 - 136*A
^2*a^6*c^4*d^8 + 136*A^2*a^6*c^5*d^7 + 32*A^2*a^6*c^6*d^6 - 32*A^2*a^6*c^7*
d^5 - 100*B^2*a^6*c^2*d^10 - 299*B^2*a^6*c^3*d^9 + 494*B^2*a^6*c^4*d^8 + 91
*B^2*a^6*c^5*d^7 - 504*B^2*a^6*c^6*d^6 + 156*B^2*a^6*c^7*d^5 + 144*B^2*a^6*
c^8*d^4 - 72*B^2*a^6*c^9*d^3 + 36*A^2*a^6*c*d^11 + 94*B^2*a^6*c*d^11 + 88*A
*B*a^6*c^2*d^10 - 628*A*B*a^6*c^3*d^9 + 208*A*B*a^6*c^4*d^8 + 572*A*B*a^6*c
^5*d^7 - 320*A*B*a^6*c^6*d^6 - 144*A*B*a^6*c^7*d^5 + 96*A*B*a^6*c^8*d^4 + 1
44*A*B*a^6*c*d^11))/(2*c*d^10 + d^11 + c^2*d^9) + (a^3*(-(c + d)^3*(c - d)^
3)^(1/2)*((8*(12*A*a^3*c*d^12 + 14*B*a^3*c*d^12 + 4*A*a^3*c^2*d^11 - 12*A*a
^3*c^3*d^10 - 4*A*a^3*c^4*d^9 - 20*B*a^3*c^3*d^10 + 6*B*a^3*c^5*d^8))/(2*c*
d^9 + d^10 + c^2*d^8) - (8*tan(e/2 + (f*x)/2)*(24*A*a^3*c*d^13 + 8*B*a^3*c*
d^13 - 8*A*a^3*c^2*d^12 - 40*A*a^3*c^3*d^11 + 8*A*a^3*c^4*d^10 + 16*A*a^3*c
^5*d^9 - 32*B*a^3*c^2*d^12 - 8*B*a^3*c^3*d^11 + 56*B*a^3*c^4*d^10 - 24*B*a^
3*c^6*d^8))/(2*c*d^10 + d^11 + c^2*d^9) + (a^3*((8*(4*c^2*d^13 + 8*c^3*d^12
 + 4*c^4*d^11))/(2*c*d^9 + d^10 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)*(12*c*d^
15 + 24*c^2*d^14 + 4*c^3*d^13 - 16*c^4*d^12 - 8*c^5*d^11))/(2*c*d^10 + d^11
 + c^2*d^9))*(-(c + d)^3*(c - d)^3)^(1/2)*(3*A*d^2 - 3*B*c^2 + B*d^2 + 2*A*
c*d - 3*B*c*d))/(3*c*d^6 + d^7 + 3*c^2*d^5 + c^3*d^4))*(3*A*d^2 - 3*B*c^2 +
 B*d^2 + 2*A*c*d - 3*B*c*d))/(3*c*d^6 + d^7 + 3*c^2*d^5 + c^3*d^4))*(3*A*d^
2 - 3*B*c^2 + B*d^2 + 2*A*c*d - 3*B*c*d))/(3*c*d^6 + d^7 + 3*c^2*d^5 + c^3*
d^4)))*(-(c + d)^3*(c - d)^3)^(1/2)*(3*A*d^2 - 3*B*c^2 + B*d^2 + 2*A*c*d -
3*B*c*d)*2i)/(f*(3*c*d^6 + d^7 + 3*c^2*d^5 + c^3*d^4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.264 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=305

$$-\frac{\left(Ad(c+4d)-B\left(3c^2+4cd-2d^2\right)\right)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{2d^2f(c+d)^2(c+d\sin(e+fx))}-\frac{a^3(c-d)\left(Ad\left(2c^2+6cd+7d^2\right)-3B\left(2c\right.\right.}{d^4f(c+}$$

```
[Out] -a^3*(-A*d+3*B*c-3*B*d)*x/d^4-1/2*a^3*(3*B*c*(2*c+3*d)-A*d*(2*c+5*d))*cos(f
*x+e)/d^3/(c+d)^2/f+1/2*a*(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))^2/d/(c+d)/
f/(c+d*sin(f*x+e))^2-1/2*(A*d*(c+4*d)-B*(3*c^2+4*c*d-2*d^2))*cos(f*x+e)*(a^
3+a^3*sin(f*x+e))/d^2/(c+d)^2/f/(c+d*sin(f*x+e))-a^3*(c-d)*(A*d*(2*c^2+6*c*
d+7*d^2)-3*B*(2*c^3+4*c^2*d+c*d^2-2*d^3))*arctan((d+c*tan(1/2*f*x+1/2*e))/(
c^2-d^2)^(1/2))/d^4/(c+d)^2/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 0.93, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2975, 2968, 3023, 2735, 2660, 618, 204}

$$-\frac{a^3(c-d)\left(Ad\left(2c^2+6cd+7d^2\right)-3B\left(4c^2d+2c^3+cd^2-2d^3\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^4f(c+d)^2\sqrt{c^2-d^2}}-\frac{\left(Ad(c+4d)-B\left(3c^2+\right.\right.}{2d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -((a^3*(3*B*c - A*d - 3*B*d)*x)/d^4) - (a^3*(c - d)*(A*d*(2*c^2 + 6*c*d + 7
*d^2) - 3*B*(2*c^3 + 4*c^2*d + c*d^2 - 2*d^3))*ArcTan[(d + c*Tan[(e + f*x)/
2])/Sqrt[c^2 - d^2]])/(d^4*(c + d)^2*Sqrt[c^2 - d^2]*f) - (a^3*(3*B*c*(2*c
+ 3*d) - A*d*(2*c + 5*d))*Cos[e + f*x])/(2*d^3*(c + d)^2*f) + (a*(B*c - A*d
)*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/(2*d*(c + d)*f*(c + d*Sin[e + f*x])^
2) - ((A*d*(c + 4*d) - B*(3*c^2 + 4*c*d - 2*d^2))*Cos[e + f*x]*(a^3 + a^3*S
in[e + f*x]))/(2*d^2*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{\int \frac{(a+a\sin(e+fx))^2(-2a}{}}{}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{\left(Ad(c+4d)-B\left(3\right.\right.}{}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{\left(Ad(c+4d)-B\left(3\right.\right.}{}$$

$$= -\frac{a^3(3Bc(2c+3d)-Ad(2c+5d))\cos(e+fx)}{2d^3(c+d)^2f} + \frac{a(Bc-Ad)\cos(}{2d(c+d)}$$

$$= -\frac{a^3(3Bc-Ad-3Bd)x}{d^4} - \frac{a^3(3Bc(2c+3d)-Ad(2c+5d))\cos(e}{2d^3(c+d)^2f}$$

$$= -\frac{a^3(3Bc-Ad-3Bd)x}{d^4} - \frac{a^3(3Bc(2c+3d)-Ad(2c+5d))\cos(e}{2d^3(c+d)^2f}$$

$$= -\frac{a^3(3Bc-Ad-3Bd)x}{d^4} - \frac{a^3(3Bc(2c+3d)-Ad(2c+5d))\cos(e}{2d^3(c+d)^2f}$$

$$= -\frac{a^3(3Bc-Ad-3Bd)x}{d^4} - \frac{a^3(c-d)\left(Ad\left(2c^2+6cd+7d^2\right)-3B\left(\right.\right.}{d^4(c+}$$

Mathematica [B] time = 3.14, size = 830, normalized size = 2.72

$$\frac{a^3(\sin(e+fx)+1)^3\left(\frac{4(c-d)\left(3B\left(2c^3+4dc^2+d^2c-2d^3\right)-Ad\left(2c^2+6dc+7d^2\right)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + \frac{-12Bec^5-12Bfxc^5+4Adec^4-12Bdec^4+4A}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^3,x]
```

```
[Out] (a^3*(1 + Sin[e + f*x])^3*((4*(c - d)*(-(A*d*(2*c^2 + 6*c*d + 7*d^2)) + 3*B
*(2*c^3 + 4*c^2*d + c*d^2 - 2*d^3))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^
2 - d^2]])/Sqrt[c^2 - d^2] + (-12*B*c^5*e + 4*A*c^4*d*e - 12*B*c^4*d*e + 8*
A*c^3*d^2*e + 6*B*c^3*d^2*e + 6*A*c^2*d^3*e + 6*B*c^2*d^3*e + 4*A*c*d^4*e +
 6*B*c*d^4*e + 2*A*d^5*e + 6*B*d^5*e - 12*B*c^5*f*x + 4*A*c^4*d*f*x - 12*B*
c^4*d*f*x + 8*A*c^3*d^2*f*x + 6*B*c^3*d^2*f*x + 6*A*c^2*d^3*f*x + 6*B*c^2*d
^3*f*x + 4*A*c*d^4*f*x + 6*B*c*d^4*f*x + 2*A*d^5*f*x + 6*B*d^5*f*x - d*(2*A
*d*(-2*c^3 - 4*c^2*d + 5*c*d^2 + d^3) + B*(12*c^4 + 12*c^3*d - 9*c^2*d^2 +
4*c*d^3 + d^4))*Cos[e + f*x] - 2*d^2*(c + d)^2*(-3*B*c + A*d + 3*B*d)*(e +
f*x)*Cos[2*(e + f*x)] + B*c^2*d^3*Cos[3*(e + f*x)] + 2*B*c*d^4*Cos[3*(e + f
*x)] + B*d^5*Cos[3*(e + f*x)] - 24*B*c^4*d*e*Sin[e + f*x] + 8*A*c^3*d^2*e*S
in[e + f*x] - 24*B*c^3*d^2*e*Sin[e + f*x] + 16*A*c^2*d^3*e*Sin[e + f*x] + 2
4*B*c^2*d^3*e*Sin[e + f*x] + 8*A*c*d^4*e*Sin[e + f*x] + 24*B*c*d^4*e*Sin[e
+ f*x] - 24*B*c^4*d*f*x*Sin[e + f*x] + 8*A*c^3*d^2*f*x*Sin[e + f*x] - 24*B*
c^3*d^2*f*x*Sin[e + f*x] + 16*A*c^2*d^3*f*x*Sin[e + f*x] + 24*B*c^2*d^3*f*x
*Sin[e + f*x] + 8*A*c*d^4*f*x*Sin[e + f*x] + 24*B*c*d^4*f*x*Sin[e + f*x] -
9*B*c^3*d^2*Sin[2*(e + f*x)] + 3*A*c^2*d^3*Sin[2*(e + f*x)] - 9*B*c^2*d^3*S
in[2*(e + f*x)] + 3*A*c*d^4*Sin[2*(e + f*x)] + 4*B*c*d^4*Sin[2*(e + f*x)] -
 6*A*d^5*Sin[2*(e + f*x)] - 2*B*d^5*Sin[2*(e + f*x)])/(c + d*Sin[e + f*x])^
2))/(4*d^4*(c + d)^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

fricas [B] time = 0.61, size = 1670, normalized size = 5.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] [-1/4*(4*(3*B*a^3*c^3*d^2 - (A - 3*B)*a^3*c^2*d^3 - (2*A + 3*B)*a^3*c*d^4 -
 (A + 3*B)*a^3*d^5)*f*x*cos(f*x + e)^2 + 4*(B*a^3*c^2*d^3 + 2*B*a^3*c*d^4 +
 B*a^3*d^5)*cos(f*x + e)^3 - 4*(3*B*a^3*c^5 - (A - 3*B)*a^3*c^4*d - 2*A*a^3
*c^3*d^2 - 2*A*a^3*c^2*d^3 - (2*A + 3*B)*a^3*c*d^4 - (A + 3*B)*a^3*d^5)*f*x
 - (6*B*a^3*c^5 - 2*(A - 6*B)*a^3*c^4*d - 3*(2*A - 3*B)*a^3*c^3*d^2 - 3*(3*
A - 2*B)*a^3*c^2*d^3 - 3*(2*A - B)*a^3*c*d^4 - (7*A + 6*B)*a^3*d^5 - (6*B*a
^3*c^3*d^2 - 2*(A - 6*B)*a^3*c^2*d^3 - 3*(2*A - B)*a^3*c*d^4 - (7*A + 6*B)*
a^3*d^5)*cos(f*x + e)^2 + 2*(6*B*a^3*c^4*d - 2*(A - 6*B)*a^3*c^3*d^2 - 3*(2
*A - B)*a^3*c^2*d^3 - (7*A + 6*B)*a^3*c*d^4)*sin(f*x + e))*sqrt(-(c - d)/(c
 + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 +
 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(
-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) -
 2*(6*B*a^3*c^4*d - 2*(A - 3*B)*a^3*c^3*d^2 - (4*A + 3*B)*a^3*c^2*d^3 + 5*(
A + B)*a^3*c*d^4 + (A + 2*B)*a^3*d^5)*cos(f*x + e) - 2*(4*(3*B*a^3*c^4*d -
(A - 3*B)*a^3*c^3*d^2 - (2*A + 3*B)*a^3*c^2*d^3 - (A + 3*B)*a^3*c*d^4)*f*x
+ (9*B*a^3*c^3*d^2 - 3*(A - 3*B)*a^3*c^2*d^3 - (3*A + 4*B)*a^3*c*d^4 + 2*(3
```

```
*A + B)*a^3*d^5)*cos(f*x + e))*sin(f*x + e))/((c^2*d^6 + 2*c*d^7 + d^8)*f*c
os(f*x + e)^2 - 2*(c^3*d^5 + 2*c^2*d^6 + c*d^7)*f*sin(f*x + e) - (c^4*d^4 +
 2*c^3*d^5 + 2*c^2*d^6 + 2*c*d^7 + d^8)*f), -1/2*(2*(3*B*a^3*c^3*d^2 - (A -
 3*B)*a^3*c^2*d^3 - (2*A + 3*B)*a^3*c*d^4 - (A + 3*B)*a^3*d^5)*f*x*cos(f*x
+ e)^2 + 2*(B*a^3*c^2*d^3 + 2*B*a^3*c*d^4 + B*a^3*d^5)*cos(f*x + e)^3 - 2*(
3*B*a^3*c^5 - (A - 3*B)*a^3*c^4*d - 2*A*a^3*c^3*d^2 - 2*A*a^3*c^2*d^3 - (2*
A + 3*B)*a^3*c*d^4 - (A + 3*B)*a^3*d^5)*f*x - (6*B*a^3*c^5 - 2*(A - 6*B)*a^
3*c^4*d - 3*(2*A - 3*B)*a^3*c^3*d^2 - 3*(3*A - 2*B)*a^3*c^2*d^3 - 3*(2*A -
B)*a^3*c*d^4 - (7*A + 6*B)*a^3*d^5 - (6*B*a^3*c^3*d^2 - 2*(A - 6*B)*a^3*c^2
*d^3 - 3*(2*A - B)*a^3*c*d^4 - (7*A + 6*B)*a^3*d^5)*cos(f*x + e)^2 + 2*(6*B
*a^3*c^4*d - 2*(A - 6*B)*a^3*c^3*d^2 - 3*(2*A - B)*a^3*c^2*d^3 - (7*A + 6*B
)*a^3*c*d^4)*sin(f*x + e))*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) +
d)*sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e))) - (6*B*a^3*c^4*d - 2*(A -
3*B)*a^3*c^3*d^2 - (4*A + 3*B)*a^3*c^2*d^3 + 5*(A + B)*a^3*c*d^4 + (A + 2*B
)*a^3*d^5)*cos(f*x + e) - (4*(3*B*a^3*c^4*d - (A - 3*B)*a^3*c^3*d^2 - (2*A
+ 3*B)*a^3*c^2*d^3 - (A + 3*B)*a^3*c*d^4)*f*x + (9*B*a^3*c^3*d^2 - 3*(A - 3
*B)*a^3*c^2*d^3 - (3*A + 4*B)*a^3*c*d^4 + 2*(3*A + B)*a^3*d^5)*cos(f*x + e)
)*sin(f*x + e))/((c^2*d^6 + 2*c*d^7 + d^8)*f*cos(f*x + e)^2 - 2*(c^3*d^5 +
2*c^2*d^6 + c*d^7)*f*sin(f*x + e) - (c^4*d^4 + 2*c^3*d^5 + 2*c^2*d^6 + 2*c*
d^7 + d^8)*f)]
```

giac [B] time = 0.30, size = 986, normalized size = 3.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] ((6*B*a^3*c^4 - 2*A*a^3*c^3*d + 6*B*a^3*c^3*d - 4*A*a^3*c^2*d^2 - 9*B*a^3*c
^2*d^2 - A*a^3*c*d^3 - 9*B*a^3*c*d^3 + 7*A*a^3*d^4 + 6*B*a^3*d^4)*(pi*floor
(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(
c^2 - d^2)))/((c^2*d^4 + 2*c*d^5 + d^6)*sqrt(c^2 - d^2)) - 2*B*a^3/((tan(1/
2*f*x + 1/2*e)^2 + 1)*d^3) - (3*B*a^3*c^5*d*tan(1/2*f*x + 1/2*e)^3 - A*a^3*
c^4*d^2*tan(1/2*f*x + 1/2*e)^3 + 3*B*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e)^3 - 5
*A*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e)^3 - 6*B*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e
)^3 + 4*A*a^3*c^2*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*A*a^3*c*d^5*tan(1/2*f*x +
1/2*e)^3 + 4*B*a^3*c^6*tan(1/2*f*x + 1/2*e)^2 - 2*A*a^3*c^5*d*tan(1/2*f*x +
 1/2*e)^2 + 2*B*a^3*c^5*d*tan(1/2*f*x + 1/2*e)^2 - 4*A*a^3*c^4*d^2*tan(1/2*
f*x + 1/2*e)^2 + B*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e)^2 + A*a^3*c^3*d^3*tan(1
/2*f*x + 1/2*e)^2 + 5*B*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e)^2 - 7*A*a^3*c^2*d^
4*tan(1/2*f*x + 1/2*e)^2 - 14*B*a^3*c^2*d^4*tan(1/2*f*x + 1/2*e)^2 + 10*A*a
^3*c*d^5*tan(1/2*f*x + 1/2*e)^2 + 2*B*a^3*c*d^5*tan(1/2*f*x + 1/2*e)^2 + 2*
A*a^3*d^6*tan(1/2*f*x + 1/2*e)^2 + 13*B*a^3*c^5*d*tan(1/2*f*x + 1/2*e) - 7*
A*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e) + 5*B*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e) -
```

```
 11*A*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e) - 22*B*a^3*c^3*d^3*tan(1/2*f*x + 1/2
*e) + 16*A*a^3*c^2*d^4*tan(1/2*f*x + 1/2*e) + 4*B*a^3*c^2*d^4*tan(1/2*f*x +
 1/2*e) + 2*A*a^3*c*d^5*tan(1/2*f*x + 1/2*e) + 4*B*a^3*c^6 - 2*A*a^3*c^5*d
+ 2*B*a^3*c^5*d - 4*A*a^3*c^4*d^2 - 7*B*a^3*c^4*d^2 + 5*A*a^3*c^3*d^3 + B*a
^3*c^3*d^3 + A*a^3*c^2*d^4)/((c^4*d^3 + 2*c^3*d^4 + c^2*d^5)*(c*tan(1/2*f*x
 + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2) - (3*B*a^3*c - A*a^3*d - 3*B
*a^3*d)*(f*x + e)/d^4)/f
```

maple [B] time = 0.56, size = 2906, normalized size = 9.53

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -3*a^3/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d
^2)*c^2*tan(1/2*f*x+1/2*e)^3*B+2*a^3/f/d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/
2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^3*tan(1/2*f*x+1/2*e)^2*A+4*a^3/f/d/(t
an(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^2*tan(1
/2*f*x+1/2*e)^2*A-10*a^3/f*d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)
*d+c)^2/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^2*A-9*a^3/f/d/(c^2+2*c*d+d^2)/
(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*
c-4*a^3/f/d^2/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1
/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c^2-2*a^3/f*d^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan
(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/2*e)^2*A-4*a^3/f/d
^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^4*
tan(1/2*f*x+1/2*e)^2*B-13*a^3/f/d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1
/2*e)*d+c)^2*c^3/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*B+6*a^3/f/d^3/(c^2+2*c*
d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1
/2))*B*c^3+a^3/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2
+2*c*d+d^2)*c^2*tan(1/2*f*x+1/2*e)^3*A-2*a^3/f*d^2/(tan(1/2*f*x+1/2*e)^2*c+
2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^3*A-9*a^3/
f/d^2/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*
d)/(c^2-d^2)^(1/2))*B*c^2-2*a^3/f/d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x
+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^3*tan(1/2*f*x+1/2*e)^2*B-3*a^3/f/d^2/(tan(
1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^3*tan(1/2*
f*x+1/2*e)^3*B-2*a^3/f/d^3/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*
tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c^3-5*a^3/f/d/(tan(1/2*f*x+1/2*e
)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*B-
a^3/f/d/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+
2*d)/(c^2-d^2)^(1/2))*A*c+6*a^3/f/d^4/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arcta
n(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c^4*B-a^3/f/d/(tan(1/2*
f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^2*tan(1/2*f*x+
1/2*e)^2*B-2*a^3/f*d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/
(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^2*B+7*a^3/f/d/(tan(1/2*f*x+1/2*e)^2*c+
```

```
2*tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*A-2*a^3/
f*d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^2+2*c*d+d^2)
*tan(1/2*f*x+1/2*e)*A-a^3/f*d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*
d+c)^2/(c^2+2*c*d+d^2)*A-6*a^3/f/d^4*B*arctan(tan(1/2*f*x+1/2*e))*c-a^3/f/(
tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*tan(1/
2*f*x+1/2*e)^2*A+22*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)
^2*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*B-5*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2
*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*tan(1/2*f*x+1/2*e)^2*B+11*a^3/
f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*tan
(1/2*f*x+1/2*e)*A+5*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)
^2/(c^2+2*c*d+d^2)*c*tan(1/2*f*x+1/2*e)^3*A+6*a^3/f/(tan(1/2*f*x+1/2*e)^2*c
+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*tan(1/2*f*x+1/2*e)^3*B-2*a^3
/f/d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*
B*c^3+7*a^3/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*
c*d+d^2)*B*c^2+2*a^3/f/d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c
)^2/(c^2+2*c*d+d^2)*A*c^3-4*a^3/f*d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1
/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*A+7*a^3/f*d/(tan(1/2*f*x+
1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2
*A+14*a^3/f*d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*
d+d^2)*tan(1/2*f*x+1/2*e)^2*B-16*a^3/f*d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*
f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*A-4*a^3/f*d/(tan(1/2*f
*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e
)*B+4*a^3/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*
d+d^2)*A*c^2-4*a^3/f/d^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^
2/(c^2+2*c*d+d^2)*c^4*B-a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*
d+c)^2/(c^2+2*c*d+d^2)*B*c-5*a^3/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/
2*e)*d+c)^2/(c^2+2*c*d+d^2)*A*c+7*a^3/f/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arc
tan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A+6*a^3/f/(c^2+2*c*d+
d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2
))*B-2*a^3/f/d^3*B/(1+tan(1/2*f*x+1/2*e)^2)+2*a^3/f/d^3*A*arctan(tan(1/2*f*
x+1/2*e))+6*a^3/f/d^3*B*arctan(tan(1/2*f*x+1/2*e))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 25.41, size = 13891, normalized size = 45.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c + d*sin(e + f*x))^3,x)

[Out] - ((A*a^3*d^4 + 6*B*a^3*c^4 + 5*A*a^3*c*d^3 - 2*A*a^3*c^3*d + B*a^3*c*d^3 +
 6*B*a^3*c^3*d - 4*A*a^3*c^2*d^2 - 5*B*a^3*c^2*d^2)/(d^3*(c + d)^2) + (4*ta
n(e/2 + (f*x)/2)^3*(A*a^3*d^4 + 6*B*a^3*c^4 + 5*A*a^3*c*d^3 - 2*A*a^3*c^3*d
 + B*a^3*c*d^3 + 6*B*a^3*c^3*d - 4*A*a^3*c^2*d^2 - 5*B*a^3*c^2*d^2))/(c*d^2
*(c + d)^2) + (tan(e/2 + (f*x)/2)^5*(2*A*a^3*d^4 + 3*B*a^3*c^4 + 4*A*a^3*c*
d^3 - A*a^3*c^3*d + 3*B*a^3*c^3*d - 5*A*a^3*c^2*d^2 - 6*B*a^3*c^2*d^2))/(c*
d^2*(c + d)^2) + (2*tan(e/2 + (f*x)/2)^2*(A*a^3*d^6 + 6*B*a^3*c^6 + 5*A*a^3
*c*d^5 - 2*A*a^3*c^5*d + B*a^3*c*d^5 + 6*B*a^3*c^5*d - 3*A*a^3*c^2*d^4 + 3*
A*a^3*c^3*d^3 - 4*A*a^3*c^4*d^2 - 3*B*a^3*c^2*d^4 + 11*B*a^3*c^3*d^3 + 3*B*
a^3*c^4*d^2))/(c^2*d^3*(c + d)^2) + (tan(e/2 + (f*x)/2)^4*(2*A*a^3*d^6 + 6*
B*a^3*c^6 + 10*A*a^3*c*d^5 - 2*A*a^3*c^5*d + 2*B*a^3*c*d^5 + 6*B*a^3*c^5*d
 - 7*A*a^3*c^2*d^4 + A*a^3*c^3*d^3 - 4*A*a^3*c^4*d^2 - 14*B*a^3*c^2*d^4 + 5*
B*a^3*c^3*d^3 + 3*B*a^3*c^4*d^2))/(c^2*d^3*(c + d)^2) + (tan(e/2 + (f*x)/2)
*(2*A*a^3*d^4 + 21*B*a^3*c^4 + 16*A*a^3*c*d^3 - 7*A*a^3*c^3*d + 4*B*a^3*c*d
^3 + 21*B*a^3*c^3*d - 11*A*a^3*c^2*d^2 - 14*B*a^3*c^2*d^2))/(c*d^2*(c + d)^
2))/(f*(tan(e/2 + (f*x)/2)^2*(3*c^2 + 4*d^2) + tan(e/2 + (f*x)/2)^4*(3*c^2
 + 4*d^2) + c^2*tan(e/2 + (f*x)/2)^6 + c^2 + 8*c*d*tan(e/2 + (f*x)/2)^3 + 4*
c*d*tan(e/2 + (f*x)/2)^5 + 4*c*d*tan(e/2 + (f*x)/2))) - (atan((((B*a^3*c*3i
 - a^3*d*(A + 3*B)*1i)*((8*(4*A^2*a^6*c^2*d^9 + 16*A^2*a^6*c^3*d^8 + 24*A^2
*a^6*c^4*d^7 + 16*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 36*B^2*a^6*c^2*d^9
 + 72*B^2*a^6*c^3*d^8 - 36*B^2*a^6*c^4*d^7 - 144*B^2*a^6*c^5*d^6 - 36*B^2*a^
6*c^6*d^5 + 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8*d^3 + 24*A*B*a^6*c^2*d^9 +
72*A*B*a^6*c^3*d^8 + 48*A*B*a^6*c^4*d^7 - 48*A*B*a^6*c^5*d^6 - 72*A*B*a^6*c
^6*d^5 - 24*A*B*a^6*c^7*d^4))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c
^4*d^8) + (8*tan(e/2 + (f*x)/2)*(46*A^2*a^6*c^2*d^10 + 99*A^2*a^6*c^3*d^9 +
 36*A^2*a^6*c^4*d^8 - 36*A^2*a^6*c^5*d^7 - 32*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c
^7*d^5 + 252*B^2*a^6*c^2*d^10 - 81*B^2*a^6*c^3*d^9 - 594*B^2*a^6*c^4*d^8 -
81*B^2*a^6*c^5*d^7 + 504*B^2*a^6*c^6*d^6 + 180*B^2*a^6*c^7*d^5 - 144*B^2*a^
6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 - 41*A^2*a^6*c*d^11 + 36*B^2*a^6*c*d^11 + 28
2*A*B*a^6*c^2*d^10 + 228*A*B*a^6*c^3*d^9 - 318*A*B*a^6*c^4*d^8 - 372*A*B*a^
6*c^5*d^7 + 24*A*B*a^6*c^6*d^6 + 144*A*B*a^6*c^7*d^5 + 48*A*B*a^6*c^8*d^4 -
 36*A*B*a^6*c*d^11))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9)
+ ((B*a^3*c*3i - a^3*d*(A + 3*B)*1i)*((8*tan(e/2 + (f*x)/2)*(28*A*a^3*c*d^1
4 + 24*B*a^3*c*d^14 + 52*A*a^3*c^2*d^13 + 4*A*a^3*c^3*d^12 - 44*A*a^3*c^4*d
^11 - 32*A*a^3*c^5*d^10 - 8*A*a^3*c^6*d^9 + 12*B*a^3*c^2*d^13 - 84*B*a^3*c^
3*d^12 - 84*B*a^3*c^4*d^11 + 36*B*a^3*c^5*d^10 + 72*B*a^3*c^6*d^9 + 24*B*a^
3*c^7*d^8))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) - (8*(4*A
```

*a^3*c*d^13 + 12*B*a^3*c*d^13 + 2*A*a^3*c^2*d^12 - 6*A*a^3*c^3*d^11 - 2*A*a
^3*c^4*d^10 + 2*A*a^3*c^5*d^9 + 24*B*a^3*c^2*d^12 + 6*B*a^3*c^3*d^11 - 18*B
*a^3*c^4*d^10 - 18*B*a^3*c^5*d^9 - 6*B*a^3*c^6*d^8))/(4*c*d^11 + d^12 + 6*c
^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (((8*(4*c^2*d^15 + 16*c^3*d^14 + 24*c^4*d^
13 + 16*c^5*d^12 + 4*c^6*d^11))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 +
c^4*d^8) + (8*tan(e/2 + (f*x)/2)*(12*c*d^17 + 48*c^2*d^16 + 64*c^3*d^15 +
16*c^4*d^14 - 36*c^5*d^13 - 32*c^6*d^12 - 8*c^7*d^11))/(4*c*d^12 + d^13 + 6
*c^2*d^11 + 4*c^3*d^10 + c^4*d^9))*(B*a^3*c*3i - a^3*d*(A + 3*B)*1i))/d^4))
/d^4)*1i)/d^4 + ((B*a^3*c*3i - a^3*d*(A + 3*B)*1i)*((8*(4*A^2*a^6*c^2*d^9 +
16*A^2*a^6*c^3*d^8 + 24*A^2*a^6*c^4*d^7 + 16*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c
^6*d^5 + 36*B^2*a^6*c^2*d^9 + 72*B^2*a^6*c^3*d^8 - 36*B^2*a^6*c^4*d^7 - 144
*B^2*a^6*c^5*d^6 - 36*B^2*a^6*c^6*d^5 + 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8
*d^3 + 24*A*B*a^6*c^2*d^9 + 72*A*B*a^6*c^3*d^8 + 48*A*B*a^6*c^4*d^7 - 48*A*
B*a^6*c^5*d^6 - 72*A*B*a^6*c^6*d^5 - 24*A*B*a^6*c^7*d^4))/(4*c*d^11 + d^12
+ 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (8*tan(e/2 + (f*x)/2)*(46*A^2*a^6*c^2
*d^10 + 99*A^2*a^6*c^3*d^9 + 36*A^2*a^6*c^4*d^8 - 36*A^2*a^6*c^5*d^7 - 32*A
^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 + 252*B^2*a^6*c^2*d^10 - 81*B^2*a^6*c^3*
d^9 - 594*B^2*a^6*c^4*d^8 - 81*B^2*a^6*c^5*d^7 + 504*B^2*a^6*c^6*d^6 + 180*
B^2*a^6*c^7*d^5 - 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 - 41*A^2*a^6*c*d
^11 + 36*B^2*a^6*c*d^11 + 282*A*B*a^6*c^2*d^10 + 228*A*B*a^6*c^3*d^9 - 318*
A*B*a^6*c^4*d^8 - 372*A*B*a^6*c^5*d^7 + 24*A*B*a^6*c^6*d^6 + 144*A*B*a^6*c^
7*d^5 + 48*A*B*a^6*c^8*d^4 - 36*A*B*a^6*c*d^11))/(4*c*d^12 + d^13 + 6*c^2*d
^11 + 4*c^3*d^10 + c^4*d^9) + ((B*a^3*c*3i - a^3*d*(A + 3*B)*1i)*((8*(4*A*a
^3*c*d^13 + 12*B*a^3*c*d^13 + 2*A*a^3*c^2*d^12 - 6*A*a^3*c^3*d^11 - 2*A*a^3
*c^4*d^10 + 2*A*a^3*c^5*d^9 + 24*B*a^3*c^2*d^12 + 6*B*a^3*c^3*d^11 - 18*B*a
^3*c^4*d^10 - 18*B*a^3*c^5*d^9 - 6*B*a^3*c^6*d^8))/(4*c*d^11 + d^12 + 6*c^2
*d^10 + 4*c^3*d^9 + c^4*d^8) - (8*tan(e/2 + (f*x)/2)*(28*A*a^3*c*d^14 + 24*
B*a^3*c*d^14 + 52*A*a^3*c^2*d^13 + 4*A*a^3*c^3*d^12 - 44*A*a^3*c^4*d^11 - 3
2*A*a^3*c^5*d^10 - 8*A*a^3*c^6*d^9 + 12*B*a^3*c^2*d^13 - 84*B*a^3*c^3*d^12
- 84*B*a^3*c^4*d^11 + 36*B*a^3*c^5*d^10 + 72*B*a^3*c^6*d^9 + 24*B*a^3*c^7*d
^8))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) + (((8*(4*c^2*d^
15 + 16*c^3*d^14 + 24*c^4*d^13 + 16*c^5*d^12 + 4*c^6*d^11))/(4*c*d^11 + d^1
2 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (8*tan(e/2 + (f*x)/2)*(12*c*d^17 +
48*c^2*d^16 + 64*c^3*d^15 + 16*c^4*d^14 - 36*c^5*d^13 - 32*c^6*d^12 - 8*c^7
*d^11))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9))*(B*a^3*c*3i
- a^3*d*(A + 3*B)*1i))/d^4))/d^4)*1i)/d^4)/((16*(54*B^3*a^9*c^8 - 29*A^3*a^
9*c^3*d^5 - 18*A^3*a^9*c^4*d^4 - 2*A^3*a^9*c^5*d^3 - 324*B^3*a^9*c^2*d^6 +
81*B^3*a^9*c^3*d^5 + 405*B^3*a^9*c^4*d^4 - 135*B^3*a^9*c^5*d^3 - 243*B^3*a^
9*c^6*d^2 + 49*A^3*a^9*c*d^7 + 108*B^3*a^9*c*d^7 + 54*B^3*a^9*c^7*d + 288*A
*B^2*a^9*c*d^7 - 54*A*B^2*a^9*c^7*d + 231*A^2*B*a^9*c*d^7 - 576*A*B^2*a^9*c
^2*d^6 - 135*A*B^2*a^9*c^3*d^5 + 540*A*B^2*a^9*c^4*d^4 + 135*A*B^2*a^9*c^5*
d^3 - 198*A*B^2*a^9*c^6*d^2 - 231*A^2*B*a^9*c^2*d^6 - 201*A^2*B*a^9*c^3*d^5
+ 69*A^2*B*a^9*c^4*d^4 + 114*A^2*B*a^9*c^5*d^3 + 18*A^2*B*a^9*c^6*d^2))/(4
*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (16*tan(e/2 + (f*x)/2)
*(216*B^3*a^9*c^9 + 52*A^3*a^9*c^2*d^7 + 4*A^3*a^9*c^3*d^6 - 44*A^3*a^9*c^4

*d^5 - 32*A^3*a^9*c^5*d^4 - 8*A^3*a^9*c^6*d^3 - 324*B^3*a^9*c^2*d^7 - 756*B
^3*a^9*c^3*d^6 + 864*B^3*a^9*c^4*d^5 + 1080*B^3*a^9*c^5*d^4 - 756*B^3*a^9*c
^6*d^3 - 756*B^3*a^9*c^7*d^2 + 28*A^3*a^9*c*d^8 + 216*B^3*a^9*c*d^8 + 216*B
^3*a^9*c^8*d + 396*A*B^2*a^9*c*d^8 - 216*A*B^2*a^9*c^8*d + 192*A^2*B*a^9*c*
d^8 - 108*A*B^2*a^9*c^2*d^7 - 1224*A*B^2*a^9*c^3*d^6 + 1260*A*B^2*a^9*c^5*d
^4 + 324*A*B^2*a^9*c^6*d^3 - 432*A*B^2*a^9*c^7*d^2 + 156*A^2*B*a^9*c^2*d^7
- 372*A^2*B*a^9*c^3*d^6 - 372*A^2*B*a^9*c^4*d^5 + 108*A^2*B*a^9*c^5*d^4 + 2
16*A^2*B*a^9*c^6*d^3 + 72*A^2*B*a^9*c^7*d^2))/(4*c*d^12 + d^13 + 6*c^2*d^11
+ 4*c^3*d^10 + c^4*d^9) + ((B*a^3*c*3i - a^3*d*(A + 3*B)*1i)*((8*(4*A^2*a^
6*c^2*d^9 + 16*A^2*a^6*c^3*d^8 + 24*A^2*a^6*c^4*d^7 + 16*A^2*a^6*c^5*d^6 +
4*A^2*a^6*c^6*d^5 + 36*B^2*a^6*c^2*d^9 + 72*B^2*a^6*c^3*d^8 - 36*B^2*a^6*c^
4*d^7 - 144*B^2*a^6*c^5*d^6 - 36*B^2*a^6*c^6*d^5 + 72*B^2*a^6*c^7*d^4 + 36*
B^2*a^6*c^8*d^3 + 24*A*B*a^6*c^2*d^9 + 72*A*B*a^6*c^3*d^8 + 48*A*B*a^6*c^4*
d^7 - 48*A*B*a^6*c^5*d^6 - 72*A*B*a^6*c^6*d^5 - 24*A*B*a^6*c^7*d^4))/(4*c*d
^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (8*tan(e/2 + (f*x)/2)*(46*
A^2*a^6*c^2*d^10 + 99*A^2*a^6*c^3*d^9 + 36*A^2*a^6*c^4*d^8 - 36*A^2*a^6*c^5
*d^7 - 32*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 + 252*B^2*a^6*c^2*d^10 - 81*B
^2*a^6*c^3*d^9 - 594*B^2*a^6*c^4*d^8 - 81*B^2*a^6*c^5*d^7 + 504*B^2*a^6*c^6
*d^6 + 180*B^2*a^6*c^7*d^5 - 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 - 41*
A^2*a^6*c*d^11 + 36*B^2*a^6*c*d^11 + 282*A*B*a^6*c^2*d^10 + 228*A*B*a^6*c^3
*d^9 - 318*A*B*a^6*c^4*d^8 - 372*A*B*a^6*c^5*d^7 + 24*A*B*a^6*c^6*d^6 + 144
*A*B*a^6*c^7*d^5 + 48*A*B*a^6*c^8*d^4 - 36*A*B*a^6*c*d^11))/(4*c*d^12 + d^1
3 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) + ((B*a^3*c*3i - a^3*d*(A + 3*B)*1i)
*((8*tan(e/2 + (f*x)/2)*(28*A*a^3*c*d^14 + 24*B*a^3*c*d^14 + 52*A*a^3*c^2*d
^13 + 4*A*a^3*c^3*d^12 - 44*A*a^3*c^4*d^11 - 32*A*a^3*c^5*d^10 - 8*A*a^3*c^
6*d^9 + 12*B*a^3*c^2*d^13 - 84*B*a^3*c^3*d^12 - 84*B*a^3*c^4*d^11 + 36*B*a^
3*c^5*d^10 + 72*B*a^3*c^6*d^9 + 24*B*a^3*c^7*d^8))/(4*c*d^12 + d^13 + 6*c^2
*d^11 + 4*c^3*d^10 + c^4*d^9) - (8*(4*A*a^3*c*d^13 + 12*B*a^3*c*d^13 + 2*A*
a^3*c^2*d^12 - 6*A*a^3*c^3*d^11 - 2*A*a^3*c^4*d^10 + 2*A*a^3*c^5*d^9 + 24*B
*a^3*c^2*d^12 + 6*B*a^3*c^3*d^11 - 18*B*a^3*c^4*d^10 - 18*B*a^3*c^5*d^9 - 6
*B*a^3*c^6*d^8))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (((
8*(4*c^2*d^15 + 16*c^3*d^14 + 24*c^4*d^13 + 16*c^5*d^12 + 4*c^6*d^11))/(4*c
*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (8*tan(e/2 + (f*x)/2)*(1
2*c*d^17 + 48*c^2*d^16 + 64*c^3*d^15 + 16*c^4*d^14 - 36*c^5*d^13 - 32*c^6*d
^12 - 8*c^7*d^11))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9))*(
B*a^3*c*3i - a^3*d*(A + 3*B)*1i))/d^4))/d^4))/d^4 - ((B*a^3*c*3i - a^3*d*(A
+ 3*B)*1i)*((8*(4*A^2*a^6*c^2*d^9 + 16*A^2*a^6*c^3*d^8 + 24*A^2*a^6*c^4*d^
7 + 16*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 36*B^2*a^6*c^2*d^9 + 72*B^2*a^
6*c^3*d^8 - 36*B^2*a^6*c^4*d^7 - 144*B^2*a^6*c^5*d^6 - 36*B^2*a^6*c^6*d^5 +
72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8*d^3 + 24*A*B*a^6*c^2*d^9 + 72*A*B*a^6*
c^3*d^8 + 48*A*B*a^6*c^4*d^7 - 48*A*B*a^6*c^5*d^6 - 72*A*B*a^6*c^6*d^5 - 24
*A*B*a^6*c^7*d^4))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (
8*tan(e/2 + (f*x)/2)*(46*A^2*a^6*c^2*d^10 + 99*A^2*a^6*c^3*d^9 + 36*A^2*a^6
*c^4*d^8 - 36*A^2*a^6*c^5*d^7 - 32*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 + 25
2*B^2*a^6*c^2*d^10 - 81*B^2*a^6*c^3*d^9 - 594*B^2*a^6*c^4*d^8 - 81*B^2*a^6*

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c^5*d^7 + 504*B^2*a^6*c^6*d^6 + 180*B^2*a^6*c^7*d^5 - 144*B^2*a^6*c^8*d^4 -
 72*B^2*a^6*c^9*d^3 - 41*A^2*a^6*c*d^11 + 36*B^2*a^6*c*d^11 + 282*A*B*a^6*c
^2*d^10 + 228*A*B*a^6*c^3*d^9 - 318*A*B*a^6*c^4*d^8 - 372*A*B*a^6*c^5*d^7 +
 24*A*B*a^6*c^6*d^6 + 144*A*B*a^6*c^7*d^5 + 48*A*B*a^6*c^8*d^4 - 36*A*B*a^6
*c*d^11))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) + ((B*a^3*c
*3i - a^3*d*(A + 3*B)*1i)*((8*(4*A*a^3*c*d^13 + 12*B*a^3*c*d^13 + 2*A*a^3*c
^2*d^12 - 6*A*a^3*c^3*d^11 - 2*A*a^3*c^4*d^10 + 2*A*a^3*c^5*d^9 + 24*B*a^3*
c^2*d^12 + 6*B*a^3*c^3*d^11 - 18*B*a^3*c^4*d^10 - 18*B*a^3*c^5*d^9 - 6*B*a^
3*c^6*d^8))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) - (8*tan(e
/2 + (f*x)/2)*(28*A*a^3*c*d^14 + 24*B*a^3*c*d^14 + 52*A*a^3*c^2*d^13 + 4*A*
a^3*c^3*d^12 - 44*A*a^3*c^4*d^11 - 32*A*a^3*c^5*d^10 - 8*A*a^3*c^6*d^9 + 12
*B*a^3*c^2*d^13 - 84*B*a^3*c^3*d^12 - 84*B*a^3*c^4*d^11 + 36*B*a^3*c^5*d^10
 + 72*B*a^3*c^6*d^9 + 24*B*a^3*c^7*d^8))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*
c^3*d^10 + c^4*d^9) + (((8*(4*c^2*d^15 + 16*c^3*d^14 + 24*c^4*d^13 + 16*c^5
*d^12 + 4*c^6*d^11))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) +
 (8*tan(e/2 + (f*x)/2)*(12*c*d^17 + 48*c^2*d^16 + 64*c^3*d^15 + 16*c^4*d^14
 - 36*c^5*d^13 - 32*c^6*d^12 - 8*c^7*d^11))/(4*c*d^12 + d^13 + 6*c^2*d^11 +
 4*c^3*d^10 + c^4*d^9))*(B*a^3*c*3i - a^3*d*(A + 3*B)*1i))/d^4))/d^4))/d^4)
)*(B*a^3*c*3i - a^3*d*(A + 3*B)*1i)*2i)/(d^4*f) - (a^3*atan(((a^3*(-(c + d)
^5*(c - d))^(1/2)*((8*(4*A^2*a^6*c^2*d^9 + 16*A^2*a^6*c^3*d^8 + 24*A^2*a^6*
c^4*d^7 + 16*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 36*B^2*a^6*c^2*d^9 + 72*
B^2*a^6*c^3*d^8 - 36*B^2*a^6*c^4*d^7 - 144*B^2*a^6*c^5*d^6 - 36*B^2*a^6*c^6
*d^5 + 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8*d^3 + 24*A*B*a^6*c^2*d^9 + 72*A*
B*a^6*c^3*d^8 + 48*A*B*a^6*c^4*d^7 - 48*A*B*a^6*c^5*d^6 - 72*A*B*a^6*c^6*d^
5 - 24*A*B*a^6*c^7*d^4))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^
8) + (8*tan(e/2 + (f*x)/2)*(46*A^2*a^6*c^2*d^10 + 99*A^2*a^6*c^3*d^9 + 36*A
^2*a^6*c^4*d^8 - 36*A^2*a^6*c^5*d^7 - 32*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^
5 + 252*B^2*a^6*c^2*d^10 - 81*B^2*a^6*c^3*d^9 - 594*B^2*a^6*c^4*d^8 - 81*B^
2*a^6*c^5*d^7 + 504*B^2*a^6*c^6*d^6 + 180*B^2*a^6*c^7*d^5 - 144*B^2*a^6*c^8
*d^4 - 72*B^2*a^6*c^9*d^3 - 41*A^2*a^6*c*d^11 + 36*B^2*a^6*c*d^11 + 282*A*B
*a^6*c^2*d^10 + 228*A*B*a^6*c^3*d^9 - 318*A*B*a^6*c^4*d^8 - 372*A*B*a^6*c^5
*d^7 + 24*A*B*a^6*c^6*d^6 + 144*A*B*a^6*c^7*d^5 + 48*A*B*a^6*c^8*d^4 - 36*A
*B*a^6*c*d^11))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) + (a^
3*(-(c + d)^5*(c - d))^(1/2)*((8*tan(e/2 + (f*x)/2)*(28*A*a^3*c*d^14 + 24*B
*a^3*c*d^14 + 52*A*a^3*c^2*d^13 + 4*A*a^3*c^3*d^12 - 44*A*a^3*c^4*d^11 - 32
*A*a^3*c^5*d^10 - 8*A*a^3*c^6*d^9 + 12*B*a^3*c^2*d^13 - 84*B*a^3*c^3*d^12 -
 84*B*a^3*c^4*d^11 + 36*B*a^3*c^5*d^10 + 72*B*a^3*c^6*d^9 + 24*B*a^3*c^7*d^
8))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) - (8*(4*A*a^3*c*d
^13 + 12*B*a^3*c*d^13 + 2*A*a^3*c^2*d^12 - 6*A*a^3*c^3*d^11 - 2*A*a^3*c^4*d
^10 + 2*A*a^3*c^5*d^9 + 24*B*a^3*c^2*d^12 + 6*B*a^3*c^3*d^11 - 18*B*a^3*c^4
*d^10 - 18*B*a^3*c^5*d^9 - 6*B*a^3*c^6*d^8))/(4*c*d^11 + d^12 + 6*c^2*d^10
+ 4*c^3*d^9 + c^4*d^8) + (a^3*(-(c + d)^5*(c - d))^(1/2)*((8*(4*c^2*d^15 +
16*c^3*d^14 + 24*c^4*d^13 + 16*c^5*d^12 + 4*c^6*d^11))/(4*c*d^11 + d^12 + 6
*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (8*tan(e/2 + (f*x)/2)*(12*c*d^17 + 48*c^
2*d^16 + 64*c^3*d^15 + 16*c^4*d^14 - 36*c^5*d^13 - 32*c^6*d^12 - 8*c^7*d^11
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))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9))*(7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d^2 + 2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d))/(2*(5*c*d^8 + d^9 + 10*c^2*d^7 + 10*c^3*d^6 + 5*c^4*d^5 + c^5*d^4))))*(7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d^2 + 2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d))/(2*(5*c*d^8 + d^9 + 10*c^2*d^7 + 10*c^3*d^6 + 5*c^4*d^5 + c^5*d^4)))*(7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d^2 + 2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d)*1i)/(2*(5*c*d^8 + d^9 + 10*c^2*d^7 + 10*c^3*d^6 + 5*c^4*d^5 + c^5*d^4)) + (a^3*(-(c + d)^5*(c - d))^(1/2)*((8*(4*A^2*a^6*c^2*d^9 + 16*A^2*a^6*c^3*d^8 + 24*A^2*a^6*c^4*d^7 + 16*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 36*B^2*a^6*c^2*d^9 + 72*B^2*a^6*c^3*d^8 - 36*B^2*a^6*c^4*d^7 - 144*B^2*a^6*c^5*d^6 - 36*B^2*a^6*c^6*d^5 + 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8*d^3 + 24*A*B*a^6*c^2*d^9 + 72*A*B*a^6*c^3*d^8 + 48*A*B*a^6*c^4*d^7 - 48*A*B*a^6*c^5*d^6 - 72*A*B*a^6*c^6*d^5 - 24*A*B*a^6*c^7*d^4))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (8*tan(e/2 + (f*x)/2)*(46*A^2*a^6*c^2*d^10 + 99*A^2*a^6*c^3*d^9 + 36*A^2*a^6*c^4*d^8 - 36*A^2*a^6*c^5*d^7 - 32*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 + 252*B^2*a^6*c^2*d^10 - 81*B^2*a^6*c^3*d^9 - 594*B^2*a^6*c^4*d^8 - 81*B^2*a^6*c^5*d^7 + 504*B^2*a^6*c^6*d^6 + 180*B^2*a^6*c^7*d^5 - 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 - 41*A^2*a^6*c*d^11 + 36*B^2*a^6*c*d^11 + 282*A*B*a^6*c^2*d^10 + 228*A*B*a^6*c^3*d^9 - 318*A*B*a^6*c^4*d^8 - 372*A*B*a^6*c^5*d^7 + 24*A*B*a^6*c^6*d^6 + 144*A*B*a^6*c^7*d^5 + 48*A*B*a^6*c^8*d^4 - 36*A*B*a^6*c*d^11))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) + (a^3*(-(c + d)^5*(c - d))^(1/2)*((8*(4*A*a^3*c*d^13 + 12*B*a^3*c*d^13 + 2*A*a^3*c^2*d^12 - 6*A*a^3*c^3*d^11 - 2*A*a^3*c^4*d^10 + 2*A*a^3*c^5*d^9 + 24*B*a^3*c^2*d^12 + 6*B*a^3*c^3*d^11 - 18*B*a^3*c^4*d^10 - 18*B*a^3*c^5*d^9 - 6*B*a^3*c^6*d^8))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) - (8*tan(e/2 + (f*x)/2)*(28*A*a^3*c*d^14 + 24*B*a^3*c*d^14 + 52*A*a^3*c^2*d^13 + 4*A*a^3*c^3*d^12 - 44*A*a^3*c^4*d^11 - 32*A*a^3*c^5*d^10 - 8*A*a^3*c^6*d^9 + 12*B*a^3*c^2*d^13 - 84*B*a^3*c^3*d^12 - 84*B*a^3*c^4*d^11 + 36*B*a^3*c^5*d^10 + 72*B*a^3*c^6*d^9 + 24*B*a^3*c^7*d^8))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) + (a^3*(-(c + d)^5*(c - d))^(1/2)*((8*(4*c^2*d^15 + 16*c^3*d^14 + 24*c^4*d^13 + 16*c^5*d^12 + 4*c^6*d^11))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (8*tan(e/2 + (f*x)/2)*(12*c*d^17 + 48*c^2*d^16 + 64*c^3*d^15 + 16*c^4*d^14 - 36*c^5*d^13 - 32*c^6*d^12 - 8*c^7*d^11))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9))*(7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d^2 + 2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d))/(2*(5*c*d^8 + d^9 + 10*c^2*d^7 + 10*c^3*d^6 + 5*c^4*d^5 + c^5*d^4))))*(7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d^2 + 2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d))/(2*(5*c*d^8 + d^9 + 10*c^2*d^7 + 10*c^3*d^6 + 5*c^4*d^5 + c^5*d^4)))*(7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d^2 + 2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d)*1i)/(2*(5*c*d^8 + d^9 + 10*c^2*d^7 + 10*c^3*d^6 + 5*c^4*d^5 + c^5*d^4)))/((16*(54*B^3*a^9*c^8 - 29*A^3*a^9*c^3*d^5 - 18*A^3*a^9*c^4*d^4 - 2*A^3*a^9*c^5*d^3 - 324*B^3*a^9*c^2*d^6 + 81*B^3*a^9*c^3*d^5 + 405*B^3*a^9*c^4*d^4 - 135*B^3*a^9*c^5*d^3 - 243*B^3*a^9*c^6*d^2 + 49*A^3*a^9*c*d^7 + 108*B^3*a^9*c*d^7 + 54*B^3*a^9*c^7*d + 288*A*B^2*a^9*c*d^7 - 54*A*B^2*a^9*c^7*d + 231*A^2*B*a^9*c*d^7 - 576*A*B^2*a^9*c^2*d^6 - 135*A*B^2*a^9*c^3*d^5 + 540*A*B^2*a^9*c^4*d^4 + 135*A

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*B^2*a^9*c^5*d^3 - 198*A*B^2*a^9*c^6*d^2 - 231*A^2*B*a^9*c^2*d^6 - 201*A^2*
B*a^9*c^3*d^5 + 69*A^2*B*a^9*c^4*d^4 + 114*A^2*B*a^9*c^5*d^3 + 18*A^2*B*a^9
*c^6*d^2))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (16*tan(e
/2 + (f*x)/2)*(216*B^3*a^9*c^9 + 52*A^3*a^9*c^2*d^7 + 4*A^3*a^9*c^3*d^6 - 4
4*A^3*a^9*c^4*d^5 - 32*A^3*a^9*c^5*d^4 - 8*A^3*a^9*c^6*d^3 - 324*B^3*a^9*c^
2*d^7 - 756*B^3*a^9*c^3*d^6 + 864*B^3*a^9*c^4*d^5 + 1080*B^3*a^9*c^5*d^4 -
756*B^3*a^9*c^6*d^3 - 756*B^3*a^9*c^7*d^2 + 28*A^3*a^9*c*d^8 + 216*B^3*a^9*
c*d^8 + 216*B^3*a^9*c^8*d + 396*A*B^2*a^9*c*d^8 - 216*A*B^2*a^9*c^8*d + 192
*A^2*B*a^9*c*d^8 - 108*A*B^2*a^9*c^2*d^7 - 1224*A*B^2*a^9*c^3*d^6 + 1260*A*
B^2*a^9*c^5*d^4 + 324*A*B^2*a^9*c^6*d^3 - 432*A*B^2*a^9*c^7*d^2 + 156*A^2*B
*a^9*c^2*d^7 - 372*A^2*B*a^9*c^3*d^6 - 372*A^2*B*a^9*c^4*d^5 + 108*A^2*B*a^
9*c^5*d^4 + 216*A^2*B*a^9*c^6*d^3 + 72*A^2*B*a^9*c^7*d^2))/(4*c*d^12 + d^13
 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) + (a^3*(-(c + d)^5*(c - d))^(1/2)*((8
*(4*A^2*a^6*c^2*d^9 + 16*A^2*a^6*c^3*d^8 + 24*A^2*a^6*c^4*d^7 + 16*A^2*a^6*
c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 36*B^2*a^6*c^2*d^9 + 72*B^2*a^6*c^3*d^8 - 36*
B^2*a^6*c^4*d^7 - 144*B^2*a^6*c^5*d^6 - 36*B^2*a^6*c^6*d^5 + 72*B^2*a^6*c^7
*d^4 + 36*B^2*a^6*c^8*d^3 + 24*A*B*a^6*c^2*d^9 + 72*A*B*a^6*c^3*d^8 + 48*A*
B*a^6*c^4*d^7 - 48*A*B*a^6*c^5*d^6 - 72*A*B*a^6*c^6*d^5 - 24*A*B*a^6*c^7*d^
4))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (8*tan(e/2 + (f*
x)/2)*(46*A^2*a^6*c^2*d^10 + 99*A^2*a^6*c^3*d^9 + 36*A^2*a^6*c^4*d^8 - 36*A
^2*a^6*c^5*d^7 - 32*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 + 252*B^2*a^6*c^2*d
^10 - 81*B^2*a^6*c^3*d^9 - 594*B^2*a^6*c^4*d^8 - 81*B^2*a^6*c^5*d^7 + 504*B
^2*a^6*c^6*d^6 + 180*B^2*a^6*c^7*d^5 - 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9
*d^3 - 41*A^2*a^6*c*d^11 + 36*B^2*a^6*c*d^11 + 282*A*B*a^6*c^2*d^10 + 228*A
*B*a^6*c^3*d^9 - 318*A*B*a^6*c^4*d^8 - 372*A*B*a^6*c^5*d^7 + 24*A*B*a^6*c^6
*d^6 + 144*A*B*a^6*c^7*d^5 + 48*A*B*a^6*c^8*d^4 - 36*A*B*a^6*c*d^11))/(4*c*
d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) + (a^3*(-(c + d)^5*(c - d)
)^(1/2)*((8*tan(e/2 + (f*x)/2)*(28*A*a^3*c*d^14 + 24*B*a^3*c*d^14 + 52*A*a^
3*c^2*d^13 + 4*A*a^3*c^3*d^12 - 44*A*a^3*c^4*d^11 - 32*A*a^3*c^5*d^10 - 8*A
*a^3*c^6*d^9 + 12*B*a^3*c^2*d^13 - 84*B*a^3*c^3*d^12 - 84*B*a^3*c^4*d^11 +
36*B*a^3*c^5*d^10 + 72*B*a^3*c^6*d^9 + 24*B*a^3*c^7*d^8))/(4*c*d^12 + d^13
+ 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) - (8*(4*A*a^3*c*d^13 + 12*B*a^3*c*d^13
 + 2*A*a^3*c^2*d^12 - 6*A*a^3*c^3*d^11 - 2*A*a^3*c^4*d^10 + 2*A*a^3*c^5*d^9
 + 24*B*a^3*c^2*d^12 + 6*B*a^3*c^3*d^11 - 18*B*a^3*c^4*d^10 - 18*B*a^3*c^5*
d^9 - 6*B*a^3*c^6*d^8))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8
) + (a^3*(-(c + d)^5*(c - d))^(1/2)*((8*(4*c^2*d^15 + 16*c^3*d^14 + 24*c^4*
d^13 + 16*c^5*d^12 + 4*c^6*d^11))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9
 + c^4*d^8) + (8*tan(e/2 + (f*x)/2)*(12*c*d^17 + 48*c^2*d^16 + 64*c^3*d^15
+ 16*c^4*d^14 - 36*c^5*d^13 - 32*c^6*d^12 - 8*c^7*d^11))/(4*c*d^12 + d^13 +
 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9))*(7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d
^2 + 2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d))/(2*(5*c*d^8 + d^9 + 10*c^2*d^7 +
10*c^3*d^6 + 5*c^4*d^5 + c^5*d^4)))*(7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d^
2 + 2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d))/(2*(5*c*d^8 + d^9 + 10*c^2*d^7 + 1
0*c^3*d^6 + 5*c^4*d^5 + c^5*d^4)))*(7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d^2
 + 2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d))/(2*(5*c*d^8 + d^9 + 10*c^2*d^7 + 10
```

```
*c^3*d^6 + 5*c^4*d^5 + c^5*d^4)) - (a^3*(-(c + d)^5*(c - d))^(1/2)*((8*(4*A
^2*a^6*c^2*d^9 + 16*A^2*a^6*c^3*d^8 + 24*A^2*a^6*c^4*d^7 + 16*A^2*a^6*c^5*d
^6 + 4*A^2*a^6*c^6*d^5 + 36*B^2*a^6*c^2*d^9 + 72*B^2*a^6*c^3*d^8 - 36*B^2*a
^6*c^4*d^7 - 144*B^2*a^6*c^5*d^6 - 36*B^2*a^6*c^6*d^5 + 72*B^2*a^6*c^7*d^4
+ 36*B^2*a^6*c^8*d^3 + 24*A*B*a^6*c^2*d^9 + 72*A*B*a^6*c^3*d^8 + 48*A*B*a^6
*c^4*d^7 - 48*A*B*a^6*c^5*d^6 - 72*A*B*a^6*c^6*d^5 - 24*A*B*a^6*c^7*d^4))/(
4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (8*tan(e/2 + (f*x)/2)
*(46*A^2*a^6*c^2*d^10 + 99*A^2*a^6*c^3*d^9 + 36*A^2*a^6*c^4*d^8 - 36*A^2*a^
6*c^5*d^7 - 32*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 + 252*B^2*a^6*c^2*d^10 -
 81*B^2*a^6*c^3*d^9 - 594*B^2*a^6*c^4*d^8 - 81*B^2*a^6*c^5*d^7 + 504*B^2*a^
6*c^6*d^6 + 180*B^2*a^6*c^7*d^5 - 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3
- 41*A^2*a^6*c*d^11 + 36*B^2*a^6*c*d^11 + 282*A*B*a^6*c^2*d^10 + 228*A*B*a^
6*c^3*d^9 - 318*A*B*a^6*c^4*d^8 - 372*A*B*a^6*c^5*d^7 + 24*A*B*a^6*c^6*d^6
+ 144*A*B*a^6*c^7*d^5 + 48*A*B*a^6*c^8*d^4 - 36*A*B*a^6*c*d^11))/(4*c*d^12
+ d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) + (a^3*(-(c + d)^5*(c - d))^(1/
2)*((8*(4*A*a^3*c*d^13 + 12*B*a^3*c*d^13 + 2*A*a^3*c^2*d^12 - 6*A*a^3*c^3*d
^11 - 2*A*a^3*c^4*d^10 + 2*A*a^3*c^5*d^9 + 24*B*a^3*c^2*d^12 + 6*B*a^3*c^3*
d^11 - 18*B*a^3*c^4*d^10 - 18*B*a^3*c^5*d^9 - 6*B*a^3*c^6*d^8))/(4*c*d^11 +
 d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) - (8*tan(e/2 + (f*x)/2)*(28*A*a^3
*c*d^14 + 24*B*a^3*c*d^14 + 52*A*a^3*c^2*d^13 + 4*A*a^3*c^3*d^12 - 44*A*a^3
*c^4*d^11 - 32*A*a^3*c^5*d^10 - 8*A*a^3*c^6*d^9 + 12*B*a^3*c^2*d^13 - 84*B*
a^3*c^3*d^12 - 84*B*a^3*c^4*d^11 + 36*B*a^3*c^5*d^10 + 72*B*a^3*c^6*d^9 + 2
4*B*a^3*c^7*d^8))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) + (
a^3*(-(c + d)^5*(c - d))^(1/2)*((8*(4*c^2*d^15 + 16*c^3*d^14 + 24*c^4*d^13
+ 16*c^5*d^12 + 4*c^6*d^11))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^
4*d^8) + (8*tan(e/2 + (f*x)/2)*(12*c*d^17 + 48*c^2*d^16 + 64*c^3*d^15 + 16*
c^4*d^14 - 36*c^5*d^13 - 32*c^6*d^12 - 8*c^7*d^11))/(4*c*d^12 + d^13 + 6*c^
2*d^11 + 4*c^3*d^10 + c^4*d^9))*(7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d^2 +
2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d))/(2*(5*c*d^8 + d^9 + 10*c^2*d^7 + 10*c^
3*d^6 + 5*c^4*d^5 + c^5*d^4)))*(7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d^2 + 2
*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d))/(2*(5*c*d^8 + d^9 + 10*c^2*d^7 + 10*c^3
*d^6 + 5*c^4*d^5 + c^5*d^4)))*(7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d^2 + 2*
A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d))/(2*(5*c*d^8 + d^9 + 10*c^2*d^7 + 10*c^3*
d^6 + 5*c^4*d^5 + c^5*d^4))))*(-(c + d)^5*(c - d))^(1/2)*(7*A*d^3 - 6*B*c^3
 + 6*B*d^3 + 6*A*c*d^2 + 2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d)*1i)/(f*(5*c*d^
8 + d^9 + 10*c^2*d^7 + 10*c^3*d^6 + 5*c^4*d^5 + c^5*d^4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.265 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=220

$$\frac{2d\left(3A\left(c^2-3cd+d^2\right)-B\left(7c^2-9cd+4d^2\right)\right)\cos(e+fx)}{3af}+\frac{x\left(3Ad\left(2c^2-2cd+d^2\right)+B\left(2c^3-6c^2d+9cd^2-3d\right.\right.}{2a}$$

```
[Out] 1/2*(3*A*d*(2*c^2-2*c*d+d^2)+B*(2*c^3-6*c^2*d+9*c*d^2-3*d^3))*x/a+2/3*d*(3*
A*(c^2-3*c*d+d^2)-B*(7*c^2-9*c*d+4*d^2))*cos(f*x+e)/a/f+1/6*d^2*(6*A*c-9*A*
d-11*B*c+9*B*d)*cos(f*x+e)*sin(f*x+e)/a/f+1/3*(3*A-4*B)*d*cos(f*x+e)*(c+d*s
in(f*x+e))^2/a/f-(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^3/f/(a+a*sin(f*x+e))
```

Rubi [A] time = 0.36, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2977, 2753, 2734}

$$\frac{2d\left(3A\left(c^2-3cd+d^2\right)-B\left(7c^2-9cd+4d^2\right)\right)\cos(e+fx)}{3af}+\frac{x\left(3Ad\left(2c^2-2cd+d^2\right)+B\left(-6c^2d+2c^3+9cd^2-3\right.\right.}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((3*A*d*(2*c^2 - 2*c*d + d^2) + B*(2*c^3 - 6*c^2*d + 9*c*d^2 - 3*d^3))*x)/(
2*a) + (2*d*(3*A*(c^2 - 3*c*d + d^2) - B*(7*c^2 - 9*c*d + 4*d^2))*Cos[e + f
*x])/(3*a*f) + (d^2*(6*A*c - 11*B*c - 9*A*d + 9*B*d)*Cos[e + f*x]*Sin[e + f
*x])/(6*a*f) + ((3*A - 4*B)*d*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*a*f)
- ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(f*(a + a*Sin[e + f*x]))
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{f(a+a\sin(e+fx))} + \frac{\int (c+d\sin(e+fx)}{}$$

$$= \frac{(3A-4B)d\cos(e+fx)(c+d\sin(e+fx))^2}{3af} - \frac{(A-B)\cos(e+}{f(a+a}$$

$$= \frac{\left(3Ad\left(2c^2-2cd+d^2\right)+B\left(2c^3-6c^2d+9cd^2-3d^3\right)\right)x}{2a} + \frac{2d\left(\right.}{}$$

Mathematica [B] time = 1.27, size = 788, normalized size = 3.58

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(9d\left(Ad(d-4c)+B\left(-4c^2+3cd-2d^2\right)\right)\cos\left(\frac{3}{2}(e+fx)\right)+3\cos\left(\frac{1}{2}(e+fx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*(4*A*d*(6*c^2*(e + f*x) - 3*c*d*(
1 + 2*e + 2*f*x) + d^2*(1 + 3*e + 3*f*x)) + B*(8*c^3*(e + f*x) - 12*c^2*d*(
1 + 2*e + 2*f*x) + 12*c*d^2*(1 + 3*e + 3*f*x) - d^3*(7 + 12*e + 12*f*x)))*C
os[(e + f*x)/2] + 9*d*(A*d*(-4*c + d) + B*(-4*c^2 + 3*c*d - 2*d^2))*Cos[(3*
(e + f*x))/2] + 9*B*c*d^2*Cos[(5*(e + f*x))/2] + 3*A*d^3*Cos[(5*(e + f*x))/
2] - 2*B*d^3*Cos[(5*(e + f*x))/2] + B*d^3*Cos[(7*(e + f*x))/2] + 48*A*c^3*S
in[(e + f*x)/2] - 48*B*c^3*Sin[(e + f*x)/2] - 144*A*c^2*d*Sin[(e + f*x)/2]
+ 180*B*c^2*d*Sin[(e + f*x)/2] + 180*A*c*d^2*Sin[(e + f*x)/2] - 180*B*c*d^2
*Sin[(e + f*x)/2] - 60*A*d^3*Sin[(e + f*x)/2] + 69*B*d^3*Sin[(e + f*x)/2] +
```

```
24*B*c^3*e*Sin[(e + f*x)/2] + 72*A*c^2*d*e*Sin[(e + f*x)/2] - 72*B*c^2*d*e
*Sin[(e + f*x)/2] - 72*A*c*d^2*e*Sin[(e + f*x)/2] + 108*B*c*d^2*e*Sin[(e +
f*x)/2] + 36*A*d^3*e*Sin[(e + f*x)/2] - 36*B*d^3*e*Sin[(e + f*x)/2] + 24*B*
c^3*f*x*Sin[(e + f*x)/2] + 72*A*c^2*d*f*x*Sin[(e + f*x)/2] - 72*B*c^2*d*f*x
*Sin[(e + f*x)/2] - 72*A*c*d^2*f*x*Sin[(e + f*x)/2] + 108*B*c*d^2*f*x*Sin[(
e + f*x)/2] + 36*A*d^3*f*x*Sin[(e + f*x)/2] - 36*B*d^3*f*x*Sin[(e + f*x)/2]
 - 36*B*c^2*d*Sin[(3*(e + f*x))/2] - 36*A*c*d^2*Sin[(3*(e + f*x))/2] + 27*B
*c*d^2*Sin[(3*(e + f*x))/2] + 9*A*d^3*Sin[(3*(e + f*x))/2] - 18*B*d^3*Sin[(
3*(e + f*x))/2] - 9*B*c*d^2*Sin[(5*(e + f*x))/2] - 3*A*d^3*Sin[(5*(e + f*x)
)/2] + 2*B*d^3*Sin[(5*(e + f*x))/2] + B*d^3*Sin[(7*(e + f*x))/2]))/(24*a*f*
(1 + Sin[e + f*x]))
```

fricas [B] time = 0.48, size = 470, normalized size = 2.14

$$\frac{2\,Bd^3\cos\left(fx+e\right)^4-6\,(A-B)c^3+18\,(A-B)c^2d-18\,(A-B)cd^2+6\,(A-B)d^3+\left(9\,Bcd^2+(3\,A-B)d^3\right)\cos\left(f\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/6*(2*B*d^3*cos(f*x + e)^4 - 6*(A - B)*c^3 + 18*(A - B)*c^2*d - 18*(A - B)
*c*d^2 + 6*(A - B)*d^3 + (9*B*c*d^2 + (3*A - B)*d^3)*cos(f*x + e)^3 + 3*(2*
B*c^3 + 6*(A - B)*c^2*d - 3*(2*A - 3*B)*c*d^2 + 3*(A - B)*d^3)*f*x - 6*(3*B
*c^2*d + 3*(A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f*x + e)^2 - 3*(2*(A - B)*c^3
 - 6*(A - 2*B)*c^2*d + 3*(4*A - 3*B)*c*d^2 - (3*A - 5*B)*d^3 - (2*B*c^3 + 6
*(A - B)*c^2*d - 3*(2*A - 3*B)*c*d^2 + 3*(A - B)*d^3)*f*x)*cos(f*x + e) + (
2*B*d^3*cos(f*x + e)^3 + 6*(A - B)*c^3 - 18*(A - B)*c^2*d + 18*(A - B)*c*d^
2 - 6*(A - B)*d^3 + 3*(2*B*c^3 + 6*(A - B)*c^2*d - 3*(2*A - 3*B)*c*d^2 + 3*
(A - B)*d^3)*f*x - 3*(3*B*c*d^2 + (A - B)*d^3)*cos(f*x + e)^2 - 3*(6*B*c^2*
d + 3*(2*A - B)*c*d^2 - (A - 3*B)*d^3)*cos(f*x + e))*sin(f*x + e))/(a*f*cos
(f*x + e) + a*f*sin(f*x + e) + a*f)
```

giac [B] time = 0.20, size = 479, normalized size = 2.18

$$\frac{\frac{3\left(2\,Bc^3+6\,Ac^2d-6\,Bc^2d-6\,Acd^2+9\,Bcd^2+3\,Ad^3-3\,Bd^3\right)\left(fx+e\right)}{a}-\frac{12\left(Ac^3-Bc^3-3\,Ac^2d+3\,Bc^2d+3\,Acd^2-3\,Bcd^2-Ad^3+Bd^3\right)}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{2\left(9\,Bcd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] 1/6*(3*(2*B*c^3 + 6*A*c^2*d - 6*B*c^2*d - 6*A*c*d^2 + 9*B*c*d^2 + 3*A*d^3 -
 3*B*d^3)*(f*x + e)/a - 12*(A*c^3 - B*c^3 - 3*A*c^2*d + 3*B*c^2*d + 3*A*c*d
^2 - 3*B*c*d^2 - A*d^3 + B*d^3)/(a*(tan(1/2*f*x + 1/2*e) + 1)) + 2*(9*B*c*d
^2*tan(1/2*f*x + 1/2*e)^5 + 3*A*d^3*tan(1/2*f*x + 1/2*e)^5 - 3*B*d^3*tan(1/
2*f*x + 1/2*e)^5 - 18*B*c^2*d*tan(1/2*f*x + 1/2*e)^4 - 18*A*c*d^2*tan(1/2*f
*x + 1/2*e)^4 + 18*B*c*d^2*tan(1/2*f*x + 1/2*e)^4 + 6*A*d^3*tan(1/2*f*x + 1
/2*e)^4 - 6*B*d^3*tan(1/2*f*x + 1/2*e)^4 - 36*B*c^2*d*tan(1/2*f*x + 1/2*e)^
2 - 36*A*c*d^2*tan(1/2*f*x + 1/2*e)^2 + 36*B*c*d^2*tan(1/2*f*x + 1/2*e)^2 +
 12*A*d^3*tan(1/2*f*x + 1/2*e)^2 - 24*B*d^3*tan(1/2*f*x + 1/2*e)^2 - 9*B*c*
d^2*tan(1/2*f*x + 1/2*e) - 3*A*d^3*tan(1/2*f*x + 1/2*e) + 3*B*d^3*tan(1/2*f
*x + 1/2*e) - 18*B*c^2*d - 18*A*c*d^2 + 18*B*c*d^2 + 6*A*d^3 - 10*B*d^3)/((
tan(1/2*f*x + 1/2*e)^2 + 1)^3*a))/f
```

maple [B] time = 0.44, size = 1110, normalized size = 5.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x)
```

```
[Out] -12/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*B*tan(1/2*f*x+1/2*e)^2*c^2*d+12/a/f/(1+t
an(1/2*f*x+1/2*e)^2)^3*B*tan(1/2*f*x+1/2*e)^2*c*d^2-3/a/f/(1+tan(1/2*f*x+1/
2*e)^2)^3*tan(1/2*f*x+1/2*e)*B*c*d^2+3/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1
/2*f*x+1/2*e)^5*B*c*d^2-6/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*A*tan(1/2*f*x+1/2*
e)^4*c*d^2-6/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*B*tan(1/2*f*x+1/2*e)^4*c^2*d+6/
a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*B*tan(1/2*f*x+1/2*e)^4*c*d^2-12/a/f/(1+tan(1
/2*f*x+1/2*e)^2)^3*A*tan(1/2*f*x+1/2*e)^2*c*d^2+1/a/f/(1+tan(1/2*f*x+1/2*e)
^2)^3*tan(1/2*f*x+1/2*e)*B*d^3-6/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*A*c*d^2-6/a
/f/(1+tan(1/2*f*x+1/2*e)^2)^3*B*c^2*d+6/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*B*c*
d^2+6/a/f*arctan(tan(1/2*f*x+1/2*e))*A*c^2*d-6/a/f*arctan(tan(1/2*f*x+1/2*e
))*A*c*d^2-6/a/f*arctan(tan(1/2*f*x+1/2*e))*B*c^2*d+9/a/f*arctan(tan(1/2*f*
x+1/2*e))*B*c*d^2+6/a/f/(tan(1/2*f*x+1/2*e)+1)*A*c^2*d-6/a/f/(tan(1/2*f*x+1
/2*e)+1)*A*c*d^2-6/a/f/(tan(1/2*f*x+1/2*e)+1)*B*c^2*d+6/a/f/(tan(1/2*f*x+1/
2*e)+1)*B*c*d^2+1/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*A*d^3
-1/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*B*d^3+2/a/f/(1+tan(1
/2*f*x+1/2*e)^2)^3*A*tan(1/2*f*x+1/2*e)^4*d^3-2/a/f/(1+tan(1/2*f*x+1/2*e)^2
)^3*B*tan(1/2*f*x+1/2*e)^4*d^3+4/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*A*tan(1/2*f
*x+1/2*e)^2*d^3-8/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*B*tan(1/2*f*x+1/2*e)^2*d^3
-1/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*A*d^3-10/3/a/f/(1+tan(
1/2*f*x+1/2*e)^2)^3*B*d^3+3/a/f*arctan(tan(1/2*f*x+1/2*e))*A*d^3+2/a/f*arct
an(tan(1/2*f*x+1/2*e))*B*c^3-3/a/f*arctan(tan(1/2*f*x+1/2*e))*B*d^3-2/a/f/(
tan(1/2*f*x+1/2*e)+1)*A*c^3+2/a/f/(tan(1/2*f*x+1/2*e)+1)*A*d^3+2/a/f/(1+tan
(1/2*f*x+1/2*e)^2)^3*A*d^3+2/a/f/(tan(1/2*f*x+1/2*e)+1)*B*c^3-2/a/f/(tan(1/
2*f*x+1/2*e)+1)*B*d^3
```

maxima [B] time = 0.73, size = 1124, normalized size = 5.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -1/3*(B*d^3*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 39*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 24*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9*sin(f*x +
e)^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 3*
a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a*sin(f*x + e)^7/(c
os(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 9*B*c*
d^2*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arct
an(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 3*A*d^3*((sin(f*x + e)/(cos(f*x +
e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4)/(a + a*sin(f*x +
e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2*a*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e)
+ 1))/a) + 18*B*c^2*d*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arct
an(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 18*A*c*d^2*((sin(f*x + e)/(cos(f*x
 + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(
cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 6*B*
c^3*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos
(f*x + e) + 1))) - 18*A*c^2*d*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a +
1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + 6*A*c^3/(a + a*sin(f*x + e)/(c
os(f*x + e) + 1)))/f
```

mupad [B] time = 14.05, size = 839, normalized size = 3.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x)),x)
```

```
[Out] -(12*A*c^3*cos(e/2 + (f*x)/2) - 18*A*d^3*cos(e/2 + (f*x)/2) - 12*B*c^3*cos(
e/2 + (f*x)/2) + 18*B*d^3*cos(e/2 + (f*x)/2) + 6*A*d^3*cos(e/2 + (f*x)/2)^3
 - 12*A*d^3*cos(e/2 + (f*x)/2)^5 - 6*B*d^3*cos(e/2 + (f*x)/2)^3 + 36*B*d^3*
cos(e/2 + (f*x)/2)^5 - 16*B*d^3*cos(e/2 + (f*x)/2)^7 - 9*A*d^3*cos(e/2 + (f
*x)/2)*(e + f*x) - 6*B*c^3*cos(e/2 + (f*x)/2)*(e + f*x) + 9*B*d^3*cos(e/2 +
 (f*x)/2)*(e + f*x) - 9*A*d^3*sin(e/2 + (f*x)/2)*(e + f*x) - 6*B*c^3*sin(e/
2 + (f*x)/2)*(e + f*x) + 9*B*d^3*sin(e/2 + (f*x)/2)*(e + f*x) - 18*A*d^3*co
s(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2) + 12*A*d^3*cos(e/2 + (f*x)/2)^4*sin(e
/2 + (f*x)/2) + 18*B*d^3*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2) + 12*B*d^3
*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2) - 16*B*d^3*cos(e/2 + (f*x)/2)^6*si
n(e/2 + (f*x)/2) + 36*A*c*d^2*cos(e/2 + (f*x)/2) - 36*A*c^2*d*cos(e/2 + (f*
x)/2) - 54*B*c*d^2*cos(e/2 + (f*x)/2) + 36*B*c^2*d*cos(e/2 + (f*x)/2) + 36*
A*c*d^2*cos(e/2 + (f*x)/2)^3 + 18*B*c*d^2*cos(e/2 + (f*x)/2)^3 + 36*B*c^2*d
*cos(e/2 + (f*x)/2)^3 - 36*B*c*d^2*cos(e/2 + (f*x)/2)^5 + 18*A*c*d^2*cos(e/
2 + (f*x)/2)*(e + f*x) - 18*A*c^2*d*cos(e/2 + (f*x)/2)*(e + f*x) - 27*B*c*d
^2*cos(e/2 + (f*x)/2)*(e + f*x) + 18*B*c^2*d*cos(e/2 + (f*x)/2)*(e + f*x) +
 18*A*c*d^2*sin(e/2 + (f*x)/2)*(e + f*x) - 18*A*c^2*d*sin(e/2 + (f*x)/2)*(e
 + f*x) - 27*B*c*d^2*sin(e/2 + (f*x)/2)*(e + f*x) + 18*B*c^2*d*sin(e/2 + (f
*x)/2)*(e + f*x) + 36*A*c*d^2*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2) - 54*
B*c*d^2*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2) + 36*B*c^2*d*cos(e/2 + (f*x
)/2)^2*sin(e/2 + (f*x)/2) + 36*B*c*d^2*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)
/2))/(6*a*f*cos(e/2 + (f*x)/2) + 6*a*f*sin(e/2 + (f*x)/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.266 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=143

$$\frac{x\left(2Ad(2c-d)+B\left(2c^2-4cd+3d^2\right)\right)}{2a}+\frac{2d(A(c-d)-B(2c-d))\cos(e+fx)}{af}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+}{f(a\sin(e+fx)+a)}$$

```
[Out] 1/2*(2*A*(2*c-d)*d+B*(2*c^2-4*c*d+3*d^2))*x/a+2*(A*(c-d)-B*(2*c-d))*d*cos(f
*x+e)/a/f+1/2*(2*A-3*B)*d^2*cos(f*x+e)*sin(f*x+e)/a/f-(A-B)*cos(f*x+e)*(c+d
*sin(f*x+e))^2/f/(a+a*sin(f*x+e))
```

Rubi [A] time = 0.21, antiderivative size = 141, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2977, 2734}

$$\frac{x\left(d^2(-(2A-3B))+4Acd+2Bc(c-2d)\right)}{2a}+\frac{2d(A(c-d)-B(2c-d))\cos(e+fx)}{af}-\frac{(A-B)\cos(e+fx)(c+d\sin(}{f(a\sin(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((2*B*c*(c - 2*d) + 4*A*c*d - (2*A - 3*B)*d^2)*x)/(2*a) + (2*(A*(c - d) - B
*(2*c - d))*d*Cos[e + f*x])/(a*f) + ((2*A - 3*B)*d^2*Cos[e + f*x]*Sin[e + f
*x])/(2*a*f) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(f*(a + a*Sin[
e + f*x]))
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
```

```
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{f(a+a\sin(e+fx))} + \frac{\int(c+d\sin(e+fx)}{}$$
$$= \frac{\left(2Bc(c-2d)+4Acd-(2A-3B)d^2\right)x}{2a} + \frac{2(A(c-d)-B(2c-d}{af}$$

Mathematica [A] time = 0.46, size = 200, normalized size = 1.40

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2(e+fx)\left(2Ad(2c-d)+B\left(2c^2-4cd+3d^2\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*(c - d)^2*Sin[(e + f*x)/2
] + 2*(2*A*(2*c - d)*d + B*(2*c^2 - 4*c*d + 3*d^2))*(e + f*x)*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2]) + 4*d*(-(A*d) + B*(-2*c + d))*Cos[e + f*x]*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]) - B*d^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*Sin[2*(e + f*x)]))/(4*a*f*(1 + Sin[e + f*x]))
```

fricas [B] time = 0.47, size = 303, normalized size = 2.12

$$\frac{Bd^2\cos\left(fx+e\right)^3-2\,(A-B)c^2+4\,(A-B)cd-2\,(A-B)d^2+\left(2\,Bc^2+4\,(A-B)cd-(2\,A-3\,B)d^2\right)fx-2\left(2\,B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/2*(B*d^2*cos(f*x + e)^3 - 2*(A - B)*c^2 + 4*(A - B)*c*d - 2*(A - B)*d^2 +
 (2*B*c^2 + 4*(A - B)*c*d - (2*A - 3*B)*d^2)*f*x - 2*(2*B*c*d + (A - B)*d^2
)*cos(f*x + e)^2 - (2*(A - B)*c^2 - 4*(A - 2*B)*c*d + (4*A - 3*B)*d^2 - (2*
B*c^2 + 4*(A - B)*c*d - (2*A - 3*B)*d^2)*f*x)*cos(f*x + e) - (B*d^2*cos(f*x
 + e)^2 - 2*(A - B)*c^2 + 4*(A - B)*c*d - 2*(A - B)*d^2 - (2*B*c^2 + 4*(A -
```

```
 B)*c*d - (2*A - 3*B)*d^2)*f*x + (4*B*c*d + (2*A - B)*d^2)*cos(f*x + e))*si
n(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

giac [A] time = 0.16, size = 222, normalized size = 1.55

$$\frac{\frac{\left(2\,Bc^2+4\,Acd-4\,Bcd-2\,Ad^2+3\,Bd^2\right)\left(fx+e\right)}{a}-\frac{4\left(Ac^2-Bc^2-2\,Acd+2\,Bcd+Ad^2-Bd^2\right)}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{2\left(Bd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-4\,Bcd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,Ad^2\tan\left(\frac{1}{2}\,f\right.}{\left(\tan\left(\right.\right.}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] 1/2*((2*B*c^2 + 4*A*c*d - 4*B*c*d - 2*A*d^2 + 3*B*d^2)*(f*x + e)/a - 4*(A*c
^2 - B*c^2 - 2*A*c*d + 2*B*c*d + A*d^2 - B*d^2)/(a*(tan(1/2*f*x + 1/2*e) +
1)) + 2*(B*d^2*tan(1/2*f*x + 1/2*e)^3 - 4*B*c*d*tan(1/2*f*x + 1/2*e)^2 - 2*
A*d^2*tan(1/2*f*x + 1/2*e)^2 + 2*B*d^2*tan(1/2*f*x + 1/2*e)^2 - B*d^2*tan(1
/2*f*x + 1/2*e) - 4*B*c*d - 2*A*d^2 + 2*B*d^2)/((tan(1/2*f*x + 1/2*e)^2 + 1
)^2*a))/f
```

maple [B] time = 0.42, size = 524, normalized size = 3.66

$$\frac{B\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d^2}{af\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{2A\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d^2}{af\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{4B\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)cd}{af\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}+\frac{2B\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d^2}{af\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}-\frac{B\tan\left(\right.}{af\left(1+\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x)
```

```
[Out] 1/a/f/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*x+1/2*e)^3*d^2-2/a/f/(1+tan(1/
2*f*x+1/2*e)^2)^2*A*tan(1/2*f*x+1/2*e)^2*d^2-4/a/f/(1+tan(1/2*f*x+1/2*e)^2)
^2*B*tan(1/2*f*x+1/2*e)^2*c*d+2/a/f/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*
x+1/2*e)^2*d^2-1/a/f/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*x+1/2*e)*d^2-2/
a/f/(1+tan(1/2*f*x+1/2*e)^2)^2*A*d^2-4/a/f/(1+tan(1/2*f*x+1/2*e)^2)^2*B*c*d
+2/a/f/(1+tan(1/2*f*x+1/2*e)^2)^2*B*d^2+4/a/f*arctan(tan(1/2*f*x+1/2*e))*A*
c*d-2/a/f*arctan(tan(1/2*f*x+1/2*e))*A*d^2+2/a/f*arctan(tan(1/2*f*x+1/2*e))
*B*c^2-4/a/f*arctan(tan(1/2*f*x+1/2*e))*B*c*d+3/a/f*arctan(tan(1/2*f*x+1/2*
e))*B*d^2-2/a/f/(tan(1/2*f*x+1/2*e)+1)*A*c^2+4/a/f/(tan(1/2*f*x+1/2*e)+1)*A
*c*d-2/a/f/(tan(1/2*f*x+1/2*e)+1)*A*d^2+2/a/f/(tan(1/2*f*x+1/2*e)+1)*B*c^2-
4/a/f/(tan(1/2*f*x+1/2*e)+1)*B*c*d+2/a/f/(tan(1/2*f*x+1/2*e)+1)*B*d^2
```

maxima [B] time = 0.73, size = 606, normalized size = 4.24

$$Bd^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+4}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{2a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{2a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)-4Bcd\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\cdots}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{a\sin\cdots}{(\cos(f\cdots}}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] (B*d^2*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*a
rctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 4*B*c*d*((sin(f*x + e)/(cos(f*x
 + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(
cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 2*A*
d^2*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(c
os(f*x + e) + 1))/a) + 2*B*c^2*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a +
 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + 4*A*c*d*(arctan(sin(f*x + e)/
(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) - 2*A*c^
2/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

mupad [B] time = 16.67, size = 297, normalized size = 2.08

$$\frac{x\left(2Bc^2-2Ad^2+3Bd^2+4Acd-4Bcd\right)}{2a}-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(2Ad^2-3Bd^2+4Bcd\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(2A\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x)),x)
```

```
[Out] (x*(2*B*c^2 - 2*A*d^2 + 3*B*d^2 + 4*A*c*d - 4*B*c*d))/(2*a) - (tan(e/2 + (f
*x)/2)^3*(2*A*d^2 - 3*B*d^2 + 4*B*c*d) + tan(e/2 + (f*x)/2)^4*(2*A*c^2 + 2*
A*d^2 - 2*B*c^2 - 3*B*d^2 - 4*A*c*d + 4*B*c*d) + tan(e/2 + (f*x)/2)^2*(4*A*
c^2 + 6*A*d^2 - 4*B*c^2 - 5*B*d^2 - 8*A*c*d + 12*B*c*d) + 2*A*c^2 + 4*A*d^2
```

```
 - 2*B*c^2 - 4*B*d^2 + tan(e/2 + (f*x)/2)*(2*A*d^2 - B*d^2 + 4*B*c*d) - 4*A
*c*d + 8*B*c*d)/(f*(a + a*tan(e/2 + (f*x)/2) + 2*a*tan(e/2 + (f*x)/2)^2 + 2
*a*tan(e/2 + (f*x)/2)^3 + a*tan(e/2 + (f*x)/2)^4 + a*tan(e/2 + (f*x)/2)^5))
```

sympy [A] time = 9.44, size = 5763, normalized size = 40.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-4*A*c**2*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f
*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**
2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 8*A*c**2*tan(e/2 + f*x/2)**2/(2*a*f*t
an(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3
+ 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*A*c**2/(2
*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/
2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 4*A*c
*d*f*x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x
/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(
e/2 + f*x/2) + 2*a*f) + 4*A*c*d*f*x*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*
x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan
(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 8*A*c*d*f*x*tan(e/2 +
f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*ta
n(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*
a*f) + 8*A*c*d*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*t
an(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2
+ 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 4*A*c*d*f*x*tan(e/2 + f*x/2)/(2*a*f*tan
(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 +
4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 4*A*c*d*f*x/(
2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x
/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 8*A*
c*d*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)
**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2
 + f*x/2) + 2*a*f) + 16*A*c*d*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**
5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 +
 f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 8*A*c*d/(2*a*f*tan(e/2 + f*x
/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(
e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 2*A*d**2*f*x*tan(e/2 +
f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*ta
n(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*
a*f) - 2*A*d**2*f*x*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*
tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2
 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*A*d**2*f*x*tan(e/2 + f*x/2)**3/(2*a*
f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)*
```

```
*3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*A*d**2
*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2
)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/
2 + f*x/2) + 2*a*f) - 2*A*d**2*f*x*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)
**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2
 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 2*A*d**2*f*x/(2*a*f*tan(e/
2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a
*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*A*d**2*tan(e/2
 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f
*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) +
 2*a*f) - 4*A*d**2*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*t
an(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2
+ 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 12*A*d**2*tan(e/2 + f*x/2)**2/(2*a*f*ta
n(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 +
 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*A*d**2*tan
(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*
f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2)
+ 2*a*f) - 8*A*d**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4
+ 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f
*x/2) + 2*a*f) + 2*B*c**2*f*x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**
5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 +
 f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 2*B*c**2*f*x*tan(e/2 + f*x/2
)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2
 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f)
+ 4*B*c**2*f*x*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e
/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*
a*f*tan(e/2 + f*x/2) + 2*a*f) + 4*B*c**2*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan
(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 +
4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 2*B*c**2*f*x*
tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4
*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/
2) + 2*a*f) + 2*B*c**2*f*x/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x
/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(
e/2 + f*x/2) + 2*a*f) + 4*B*c**2*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2
)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/
2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 8*B*c**2*tan(e/2 + f*x/2)
**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2
+ f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) +
 4*B*c**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*ta
n(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*
a*f) - 4*B*c*d*f*x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*t
an(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2
+ 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*B*c*d*f*x*tan(e/2 + f*x/2)**4/(2*a*f*
tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3
 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 8*B*c*d*f*
```

```
x*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**
4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 +
 f*x/2) + 2*a*f) - 8*B*c*d*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)*
*5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2
+ f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*B*c*d*f*x*tan(e/2 + f*x/2
)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 +
f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4
*B*c*d*f*x/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*t
an(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2
*a*f) - 8*B*c*d*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(
e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2
*a*f*tan(e/2 + f*x/2) + 2*a*f) - 8*B*c*d*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2
 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*
f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 24*B*c*d*tan(e/2
+ f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*
tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) +
2*a*f) - 8*B*c*d*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/
2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a
*f*tan(e/2 + f*x/2) + 2*a*f) - 16*B*c*d/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*
tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2
 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 3*B*d**2*f*x*tan(e/2 + f*x/2)**5/(2*a*
f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)*
*3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 3*B*d**2
*f*x*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2
)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/
2 + f*x/2) + 2*a*f) + 6*B*d**2*f*x*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x
/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(
e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 6*B*d**2*f*x*tan(e/2 +
f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*ta
n(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*
a*f) + 3*B*d**2*f*x*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan
(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 +
2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 3*B*d**2*f*x/(2*a*f*tan(e/2 + f*x/2)**5 +
 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*
x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 6*B*d**2*tan(e/2 + f*x/2)**4/(2
*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/
2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 6*B*d
**2*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)
**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2
 + f*x/2) + 2*a*f) + 10*B*d**2*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)*
*5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2
+ f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 2*B*d**2*tan(e/2 + f*x/2)/(
2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x
/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 8*B*
d**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2
```

```
+ f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f),
Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))**2/(a*sin(e) + a), True))
```

3.267 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=67

$$-\frac{(A-B)(c-d)\cos(e+fx)}{af(\sin(e+fx)+1)}+\frac{x(Ad+B(c-d))}{a}-\frac{Bd\cos(e+fx)}{af}$$

```
[Out] (B*(c-d)+A*d)*x/a-B*d*cos(f*x+e)/a/f-(A-B)*(c-d)*cos(f*x+e)/a/f/(1+sin(f*x+
e))
```

Rubi [A] time = 0.20, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2968, 3023, 2735, 2648}

$$-\frac{(A-B)(c-d)\cos(e+fx)}{af(\sin(e+fx)+1)}+\frac{x(Ad+B(c-d))}{a}-\frac{Bd\cos(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((B*(c - d) + A*d)*x)/a - (B*d*Cos[e + f*x])/(a*f) - ((A - B)*(c - d)*Cos[e
 + f*x])/(a*f*(1 + Sin[e + f*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \int \frac{Ac+(Bc+Ad)\sin(e+fx)+Bd\sin^2(e+fx)}{a+a\sin(e+fx)}\,dx$$

$$= -\frac{Bd\cos(e+fx)}{af} + \frac{\int \frac{aAc+a(B(c-d)+Ad)\sin(e+fx)}{a+a\sin(e+fx)}\,dx}{a}$$

$$= \frac{(B(c-d)+Ad)x}{a} - \frac{Bd\cos(e+fx)}{af} + ((A-B)(c-d))\int \frac{\quad}{a+a\,\mathrm{s}\cdots}$$

$$= \frac{(B(c-d)+Ad)x}{a} - \frac{Bd\cos(e+fx)}{af} - \frac{(A-B)(c-d)\cos(e+fx\cdots}{f(a+a\sin(e+fx))}$$

Mathematica [A] time = 0.47, size = 126, normalized size = 1.88

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)((e+fx)(Ad+B(c-d))-Bd\cos(e+fx))+\sin\left(\frac{1}{2}(e+f\cdots\right.\right.}{af(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x]),
x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(Cos[(e + f*x)/2]*((B*(c - d) + A*d)
*(e + f*x) - B*d*Cos[e + f*x]) + (2*A*c + B*(c - d)*(-2 + e + f*x) + A*d*(-
2 + e + f*x) - B*d*Cos[e + f*x])*Sin[(e + f*x)/2]))/(a*f*(1 + Sin[e + f*x])
)
```

fricas [B] time = 0.47, size = 154, normalized size = 2.30

$$-\frac{Bd\cos\left(fx+e\right)^2-(Bc+(A-B)d)fx+(A-B)c-(A-B)d-\left((Bc+(A-B)d)fx-(A-B)c+(A-2B)d\right)\cdots}{af\cos\left(fx+e\right)+af\sin\left(fx\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -(B*d*cos(f*x + e)^2 - (B*c + (A - B)*d)*f*x + (A - B)*c - (A - B)*d - ((B*
c + (A - B)*d)*f*x - (A - B)*c + (A - 2*B)*d)*cos(f*x + e) - ((B*c + (A - B
)*d)*f*x - B*d*cos(f*x + e) + (A - B)*c - (A - B)*d)*sin(f*x + e))/(a*f*cos
(f*x + e) + a*f*sin(f*x + e) + a*f)
```

giac [B] time = 0.14, size = 160, normalized size = 2.39

$$\frac{\frac{(Bc+Ad-Bd)(fx+e)}{a}-\frac{2\left(Ac\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-Bc\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-Ad\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+Bd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+Bd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+Ac-Bc-Ad+2Bd\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] ((B*c + A*d - B*d)*(f*x + e)/a - 2*(A*c*tan(1/2*f*x + 1/2*e)^2 - B*c*tan(1/
2*f*x + 1/2*e)^2 - A*d*tan(1/2*f*x + 1/2*e)^2 + B*d*tan(1/2*f*x + 1/2*e)^2
+ B*d*tan(1/2*f*x + 1/2*e) + A*c - B*c - A*d + 2*B*d)/((tan(1/2*f*x + 1/2*e
)^3 + tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e) + 1)*a))/f
```

maple [B] time = 0.36, size = 179, normalized size = 2.67

$$-\frac{2Bd}{af\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}+\frac{2A\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d}{af}+\frac{2B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c}{af}-\frac{2B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d}{af}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] -2/a/f*B*d/(1+tan(1/2*f*x+1/2*e)^2)+2/a/f*A*arctan(tan(1/2*f*x+1/2*e))*d+2/
a/f*B*arctan(tan(1/2*f*x+1/2*e))*c-2/a/f*B*arctan(tan(1/2*f*x+1/2*e))*d-2/a
/f/(tan(1/2*f*x+1/2*e)+1)*A*c+2/a/f/(tan(1/2*f*x+1/2*e)+1)*A*d+2/a/f/(tan(1
/2*f*x+1/2*e)+1)*B*c-2/a/f/(tan(1/2*f*x+1/2*e)+1)*B*d
```

maxima [B] time = 0.49, size = 256, normalized size = 3.82

$$-\frac{2\left(Bd\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)-Bc\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)-Ad\left(\frac{\arctan(\ldots}{\ldots}\right.\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -2*(B*d*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/a) - B*c*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a +
 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) - A*d*(arctan(sin(f*x + e)/(cos
(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + A*c/(a + a
*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

mupad [B] time = 13.68, size = 122, normalized size = 1.82

$$\frac{x\,\left(A\,d+B\,c-B\,d\right)}{a}-\frac{\left(2\,A\,c-2\,A\,d-2\,B\,c+2\,B\,d\right)\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+2\,B\,d\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)+2\,A\,c-2\,A\,d-2\,B\,c+}{f\,\left(a\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+a\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+a\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x)),x)
```

```
[Out] (x*(A*d + B*c - B*d))/a - (2*A*c - 2*A*d - 2*B*c + 4*B*d + tan(e/2 + (f*x)/
2)^2*(2*A*c - 2*A*d - 2*B*c + 2*B*d) + 2*B*d*tan(e/2 + (f*x)/2))/(f*(a + a*
tan(e/2 + (f*x)/2) + a*tan(e/2 + (f*x)/2)^2 + a*tan(e/2 + (f*x)/2)^3))
```

sympy [A] time = 4.18, size = 1307, normalized size = 19.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-2*A*c*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/
2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 2*A*c/(a*f*tan(e/2 + f*x/2)**
3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + A*d*f*x*tan(e/2
 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e
/2 + f*x/2) + a*f) + A*d*f*x*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 +
 a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + A*d*f*x*tan(e/2 +
f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f
*x/2) + a*f) + A*d*f*x/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 +
 a*f*tan(e/2 + f*x/2) + a*f) + 2*A*d*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x
/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*A*d/(a*f
*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f
```

```
) + B*c*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*
x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + B*c*f*x*tan(e/2 + f*x/2)**2/(a*f*ta
n(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) +
 B*c*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**
2 + a*f*tan(e/2 + f*x/2) + a*f) + B*c*f*x/(a*f*tan(e/2 + f*x/2)**3 + a*f*ta
n(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*tan(e/2 + f*x/2)**2
/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2)
+ a*f) + 2*B*c/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan
(e/2 + f*x/2) + a*f) - B*d*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3
 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - B*d*f*x*tan(e/2
+ f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/
2 + f*x/2) + a*f) - B*d*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f
*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - B*d*f*x/(a*f*tan(e/2 +
 f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 2*B*d*
tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*
f*tan(e/2 + f*x/2) + a*f) - 2*B*d*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3
 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 4*B*d/(a*f*tan(e
/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f), Ne(
f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))/(a*sin(e) + a), True))
```

3.268 $\int \frac{A+B\sin(e+fx)}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=35

$$\frac{Bx}{a}-\frac{(A-B)\cos(e+fx)}{f(a\sin(e+fx)+a)}$$

[Out] B*x/a-(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))

Rubi [A] time = 0.05, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2735, 2648}

$$\frac{Bx}{a}-\frac{(A-B)\cos(e+fx)}{f(a\sin(e+fx)+a)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x]),x]

[Out] (B*x)/a - ((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x]))

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{a+a\sin(e+fx)}\,dx &= \frac{Bx}{a}-(-A+B)\int \frac{1}{a+a\sin(e+fx)}\,dx\\ &= \frac{Bx}{a}-\frac{(A-B)\cos(e+fx)}{f(a+a\sin(e+fx))}\end{aligned}$$

Mathematica [B] time = 0.16, size = 79, normalized size = 2.26

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)(2A+B(e+fx-2))+B(e+fx)\cos\left(\frac{1}{2}(e+fx)\right)\right)}{af(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(B*(e + f*x)*Cos[(e + f*x)/2] + (2*A
 + B*(-2 + e + f*x))*Sin[(e + f*x)/2]))/(a*f*(1 + Sin[e + f*x]))
```

fricas [A] time = 0.44, size = 66, normalized size = 1.89

$$\frac{Bfx+\left(Bfx-A+B\right)\cos\left(fx+e\right)+\left(Bfx+A-B\right)\sin\left(fx+e\right)-A+B}{af\cos\left(fx+e\right)+af\sin\left(fx+e\right)+af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] (B*f*x + (B*f*x - A + B)*cos(f*x + e) + (B*f*x + A - B)*sin(f*x + e) - A +
B)/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

giac [A] time = 0.15, size = 40, normalized size = 1.14

$$\frac{\frac{\left(fx+e\right)B}{a}-\frac{2\,(A-B)}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] ((f*x + e)*B/a - 2*(A - B)/(a*(tan(1/2*f*x + 1/2*e) + 1)))/f
```

maple [A] time = 0.23, size = 65, normalized size = 1.86

$$\frac{2B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{2A}{af\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{2B}{af\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

[Out] 2/a/f*B*arctan(tan(1/2*f*x+1/2*e))-2/a/f/(tan(1/2*f*x+1/2*e)+1)*A+2/a/f/(tan(1/2*f*x+1/2*e)+1)*B

maxima [B] time = 0.52, size = 78, normalized size = 2.23

$$\frac{2\left(B\left(\frac{\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{a}+\frac{1}{a+\frac{a\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\right)-\frac{A}{a+\frac{a\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="maxima")

[Out] 2*(B*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) - A/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f

mupad [B] time = 12.92, size = 35, normalized size = 1.00

$$\frac{B\,x}{a}-\frac{2\,A-2\,B}{a\,f\,\left(\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x)),x)

[Out] (B*x)/a - (2*A - 2*B)/(a*f*(tan(e/2 + (f*x)/2) + 1))

sympy [A] time = 1.83, size = 109, normalized size = 3.11

$$\begin{cases} -\frac{2A}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}+\frac{Bfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}+\frac{Bfx}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}+\frac{2B}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} & \text{for}\; f\neq 0 \\ \frac{x(A+B\sin(e))}{a\sin(e)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)

[Out] Piecewise((-2*A/(a*f*tan(e/2 + f*x/2) + a*f) + B*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2) + a*f) + B*f*x/(a*f*tan(e/2 + f*x/2) + a*f) + 2*B/(a*f*tan(e/2 + f*x/2) + a*f), Ne(f, 0)), (x*(A + B*sin(e))/(a*sin(e) + a), True))

3.269 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=101

$$\frac{2(Bc-Ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\sqrt{c^2-d^2}}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}$$

```
[Out] -(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))+2*(-A*d+B*c)*arctan((d+c*tan(1/2
*f*x+1/2*e))/(c^2-d^2)^(1/2))/a/(c-d)/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2978, 12, 2660, 618, 204}

$$\frac{2(Bc-Ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\sqrt{c^2-d^2}}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
```

```
[Out] (2*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a*(c - d)
*Sqrt[c^2 - d^2]*f) - ((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x]
))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx = -\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))}+\frac{\int \frac{a(Bc-Ad)}{c+d\sin(e+fx)}\,dx}{a^2(c-d)}$$

$$= -\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))}+\frac{(Bc-Ad)\int \frac{1}{c+d\sin(e+fx)}\,dx}{a(c-d)}$$

$$= -\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))}+\frac{(2(Bc-Ad))\,\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,d\right.}{a(c-d)f}$$

$$= -\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))}-\frac{(4(Bc-Ad))\,\text{Subst}\left(\int \frac{1}{-4\left(c^2-d^2\right)-x^2}\right.}{a(c-}$$

$$= \frac{2(Bc-Ad)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)\sqrt{c^2-d^2}\,f}-\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))}$$

Mathematica [A] time = 0.33, size = 148, normalized size = 1.47

$$\frac{2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left((A-B)\sqrt{c^2-d^2}\sin\left(\frac{1}{2}(e+fx)\right)+(Bc-Ad)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e\right.\right.\right.}{af(c-d)\sqrt{c^2-d^2}(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])),
x]
```

```
[Out] (2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((A - B)*Sqrt[c^2 - d^2]*Sin[(e +
f*x)/2] + (B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])))/(a*(c - d)*Sqrt[c^2 - d^2]*f*(1 + Sin[e
 + f*x]))
```

fricas [B] time = 0.48, size = 595, normalized size = 5.89

$$\left[-\frac{2(A-B)c^2-2(A-B)d^2+\left(Bc-Ad+(Bc-Ad)\cos\left(fx+e\right)+(Bc-Ad)\sin\left(fx+e\right)\right)\sqrt{-c^2+d^2}\log\left(\frac{(2c^2-\ldots}{}\right.}{2\left(\left(ac^3-ac^2d-acd^2+ad^3\right)f\cos\left(f\ldots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] [-1/2*(2*(A - B)*c^2 - 2*(A - B)*d^2 + (B*c - A*d + (B*c - A*d)*cos(f*x + e
) + (B*c - A*d)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x +
 e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d
*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) -
 c^2 - d^2)) + 2*((A - B)*c^2 - (A - B)*d^2)*cos(f*x + e) - 2*((A - B)*c^2
- (A - B)*d^2)*sin(f*x + e))/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*cos(f*x
 + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*sin(f*x + e) + (a*c^3 - a*c^2
*d - a*c*d^2 + a*d^3)*f), -((A - B)*c^2 - (A - B)*d^2 + (B*c - A*d + (B*c -
 A*d)*cos(f*x + e) + (B*c - A*d)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*s
in(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + ((A - B)*c^2 - (A - B)*d
^2)*cos(f*x + e) - ((A - B)*c^2 - (A - B)*d^2)*sin(f*x + e))/((a*c^3 - a*c^
2*d - a*c*d^2 + a*d^3)*f*cos(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)
*f*sin(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f)]
```

giac [A] time = 0.17, size = 113, normalized size = 1.12

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)(Bc-Ad)}{(ac-ad)\sqrt{c^2-d^2}}-\frac{A-B}{(ac-ad)\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e
) + d)/sqrt(c^2 - d^2)))*(B*c - A*d)/((a*c - a*d)*sqrt(c^2 - d^2)) - (A - B
)/((a*c - a*d)*(tan(1/2*f*x + 1/2*e) + 1)))/f
```

maple [A] time = 0.54, size = 176, normalized size = 1.74

$$-\frac{2\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)Ad}{af\,(c-d)\sqrt{c^2-d^2}}+\frac{2\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)Bc}{af\,(c-d)\sqrt{c^2-d^2}}-\frac{2A}{af\,(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{2B}{af\,(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] -2/a/f/(c-d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d
^2)^(1/2))*A*d+2/a/f/(c-d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*
e)+2*d)/(c^2-d^2)^(1/2))*B*c-2/a/f/(c-d)/(tan(1/2*f*x+1/2*e)+1)*A+2/a/f/(c-
d)/(tan(1/2*f*x+1/2*e)+1)*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 13.32, size = 154, normalized size = 1.52

$$\frac{2\,\mathrm{atan}\left(\frac{\frac{(A\,d-B\,c)\left(2\,a\,d^2-2\,a\,c\,d\right)}{a\,\sqrt{c+d}\,(c-d)^{3/2}}-\frac{2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(A\,d-B\,c)\,(a\,c-a\,d)}{a\,\sqrt{c+d}\,(c-d)^{3/2}}}{2\,A\,d-2\,B\,c}\right)(A\,d-B\,c)}{a\,f\,\sqrt{c+d}\,(c-d)^{3/2}}-\frac{2\,(A-B)}{f\,\left(a+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,(c-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))),x)
```

```
[Out] (2*atan((((A*d - B*c)*(2*a*d^2 - 2*a*c*d))/(a*(c + d)^(1/2)*(c - d)^(3/2))
- (2*c*tan(e/2 + (f*x)/2)*(A*d - B*c)*(a*c - a*d))/(a*(c + d)^(1/2)*(c - d)
^(3/2)))/(2*A*d - 2*B*c))*(A*d - B*c))/(a*f*(c + d)^(1/2)*(c - d)^(3/2)) -
(2*(A - B))/(f*(a + a*tan(e/2 + (f*x)/2))*(c - d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.270 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=181

$$-\frac{2\left(Ad(2c+d)-B\left(c^2+cd+d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\left(c^2-d^2\right)^{3/2}}+\frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{af(c-d)^2(c+d)(c+d\sin(e+fx))}-\frac{}{f(c-d)(a\sin}$$

```
[Out] -2*(A*d*(2*c+d)-B*(c^2+c*d+d^2))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^
(1/2))/a/(c-d)/(c^2-d^2)^(3/2)/f+d*(B*(2*c+d)-A*(c+2*d))*cos(f*x+e)/a/(c-d)
^2/(c+d)/f/(c+d*sin(f*x+e))-(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))/(c+d*
sin(f*x+e))
```

Rubi [A] time = 0.35, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2978, 2754, 12, 2660, 618, 204}

$$-\frac{2\left(Ad(2c+d)-B\left(c^2+cd+d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\left(c^2-d^2\right)^{3/2}}+\frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{af(c-d)^2(c+d)(c+d\sin(e+fx))}-\frac{}{f(c-d)(a\sin}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (-2*(A*d*(2*c + d) - B*(c^2 + c*d + d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/S
qrt[c^2 - d^2]])/(a*(c - d)*(c^2 - d^2)^(3/2)*f) + (d*(B*(2*c + d) - A*(c +
 2*d))*Cos[e + f*x])/(a*(c - d)^2*(c + d)*f*(c + d*Sin[e + f*x])) - ((A - B
)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx = -\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))} - \frac{\int \frac{a(2Ad-B(c+d))\cdots}{(c+d\,\mathrm{s}\cdots}}{a^2\cdots}$$

$$= \frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} - \frac{(A-B)\cos\cdots}{(c-d)f(a+a\sin(e+f\cdots}$$

$$= \frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} - \frac{(A-B)\cos\cdots}{(c-d)f(a+a\sin(e+f\cdots}$$

$$= \frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} - \frac{(A-B)\cos\cdots}{(c-d)f(a+a\sin(e+f\cdots}$$

$$= \frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} - \frac{(A-B)\cos\cdots}{(c-d)f(a+a\sin(e+f\cdots}$$

$$= -\frac{2\left(Ad(2c+d)-B\left(c^2+cd+d^2\right)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)^2(c+d)\sqrt{c^2-d^2}\,f} + \frac{d(B\cdots}{a(\cdots}$$

Mathematica [A] time = 1.22, size = 209, normalized size = 1.15

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{2\left(B\left(c^2+cd+d^2\right)-Ad(2c+d)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}} + \frac{d(Bc-Ad)\,\mathrm{c}\cdots}{\cdots}\right.}{af(c-d)^2(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^2
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*Sin[(e + f*x)/2] + (2*(-(
A*d*(2*c + d)) + B*(c^2 + c*d + d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[
c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/((c + d)*Sqrt[c^2 - d^2]
) + (d*(B*c - A*d)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/((c
+ d)*(c + d*Sin[e + f*x]))))/(a*(c - d)^2*f*(1 + Sin[e + f*x]))
```

fricas [B] time = 0.51, size = 1538, normalized size = 8.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] [1/2*(2*(A - B)*c^4 - 4*(A - B)*c^2*d^2 + 2*(A - B)*d^4 + 2*((A - 2*B)*c^3*
d + (2*A - B)*c^2*d^2 - (A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)^2 + (
B*c^3 - 2*(A - B)*c^2*d - (3*A - 2*B)*c*d^2 - (A - B)*d^3 - (B*c^2*d - (2*A
 - B)*c*d^2 - (A - B)*d^3)*cos(f*x + e)^2 + (B*c^3 - (2*A - B)*c^2*d - (A -
 B)*c*d^2)*cos(f*x + e) + (B*c^3 - 2*(A - B)*c^2*d - (3*A - 2*B)*c*d^2 - (A
 - B)*d^3 + (B*c^2*d - (2*A - B)*c*d^2 - (A - B)*d^3)*cos(f*x + e))*sin(f*x
 + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x +
 e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^
2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*((A -
B)*c^4 + (A - 2*B)*c^3*d + B*c^2*d^2 - (A - 2*B)*c*d^3 - A*d^4)*cos(f*x + e
) - 2*((A - B)*c^4 - 2*(A - B)*c^2*d^2 + (A - B)*d^4 - ((A - 2*B)*c^3*d + (
2*A - B)*c^2*d^2 - (A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e))*sin(f*x +
 e))/((a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f
*cos(f*x + e)^2 - (a*c^6 - a*c^5*d - 2*a*c^4*d^2 + 2*a*c^3*d^3 + a*c^2*d^4
- a*c*d^5)*f*cos(f*x + e) - (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f -
 ((a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f*cos
(f*x + e) + (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f)*sin(f*x + e)), (
(A - B)*c^4 - 2*(A - B)*c^2*d^2 + (A - B)*d^4 + ((A - 2*B)*c^3*d + (2*A - B
)*c^2*d^2 - (A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)^2 + (B*c^3 - 2*(A
 - B)*c^2*d - (3*A - 2*B)*c*d^2 - (A - B)*d^3 - (B*c^2*d - (2*A - B)*c*d^2
- (A - B)*d^3)*cos(f*x + e)^2 + (B*c^3 - (2*A - B)*c^2*d - (A - B)*c*d^2)*c
os(f*x + e) + (B*c^3 - 2*(A - B)*c^2*d - (3*A - 2*B)*c*d^2 - (A - B)*d^3 +
(B*c^2*d - (2*A - B)*c*d^2 - (A - B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(
c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (
(A - B)*c^4 + (A - 2*B)*c^3*d + B*c^2*d^2 - (A - 2*B)*c*d^3 - A*d^4)*cos(f*
x + e) - ((A - B)*c^4 - 2*(A - B)*c^2*d^2 + (A - B)*d^4 - ((A - 2*B)*c^3*d
+ (2*A - B)*c^2*d^2 - (A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e))*sin(f*
x + e))/((a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6
)*f*cos(f*x + e)^2 - (a*c^6 - a*c^5*d - 2*a*c^4*d^2 + 2*a*c^3*d^3 + a*c^2*d
^4 - a*c*d^5)*f*cos(f*x + e) - (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*
f - ((a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f*
cos(f*x + e) + (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f)*sin(f*x + e))
]
```

giac [B] time = 0.19, size = 443, normalized size = 2.45

$$2\left(\frac{\left(Bc^2-2Acd+Bcd-Ad^2+Bd^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(ac^3-ac^2d-acd^2+ad^3\right)\sqrt{c^2-d^2}}-\frac{Ac^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-Bc^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+Ac^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] 2*((B*c^2 - 2*A*c*d + B*c*d - A*d^2 + B*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1
/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a*c^3
- a*c^2*d - a*c*d^2 + a*d^3)*sqrt(c^2 - d^2)) - (A*c^3*tan(1/2*f*x + 1/2*e)
^2 - B*c^3*tan(1/2*f*x + 1/2*e)^2 + A*c^2*d*tan(1/2*f*x + 1/2*e)^2 - B*c^2*
d*tan(1/2*f*x + 1/2*e)^2 - B*c*d^2*tan(1/2*f*x + 1/2*e)^2 + A*d^3*tan(1/2*f
*x + 1/2*e)^2 + 2*A*c^2*d*tan(1/2*f*x + 1/2*e) - 3*B*c^2*d*tan(1/2*f*x + 1/
2*e) + 3*A*c*d^2*tan(1/2*f*x + 1/2*e) - 3*B*c*d^2*tan(1/2*f*x + 1/2*e) + A*
d^3*tan(1/2*f*x + 1/2*e) + A*c^3 - B*c^3 + A*c^2*d - 2*B*c^2*d + A*c*d^2)/(
(a*c^4 - a*c^3*d - a*c^2*d^2 + a*c*d^3)*(c*tan(1/2*f*x + 1/2*e)^3 + c*tan(1
/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e)^2 + c*tan(1/2*f*x + 1/2*e) + 2
*d*tan(1/2*f*x + 1/2*e) + c)))/f
```

maple [B] time = 0.58, size = 615, normalized size = 3.40

$$-\frac{2d^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)A}{af\,(c-d)^2\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d+c\right)(c+d)\,c}+\frac{2d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)B}{af\,(c-d)^2\left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -2/a/f/(c-d)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)*d^3/(c+d)/
c*tan(1/2*f*x+1/2*e)*A+2/a/f/(c-d)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+
1/2*e)*d+c)*d^2/(c+d)*tan(1/2*f*x+1/2*e)*B-2/a/f/(c-d)^2/(tan(1/2*f*x+1/2*e
)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)*d^2/(c+d)*A+2/a/f/(c-d)^2/(tan(1/2*f*x+1/2*
e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)*d/(c+d)*B*c-4/a/f/(c-d)^2/(c+d)/(c^2-d^2)^
(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c*d-2/a/f/
(c-d)^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-
d^2)^(1/2))*A*d^2+2/a/f/(c-d)^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1
/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^2+2/a/f/(c-d)^2/(c+d)/(c^2-d^2)^(1/
2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c*d+2/a/f/(c-
d)^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2
)^(1/2))*B*d^2-2/a/f/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)*A+2/a/f/(c-d)^2/(tan(1/
2*f*x+1/2*e)+1)*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 15.53, size = 437, normalized size = 2.41

$$\frac{2\,\mathrm{atan}\left(\frac{\frac{\left(2\,a\,c^3\,d-2\,a\,c^2\,d^2-2\,a\,c\,d^3+2\,a\,d^4\right)\,\left(B\,c^2-A\,d^2+B\,d^2-2\,A\,c\,d+B\,c\,d\right)}{a\,{\left(c+d\right)}^{3/2}\,{\left(c-d\right)}^{5/2}}+\frac{2\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(a\,c^3-a\,c^2\,d-a\,c\,d^2+a\,d^3\right)\,\left(B\,c^2-A\,d^2+B\,d^2-2\,A\,c\,d+B\,c\,d\right)}{a\,{\left(c+d\right)}^{3/2}\,{\left(c-d\right)}^{5/2}}}{2\,B\,c^2-2\,A\,d^2+2\,B\,d^2-4\,A\,c\,d+2\,B\,c\,d}\right)\,\left(B\,c^2-A\,d^2\right.}{a\,f\,{\left(c+d\right)}^{3/2}\,{\left(c-d\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^2),x)
```

```
[Out] (2*atan((((2*a*d^4 - 2*a*c^2*d^2 - 2*a*c*d^3 + 2*a*c^3*d)*(B*c^2 - A*d^2 +
B*d^2 - 2*A*c*d + B*c*d))/(a*(c + d)^(3/2)*(c - d)^(5/2)) + (2*c*tan(e/2 +
(f*x)/2)*(a*c^3 + a*d^3 - a*c*d^2 - a*c^2*d)*(B*c^2 - A*d^2 + B*d^2 - 2*A*c
*d + B*c*d))/(a*(c + d)^(3/2)*(c - d)^(5/2)))/(2*B*c^2 - 2*A*d^2 + 2*B*d^2
- 4*A*c*d + 2*B*c*d))*(B*c^2 - A*d^2 + B*d^2 - 2*A*c*d + B*c*d))/(a*f*(c +
d)^(3/2)*(c - d)^(5/2)) - ((2*(A*c^2 + A*d^2 - B*c^2 + A*c*d - 2*B*c*d))/((
c + d)*(c - d)^2) + (2*tan(e/2 + (f*x)/2)*(A*d^2 + 2*A*c*d - 3*B*c*d))/(c*(
c - d)^2) + (2*tan(e/2 + (f*x)/2)^2*(A*c^3 + A*d^3 - B*c^3 + A*c^2*d - B*c*
d^2 - B*c^2*d))/(c*(c + d)*(c - d)^2))/(f*(a*c + tan(e/2 + (f*x)/2)^2*(a*c
+ 2*a*d) + tan(e/2 + (f*x)/2)*(a*c + 2*a*d) + a*c*tan(e/2 + (f*x)/2)^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.271 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=283

$$-\frac{d\left(2Ac^2+9Acd+4Ad^2-5Bc^2-6Bcd-4Bd^2\right)\cos(e+fx)}{2af(c-d)^3(c+d)^2(c+d\sin(e+fx))}-\frac{\left(3Ad\left(2c^2+2cd+d^2\right)-B\left(2c^3+4c^2d+7cd^2+\right.\right.}{af(c-d)\left(c^2-d^2\right)^{5/}}$$

```
[Out] -(3*A*d*(2*c^2+2*c*d+d^2)-B*(2*c^3+4*c^2*d+7*c*d^2+2*d^3))*arctan((d+c*tan(
1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a/(c-d)/(c^2-d^2)^(5/2)/f-1/2*d*(2*A*c+3*A
*d-3*B*c-2*B*d)*cos(f*x+e)/a/(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))^2-(A-B)*cos(f
*x+e)/(c-d)/f/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2-1/2*d*(2*A*c^2+9*A*c*d+4*
A*d^2-5*B*c^2-6*B*c*d-4*B*d^2)*cos(f*x+e)/a/(c-d)^3/(c+d)^2/f/(c+d*sin(f*x+
e))
```

Rubi [A] time = 0.55, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2978, 2754, 12, 2660, 618, 204}

$$-\frac{\left(3Ad\left(2c^2+2cd+d^2\right)-B\left(4c^2d+2c^3+7cd^2+2d^3\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\left(c^2-d^2\right)^{5/2}}-\frac{d\left(2Ac^2+9Acd+4Ad^2-5Bc\right.}{2af(c-d)^3(c+d)^2(}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] -(((3*A*d*(2*c^2 + 2*c*d + d^2) - B*(2*c^3 + 4*c^2*d + 7*c*d^2 + 2*d^3))*Ar
cTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a*(c - d)*(c^2 - d^2)^(5/2
)*f)) - (d*(2*A*c - 3*B*c + 3*A*d - 2*B*d)*Cos[e + f*x])/(2*a*(c - d)^2*(c
+ d)*f*(c + d*Sin[e + f*x])^2) - ((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*S
in[e + f*x])*(c + d*Sin[e + f*x])^2) - (d*(2*A*c^2 - 5*B*c^2 + 9*A*c*d - 6*
B*c*d + 4*A*d^2 - 4*B*d^2)*Cos[e + f*x])/(2*a*(c - d)^3*(c + d)^2*f*(c + d*
Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx = -\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))^2} - \frac{\int \frac{a(3Ad-B(c+2d\ldots}{(c+\ldots}}{a\ldots}$$

$$= -\frac{d(2Ac-3Bc+3Ad-2Bd)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{(A-B\ldots}{(c-d)f(a+a\sin(e\ldots}$$

$$= -\frac{d(2Ac-3Bc+3Ad-2Bd)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{(A-B\ldots}{(c-d)f(a+a\sin(e\ldots}$$

$$= -\frac{d(2Ac-3Bc+3Ad-2Bd)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{(A-B\ldots}{(c-d)f(a+a\sin(e\ldots}$$

$$= -\frac{d(2Ac-3Bc+3Ad-2Bd)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{(A-B\ldots}{(c-d)f(a+a\sin(e\ldots}$$

$$= -\frac{d(2Ac-3Bc+3Ad-2Bd)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{(A-B\ldots}{(c-d)f(a+a\sin(e\ldots}$$

$$= -\frac{\left(3Ad\left(2c^2+2cd+d^2\right)-B\left(2c^3+4c^2d+7cd^2+2d^3\right)\right)\tan^{-1}\left(\ldots\right.}{a(c-d)^3(c+d)^2\sqrt{c^2-d^2}\,f}$$

Mathematica [A] time = 1.61, size = 313, normalized size = 1.11

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{d\left(B\left(3c^2+2cd+2d^2\right)-Ad(5c+2d)\right)\cos(e+fx)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)^2(c+d\sin(e+fx))}+\frac{2\left(B\left(2c^3+4c^2d+7cd^2\ldots\right.\right.}{\ldots}\right.}{2\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^3
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(4*(A - B)*Sin[(e + f*x)/2] + (2*(-3
*A*d*(2*c^2 + 2*c*d + d^2) + B*(2*c^3 + 4*c^2*d + 7*c*d^2 + 2*d^3))*ArcTan[
(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2]))/((c + d)^2*Sqrt[c^2 - d^2]) + ((c - d)*d*(B*c - A*d)*Cos[e + f*x]*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2]))/((c + d)*(c + d*Sin[e + f*x])^2) + (d*(
```

```
-(A*d*(5*c + 2*d)) + B*(3*c^2 + 2*c*d + 2*d^2))*Cos[e + f*x]*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]))/((c + d)^2*(c + d*Sin[e + f*x]))))/(2*a*(c - d)^3*
f*(1 + Sin[e + f*x]))
```

fricas [B] time = 0.61, size = 3303, normalized size = 11.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] [1/4*(4*(A - B)*c^6 - 12*(A - B)*c^4*d^2 + 12*(A - B)*c^2*d^4 - 4*(A - B)*d
^6 - 2*((2*A - 5*B)*c^4*d^2 + 3*(3*A - 2*B)*c^3*d^3 + (2*A + B)*c^2*d^4 - 3
*(3*A - 2*B)*c*d^5 - 4*(A - B)*d^6)*cos(f*x + e)^3 + 2*(4*(A - 2*B)*c^5*d +
 4*(3*A - 2*B)*c^4*d^2 - (2*A - 7*B)*c^3*d^3 - 5*(3*A - 2*B)*c^2*d^4 - (2*A
 - B)*c*d^5 + (3*A - 2*B)*d^6)*cos(f*x + e)^2 - (2*B*c^5 - 2*(3*A - 4*B)*c^
4*d - (18*A - 17*B)*c^3*d^2 - (21*A - 20*B)*c^2*d^3 - (12*A - 11*B)*c*d^4 -
 (3*A - 2*B)*d^5 - (2*B*c^3*d^2 - 2*(3*A - 2*B)*c^2*d^3 - (6*A - 7*B)*c*d^4
 - (3*A - 2*B)*d^5)*cos(f*x + e)^3 - (4*B*c^4*d - 2*(6*A - 5*B)*c^3*d^2 - 1
8*(A - B)*c^2*d^3 - (12*A - 11*B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e)^2 +
 (2*B*c^5 - 2*(3*A - 2*B)*c^4*d - 3*(2*A - 3*B)*c^3*d^2 - 3*(3*A - 2*B)*c^2
*d^3 - (6*A - 7*B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e) + (2*B*c^5 - 2*(3*
A - 4*B)*c^4*d - (18*A - 17*B)*c^3*d^2 - (21*A - 20*B)*c^2*d^3 - (12*A - 11
*B)*c*d^4 - (3*A - 2*B)*d^5 - (2*B*c^3*d^2 - 2*(3*A - 2*B)*c^2*d^3 - (6*A -
 7*B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e)^2 + 2*(2*B*c^4*d - 2*(3*A - 2*B
)*c^3*d^2 - (6*A - 7*B)*c^2*d^3 - (3*A - 2*B)*c*d^4)*cos(f*x + e))*sin(f*x
+ e))*sqrt(-c^2 + d^2)*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x +
 e) - c^2 - d^2 - 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^
2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(2*(A
- B)*c^6 + 4*(A - 2*B)*c^5*d + (8*A - 7*B)*c^4*d^2 + (7*A + B)*c^3*d^3 - (7
*A - 5*B)*c^2*d^4 - (11*A - 7*B)*c*d^5 - (3*A - 4*B)*d^6)*cos(f*x + e) - 2*
(2*(A - B)*c^6 - 6*(A - B)*c^4*d^2 + 6*(A - B)*c^2*d^4 - 2*(A - B)*d^6 - ((
2*A - 5*B)*c^4*d^2 + 3*(3*A - 2*B)*c^3*d^3 + (2*A + B)*c^2*d^4 - 3*(3*A - 2
*B)*c*d^5 - 4*(A - B)*d^6)*cos(f*x + e)^2 - (4*(A - 2*B)*c^5*d + (14*A - 13
*B)*c^4*d^2 + (7*A + B)*c^3*d^3 - (13*A - 11*B)*c^2*d^4 - (11*A - 7*B)*c*d^
5 - (A - 2*B)*d^6)*cos(f*x + e))*sin(f*x + e))/((a*c^7*d^2 - a*c^6*d^3 - 3*
a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a*c^2*d^7 - a*c*d^8 + a*d^9)*f*co
s(f*x + e)^3 + (2*a*c^8*d - a*c^7*d^2 - 7*a*c^6*d^3 + 3*a*c^5*d^4 + 9*a*c^4
*d^5 - 3*a*c^3*d^6 - 5*a*c^2*d^7 + a*c*d^8 + a*d^9)*f*cos(f*x + e)^2 - (a*c
^9 - a*c^8*d - 2*a*c^7*d^2 + 2*a*c^6*d^3 + 2*a*c^3*d^6 - 2*a*c^2*d^7 - a*c*
d^8 + a*d^9)*f*cos(f*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4*a*c^6*d^3
+ 6*a*c^5*d^4 + 6*a*c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^8 + a*d^9)*
f + ((a*c^7*d^2 - a*c^6*d^3 - 3*a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a
*c^2*d^7 - a*c*d^8 + a*d^9)*f*cos(f*x + e)^2 - 2*(a*c^8*d - a*c^7*d^2 - 3*a
```

*c^6*d^3 + 3*a*c^5*d^4 + 3*a*c^4*d^5 - 3*a*c^3*d^6 - a*c^2*d^7 + a*c*d^8)*f
*cos(f*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4*a*c^6*d^3 + 6*a*c^5*d^4
+ 6*a*c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^8 + a*d^9)*f)*sin(f*x + e
)), 1/2*(2*(A - B)*c^6 - 6*(A - B)*c^4*d^2 + 6*(A - B)*c^2*d^4 - 2*(A - B)*
d^6 - ((2*A - 5*B)*c^4*d^2 + 3*(3*A - 2*B)*c^3*d^3 + (2*A + B)*c^2*d^4 - 3*
(3*A - 2*B)*c*d^5 - 4*(A - B)*d^6)*cos(f*x + e)^3 + (4*(A - 2*B)*c^5*d + 4*
(3*A - 2*B)*c^4*d^2 - (2*A - 7*B)*c^3*d^3 - 5*(3*A - 2*B)*c^2*d^4 - (2*A -
B)*c*d^5 + (3*A - 2*B)*d^6)*cos(f*x + e)^2 + (2*B*c^5 - 2*(3*A - 4*B)*c^4*d
 - (18*A - 17*B)*c^3*d^2 - (21*A - 20*B)*c^2*d^3 - (12*A - 11*B)*c*d^4 - (3
*A - 2*B)*d^5 - (2*B*c^3*d^2 - 2*(3*A - 2*B)*c^2*d^3 - (6*A - 7*B)*c*d^4 -
(3*A - 2*B)*d^5)*cos(f*x + e)^3 - (4*B*c^4*d - 2*(6*A - 5*B)*c^3*d^2 - 18*(
A - B)*c^2*d^3 - (12*A - 11*B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e)^2 + (2
*B*c^5 - 2*(3*A - 2*B)*c^4*d - 3*(2*A - 3*B)*c^3*d^2 - 3*(3*A - 2*B)*c^2*d^
3 - (6*A - 7*B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e) + (2*B*c^5 - 2*(3*A -
 4*B)*c^4*d - (18*A - 17*B)*c^3*d^2 - (21*A - 20*B)*c^2*d^3 - (12*A - 11*B)
*c*d^4 - (3*A - 2*B)*d^5 - (2*B*c^3*d^2 - 2*(3*A - 2*B)*c^2*d^3 - (6*A - 7*
B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e)^2 + 2*(2*B*c^4*d - 2*(3*A - 2*B)*c
^3*d^2 - (6*A - 7*B)*c^2*d^3 - (3*A - 2*B)*c*d^4)*cos(f*x + e))*sin(f*x + e
))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x +
e))) + (2*(A - B)*c^6 + 4*(A - 2*B)*c^5*d + (8*A - 7*B)*c^4*d^2 + (7*A + B)
*c^3*d^3 - (7*A - 5*B)*c^2*d^4 - (11*A - 7*B)*c*d^5 - (3*A - 4*B)*d^6)*cos(
f*x + e) - (2*(A - B)*c^6 - 6*(A - B)*c^4*d^2 + 6*(A - B)*c^2*d^4 - 2*(A -
B)*d^6 - ((2*A - 5*B)*c^4*d^2 + 3*(3*A - 2*B)*c^3*d^3 + (2*A + B)*c^2*d^4 -
 3*(3*A - 2*B)*c*d^5 - 4*(A - B)*d^6)*cos(f*x + e)^2 - (4*(A - 2*B)*c^5*d +
 (14*A - 13*B)*c^4*d^2 + (7*A + B)*c^3*d^3 - (13*A - 11*B)*c^2*d^4 - (11*A
- 7*B)*c*d^5 - (A - 2*B)*d^6)*cos(f*x + e))*sin(f*x + e))/((a*c^7*d^2 - a*c
^6*d^3 - 3*a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a*c^2*d^7 - a*c*d^8 +
a*d^9)*f*cos(f*x + e)^3 + (2*a*c^8*d - a*c^7*d^2 - 7*a*c^6*d^3 + 3*a*c^5*d^
4 + 9*a*c^4*d^5 - 3*a*c^3*d^6 - 5*a*c^2*d^7 + a*c*d^8 + a*d^9)*f*cos(f*x +
e)^2 - (a*c^9 - a*c^8*d - 2*a*c^7*d^2 + 2*a*c^6*d^3 + 2*a*c^3*d^6 - 2*a*c^2
*d^7 - a*c*d^8 + a*d^9)*f*cos(f*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4
*a*c^6*d^3 + 6*a*c^5*d^4 + 6*a*c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^
8 + a*d^9)*f + ((a*c^7*d^2 - a*c^6*d^3 - 3*a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^
3*d^6 - 3*a*c^2*d^7 - a*c*d^8 + a*d^9)*f*cos(f*x + e)^2 - 2*(a*c^8*d - a*c^
7*d^2 - 3*a*c^6*d^3 + 3*a*c^5*d^4 + 3*a*c^4*d^5 - 3*a*c^3*d^6 - a*c^2*d^7 +
 a*c*d^8)*f*cos(f*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4*a*c^6*d^3 + 6
*a*c^5*d^4 + 6*a*c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^8 + a*d^9)*f)*
sin(f*x + e))]

giac [B] time = 0.30, size = 753, normalized size = 2.66

$$\frac{\frac{\left(2\,Bc^3-6\,Ac^2d+4\,Bc^2d-6\,Acd^2+7\,Bcd^2-3\,Ad^3+2\,Bd^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(ac^5-ac^4d-2\,ac^3d^2+2\,ac^2d^3+acd^4-ad^5\right)\sqrt{c^2-d^2}}-\frac{2\,(A-B)}{\left(ac^3-3\,ac^2d+3\,acd^2-ad^3\right)\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] ((2*B*c^3 - 6*A*c^2*d + 4*B*c^2*d - 6*A*c*d^2 + 7*B*c*d^2 - 3*A*d^3 + 2*B*d
^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*
e) + d)/sqrt(c^2 - d^2)))/((a*c^5 - a*c^4*d - 2*a*c^3*d^2 + 2*a*c^2*d^3 + a
*c*d^4 - a*d^5)*sqrt(c^2 - d^2)) - 2*(A - B)/((a*c^3 - 3*a*c^2*d + 3*a*c*d^
2 - a*d^3)*(tan(1/2*f*x + 1/2*e) + 1)) + (5*B*c^4*d^2*tan(1/2*f*x + 1/2*e)^
3 - 7*A*c^3*d^3*tan(1/2*f*x + 1/2*e)^3 + 2*B*c^3*d^3*tan(1/2*f*x + 1/2*e)^3
 - 2*A*c^2*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*A*c*d^5*tan(1/2*f*x + 1/2*e)^3 +
4*B*c^5*d*tan(1/2*f*x + 1/2*e)^2 - 6*A*c^4*d^2*tan(1/2*f*x + 1/2*e)^2 + 2*B
*c^4*d^2*tan(1/2*f*x + 1/2*e)^2 - 2*A*c^3*d^3*tan(1/2*f*x + 1/2*e)^2 + 9*B*
c^3*d^3*tan(1/2*f*x + 1/2*e)^2 - 11*A*c^2*d^4*tan(1/2*f*x + 1/2*e)^2 + 4*B*
c^2*d^4*tan(1/2*f*x + 1/2*e)^2 - 4*A*c*d^5*tan(1/2*f*x + 1/2*e)^2 + 2*B*c*d
^5*tan(1/2*f*x + 1/2*e)^2 + 2*A*d^6*tan(1/2*f*x + 1/2*e)^2 + 11*B*c^4*d^2*t
an(1/2*f*x + 1/2*e) - 17*A*c^3*d^3*tan(1/2*f*x + 1/2*e) + 6*B*c^3*d^3*tan(1
/2*f*x + 1/2*e) - 6*A*c^2*d^4*tan(1/2*f*x + 1/2*e) + 4*B*c^2*d^4*tan(1/2*f*
x + 1/2*e) + 2*A*c*d^5*tan(1/2*f*x + 1/2*e) + 4*B*c^5*d - 6*A*c^4*d^2 + 2*B
*c^4*d^2 - 2*A*c^3*d^3 + B*c^3*d^3 + A*c^2*d^4)/((a*c^7 - a*c^6*d - 2*a*c^5
*d^2 + 2*a*c^4*d^3 + a*c^3*d^4 - a*c^2*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d
*tan(1/2*f*x + 1/2*e) + c)^2))/f
```

maple [B] time = 0.56, size = 2482, normalized size = 8.77

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -4/a/f/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^5/(c^2
+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^2*A+2/a/f/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+
2*tan(1/2*f*x+1/2*e)*d+c)^2*d^6/(c^2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/2*e)^2*A+
4/a/f/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*d/(c^2+2*
c*d+d^2)*c^3*tan(1/2*f*x+1/2*e)^2*B+2/a/f/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2
*tan(1/2*f*x+1/2*e)*d+c)^2*d^2/(c^2+2*c*d+d^2)*c^2*tan(1/2*f*x+1/2*e)^2*B+9
```

/a/f/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^3/(c^2+2
*c*d+d^2)*c*tan(1/2*f*x+1/2*e)^2*B+6/a/f/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*
tan(1/2*f*x+1/2*e)*d+c)^2*d^3*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*B+11/a/f
/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^2*c^2/(c^2+2
*c*d+d^2)*tan(1/2*f*x+1/2*e)*B-7/a/f/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(
1/2*f*x+1/2*e)*d+c)^2*d^3*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*A+2/a/f/(c
-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^5/c/(c^2+2*c*d+
d^2)*tan(1/2*f*x+1/2*e)^3*A+4/a/f/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2
*f*x+1/2*e)*d+c)^2*d^4/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2*B-6/a/f/(c-d)^3
/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^4/(c^2+2*c*d+d^2)*ta
n(1/2*f*x+1/2*e)*A-11/a/f/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2
*e)*d+c)^2*d^4/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2*A-6/a/f/(c-d)^3/(tan(1/
2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^2/(c^2+2*c*d+d^2)*A*c^2-2/a/
f/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^3/(c^2+2*c*
d+d^2)*A*c+4/a/f/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^
2*d/(c^2+2*c*d+d^2)*B*c^3+2/a/f/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f
*x+1/2*e)*d+c)^2*d^2/(c^2+2*c*d+d^2)*B*c^2+1/a/f/(c-d)^3/(tan(1/2*f*x+1/2*e
)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^3/(c^2+2*c*d+d^2)*B*c+4/a/f/(c-d)^3/(ta
n(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^4/(c^2+2*c*d+d^2)*tan(1/
2*f*x+1/2*e)*B-2/a/f/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d
+c)^2*d^4/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*A-3/a/f/(c-d)^3/(c^2+2*c*d+d
^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)
)*A*d^3+2/a/f/(c-d)^3/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1
/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^3+2/a/f/(c-d)^3/(c^2+2*c*d+d^2)/(c^
2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*d^3
+1/a/f/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^4/(c^2
+2*c*d+d^2)*A+5/a/f/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+
c)^2*d^2*c^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*B+2/a/f/(c-d)^3/(tan(1/2*
f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^3*c/(c^2+2*c*d+d^2)*tan(1/2*f*
x+1/2*e)^3*B+4/a/f/(c-d)^3/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*
tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^2*d+7/a/f/(c-d)^3/(c^2+2*c*d+d
^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)
)*B*c*d^2-6/a/f/(c-d)^3/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan
(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c*d^2+2/a/f/(c-d)^3/(tan(1/2*f*x+1/
2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^5/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*
e)^2*B-17/a/f/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*d
^3*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*A+2/a/f/(c-d)^3/(tan(1/2*f*x+1/2*e)
^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^5/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*A
-6/a/f/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^2/(c^2
+2*c*d+d^2)*c^2*tan(1/2*f*x+1/2*e)^2*A-6/a/f/(c-d)^3/(c^2+2*c*d+d^2)/(c^2-d
^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c^2*d-
2/a/f/(c-d)^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^3/(c^2+
2*c*d+d^2)*c*tan(1/2*f*x+1/2*e)^2*A+2/a/f/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)*B-
2/a/f/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)*A

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 17.44, size = 1076, normalized size = 3.80

$$\frac{\frac{A\,d^4-2\,A\,c^4+2\,B\,c^4-8\,A\,c^2\,d^2+4\,B\,c^2\,d^2-2\,A\,c\,d^3-4\,A\,c^3\,d+B\,c\,d^3+8\,B\,c^3\,d}{(c+d)\left(c^2-d^2\right)\left(c^2-2\,c\,d+d^2\right)}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\left(2\,A\,d^6-13\,A\,c^2\,d^4-17\,A\,c^3\,d^3-22\,A\,c^4\,d^2+4\,B\,c^2\,d^4+19\,B\,c^3\cdots\right.}{c^2\left(c^2-2\,c\,d+d^2\right)\left(-c^3-c^2\,d+\cdots\right.}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^3),x)
```

```
[Out] ((A*d^4 - 2*A*c^4 + 2*B*c^4 - 8*A*c^2*d^2 + 4*B*c^2*d^2 - 2*A*c*d^3 - 4*A*c
^3*d + B*c*d^3 + 8*B*c^3*d)/((c + d)*(c^2 - d^2)*(c^2 - 2*c*d + d^2)) - (ta
n(e/2 + (f*x)/2)^3*(2*A*d^6 - 13*A*c^2*d^4 - 17*A*c^3*d^3 - 22*A*c^4*d^2 +
4*B*c^2*d^4 + 19*B*c^3*d^3 + 23*B*c^4*d^2 - 2*A*c*d^5 - 8*A*c^5*d + 2*B*c*d
^5 + 12*B*c^5*d))/(c^2*(c^2 - 2*c*d + d^2)*(c*d^2 - c^2*d - c^3 + d^3)) + (
tan(e/2 + (f*x)/2)^2*(2*A*d^5 - 4*A*c^5 + 4*B*c^5 - 21*A*c^2*d^3 - 14*A*c^3
*d^2 + 14*B*c^2*d^3 + 17*B*c^3*d^2 - 4*A*c*d^4 - 4*A*c^4*d + 2*B*c*d^4 + 8*
B*c^4*d))/(c^2*(c^2 - d^2)*(c^2 - 2*c*d + d^2)) + (tan(e/2 + (f*x)/2)^4*(2*
A*c^5 - 2*A*d^5 - 2*B*c^5 + 7*A*c^2*d^3 + 2*A*c^3*d^2 - 2*B*c^2*d^3 - 7*B*c
^3*d^2 + 2*A*c*d^4 + 4*A*c^4*d - 4*B*c^4*d))/(c*(c^2 - 2*c*d + d^2)*(c*d^2
- c^2*d - c^3 + d^3)) + (tan(e/2 + (f*x)/2)*(2*A*d^5 - 27*A*c^2*d^3 - 22*A*
c^3*d^2 + 15*B*c^2*d^3 + 29*B*c^3*d^2 - 5*A*c*d^4 - 8*A*c^4*d + 4*B*c*d^4 +
 12*B*c^4*d))/(c*(c + d)*(c^2 - d^2)*(c^2 - 2*c*d + d^2)))/(f*(tan(e/2 + (f
*x)/2)^2*(2*a*c^2 + 4*a*d^2 + 4*a*c*d) + tan(e/2 + (f*x)/2)^3*(2*a*c^2 + 4*
a*d^2 + 4*a*c*d) + a*c^2 + tan(e/2 + (f*x)/2)*(a*c^2 + 4*a*c*d) + tan(e/2 +
 (f*x)/2)^4*(a*c^2 + 4*a*c*d) + a*c^2*tan(e/2 + (f*x)/2)^5)) - (atan((((2*a
*d^6 - 4*a*c^2*d^4 + 4*a*c^3*d^3 + 2*a*c^4*d^2 - 2*a*c*d^5 - 2*a*c^5*d)*(2*
B*c^3 - 3*A*d^3 + 2*B*d^3 - 6*A*c*d^2 - 6*A*c^2*d + 7*B*c*d^2 + 4*B*c^2*d))
```

```
/(2*a*(c + d)^(5/2)*(c - d)^(7/2)) - (c*tan(e/2 + (f*x)/2)*(a*c^5 - a*d^5 +
 2*a*c^2*d^3 - 2*a*c^3*d^2 + a*c*d^4 - a*c^4*d)*(2*B*c^3 - 3*A*d^3 + 2*B*d^
3 - 6*A*c*d^2 - 6*A*c^2*d + 7*B*c*d^2 + 4*B*c^2*d))/(a*(c + d)^(5/2)*(c - d
)^(7/2)))/(2*B*c^3 - 3*A*d^3 + 2*B*d^3 - 6*A*c*d^2 - 6*A*c^2*d + 7*B*c*d^2
+ 4*B*c^2*d))*(2*B*c^3 - 3*A*d^3 + 2*B*d^3 - 6*A*c*d^2 - 6*A*c^2*d + 7*B*c*
d^2 + 4*B*c^2*d))/(a*f*(c + d)^(5/2)*(c - d)^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.272 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=228

$$\frac{2d\left(A\left(c^2+6cd-5d^2\right)+B\left(2c^2-15cd+8d^2\right)\right)\cos(e+fx)}{3a^2f}+\frac{dx\left(2Ad(3c-2d)+B\left(6c^2-12cd+7d^2\right)\right)}{2a^2}+\frac{d^2(2A(\cdots}{}$$

```
[Out] 1/2*d*(2*A*(3*c-2*d)*d+B*(6*c^2-12*c*d+7*d^2))*x/a^2+2/3*d*(A*(c^2+6*c*d-5*
d^2)+B*(2*c^2-15*c*d+8*d^2))*cos(f*x+e)/a^2/f+1/6*d^2*(B*(4*c-21*d)+2*A*(c+
6*d))*cos(f*x+e)*sin(f*x+e)/a^2/f-1/3*(2*B*(c-4*d)+A*(c+5*d))*cos(f*x+e)*(c
+d*sin(f*x+e))^2/a^2/f/(1+sin(f*x+e))-1/3*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))
^3/f/(a+a*sin(f*x+e))^2
```

Rubi [A] time = 0.52, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2977, 2734}

$$\frac{2d\left(A\left(c^2+6cd-5d^2\right)+B\left(2c^2-15cd+8d^2\right)\right)\cos(e+fx)}{3a^2f}+\frac{dx\left(2Ad(3c-2d)+B\left(6c^2-12cd+7d^2\right)\right)}{2a^2}+\frac{d^2(2A(\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (d*(2*A*(3*c - 2*d)*d + B*(6*c^2 - 12*c*d + 7*d^2))*x)/(2*a^2) + (2*d*(A*(c
^2 + 6*c*d - 5*d^2) + B*(2*c^2 - 15*c*d + 8*d^2))*Cos[e + f*x])/(3*a^2*f) +
 (d^2*(B*(4*c - 21*d) + 2*A*(c + 6*d))*Cos[e + f*x]*Sin[e + f*x])/(6*a^2*f)
 - ((2*B*(c - 4*d) + A*(c + 5*d))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*a
^2*f*(1 + Sin[e + f*x])) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(3
*f*(a + a*Sin[e + f*x])^2)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
```

```
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{3f(a+a\sin(e+fx))^2} + \frac{\int \frac{(c+d\sin(e+fx))^2(a(A\cdots}{\cdots}}{\cdots}$$

$$= -\frac{(2B(c-4d)+A(c+5d))\cos(e+fx)(c+d\sin(e+fx))^2}{3a^2f(1+\sin(e+fx))} - \frac{(A\cdots}{\cdots}$$

$$= \frac{d\left(2A(3c-2d)d+B\left(6c^2-12cd+7d^2\right)\right)x}{2a^2} + \frac{2d\left(A\left(c^2+6cd-\cdots\right.\right.}{\cdots}$$

Mathematica [B] time = 3.57, size = 547, normalized size = 2.40

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(3\cos\left(\frac{1}{2}(e+fx)\right)\left(8Ad\left(6c^2+3cd(3e+3fx-4)+d^2(-6e-6fx+5)\right)+B\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*(8*A*d*(6*c^2 + d^2*(5 - 6*e - 6*
f*x) + 3*c*d*(-4 + 3*e + 3*f*x)) + B*(16*c^3 + 24*c^2*d*(-4 + 3*e + 3*f*x)
- 24*c*d^2*(-5 + 6*e + 6*f*x) + 7*d^3*(-7 + 12*e + 12*f*x)))*Cos[(e + f*x)/
2] - (4*A*(4*c^3 + 24*c^2*d + d^3*(41 - 12*e - 12*f*x) + 6*c*d^2*(-10 + 3*e
 + 3*f*x)) + B*(32*c^3 + 24*c^2*d*(-10 + 3*e + 3*f*x) - 12*c*d^2*(-41 + 12*
e + 12*f*x) + d^3*(-239 + 84*e + 84*f*x)))*Cos[(3*(e + f*x))/2] + 3*(d^2*(1
2*B*c + 4*A*d - 5*B*d)*Cos[(5*(e + f*x))/2] + B*d^3*Cos[(7*(e + f*x))/2] +
2*(8*A*c^3 + 8*B*c^3 + 24*A*c^2*d - 72*B*c^2*d - 72*A*c*d^2 + 108*B*c*d^2 +
 36*A*d^3 - 50*B*d^3 + 48*B*c^2*d*e + 48*A*c*d^2*e - 96*B*c*d^2*e - 32*A*d^
3*e + 56*B*d^3*e + 48*B*c^2*d*f*x + 48*A*c*d^2*f*x - 96*B*c*d^2*f*x - 32*A*
d^3*f*x + 56*B*d^3*f*x + d*(8*A*d*(3*c*(e + f*x) - 2*d*(1 + e + f*x)) + B*(
24*c^2*(e + f*x) - 48*c*d*(1 + e + f*x) + d^2*(27 + 28*e + 28*f*x)))*Cos[e
+ f*x] + 2*d^2*(-6*B*c - 2*A*d + 3*B*d)*Cos[2*(e + f*x)] + B*d^3*Cos[3*(e +
 f*x)])*Sin[(e + f*x)/2])))/(48*a^2*f*(1 + Sin[e + f*x])^2)
```

fricas [B] time = 0.46, size = 584, normalized size = 2.56

$$-\frac{3\,Bd^3\cos\left(fx+e\right)^4-2\,(A-B)c^3+6\,(A-B)c^2d-6\,(A-B)cd^2+2\,(A-B)d^3+6\left(3\,Bcd^2+(A-B)d^3\right)\cos\left(fx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/6*(3*B*d^3*cos(f*x + e)^4 - 2*(A - B)*c^3 + 6*(A - B)*c^2*d - 6*(A - B)*
c*d^2 + 2*(A - B)*d^3 + 6*(3*B*c*d^2 + (A - B)*d^3)*cos(f*x + e)^3 + 6*(6*B
*c^2*d + 6*(A - 2*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x - (2*(A + 2*B)*c^3 + 6*(2
*A - 5*B)*c^2*d - 6*(5*A - 11*B)*c*d^2 + (22*A - 31*B)*d^3 + 3*(6*B*c^2*d +
 6*(A - 2*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x)*cos(f*x + e)^2 - (2*(2*A + B)*c^
3 + 6*(A - 4*B)*c^2*d - 6*(4*A - 13*B)*c*d^2 + 2*(13*A - 19*B)*d^3 - 3*(6*B
*c^2*d + 6*(A - 2*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x)*cos(f*x + e) + (3*B*d^3*
cos(f*x + e)^3 + 2*(A - B)*c^3 - 6*(A - B)*c^2*d + 6*(A - B)*c*d^2 - 2*(A -
 B)*d^3 + 6*(6*B*c^2*d + 6*(A - 2*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x - 3*(6*B*
c*d^2 + (2*A - 3*B)*d^3)*cos(f*x + e)^2 - (2*(A + 2*B)*c^3 + 6*(2*A - 5*B)*
c^2*d - 6*(5*A - 14*B)*c*d^2 + 4*(7*A - 10*B)*d^3 - 3*(6*B*c^2*d + 6*(A - 2
*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*
x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*si
n(f*x + e))
```

giac [B] time = 0.22, size = 494, normalized size = 2.17

$$\frac{\frac{3\left(6\,Bc^2d+6\,Acd^2-12\,Bcd^2-4\,Ad^3+7\,Bd^3\right)\left(fx+e\right)}{a^2}+\frac{6\left(Bd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-6\,Bcd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,Ad^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+4\,Bd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-B\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/6*(3*(6*B*c^2*d + 6*A*c*d^2 - 12*B*c*d^2 - 4*A*d^3 + 7*B*d^3)*(f*x + e)/a
^2 + 6*(B*d^3*tan(1/2*f*x + 1/2*e)^3 - 6*B*c*d^2*tan(1/2*f*x + 1/2*e)^2 - 2
*A*d^3*tan(1/2*f*x + 1/2*e)^2 + 4*B*d^3*tan(1/2*f*x + 1/2*e)^2 - B*d^3*tan(
1/2*f*x + 1/2*e) - 6*B*c*d^2 - 2*A*d^3 + 4*B*d^3)/((tan(1/2*f*x + 1/2*e)^2
+ 1)^2*a^2) - 4*(3*A*c^3*tan(1/2*f*x + 1/2*e)^2 - 9*B*c^2*d*tan(1/2*f*x + 1
/2*e)^2 - 9*A*c*d^2*tan(1/2*f*x + 1/2*e)^2 + 18*B*c*d^2*tan(1/2*f*x + 1/2*e
)^2 + 6*A*d^3*tan(1/2*f*x + 1/2*e)^2 - 9*B*d^3*tan(1/2*f*x + 1/2*e)^2 + 3*A
```

```
*c^3*tan(1/2*f*x + 1/2*e) + 3*B*c^3*tan(1/2*f*x + 1/2*e) + 9*A*c^2*d*tan(1/
2*f*x + 1/2*e) - 27*B*c^2*d*tan(1/2*f*x + 1/2*e) - 27*A*c*d^2*tan(1/2*f*x +
 1/2*e) + 45*B*c*d^2*tan(1/2*f*x + 1/2*e) + 15*A*d^3*tan(1/2*f*x + 1/2*e) -
 21*B*d^3*tan(1/2*f*x + 1/2*e) + 2*A*c^3 + B*c^3 + 3*A*c^2*d - 12*B*c^2*d -
 12*A*c*d^2 + 21*B*c*d^2 + 7*A*d^3 - 10*B*d^3)/(a^2*(tan(1/2*f*x + 1/2*e) +
 1)^3))/f
```

maple [B] time = 0.43, size = 946, normalized size = 4.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x)
```

```
[Out] -6/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^2*B*c*d^2+6/a^2/f*arctan(tan(1/2*f*x+1/2*
e))*A*c*d^2+6/a^2/f*d*arctan(tan(1/2*f*x+1/2*e))*B*c^2-12/a^2/f*arctan(tan(
1/2*f*x+1/2*e))*B*c*d^2+1/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*x+1/
2*e)^3*d^3-2/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^2*A*tan(1/2*f*x+1/2*e)^2*d^3-6/
a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*x+1/2*e)^2*c*d^2+4/a^2/f/(1+ta
n(1/2*f*x+1/2*e)^2)^2*B*d^3-2/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^2*A*d^3-6/a^2/
f/(tan(1/2*f*x+1/2*e)+1)^2*A*c^2*d+6/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*A*c*d^2
+6/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*B*c^2*d-6/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*
B*c*d^2+4/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*A*c^2*d-4/a^2/f/(tan(1/2*f*x+1/2*e
)+1)^3*A*c*d^2-4/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*B*c^2*d+4/a^2/f/(tan(1/2*f*
x+1/2*e)+1)^3*B*c*d^2+4/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*x+1/2*
e)^2*d^3-1/a^2/f/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*x+1/2*e)*d^3+6/a^2/
f/(tan(1/2*f*x+1/2*e)+1)*A*c*d^2+6/a^2/f/(tan(1/2*f*x+1/2*e)+1)*B*c^2*d-12/
a^2/f/(tan(1/2*f*x+1/2*e)+1)*B*c*d^2-2/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*A*d^3
-2/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*B*c^3+2/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*B*
d^3-4/3/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*A*c^3-4/a^2/f*arctan(tan(1/2*f*x+1/2
*e))*A*d^3+7/a^2/f*arctan(tan(1/2*f*x+1/2*e))*B*d^3-2/a^2/f/(tan(1/2*f*x+1/
2*e)+1)*A*c^3-4/a^2/f/(tan(1/2*f*x+1/2*e)+1)*A*d^3+6/a^2/f/(tan(1/2*f*x+1/2
*e)+1)*B*d^3+2/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*A*c^3+4/3/a^2/f/(tan(1/2*f*x+
1/2*e)+1)^3*A*d^3+4/3/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*B*c^3-4/3/a^2/f/(tan(1
/2*f*x+1/2*e)+1)^3*B*d^3
```

maxima [B] time = 0.57, size = 1382, normalized size = 6.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/3*(B*d^3*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e)
+ 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
+ a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f*
x + e) + 1))/a^2) - 12*B*c*d^2*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(
f*x + e) + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^
2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e
) + 1))/a^2) - 4*A*d^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e)
 + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/
a^2) + 6*B*c^2*d*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 6*A*c*d^2*((9*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^
2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x +
e)/(cos(f*x + e) + 1))/a^2) - 2*A*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) +
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f
*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3) - 2*B*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/
(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - 6*A*c^2*d*(3*sin
(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) +
 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3))/f
```

mupad [B] time = 16.35, size = 663, normalized size = 2.91

$$\frac{d\,\mathrm{atan}\left(\frac{d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(6\,B\,c^2-4\,A\,d^2+7\,B\,d^2+6\,A\,c\,d-12\,B\,c\,d\right)}{7\,B\,d^3-4\,A\,d^3+6\,A\,c\,d^2-12\,B\,c\,d^2+6\,B\,c^2\,d}\right)\left(6\,B\,c^2-4\,A\,d^2+7\,B\,d^2+6\,A\,c\,d-12\,B\,c\,d\right)}{a^2\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (d*atan((d*tan(e/2 + (f*x)/2)*(6*B*c^2 - 4*A*d^2 + 7*B*d^2 + 6*A*c*d - 12*B
*c*d))/(7*B*d^3 - 4*A*d^3 + 6*A*c*d^2 - 12*B*c*d^2 + 6*B*c^2*d))*(6*B*c^2 -
 4*A*d^2 + 7*B*d^2 + 6*A*c*d - 12*B*c*d))/(a^2*f) - (tan(e/2 + (f*x)/2)*(2*
A*c^3 + 16*A*d^3 + 2*B*c^3 - 25*B*d^3 - 18*A*c*d^2 + 6*A*c^2*d + 48*B*c*d^2
 - 18*B*c^2*d) + (4*A*c^3)/3 + (20*A*d^3)/3 + (2*B*c^3)/3 - (32*B*d^3)/3 +
tan(e/2 + (f*x)/2)^6*(2*A*c^3 + 4*A*d^3 - 7*B*d^3 - 6*A*c*d^2 + 12*B*c*d^2
- 6*B*c^2*d) + tan(e/2 + (f*x)/2)^5*(2*A*c^3 + 12*A*d^3 + 2*B*c^3 - 21*B*d^
3 - 18*A*c*d^2 + 6*A*c^2*d + 36*B*c*d^2 - 18*B*c^2*d) + tan(e/2 + (f*x)/2)^
3*(4*A*c^3 + 28*A*d^3 + 4*B*c^3 - 42*B*d^3 - 36*A*c*d^2 + 12*A*c^2*d + 84*B
*c*d^2 - 36*B*c^2*d) + tan(e/2 + (f*x)/2)^4*((16*A*c^3)/3 + (56*A*d^3)/3 +
(2*B*c^3)/3 - (98*B*d^3)/3 - 20*A*c*d^2 + 2*A*c^2*d + 56*B*c*d^2 - 20*B*c^2
*d) + tan(e/2 + (f*x)/2)^2*((14*A*c^3)/3 + (64*A*d^3)/3 + (4*B*c^3)/3 - (97
*B*d^3)/3 - 22*A*c*d^2 + 4*A*c^2*d + 64*B*c*d^2 - 22*B*c^2*d) - 8*A*c*d^2 +
 2*A*c^2*d + 20*B*c*d^2 - 8*B*c^2*d)/(f*(5*a^2*tan(e/2 + (f*x)/2)^2 + 7*a^2
*tan(e/2 + (f*x)/2)^3 + 7*a^2*tan(e/2 + (f*x)/2)^4 + 5*a^2*tan(e/2 + (f*x)/
2)^5 + 3*a^2*tan(e/2 + (f*x)/2)^6 + a^2*tan(e/2 + (f*x)/2)^7 + a^2 + 3*a^2*
tan(e/2 + (f*x)/2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.273 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=132

$$-\frac{(c-d)(A(c+3d)+2B(c-3d))\cos(e+fx)}{3a^2f(\sin(e+fx)+1)}+\frac{dx(Ad+2B(c-d))}{a^2}+\frac{d^2(A-4B)\cos(e+fx)}{3a^2f}-\frac{(A-B)\cos(e+fx)}{3f(a\sin(e$$

```
[Out] d*(2*B*(c-d)+A*d)*x/a^2+1/3*(A-4*B)*d^2*cos(f*x+e)/a^2/f-1/3*(c-d)*(2*B*(c-
3*d)+A*(c+3*d))*cos(f*x+e)/a^2/f/(1+sin(f*x+e))-1/3*(A-B)*cos(f*x+e)*(c+d*s
in(f*x+e))^2/f/(a+a*sin(f*x+e))^2
```

Rubi [A] time = 0.51, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2977, 2968, 3023, 2735, 2648}

$$-\frac{(c-d)(A(c+3d)+2B(c-3d))\cos(e+fx)}{3a^2f(\sin(e+fx)+1)}+\frac{dx(Ad+2B(c-d))}{a^2}+\frac{d^2(A-4B)\cos(e+fx)}{3a^2f}-\frac{(A-B)\cos(e+fx)}{3f(a\sin(e$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (d*(2*B*(c - d) + A*d)*x)/a^2 + ((A - 4*B)*d^2*Cos[e + f*x])/(3*a^2*f) - ((
c - d)*(2*B*(c - 3*d) + A*(c + 3*d))*Cos[e + f*x])/(3*a^2*f*(1 + Sin[e + f*
x])) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*f*(a + a*Sin[e + f*
x])^2)
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a+a\sin(e+fx))^2} + \frac{\int \frac{(c+d\sin(e+fx))(a(2B(}{}}{}$$

$$= -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a+a\sin(e+fx))^2} + \frac{\int \frac{ac(2B(c-d)+A(c+2d))}{}}{}$$

$$= \frac{(A-4B)d^2\cos(e+fx)}{3a^2f} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx}{3f(a+a\sin(e+fx))^2}$$

$$= \frac{d(2B(c-d)+Ad)x}{a^2} + \frac{(A-4B)d^2\cos(e+fx)}{3a^2f} - \frac{(A-B)\cos(e}{3f(a}$$

$$= \frac{d(2B(c-d)+Ad)x}{a^2} + \frac{(A-4B)d^2\cos(e+fx)}{3a^2f} - \frac{(c-d)(2B(c-}{3f\Big(}$$

Mathematica [B] time = 1.67, size = 338, normalized size = 2.56

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(6\cos\left(\frac{1}{2}(e+fx)\right)\left(Ad(4c+d(3e+3fx-4))+B\left(2c^2+2cd(3e+3fx-4)+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(A*d*(4*c + d*(-4 + 3*e + 3*f*x))
 + B*(2*c^2 + d^2*(5 - 6*e - 6*f*x) + 2*c*d*(-4 + 3*e + 3*f*x)))*Cos[(e + f
*x)/2] - (B*(8*c^2 + d^2*(41 - 12*e - 12*f*x) + 4*c*d*(-10 + 3*e + 3*f*x))
+ 2*A*(2*c^2 + 8*c*d + d^2*(-10 + 3*e + 3*f*x)))*Cos[(3*(e + f*x))/2] + 3*B
*d^2*Cos[(5*(e + f*x))/2] + 6*(2*A*c^2 + 2*B*c^2 + 4*A*c*d - 12*B*c*d - 6*A
*d^2 + 9*B*d^2 + 8*B*c*d*e + 4*A*d^2*e - 8*B*d^2*e + 8*B*c*d*f*x + 4*A*d^2*
f*x - 8*B*d^2*f*x - 2*d*(-2*B*c*(e + f*x) - A*d*(e + f*x) + 2*B*d*(1 + e +
f*x))*Cos[e + f*x] - B*d^2*Cos[2*(e + f*x)])*Sin[(e + f*x)/2]))/(12*a^2*f*(
1 + Sin[e + f*x])^2)
```

fricas [B] time = 0.46, size = 375, normalized size = 2.84

$$-\frac{3\,Bd^2\cos\left(fx+e\right)^3-(A-B)c^2+2\,(A-B)cd-(A-B)d^2+6\left(2\,Bcd+(A-2\,B)d^2\right)fx-\left((A+2\,B)c^2+2\,(2\,A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/3*(3*B*d^2*cos(f*x + e)^3 - (A - B)*c^2 + 2*(A - B)*c*d - (A - B)*d^2 +
6*(2*B*c*d + (A - 2*B)*d^2)*f*x - ((A + 2*B)*c^2 + 2*(2*A - 5*B)*c*d - (5*A
 - 11*B)*d^2 + 3*(2*B*c*d + (A - 2*B)*d^2)*f*x)*cos(f*x + e)^2 - ((2*A + B)
*c^2 + 2*(A - 4*B)*c*d - (4*A - 13*B)*d^2 - 3*(2*B*c*d + (A - 2*B)*d^2)*f*x
)*cos(f*x + e) - (3*B*d^2*cos(f*x + e)^2 - (A - B)*c^2 + 2*(A - B)*c*d - (A
 - B)*d^2 - 6*(2*B*c*d + (A - 2*B)*d^2)*f*x + ((A + 2*B)*c^2 + 2*(2*A - 5*B
)*c*d - (5*A - 14*B)*d^2 - 3*(2*B*c*d + (A - 2*B)*d^2)*f*x)*cos(f*x + e))*s
in(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*
cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

giac [B] time = 0.19, size = 277, normalized size = 2.10

$$\frac{\frac{3\left(2\,Bcd+Ad^2-2\,Bd^2\right)\left(fx+e\right)}{a^2}-\frac{6\,Bd^2}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^2}-\frac{2\left(3\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-6\,Bcd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,Ad^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+6\,Bd^2\tan\left(\frac{1}{2}\,fx+\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/3*(3*(2*B*c*d + A*d^2 - 2*B*d^2)*(f*x + e)/a^2 - 6*B*d^2/((tan(1/2*f*x +
1/2*e)^2 + 1)*a^2) - 2*(3*A*c^2*tan(1/2*f*x + 1/2*e)^2 - 6*B*c*d*tan(1/2*f*
x + 1/2*e)^2 - 3*A*d^2*tan(1/2*f*x + 1/2*e)^2 + 6*B*d^2*tan(1/2*f*x + 1/2*e
)^2 + 3*A*c^2*tan(1/2*f*x + 1/2*e) + 3*B*c^2*tan(1/2*f*x + 1/2*e) + 6*A*c*d
*tan(1/2*f*x + 1/2*e) - 18*B*c*d*tan(1/2*f*x + 1/2*e) - 9*A*d^2*tan(1/2*f*x
 + 1/2*e) + 15*B*d^2*tan(1/2*f*x + 1/2*e) + 2*A*c^2 + B*c^2 + 2*A*c*d - 8*B
*c*d - 4*A*d^2 + 7*B*d^2)/(a^2*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

maple [B] time = 0.41, size = 489, normalized size = 3.70

$$-\frac{2B\,d^2}{a^2 f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}+\frac{2A\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d^2}{a^2 f}+\frac{4dB\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c}{a^2 f}-\frac{4B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{a^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x)
```

```
[Out] -2/a^2/f*B*d^2/(1+tan(1/2*f*x+1/2*e)^2)+2/a^2/f*A*arctan(tan(1/2*f*x+1/2*e)
)*d^2+4/a^2/f*d*B*arctan(tan(1/2*f*x+1/2*e))*c-4/a^2/f*B*arctan(tan(1/2*f*x
+1/2*e))*d^2-2/a^2/f/(tan(1/2*f*x+1/2*e)+1)*A*c^2+2/a^2/f/(tan(1/2*f*x+1/2*
e)+1)*A*d^2+4/a^2/f/(tan(1/2*f*x+1/2*e)+1)*B*c*d-4/a^2/f/(tan(1/2*f*x+1/2*e
)+1)*B*d^2+2/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*A*c^2-4/a^2/f/(tan(1/2*f*x+1/2*
e)+1)^2*A*c*d+2/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*A*d^2-2/a^2/f/(tan(1/2*f*x+1
/2*e)+1)^2*B*c^2+4/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*B*c*d-2/a^2/f/(tan(1/2*f*
x+1/2*e)+1)^2*B*d^2-4/3/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*A*c^2+8/3/a^2/f/(tan
(1/2*f*x+1/2*e)+1)^3*A*c*d-4/3/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*A*d^2+4/3/a^2
/f/(tan(1/2*f*x+1/2*e)+1)^3*B*c^2-8/3/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*B*c*d+
4/3/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*B*d^2
```

maxima [B] time = 0.58, size = 831, normalized size = 6.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] -2/3*(2*B*d^2*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4
```

```
/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4
*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 2*
B*c*d*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*ar
ctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - A*d^2*((9*sin(f*x + e)/(cos(f*
x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e
) + 1))/a^2) + A*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*
a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3) + B*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*
x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 2*A*c*d*(3*sin(f*x + e)/(cos(f*x +
e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

mupad [B] time = 16.04, size = 365, normalized size = 2.77

$$\frac{2\,d\,\mathrm{atan}\left(\frac{2\,d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(A\,d+2\,B\,c-2\,B\,d\right)}{2\,A\,d^2-4\,B\,d^2+4\,B\,c\,d}\right)\,\left(A\,d+2\,B\,c-2\,B\,d\right)}{a^2\,f}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(2\,A\,c^2-6\,A\,d^2+2\,B\,c^2+12\,B\,d^2+4\,A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (2*d*atan((2*d*tan(e/2 + (f*x)/2)*(A*d + 2*B*c - 2*B*d))/(2*A*d^2 - 4*B*d^2
 + 4*B*c*d))*(A*d + 2*B*c - 2*B*d))/(a^2*f) - (tan(e/2 + (f*x)/2)^3*(2*A*c^
2 - 6*A*d^2 + 2*B*c^2 + 12*B*d^2 + 4*A*c*d - 12*B*c*d) + tan(e/2 + (f*x)/2)
^2*((10*A*c^2)/3 - (14*A*d^2)/3 + (2*B*c^2)/3 + (44*B*d^2)/3 + (4*A*c*d)/3
- (28*B*c*d)/3) + (4*A*c^2)/3 - (8*A*d^2)/3 + (2*B*c^2)/3 + (20*B*d^2)/3 +
tan(e/2 + (f*x)/2)^4*(2*A*c^2 - 2*A*d^2 + 4*B*d^2 - 4*B*c*d) + tan(e/2 + (f
*x)/2)*(2*A*c^2 - 6*A*d^2 + 2*B*c^2 + 16*B*d^2 + 4*A*c*d - 12*B*c*d) + (4*A
*c*d)/3 - (16*B*c*d)/3)/(f*(4*a^2*tan(e/2 + (f*x)/2)^2 + 4*a^2*tan(e/2 + (f
*x)/2)^3 + 3*a^2*tan(e/2 + (f*x)/2)^4 + a^2*tan(e/2 + (f*x)/2)^5 + a^2 + 3*
a^2*tan(e/2 + (f*x)/2)))
```

sympy [A] time = 17.79, size = 5358, normalized size = 40.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*A*c**2*tan(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*
a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(
e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*A*c**2*tan(e/2
+ f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 +
12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*ta
n(e/2 + f*x/2) + 3*a**2*f) - 10*A*c**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/
2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**
3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) -
 6*A*c**2*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2
 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**
2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A*c**2/(3*a**2*f*tan(e/2 + f*
x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12
*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 12*A*
c*d*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 +
f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 +
 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A*c*d*tan(e/2 + f*x/2)**2/(3*a**
2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2
+ f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3
*a**2*f) - 12*A*c*d*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2
*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A*c*d/(3*a**2*f*tan
(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2
)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f
) + 3*A*d**2*f*x*tan(e/2 + f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2
*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*A*d**2*f*x*tan(e/2
+ f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 +
12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*ta
n(e/2 + f*x/2) + 3*a**2*f) + 12*A*d**2*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*ta
n(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/
2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*
f) + 12*A*d**2*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a*
*2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/
2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*A*d**2*f*x*tan(e/
2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 1
2*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan
(e/2 + f*x/2) + 3*a**2*f) + 3*A*d**2*f*x/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*
a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(
e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 6*A*d**2*tan(e/2
+ f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 +
12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*ta
```

```
n(e/2 + f*x/2) + 3*a**2*f) + 18*A*d**2*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/
2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**
3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) +
 14*A*d**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan
(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/
2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 18*A*d**2*tan(e/2 + f*x/2)/
(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*ta
n(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/
2) + 3*a**2*f) + 8*A*d**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2
+ f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2
 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*B*c**2*tan(e/2 + f*x/2)**3/(3*
a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e
/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2)
+ 3*a**2*f) - 2*B*c**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 +
9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*ta
n(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*B*c**2*tan(e/
2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 1
2*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan
(e/2 + f*x/2) + 3*a**2*f) - 2*B*c**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2
*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 6*B*c*d*f*x*tan(e/2 +
 f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 1
2*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan
(e/2 + f*x/2) + 3*a**2*f) + 18*B*c*d*f*x*tan(e/2 + f*x/2)**4/(3*a**2*f*tan(
e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)
**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f)
 + 24*B*c*d*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*
f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 +
 f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 24*B*c*d*f*x*tan(e/2 +
 f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 1
2*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan
(e/2 + f*x/2) + 3*a**2*f) + 18*B*c*d*f*x*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2
 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3
 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) +
6*B*c*d*f*x/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 +
12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*ta
n(e/2 + f*x/2) + 3*a**2*f) + 12*B*c*d*tan(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2
 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3
 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) +
36*B*c*d*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e
/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)
**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 28*B*c*d*tan(e/2 + f*x/2)**2/
(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*ta
n(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/
2) + 3*a**2*f) + 36*B*c*d*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 +
```

```
9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*ta
n(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 16*B*c*d/(3*a**
2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2
+ f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3
*a**2*f) - 6*B*d**2*f*x*tan(e/2 + f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 +
 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*t
an(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 18*B*d**2*f*x*
tan(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/
2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a
**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 24*B*d**2*f*x*tan(e/2 + f*x/2)**3/(3*a
**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/
2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) +
 3*a**2*f) - 24*B*d**2*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**
5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*
f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 18*B*d**2*f
*x*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/
2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a
**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*B*d**2*f*x/(3*a**2*f*tan(e/2 + f*x/2
)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a*
*2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 12*B*d**
2*tan(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*
x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9
*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 36*B*d**2*tan(e/2 + f*x/2)**3/(3*a**
2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2
+ f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3
*a**2*f) - 44*B*d**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*
a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(
e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 48*B*d**2*tan(e/2
 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12
*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(
e/2 + f*x/2) + 3*a**2*f) - 20*B*d**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2
*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f), Ne(f, 0)), (x*(A + B*s
in(e))*(c + d*sin(e))**2/(a*sin(e) + a)**2, True))
```

3.274 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=85

$$-\frac{(Ac+2Ad+2Bc-5Bd)\cos(e+fx)}{3a^2f(\sin(e+fx)+1)}+\frac{Bdx}{a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

```
[Out] B*d*x/a^2-1/3*(A*c+2*A*d+2*B*c-5*B*d)*cos(f*x+e)/a^2/f/(1+sin(f*x+e))-1/3*(
A-B)*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^2
```

Rubi [A] time = 0.21, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2968, 3019, 2735, 2648}

$$-\frac{(Ac+2Ad+2Bc-5Bd)\cos(e+fx)}{3a^2f(\sin(e+fx)+1)}+\frac{Bdx}{a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (B*d*x)/a^2 - ((A*c + 2*B*c + 2*A*d - 5*B*d)*Cos[e + f*x])/(3*a^2*f*(1 + Si
n[e + f*x])) - ((A - B)*(c - d)*Cos[e + f*x])/(3*f*(a + a*Sin[e + f*x])^2)
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \int \frac{Ac+(Bc+Ad)\sin(e+fx)+Bd\sin^2(e+fx)}{(a+a\sin(e+fx))^2}\,dx$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{3f(a+a\sin(e+fx))^2} - \frac{\int \frac{-a(2B(c-d)+A(c+2d))-3aBd\sin(e+fx)}{a+a\sin(e+fx)}\,d}{3a^2}$$

$$= \frac{Bdx}{a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{3f(a+a\sin(e+fx))^2} + \frac{(Ac+2Bc+2Ad-5Bd)\int}{3a}$$

$$= \frac{Bdx}{a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{3f(a+a\sin(e+fx))^2} - \frac{(Ac+2Bc+2Ad-5Bd)\,\mathrm{co}}{3f\left(a^2+a^2\sin(e+f\right.}$$

Mathematica [B] time = 0.34, size = 180, normalized size = 2.12

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)(c-d)\sin\left(\frac{1}{2}(e+fx)\right)+2(Ac+2Ad+2Bc-5Bd)\sin\left(\frac{1}{2}(e+fx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^
2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
- (A - B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(A*c + 2*B*c +
2*A*d - 5*B*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 3
*B*d*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3))/(3*a^2*f*(1 + Sin[
e + f*x])^2)
```

fricas [B] time = 0.43, size = 208, normalized size = 2.45

$$-\frac{6\,Bdfx-\left(3\,Bdfx+(A+2\,B)c+(2\,A-5\,B)d\right)\cos\left(fx+e\right)^2-(A-B)c+(A-B)d+\left(3\,Bdfx-(2\,A+B)c-\right.}{3\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] -1/3*(6*B*d*f*x - (3*B*d*f*x + (A + 2*B)*c + (2*A - 5*B)*d)*cos(f*x + e)^2
- (A - B)*c + (A - B)*d + (3*B*d*f*x - (2*A + B)*c - (A - 4*B)*d)*cos(f*x +
 e) + (6*B*d*f*x + (A - B)*c - (A - B)*d + (3*B*d*f*x - (A + 2*B)*c - (2*A
- 5*B)*d)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x
 + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

giac [A] time = 0.15, size = 141, normalized size = 1.66

$$\frac{\frac{3\left(fx+e\right)Bd}{a^2} - \frac{2\left(3\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,Bd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3\,Ad\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-9\,Bd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,A}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] 1/3*(3*(f*x + e)*B*d/a^2 - 2*(3*A*c*tan(1/2*f*x + 1/2*e)^2 - 3*B*d*tan(1/2*
f*x + 1/2*e)^2 + 3*A*c*tan(1/2*f*x + 1/2*e) + 3*B*c*tan(1/2*f*x + 1/2*e) +
3*A*d*tan(1/2*f*x + 1/2*e) - 9*B*d*tan(1/2*f*x + 1/2*e) + 2*A*c + B*c + A*d
 - 4*B*d)/(a^2*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

maple [B] time = 0.43, size = 252, normalized size = 2.96

$$\frac{2Bd\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^2f}-\frac{2Ac}{a^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{2Bd}{a^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{2Ac}{a^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{}{a^2f\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 2/a^2/f*B*d*arctan(tan(1/2*f*x+1/2*e))-2/a^2/f/(tan(1/2*f*x+1/2*e)+1)*A*c+2
/a^2/f/(tan(1/2*f*x+1/2*e)+1)*B*d+2/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*A*c-2/a^
2/f/(tan(1/2*f*x+1/2*e)+1)^2*A*d-2/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*B*c+2/a^2
/f/(tan(1/2*f*x+1/2*e)+1)^2*B*d-4/3/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*A*c+4/3/
a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*A*d+4/3/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*B*c-4
/3/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*B*d
```

maxima [B] time = 0.50, size = 454, normalized size = 5.34

$$\frac{2\left(Bd\left(\frac{\frac{9\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+4}{a^{2}+\frac{3a^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3a^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a^{2}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}}+\frac{3\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{a^{2}}\right)-\frac{Ac\left(\frac{3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+2\right)}{a^{2}+\frac{3a^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3a^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a^{2}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}}-\cdots}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] 2/3*(B*d*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3
*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - A*c*(3*sin(f*x + e)/(cos(f*
x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - B*c*(3*sin(f*x + e)/(cos(f*x + e)
+ 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - A*d*(3
*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3))/f
```

mupad [B] time = 13.74, size = 94, normalized size = 1.11

$$\frac{B\,d\,x}{a^2}-\frac{\left(2\,A\,c-2\,B\,d\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+\left(2\,A\,c+2\,A\,d+2\,B\,c-6\,B\,d\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\frac{4\,A\,c}{3}+\frac{2\,A\,d}{3}+\frac{2\,B\,c}{3}-\frac{8\,B\,d}{3}}{a^2\,f\,{\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^2,x)
```

```
[Out] (B*d*x)/a^2 - ((4*A*c)/3 + (2*A*d)/3 + (2*B*c)/3 - (8*B*d)/3 + tan(e/2 + (f
*x)/2)*(2*A*c + 2*A*d + 2*B*c - 6*B*d) + tan(e/2 + (f*x)/2)^2*(2*A*c - 2*B*
d))/(a^2*f*(tan(e/2 + (f*x)/2) + 1)^3)
```

sympy [A] time = 8.34, size = 1062, normalized size = 12.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*A*c*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**
2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*A*c*tan
(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2
+ 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A*c/(3*a**2*f*tan(e/2 + f*x/2)*
*3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) -
 6*A*d*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 +
f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*A*d/(3*a**2*f*tan(e/2
 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3
*a**2*f) - 6*B*c*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*
tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*B*c/(3*a**2
*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 +
f*x/2) + 3*a**2*f) + 3*B*d*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/
2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f
) + 9*B*d*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*
tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*B*d*f*x*tan
(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2
+ 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 3*B*d*f*x/(3*a**2*f*tan(e/2 + f*x
/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*
f) + 6*B*d*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan
(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 18*B*d*tan(e/2 +
 f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a*
*2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 8*B*d/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9
*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f), Ne(f,
0)), (x*(A + B*sin(e))*(c + d*sin(e))/(a*sin(e) + a)**2, True))
```

3.275 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=65

$$-\frac{(A+2B)\cos(e+fx)}{3f\left(a^2\sin(e+fx)+a^2\right)}-\frac{(A-B)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

```
[Out] -1/3*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^2-1/3*(A+2*B)*cos(f*x+e)/f/(a^2+a^
2*sin(f*x+e))
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2750, 2648}

$$-\frac{(A+2B)\cos(e+fx)}{3f\left(a^2\sin(e+fx)+a^2\right)}-\frac{(A-B)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -((A - B)*Cos[e + f*x])/(3*f*(a + a*Sin[e + f*x])^2) - ((A + 2*B)*Cos[e + f
*x])/(3*f*(a^2 + a^2*Sin[e + f*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2}\, dx = -\frac{(A - B)\cos(e + fx)}{3f(a + a\sin(e + fx))^2} + \frac{(A + 2B)\int \frac{1}{a + a\sin(e + fx)}\, dx}{3a}$$
$$= -\frac{(A - B)\cos(e + fx)}{3f(a + a\sin(e + fx))^2} - \frac{(A + 2B)\cos(e + fx)}{3f\left(a^2 + a^2\sin(e + fx)\right)}$$

Mathematica [**A**] time = 0.06, size = 43, normalized size = 0.66

$$-\frac{\cos(e + fx)((A + 2B)\sin(e + fx) + 2A + B)}{3a^2 f(\sin(e + fx) + 1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -1/3*(Cos[e + f*x]*(2*A + B + (A + 2*B)*Sin[e + f*x]))/(a^2*f*(1 + Sin[e +
f*x])^2)
```

fricas [**A**] time = 0.42, size = 117, normalized size = 1.80

$$\frac{(A + 2B)\cos\left(fx + e\right)^2 + (2A + B)\cos\left(fx + e\right) + \left((A + 2B)\cos\left(fx + e\right) - A + B\right)\sin\left(fx + e\right) + A - B}{3\left(a^2 f\cos\left(fx + e\right)^2 - a^2 f\cos\left(fx + e\right) - 2a^2 f - \left(a^2 f\cos\left(fx + e\right) + 2a^2 f\right)\sin\left(fx + e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*((A + 2*B)*cos(f*x + e)^2 + (2*A + B)*cos(f*x + e) + ((A + 2*B)*cos(f*x
 + e) - A + B)*sin(f*x + e) + A - B)/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x
+ e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

giac [**A**] time = 0.19, size = 68, normalized size = 1.05

$$-\frac{2\left(3A\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 3A\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 3B\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 2A + B\right)}{3a^2 f\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*A*tan(1/2*f*x + 1/2*e)^2 + 3*A*tan(1/2*f*x + 1/2*e) + 3*B*tan(1/2*f
*x + 1/2*e) + 2*A + B)/(a^2*f*(tan(1/2*f*x + 1/2*e) + 1)^3)
```

maple [A] time = 0.33, size = 70, normalized size = 1.08

$$\frac{-\frac{2(2A-2B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2A+2B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}}{f\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 2/f/a^2*(-1/3*(2*A-2*B)/(tan(1/2*f*x+1/2*e)+1)^3-A/(tan(1/2*f*x+1/2*e)+1)-1
/2*(-2*A+2*B)/(tan(1/2*f*x+1/2*e)+1)^2)
```

maxima [B] time = 0.35, size = 214, normalized size = 3.29

$$-\frac{2\left(\frac{A\left(\frac{3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+2\right)}{a^{2}+\frac{3a^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3a^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a^{2}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}}+\frac{B\left(\frac{3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{a^{2}+\frac{3a^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3a^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a^{2}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(A*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + B*(
3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
 e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3))/f
```

mupad [B] time = 13.39, size = 97, normalized size = 1.49

$$-\frac{2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{5\,A}{2}+\frac{B}{2}-\frac{A\,\cos\left(e+f\,x\right)}{2}+\frac{B\,\cos\left(e+f\,x\right)}{2}+\frac{3\,A\,\sin\left(e+f\,x\right)}{2}+\frac{3\,B\,\sin\left(e+f\,x\right)}{2}\right)}{3\,a^2\,f\,\left(\frac{3\,\sqrt{2}\,\cos\left(\frac{e}{2}-\frac{\pi}{4}+\frac{f\,x}{2}\right)}{2}-\frac{\sqrt{2}\,\cos\left(\frac{3\,e}{2}+\frac{\pi}{4}+\frac{3\,f\,x}{2}\right)}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^2,x)
```

```
[Out] -(2*cos(e/2 + (f*x)/2)*((5*A)/2 + B/2 - (A*cos(e + f*x))/2 + (B*cos(e + f*x
))/2 + (3*A*sin(e + f*x))/2 + (3*B*sin(e + f*x))/2))/(3*a^2*f*((3*2^(1/2)*c
os(e/2 - pi/4 + (f*x)/2))/2 - (2^(1/2)*cos((3*e)/2 + pi/4 + (3*f*x)/2))/2))
```

sympy [A] time = 4.58, size = 372, normalized size = 5.72

$$\begin{cases} -\frac{6A\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} - \frac{6A\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} - \frac{}{3a^2f\tan^3\left(\right.} \\ \frac{x(A+B\sin(e))}{(a\sin(e)+a)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*A*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*
f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*A*tan(e/2
 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*
a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9
*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*B*t
an(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**
2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*B/(3*a**2*f*tan(e/2 + f*x/2)*
*3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f),
Ne(f, 0)), (x*(A + B*sin(e))/(a*sin(e) + a)**2, True))
```

3.276 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=152

$$-\frac{2d(Bc-Ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^2\sqrt{c^2-d^2}}-\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{3a^2f(c-d)^2(\sin(e+fx)+1)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

```
[Out] -1/3*(A*(c-4*d)+B*(2*c+d))*cos(f*x+e)/a^2/(c-d)^2/f/(1+sin(f*x+e))-1/3*(A-B
)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^2-2*d*(-A*d+B*c)*arctan((d+c*tan(1/2*
f*x+1/2*e))/(c^2-d^2)^(1/2))/a^2/(c-d)^2/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2978, 12, 2660, 618, 204}

$$-\frac{2d(Bc-Ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^2\sqrt{c^2-d^2}}-\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{3a^2f(c-d)^2(\sin(e+fx)+1)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])),x]
```

```
[Out] (-2*d*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^2*(c
 - d)^2*Sqrt[c^2 - d^2]*f) - ((A*(c - 4*d) + B*(2*c + d))*Cos[e + f*x])/(3*
a^2*(c - d)^2*f*(1 + Sin[e + f*x])) - ((A - B)*Cos[e + f*x])/(3*(c - d)*f*(
a + a*Sin[e + f*x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx = -\frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2} - \frac{\int \frac{-a(2Bc+A(c-3d))-a(A-B)d\sin(e+fx}{(a+a\sin(e+fx))(c+d\sin(e+fx))}}{3a^2(c-d)}$$

$$= -\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{(A-B)\cos(e+fx}{3(c-d)f(a+a\sin(e+}$$

$$= -\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{(A-B)\cos(e+fx}{3(c-d)f(a+a\sin(e+}$$

$$= -\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{(A-B)\cos(e+fx}{3(c-d)f(a+a\sin(e+}$$

$$= -\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{(A-B)\cos(e+fx}{3(c-d)f(a+a\sin(e+}$$

$$= -\frac{2d(Bc-Ad)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^2\sqrt{c^2-d^2}\,f} - \frac{(A(c-4d)+B(2c+d))}{3a^2(c-d)^2f(1+\sin(}$$

Mathematica [A] time = 0.63, size = 229, normalized size = 1.51

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{6d(Ad-Bc)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + 2(A-B)(c-d)\sin\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
+ (-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(A*(c - 4*d) +
 B*(2*c + d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (6
*d*(-(B*c) + A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^3)/Sqrt[c^2 - d^2]))/(3*a^2*(c - d)^2*f*(1 +
Sin[e + f*x])^2)
```

fricas [B] time = 0.50, size = 1285, normalized size = 8.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] [1/6*(2*(A - B)*c^3 - 2*(A - B)*c^2*d - 2*(A - B)*c*d^2 + 2*(A - B)*d^3 + 2
*((A + 2*B)*c^3 - (4*A - B)*c^2*d - (A + 2*B)*c*d^2 + (4*A - B)*d^3)*cos(f*
x + e)^2 - 3*(2*B*c*d - 2*A*d^2 - (B*c*d - A*d^2)*cos(f*x + e)^2 + (B*c*d -
 A*d^2)*cos(f*x + e) + (2*B*c*d - 2*A*d^2 + (B*c*d - A*d^2)*cos(f*x + e))*s
in(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin
(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sq
rt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*
((2*A + B)*c^3 - (5*A - 2*B)*c^2*d - (2*A + B)*c*d^2 + (5*A - 2*B)*d^3)*cos
(f*x + e) - 2*((A - B)*c^3 - (A - B)*c^2*d - (A - B)*c*d^2 + (A - B)*d^3 -
((A + 2*B)*c^3 - (4*A - B)*c^2*d - (A + 2*B)*c*d^2 + (4*A - B)*d^3)*cos(f*x
 + e))*sin(f*x + e))/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos
(f*x + e)^2 - (a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e
) - 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f - ((a^2*c^4 - 2*a^2
*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e) + 2*(a^2*c^4 - 2*a^2*c^3*d +
 2*a^2*c*d^3 - a^2*d^4)*f)*sin(f*x + e)), 1/3*((A - B)*c^3 - (A - B)*c^2*d
- (A - B)*c*d^2 + (A - B)*d^3 + ((A + 2*B)*c^3 - (4*A - B)*c^2*d - (A + 2*B
)*c*d^2 + (4*A - B)*d^3)*cos(f*x + e)^2 - 3*(2*B*c*d - 2*A*d^2 - (B*c*d - A
*d^2)*cos(f*x + e)^2 + (B*c*d - A*d^2)*cos(f*x + e) + (2*B*c*d - 2*A*d^2 +
(B*c*d - A*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(
f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + ((2*A + B)*c^3 - (5*A - 2*B
)*c^2*d - (2*A + B)*c*d^2 + (5*A - 2*B)*d^3)*cos(f*x + e) - ((A - B)*c^3 -
(A - B)*c^2*d - (A - B)*c*d^2 + (A - B)*d^3 - ((A + 2*B)*c^3 - (4*A - B)*c^
2*d - (A + 2*B)*c*d^2 + (4*A - B)*d^3)*cos(f*x + e))*sin(f*x + e))/((a^2*c^
4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e)^2 - (a^2*c^4 - 2*a^
2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e) - 2*(a^2*c^4 - 2*a^2*c^3*d
+ 2*a^2*c*d^3 - a^2*d^4)*f - ((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^
4)*f*cos(f*x + e) + 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f)*si
n(f*x + e))]
```

giac [A] time = 0.22, size = 259, normalized size = 1.70

$$-\frac{2\left(\frac{3\left(Bcd-Ad^{2}\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^{2}-d^{2}}}\right)\right)}{\left(a^{2}c^{2}-2a^{2}cd+a^{2}d^{2}\right)\sqrt{c^{2}-d^{2}}}+\frac{3Ac\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-6Ad\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+3Bd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+3Ac\tan\left(\frac{1}{2}f\cdots\right.}{\left(a^{2}c^{2}-2a^{2}cd+\cdots\right.}\right.}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -2/3*(3*(B*c*d - A*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((
c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^2*c^2 - 2*a^2*c*d + a^2*d
^2)*sqrt(c^2 - d^2)) + (3*A*c*tan(1/2*f*x + 1/2*e)^2 - 6*A*d*tan(1/2*f*x +
1/2*e)^2 + 3*B*d*tan(1/2*f*x + 1/2*e)^2 + 3*A*c*tan(1/2*f*x + 1/2*e) + 3*B*
c*tan(1/2*f*x + 1/2*e) - 9*A*d*tan(1/2*f*x + 1/2*e) + 3*B*d*tan(1/2*f*x + 1
/2*e) + 2*A*c + B*c - 5*A*d + 2*B*d)/((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*(tan(
1/2*f*x + 1/2*e) + 1)^3))/f
```

maple [B] time = 0.56, size = 327, normalized size = 2.15

$$\frac{2d^2 \arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)A}{f\,a^2\,(c-d)^2\sqrt{c^2-d^2}}-\frac{2d \arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)Bc}{f\,a^2\,(c-d)^2\sqrt{c^2-d^2}}+\frac{2A}{f\,a^2\,(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2}{f\,a^2\,(c-d)\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/f/a^2*d^2/(c-d)^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)
/(c^2-d^2)^(1/2))*A-2/f/a^2*d/(c-d)^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1
/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c+2/f/a^2/(c-d)/(tan(1/2*f*x+1/2*e)+1
)^2*A-2/f/a^2/(c-d)/(tan(1/2*f*x+1/2*e)+1)^2*B-4/3/f/a^2/(c-d)/(tan(1/2*f*x
+1/2*e)+1)^3*A+4/3/f/a^2/(c-d)/(tan(1/2*f*x+1/2*e)+1)^3*B-2/f/a^2/(c-d)^2/(
tan(1/2*f*x+1/2*e)+1)*A*c+4/f/a^2/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)*A*d-2/f/a^
2/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)*B*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 14.78, size = 302, normalized size = 1.99

$$\frac{2\,d\,\mathrm{atan}\left(\frac{\frac{d\,(A\,d-B\,c)\,\left(2\,a^2\,c^2\,d-4\,a^2\,c\,d^2+2\,a^2\,d^3\right)}{a^2\,\sqrt{c+d}\,(c-d)^{5/2}}+\frac{2\,c\,d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(A\,d-B\,c)\,\left(a^2\,c^2-2\,a^2\,c\,d+a^2\,d^2\right)}{a^2\,\sqrt{c+d}\,(c-d)^{5/2}}}{2\,A\,d^2-2\,B\,c\,d}\right)\,(A\,d-B\,c)}{a^2\,f\,\sqrt{c+d}\,(c-d)^{5/2}}-\frac{\frac{2\,(2\,A\,c-5\,A\,d+B\,c+2\,B\,d)}{3\,(c-d)^2}+\frac{2\tan\left(\frac{e}{2}+\cdots\right.}{}}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+3\,a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))),x)
```

```
[Out] (2*d*atan(((d*(A*d - B*c)*(2*a^2*d^3 - 4*a^2*c*d^2 + 2*a^2*c^2*d))/(a^2*(c
+ d)^(1/2)*(c - d)^(5/2)) + (2*c*d*tan(e/2 + (f*x)/2)*(A*d - B*c)*(a^2*c^2
+ a^2*d^2 - 2*a^2*c*d))/(a^2*(c + d)^(1/2)*(c - d)^(5/2)))/(2*A*d^2 - 2*B*c
*d))*(A*d - B*c))/(a^2*f*(c + d)^(1/2)*(c - d)^(5/2)) - ((2*(2*A*c - 5*A*d
+ B*c + 2*B*d))/(3*(c - d)^2) + (2*tan(e/2 + (f*x)/2)*(A*c - 3*A*d + B*c +
B*d))/(c - d)^2 + (2*tan(e/2 + (f*x)/2)^2*(A*c - 2*A*d + B*d))/(c - d)^2)/(
f*(3*a^2*tan(e/2 + (f*x)/2)^2 + a^2*tan(e/2 + (f*x)/2)^3 + a^2 + 3*a^2*tan(
e/2 + (f*x)/2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.277 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=275

$$\frac{2d\left(Ad(3c+2d)-B\left(2c^2+2cd+d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^3(c+d)\sqrt{c^2-d^2}}-\frac{d\left(A\left(c^2-6cd-10d^2\right)+B\left(2c^2+9cd+4d^2\right)\right)\text{co}}{3a^2f(c-d)^3(c+d)(c+d\sin(e+fx))}$$

```
[Out] -1/3*d*(A*(c^2-6*c*d-10*d^2)+B*(2*c^2+9*c*d+4*d^2))*cos(f*x+e)/a^2/(c-d)^3/
(c+d)/f/(c+d*sin(f*x+e))-1/3*(A*c-6*A*d+2*B*c+3*B*d)*cos(f*x+e)/a^2/(c-d)^2
/f/(1+sin(f*x+e))/(c+d*sin(f*x+e))-1/3*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*
x+e))^2/(c+d*sin(f*x+e))+2*d*(A*d*(3*c+2*d)-B*(2*c^2+2*c*d+d^2))*arctan((d+
c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^2/(c-d)^3/(c+d)/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2978, 2754, 12, 2660, 618, 204}

$$\frac{2d\left(Ad(3c+2d)-B\left(2c^2+2cd+d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^3(c+d)\sqrt{c^2-d^2}}-\frac{d\left(A\left(c^2-6cd-10d^2\right)+B\left(2c^2+9cd+4d^2\right)\right)\text{co}}{3a^2f(c-d)^3(c+d)(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (2*d*(A*d*(3*c + 2*d) - B*(2*c^2 + 2*c*d + d^2))*ArcTan[(d + c*Tan[(e + f*x
)/2])/Sqrt[c^2 - d^2]])/(a^2*(c - d)^3*(c + d)*Sqrt[c^2 - d^2]*f) - (d*(A*(
c^2 - 6*c*d - 10*d^2) + B*(2*c^2 + 9*c*d + 4*d^2))*Cos[e + f*x])/(3*a^2*(c
- d)^3*(c + d)*f*(c + d*Sin[e + f*x])) - ((A*c + 2*B*c - 6*A*d + 3*B*d)*Cos
[e + f*x])/(3*a^2*(c - d)^2*f*(1 + Sin[e + f*x])*(c + d*Sin[e + f*x])) - ((
A - B)*Cos[e + f*x])/(3*(c - d)*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x
]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx = -\frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))} - \frac{\int \frac{-a(A(c-4d)}{(a+a\,s}}{}$$

$$= -\frac{(Ac+2Bc-6Ad+3Bd)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))} - \frac{}{3(c-d)f(}$$

$$= -\frac{d\left(A\left(c^2-6cd-10d^2\right)+B\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))} -$$

$$= -\frac{d\left(A\left(c^2-6cd-10d^2\right)+B\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))} -$$

$$= -\frac{d\left(A\left(c^2-6cd-10d^2\right)+B\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))} -$$

$$= -\frac{d\left(A\left(c^2-6cd-10d^2\right)+B\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))} -$$

$$= \frac{2d\left(Ad(3c+2d)-B\left(2c^2+2cd+d^2\right)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^3(c+d)\sqrt{c^2-d^2}\,f} -$$

Mathematica [A] time = 2.83, size = 313, normalized size = 1.14

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-\frac{6d\left(B\left(2c^2+2cd+d^2\right)-Ad(3c+2d)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}} + \frac{3d^2}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])
^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
+ (-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(A*(c - 7*d) +
 2*B*(c + 2*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 -
(6*d*(-(A*d*(3*c + 2*d)) + B*(2*c^2 + 2*c*d + d^2))*ArcTan[(d + c*Tan[(e +
f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/((c + d)
```

```
*Sqrt[c^2 - d^2]) + (3*d^2*(-(B*c) + A*d)*Cos[e + f*x]*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^3)/((c + d)*(c + d*Sin[e + f*x]))))/(3*a^2*(c - d)^3*f*(1
 + Sin[e + f*x])^2)
```

fricas [B] time = 0.59, size = 3123, normalized size = 11.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] [1/6*(2*(A - B)*c^5 - 2*(A - B)*c^4*d - 4*(A - B)*c^3*d^2 + 4*(A - B)*c^2*d
^3 + 2*(A - B)*c*d^4 - 2*(A - B)*d^5 - 2*((A + 2*B)*c^4*d - 3*(2*A - 3*B)*c
^3*d^2 - (11*A - 2*B)*c^2*d^3 + 3*(2*A - 3*B)*c*d^4 + 2*(5*A - 2*B)*d^5)*co
s(f*x + e)^3 + 2*((A + 2*B)*c^5 - 5*(A - B)*c^4*d - (8*A - 5*B)*c^3*d^2 + (
A - 4*B)*c^2*d^3 + 7*(A - B)*c*d^4 + (4*A - B)*d^5)*cos(f*x + e)^2 - 3*(4*B
*c^3*d - 2*(3*A - 4*B)*c^2*d^2 - 2*(5*A - 3*B)*c*d^3 - 2*(2*A - B)*d^4 - (2
*B*c^2*d^2 - (3*A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)^3 - (2*B*c^3*d
 - 3*(A - 2*B)*c^2*d^2 - (8*A - 5*B)*c*d^3 - 2*(2*A - B)*d^4)*cos(f*x + e)^
2 + (2*B*c^3*d - (3*A - 4*B)*c^2*d^2 - (5*A - 3*B)*c*d^3 - (2*A - B)*d^4)*c
os(f*x + e) + (4*B*c^3*d - 2*(3*A - 4*B)*c^2*d^2 - 2*(5*A - 3*B)*c*d^3 - 2*
(2*A - B)*d^4 - (2*B*c^2*d^2 - (3*A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x +
 e)^2 + (2*B*c^3*d - (3*A - 4*B)*c^2*d^2 - (5*A - 3*B)*c*d^3 - (2*A - B)*d^
4)*cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x
+ e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) +
d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e)
- c^2 - d^2)) + 2*((2*A + B)*c^5 - (5*A - 8*B)*c^4*d - 16*(A - B)*c^3*d^2 -
 4*(2*A + B)*c^2*d^3 + (14*A - 17*B)*c*d^4 + (13*A - 4*B)*d^5)*cos(f*x + e)
 - 2*((A - B)*c^5 - (A - B)*c^4*d - 2*(A - B)*c^3*d^2 + 2*(A - B)*c^2*d^3 +
 (A - B)*c*d^4 - (A - B)*d^5 - ((A + 2*B)*c^4*d - 3*(2*A - 3*B)*c^3*d^2 - (
11*A - 2*B)*c^2*d^3 + 3*(2*A - 3*B)*c*d^4 + 2*(5*A - 2*B)*d^5)*cos(f*x + e)
^2 - ((A + 2*B)*c^5 - (4*A - 7*B)*c^4*d - 14*(A - B)*c^3*d^2 - 2*(5*A + B)*
c^2*d^3 + (13*A - 16*B)*c*d^4 + (14*A - 5*B)*d^5)*cos(f*x + e))*sin(f*x + e
))/((a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a^2*c^3*d^4 - a^2*c^2*d^5
- 2*a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e)^3 + (a^2*c^7 - 5*a^2*c^5*d^2 + 2*a^
2*c^4*d^3 + 7*a^2*c^3*d^4 - 4*a^2*c^2*d^5 - 3*a^2*c*d^6 + 2*a^2*d^7)*f*cos(
f*x + e)^2 - (a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c
^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e) - 2*(a^2*c^7 -
 a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5
- a^2*c*d^6 + a^2*d^7)*f + ((a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a^
2*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e)^2 - (a^2*c^
7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d
^5 - a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e) - 2*(a^2*c^7 - a^2*c^6*d - 3*a^2*c
^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^
```

```
7)*f)*sin(f*x + e)), 1/3*((A - B)*c^5 - (A - B)*c^4*d - 2*(A - B)*c^3*d^2 +
 2*(A - B)*c^2*d^3 + (A - B)*c*d^4 - (A - B)*d^5 - ((A + 2*B)*c^4*d - 3*(2*
A - 3*B)*c^3*d^2 - (11*A - 2*B)*c^2*d^3 + 3*(2*A - 3*B)*c*d^4 + 2*(5*A - 2*
B)*d^5)*cos(f*x + e)^3 + ((A + 2*B)*c^5 - 5*(A - B)*c^4*d - (8*A - 5*B)*c^3
*d^2 + (A - 4*B)*c^2*d^3 + 7*(A - B)*c*d^4 + (4*A - B)*d^5)*cos(f*x + e)^2
- 3*(4*B*c^3*d - 2*(3*A - 4*B)*c^2*d^2 - 2*(5*A - 3*B)*c*d^3 - 2*(2*A - B)*
d^4 - (2*B*c^2*d^2 - (3*A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)^3 - (2
*B*c^3*d - 3*(A - 2*B)*c^2*d^2 - (8*A - 5*B)*c*d^3 - 2*(2*A - B)*d^4)*cos(f
*x + e)^2 + (2*B*c^3*d - (3*A - 4*B)*c^2*d^2 - (5*A - 3*B)*c*d^3 - (2*A - B
)*d^4)*cos(f*x + e) + (4*B*c^3*d - 2*(3*A - 4*B)*c^2*d^2 - 2*(5*A - 3*B)*c*
d^3 - 2*(2*A - B)*d^4 - (2*B*c^2*d^2 - (3*A - 2*B)*c*d^3 - (2*A - B)*d^4)*c
os(f*x + e)^2 + (2*B*c^3*d - (3*A - 4*B)*c^2*d^2 - (5*A - 3*B)*c*d^3 - (2*A
 - B)*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x +
 e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + ((2*A + B)*c^5 - (5*A - 8*B)*c^4
*d - 16*(A - B)*c^3*d^2 - 4*(2*A + B)*c^2*d^3 + (14*A - 17*B)*c*d^4 + (13*A
 - 4*B)*d^5)*cos(f*x + e) - ((A - B)*c^5 - (A - B)*c^4*d - 2*(A - B)*c^3*d^
2 + 2*(A - B)*c^2*d^3 + (A - B)*c*d^4 - (A - B)*d^5 - ((A + 2*B)*c^4*d - 3*
(2*A - 3*B)*c^3*d^2 - (11*A - 2*B)*c^2*d^3 + 3*(2*A - 3*B)*c*d^4 + 2*(5*A -
 2*B)*d^5)*cos(f*x + e)^2 - ((A + 2*B)*c^5 - (4*A - 7*B)*c^4*d - 14*(A - B)
*c^3*d^2 - 2*(5*A + B)*c^2*d^3 + (13*A - 16*B)*c*d^4 + (14*A - 5*B)*d^5)*co
s(f*x + e))*sin(f*x + e))/((a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a^2
*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e)^3 + (a^2*c^7
 - 5*a^2*c^5*d^2 + 2*a^2*c^4*d^3 + 7*a^2*c^3*d^4 - 4*a^2*c^2*d^5 - 3*a^2*c*
d^6 + 2*a^2*d^7)*f*cos(f*x + e)^2 - (a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 +
3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f*cos(
f*x + e) - 2*(a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c
^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f + ((a^2*c^6*d - 2*a^2*c^5*d
^2 - a^2*c^4*d^3 + 4*a^2*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f*c
os(f*x + e)^2 - (a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^
2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e) - 2*(a^2*c^
7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d
^5 - a^2*c*d^6 + a^2*d^7)*f)*sin(f*x + e))]
```

giac [A] time = 0.23, size = 425, normalized size = 1.55

$$-\frac{2\left(\frac{3\left(2\,Bc^2d-3\,Acd^2+2\,Bcd^2-2\,Ad^3+Bd^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(a^2c^4-2\,a^2c^3d+2\,a^2cd^3-a^2d^4\right)\sqrt{c^2-d^2}}+\frac{3\left(Bcd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-Ad^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\cdots\right.}{\left(a^2c^5-2\,a^2c^4d+2\,a^2c^2d^3-a^2cd^4\right)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+\cdots\right.}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] -2/3*(3*(2*B*c^2*d - 3*A*c*d^2 + 2*B*c*d^2 - 2*A*d^3 + B*d^3)*(pi*floor(1/2
*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2
- d^2)))/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*sqrt(c^2 - d^2))
+ 3*(B*c*d^3*tan(1/2*f*x + 1/2*e) - A*d^4*tan(1/2*f*x + 1/2*e) + B*c^2*d^2
- A*c*d^3)/((a^2*c^5 - 2*a^2*c^4*d + 2*a^2*c^2*d^3 - a^2*c*d^4)*(c*tan(1/2*
f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) + (3*A*c*tan(1/2*f*x + 1/2*
e)^2 - 9*A*d*tan(1/2*f*x + 1/2*e)^2 + 6*B*d*tan(1/2*f*x + 1/2*e)^2 + 3*A*c*
tan(1/2*f*x + 1/2*e) + 3*B*c*tan(1/2*f*x + 1/2*e) - 15*A*d*tan(1/2*f*x + 1/
2*e) + 9*B*d*tan(1/2*f*x + 1/2*e) + 2*A*c + B*c - 8*A*d + 5*B*d)/((a^2*c^3
- 3*a^2*c^2*d + 3*a^2*c*d^2 - a^2*d^3)*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

maple [B] time = 0.55, size = 770, normalized size = 2.80

$$\frac{2d^4 \tan\left(\frac{fx}{2}+\frac{e}{2}\right) A}{f\,a^2 (c-d)^3 \left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right) c + 2\tan\left(\frac{fx}{2}+\frac{e}{2}\right) d + c\right)(c+d)\,c} - \frac{2d^3 \tan\left(\frac{fx}{2}+\frac{e}{2}\right) B}{f\,a^2 (c-d)^3 \left(\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right) c + 2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/f/a^2/(c-d)^3*d^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)
/c*tan(1/2*f*x+1/2*e)*A-2/f/a^2/(c-d)^3*d^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1
/2*f*x+1/2*e)*d+c)/(c+d)*tan(1/2*f*x+1/2*e)*B+2/f/a^2/(c-d)^3*d^3/(tan(1/2*
f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*A-2/f/a^2/(c-d)^3*d^2/(tan(1
/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*B*c+6/f/a^2/(c-d)^3*d^2/(
c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2
))*A*c+4/f/a^2/(c-d)^3*d^3/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*
x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A-4/f/a^2/(c-d)^3*d/(c+d)/(c^2-d^2)^(1/2)*ar
ctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^2-4/f/a^2/(c-d)^
3*d^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^
2)^(1/2))*B*c-2/f/a^2/(c-d)^3*d^3/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan
(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B+2/f/a^2/(c-d)^2/(tan(1/2*f*x+1/2*e)
+1)^2*A-2/f/a^2/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^2*B-4/3/f/a^2/(c-d)^2/(tan(1
/2*f*x+1/2*e)+1)^3*A+4/3/f/a^2/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^3*B-2/f/a^2/(
c-d)^3/(tan(1/2*f*x+1/2*e)+1)*A*c+6/f/a^2/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)*A*
d-4/f/a^2/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)*B*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 16.76, size = 844, normalized size = 3.07

$$\frac{2\,d\,\mathrm{atan}\left(\frac{\frac{d\left(-2\,a^2\,c^4\,d+4\,a^2\,c^3\,d^2-4\,a^2\,c\,d^4+2\,a^2\,d^5\right)\left(2\,B\,c^2-2\,A\,d^2+B\,d^2-3\,A\,c\,d+2\,B\,c\,d\right)}{a^2\,{\left(c+d\right)}^{3/2}\,{\left(c-d\right)}^{7/2}}-\frac{2\,c\,d\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(a^2\,c^4-2\,a^2\,c^3\,d+2\,a^2\,c\,d^3-a^2\,d^4\right)\left(2\,B\,c^2-2\,A\,d^2+B\,d^2-3\,A\,c\,c}{a^2\,{\left(c+d\right)}^{3/2}\,{\left(c-d\right)}^{7/2}}}{2\,B\,d^3-4\,A\,d^3-6\,A\,c\,d^2+4\,B\,c\,d^2+4\,B\,c^2\,d}\right.}{a^2\,f\,{\left(c+d\right)}^{3/2}\,{\left(c-d\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^2),x)
```

```
[Out] (2*d*atan(((d*(2*a^2*d^5 - 4*a^2*c*d^4 - 2*a^2*c^4*d + 4*a^2*c^3*d^2)*(2*B*
c^2 - 2*A*d^2 + B*d^2 - 3*A*c*d + 2*B*c*d))/(a^2*(c + d)^(3/2)*(c - d)^(7/2
)) - (2*c*d*tan(e/2 + (f*x)/2)*(a^2*c^4 - a^2*d^4 + 2*a^2*c*d^3 - 2*a^2*c^3
*d)*(2*B*c^2 - 2*A*d^2 + B*d^2 - 3*A*c*d + 2*B*c*d))/(a^2*(c + d)^(3/2)*(c
- d)^(7/2)))/(2*B*d^3 - 4*A*d^3 - 6*A*c*d^2 + 4*B*c*d^2 + 4*B*c^2*d))*(2*B*
c^2 - 2*A*d^2 + B*d^2 - 3*A*c*d + 2*B*c*d))/(a^2*f*(c + d)^(3/2)*(c - d)^(7
/2)) - ((2*(2*A*c^3 - 3*A*d^3 + B*c^3 - 8*A*c*d^2 - 6*A*c^2*d + 8*B*c*d^2 +
 6*B*c^2*d))/(3*(c + d)*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2
)^2*(5*A*c^3 - 9*A*d^3 + B*c^3 - 30*A*c*d^2 - 11*A*c^2*d + 27*B*c*d^2 + 17*
B*c^2*d))/(3*c*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^3*(A*c^
4 - 3*A*d^4 + B*c^4 - 9*A*c^2*d^2 + 8*B*c^2*d^2 - 7*A*c*d^3 - 2*A*c^3*d + 7
*B*c*d^3 + 4*B*c^3*d))/(c*(c + d)*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2
 + (f*x)/2)*(3*A*c^4 - 3*A*d^4 + 3*B*c^4 - 27*A*c^2*d^2 + 30*B*c^2*d^2 - 25
*A*c*d^3 - 8*A*c^3*d + 13*B*c*d^3 + 14*B*c^3*d))/(3*c*(c + d)*(c - d)*(c^2
- 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^4*(A*c^4 - A*d^4 - 3*A*c^2*d^2 + 2*
B*c^2*d^2 - 2*A*c^3*d + B*c*d^3 + 2*B*c^3*d))/(c*(c + d)*(c - d)*(c^2 - 2*c
*d + d^2)))/(f*(a^2*c + tan(e/2 + (f*x)/2)*(3*a^2*c + 2*a^2*d) + tan(e/2 +
(f*x)/2)^4*(3*a^2*c + 2*a^2*d) + tan(e/2 + (f*x)/2)^2*(4*a^2*c + 6*a^2*d) +
 tan(e/2 + (f*x)/2)^3*(4*a^2*c + 6*a^2*d) + a^2*c*tan(e/2 + (f*x)/2)^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.278 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=386

$$-\frac{d\left(A\left(2c^2-16cd-21d^2\right)+B\left(4c^2+19cd+12d^2\right)\right)\cos(e+fx)}{6a^2f(c-d)^3(c+d)(c+d\sin(e+fx))^2}+\frac{d\left(Ad\left(12c^2+16cd+7d^2\right)-B\left(6c^3+12c^2d+\right.\right.}{a^2f(c-d)^4(c+d)}$$

```
[Out] -1/6*d*(A*(2*c^2-16*c*d-21*d^2)+B*(4*c^2+19*c*d+12*d^2))*cos(f*x+e)/a^2/(c-
d)^3/(c+d)/f/(c+d*sin(f*x+e))^2-1/3*(A*c-8*A*d+2*B*c+5*B*d)*cos(f*x+e)/a^2/
(c-d)^2/f/(1+sin(f*x+e))/(c+d*sin(f*x+e))^2-1/3*(A-B)*cos(f*x+e)/(c-d)/f/(a
+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2-1/6*d*(A*(2*c^3-16*c^2*d-59*c*d^2-32*d^
3)+B*(4*c^3+37*c^2*d+44*c*d^2+20*d^3))*cos(f*x+e)/a^2/(c-d)^4/(c+d)^2/f/(c+
d*sin(f*x+e))+d*(A*d*(12*c^2+16*c*d+7*d^2)-B*(6*c^3+12*c^2*d+13*c*d^2+4*d^3
))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^2/(c-d)^4/(c+d)^2/f/(
c^2-d^2)^(1/2)
```

Rubi [A] time = 0.96, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2978, 2754, 12, 2660, 618, 204}

$$\frac{d\left(Ad\left(12c^2+16cd+7d^2\right)-B\left(12c^2d+6c^3+13cd^2+4d^3\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^4(c+d)^2\sqrt{c^2-d^2}}-\frac{d\left(A\left(-16c^2d+2c^3-59cd\right.\right.}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] (d*(A*d*(12*c^2 + 16*c*d + 7*d^2) - B*(6*c^3 + 12*c^2*d + 13*c*d^2 + 4*d^3)
)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^2*(c - d)^4*(c + d)^
2*Sqrt[c^2 - d^2]*f) - (d*(A*(2*c^2 - 16*c*d - 21*d^2) + B*(4*c^2 + 19*c*d
+ 12*d^2))*Cos[e + f*x])/(6*a^2*(c - d)^3*(c + d)*f*(c + d*Sin[e + f*x])^2)
 - ((A*c + 2*B*c - 8*A*d + 5*B*d)*Cos[e + f*x])/(3*a^2*(c - d)^2*f*(1 + Sin
[e + f*x])*(c + d*Sin[e + f*x])^2) - ((A - B)*Cos[e + f*x])/(3*(c - d)*f*(a
 + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2) - (d*(A*(2*c^3 - 16*c^2*d - 59
*c*d^2 - 32*d^3) + B*(4*c^3 + 37*c^2*d + 44*c*d^2 + 20*d^3))*Cos[e + f*x])/
(6*a^2*(c - d)^4*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx = -\frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2} - \frac{\int \frac{-a(A(c-5\ldots}{(a+\ldots}}{}$$

$$= -\frac{(Ac+2Bc-8Ad+5Bd)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))^2} - \frac{}{3(c-d)f}$$

$$= -\frac{d\left(A\left(2c^2-16cd-21d^2\right)+B\left(4c^2+19cd+12d^2\right)\right)\cos(e+fx}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2}$$

$$= -\frac{d\left(A\left(2c^2-16cd-21d^2\right)+B\left(4c^2+19cd+12d^2\right)\right)\cos(e+fx}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2}$$

$$= -\frac{d\left(A\left(2c^2-16cd-21d^2\right)+B\left(4c^2+19cd+12d^2\right)\right)\cos(e+fx}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2}$$

$$= -\frac{d\left(A\left(2c^2-16cd-21d^2\right)+B\left(4c^2+19cd+12d^2\right)\right)\cos(e+fx}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2}$$

$$= -\frac{d\left(A\left(2c^2-16cd-21d^2\right)+B\left(4c^2+19cd+12d^2\right)\right)\cos(e+fx}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2}$$

$$= \frac{d\left(Ad\left(12c^2+16cd+7d^2\right)-B\left(6c^3+12c^2d+13cd^2+4d^3\right)\right)\mathrm{ta}}{a^2(c-d)^4(c+d)^2\sqrt{c^2-d^2}\,f}$$

Mathematica [B] time = 6.37, size = 1522, normalized size = 3.94

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])
^3),x]
```

```
[Out] -((d*(6*B*c^3 - 12*A*c^2*d + 12*B*c^2*d - 16*A*c*d^2 + 13*B*c*d^2 - 7*A*d^3
 + 4*B*d^3)*ArcTan[(Sec[(e + f*x)/2]*(d*Cos[(e + f*x)/2] + c*Sin[(e + f*x)/
2]))/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)/((c - d)^4*(
c + d)^2*Sqrt[c^2 - d^2]*f*(a + a*Sin[e + f*x])^2)) + ((Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])*(48*B*c^5*Cos[(e + f*x)/2] - 96*A*c^4*d*Cos[(e + f*x)/2]
+ 240*B*c^4*d*Cos[(e + f*x)/2] - 524*A*c^3*d^2*Cos[(e + f*x)/2] + 536*B*c^3
```

```
*d^2*Cos[(e + f*x)/2] - 776*A*c^2*d^3*Cos[(e + f*x)/2] + 701*B*c^2*d^3*Cos[
(e + f*x)/2] - 487*A*c*d^4*Cos[(e + f*x)/2] + 400*B*c*d^4*Cos[(e + f*x)/2]
- 112*A*d^5*Cos[(e + f*x)/2] + 70*B*d^5*Cos[(e + f*x)/2] - 16*A*c^5*Cos[(3*
(e + f*x))/2] - 32*B*c^5*Cos[(3*(e + f*x))/2] + 80*A*c^4*d*Cos[(3*(e + f*x)
)/2] - 224*B*c^4*d*Cos[(3*(e + f*x))/2] + 536*A*c^3*d^2*Cos[(3*(e + f*x))/2
] - 728*B*c^3*d^2*Cos[(3*(e + f*x))/2] + 1028*A*c^2*d^3*Cos[(3*(e + f*x))/2
] - 893*B*c^2*d^3*Cos[(3*(e + f*x))/2] + 695*A*c*d^4*Cos[(3*(e + f*x))/2] -
 482*B*c*d^4*Cos[(3*(e + f*x))/2] + 134*A*d^5*Cos[(3*(e + f*x))/2] - 98*B*d
^5*Cos[(3*(e + f*x))/2] + 24*B*c^3*d^2*Cos[(5*(e + f*x))/2] - 12*A*c^2*d^3*
Cos[(5*(e + f*x))/2] + 21*B*c^2*d^3*Cos[(5*(e + f*x))/2] - 15*A*c*d^4*Cos[(
5*(e + f*x))/2] - 18*B*c*d^4*Cos[(5*(e + f*x))/2] + 6*A*d^5*Cos[(5*(e + f*x
))/2] - 6*B*d^5*Cos[(5*(e + f*x))/2] + 4*A*c^3*d^2*Cos[(7*(e + f*x))/2] + 8
*B*c^3*d^2*Cos[(7*(e + f*x))/2] - 32*A*c^2*d^3*Cos[(7*(e + f*x))/2] + 59*B*
c^2*d^3*Cos[(7*(e + f*x))/2] - 97*A*c*d^4*Cos[(7*(e + f*x))/2] + 76*B*c*d^4
*Cos[(7*(e + f*x))/2] - 52*A*d^5*Cos[(7*(e + f*x))/2] + 34*B*d^5*Cos[(7*(e
+ f*x))/2] + 48*A*c^5*Sin[(e + f*x)/2] + 48*B*c^5*Sin[(e + f*x)/2] - 224*A*
c^4*d*Sin[(e + f*x)/2] + 416*B*c^4*d*Sin[(e + f*x)/2] - 872*A*c^3*d^2*Sin[(
e + f*x)/2] + 992*B*c^3*d^2*Sin[(e + f*x)/2] - 1144*A*c^2*d^3*Sin[(e + f*x)
/2] + 967*B*c^2*d^3*Sin[(e + f*x)/2] - 685*A*c*d^4*Sin[(e + f*x)/2] + 496*B
*c*d^4*Sin[(e + f*x)/2] - 168*A*d^5*Sin[(e + f*x)/2] + 126*B*d^5*Sin[(e + f
*x)/2] + 48*B*c^4*d*Sin[(3*(e + f*x))/2] - 132*A*c^3*d^2*Sin[(3*(e + f*x))/
2] + 96*B*c^3*d^2*Sin[(3*(e + f*x))/2] - 204*A*c^2*d^3*Sin[(3*(e + f*x))/2]
 + 207*B*c^2*d^3*Sin[(3*(e + f*x))/2] - 165*A*c*d^4*Sin[(3*(e + f*x))/2] +
174*B*c*d^4*Sin[(3*(e + f*x))/2] - 66*A*d^5*Sin[(3*(e + f*x))/2] + 42*B*d^5
*Sin[(3*(e + f*x))/2] - 16*A*c^4*d*Sin[(5*(e + f*x))/2] - 32*B*c^4*d*Sin[(5
*(e + f*x))/2] + 116*A*c^3*d^2*Sin[(5*(e + f*x))/2] - 224*B*c^3*d^2*Sin[(5*
(e + f*x))/2] + 412*A*c^2*d^3*Sin[(5*(e + f*x))/2] - 409*B*c^2*d^3*Sin[(5*(
e + f*x))/2] + 403*A*c*d^4*Sin[(5*(e + f*x))/2] - 286*B*c*d^4*Sin[(5*(e + f
*x))/2] + 114*A*d^5*Sin[(5*(e + f*x))/2] - 78*B*d^5*Sin[(5*(e + f*x))/2] +
15*B*c^2*d^3*Sin[(7*(e + f*x))/2] - 21*A*c*d^4*Sin[(7*(e + f*x))/2] + 12*B*
c*d^4*Sin[(7*(e + f*x))/2] - 12*A*d^5*Sin[(7*(e + f*x))/2] + 6*B*d^5*Sin[(7
*(e + f*x))/2]))/(48*(c - d)^4*(c + d)^2*f*(a + a*Sin[e + f*x])^2*(c + d*Si
n[e + f*x])^2)
```

fricas [B] time = 0.70, size = 4997, normalized size = 12.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] [-1/12*(4*(A - B)*c^7 - 4*(A - B)*c^6*d - 12*(A - B)*c^5*d^2 + 12*(A - B)*c
^4*d^3 + 12*(A - B)*c^3*d^4 - 12*(A - B)*c^2*d^5 - 4*(A - B)*c*d^6 + 4*(A -
 B)*d^7 - 2*(2*(A + 2*B)*c^5*d^2 - (16*A - 37*B)*c^4*d^3 - (61*A - 40*B)*c^
```

3*d^4 - (16*A + 17*B)*c^2*d^5 + (59*A - 44*B)*c*d^6 + 4*(8*A - 5*B)*d^7)*co
s(f*x + e)^4 - 2*(4*(A + 2*B)*c^6*d - 4*(7*A - 16*B)*c^5*d^2 - 118*(A - B)*
c^4*d^3 - (106*A - 25*B)*c^3*d^4 + (71*A - 98*B)*c^2*d^5 + (134*A - 89*B)*c
*d^6 + (43*A - 28*B)*d^7)*cos(f*x + e)^3 + 2*(2*(A + 2*B)*c^7 - 6*(2*A - 3*
B)*c^6*d - 12*(3*A - 4*B)*c^5*d^2 - 3*(18*A - 17*B)*c^4*d^3 - 3*(13*A + B)*
c^3*d^4 + 3*(13*A - 17*B)*c^2*d^5 + (73*A - 49*B)*c*d^6 + 9*(3*A - 2*B)*d^7
)*cos(f*x + e)^2 + 3*(12*B*c^5*d - 24*(A - 2*B)*c^4*d^2 - 2*(40*A - 43*B)*c
^3*d^3 - 6*(17*A - 14*B)*c^2*d^4 - 6*(10*A - 7*B)*c*d^5 - 2*(7*A - 4*B)*d^6
 + (6*B*c^3*d^3 - 12*(A - B)*c^2*d^4 - (16*A - 13*B)*c*d^5 - (7*A - 4*B)*d^
6)*cos(f*x + e)^4 - (12*B*c^4*d^2 - 6*(4*A - 5*B)*c^3*d^3 - 2*(22*A - 19*B)
*c^2*d^4 - 3*(10*A - 7*B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x + e)^3 - (6*B*c^
5*d - 12*(A - 3*B)*c^4*d^2 - (64*A - 79*B)*c^3*d^3 - (107*A - 92*B)*c^2*d^4
 - (76*A - 55*B)*c*d^5 - 3*(7*A - 4*B)*d^6)*cos(f*x + e)^2 + (6*B*c^5*d - 1
2*(A - 2*B)*c^4*d^2 - (40*A - 43*B)*c^3*d^3 - 3*(17*A - 14*B)*c^2*d^4 - 3*(
10*A - 7*B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x + e) + (12*B*c^5*d - 24*(A - 2
*B)*c^4*d^2 - 2*(40*A - 43*B)*c^3*d^3 - 6*(17*A - 14*B)*c^2*d^4 - 6*(10*A -
 7*B)*c*d^5 - 2*(7*A - 4*B)*d^6 - (6*B*c^3*d^3 - 12*(A - B)*c^2*d^4 - (16*A
 - 13*B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x + e)^3 - 2*(6*B*c^4*d^2 - 6*(2*A
 - 3*B)*c^3*d^3 - (28*A - 25*B)*c^2*d^4 - (23*A - 17*B)*c*d^5 - (7*A - 4*B)*
d^6)*cos(f*x + e)^2 + (6*B*c^5*d - 12*(A - 2*B)*c^4*d^2 - (40*A - 43*B)*c^3
d^3 - 3(17*A - 14*B)*c^2*d^4 - 3*(10*A - 7*B)*c*d^5 - (7*A - 4*B)*d^6)*co
s(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(-((2*c^2 - d^2)*cos(f*x + e)
^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 - 2*(c*cos(f*x + e)*sin(f*x + e) + d*co
s(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^
2 - d^2)) + 4*((2*A + B)*c^7 - (5*A - 14*B)*c^6*d - 3*(12*A - 19*B)*c^5*d^2
 - 3*(25*A - 21*B)*c^4*d^3 - 3*(13*A + 4*B)*c^3*d^4 + 3*(20*A - 21*B)*c^2*d
^5 + (73*A - 46*B)*c*d^6 + 2*(10*A - 7*B)*d^7)*cos(f*x + e) - 2*(2*(A - B)*
c^7 - 2*(A - B)*c^6*d - 6*(A - B)*c^5*d^2 + 6*(A - B)*c^4*d^3 + 6*(A - B)*c
^3*d^4 - 6*(A - B)*c^2*d^5 - 2*(A - B)*c*d^6 + 2*(A - B)*d^7 + (2*(A + 2*B)
*c^5*d^2 - (16*A - 37*B)*c^4*d^3 - (61*A - 40*B)*c^3*d^4 - (16*A + 17*B)*c^
2*d^5 + (59*A - 44*B)*c*d^6 + 4*(8*A - 5*B)*d^7)*cos(f*x + e)^3 - (4*(A + 2
*B)*c^6*d - 30*(A - 2*B)*c^5*d^2 - 3*(34*A - 27*B)*c^4*d^3 - 15*(3*A + B)*c
^3*d^4 + 3*(29*A - 27*B)*c^2*d^5 + 15*(5*A - 3*B)*c*d^6 + (11*A - 8*B)*d^7)
*cos(f*x + e)^2 - 2*((A + 2*B)*c^7 - (4*A - 13*B)*c^6*d - 3*(11*A - 18*B)*c
^5*d^2 - 6*(13*A - 11*B)*c^4*d^3 - 3*(14*A + 3*B)*c^3*d^4 + 3*(21*A - 22*B)
*c^2*d^5 + (74*A - 47*B)*c*d^6 + (19*A - 13*B)*d^7)*cos(f*x + e))*sin(f*x +
 e))/((a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*
c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f*cos(f*x + e)^4 - (2*a^2
*c^9*d - 3*a^2*c^8*d^2 - 6*a^2*c^7*d^3 + 10*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 1
2*a^2*c^4*d^6 - 2*a^2*c^3*d^7 + 6*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e)^3
 - (a^2*c^10 + 2*a^2*c^9*d - 7*a^2*c^8*d^2 - 8*a^2*c^7*d^3 + 18*a^2*c^6*d^4
 + 12*a^2*c^5*d^5 - 22*a^2*c^4*d^6 - 8*a^2*c^3*d^7 + 13*a^2*c^2*d^8 + 2*a^2*
c*d^9 - 3*a^2*d^10)*f*cos(f*x + e)^2 + (a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c
^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e) + 2*(a^2
*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a

```
^2*d^10)*f - ((a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^2*c^5*d^5
- 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f*cos(f*x + e)^3
+ 2*(a^2*c^9*d - a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 4*a^2*c^6*d^4 + 6*a^2*c^5*d^
5 - 6*a^2*c^4*d^6 - 4*a^2*c^3*d^7 + 4*a^2*c^2*d^8 + a^2*c*d^9 - a^2*d^10)*f
*cos(f*x + e)^2 - (a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d
^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e) - 2*(a^2*c^10 - 5*a^2*c^8*d^2
 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f)*sin(f*x +
 e)), -1/6*(2*(A - B)*c^7 - 2*(A - B)*c^6*d - 6*(A - B)*c^5*d^2 + 6*(A - B)
*c^4*d^3 + 6*(A - B)*c^3*d^4 - 6*(A - B)*c^2*d^5 - 2*(A - B)*c*d^6 + 2*(A -
 B)*d^7 - (2*(A + 2*B)*c^5*d^2 - (16*A - 37*B)*c^4*d^3 - (61*A - 40*B)*c^3*
d^4 - (16*A + 17*B)*c^2*d^5 + (59*A - 44*B)*c*d^6 + 4*(8*A - 5*B)*d^7)*cos(
f*x + e)^4 - (4*(A + 2*B)*c^6*d - 4*(7*A - 16*B)*c^5*d^2 - 118*(A - B)*c^4*
d^3 - (106*A - 25*B)*c^3*d^4 + (71*A - 98*B)*c^2*d^5 + (134*A - 89*B)*c*d^6
 + (43*A - 28*B)*d^7)*cos(f*x + e)^3 + (2*(A + 2*B)*c^7 - 6*(2*A - 3*B)*c^6
*d - 12*(3*A - 4*B)*c^5*d^2 - 3*(18*A - 17*B)*c^4*d^3 - 3*(13*A + B)*c^3*d^
4 + 3*(13*A - 17*B)*c^2*d^5 + (73*A - 49*B)*c*d^6 + 9*(3*A - 2*B)*d^7)*cos(
f*x + e)^2 - 3*(12*B*c^5*d - 24*(A - 2*B)*c^4*d^2 - 2*(40*A - 43*B)*c^3*d^3
 - 6*(17*A - 14*B)*c^2*d^4 - 6*(10*A - 7*B)*c*d^5 - 2*(7*A - 4*B)*d^6 + (6*
B*c^3*d^3 - 12*(A - B)*c^2*d^4 - (16*A - 13*B)*c*d^5 - (7*A - 4*B)*d^6)*cos
(f*x + e)^4 - (12*B*c^4*d^2 - 6*(4*A - 5*B)*c^3*d^3 - 2*(22*A - 19*B)*c^2*d
^4 - 3*(10*A - 7*B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x + e)^3 - (6*B*c^5*d -
12*(A - 3*B)*c^4*d^2 - (64*A - 79*B)*c^3*d^3 - (107*A - 92*B)*c^2*d^4 - (76
*A - 55*B)*c*d^5 - 3*(7*A - 4*B)*d^6)*cos(f*x + e)^2 + (6*B*c^5*d - 12*(A -
 2*B)*c^4*d^2 - (40*A - 43*B)*c^3*d^3 - 3*(17*A - 14*B)*c^2*d^4 - 3*(10*A -
 7*B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x + e) + (12*B*c^5*d - 24*(A - 2*B)*c^
4*d^2 - 2*(40*A - 43*B)*c^3*d^3 - 6*(17*A - 14*B)*c^2*d^4 - 6*(10*A - 7*B)*
c*d^5 - 2*(7*A - 4*B)*d^6 - (6*B*c^3*d^3 - 12*(A - B)*c^2*d^4 - (16*A - 13*
B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x + e)^3 - 2*(6*B*c^4*d^2 - 6*(2*A - 3*B)
*c^3*d^3 - (28*A - 25*B)*c^2*d^4 - (23*A - 17*B)*c*d^5 - (7*A - 4*B)*d^6)*c
os(f*x + e)^2 + (6*B*c^5*d - 12*(A - 2*B)*c^4*d^2 - (40*A - 43*B)*c^3*d^3 -
 3*(17*A - 14*B)*c^2*d^4 - 3*(10*A - 7*B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x
+ e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2
- d^2)*cos(f*x + e))) + 2*((2*A + B)*c^7 - (5*A - 14*B)*c^6*d - 3*(12*A - 1
9*B)*c^5*d^2 - 3*(25*A - 21*B)*c^4*d^3 - 3*(13*A + 4*B)*c^3*d^4 + 3*(20*A -
 21*B)*c^2*d^5 + (73*A - 46*B)*c*d^6 + 2*(10*A - 7*B)*d^7)*cos(f*x + e) - (
2*(A - B)*c^7 - 2*(A - B)*c^6*d - 6*(A - B)*c^5*d^2 + 6*(A - B)*c^4*d^3 + 6
*(A - B)*c^3*d^4 - 6*(A - B)*c^2*d^5 - 2*(A - B)*c*d^6 + 2*(A - B)*d^7 + (2
*(A + 2*B)*c^5*d^2 - (16*A - 37*B)*c^4*d^3 - (61*A - 40*B)*c^3*d^4 - (16*A
+ 17*B)*c^2*d^5 + (59*A - 44*B)*c*d^6 + 4*(8*A - 5*B)*d^7)*cos(f*x + e)^3 -
 (4*(A + 2*B)*c^6*d - 30*(A - 2*B)*c^5*d^2 - 3*(34*A - 27*B)*c^4*d^3 - 15*(
3*A + B)*c^3*d^4 + 3*(29*A - 27*B)*c^2*d^5 + 15*(5*A - 3*B)*c*d^6 + (11*A -
 8*B)*d^7)*cos(f*x + e)^2 - 2*((A + 2*B)*c^7 - (4*A - 13*B)*c^6*d - 3*(11*A
 - 18*B)*c^5*d^2 - 6*(13*A - 11*B)*c^4*d^3 - 3*(14*A + 3*B)*c^3*d^4 + 3*(21
*A - 22*B)*c^2*d^5 + (74*A - 47*B)*c*d^6 + (19*A - 13*B)*d^7)*cos(f*x + e))
*sin(f*x + e))/((a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^2*c^5*d^
```

```
5 - 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f*cos(f*x + e)^
4 - (2*a^2*c^9*d - 3*a^2*c^8*d^2 - 6*a^2*c^7*d^3 + 10*a^2*c^6*d^4 + 6*a^2*c
^5*d^5 - 12*a^2*c^4*d^6 - 2*a^2*c^3*d^7 + 6*a^2*c^2*d^8 - a^2*d^10)*f*cos(f
*x + e)^3 - (a^2*c^10 + 2*a^2*c^9*d - 7*a^2*c^8*d^2 - 8*a^2*c^7*d^3 + 18*a^
2*c^6*d^4 + 12*a^2*c^5*d^5 - 22*a^2*c^4*d^6 - 8*a^2*c^3*d^7 + 13*a^2*c^2*d^
8 + 2*a^2*c*d^9 - 3*a^2*d^10)*f*cos(f*x + e)^2 + (a^2*c^10 - 5*a^2*c^8*d^2
+ 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e
) + 2*(a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c
^2*d^8 - a^2*d^10)*f - ((a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^
2*c^5*d^5 - 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f*cos(f
*x + e)^3 + 2*(a^2*c^9*d - a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 4*a^2*c^6*d^4 + 6*
a^2*c^5*d^5 - 6*a^2*c^4*d^6 - 4*a^2*c^3*d^7 + 4*a^2*c^2*d^8 + a^2*c*d^9 - a
^2*d^10)*f*cos(f*x + e)^2 - (a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10
*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e) - 2*(a^2*c^10 - 5*a
^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f)
*sin(f*x + e))]
```

giac [B] time = 0.31, size = 944, normalized size = 2.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -1/3*(3*(6*B*c^3*d - 12*A*c^2*d^2 + 12*B*c^2*d^2 - 16*A*c*d^3 + 13*B*c*d^3
- 7*A*d^4 + 4*B*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*t
an(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^2*c^6 - 2*a^2*c^5*d - a^2*c^
4*d^2 + 4*a^2*c^3*d^3 - a^2*c^2*d^4 - 2*a^2*c*d^5 + a^2*d^6)*sqrt(c^2 - d^2
)) + 3*(7*B*c^4*d^3*tan(1/2*f*x + 1/2*e)^3 - 9*A*c^3*d^4*tan(1/2*f*x + 1/2*
e)^3 + 4*B*c^3*d^4*tan(1/2*f*x + 1/2*e)^3 - 4*A*c^2*d^5*tan(1/2*f*x + 1/2*e
)^3 + 2*A*c*d^6*tan(1/2*f*x + 1/2*e)^3 + 6*B*c^5*d^2*tan(1/2*f*x + 1/2*e)^2
 - 8*A*c^4*d^3*tan(1/2*f*x + 1/2*e)^2 + 4*B*c^4*d^3*tan(1/2*f*x + 1/2*e)^2
- 4*A*c^3*d^4*tan(1/2*f*x + 1/2*e)^2 + 13*B*c^3*d^4*tan(1/2*f*x + 1/2*e)^2
- 15*A*c^2*d^5*tan(1/2*f*x + 1/2*e)^2 + 8*B*c^2*d^5*tan(1/2*f*x + 1/2*e)^2
- 8*A*c*d^6*tan(1/2*f*x + 1/2*e)^2 + 2*B*c*d^6*tan(1/2*f*x + 1/2*e)^2 + 2*A
*d^7*tan(1/2*f*x + 1/2*e)^2 + 17*B*c^4*d^3*tan(1/2*f*x + 1/2*e) - 23*A*c^3*
d^4*tan(1/2*f*x + 1/2*e) + 12*B*c^3*d^4*tan(1/2*f*x + 1/2*e) - 12*A*c^2*d^5
*tan(1/2*f*x + 1/2*e) + 4*B*c^2*d^5*tan(1/2*f*x + 1/2*e) + 2*A*c*d^6*tan(1/
2*f*x + 1/2*e) + 6*B*c^5*d^2 - 8*A*c^4*d^3 + 4*B*c^4*d^3 - 4*A*c^3*d^4 + B*
c^3*d^4 + A*c^2*d^5)/((a^2*c^8 - 2*a^2*c^7*d - a^2*c^6*d^2 + 4*a^2*c^5*d^3
- a^2*c^4*d^4 - 2*a^2*c^3*d^5 + a^2*c^2*d^6)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*
d*tan(1/2*f*x + 1/2*e) + c)^2) + 2*(3*A*c*tan(1/2*f*x + 1/2*e)^2 - 12*A*d*t
an(1/2*f*x + 1/2*e)^2 + 9*B*d*tan(1/2*f*x + 1/2*e)^2 + 3*A*c*tan(1/2*f*x +
1/2*e) + 3*B*c*tan(1/2*f*x + 1/2*e) - 21*A*d*tan(1/2*f*x + 1/2*e) + 15*B*d*
```

```
tan(1/2*f*x + 1/2*e) + 2*A*c + B*c - 11*A*d + 8*B*d)/((a^2*c^4 - 4*a^2*c^3*
d + 6*a^2*c^2*d^2 - 4*a^2*c*d^3 + a^2*d^4)*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

maple [B] time = 0.59, size = 2641, normalized size = 6.84

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -4/f/a^2*d^4/(c-d)^4/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/
2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B-1/f/a^2*d^4/(c-d)^4/(tan(1/2*f*x+1/2*e
)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*B*c+4/f/a^2*d^5/(c-d)^4/(
tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*
f*x+1/2*e)^3*A+15/f/a^2*d^5/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1
/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2*A-8/f/a^2*d^5/(c-d)^4/(ta
n(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*
x+1/2*e)^2*B+12/f/a^2*d^5/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2
*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*A-4/f/a^2*d^5/(c-d)^4/(tan(1/
2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/
2*e)*B+8/f/a^2*d^3/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c
)^2/(c^2+2*c*d+d^2)*A*c^2+4/f/a^2*d^4/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan
(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*A*c-6/f/a^2*d^2/(c-d)^4/(tan(1/2*f*x
+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*B*c^3+7/f/a^2*d^4/(
c-d)^4/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2
*d)/(c^2-d^2)^(1/2))*A-4/f/a^2*d^3/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/
2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*B*c^2-2/f/a^2/(c-d)^4/(tan(1/2*f*x+1/2*
e)+1)*A*c+8/f/a^2/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)*A*d-6/f/a^2/(c-d)^4/(tan(1
/2*f*x+1/2*e)+1)*B*d+2/f/a^2/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^2*A-2/f/a^2/(c-
d)^3/(tan(1/2*f*x+1/2*e)+1)^2*B-4/3/f/a^2/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^3*
A-4/f/a^2*d^3/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(
c^2+2*c*d+d^2)*c^2*tan(1/2*f*x+1/2*e)^2*B-13/f/a^2*d^4/(c-d)^4/(tan(1/2*f*x
+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*tan(1/2*f*x+1/2*e
)^2*B-2/f/a^2*d^6/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)
^2/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^2*B+23/f/a^2*d^4/(c-d)^4/(tan(1/2*f
*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2
*e)*A-13/f/a^2*d^3/(c-d)^4/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*
tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c+9/f/a^2*d^4/(c-d)^4/(tan(1/2*f
*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2
*e)^3*A-2/f/a^2*d^6/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+
c)^2/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*A-7/f/a^2*d^3/(c-d)^4/(tan(1/2*
f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+
1/2*e)^3*B-4/f/a^2*d^4/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)
*d+c)^2*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*B+8/f/a^2*d^3/(c-d)^4/(tan(1
/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^2*tan(1/2*f
```

```
*x+1/2*e)^2*A-2/f/a^2*d^6/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2
*e)*d+c)^2/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*A-1/f/a^2*d^5/(c-d)^4/(tan(
1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*A-17/f/a^2*d
^3/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^2+2*c
*d+d^2)*tan(1/2*f*x+1/2*e)*B+16/f/a^2*d^3/(c-d)^4/(c^2+2*c*d+d^2)/(c^2-d^2)
^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c+4/f/a^2
*d^4/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d
+d^2)*c*tan(1/2*f*x+1/2*e)^2*A+8/f/a^2*d^6/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+
2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^2*A-2/f/a^
2*d^7/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*
d+d^2)/c^2*tan(1/2*f*x+1/2*e)^2*A-6/f/a^2*d^2/(c-d)^4/(tan(1/2*f*x+1/2*e)^2
*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^3*tan(1/2*f*x+1/2*e)^2*B-1
2/f/a^2*d^4/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(
c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*B-6/f/a^2*d/(c-d)^4/(c^2+2*c*d+d^2)/(c^2-
d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^3-1
2/f/a^2*d^2/(c-d)^4/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2
*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^2+12/f/a^2*d^2/(c-d)^4/(c^2+2*c*d+d^2
)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*
A*c^2+4/3/f/a^2/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^3*B
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [**B**] time = 17.69, size = 1686, normalized size = 4.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^3),x)
```

```
[Out] (d*atan(((d*(4*a^2*c*d^6 - 2*a^2*d^7 - 2*a^2*c^6*d + 2*a^2*c^2*d^5 - 8*a^2*
c^3*d^4 + 2*a^2*c^4*d^3 + 4*a^2*c^5*d^2)*(6*B*c^3 - 7*A*d^3 + 4*B*d^3 - 16*
A*c*d^2 - 12*A*c^2*d + 13*B*c*d^2 + 12*B*c^2*d))/(2*a^2*(c + d)^(5/2)*(c -
d)^(9/2)) + (c*d*tan(e/2 + (f*x)/2)*(2*a^2*c*d^5 - a^2*d^6 - a^2*c^6 + 2*a^
2*c^5*d + a^2*c^2*d^4 - 4*a^2*c^3*d^3 + a^2*c^4*d^2)*(6*B*c^3 - 7*A*d^3 + 4
```

```
*B*d^3 - 16*A*c*d^2 - 12*A*c^2*d + 13*B*c*d^2 + 12*B*c^2*d))/(a^2*(c + d)^(
5/2)*(c - d)^(9/2)))/(4*B*d^4 - 7*A*d^4 - 12*A*c^2*d^2 + 12*B*c^2*d^2 - 16*
A*c*d^3 + 13*B*c*d^3 + 6*B*c^3*d))*(6*B*c^3 - 7*A*d^3 + 4*B*d^3 - 16*A*c*d^
2 - 12*A*c^2*d + 13*B*c*d^2 + 12*B*c^2*d))/(a^2*f*(c + d)^(5/2)*(c - d)^(9/
2)) - ((tan(e/2 + (f*x)/2)^5*(2*A*c^6 + 2*A*d^6 + 2*B*c^6 - 23*A*c^2*d^4 -
40*A*c^3*d^3 - 38*A*c^4*d^2 + 6*B*c^2*d^4 + 43*B*c^3*d^3 + 40*B*c^4*d^2 - 4
*A*c*d^5 - 4*A*c^5*d + 2*B*c*d^5 + 12*B*c^5*d))/(c^2*(c^5 - 3*c^4*d - 3*c*d
^4 + d^5 + 2*c^2*d^3 + 2*c^3*d^2)) + (4*A*c^5 + 3*A*d^5 + 2*B*c^5 - 46*A*c^
2*d^3 - 40*A*c^3*d^2 + 28*B*c^2*d^3 + 52*B*c^3*d^2 - 12*A*c*d^4 - 14*A*c^4*
d + 3*B*c*d^4 + 20*B*c^4*d)/(3*(c + d)*(c^2 - d^2)*(3*c*d^2 - 3*c^2*d + c^3
 - d^3)) + (2*tan(e/2 + (f*x)/2)^3*(6*A*c^6 + 9*A*d^6 + 6*B*c^6 - 177*A*c^2
*d^4 - 212*A*c^3*d^3 - 102*A*c^4*d^2 + 105*B*c^2*d^4 + 215*B*c^3*d^3 + 150*
B*c^4*d^2 - 33*A*c*d^5 - 16*A*c^5*d + 9*B*c*d^5 + 40*B*c^5*d))/(3*c^2*(c^2
- d^2)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (tan(e/2 + (f*x)/2)*(6*A*c^5 + 6*
A*d^5 + 6*B*c^5 - 160*A*c^2*d^3 - 114*A*c^3*d^2 + 97*B*c^2*d^3 + 156*B*c^3*
d^2 - 33*A*c*d^4 - 20*A*c^4*d + 12*B*c*d^4 + 44*B*c^4*d))/(3*c*(c^2 - d^2)*
(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (tan(e/2 + (f*x)/2)^2*(14*A*c^7 + 6*A*d^
7 + 4*B*c^7 - 232*A*c^2*d^5 - 583*A*c^3*d^4 - 532*A*c^4*d^3 - 226*A*c^5*d^2
 + 124*B*c^2*d^5 + 412*B*c^3*d^4 + 595*B*c^4*d^3 + 352*B*c^5*d^2 - 6*A*c*d^
6 - 16*A*c^6*d + 6*B*c*d^6 + 82*B*c^6*d))/(3*c^2*(c + d)*(c^2 - d^2)*(3*c*d
^2 - 3*c^2*d + c^3 - d^3)) + (tan(e/2 + (f*x)/2)^4*(16*A*c^7 + 18*A*d^7 + 2
*B*c^7 - 303*A*c^2*d^5 - 522*A*c^3*d^4 - 502*A*c^4*d^3 - 220*A*c^5*d^2 + 15
6*B*c^2*d^5 + 453*B*c^3*d^4 + 538*B*c^4*d^3 + 328*B*c^5*d^2 - 48*A*c*d^6 -
14*A*c^6*d + 18*B*c*d^6 + 80*B*c^6*d))/(3*c^2*(c + d)*(c^2 - d^2)*(3*c*d^2
- 3*c^2*d + c^3 - d^3)) + (tan(e/2 + (f*x)/2)^6*(2*A*c^6 + 2*A*d^6 - 9*A*c^
2*d^4 - 8*A*c^3*d^3 - 14*A*c^4*d^2 + 4*B*c^2*d^4 + 13*B*c^3*d^3 + 12*B*c^4*
d^2 - 4*A*c*d^5 - 4*A*c^5*d + 6*B*c^5*d))/(c*(c - d)*(2*c*d + c^2 + d^2)*(3
*c*d^2 - 3*c^2*d + c^3 - d^3)))/(f*(tan(e/2 + (f*x)/2)*(3*a^2*c^2 + 4*a^2*c
*d) + tan(e/2 + (f*x)/2)^2*(5*a^2*c^2 + 4*a^2*d^2 + 12*a^2*c*d) + tan(e/2 +
 (f*x)/2)^5*(5*a^2*c^2 + 4*a^2*d^2 + 12*a^2*c*d) + tan(e/2 + (f*x)/2)^3*(7*
a^2*c^2 + 12*a^2*d^2 + 16*a^2*c*d) + tan(e/2 + (f*x)/2)^4*(7*a^2*c^2 + 12*a
^2*d^2 + 16*a^2*c*d) + tan(e/2 + (f*x)/2)^6*(3*a^2*c^2 + 4*a^2*c*d) + a^2*c
^2 + a^2*c^2*tan(e/2 + (f*x)/2)^7))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.279 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=225

$$-\frac{(c-d)\left(A\left(2c^2+7cd+15d^2\right)+3B\left(c^2+6cd-15d^2\right)\right)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}+\frac{d^2(A(2c+7d)+3B(c-9d))\cos(e+fx)}{15a^3f}+$$

```
[Out] d^2*(3*B*(c-d)+A*d)*x/a^3+1/15*d^2*(3*B*(c-9*d)+A*(2*c+7*d))*cos(f*x+e)/a^3
/f-1/15*(c-d)*(3*B*(c^2+6*c*d-15*d^2)+A*(2*c^2+7*c*d+15*d^2))*cos(f*x+e)/f/
(a^3+a^3*sin(f*x+e))-1/15*(3*B*(c-3*d)+2*A*(c+2*d))*cos(f*x+e)*(c+d*sin(f*x
+e))^2/a/f/(a+a*sin(f*x+e))^2-1/5*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^3/f/(a+
a*sin(f*x+e))^3
```

Rubi [A] time = 0.81, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2977, 2968, 3023, 2735, 2648}

$$-\frac{(c-d)\left(A\left(2c^2+7cd+15d^2\right)+3B\left(c^2+6cd-15d^2\right)\right)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}+\frac{d^2(A(2c+7d)+3B(c-9d))\cos(e+fx)}{15a^3f}+$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (d^2*(3*B*(c - d) + A*d)*x)/a^3 + (d^2*(3*B*(c - 9*d) + A*(2*c + 7*d))*Cos[
e + f*x])/(15*a^3*f) - ((c - d)*(3*B*(c^2 + 6*c*d - 15*d^2) + A*(2*c^2 + 7*
c*d + 15*d^2))*Cos[e + f*x])/(15*f*(a^3 + a^3*Sin[e + f*x])) - ((3*B*(c - 3
*d) + 2*A*(c + 2*d))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(15*a*f*(a + a*Si
n[e + f*x])^2) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(5*f*(a + a*
Sin[e + f*x])^3)
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a+a\sin(e+fx))^3} + \frac{\int \frac{(c+d\sin(e+fx))^2(a(2A}{}}{}$$

$$= -\frac{(3B(c-3d)+2A(c+2d))\cos(e+fx)(c+d\sin(e+fx))^2}{15af(a+a\sin(e+fx))^2} - ($$

$$= -\frac{(3B(c-3d)+2A(c+2d))\cos(e+fx)(c+d\sin(e+fx))^2}{15af(a+a\sin(e+fx))^2} - ($$

$$= \frac{d^2(3B(c-9d)+A(2c+7d))\cos(e+fx)}{15a^3f} - \frac{(3B(c-3d)+2A(c}{15a}$$

$$= \frac{d^2(3B(c-d)+Ad)x}{a^3} + \frac{d^2(3B(c-9d)+A(2c+7d))\cos(e+fx}{15a^3f}$$

$$= \frac{d^2(3B(c-d)+Ad)x}{a^3} + \frac{d^2(3B(c-9d)+A(2c+7d))\cos(e+fx}{15a^3f}$$

Mathematica [A] time = 2.97, size = 366, normalized size = 1.63

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2(c-d)\left(A\left(2c^2+11cd+32d^2\right)+3B\left(c^2+8cd-24d^2\right)\right)\sin\left(\frac{1}{2}(e+fx)\right)\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
)^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(A - B)*(c - d)^3*Sin[(e + f*x)/2
] - 3*(A - B)*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(c - d)^2
*(3*B*(c - 6*d) + A*(2*c + 13*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^2 - (c - d)^2*(3*B*(c - 6*d) + A*(2*c + 13*d))*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^3 + 2*(c - d)*(3*B*(c^2 + 8*c*d - 24*d^2) + A*(2*c^
2 + 11*c*d + 32*d^2))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^4 - 15*d^2*(-3*B*c - A*d + 3*B*d)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2])^5 - 15*B*d^3*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
)/(15*a^3*f*(1 + Sin[e + f*x])^3)
```

fricas [B] time = 0.48, size = 649, normalized size = 2.88

$$-\frac{15\,Bd^3\cos\left(fx+e\right)^4-3\,(A-B)c^3+9\,(A-B)c^2d-9\,(A-B)cd^2+3\,(A-B)d^3+\left((2\,A+3\,B)c^3+3\,(3\,A+7\,B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/15*(15*B*d^3*cos(f*x + e)^4 - 3*(A - B)*c^3 + 9*(A - B)*c^2*d - 9*(A - B
)*c*d^2 + 3*(A - B)*d^3 + ((2*A + 3*B)*c^3 + 3*(3*A + 7*B)*c^2*d + 3*(7*A -
 32*B)*c*d^2 - (32*A - 117*B)*d^3 - 15*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x)*cos
(f*x + e)^3 + 60*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x - (2*(2*A + 3*B)*c^3 + 3*(
6*A - B)*c^2*d - 3*(A + 19*B)*c*d^2 - (19*A - 84*B)*d^3 + 45*(3*B*c*d^2 + (
A - 3*B)*d^3)*f*x)*cos(f*x + e)^2 - 3*((3*A + 2*B)*c^3 + 3*(2*A + 3*B)*c^2*
d + 9*(A - 6*B)*c*d^2 - 9*(2*A - 7*B)*d^3 - 10*(3*B*c*d^2 + (A - 3*B)*d^3)*
f*x)*cos(f*x + e) + (15*B*d^3*cos(f*x + e)^3 + 3*(A - B)*c^3 - 9*(A - B)*c^
2*d + 9*(A - B)*c*d^2 - 3*(A - B)*d^3 + 60*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x
- ((2*A + 3*B)*c^3 + 3*(3*A + 7*B)*c^2*d + 3*(7*A - 32*B)*c*d^2 - 2*(16*A -
 51*B)*d^3 + 15*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x)*cos(f*x + e)^2 - 3*((2*A +
 3*B)*c^3 + 3*(3*A + 2*B)*c^2*d + 3*(2*A - 17*B)*c*d^2 - (17*A - 62*B)*d^3
- 10*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x)*cos(f*x + e))*sin(f*x + e))/(a^3*f*co
s(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a
^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

giac [B] time = 0.22, size = 596, normalized size = 2.65

$$-\frac{\frac{30\,Bd^3}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^3}-\frac{15\left(3\,Bcd^2+Ad^3-3\,Bd^3\right)\left(fx+e\right)}{a^3}+\frac{2\left(15\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-45\,Bcd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-15\,Ad^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+45\,Bd^3\,\mathrm{t}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -1/15*(30*B*d^3/((tan(1/2*f*x + 1/2*e)^2 + 1)*a^3) - 15*(3*B*c*d^2 + A*d^3
- 3*B*d^3)*(f*x + e)/a^3 + 2*(15*A*c^3*tan(1/2*f*x + 1/2*e)^4 - 45*B*c*d^2*
tan(1/2*f*x + 1/2*e)^4 - 15*A*d^3*tan(1/2*f*x + 1/2*e)^4 + 45*B*d^3*tan(1/2
*f*x + 1/2*e)^4 + 30*A*c^3*tan(1/2*f*x + 1/2*e)^3 + 15*B*c^3*tan(1/2*f*x +
1/2*e)^3 + 45*A*c^2*d*tan(1/2*f*x + 1/2*e)^3 - 225*B*c*d^2*tan(1/2*f*x + 1/
2*e)^3 - 75*A*d^3*tan(1/2*f*x + 1/2*e)^3 + 210*B*d^3*tan(1/2*f*x + 1/2*e)^3
```

```
 + 40*A*c^3*tan(1/2*f*x + 1/2*e)^2 + 15*B*c^3*tan(1/2*f*x + 1/2*e)^2 + 45*A
*c^2*d*tan(1/2*f*x + 1/2*e)^2 + 60*B*c^2*d*tan(1/2*f*x + 1/2*e)^2 + 60*A*c*
d^2*tan(1/2*f*x + 1/2*e)^2 - 435*B*c*d^2*tan(1/2*f*x + 1/2*e)^2 - 145*A*d^3
*tan(1/2*f*x + 1/2*e)^2 + 360*B*d^3*tan(1/2*f*x + 1/2*e)^2 + 20*A*c^3*tan(1
/2*f*x + 1/2*e) + 15*B*c^3*tan(1/2*f*x + 1/2*e) + 45*A*c^2*d*tan(1/2*f*x +
1/2*e) + 30*B*c^2*d*tan(1/2*f*x + 1/2*e) + 30*A*c*d^2*tan(1/2*f*x + 1/2*e)
- 285*B*c*d^2*tan(1/2*f*x + 1/2*e) - 95*A*d^3*tan(1/2*f*x + 1/2*e) + 240*B*
d^3*tan(1/2*f*x + 1/2*e) + 7*A*c^3 + 3*B*c^3 + 9*A*c^2*d + 6*B*c^2*d + 6*A*
c*d^2 - 66*B*c*d^2 - 22*A*d^3 + 57*B*d^3)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5
))/f
```

maple [B] time = 0.43, size = 936, normalized size = 4.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 6/a^3/f*d^2*B*arctan(tan(1/2*f*x+1/2*e))*c+6/a^3/f/(tan(1/2*f*x+1/2*e)+1)*B
*c*d^2-24/5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5*B*c^2*d+24/5/a^3/f/(tan(1/2*f*x+
1/2*e)+1)^5*B*c*d^2-6/a^3/f/(tan(1/2*f*x+1/2*e)+1)^2*A*c^2*d-6/a^3/f/(tan(1
/2*f*x+1/2*e)+1)*B*d^3+4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^2*A*c^3+2/a^3/f/(tan(
1/2*f*x+1/2*e)+1)^2*A*d^3-2/a^3/f/(tan(1/2*f*x+1/2*e)+1)^2*B*c^3-4/a^3/f/(t
an(1/2*f*x+1/2*e)+1)^2*B*d^3-16/3/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3*A*c^3+4/3/
a^3/f/(tan(1/2*f*x+1/2*e)+1)^3*A*d^3+4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3*B*c^3
-4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*B*c^3+4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*B*
d^3-8/5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5*A*c^3+8/5/a^3/f/(tan(1/2*f*x+1/2*e)+
1)^5*A*d^3+8/5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5*B*c^3-8/5/a^3/f/(tan(1/2*f*x+
1/2*e)+1)^5*B*d^3-2/a^3/f*d^3*B/(1+tan(1/2*f*x+1/2*e)^2)+2/a^3/f*d^3*A*arct
an(tan(1/2*f*x+1/2*e))+6/a^3/f/(tan(1/2*f*x+1/2*e)+1)^2*B*c*d^2+12/a^3/f/(t
an(1/2*f*x+1/2*e)+1)^3*A*c^2*d-8/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3*A*c*d^2-8/a
^3/f/(tan(1/2*f*x+1/2*e)+1)^3*B*c^2*d+4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3*B*c*
d^2-12/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*A*c^2*d+2/a^3/f/(tan(1/2*f*x+1/2*e)+1
)*A*d^3+12/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*A*c*d^2+12/a^3/f/(tan(1/2*f*x+1/2
*e)+1)^4*B*c^2*d-12/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*B*c*d^2+24/5/a^3/f/(tan(
1/2*f*x+1/2*e)+1)^5*A*c^2*d-24/5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5*A*c*d^2-6/a
^3/f*d^3*B*arctan(tan(1/2*f*x+1/2*e))-2/a^3/f/(tan(1/2*f*x+1/2*e)+1)*A*c^3+
4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*A*c^3-4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*A*d
^3
```

maxima [B] time = 0.58, size = 1682, normalized size = 7.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorit
hm="maxima")

[Out] -2/15*(3*B*d^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*
x + e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1
1*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x +
 e)/(cos(f*x + e) + 1))/a^3) - 3*B*c*d^2*((95*sin(f*x + e)/(cos(f*x + e) +
1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(
f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x
+ e)/(cos(f*x + e) + 1))/a^3) - A*d^3*((95*sin(f*x + e)/(cos(f*x + e) + 1)
+ 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x
 + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*
a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/a^3) + A*c^3*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/
(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 6*B*c^2*d*(5*sin(f*x + e)/(
cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^
3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 6*A*c*d^2*(5
*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
 + 3*B*c^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*
x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10
*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 9*A*c^2*d*(5*sin(f*x
 + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e)
+ 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(
```

```
cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

mupad [B] time = 15.70, size = 593, normalized size = 2.64

$$\frac{2\,d^2\,\mathrm{atan}\left(\frac{2\,d^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(A\,d+3\,B\,c-3\,B\,d\right)}{2\,A\,d^3-6\,B\,d^3+6\,B\,c\,d^2}\right)\,\left(A\,d+3\,B\,c-3\,B\,d\right)}{a^3\,f}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,\left(4\,A\,c^3-10\,A\,d^3+2\,B\,c^3+30\,B\,d^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (2*d^2*atan((2*d^2*tan(e/2 + (f*x)/2)*(A*d + 3*B*c - 3*B*d))/(2*A*d^3 - 6*B
*d^3 + 6*B*c*d^2))*(A*d + 3*B*c - 3*B*d))/(a^3*f) - (tan(e/2 + (f*x)/2)^5*(
4*A*c^3 - 10*A*d^3 + 2*B*c^3 + 30*B*d^3 + 6*A*c^2*d - 30*B*c*d^2) + tan(e/2
 + (f*x)/2)*((8*A*c^3)/3 - (38*A*d^3)/3 + 2*B*c^3 + 42*B*d^3 + 4*A*c*d^2 +
6*A*c^2*d - 38*B*c*d^2 + 4*B*c^2*d) + (14*A*c^3)/15 - (44*A*d^3)/15 + (2*B*
c^3)/5 + (48*B*d^3)/5 + tan(e/2 + (f*x)/2)^4*((22*A*c^3)/3 - (64*A*d^3)/3 +
 2*B*c^3 + 64*B*d^3 + 8*A*c*d^2 + 6*A*c^2*d - 64*B*c*d^2 + 8*B*c^2*d) + tan
(e/2 + (f*x)/2)^3*((20*A*c^3)/3 - (68*A*d^3)/3 + 4*B*c^3 + 80*B*d^3 + 4*A*c
*d^2 + 12*A*c^2*d - 68*B*c*d^2 + 4*B*c^2*d) + tan(e/2 + (f*x)/2)^2*((94*A*c
^3)/15 - (334*A*d^3)/15 + (12*B*c^3)/5 + (378*B*d^3)/5 + (44*A*c*d^2)/5 + (
36*A*c^2*d)/5 - (334*B*c*d^2)/5 + (44*B*c^2*d)/5) + tan(e/2 + (f*x)/2)^6*(2
*A*c^3 - 2*A*d^3 + 6*B*d^3 - 6*B*c*d^2) + (4*A*c*d^2)/5 + (6*A*c^2*d)/5 - (
44*B*c*d^2)/5 + (4*B*c^2*d)/5)/(f*(11*a^3*tan(e/2 + (f*x)/2)^2 + 15*a^3*tan
(e/2 + (f*x)/2)^3 + 15*a^3*tan(e/2 + (f*x)/2)^4 + 11*a^3*tan(e/2 + (f*x)/2)
^5 + 5*a^3*tan(e/2 + (f*x)/2)^6 + a^3*tan(e/2 + (f*x)/2)^7 + a^3 + 5*a^3*ta
n(e/2 + (f*x)/2)))
```

sympy [A] time = 56.43, size = 11456, normalized size = 50.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*c**3*tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 +
75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f
*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2
+ f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c**3*tan(e/2 +
 f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 +
 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3
```

```
*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2
 + f*x/2) + 15*a**3*f) - 110*A*c**3*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2
+ f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**
5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a
**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 100*A
*c**3*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/
2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2
)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75
*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 94*A*c**3*tan(e/2 + f*x/2)**2/(15*a
**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(
e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x
/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a
**3*f) - 40*A*c**3*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**
3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e
/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/
2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*A*c**3/(15*a**3*f*tan(
e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/
2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 1
65*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 9
0*A*c**2*d*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*t
an(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 +
f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2
 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 90*A*c**2*d*tan(e/2 + f*x/2)**
4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3
*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/
2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)
 + 15*a**3*f) - 180*A*c**2*d*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2
)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225
*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*t
an(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 108*A*c**2*d
*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f
*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4
+ 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3
*f*tan(e/2 + f*x/2) + 15*a**3*f) - 90*A*c**2*d*tan(e/2 + f*x/2)/(15*a**3*f*
tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 +
f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3
 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f)
 - 18*A*c**2*d/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)*
*6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*
a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan
(e/2 + f*x/2) + 15*a**3*f) - 120*A*c*d**2*tan(e/2 + f*x/2)**4/(15*a**3*f*ta
n(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*
x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 +
 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) -
 60*A*c*d**2*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f
```

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*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2
+ f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)*
*2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 132*A*c*d**2*tan(e/2 + f*x/2
)**2/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a
**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan
(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x
/2) + 15*a**3*f) - 60*A*c*d**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)
**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*
a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*ta
n(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 12*A*c*d**2/(
15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*
tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 +
 f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
15*a**3*f) + 15*A*d**3*f*x*tan(e/2 + f*x/2)**7/(15*a**3*f*tan(e/2 + f*x/2)*
*7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a
**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan
(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 75*A*d**3*f*x*
tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*
x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 +
 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*
f*tan(e/2 + f*x/2) + 15*a**3*f) + 165*A*d**3*f*x*tan(e/2 + f*x/2)**5/(15*a*
*3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e
/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/
2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a*
*3*f) + 225*A*d**3*f*x*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 +
 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*
f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2
 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 225*A*d**3*f*x*tan
(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2
)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 22
5*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*t
an(e/2 + f*x/2) + 15*a**3*f) + 165*A*d**3*f*x*tan(e/2 + f*x/2)**2/(15*a**3*
f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2
+ f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)*
*3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*
f) + 75*A*d**3*f*x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**
3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e
/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/
2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 15*A*d**3*f*x/(15*a**3*f*
tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 +
f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3
 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f)
 + 30*A*d**3*tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f
*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2
+ f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)*
```

```
*2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 150*A*d**3*tan(e/2 + f*x/2)*
*5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**
3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e
/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2
) + 15*a**3*f) + 320*A*d**3*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)
**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*
a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*ta
n(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 340*A*d**3*ta
n(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/
2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 2
25*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*
tan(e/2 + f*x/2) + 15*a**3*f) + 334*A*d**3*tan(e/2 + f*x/2)**2/(15*a**3*f*t
an(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f
*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3
+ 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f)
+ 190*A*d**3*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*ta
n(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f
*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2
+ 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 44*A*d**3/(15*a**3*f*tan(e/2 +
f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5
+ 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**
3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*c*
*3*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 +
 f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**
4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a*
*3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*c**3*tan(e/2 + f*x/2)**4/(15*a**3
*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2
 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)
**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3
*f) - 60*B*c**3*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**
3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e
/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/
2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 36*B*c**3*tan(e/2 + f*x/2
)**2/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a
**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan
(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x
/2) + 15*a**3*f) - 30*B*c**3*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**
7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a*
*3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(
e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 6*B*c**3/(15*a*
*3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e
/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/
2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a*
*3*f) - 120*B*c**2*d*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 7
5*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*
```

```
tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 +
 f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*B*c**2*d*tan(e/2
+ f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6
+ 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**
3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/
2 + f*x/2) + 15*a**3*f) - 132*B*c**2*d*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e
/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2
)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 16
5*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60
*B*c**2*d*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e
/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/
2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 7
5*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 12*B*c**2*d/(15*a**3*f*tan(e/2 + f
*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 +
 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3
*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 45*B*c*d
**2*f*x*tan(e/2 + f*x/2)**7/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(
e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x
/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 +
75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 225*B*c*d**2*f*x*tan(e/2 + f*x/2)
**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a*
*3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(
e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/
2) + 15*a**3*f) + 495*B*c*d**2*f*x*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 +
 f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5
 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a*
*3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 675*B*
c*d**2*f*x*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*t
an(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 +
f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2
 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 675*B*c*d**2*f*x*tan(e/2 + f*x
/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165
*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*t
an(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f
*x/2) + 15*a**3*f) + 495*B*c*d**2*f*x*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/
2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)
**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165
*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 225
*B*c*d**2*f*x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*t
an(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 +
f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2
 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 45*B*c*d**2*f*x/(15*a**3*f*tan
(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x
/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 +
165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) +
```

90*B*c*d**2*tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 450*B*c*d**2*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 960*B*c*d**2*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 1020*B*c*d**2*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 1002*B*c*d**2*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 570*B*c*d**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 132*B*c*d**2/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 45*B*d**3*f*x*tan(e/2 + f*x/2)**7/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 225*B*d**3*f*x*tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 495*B*d**3*f*x*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 675*B*d**3*f*x*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 675*B*d**3*f*x*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 495*B*d**3*f*x*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x

```
/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 +
165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) -
225*B*d**3*f*x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*
tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 +
 f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**
2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 45*B*d**3*f*x/(15*a**3*f*tan(
e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/
2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 1
65*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 9
0*B*d**3*tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan
(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*
x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 +
 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 450*B*d**3*tan(e/2 + f*x/2)**5/(
15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*
tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 +
 f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
15*a**3*f) - 960*B*d**3*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7
+ 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3
*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/
2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 1200*B*d**3*tan(e
/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)*
*6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*
a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan
(e/2 + f*x/2) + 15*a**3*f) - 1134*B*d**3*tan(e/2 + f*x/2)**2/(15*a**3*f*tan
(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x
/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 +
165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) -
630*B*d**3*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(
e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x
/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 +
75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 144*B*d**3/(15*a**3*f*tan(e/2 + f
*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 +
 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3
*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0))
, (x*(A + B*sin(e))*(c + d*sin(e))**3/(a*sin(e) + a)**3, True))
```

3.280 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=164

$$-\frac{\left(2A\left(c^2+3cd+2d^2\right)+B\left(3c^2+14cd-29d^2\right)\right)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}+\frac{Bd^2x}{a^3}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}-\frac{(c$$

```
[Out] B*d^2*x/a^3-1/15*(c-d)*(B*(3*c-7*d)+2*A*(c+d))*cos(f*x+e)/a/f/(a+a*sin(f*x+
e))^2-1/15*(B*(3*c^2+14*c*d-29*d^2)+2*A*(c^2+3*c*d+2*d^2))*cos(f*x+e)/f/(a^
3+a^3*sin(f*x+e))-1/5*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e)
)^3
```

Rubi [A] time = 0.46, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2977, 2968, 3019, 2735, 2648}

$$-\frac{\left(2A\left(c^2+3cd+2d^2\right)+B\left(3c^2+14cd-29d^2\right)\right)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}+\frac{Bd^2x}{a^3}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}-\frac{(c$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (B*d^2*x)/a^3 - ((c - d)*(B*(3*c - 7*d) + 2*A*(c + d))*Cos[e + f*x])/(15*a*
f*(a + a*Sin[e + f*x])^2) - ((B*(3*c^2 + 14*c*d - 29*d^2) + 2*A*(c^2 + 3*c*
d + 2*d^2))*Cos[e + f*x])/(15*f*(a^3 + a^3*Sin[e + f*x])) - ((A - B)*Cos[e
+ f*x]*(c + d*Sin[e + f*x])^2)/(5*f*(a + a*Sin[e + f*x])^3)
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a+a\sin(e+fx))^3} + \frac{\int \frac{(c+d\sin(e+fx))(a(B(3}{(a}}{}$$

$$= -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a+a\sin(e+fx))^3} + \frac{\int \frac{ac(B(3c-2d)+2A(c+d)}{}}{}$$

$$= -\frac{(c-d)(B(3c-7d)+2A(c+d))\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(A-B)\cos(e}{5f(a+}$$

$$= \frac{Bd^2x}{a^3} - \frac{(c-d)(B(3c-7d)+2A(c+d))\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(A-B)}{}$$

$$= \frac{Bd^2x}{a^3} - \frac{(c-d)(B(3c-7d)+2A(c+d))\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{\left(B\left(3c^2\right.\right.}{}$$

Mathematica [B] time = 0.90, size = 514, normalized size = 3.13

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(30\cos\left(\frac{1}{2}(e+fx)\right)\left(2Ad(c+d)+B\left(c^2+4cd+d^2(5e+5fx-9)\right)\right)-5\cos\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
)^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(30*(2*A*d*(c + d) + B*(c^2 + 4*c*d
+ d^2*(-9 + 5*e + 5*f*x)))*Cos[(e + f*x)/2] - 5*(4*A*(c^2 + 3*c*d + 2*d^2)
+ B*(6*c^2 + 16*c*d + d^2*(-46 + 15*e + 15*f*x)))*Cos[(3*(e + f*x))/2] - 15
*B*d^2*e*Cos[(5*(e + f*x))/2] - 15*B*d^2*f*x*Cos[(5*(e + f*x))/2] + 40*A*c^
2*Sin[(e + f*x)/2] + 30*B*c^2*Sin[(e + f*x)/2] + 60*A*c*d*Sin[(e + f*x)/2]
+ 160*B*c*d*Sin[(e + f*x)/2] + 80*A*d^2*Sin[(e + f*x)/2] - 370*B*d^2*Sin[(e
 + f*x)/2] + 150*B*d^2*e*Sin[(e + f*x)/2] + 150*B*d^2*f*x*Sin[(e + f*x)/2]
+ 60*B*c*d*Sin[(3*(e + f*x))/2] + 30*A*d^2*Sin[(3*(e + f*x))/2] - 90*B*d^2*
Sin[(3*(e + f*x))/2] + 75*B*d^2*e*Sin[(3*(e + f*x))/2] + 75*B*d^2*f*x*Sin[(
3*(e + f*x))/2] - 4*A*c^2*Sin[(5*(e + f*x))/2] - 6*B*c^2*Sin[(5*(e + f*x))/
2] - 12*A*c*d*Sin[(5*(e + f*x))/2] - 28*B*c*d*Sin[(5*(e + f*x))/2] - 14*A*d
^2*Sin[(5*(e + f*x))/2] + 64*B*d^2*Sin[(5*(e + f*x))/2] - 15*B*d^2*e*Sin[(5
*(e + f*x))/2] - 15*B*d^2*f*x*Sin[(5*(e + f*x))/2]))/(60*a^3*f*(1 + Sin[e +
 f*x])^3)
```

fricas [B] time = 0.46, size = 432, normalized size = 2.63

$$-\frac{60\,Bd^2fx-\left(15\,Bd^2fx-(2\,A+3\,B)c^2-2\,(3\,A+7\,B)cd-(7\,A-32\,B)d^2\right)\cos\left(fx+e\right)^3-3\,(A-B)c^2+6\,(A-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/15*(60*B*d^2*f*x - (15*B*d^2*f*x - (2*A + 3*B)*c^2 - 2*(3*A + 7*B)*c*d -
 (7*A - 32*B)*d^2)*cos(f*x + e)^3 - 3*(A - B)*c^2 + 6*(A - B)*c*d - 3*(A -
B)*d^2 - (45*B*d^2*f*x + 2*(2*A + 3*B)*c^2 + 2*(6*A - B)*c*d - (A + 19*B)*d
^2)*cos(f*x + e)^2 + 3*(10*B*d^2*f*x - (3*A + 2*B)*c^2 - 2*(2*A + 3*B)*c*d
- 3*(A - 6*B)*d^2)*cos(f*x + e) + (60*B*d^2*f*x + 3*(A - B)*c^2 - 6*(A - B)
*c*d + 3*(A - B)*d^2 - (15*B*d^2*f*x + (2*A + 3*B)*c^2 + 2*(3*A + 7*B)*c*d
+ (7*A - 32*B)*d^2)*cos(f*x + e)^2 + 3*(10*B*d^2*f*x - (2*A + 3*B)*c^2 - 2*
(3*A + 2*B)*c*d - (2*A - 17*B)*d^2)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(
```

```
f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3
*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

giac [B] time = 0.19, size = 382, normalized size = 2.33

$$\frac{\frac{15\left(fx+e\right)Bd^2}{a^3} - \frac{2\left(15\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 - 15\,Bd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 + 30\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 15\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 30\,Acd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 - 75\,B\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/15*(15*(f*x + e)*B*d^2/a^3 - 2*(15*A*c^2*tan(1/2*f*x + 1/2*e)^4 - 15*B*d^
2*tan(1/2*f*x + 1/2*e)^4 + 30*A*c^2*tan(1/2*f*x + 1/2*e)^3 + 15*B*c^2*tan(1
/2*f*x + 1/2*e)^3 + 30*A*c*d*tan(1/2*f*x + 1/2*e)^3 - 75*B*d^2*tan(1/2*f*x
+ 1/2*e)^3 + 40*A*c^2*tan(1/2*f*x + 1/2*e)^2 + 15*B*c^2*tan(1/2*f*x + 1/2*e
)^2 + 30*A*c*d*tan(1/2*f*x + 1/2*e)^2 + 40*B*c*d*tan(1/2*f*x + 1/2*e)^2 + 2
0*A*d^2*tan(1/2*f*x + 1/2*e)^2 - 145*B*d^2*tan(1/2*f*x + 1/2*e)^2 + 20*A*c^
2*tan(1/2*f*x + 1/2*e) + 15*B*c^2*tan(1/2*f*x + 1/2*e) + 30*A*c*d*tan(1/2*f
*x + 1/2*e) + 20*B*c*d*tan(1/2*f*x + 1/2*e) + 10*A*d^2*tan(1/2*f*x + 1/2*e)
 - 95*B*d^2*tan(1/2*f*x + 1/2*e) + 7*A*c^2 + 3*B*c^2 + 6*A*c*d + 4*B*c*d +
2*A*d^2 - 22*B*d^2)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

maple [B] time = 0.42, size = 617, normalized size = 3.76

$$-\frac{2B\,c^2}{a^3 f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{2B\,d^2}{a^3 f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{2B\,d^2\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^3 f}-\frac{2A\,c^2}{a^3 f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x)
```

```
[Out] -2/a^3/f/(tan(1/2*f*x+1/2*e)+1)^2*B*c^2+2/a^3/f/(tan(1/2*f*x+1/2*e)+1)^2*B*
d^2+2/a^3/f*B*d^2*arctan(tan(1/2*f*x+1/2*e))-2/a^3/f/(tan(1/2*f*x+1/2*e)+1)
*A*c^2+2/a^3/f/(tan(1/2*f*x+1/2*e)+1)*B*d^2+4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^
2*A*c^2-4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*B*d^2-8/5/a^3/f/(tan(1/2*f*x+1/2*e
)+1)^5*A*c^2-8/5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5*A*d^2+8/5/a^3/f/(tan(1/2*f*
x+1/2*e)+1)^5*B*c^2+8/5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5*B*d^2-16/3/a^3/f/(ta
n(1/2*f*x+1/2*e)+1)^3*A*c^2-8/3/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3*A*d^2+4/a^3/
f/(tan(1/2*f*x+1/2*e)+1)^3*B*c^2+4/3/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3*B*d^2+4
/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*A*c^2+4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*A*d^
```

```
2-16/5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5*B*c*d-4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^
4*B*c^2+8/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3*A*c*d-16/3/a^3/f/(tan(1/2*f*x+1/2*
e)+1)^3*B*c*d-4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^2*A*c*d-8/a^3/f/(tan(1/2*f*x+1
/2*e)+1)^4*A*c*d+8/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*B*c*d+16/5/a^3/f/(tan(1/2
*f*x+1/2*e)+1)^5*A*c*d
```

maxima [B] time = 0.55, size = 1132, normalized size = 6.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 2/15*(B*d^2*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) +
 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e
)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3)
 - A*c^2*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5) - 4*B*c*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5) - 2*A*d^2*(5*sin(f*x + e)/(cos(f*x + e) +
1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/
(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 3*B*c^2*(5*sin(f*x + e)/(co
s(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10
*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5) - 6*A*c*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))
/f
```

mupad [B] time = 16.54, size = 286, normalized size = 1.74

$$\frac{B\,d^2\,x}{a^3} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{16\,A\,c^2}{3}+\frac{8\,A\,d^2}{3}+2\,B\,c^2-\frac{58\,B\,d^2}{3}+4\,A\,c\,d+\frac{16\,B\,c\,d}{3}\right)+\frac{14\,A\,c^2}{15}+\frac{4\,A\,d^2}{15}+\frac{2\,B\,c^2}{5}-\frac{44\,B\,d^2}{15}+\tan}{f\,\left(a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+5\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (B*d^2*x)/a^3 - (tan(e/2 + (f*x)/2)^2*((16*A*c^2)/3 + (8*A*d^2)/3 + 2*B*c^2
 - (58*B*d^2)/3 + 4*A*c*d + (16*B*c*d)/3) + (14*A*c^2)/15 + (4*A*d^2)/15 +
(2*B*c^2)/5 - (44*B*d^2)/15 + tan(e/2 + (f*x)/2)^3*(4*A*c^2 + 2*B*c^2 - 10*
B*d^2 + 4*A*c*d) + tan(e/2 + (f*x)/2)^4*(2*A*c^2 - 2*B*d^2) + tan(e/2 + (f*
x)/2)*((8*A*c^2)/3 + (4*A*d^2)/3 + 2*B*c^2 - (38*B*d^2)/3 + 4*A*c*d + (8*B*
c*d)/3) + (4*A*c*d)/5 + (8*B*c*d)/15)/(f*(10*a^3*tan(e/2 + (f*x)/2)^2 + 10*
a^3*tan(e/2 + (f*x)/2)^3 + 5*a^3*tan(e/2 + (f*x)/2)^4 + a^3*tan(e/2 + (f*x)
/2)^5 + a^3 + 5*a^3*tan(e/2 + (f*x)/2)))
```

sympy [A] time = 33.40, size = 3468, normalized size = 21.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*c**2*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 +
75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f
*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c**2*
tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*
x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 +
 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 80*A*c**2*tan(e/2 + f*x/2)**2/(1
5*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*t
an(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f
*x/2) + 15*a**3*f) - 40*A*c**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)
**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*
a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*A
*c**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*
a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan
(e/2 + f*x/2) + 15*a**3*f) - 60*A*c*d*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/
2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)
**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3
*f) - 60*A*c*d*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3
*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/
```

2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c*d*tan(e/2
+ f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 1
50*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*
tan(e/2 + f*x/2) + 15*a**3*f) - 12*A*c*d/(15*a**3*f*tan(e/2 + f*x/2)**5 + 7
5*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*
tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 40*A*d**2*t
an(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x
/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 +
75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 20*A*d**2*tan(e/2 + f*x/2)/(15*a*
*3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e
/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2
) + 15*a**3*f) - 4*A*d**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/
2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2
)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*c**2*tan(e/2 + f*x/2)
3/(15*a3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a*
*3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e
/2 + f*x/2) + 15*a**3*f) - 30*B*c**2*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2
 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)*
*3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*
f) - 30*B*c**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*
tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 +
 f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 6*B*c**2/(15*a**3*f*
tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 +
f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 1
5*a**3*f) - 80*B*c*d*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 7
5*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*
tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 40*B*c*d*ta
n(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)*
*4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a
3*f*tan(e/2 + f*x/2) + 15*a3*f) - 8*B*c*d/(15*a**3*f*tan(e/2 + f*x/2)**
5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a*
*3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 15*B*d
2*f*x*tan(e/2 + f*x/2)5/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(
e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x
/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 75*B*d**2*f*x*tan(e/2 +
f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 +
150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f
*tan(e/2 + f*x/2) + 15*a**3*f) + 150*B*d**2*f*x*tan(e/2 + f*x/2)**3/(15*a**
3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/
2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)
 + 15*a**3*f) + 150*B*d**2*f*x*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x
/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 1
50*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 7
5*B*d**2*f*x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*ta
n(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f

```
*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 15*B*d**2*f*x/(15*a**3
*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2
 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)
+ 15*a**3*f) + 30*B*d**2*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5
 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**
3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 150*B*d
**2*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2
+ f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)*
*2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 290*B*d**2*tan(e/2 + f*x/2)*
*2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**
3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/
2 + f*x/2) + 15*a**3*f) + 190*B*d**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 +
f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3
+ 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f)
+ 44*B*d**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4
+ 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3
*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e
))**2/(a*sin(e) + a)**3, True))
```

3.281 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=127

$$-\frac{(2Ac+3Ad+3Bc+7Bd)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}-\frac{(2Ac+3Ad+3Bc-8Bd)\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}-\frac{(A-B)(c-d)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

```
[Out] -1/5*(A-B)*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^3-1/15*(2*A*c+3*A*d+3*B*c-8*
B*d)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^2-1/15*(2*A*c+3*A*d+3*B*c+7*B*d)*cos(f
*x+e)/f/(a^3+a^3*sin(f*x+e))
```

Rubi [A] time = 0.23, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2968, 3019, 2750, 2648}

$$-\frac{(2Ac+3Ad+3Bc+7Bd)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}-\frac{(2Ac+3Ad+3Bc-8Bd)\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}-\frac{(A-B)(c-d)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -((A - B)*(c - d)*Cos[e + f*x])/(5*f*(a + a*Sin[e + f*x])^3) - ((2*A*c + 3*
B*c + 3*A*d - 8*B*d)*Cos[e + f*x])/(15*a*f*(a + a*Sin[e + f*x])^2) - ((2*A*
c + 3*B*c + 3*A*d + 7*B*d)*Cos[e + f*x])/(15*f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \int \frac{Ac+(Bc+Ad)\sin(e+fx)+Bd\sin^2(e+fx)}{(a+a\sin(e+fx))^3}\,dx$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{-a(2Ac+3Bc+3Ad-3Bd)-5aBd\sin(e+fx}{(a+a\sin(e+fx))^2}}{5a^2}$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc+3Ad-8Bd)\cos(e}{15af(a+a\sin(e+fx))^2}$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc+3Ad-8Bd)\cos(e}{15af(a+a\sin(e+fx))^2}$$

Mathematica [A] time = 0.69, size = 176, normalized size = 1.39

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(15(Ad+B(c+2d))\cos\left(\frac{1}{2}(e+fx)\right)-5(2Ac+3Ad+3Bc+4Bd)\cos\left(\frac{3}{2}(e\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^
3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(15*(A*d + B*(c + 2*d))*Cos[(e + f*x
)/2] - 5*(2*A*c + 3*B*c + 3*A*d + 4*B*d)*Cos[(3*(e + f*x))/2] - 2*(-3*(3*A*
c + 2*B*c + 2*A*d + 8*B*d) + (2*A*c + 3*B*c + 3*A*d - 8*B*d)*Cos[e + f*x] +
 (2*A*c + 3*B*c + 3*A*d + 7*B*d)*Cos[2*(e + f*x)])*Sin[(e + f*x)/2]))/(30*a
^3*f*(1 + Sin[e + f*x])^3)
```

fricas [B] time = 0.43, size = 271, normalized size = 2.13

$$-\frac{((2\,A+3\,B)c+(3\,A+7\,B)d)\cos\left(fx+e\right)^{3}-(2\,(2\,A+3\,B)c+(6\,A-B)d)\cos\left(fx+e\right)^{2}-3\,(A-B)c+3\,(A-\ldots}{15\,\left(a^{3}f\cos\left(fx+e\right)^{3}+3\,a^{3}f\cos\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -1/15*(((2*A + 3*B)*c + (3*A + 7*B)*d)*cos(f*x + e)^3 - (2*(2*A + 3*B)*c +
(6*A - B)*d)*cos(f*x + e)^2 - 3*(A - B)*c + 3*(A - B)*d - 3*((3*A + 2*B)*c
+ (2*A + 3*B)*d)*cos(f*x + e) - (((2*A + 3*B)*c + (3*A + 7*B)*d)*cos(f*x +
e)^2 - 3*(A - B)*c + 3*(A - B)*d + 3*((2*A + 3*B)*c + (3*A + 2*B)*d)*cos(f*
x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^
3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) -
 4*a^3*f)*sin(f*x + e))
```

giac [A] time = 0.18, size = 223, normalized size = 1.76

$$-\frac{2\left(15\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+30\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+15\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+15\,Ad\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+40\,A\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -2/15*(15*A*c*tan(1/2*f*x + 1/2*e)^4 + 30*A*c*tan(1/2*f*x + 1/2*e)^3 + 15*B
*c*tan(1/2*f*x + 1/2*e)^3 + 15*A*d*tan(1/2*f*x + 1/2*e)^3 + 40*A*c*tan(1/2*
f*x + 1/2*e)^2 + 15*B*c*tan(1/2*f*x + 1/2*e)^2 + 15*A*d*tan(1/2*f*x + 1/2*e
)^2 + 20*B*d*tan(1/2*f*x + 1/2*e)^2 + 20*A*c*tan(1/2*f*x + 1/2*e) + 15*B*c*
tan(1/2*f*x + 1/2*e) + 15*A*d*tan(1/2*f*x + 1/2*e) + 10*B*d*tan(1/2*f*x + 1
/2*e) + 7*A*c + 3*B*c + 3*A*d + 2*B*d)/(a^3*f*(tan(1/2*f*x + 1/2*e) + 1)^5)
```

maple [A] time = 0.44, size = 151, normalized size = 1.19

$$\frac{-\frac{2(8Ac-6Ad-6Bc+4Bd)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{-8Ac+8Ad+8Bc-8Bd}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{-4Ac+2Ad+2Bc}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2(4Ac-4Ad-4Bc+4Bd)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{2Ac}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{f\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/f/a^3*(-1/3*(8*A*c-6*A*d-6*B*c+4*B*d)/(tan(1/2*f*x+1/2*e)+1)^3-1/4*(-8*A*
c+8*A*d+8*B*c-8*B*d)/(tan(1/2*f*x+1/2*e)+1)^4-1/2*(-4*A*c+2*A*d+2*B*c)/(tan
(1/2*f*x+1/2*e)+1)^2-1/5*(4*A*c-4*A*d-4*B*c+4*B*d)/(tan(1/2*f*x+1/2*e)+1)^5
-A*c/(tan(1/2*f*x+1/2*e)+1))
```

maxima [B] time = 0.46, size = 733, normalized size = 5.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] -2/15*(A*c*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*
a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) + 2*B*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*B*c*(5*sin(f*x + e)/(cos(f*x + e) + 1)
 + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*
a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5) + 3*A*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*s
in(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

mupad [B] time = 14.26, size = 245, normalized size = 1.93

$$\frac{2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{53\,A\,c}{4}+3\,A\,d+3\,B\,c+\frac{13\,B\,d}{4}-4\,A\,c\,\cos\left(e+f\,x\right)+\frac{3\,A\,d\,\cos\left(e+f\,x\right)}{2}+\frac{3\,B\,c\,\cos\left(e+f\,x\right)}{2}+B\,d\,\cos\left(e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((53*A*c)/4 + 3*A*d + 3*B*c + (13*B*d)/4 - 4*A*c*cos(
e + f*x) + (3*A*d*cos(e + f*x))/2 + (3*B*c*cos(e + f*x))/2 + B*d*cos(e + f*
x) + (25*A*c*sin(e + f*x))/2 + (15*A*d*sin(e + f*x))/2 + (15*B*c*sin(e + f*
x))/2 + (5*B*d*sin(e + f*x))/2 - (9*A*c*cos(2*e + 2*f*x))/4 - (3*A*d*cos(2*
e + 2*f*x))/2 - (3*B*c*cos(2*e + 2*f*x))/2 - (9*B*d*cos(2*e + 2*f*x))/4 - (
5*A*c*sin(2*e + 2*f*x))/4 + (5*B*d*sin(2*e + 2*f*x))/4))/(15*a^3*f*((5*2^(1
/2)*cos((3*e)/2 + pi/4 + (3*f*x)/2))/4 - (5*2^(1/2)*cos(e/2 - pi/4 + (f*x)/
2))/2 + (2^(1/2)*cos((5*e)/2 - pi/4 + (5*f*x)/2))/4))
```

sympy [A] time = 15.10, size = 1819, normalized size = 14.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*c*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*
a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*ta
n(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c*tan(e/
2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**
4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a*
*3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 80*A*c*tan(e/2 + f*x/2)**2/(15*a**3*f*
tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 +
f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 1
5*a**3*f) - 40*A*c*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**
3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e
/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*A*c/(15*a**3*
f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2
+ f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
 15*a**3*f) - 30*A*d*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 7
5*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*
tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*A*d*tan(
e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)
**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*
a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*A*d*tan(e/2 + f*x/2)/(15*a**3*f*t
an(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f
*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15
*a**3*f) - 6*A*d/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2
)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75
*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*c*tan(e/2 + f*x/2)**3/(15*a**3
*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2
 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)
+ 15*a**3*f) - 30*B*c*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 +
75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f
*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*c*tan
```

```
(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**
4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a*
*3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 6*B*c/(15*a**3*f*tan(e/2 + f*x/2)**5 +
 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*
f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 40*B*d*ta
n(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/
2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 7
5*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 20*B*d*tan(e/2 + f*x/2)/(15*a**3*f
*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 +
 f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
15*a**3*f) - 4*B*d/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x
/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 +
75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(A + B*sin(e))*(c +
d*sin(e))/(a*sin(e) + a)**3, True))
```

3.282 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=102

$$-\frac{(2A+3B)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}-\frac{(2A+3B)\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}-\frac{(A-B)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

```
[Out] -1/5*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^3-1/15*(2*A+3*B)*cos(f*x+e)/a/f/(a
+a*sin(f*x+e))^2-1/15*(2*A+3*B)*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e))
```

Rubi [A] time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2750, 2650, 2648}

$$-\frac{(2A+3B)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}-\frac{(2A+3B)\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}-\frac{(A-B)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -((A - B)*Cos[e + f*x])/(5*f*(a + a*Sin[e + f*x])^3) - ((2*A + 3*B)*Cos[e +
 f*x])/(15*a*f*(a + a*Sin[e + f*x])^2) - ((2*A + 3*B)*Cos[e + f*x])/(15*f*(
a^3 + a^3*Sin[e + f*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
```

```
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx = -\frac{(A-B)\cos(e+fx)}{5f(a+a\sin(e+fx))^3} + \frac{(2A+3B)\int \frac{1}{(a+a\sin(e+fx))^2}\,dx}{5a}$$

$$= -\frac{(A-B)\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{(2A+3B)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} + \frac{(2A+3B)\int \frac{1}{a+a\sin(e+fx)}\,dx}{15a^2}$$

$$= -\frac{(A-B)\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{(2A+3B)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(2A+3B)\cos(e+fx)}{15f\left(a^3+a^3\sin(e+fx)\right)}$$

Mathematica [A] time = 0.09, size = 63, normalized size = 0.62

$$-\frac{\cos(e+fx)\left((2A+3B)\sin^2(e+fx)+(6A+9B)\sin(e+fx)+7A+3B\right)}{15a^3f(\sin(e+fx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -1/15*(Cos[e + f*x]*(7*A + 3*B + (6*A + 9*B)*Sin[e + f*x] + (2*A + 3*B)*Sin
[e + f*x]^2))/(a^3*f*(1 + Sin[e + f*x])^3)
```

fricas [A] time = 0.42, size = 190, normalized size = 1.86

$$-\frac{(2\,A+3\,B)\cos\left(fx+e\right)^3-2\,(2\,A+3\,B)\cos\left(fx+e\right)^2-3\,(3\,A+2\,B)\cos\left(fx+e\right)-\left((2\,A+3\,B)\cos\left(fx+e\right)\right.}{15\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2-2\,a^3f\cos\left(fx+e\right)-4\,a^3f+\left(a^3f\cos\left(fx+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*((2*A + 3*B)*cos(f*x + e)^3 - 2*(2*A + 3*B)*cos(f*x + e)^2 - 3*(3*A +
 2*B)*cos(f*x + e) - ((2*A + 3*B)*cos(f*x + e)^2 + 3*(2*A + 3*B)*cos(f*x +
e) - 3*A + 3*B)*sin(f*x + e) - 3*A + 3*B)/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*c
os(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*
a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

giac [A] time = 0.17, size = 130, normalized size = 1.27

$$-\frac{2\left(15\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+30\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+15\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+40\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+15\,B\tan\cdots\right.}{15\,a^{3}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*A*tan(1/2*f*x + 1/2*e)^4 + 30*A*tan(1/2*f*x + 1/2*e)^3 + 15*B*tan
(1/2*f*x + 1/2*e)^3 + 40*A*tan(1/2*f*x + 1/2*e)^2 + 15*B*tan(1/2*f*x + 1/2*
e)^2 + 20*A*tan(1/2*f*x + 1/2*e) + 15*B*tan(1/2*f*x + 1/2*e) + 7*A + 3*B)/(
a^3*f*(tan(1/2*f*x + 1/2*e) + 1)^5)
```

maple [A] time = 0.32, size = 114, normalized size = 1.12

$$\frac{-\frac{2(8A-6B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2(4A-4B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{2A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-4A+2B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{-8A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}}{f\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/f/a^3*(-1/3*(8*A-6*B)/(tan(1/2*f*x+1/2*e)+1)^3-1/5*(4*A-4*B)/(tan(1/2*f*x
+1/2*e)+1)^5-A/(tan(1/2*f*x+1/2*e)+1)-1/2*(-4*A+2*B)/(tan(1/2*f*x+1/2*e)+1)
^2-1/4*(-8*A+8*B)/(tan(1/2*f*x+1/2*e)+1)^4)
```

maxima [B] time = 0.39, size = 387, normalized size = 3.79

$$-\frac{2\left(\frac{A\left(\frac{20\sin(fx+e)}{\cos(fx+e)+1}+\frac{40\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{30\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{15\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+7\right)}{a^{3}+\frac{5a^{3}\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^{3}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{10a^{3}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{5a^{3}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{a^{3}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}}+\frac{3B\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}+\frac{5\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{5\sin(fx+\cdots}{(\cos(fx+e)\cdots}\right.}{a^{3}+\frac{5a^{3}\sin(fx+e)}{\cos(fx+e)+1}+\frac{10a^{3}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{10a^{3}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{5a^{3}\sin\cdots}{(\cos(fx\cdots}}\right.}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(A*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^
```

```
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5) + 3*B*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3
 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

mupad [B] time = 13.80, size = 150, normalized size = 1.47

$$\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{53A}{4}+3B-4A\cos\left(e+fx\right)+\frac{3B\cos\left(e+fx\right)}{2}+\frac{25A\sin\left(e+fx\right)}{2}+\frac{15B\sin\left(e+fx\right)}{2}-\frac{9A\cos\left(2e+2fx\right)}{4}-\right.}{15a^3f\left(\frac{5\sqrt{2}\cos\left(\frac{3e}{2}+\frac{\pi}{4}+\frac{3fx}{2}\right)}{4}-\frac{5\sqrt{2}\cos\left(\frac{e}{2}-\frac{\pi}{4}+\frac{fx}{2}\right)}{2}+\frac{\sqrt{2}\cos\left(\frac{5e}{2}-\frac{\pi}{4}+\frac{5fx}{2}\right)}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((53*A)/4 + 3*B - 4*A*cos(e + f*x) + (3*B*cos(e + f*x
))/2 + (25*A*sin(e + f*x))/2 + (15*B*sin(e + f*x))/2 - (9*A*cos(2*e + 2*f*x
))/4 - (3*B*cos(2*e + 2*f*x))/2 - (5*A*sin(2*e + 2*f*x))/4))/(15*a^3*f*((5*
2^(1/2)*cos((3*e)/2 + pi/4 + (3*f*x)/2))/4 - (5*2^(1/2)*cos(e/2 - pi/4 + (f
*x)/2))/2 + (2^(1/2)*cos((5*e)/2 - pi/4 + (5*f*x)/2))/4))
```

sympy [A] time = 8.34, size = 1015, normalized size = 9.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a*
*3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(
e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*tan(e/2 +
f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 +
150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f
*tan(e/2 + f*x/2) + 15*a**3*f) - 80*A*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/
2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)
**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3
*f) - 40*A*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(
e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x
/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*A/(15*a**3*f*tan(e/2
+ f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**
```

```
3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f
) - 30*B*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan
(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*
x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*tan(e/2 + f*x/2)**
2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3
*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2
 + f*x/2) + 15*a**3*f) - 30*B*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)*
*5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a
**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 6*B/(
15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*
tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 +
f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(A + B*sin(e))/(a*sin(e) + a)**3, True))
```

3.283 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=229

$$\frac{2d^2(Bc-Ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^3\sqrt{c^2-d^2}}-\frac{\left(A\left(2c^2-9cd+22d^2\right)+B\left(3c^2-16cd-2d^2\right)\right)\cos(e+fx)}{15f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)}-\frac{(2Ac-7Ad}{15af(c-}$$

```
[Out] -1/5*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^3-1/15*(2*A*c-7*A*d+3*B*c+2*
B*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^2-1/15*(B*(3*c^2-16*c*d-2*d^2)
+A*(2*c^2-9*c*d+22*d^2))*cos(f*x+e)/(c-d)^3/f/(a^3+a^3*sin(f*x+e))+2*d^2*(-
A*d+B*c)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^3/(c-d)^3/f/(c^
2-d^2)^(1/2)
```

Rubi [A] time = 0.72, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2978, 12, 2660, 618, 204}

$$\frac{2d^2(Bc-Ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^3\sqrt{c^2-d^2}}-\frac{\left(A\left(2c^2-9cd+22d^2\right)+B\left(3c^2-16cd-2d^2\right)\right)\cos(e+fx)}{15f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)}-\frac{(2Ac-7Ad}{15af(c-}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])),x]
```

```
[Out] (2*d^2*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^3*(
c - d)^3*Sqrt[c^2 - d^2]*f) - ((A - B)*Cos[e + f*x])/(5*(c - d)*f*(a + a*Si
n[e + f*x])^3) - ((2*A*c + 3*B*c - 7*A*d + 2*B*d)*Cos[e + f*x])/(15*a*(c -
d)^2*f*(a + a*Sin[e + f*x])^2) - ((B*(3*c^2 - 16*c*d - 2*d^2) + A*(2*c^2 -
9*c*d + 22*d^2))*Cos[e + f*x])/(15*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx = -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{\int \frac{-a(2Ac+3Bc-5Ad)-2a(A-B)d\sin(e+}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}}{5a^2(c-d)}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc-7Ad+2Bd)\cos}{15a(c-d)^2f(a+a\sin(e+}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc-7Ad+2Bd)\cos}{15a(c-d)^2f(a+a\sin(e+}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc-7Ad+2Bd)\cos}{15a(c-d)^2f(a+a\sin(e+}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc-7Ad+2Bd)\cos}{15a(c-d)^2f(a+a\sin(e+}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc-7Ad+2Bd)\cos}{15a(c-d)^2f(a+a\sin(e+}$$

$$= \frac{2d^2(Bc-Ad)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^3\sqrt{c^2-d^2}\,f} - \frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+}$$

Mathematica [B] time = 1.25, size = 502, normalized size = 2.19

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-\frac{60d^2(Ad-Bc)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + 20Ac^2\sin\left(\frac{1}{2}(e+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(15*B*c^2*Cos[(e + f*x)/2] - 15*A*c*
d*Cos[(e + f*x)/2] - 75*B*c*d*Cos[(e + f*x)/2] + 75*A*d^2*Cos[(e + f*x)/2]
- 10*A*c^2*Cos[(3*(e + f*x))/2] - 15*B*c^2*Cos[(3*(e + f*x))/2] + 45*A*c*d*
```

```
Cos[(3*(e + f*x))/2] + 65*B*c*d*Cos[(3*(e + f*x))/2] - 95*A*d^2*Cos[(3*(e +
 f*x))/2] + 10*B*d^2*Cos[(3*(e + f*x))/2] + 20*A*c^2*Sin[(e + f*x)/2] + 15*
B*c^2*Sin[(e + f*x)/2] - 75*A*c*d*Sin[(e + f*x)/2] - 85*B*c*d*Sin[(e + f*x)
/2] + 145*A*d^2*Sin[(e + f*x)/2] - 20*B*d^2*Sin[(e + f*x)/2] - (60*d^2*(-(B
*c) + A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^5)/Sqrt[c^2 - d^2] - 15*B*c*d*Sin[(3*(e + f*x))/2] +
 15*A*d^2*Sin[(3*(e + f*x))/2] - 2*A*c^2*Sin[(5*(e + f*x))/2] - 3*B*c^2*Sin
[(5*(e + f*x))/2] + 9*A*c*d*Sin[(5*(e + f*x))/2] + 16*B*c*d*Sin[(5*(e + f*x
))/2] - 22*A*d^2*Sin[(5*(e + f*x))/2] + 2*B*d^2*Sin[(5*(e + f*x))/2]))/(30*
a^3*(c - d)^3*f*(1 + Sin[e + f*x])^3)
```

fricas [B] time = 0.54, size = 2292, normalized size = 10.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] [1/30*(6*(A - B)*c^4 - 12*(A - B)*c^3*d + 12*(A - B)*c*d^3 - 6*(A - B)*d^4
- 2*((2*A + 3*B)*c^4 - (9*A + 16*B)*c^3*d + 5*(4*A - B)*c^2*d^2 + (9*A + 16
*B)*c*d^3 - 2*(11*A - B)*d^4)*cos(f*x + e)^3 + 2*(2*(2*A + 3*B)*c^4 - (18*A
 + 17*B)*c^3*d + 5*(5*A - 2*B)*c^2*d^2 + (18*A + 17*B)*c*d^3 - (29*A - 4*B)
*d^4)*cos(f*x + e)^2 + 15*(4*B*c*d^2 - 4*A*d^3 - (B*c*d^2 - A*d^3)*cos(f*x
+ e)^3 - 3*(B*c*d^2 - A*d^3)*cos(f*x + e)^2 + 2*(B*c*d^2 - A*d^3)*cos(f*x +
 e) + (4*B*c*d^2 - 4*A*d^3 - (B*c*d^2 - A*d^3)*cos(f*x + e)^2 + 2*(B*c*d^2
- A*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*co
s(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x +
 e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x
 + e) - c^2 - d^2)) + 6*((3*A + 2*B)*c^4 - (11*A + 9*B)*c^3*d + 5*(3*A - B)
*c^2*d^2 + (11*A + 9*B)*c*d^3 - 3*(6*A - B)*d^4)*cos(f*x + e) - 2*(3*(A - B
)*c^4 - 6*(A - B)*c^3*d + 6*(A - B)*c*d^3 - 3*(A - B)*d^4 - ((2*A + 3*B)*c^
4 - (9*A + 16*B)*c^3*d + 5*(4*A - B)*c^2*d^2 + (9*A + 16*B)*c*d^3 - 2*(11*A
 - B)*d^4)*cos(f*x + e)^2 - 3*((2*A + 3*B)*c^4 - (9*A + 11*B)*c^3*d + 5*(3*
A - B)*c^2*d^2 + (9*A + 11*B)*c*d^3 - (17*A - 2*B)*d^4)*cos(f*x + e))*sin(f
*x + e))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*
d^4 + a^3*d^5)*f*cos(f*x + e)^3 + 3*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2
+ 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2 - 2*(a^3*c^5 - 3*
a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*
x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c
*d^4 + a^3*d^5)*f + ((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3
 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2 - 2*(a^3*c^5 - 3*a^3*c^4*d + 2*a
^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e) - 4*(a^3
*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)
*f)*sin(f*x + e)), 1/15*(3*(A - B)*c^4 - 6*(A - B)*c^3*d + 6*(A - B)*c*d^3
```

```
- 3*(A - B)*d^4 - ((2*A + 3*B)*c^4 - (9*A + 16*B)*c^3*d + 5*(4*A - B)*c^2*d
^2 + (9*A + 16*B)*c*d^3 - 2*(11*A - B)*d^4)*cos(f*x + e)^3 + (2*(2*A + 3*B)
*c^4 - (18*A + 17*B)*c^3*d + 5*(5*A - 2*B)*c^2*d^2 + (18*A + 17*B)*c*d^3 -
(29*A - 4*B)*d^4)*cos(f*x + e)^2 + 15*(4*B*c*d^2 - 4*A*d^3 - (B*c*d^2 - A*d
^3)*cos(f*x + e)^3 - 3*(B*c*d^2 - A*d^3)*cos(f*x + e)^2 + 2*(B*c*d^2 - A*d^
3)*cos(f*x + e) + (4*B*c*d^2 - 4*A*d^3 - (B*c*d^2 - A*d^3)*cos(f*x + e)^2 +
 2*(B*c*d^2 - A*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c
*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + 3*((3*A + 2*B)*c^4 - (
11*A + 9*B)*c^3*d + 5*(3*A - B)*c^2*d^2 + (11*A + 9*B)*c*d^3 - 3*(6*A - B)*
d^4)*cos(f*x + e) - (3*(A - B)*c^4 - 6*(A - B)*c^3*d + 6*(A - B)*c*d^3 - 3*
(A - B)*d^4 - ((2*A + 3*B)*c^4 - (9*A + 16*B)*c^3*d + 5*(4*A - B)*c^2*d^2 +
 (9*A + 16*B)*c*d^3 - 2*(11*A - B)*d^4)*cos(f*x + e)^2 - 3*((2*A + 3*B)*c^4
 - (9*A + 11*B)*c^3*d + 5*(3*A - B)*c^2*d^2 + (9*A + 11*B)*c*d^3 - (17*A -
2*B)*d^4)*cos(f*x + e))*sin(f*x + e))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d
^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^3 + 3*(a^3*c^5 -
 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos
(f*x + e)^2 - 2*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*
a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*
d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f + ((a^3*c^5 - 3*a^3*c^4*d +
2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2 - 2
*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3
*d^5)*f*cos(f*x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2
*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f)*sin(f*x + e))]
```

giac [B] time = 0.26, size = 578, normalized size = 2.52

$$2\left(\frac{15\left(Bcd^2-Ad^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(a^3c^3-3a^3c^2d+3a^3cd^2-a^3d^3\right)\sqrt{c^2-d^2}}-\frac{15Ac^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-45Acd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+45Ad^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-15\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] 2/15*(15*(B*c*d^2 - A*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arcta
n((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^3*c^3 - 3*a^3*c^2*d +
3*a^3*c*d^2 - a^3*d^3)*sqrt(c^2 - d^2)) - (15*A*c^2*tan(1/2*f*x + 1/2*e)^4
- 45*A*c*d*tan(1/2*f*x + 1/2*e)^4 + 45*A*d^2*tan(1/2*f*x + 1/2*e)^4 - 15*B*
d^2*tan(1/2*f*x + 1/2*e)^4 + 30*A*c^2*tan(1/2*f*x + 1/2*e)^3 + 15*B*c^2*tan
(1/2*f*x + 1/2*e)^3 - 105*A*c*d*tan(1/2*f*x + 1/2*e)^3 - 45*B*c*d*tan(1/2*f
*x + 1/2*e)^3 + 135*A*d^2*tan(1/2*f*x + 1/2*e)^3 - 30*B*d^2*tan(1/2*f*x + 1
/2*e)^3 + 40*A*c^2*tan(1/2*f*x + 1/2*e)^2 + 15*B*c^2*tan(1/2*f*x + 1/2*e)^2
```

```
 - 135*A*c*d*tan(1/2*f*x + 1/2*e)^2 - 65*B*c*d*tan(1/2*f*x + 1/2*e)^2 + 185
*A*d^2*tan(1/2*f*x + 1/2*e)^2 - 40*B*d^2*tan(1/2*f*x + 1/2*e)^2 + 20*A*c^2*
tan(1/2*f*x + 1/2*e) + 15*B*c^2*tan(1/2*f*x + 1/2*e) - 75*A*c*d*tan(1/2*f*x
 + 1/2*e) - 55*B*c*d*tan(1/2*f*x + 1/2*e) + 115*A*d^2*tan(1/2*f*x + 1/2*e)
- 20*B*d^2*tan(1/2*f*x + 1/2*e) + 7*A*c^2 + 3*B*c^2 - 24*A*c*d - 11*B*c*d +
 32*A*d^2 - 7*B*d^2)/((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*(tan(
1/2*f*x + 1/2*e) + 1)^5))/f
```

maple [B] time = 0.54, size = 606, normalized size = 2.65

$$-\frac{2d^3 \arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)A}{f\,a^3\,(c-d)^3\sqrt{c^2-d^2}}+\frac{2d^2 \arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)Bc}{f\,a^3\,(c-d)^3\sqrt{c^2-d^2}}+\frac{4A}{f\,a^3\,(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{}{f\,a^3\,(c-d)\,(\mathrm{t}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x)
```

```
[Out] -2/f/a^3*d^3/(c-d)^3/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d
)/(c^2-d^2)^(1/2))*A+2/f/a^3*d^2/(c-d)^3/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*ta
n(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c+4/f/a^3/(c-d)/(tan(1/2*f*x+1/2*e
)+1)^4*A-4/f/a^3/(c-d)/(tan(1/2*f*x+1/2*e)+1)^4*B-8/5/f/a^3/(c-d)/(tan(1/2*
f*x+1/2*e)+1)^5*A+8/5/f/a^3/(c-d)/(tan(1/2*f*x+1/2*e)+1)^5*B+4/f/a^3/(c-d)^
2/(tan(1/2*f*x+1/2*e)+1)^2*A*c-6/f/a^3/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^2*A*d
-2/f/a^3/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^2*B*c+4/f/a^3/(c-d)^2/(tan(1/2*f*x+
1/2*e)+1)^2*B*d-16/3/f/a^3/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^3*A*c+20/3/f/a^3/
(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^3*A*d+4/f/a^3/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)
^3*B*c-16/3/f/a^3/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^3*B*d-2/f/a^3/(c-d)^3/(tan
(1/2*f*x+1/2*e)+1)*A*c^2+6/f/a^3/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)*A*c*d-6/f/a
^3/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)*A*d^2+2/f/a^3/(c-d)^3/(tan(1/2*f*x+1/2*e)
+1)*B*d^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 17.18, size = 591, normalized size = 2.58

$$\frac{2 d^2 \operatorname{atan}\left(\frac{\frac{d^2 (A d-B c)\left(-2 a^3 c^3 d+6 a^3 c^2 d^2-6 a^3 c d^3+2 a^3 d^4\right)}{a^3 \sqrt{c+d}\,(c-d)^{7/2}}-\frac{2 c d^2 \tan\left(\frac{e}{2}+\frac{f x}{2}\right)(A d-B c)\left(a^3 c^3-3 a^3 c^2 d+3 a^3 c d^2-a^3 d^3\right)}{a^3 \sqrt{c+d}\,(c-d)^{7/2}}}{2 A d^3-2 B c d^2}\right)(A d-B c)}{a^3 f \sqrt{c+d}\,(c-d)^{7/2}} - \frac{2\left(7 A c^2+32 A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))),x)
```

```
[Out] (2*d^2*atan(((d^2*(A*d - B*c)*(2*a^3*d^4 - 6*a^3*c*d^3 - 2*a^3*c^3*d + 6*a^
3*c^2*d^2))/(a^3*(c + d)^(1/2)*(c - d)^(7/2)) - (2*c*d^2*tan(e/2 + (f*x)/2)
*(A*d - B*c)*(a^3*c^3 - a^3*d^3 + 3*a^3*c*d^2 - 3*a^3*c^2*d))/(a^3*(c + d)^
(1/2)*(c - d)^(7/2)))/(2*A*d^3 - 2*B*c*d^2))*(A*d - B*c))/(a^3*f*(c + d)^(1
/2)*(c - d)^(7/2)) - ((2*(7*A*c^2 + 32*A*d^2 + 3*B*c^2 - 7*B*d^2 - 24*A*c*d
 - 11*B*c*d))/(15*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)*(4*A
*c^2 + 23*A*d^2 + 3*B*c^2 - 4*B*d^2 - 15*A*c*d - 11*B*c*d))/(3*(c - d)*(c^2
 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^3*(2*A*c^2 + 9*A*d^2 + B*c^2 - 2*B
*d^2 - 7*A*c*d - 3*B*c*d))/((c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*
x)/2)^2*(8*A*c^2 + 37*A*d^2 + 3*B*c^2 - 8*B*d^2 - 27*A*c*d - 13*B*c*d))/(3*
(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^4*(A*c^2 + 3*A*d^2 - B
*d^2 - 3*A*c*d))/((c - d)*(c^2 - 2*c*d + d^2)))/(f*(10*a^3*tan(e/2 + (f*x)/
2)^2 + 10*a^3*tan(e/2 + (f*x)/2)^3 + 5*a^3*tan(e/2 + (f*x)/2)^4 + a^3*tan(e
/2 + (f*x)/2)^5 + a^3 + 5*a^3*tan(e/2 + (f*x)/2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.284 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=381

$$-\frac{2d^2\left(Ad(4c+3d)-B\left(3c^2+3cd+d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^4(c+d)\sqrt{c^2-d^2}}-\frac{\left(A\left(2c^2-12cd+45d^2\right)+B\left(3c^2-23cd-15d^2\right.\right.}{15f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)(c+d\sin}$$

```
[Out] -1/15*d*(B*(3*c^3-23*c^2*d-63*c*d^2-22*d^3)+A*(2*c^3-12*c^2*d+43*c*d^2+72*d
^3))*cos(f*x+e)/a^3/(c-d)^4/(c+d)/f/(c+d*sin(f*x+e))-1/5*(A-B)*cos(f*x+e)/(
c-d)/f/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))-1/15*(2*A*c-9*A*d+3*B*c+4*B*d)*c
os(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))-1/15*(B*(3*c^2-23
*c*d-15*d^2)+A*(2*c^2-12*c*d+45*d^2))*cos(f*x+e)/(c-d)^3/f/(a^3+a^3*sin(f*x
+e))/(c+d*sin(f*x+e))-2*d^2*(A*d*(4*c+3*d)-B*(3*c^2+3*c*d+d^2))*arctan((d+c
*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^3/(c-d)^4/(c+d)/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 1.08, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2978, 2754, 12, 2660, 618, 204}

$$-\frac{2d^2\left(Ad(4c+3d)-B\left(3c^2+3cd+d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^4(c+d)\sqrt{c^2-d^2}}-\frac{d\left(A\left(-12c^2d+2c^3+43cd^2+72d^3\right)+B\left(-23c\right.\right.}{15a^3f(c-d)^4(c+d)(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (-2*d^2*(A*d*(4*c + 3*d) - B*(3*c^2 + 3*c*d + d^2))*ArcTan[(d + c*Tan[(e +
f*x)/2])/Sqrt[c^2 - d^2]])/(a^3*(c - d)^4*(c + d)*Sqrt[c^2 - d^2]*f) - (d*(
B*(3*c^3 - 23*c^2*d - 63*c*d^2 - 22*d^3) + A*(2*c^3 - 12*c^2*d + 43*c*d^2 +
 72*d^3))*Cos[e + f*x])/(15*a^3*(c - d)^4*(c + d)*f*(c + d*Sin[e + f*x])) -
 ((A - B)*Cos[e + f*x])/(5*(c - d)*f*(a + a*Sin[e + f*x])^3*(c + d*Sin[e +
f*x])) - ((2*A*c + 3*B*c - 9*A*d + 4*B*d)*Cos[e + f*x])/(15*a*(c - d)^2*f*(
a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])) - ((B*(3*c^2 - 23*c*d - 15*d^2)
 + A*(2*c^2 - 12*c*d + 45*d^2))*Cos[e + f*x])/(15*(c - d)^3*f*(a^3 + a^3*Si
n[e + f*x])*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx = -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))} - \frac{\int \frac{-a(2A(c-3d)\cdots}{(a+a\sin\cdots}}{\cdots}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))} - \frac{(2Ac\cdots}{15a(c-d)^2f\cdots}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))} - \frac{(2Ac\cdots}{15a(c-d)^2f\cdots}$$

$$= -\frac{d\left(B\left(3c^3-23c^2d-63cd^2-22d^3\right)+A\left(2c^3-12c^2d+43cd^2+\cdots\right.\right.}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))}$$

$$= -\frac{d\left(B\left(3c^3-23c^2d-63cd^2-22d^3\right)+A\left(2c^3-12c^2d+43cd^2+\cdots\right.\right.}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))}$$

$$= -\frac{d\left(B\left(3c^3-23c^2d-63cd^2-22d^3\right)+A\left(2c^3-12c^2d+43cd^2+\cdots\right.\right.}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))}$$

$$= -\frac{d\left(B\left(3c^3-23c^2d-63cd^2-22d^3\right)+A\left(2c^3-12c^2d+43cd^2+\cdots\right.\right.}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))}$$

$$= -\frac{2d^2\left(Ad(4c+3d)-B\left(3c^2+3cd+d^2\right)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^4(c+d)\sqrt{c^2-d^2}\,f}$$

Mathematica [B] time = 6.38, size = 1253, normalized size = 3.29

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])
^2),x]
```

```
[Out] (2*d^2*(3*B*c^2 - 4*A*c*d + 3*B*c*d - 3*A*d^2 + B*d^2)*ArcTan[(Sec[(e + f*x
)/2]*(d*Cos[(e + f*x)/2] + c*Sin[(e + f*x)/2]))/Sqrt[c^2 - d^2]]*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^6)/((c - d)^4*(c + d)*Sqrt[c^2 - d^2]*f*(a + a*
Sin[e + f*x])^3) + ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(60*B*c^4*Cos[(e
```

```
+ f*x)/2] - 80*A*c^3*d*Cos[(e + f*x)/2] - 390*B*c^3*d*Cos[(e + f*x)/2] + 54
0*A*c^2*d^2*Cos[(e + f*x)/2] - 1090*B*c^2*d^2*Cos[(e + f*x)/2] + 1430*A*c*d
^3*Cos[(e + f*x)/2] - 885*B*c*d^3*Cos[(e + f*x)/2] + 735*A*d^4*Cos[(e + f*x
)/2] - 320*B*d^4*Cos[(e + f*x)/2] - 40*A*c^4*Cos[(3*(e + f*x))/2] - 60*B*c^
4*Cos[(3*(e + f*x))/2] + 196*A*c^3*d*Cos[(3*(e + f*x))/2] + 304*B*c^3*d*Cos
[(3*(e + f*x))/2] - 476*A*c^2*d^2*Cos[(3*(e + f*x))/2] + 1076*B*c^2*d^2*Cos
[(3*(e + f*x))/2] - 1546*A*c*d^3*Cos[(3*(e + f*x))/2] + 1181*B*c*d^3*Cos[(3
*(e + f*x))/2] - 969*A*d^4*Cos[(3*(e + f*x))/2] + 334*B*d^4*Cos[(3*(e + f*x
))/2] + 60*B*c^2*d^2*Cos[(5*(e + f*x))/2] - 90*A*c*d^3*Cos[(5*(e + f*x))/2]
 + 15*B*c*d^3*Cos[(5*(e + f*x))/2] - 15*A*d^4*Cos[(5*(e + f*x))/2] + 30*B*d
^4*Cos[(5*(e + f*x))/2] + 4*A*c^3*d*Cos[(7*(e + f*x))/2] + 6*B*c^3*d*Cos[(7
*(e + f*x))/2] - 24*A*c^2*d^2*Cos[(7*(e + f*x))/2] - 46*B*c^2*d^2*Cos[(7*(e
 + f*x))/2] + 86*A*c*d^3*Cos[(7*(e + f*x))/2] - 111*B*c*d^3*Cos[(7*(e + f*x
))/2] + 129*A*d^4*Cos[(7*(e + f*x))/2] - 44*B*d^4*Cos[(7*(e + f*x))/2] + 80
*A*c^4*Sin[(e + f*x)/2] + 60*B*c^4*Sin[(e + f*x)/2] - 340*A*c^3*d*Sin[(e +
f*x)/2] - 440*B*c^3*d*Sin[(e + f*x)/2] + 820*A*c^2*d^2*Sin[(e + f*x)/2] - 1
520*B*c^2*d^2*Sin[(e + f*x)/2] + 2140*A*c*d^3*Sin[(e + f*x)/2] - 1435*B*c*d
^3*Sin[(e + f*x)/2] + 975*A*d^4*Sin[(e + f*x)/2] - 340*B*d^4*Sin[(e + f*x)/
2] - 90*B*c^3*d*Sin[(3*(e + f*x))/2] + 120*A*c^2*d^2*Sin[(3*(e + f*x))/2] -
 390*B*c^2*d^2*Sin[(3*(e + f*x))/2] + 540*A*c*d^3*Sin[(3*(e + f*x))/2] - 31
5*B*c*d^3*Sin[(3*(e + f*x))/2] + 285*A*d^4*Sin[(3*(e + f*x))/2] - 150*B*d^4
*Sin[(3*(e + f*x))/2] - 8*A*c^4*Sin[(5*(e + f*x))/2] - 12*B*c^4*Sin[(5*(e +
 f*x))/2] + 28*A*c^3*d*Sin[(5*(e + f*x))/2] + 62*B*c^3*d*Sin[(5*(e + f*x))/
2] - 52*A*c^2*d^2*Sin[(5*(e + f*x))/2] + 362*B*c^2*d^2*Sin[(5*(e + f*x))/2]
 - 568*A*c*d^3*Sin[(5*(e + f*x))/2] + 553*B*c*d^3*Sin[(5*(e + f*x))/2] - 55
5*A*d^4*Sin[(5*(e + f*x))/2] + 190*B*d^4*Sin[(5*(e + f*x))/2] - 15*B*c*d^3*
Sin[(7*(e + f*x))/2] + 15*A*d^4*Sin[(7*(e + f*x))/2]))/(120*(c - d)^4*(c +
d)*f*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x]))
```

fricas [B] time = 0.66, size = 4486, normalized size = 11.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] [-1/30*(6*(A - B)*c^6 - 12*(A - B)*c^5*d - 6*(A - B)*c^4*d^2 + 24*(A - B)*c
^3*d^3 - 6*(A - B)*c^2*d^4 - 12*(A - B)*c*d^5 + 6*(A - B)*d^6 - 2*((2*A + 3
*B)*c^5*d - (12*A + 23*B)*c^4*d^2 + (41*A - 66*B)*c^3*d^3 + (84*A + B)*c^2*
d^4 - (43*A - 63*B)*c*d^5 - 2*(36*A - 11*B)*d^6)*cos(f*x + e)^4 - 2*((2*A +
 3*B)*c^6 - 2*(3*A + 7*B)*c^5*d + 5*(A - 18*B)*c^4*d^2 + (147*A - 152*B)*c^
3*d^3 + 4*(41*A + 9*B)*c^2*d^4 - (141*A - 166*B)*c*d^5 - 3*(57*A - 17*B)*d^
6)*cos(f*x + e)^3 + 2*(2*(2*A + 3*B)*c^6 - (19*A + 16*B)*c^5*d + 11*(2*A -
7*B)*c^4*d^2 + 8*(16*A - 11*B)*c^3*d^3 + 2*(32*A + 23*B)*c^2*d^4 - (109*A -
```

104*B)*c*d^5 - 5*(18*A - 5*B)*d^6)*cos(f*x + e)^2 + 15*(12*B*c^3*d^2 - 8*(
2*A - 3*B)*c^2*d^3 - 4*(7*A - 4*B)*c*d^4 - 4*(3*A - B)*d^5 + (3*B*c^2*d^3 -
(4*A - 3*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e)^4 - (3*B*c^3*d^2 - (4*A -
9*B)*c^2*d^3 - (11*A - 7*B)*c*d^4 - 2*(3*A - B)*d^5)*cos(f*x + e)^3 - (9*B*
c^3*d^2 - 12*(A - 2*B)*c^2*d^3 - (29*A - 18*B)*c*d^4 - 5*(3*A - B)*d^5)*cos
(f*x + e)^2 + 2*(3*B*c^3*d^2 - 2*(2*A - 3*B)*c^2*d^3 - (7*A - 4*B)*c*d^4 -
(3*A - B)*d^5)*cos(f*x + e) + (12*B*c^3*d^2 - 8*(2*A - 3*B)*c^2*d^3 - 4*(7*
A - 4*B)*c*d^4 - 4*(3*A - B)*d^5 - (3*B*c^2*d^3 - (4*A - 3*B)*c*d^4 - (3*A
- B)*d^5)*cos(f*x + e)^3 - (3*B*c^3*d^2 - 4*(A - 3*B)*c^2*d^3 - 5*(3*A - 2*
B)*c*d^4 - 3*(3*A - B)*d^5)*cos(f*x + e)^2 + 2*(3*B*c^3*d^2 - 2*(2*A - 3*B)
*c^2*d^3 - (7*A - 4*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e))*sin(f*x + e))*s
qrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^
2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2)
)/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 6*((3*A + 2*B)*c
^6 - (11*A + 9*B)*c^5*d + (12*A - 47*B)*c^4*d^2 + 2*(41*A - 31*B)*c^3*d^3 +
(47*A + 28*B)*c^2*d^4 - 71*(A - B)*c*d^5 - (62*A - 17*B)*d^6)*cos(f*x + e)
- 2*(3*(A - B)*c^6 - 6*(A - B)*c^5*d - 3*(A - B)*c^4*d^2 + 12*(A - B)*c^3*
d^3 - 3*(A - B)*c^2*d^4 - 6*(A - B)*c*d^5 + 3*(A - B)*d^6 + ((2*A + 3*B)*c^
5*d - (12*A + 23*B)*c^4*d^2 + (41*A - 66*B)*c^3*d^3 + (84*A + B)*c^2*d^4 -
(43*A - 63*B)*c*d^5 - 2*(36*A - 11*B)*d^6)*cos(f*x + e)^3 - ((2*A + 3*B)*c^
6 - (8*A + 17*B)*c^5*d + (17*A - 67*B)*c^4*d^2 + 2*(53*A - 43*B)*c^3*d^3 +
5*(16*A + 7*B)*c^2*d^4 - (98*A - 103*B)*c*d^5 - (99*A - 29*B)*d^6)*cos(f*x
+ e)^2 - 3*((2*A + 3*B)*c^6 - (9*A + 11*B)*c^5*d + (13*A - 48*B)*c^4*d^2 +
2*(39*A - 29*B)*c^3*d^3 + 3*(16*A + 9*B)*c^2*d^4 - 69*(A - B)*c*d^5 - 9*(7*
A - 2*B)*d^6)*cos(f*x + e))*sin(f*x + e))/((a^3*c^7*d - 3*a^3*c^6*d^2 + a^3
*c^5*d^3 + 5*a^3*c^4*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*
d^8)*f*cos(f*x + e)^4 - (a^3*c^8 - a^3*c^7*d - 5*a^3*c^6*d^2 + 7*a^3*c^5*d^
3 + 5*a^3*c^4*d^4 - 11*a^3*c^3*d^5 + a^3*c^2*d^6 + 5*a^3*c*d^7 - 2*a^3*d^8)
*f*cos(f*x + e)^3 - (3*a^3*c^8 - 4*a^3*c^7*d - 12*a^3*c^6*d^2 + 20*a^3*c^5*
d^3 + 10*a^3*c^4*d^4 - 28*a^3*c^3*d^5 + 4*a^3*c^2*d^6 + 12*a^3*c*d^7 - 5*a^
3*d^8)*f*cos(f*x + e)^2 + 2*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*
c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*cos(f*x
+ e) + 4*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3
*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f - ((a^3*c^7*d - 3*a^3*c^6*d
^2 + a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^
7 - a^3*d^8)*f*cos(f*x + e)^3 + (a^3*c^8 - 8*a^3*c^6*d^2 + 8*a^3*c^5*d^3 +
10*a^3*c^4*d^4 - 16*a^3*c^3*d^5 + 8*a^3*c*d^7 - 3*a^3*d^8)*f*cos(f*x + e)^2
- 2*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5
+ 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e) - 4*(a^3*c^8 - 2*a
^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 +
2*a^3*c*d^7 - a^3*d^8)*f)*sin(f*x + e)), -1/15*(3*(A - B)*c^6 - 6*(A - B)*c
^5*d - 3*(A - B)*c^4*d^2 + 12*(A - B)*c^3*d^3 - 3*(A - B)*c^2*d^4 - 6*(A -
B)*c*d^5 + 3*(A - B)*d^6 - ((2*A + 3*B)*c^5*d - (12*A + 23*B)*c^4*d^2 + (41
*A - 66*B)*c^3*d^3 + (84*A + B)*c^2*d^4 - (43*A - 63*B)*c*d^5 - 2*(36*A - 1
1*B)*d^6)*cos(f*x + e)^4 - ((2*A + 3*B)*c^6 - 2*(3*A + 7*B)*c^5*d + 5*(A -

18*B)*c^4*d^2 + (147*A - 152*B)*c^3*d^3 + 4*(41*A + 9*B)*c^2*d^4 - (141*A - 166*B)*c*d^5 - 3*(57*A - 17*B)*d^6)*cos(f*x + e)^3 + (2*(2*A + 3*B)*c^6 - (19*A + 16*B)*c^5*d + 11*(2*A - 7*B)*c^4*d^2 + 8*(16*A - 11*B)*c^3*d^3 + 2*(32*A + 23*B)*c^2*d^4 - (109*A - 104*B)*c*d^5 - 5*(18*A - 5*B)*d^6)*cos(f*x + e)^2 + 15*(12*B*c^3*d^2 - 8*(2*A - 3*B)*c^2*d^3 - 4*(7*A - 4*B)*c*d^4 - 4*(3*A - B)*d^5 + (3*B*c^2*d^3 - (4*A - 3*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e)^4 - (3*B*c^3*d^2 - (4*A - 9*B)*c^2*d^3 - (11*A - 7*B)*c*d^4 - 2*(3*A - B)*d^5)*cos(f*x + e)^3 - (9*B*c^3*d^2 - 12*(A - 2*B)*c^2*d^3 - (29*A - 18*B)*c*d^4 - 5*(3*A - B)*d^5)*cos(f*x + e)^2 + 2*(3*B*c^3*d^2 - 2*(2*A - 3*B)*c^2*d^3 - (7*A - 4*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e) + (12*B*c^3*d^2 - 8*(2*A - 3*B)*c^2*d^3 - 4*(7*A - 4*B)*c*d^4 - 4*(3*A - B)*d^5 - (3*B*c^2*d^3 - (4*A - 3*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e)^3 - (3*B*c^3*d^2 - 4*(A - 3*B)*c^2*d^3 - 5*(3*A - 2*B)*c*d^4 - 3*(3*A - B)*d^5)*cos(f*x + e)^2 + 2*(3*B*c^3*d^2 - 2*(2*A - 3*B)*c^2*d^3 - (7*A - 4*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + 3*((3*A + 2*B)*c^6 - (11*A + 9*B)*c^5*d + (12*A - 47*B)*c^4*d^2 + 2*(41*A - 31*B)*c^3*d^3 + (47*A + 28*B)*c^2*d^4 - 71*(A - B)*c*d^5 - (62*A - 17*B)*d^6)*cos(f*x + e) - (3*(A - B)*c^6 - 6*(A - B)*c^5*d - 3*(A - B)*c^4*d^2 + 12*(A - B)*c^3*d^3 - 3*(A - B)*c^2*d^4 - 6*(A - B)*c*d^5 + 3*(A - B)*d^6 + ((2*A + 3*B)*c^5*d - (12*A + 23*B)*c^4*d^2 + (41*A - 66*B)*c^3*d^3 + (84*A + B)*c^2*d^4 - (43*A - 63*B)*c*d^5 - 2*(36*A - 11*B)*d^6)*cos(f*x + e)^3 - ((2*A + 3*B)*c^6 - (8*A + 17*B)*c^5*d + (17*A - 67*B)*c^4*d^2 + 2*(53*A - 43*B)*c^3*d^3 + 5*(16*A + 7*B)*c^2*d^4 - (98*A - 103*B)*c*d^5 - (99*A - 29*B)*d^6)*cos(f*x + e)^2 - 3*((2*A + 3*B)*c^6 - (9*A + 11*B)*c^5*d + (13*A - 48*B)*c^4*d^2 + 2*(39*A - 29*B)*c^3*d^3 + 3*(16*A + 9*B)*c^2*d^4 - 69*(A - B)*c*d^5 - 9*(7*A - 2*B)*d^6)*cos(f*x + e))*sin(f*x + e))/((a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e)^4 - (a^3*c^8 - a^3*c^7*d - 5*a^3*c^6*d^2 + 7*a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 11*a^3*c^3*d^5 + a^3*c^2*d^6 + 5*a^3*c*d^7 - 2*a^3*d^8)*f*cos(f*x + e)^3 - (3*a^3*c^8 - 4*a^3*c^7*d - 12*a^3*c^6*d^2 + 20*a^3*c^5*d^3 + 10*a^3*c^4*d^4 - 28*a^3*c^3*d^5 + 4*a^3*c^2*d^6 + 12*a^3*c*d^7 - 5*a^3*d^8)*f*cos(f*x + e)^2 + 2*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e) + 4*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f - ((a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e)^3 + (a^3*c^8 - 8*a^3*c^6*d^2 + 8*a^3*c^5*d^3 + 10*a^3*c^4*d^4 - 16*a^3*c^3*d^5 + 8*a^3*c*d^7 - 3*a^3*d^8)*f*cos(f*x + e)^2 - 2*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e) - 4*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f)*sin(f*x + e))]

giac [B] time = 0.30, size = 772, normalized size = 2.03

$$2\left(\frac{15\left(3Bc^2d^2-4Acd^3+3Bcd^3-3Ad^4+Bd^4\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(a^3c^5-3a^3c^4d+2a^3c^3d^2+2a^3c^2d^3-3a^3cd^4+a^3d^5\right)\sqrt{c^2-d^2}}+\frac{15\left(Bcd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-Ad^5\tan\left(\frac{1}{2}\right.\right.}{\left(a^3c^6-3a^3c^5d+2a^3c^4d^2+2a^3c^3d^3-3a^3c^2d^4+a^3cd^5\right)\left(c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 2/15*(15*(3*B*c^2*d^2 - 4*A*c*d^3 + 3*B*c*d^3 - 3*A*d^4 + B*d^4)*(pi*floor(
1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c
^2 - d^2)))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3
*c*d^4 + a^3*d^5)*sqrt(c^2 - d^2)) + 15*(B*c*d^4*tan(1/2*f*x + 1/2*e) - A*d
^5*tan(1/2*f*x + 1/2*e) + B*c^2*d^3 - A*c*d^4)/((a^3*c^6 - 3*a^3*c^5*d + 2*
a^3*c^4*d^2 + 2*a^3*c^3*d^3 - 3*a^3*c^2*d^4 + a^3*c*d^5)*(c*tan(1/2*f*x + 1
/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) - (15*A*c^2*tan(1/2*f*x + 1/2*e)^4
 - 60*A*c*d*tan(1/2*f*x + 1/2*e)^4 + 90*A*d^2*tan(1/2*f*x + 1/2*e)^4 - 45*B
*d^2*tan(1/2*f*x + 1/2*e)^4 + 30*A*c^2*tan(1/2*f*x + 1/2*e)^3 + 15*B*c^2*ta
n(1/2*f*x + 1/2*e)^3 - 150*A*c*d*tan(1/2*f*x + 1/2*e)^3 - 60*B*c*d*tan(1/2*
f*x + 1/2*e)^3 + 300*A*d^2*tan(1/2*f*x + 1/2*e)^3 - 135*B*d^2*tan(1/2*f*x +
 1/2*e)^3 + 40*A*c^2*tan(1/2*f*x + 1/2*e)^2 + 15*B*c^2*tan(1/2*f*x + 1/2*e)
^2 - 190*A*c*d*tan(1/2*f*x + 1/2*e)^2 - 100*B*c*d*tan(1/2*f*x + 1/2*e)^2 +
420*A*d^2*tan(1/2*f*x + 1/2*e)^2 - 185*B*d^2*tan(1/2*f*x + 1/2*e)^2 + 20*A*
c^2*tan(1/2*f*x + 1/2*e) + 15*B*c^2*tan(1/2*f*x + 1/2*e) - 110*A*c*d*tan(1/
2*f*x + 1/2*e) - 80*B*c*d*tan(1/2*f*x + 1/2*e) + 270*A*d^2*tan(1/2*f*x + 1/
2*e) - 115*B*d^2*tan(1/2*f*x + 1/2*e) + 7*A*c^2 + 3*B*c^2 - 34*A*c*d - 16*B
*c*d + 72*A*d^2 - 32*B*d^2)/((a^3*c^4 - 4*a^3*c^3*d + 6*a^3*c^2*d^2 - 4*a^3
*c*d^3 + a^3*d^4)*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

maple [B] time = 0.57, size = 1049, normalized size = 2.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/f/a^3*d^4/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)
*tan(1/2*f*x+1/2*e)*B-6/f/a^3*d^4/(c-d)^4/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*
(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A+2/f/a^3*d^4/(c-d)^4/(c+d)/(
c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B+2
```

```
/f/a^3*d^3/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*
B*c+6/f/a^3/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)*B*d^2+4/f/a^3/(c-d)^3/(tan(1/2*f
*x+1/2*e)+1)^2*A*c-8/f/a^3/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^2*A*d-2/f/a^3/(c-
d)^3/(tan(1/2*f*x+1/2*e)+1)^2*B*c+6/f/a^3/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^2*
B*d-2/f/a^3*d^4/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(
c+d)*A+8/f/a^3/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)*A*c*d-16/3/f/a^3/(c-d)^3/(tan
(1/2*f*x+1/2*e)+1)^3*A*c+8/f/a^3/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^3*A*d+4/f/a
^3/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^3*B*c-20/3/f/a^3/(c-d)^3/(tan(1/2*f*x+1/2
*e)+1)^3*B*d-2/f/a^3/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)*A*c^2-12/f/a^3/(c-d)^4/
(tan(1/2*f*x+1/2*e)+1)*A*d^2+4/f/a^3/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^4*A-2/f
/a^3*d^5/(c-d)^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)/c*
tan(1/2*f*x+1/2*e)*A-8/f/a^3*d^3/(c-d)^4/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(
2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c+6/f/a^3*d^2/(c-d)^4/(c+d)/
(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*
c^2+6/f/a^3*d^3/(c-d)^4/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1
/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c-8/5/f/a^3/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^5*
A+8/5/f/a^3/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^5*B-4/f/a^3/(c-d)^2/(tan(1/2*f*x
+1/2*e)+1)^4*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 17.67, size = 1349, normalized size = 3.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^2),x)
```

```
[Out] (2*d^2*atan(((d^2*(3*B*c^2 - 3*A*d^2 + B*d^2 - 4*A*c*d + 3*B*c*d)*(2*a^3*d^
6 - 6*a^3*c*d^5 + 2*a^3*c^5*d + 4*a^3*c^2*d^4 + 4*a^3*c^3*d^3 - 6*a^3*c^4*d
^2))/(a^3*(c + d)^(3/2)*(c - d)^(9/2)) + (2*c*d^2*tan(e/2 + (f*x)/2)*(3*B*c
^2 - 3*A*d^2 + B*d^2 - 4*A*c*d + 3*B*c*d)*(a^3*c^5 + a^3*d^5 - 3*a^3*c*d^4
- 3*a^3*c^4*d + 2*a^3*c^2*d^3 + 2*a^3*c^3*d^2))/(a^3*(c + d)^(3/2)*(c - d)^
(9/2)))/(2*B*d^4 - 6*A*d^4 + 6*B*c^2*d^2 - 8*A*c*d^3 + 6*B*c*d^3))*(3*B*c^2
```

```
 - 3*A*d^2 + B*d^2 - 4*A*c*d + 3*B*c*d))/(a^3*f*(c + d)^(3/2)*(c - d)^(9/2)
) - ((2*(7*A*c^4 + 15*A*d^4 + 3*B*c^4 + 38*A*c^2*d^2 - 48*B*c^2*d^2 + 72*A*
c*d^3 - 27*A*c^3*d - 47*B*c*d^3 - 13*B*c^3*d))/(15*(c + d)*(c - d)*(3*c*d^2
 - 3*c^2*d + c^3 - d^3)) + (4*tan(e/2 + (f*x)/2)^3*(5*A*c^4 + 15*A*d^4 + 3*
B*c^4 + 19*A*c^2*d^2 - 45*B*c^2*d^2 + 84*A*c*d^3 - 18*A*c^3*d - 52*B*c*d^3
- 11*B*c^3*d))/(3*c*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (2*tan(e/2 +
 (f*x)/2)*(20*A*c^5 + 15*A*d^5 + 15*B*c^5 + 346*A*c^2*d^3 + 106*A*c^3*d^2 -
 286*B*c^2*d^3 - 221*B*c^3*d^2 + 219*A*c*d^4 - 76*A*c^4*d - 79*B*c*d^4 - 59
*B*c^4*d))/(15*c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (2*tan(
e/2 + (f*x)/2)^5*(2*A*c^5 + 5*A*d^5 + B*c^5 + 24*A*c^2*d^3 + 4*A*c^3*d^2 -
16*B*c^2*d^3 - 13*B*c^3*d^2 + 13*A*c*d^4 - 6*A*c^4*d - 11*B*c*d^4 - 3*B*c^4
*d))/(c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (2*tan(e/2 + (f*
x)/2)^4*(11*A*c^5 + 30*A*d^5 + 3*B*c^5 + 162*A*c^2*d^3 + 4*A*c^3*d^2 - 139*
B*c^2*d^3 - 84*B*c^3*d^2 + 135*A*c*d^4 - 27*A*c^4*d - 84*B*c*d^4 - 11*B*c^4
*d))/(3*c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (2*tan(e/2 + (
f*x)/2)^2*(47*A*c^5 + 75*A*d^5 + 18*B*c^5 + 812*A*c^2*d^3 + 88*A*c^3*d^2 -
757*B*c^2*d^3 - 463*B*c^3*d^2 + 690*A*c*d^4 - 137*A*c^4*d - 305*B*c*d^4 - 6
8*B*c^4*d))/(15*c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (2*tan
(e/2 + (f*x)/2)^6*(A*c^5 + A*d^5 + 6*A*c^2*d^3 + 2*A*c^3*d^2 - 3*B*c^2*d^3
- 3*B*c^3*d^2 - 3*A*c^4*d - B*c*d^4))/(c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d
 + c^3 - d^3)))/(f*(a^3*c + tan(e/2 + (f*x)/2)*(5*a^3*c + 2*a^3*d) + tan(e/
2 + (f*x)/2)^6*(5*a^3*c + 2*a^3*d) + tan(e/2 + (f*x)/2)^2*(11*a^3*c + 10*a^
3*d) + tan(e/2 + (f*x)/2)^5*(11*a^3*c + 10*a^3*d) + tan(e/2 + (f*x)/2)^3*(1
5*a^3*c + 20*a^3*d) + tan(e/2 + (f*x)/2)^4*(15*a^3*c + 20*a^3*d) + a^3*c*ta
n(e/2 + (f*x)/2)^7))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.285 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=508

$$-\frac{\left(A\left(2c^2-15cd+76d^2\right)+3B\left(c^2-10cd-12d^2\right)\right)\cos(e+fx)}{15f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)(c+d\sin(e+fx))^2}-\frac{d^2\left(Ad\left(20c^2+30cd+13d^2\right)-3B\left(4c^3+8c^2d\right.\right.}{a^3f(c-d)^5(c+d)^2}$$

```
[Out] -1/30*d*(3*B*(2*c^3-20*c^2*d-57*c*d^2-30*d^3)+A*(4*c^3-30*c^2*d+146*c*d^2+1
95*d^3))*cos(f*x+e)/a^3/(c-d)^4/(c+d)/f/(c+d*sin(f*x+e))^2-1/5*(A-B)*cos(f*
x+e)/(c-d)/f/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2-1/15*(2*A*c-11*A*d+3*B*c
+6*B*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2-1/15*(
3*B*(c^2-10*c*d-12*d^2)+A*(2*c^2-15*c*d+76*d^2))*cos(f*x+e)/(c-d)^3/f/(a^3+
a^3*sin(f*x+e))/(c+d*sin(f*x+e))^2-1/30*d*(3*B*(2*c^4-20*c^3*d-119*c^2*d^2-
130*c*d^3-48*d^4)+A*(4*c^4-30*c^3*d+142*c^2*d^2+525*c*d^3+304*d^4))*cos(f*x
+e)/a^3/(c-d)^5/(c+d)^2/f/(c+d*sin(f*x+e))-d^2*(A*d*(20*c^2+30*c*d+13*d^2)-
3*B*(4*c^3+8*c^2*d+7*c*d^2+2*d^3))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2
)^(1/2))/a^3/(c-d)^5/(c+d)^2/f/(c^2-d^2)^(1/2)
```

Rubi [A] time = 1.45, antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2978, 2754, 12, 2660, 618, 204}

$$-\frac{d^2\left(Ad\left(20c^2+30cd+13d^2\right)-3B\left(8c^2d+4c^3+7cd^2+2d^3\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^5(c+d)^2\sqrt{c^2-d^2}}-\frac{d\left(A\left(142c^2d^2-30c^3d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] -((d^2*(A*d*(20*c^2 + 30*c*d + 13*d^2) - 3*B*(4*c^3 + 8*c^2*d + 7*c*d^2 + 2
*d^3))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^3*(c - d)^5*(c
+ d)^2*Sqrt[c^2 - d^2]*f)) - (d*(3*B*(2*c^3 - 20*c^2*d - 57*c*d^2 - 30*d^3)
 + A*(4*c^3 - 30*c^2*d + 146*c*d^2 + 195*d^3))*Cos[e + f*x])/(30*a^3*(c - d
)^4*(c + d)*f*(c + d*Sin[e + f*x])^2) - ((A - B)*Cos[e + f*x])/(5*(c - d)*f
*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2) - ((2*A*c + 3*B*c - 11*A*d
+ 6*B*d)*Cos[e + f*x])/(15*a*(c - d)^2*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[
e + f*x])^2) - ((3*B*(c^2 - 10*c*d - 12*d^2) + A*(2*c^2 - 15*c*d + 76*d^2))
*Cos[e + f*x])/(15*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^2) - (d*(3*B*(2*c^4 - 20*c^3*d - 119*c^2*d^2 - 130*c*d^3 - 48*d^4) + A*(4
*c^4 - 30*c^3*d + 142*c^2*d^2 + 525*c*d^3 + 304*d^4))*Cos[e + f*x])/(30*a^3
*(c - d)^5*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx = -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} - \frac{\int \frac{-a(2Ac+3}{(a+}}{}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} - \frac{(2A}{15a(c-d)}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} - \frac{(2A}{15a(c-d)}$$

$$= -\frac{d\left(3B\left(2c^3-20c^2d-57cd^2-30d^3\right)+A\left(4c^3-30c^2d+146cd\right.\right.}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))}$$

$$= -\frac{d\left(3B\left(2c^3-20c^2d-57cd^2-30d^3\right)+A\left(4c^3-30c^2d+146cd\right.\right.}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))}$$

$$= -\frac{d\left(3B\left(2c^3-20c^2d-57cd^2-30d^3\right)+A\left(4c^3-30c^2d+146cd\right.\right.}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))}$$

$$= -\frac{d\left(3B\left(2c^3-20c^2d-57cd^2-30d^3\right)+A\left(4c^3-30c^2d+146cd\right.\right.}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))}$$

$$= -\frac{d\left(3B\left(2c^3-20c^2d-57cd^2-30d^3\right)+A\left(4c^3-30c^2d+146cd\right.\right.}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))}$$

$$= -\frac{d^2\left(Ad\left(20c^2+30cd+13d^2\right)-3B\left(4c^3+8c^2d+7cd^2+2d^3\right)\right)}{a^3(c-d)^5(c+d)^2\sqrt{c^2-d^2}\,f}$$

Mathematica [A] time = 4.96, size = 548, normalized size = 1.08

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(4\left(A\left(2c^2-19cd+107d^2\right)+3B\left(c^2-12cd-19d^2\right)\right)\sin\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])
^3),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(12*(A - B)*(c - d)^2*Sin[(e + f*x)/
2] + 6*(-A + B)*(c - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 4*(c - d)
*(A*(2*c - 17*d) + 3*B*(c + 4*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^2 - 2*(c - d)*(A*(2*c - 17*d) + 3*B*(c + 4*d))*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^3 + 4*(3*B*(c^2 - 12*c*d - 19*d^2) + A*(2*c^2 - 19*
c*d + 107*d^2))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 +
(30*d^2*(-(A*d*(20*c^2 + 30*c*d + 13*d^2)) + 3*B*(4*c^3 + 8*c^2*d + 7*c*d^2
 + 2*d^3))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^5)/((c + d)^2*Sqrt[c^2 - d^2]) + (15*(c - d)*d^3*(B*
c - A*d)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/((c + d)*(c
+ d*Sin[e + f*x])^2) + (15*d^3*(-3*A*d*(3*c + 2*d) + B*(7*c^2 + 6*c*d + 2*d
^2))*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/((c + d)^2*(c +
d*Sin[e + f*x]))))/(30*a^3*(c - d)^5*f*(1 + Sin[e + f*x])^3)
```

fricas [B] time = 0.85, size = 7283, normalized size = 14.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] [-1/60*(12*(A - B)*c^8 - 24*(A - B)*c^7*d - 24*(A - B)*c^6*d^2 + 72*(A - B)
*c^5*d^3 - 72*(A - B)*c^3*d^5 + 24*(A - B)*c^2*d^6 + 24*(A - B)*c*d^7 - 12*
(A - B)*d^8 + 2*(2*(2*A + 3*B)*c^6*d^2 - 30*(A + 2*B)*c^5*d^3 + 3*(46*A - 1
21*B)*c^4*d^4 + 15*(37*A - 22*B)*c^3*d^5 + 3*(54*A + 71*B)*c^2*d^6 - 15*(35
*A - 26*B)*c*d^7 - 16*(19*A - 9*B)*d^8)*cos(f*x + e)^5 - 2*(4*(2*A + 3*B)*c
^7*d - 4*(13*A + 27*B)*c^6*d^2 + 18*(12*A - 37*B)*c^5*d^3 + 6*(181*A - 171*
B)*c^4*d^4 + 3*(328*A - 33*B)*c^3*d^5 - 9*(69*A - 104*B)*c^2*d^6 - (1208*A
- 753*B)*c*d^7 - (413*A - 198*B)*d^8)*cos(f*x + e)^4 - 2*(2*(2*A + 3*B)*c^8
 - 6*(A + 4*B)*c^7*d - 20*(A + 21*B)*c^6*d^2 + 6*(128*A - 293*B)*c^5*d^3 +
3*(892*A - 827*B)*c^4*d^4 + 3*(769*A - 49*B)*c^3*d^5 - (1573*A - 2373*B)*c^
2*d^6 - 3*(1023*A - 643*B)*c*d^7 - (1087*A - 522*B)*d^8)*cos(f*x + e)^3 + 4
*(2*(2*A + 3*B)*c^8 - 5*(4*A + 3*B)*c^7*d + (19*A - 174*B)*c^6*d^2 + 15*(22
*A - 35*B)*c^5*d^3 + 3*(233*A - 173*B)*c^4*d^4 + 15*(23*A + 10*B)*c^3*d^5 -
 (526*A - 591*B)*c^2*d^6 - 5*(131*A - 78*B)*c*d^7 - 4*(49*A - 24*B)*d^8)*co
s(f*x + e)^2 - 15*(48*B*c^5*d^2 - 16*(5*A - 12*B)*c^4*d^3 - 4*(70*A - 81*B)
*c^3*d^4 - 12*(31*A - 24*B)*c^2*d^5 - 4*(56*A - 33*B)*c*d^6 - 4*(13*A - 6*B
)*d^7 + (12*B*c^3*d^4 - 4*(5*A - 6*B)*c^2*d^5 - 3*(10*A - 7*B)*c*d^6 - (13*
A - 6*B)*d^7)*cos(f*x + e)^5 + (24*B*c^4*d^3 - 4*(10*A - 21*B)*c^3*d^4 - 6*
```

(20*A - 19*B)*c^2*d^5 - (116*A - 75*B)*c*d^6 - 3*(13*A - 6*B)*d^7)*cos(f*x + e)^4 - (12*B*c^5*d^2 - 4*(5*A - 18*B)*c^4*d^3 - (110*A - 153*B)*c^3*d^4 - (193*A - 162*B)*c^2*d^5 - (142*A - 87*B)*c*d^6 - 3*(13*A - 6*B)*d^7)*cos(f*x + e)^3 - (36*B*c^5*d^2 - 12*(5*A - 16*B)*c^4*d^3 - (290*A - 387*B)*c^3*d^4 - (479*A - 396*B)*c^2*d^5 - (340*A - 207*B)*c*d^6 - 7*(13*A - 6*B)*d^7)*cos(f*x + e)^2 + 2*(12*B*c^5*d^2 - 4*(5*A - 12*B)*c^4*d^3 - (70*A - 81*B)*c^3*d^4 - 3*(31*A - 24*B)*c^2*d^5 - (56*A - 33*B)*c*d^6 - (13*A - 6*B)*d^7)*cos(f*x + e) + (48*B*c^5*d^2 - 16*(5*A - 12*B)*c^4*d^3 - 4*(70*A - 81*B)*c^3*d^4 - 12*(31*A - 24*B)*c^2*d^5 - 4*(56*A - 33*B)*c*d^6 - 4*(13*A - 6*B)*d^7 + (12*B*c^3*d^4 - 4*(5*A - 6*B)*c^2*d^5 - 3*(10*A - 7*B)*c*d^6 - (13*A - 6*B)*d^7)*cos(f*x + e)^4 - 2*(12*B*c^4*d^3 - 4*(5*A - 9*B)*c^3*d^4 - 5*(10*A - 9*B)*c^2*d^5 - (43*A - 27*B)*c*d^6 - (13*A - 6*B)*d^7)*cos(f*x + e)^3 - (12*B*c^5*d^2 - 4*(5*A - 24*B)*c^4*d^3 - 75*(2*A - 3*B)*c^3*d^4 - (293*A - 252*B)*c^2*d^5 - 3*(76*A - 47*B)*c*d^6 - 5*(13*A - 6*B)*d^7)*cos(f*x + e)^2 + 2*(12*B*c^5*d^2 - 4*(5*A - 12*B)*c^4*d^3 - (70*A - 81*B)*c^3*d^4 - 3*(31*A - 24*B)*c^2*d^5 - (56*A - 33*B)*c*d^6 - (13*A - 6*B)*d^7)*cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 - 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 12*((3*A + 2*B)*c^8 - (11*A + 9*B)*c^7*d + (9*A - 109*B)*c^6*d^2 + (213*A - 353*B)*c^5*d^3 + 5*(95*A - 71*B)*c^4*d^4 + (237*A + 103*B)*c^3*d^5 - (359*A - 399*B)*c^2*d^6 - (439*A - 259*B)*c*d^7 - (128*A - 63*B)*d^8)*cos(f*x + e) - 2*(6*(A - B)*c^8 - 12*(A - B)*c^7*d - 12*(A - B)*c^6*d^2 + 36*(A - B)*c^5*d^3 - 36*(A - B)*c^3*d^5 + 12*(A - B)*c^2*d^6 + 12*(A - B)*c*d^7 - 6*(A - B)*d^8 + (2*(2*A + 3*B)*c^6*d^2 - 30*(A + 2*B)*c^5*d^3 + 3*(46*A - 121*B)*c^4*d^4 + 15*(37*A - 22*B)*c^3*d^5 + 3*(54*A + 71*B)*c^2*d^6 - 15*(35*A - 26*B)*c*d^7 - 16*(19*A - 9*B)*d^8)*cos(f*x + e)^4 + (4*(2*A + 3*B)*c^7*d - 6*(8*A + 17*B)*c^6*d^2 + 6*(31*A - 121*B)*c^5*d^3 + 3*(408*A - 463*B)*c^4*d^4 + 3*(513*A - 143*B)*c^3*d^5 - 3*(153*A - 383*B)*c^2*d^6 - (1733*A - 1143*B)*c*d^7 - 3*(239*A - 114*B)*d^8)*cos(f*x + e)^3 - 2*((2*A + 3*B)*c^8 - (7*A + 18*B)*c^7*d + (14*A - 159*B)*c^6*d^2 + 3*(97*A - 172*B)*c^5*d^3 + 6*(121*A - 91*B)*c^4*d^4 + 3*(128*A + 47*B)*c^3*d^5 - (557*A - 612*B)*c^2*d^6 - (668*A - 393*B)*c*d^7 - 5*(37*A - 18*B)*d^8)*cos(f*x + e)^2 - 6*((2*A + 3*B)*c^8 - (9*A + 11*B)*c^7*d + (11*A - 111*B)*c^6*d^2 + (207*A - 347*B)*c^5*d^3 + 5*(95*A - 71*B)*c^4*d^4 + (243*A + 97*B)*c^3*d^5 - (361*A - 401*B)*c^2*d^6 - 9*(49*A - 29*B)*c*d^7 - (127*A - 62*B)*d^8)*cos(f*x + e))*sin(f*x + e))/((a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d^5 - 6*a^3*c^5*d^6 - 6*a^3*c^4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e)^5 + (2*a^3*c^10*d - 3*a^3*c^9*d^2 - 9*a^3*c^8*d^3 + 16*a^3*c^7*d^4 + 12*a^3*c^6*d^5 - 30*a^3*c^5*d^6 - 2*a^3*c^4*d^7 + 24*a^3*c^3*d^8 - 6*a^3*c^2*d^9 - 7*a^3*c*d^10 + 3*a^3*d^11)*f*cos(f*x + e)^4 - (a^3*c^11 + a^3*c^10*d - 9*a^3*c^9*d^2 - a^3*c^8*d^3 + 26*a^3*c^7*d^4 - 6*a^3*c^6*d^5 - 34*a^3*c^5*d^6 + 14*a^3*c^4*d^7 + 21*a^3*c^3*d^8 - 11*a^3*c^2*d^9 - 5*a^3*c*d^10 + 3*a^3*d^11)*f*cos(f*x + e)^3 - (3*a^3*c^11 + a^3*c^10*d - 23*a^3*c^9*d^2 + 3*a^3*c^8*d^3 + 62*a^3*c^7*d^4 - 22*a^3*c^6*d^5 - 78*a^3*c^5*d^6 + 38*a^3*c^4*d^7 + 47*

a^3*c^3*d^8 - 27*a^3*c^2*d^9 - 11*a^3*c*d^10 + 7*a^3*d^11)*f*cos(f*x + e)^2
 + 2*(a^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^
4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^
3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e) + 4*(a^3*c^11 - a^3*c^10*
d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^
3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a
^3*d^11)*f + ((a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d^5 - 6*a^3*c^5*d^6
- 6*a^3*c^4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e)^4
 - 2*(a^3*c^10*d - 2*a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^7*d^4 + 2*a^3*c^
6*d^5 - 12*a^3*c^5*d^6 + 2*a^3*c^4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c^2*d^9 - 2*
a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e)^3 - (a^3*c^11 + 3*a^3*c^10*d - 13*a^3
*c^9*d^2 - 7*a^3*c^8*d^3 + 42*a^3*c^7*d^4 - 2*a^3*c^6*d^5 - 58*a^3*c^5*d^6
+ 18*a^3*c^4*d^7 + 37*a^3*c^3*d^8 - 17*a^3*c^2*d^9 - 9*a^3*c*d^10 + 5*a^3*d
^11)*f*cos(f*x + e)^2 + 2*(a^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^
8*d^3 + 10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 +
 5*a^3*c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e) + 4*
(a^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^4 - 1
0*a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^3*c^2
*d^9 - a^3*c*d^10 + a^3*d^11)*f)*sin(f*x + e)), -1/30*(6*(A - B)*c^8 - 12*(
A - B)*c^7*d - 12*(A - B)*c^6*d^2 + 36*(A - B)*c^5*d^3 - 36*(A - B)*c^3*d^5
 + 12*(A - B)*c^2*d^6 + 12*(A - B)*c*d^7 - 6*(A - B)*d^8 + (2*(2*A + 3*B)*c
^6*d^2 - 30*(A + 2*B)*c^5*d^3 + 3*(46*A - 121*B)*c^4*d^4 + 15*(37*A - 22*B)
*c^3*d^5 + 3*(54*A + 71*B)*c^2*d^6 - 15*(35*A - 26*B)*c*d^7 - 16*(19*A - 9*
B)*d^8)*cos(f*x + e)^5 - (4*(2*A + 3*B)*c^7*d - 4*(13*A + 27*B)*c^6*d^2 + 1
8*(12*A - 37*B)*c^5*d^3 + 6*(181*A - 171*B)*c^4*d^4 + 3*(328*A - 33*B)*c^3*
d^5 - 9*(69*A - 104*B)*c^2*d^6 - (1208*A - 753*B)*c*d^7 - (413*A - 198*B)*d
^8)*cos(f*x + e)^4 - (2*(2*A + 3*B)*c^8 - 6*(A + 4*B)*c^7*d - 20*(A + 21*B)
*c^6*d^2 + 6*(128*A - 293*B)*c^5*d^3 + 3*(892*A - 827*B)*c^4*d^4 + 3*(769*A
 - 49*B)*c^3*d^5 - (1573*A - 2373*B)*c^2*d^6 - 3*(1023*A - 643*B)*c*d^7 - (
1087*A - 522*B)*d^8)*cos(f*x + e)^3 + 2*(2*(2*A + 3*B)*c^8 - 5*(4*A + 3*B)*
c^7*d + (19*A - 174*B)*c^6*d^2 + 15*(22*A - 35*B)*c^5*d^3 + 3*(233*A - 173*
B)*c^4*d^4 + 15*(23*A + 10*B)*c^3*d^5 - (526*A - 591*B)*c^2*d^6 - 5*(131*A
 - 78*B)*c*d^7 - 4*(49*A - 24*B)*d^8)*cos(f*x + e)^2 + 15*(48*B*c^5*d^2 - 16
*(5*A - 12*B)*c^4*d^3 - 4*(70*A - 81*B)*c^3*d^4 - 12*(31*A - 24*B)*c^2*d^5
 - 4*(56*A - 33*B)*c*d^6 - 4*(13*A - 6*B)*d^7 + (12*B*c^3*d^4 - 4*(5*A - 6*B
)*c^2*d^5 - 3*(10*A - 7*B)*c*d^6 - (13*A - 6*B)*d^7)*cos(f*x + e)^5 + (24*B
*c^4*d^3 - 4*(10*A - 21*B)*c^3*d^4 - 6*(20*A - 19*B)*c^2*d^5 - (116*A - 75*
B)*c*d^6 - 3*(13*A - 6*B)*d^7)*cos(f*x + e)^4 - (12*B*c^5*d^2 - 4*(5*A - 18
*B)*c^4*d^3 - (110*A - 153*B)*c^3*d^4 - (193*A - 162*B)*c^2*d^5 - (142*A -
87*B)*c*d^6 - 3*(13*A - 6*B)*d^7)*cos(f*x + e)^3 - (36*B*c^5*d^2 - 12*(5*A
 - 16*B)*c^4*d^3 - (290*A - 387*B)*c^3*d^4 - (479*A - 396*B)*c^2*d^5 - (340*
A - 207*B)*c*d^6 - 7*(13*A - 6*B)*d^7)*cos(f*x + e)^2 + 2*(12*B*c^5*d^2 - 4
*(5*A - 12*B)*c^4*d^3 - (70*A - 81*B)*c^3*d^4 - 3*(31*A - 24*B)*c^2*d^5 - (
56*A - 33*B)*c*d^6 - (13*A - 6*B)*d^7)*cos(f*x + e) + (48*B*c^5*d^2 - 16*(5
*A - 12*B)*c^4*d^3 - 4*(70*A - 81*B)*c^3*d^4 - 12*(31*A - 24*B)*c^2*d^5 - 4

*(56*A - 33*B)*c*d^6 - 4*(13*A - 6*B)*d^7 + (12*B*c^3*d^4 - 4*(5*A - 6*B)*c
^2*d^5 - 3*(10*A - 7*B)*c*d^6 - (13*A - 6*B)*d^7)*cos(f*x + e)^4 - 2*(12*B*
c^4*d^3 - 4*(5*A - 9*B)*c^3*d^4 - 5*(10*A - 9*B)*c^2*d^5 - (43*A - 27*B)*c*
d^6 - (13*A - 6*B)*d^7)*cos(f*x + e)^3 - (12*B*c^5*d^2 - 4*(5*A - 24*B)*c^4
d^3 - 75(2*A - 3*B)*c^3*d^4 - (293*A - 252*B)*c^2*d^5 - 3*(76*A - 47*B)*c
d^6 - 5(13*A - 6*B)*d^7)*cos(f*x + e)^2 + 2*(12*B*c^5*d^2 - 4*(5*A - 12*B
)*c^4*d^3 - (70*A - 81*B)*c^3*d^4 - 3*(31*A - 24*B)*c^2*d^5 - (56*A - 33*B)
*c*d^6 - (13*A - 6*B)*d^7)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2)*arct
an(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + 6*((3*A + 2*B)*c
^8 - (11*A + 9*B)*c^7*d + (9*A - 109*B)*c^6*d^2 + (213*A - 353*B)*c^5*d^3 +
5*(95*A - 71*B)*c^4*d^4 + (237*A + 103*B)*c^3*d^5 - (359*A - 399*B)*c^2*d^
6 - (439*A - 259*B)*c*d^7 - (128*A - 63*B)*d^8)*cos(f*x + e) - (6*(A - B)*c
^8 - 12*(A - B)*c^7*d - 12*(A - B)*c^6*d^2 + 36*(A - B)*c^5*d^3 - 36*(A - B
)*c^3*d^5 + 12*(A - B)*c^2*d^6 + 12*(A - B)*c*d^7 - 6*(A - B)*d^8 + (2*(2*A
+ 3*B)*c^6*d^2 - 30*(A + 2*B)*c^5*d^3 + 3*(46*A - 121*B)*c^4*d^4 + 15*(37*
A - 22*B)*c^3*d^5 + 3*(54*A + 71*B)*c^2*d^6 - 15*(35*A - 26*B)*c*d^7 - 16*(
19*A - 9*B)*d^8)*cos(f*x + e)^4 + (4*(2*A + 3*B)*c^7*d - 6*(8*A + 17*B)*c^6
d^2 + 6(31*A - 121*B)*c^5*d^3 + 3*(408*A - 463*B)*c^4*d^4 + 3*(513*A - 14
3*B)*c^3*d^5 - 3*(153*A - 383*B)*c^2*d^6 - (1733*A - 1143*B)*c*d^7 - 3*(239
*A - 114*B)*d^8)*cos(f*x + e)^3 - 2*((2*A + 3*B)*c^8 - (7*A + 18*B)*c^7*d +
(14*A - 159*B)*c^6*d^2 + 3*(97*A - 172*B)*c^5*d^3 + 6*(121*A - 91*B)*c^4*d
^4 + 3*(128*A + 47*B)*c^3*d^5 - (557*A - 612*B)*c^2*d^6 - (668*A - 393*B)*c
d^7 - 5(37*A - 18*B)*d^8)*cos(f*x + e)^2 - 6*((2*A + 3*B)*c^8 - (9*A + 11
*B)*c^7*d + (11*A - 111*B)*c^6*d^2 + (207*A - 347*B)*c^5*d^3 + 5*(95*A - 71
*B)*c^4*d^4 + (243*A + 97*B)*c^3*d^5 - (361*A - 401*B)*c^2*d^6 - 9*(49*A -
29*B)*c*d^7 - (127*A - 62*B)*d^8)*cos(f*x + e))*sin(f*x + e))/((a^3*c^9*d^2
- 3*a^3*c^8*d^3 + 8*a^3*c^6*d^5 - 6*a^3*c^5*d^6 - 6*a^3*c^4*d^7 + 8*a^3*c^
3*d^8 - 3*a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e)^5 + (2*a^3*c^10*d - 3*a^3*c
^9*d^2 - 9*a^3*c^8*d^3 + 16*a^3*c^7*d^4 + 12*a^3*c^6*d^5 - 30*a^3*c^5*d^6 -
2*a^3*c^4*d^7 + 24*a^3*c^3*d^8 - 6*a^3*c^2*d^9 - 7*a^3*c*d^10 + 3*a^3*d^11
)*f*cos(f*x + e)^4 - (a^3*c^11 + a^3*c^10*d - 9*a^3*c^9*d^2 - a^3*c^8*d^3 +
26*a^3*c^7*d^4 - 6*a^3*c^6*d^5 - 34*a^3*c^5*d^6 + 14*a^3*c^4*d^7 + 21*a^3*
c^3*d^8 - 11*a^3*c^2*d^9 - 5*a^3*c*d^10 + 3*a^3*d^11)*f*cos(f*x + e)^3 - (3
*a^3*c^11 + a^3*c^10*d - 23*a^3*c^9*d^2 + 3*a^3*c^8*d^3 + 62*a^3*c^7*d^4 -
22*a^3*c^6*d^5 - 78*a^3*c^5*d^6 + 38*a^3*c^4*d^7 + 47*a^3*c^3*d^8 - 27*a^3*
c^2*d^9 - 11*a^3*c*d^10 + 7*a^3*d^11)*f*cos(f*x + e)^2 + 2*(a^3*c^11 - a^3*
c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 -
10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^1
0 + a^3*d^11)*f*cos(f*x + e) + 4*(a^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5
*a^3*c^8*d^3 + 10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^
4*d^7 + 5*a^3*c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*f + ((a^3*c^
9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d^5 - 6*a^3*c^5*d^6 - 6*a^3*c^4*d^7 + 8*a
^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e)^4 - 2*(a^3*c^10*d - 2*
a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^7*d^4 + 2*a^3*c^6*d^5 - 12*a^3*c^5*d^
6 + 2*a^3*c^4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c^2*d^9 - 2*a^3*c*d^10 + a^3*d^11

```
)*f*cos(f*x + e)^3 - (a^3*c^11 + 3*a^3*c^10*d - 13*a^3*c^9*d^2 - 7*a^3*c^8*
d^3 + 42*a^3*c^7*d^4 - 2*a^3*c^6*d^5 - 58*a^3*c^5*d^6 + 18*a^3*c^4*d^7 + 37
*a^3*c^3*d^8 - 17*a^3*c^2*d^9 - 9*a^3*c*d^10 + 5*a^3*d^11)*f*cos(f*x + e)^2
 + 2*(a^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^
4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^
3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e) + 4*(a^3*c^11 - a^3*c^10*
d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^
3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a
^3*d^11)*f)*sin(f*x + e))]
```

giac [B] time = 0.59, size = 1272, normalized size = 2.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/15*(15*(12*B*c^3*d^2 - 20*A*c^2*d^3 + 24*B*c^2*d^3 - 30*A*c*d^4 + 21*B*c*
d^4 - 13*A*d^5 + 6*B*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan
((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^3*c^7 - 3*a^3*c^6*d + a
^3*c^5*d^2 + 5*a^3*c^4*d^3 - 5*a^3*c^3*d^4 - a^3*c^2*d^5 + 3*a^3*c*d^6 - a^
3*d^7)*sqrt(c^2 - d^2)) + 15*(9*B*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 - 11*A*c^3
*d^5*tan(1/2*f*x + 1/2*e)^3 + 6*B*c^3*d^5*tan(1/2*f*x + 1/2*e)^3 - 6*A*c^2*
d^6*tan(1/2*f*x + 1/2*e)^3 + 2*A*c*d^7*tan(1/2*f*x + 1/2*e)^3 + 8*B*c^5*d^3
*tan(1/2*f*x + 1/2*e)^2 - 10*A*c^4*d^4*tan(1/2*f*x + 1/2*e)^2 + 6*B*c^4*d^4
*tan(1/2*f*x + 1/2*e)^2 - 6*A*c^3*d^5*tan(1/2*f*x + 1/2*e)^2 + 17*B*c^3*d^5
*tan(1/2*f*x + 1/2*e)^2 - 19*A*c^2*d^6*tan(1/2*f*x + 1/2*e)^2 + 12*B*c^2*d^
6*tan(1/2*f*x + 1/2*e)^2 - 12*A*c*d^7*tan(1/2*f*x + 1/2*e)^2 + 2*B*c*d^7*ta
n(1/2*f*x + 1/2*e)^2 + 2*A*d^8*tan(1/2*f*x + 1/2*e)^2 + 23*B*c^4*d^4*tan(1/
2*f*x + 1/2*e) - 29*A*c^3*d^5*tan(1/2*f*x + 1/2*e) + 18*B*c^3*d^5*tan(1/2*f
*x + 1/2*e) - 18*A*c^2*d^6*tan(1/2*f*x + 1/2*e) + 4*B*c^2*d^6*tan(1/2*f*x +
 1/2*e) + 2*A*c*d^7*tan(1/2*f*x + 1/2*e) + 8*B*c^5*d^3 - 10*A*c^4*d^4 + 6*B
*c^4*d^4 - 6*A*c^3*d^5 + B*c^3*d^5 + A*c^2*d^6)/((a^3*c^9 - 3*a^3*c^8*d + a
^3*c^7*d^2 + 5*a^3*c^6*d^3 - 5*a^3*c^5*d^4 - a^3*c^4*d^5 + 3*a^3*c^3*d^6 -
a^3*c^2*d^7)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2) -
 2*(15*A*c^2*tan(1/2*f*x + 1/2*e)^4 - 75*A*c*d*tan(1/2*f*x + 1/2*e)^4 + 150
*A*d^2*tan(1/2*f*x + 1/2*e)^4 - 90*B*d^2*tan(1/2*f*x + 1/2*e)^4 + 30*A*c^2*
tan(1/2*f*x + 1/2*e)^3 + 15*B*c^2*tan(1/2*f*x + 1/2*e)^3 - 195*A*c*d*tan(1/
2*f*x + 1/2*e)^3 - 75*B*c*d*tan(1/2*f*x + 1/2*e)^3 + 525*A*d^2*tan(1/2*f*x
+ 1/2*e)^3 - 300*B*d^2*tan(1/2*f*x + 1/2*e)^3 + 40*A*c^2*tan(1/2*f*x + 1/2*
e)^2 + 15*B*c^2*tan(1/2*f*x + 1/2*e)^2 - 245*A*c*d*tan(1/2*f*x + 1/2*e)^2 -
 135*B*c*d*tan(1/2*f*x + 1/2*e)^2 + 745*A*d^2*tan(1/2*f*x + 1/2*e)^2 - 420*
B*d^2*tan(1/2*f*x + 1/2*e)^2 + 20*A*c^2*tan(1/2*f*x + 1/2*e) + 15*B*c^2*tan
(1/2*f*x + 1/2*e) - 145*A*c*d*tan(1/2*f*x + 1/2*e) - 105*B*c*d*tan(1/2*f*x
```

```
+ 1/2*e) + 485*A*d^2*tan(1/2*f*x + 1/2*e) - 270*B*d^2*tan(1/2*f*x + 1/2*e)
+ 7*A*c^2 + 3*B*c^2 - 44*A*c*d - 21*B*c*d + 127*A*d^2 - 72*B*d^2)/((a^3*c^5
 - 5*a^3*c^4*d + 10*a^3*c^3*d^2 - 10*a^3*c^2*d^3 + 5*a^3*c*d^4 - a^3*d^5)*(
tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

maple [B] time = 0.65, size = 2918, normalized size = 5.74

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -10/f/a^3*d^4/(c-d)^5/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(
c^2+2*c*d+d^2)*A*c^2-19/f/a^3*d^6/(c-d)^5/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2
*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2*A-6/f/a^3*d^5/(c-d)
^5/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*A*c-
6/f/a^3*d^6/(c-d)^5/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^
2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*A+8/f/a^3*d^3/(c-d)^5/(tan(1/2*f*x+1/2*e)
^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*B*c^3+6/f/a^3*d^4/(c-d)^5/
(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*B*c^2-1
8/f/a^3*d^6/(c-d)^5/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^
2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*A+4/f/a^3*d^6/(c-d)^5/(tan(1/2*f*x+1/2*e)^2
*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*B-13/f/a^
3*d^5/(c-d)^5/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1
/2*e)+2*d)/(c^2-d^2)^(1/2))*A+6/f/a^3*d^5/(c-d)^5/(c^2+2*c*d+d^2)/(c^2-d^2)
^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B+12/f/a^3*
d^6/(c-d)^5/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+
d^2)*tan(1/2*f*x+1/2*e)^2*B+1/f/a^3*d^5/(c-d)^5/(tan(1/2*f*x+1/2*e)^2*c+2*t
an(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*B*c-2/f/a^3/(c-d)^4/(tan(1/2*f*x+1
/2*e)+1)^2*B*c+8/f/a^3/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)^2*B*d-16/3/f/a^3/(c-d
)^4/(tan(1/2*f*x+1/2*e)+1)^3*A*c+28/3/f/a^3/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)^
3*A*d+4/f/a^3/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)^3*B*c-8/f/a^3/(c-d)^4/(tan(1/2
*f*x+1/2*e)+1)^3*B*d-2/f/a^3/(c-d)^5/(tan(1/2*f*x+1/2*e)+1)*A*c^2-20/f/a^3/
(c-d)^5/(tan(1/2*f*x+1/2*e)+1)*A*d^2+2/f/a^3*d^7/(c-d)^5/(tan(1/2*f*x+1/2*e
)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*A+23
/f/a^3*d^4/(c-d)^5/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^2/
(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*B+18/f/a^3*d^5/(c-d)^5/(tan(1/2*f*x+1/2*
e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*B+2
4/f/a^3*d^3/(c-d)^5/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2
*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^2+1/f/a^3*d^6/(c-d)^5/(tan(1/2*f*x+1/
2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*A+10/f/a^3/(c-d)^5/(ta
n(1/2*f*x+1/2*e)+1)*A*c*d+2/f/a^3*d^7/(c-d)^5/(tan(1/2*f*x+1/2*e)^2*c+2*tan
(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^2*B-29/f/a^3*d^
5/(c-d)^5/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+
d^2)*tan(1/2*f*x+1/2*e)*A+12/f/a^3*d^2/(c-d)^5/(c^2+2*c*d+d^2)/(c^2-d^2)^(1
```

```
/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^3-20/f/a^3
*d^3/(c-d)^5/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/
2*e)+2*d)/(c^2-d^2)^(1/2))*A*c^2-12/f/a^3*d^7/(c-d)^5/(tan(1/2*f*x+1/2*e)^2
*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^2*A-30/
f/a^3*d^4/(c-d)^5/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f
*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c+21/f/a^3*d^4/(c-d)^5/(c^2+2*c*d+d^2)/(c
^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c+
2/f/a^3*d^8/(c-d)^5/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^
2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/2*e)^2*A+8/f/a^3*d^3/(c-d)^5/(tan(1/2*f*x+1/
2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^3*tan(1/2*f*x+1/2*e)
^2*B+6/f/a^3*d^4/(c-d)^5/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^
2/(c^2+2*c*d+d^2)*c^2*tan(1/2*f*x+1/2*e)^2*B-11/f/a^3*d^5/(c-d)^5/(tan(1/2*
f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/
2*e)^3*A+2/f/a^3*d^7/(c-d)^5/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d
+c)^2/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*A+9/f/a^3*d^4/(c-d)^5/(tan(1/2
*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^2+2*c*d+d^2)*tan(1/2*f*x
+1/2*e)^3*B+6/f/a^3*d^5/(c-d)^5/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e
)*d+c)^2*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*B-10/f/a^3*d^4/(c-d)^5/(tan
(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^2*tan(1/2
*f*x+1/2*e)^2*A-6/f/a^3*d^5/(c-d)^5/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1
/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*tan(1/2*f*x+1/2*e)^2*A+17/f/a^3*d^5/(c-d)^5/
(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*tan(1
/2*f*x+1/2*e)^2*B+12/f/a^3/(c-d)^5/(tan(1/2*f*x+1/2*e)+1)*B*d^2+4/f/a^3/(c-
d)^4/(tan(1/2*f*x+1/2*e)+1)^2*A*c-10/f/a^3/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)^2
*A*d+4/f/a^3/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^4*A-4/f/a^3/(c-d)^3/(tan(1/2*f*
x+1/2*e)+1)^4*B-8/5/f/a^3/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^5*A+8/5/f/a^3/(c-d
)^3/(tan(1/2*f*x+1/2*e)+1)^5*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 19.88, size = 2387, normalized size = 4.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^3),x)

[Out] ((15*A*d^6 - 14*A*c^6 - 6*B*c^6 - 404*A*c^2*d^4 - 420*A*c^3*d^3 - 92*A*c^4*
d^2 + 234*B*c^2*d^4 + 450*B*c^3*d^3 + 222*B*c^4*d^2 - 90*A*c*d^5 + 60*A*c^5
*d + 15*B*c*d^5 + 30*B*c^5*d)/(15*(c + d)^2*(c - d)*(c^4 - 4*c^3*d - 4*c*d^
3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^7*(2*A*d^8 - 4*A*c^8 - 2*B*c^8
- 49*A*c^2*d^6 - 141*A*c^3*d^5 - 200*A*c^4*d^4 - 122*A*c^5*d^3 + 2*A*c^6*d^
2 + 12*B*c^2*d^6 + 95*B*c^3*d^5 + 187*B*c^4*d^4 + 146*B*c^5*d^3 + 58*B*c^6*
d^2 - 2*A*c*d^7 + 10*A*c^7*d + 2*B*c*d^7 + 6*B*c^7*d))/(c^2*(c - d)*(2*c*d
+ c^2 + d^2)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x
)/2)^6*(30*A*d^8 - 28*A*c^8 - 6*B*c^8 - 759*A*c^2*d^6 - 1707*A*c^3*d^5 - 19
60*A*c^4*d^4 - 870*A*c^5*d^3 + 62*A*c^6*d^2 + 336*B*c^2*d^6 + 1257*B*c^3*d^
5 + 1893*B*c^4*d^4 + 1350*B*c^5*d^3 + 414*B*c^6*d^2 - 114*A*c*d^7 + 54*A*c^
7*d + 30*B*c*d^7 + 18*B*c^7*d))/(3*c^2*(c - d)*(2*c*d + c^2 + d^2)*(c^4 - 4
*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^5*(60*A*d^8 - 32
*A*c^8 - 18*B*c^8 - 1857*A*c^2*d^6 - 3763*A*c^3*d^5 - 3560*A*c^4*d^4 - 1294
*A*c^5*d^3 + 70*A*c^6*d^2 + 900*B*c^2*d^6 + 2859*B*c^3*d^5 + 3705*B*c^4*d^4
 + 2358*B*c^5*d^3 + 678*B*c^6*d^2 - 270*A*c*d^7 + 62*A*c^7*d + 60*B*c*d^7 +
 42*B*c^7*d))/(3*c^2*(c - d)*(2*c*d + c^2 + d^2)*(c^4 - 4*c^3*d - 4*c*d^3 +
 d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^2*(30*A*d^8 - 108*A*c^8 - 42*B*c^8
 - 2501*A*c^2*d^6 - 8725*A*c^3*d^5 - 10616*A*c^4*d^4 - 4810*A*c^5*d^3 + 10*
A*c^6*d^2 + 1056*B*c^2*d^6 + 5235*B*c^3*d^5 + 9891*B*c^4*d^4 + 7770*B*c^5*d
^3 + 2370*B*c^6*d^2 - 30*A*c*d^7 + 290*A*c^7*d + 30*B*c*d^7 + 150*B*c^7*d))
/(15*c^2*(c + d)^2*(c - d)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (
tan(e/2 + (f*x)/2)^3*(150*A*d^8 - 140*A*c^8 - 90*B*c^8 - 7945*A*c^2*d^6 - 1
9441*A*c^3*d^5 - 18600*A*c^4*d^4 - 6898*A*c^5*d^3 + 210*A*c^6*d^2 + 3660*B*
c^2*d^6 + 13311*B*c^3*d^5 + 19455*B*c^4*d^4 + 12618*B*c^5*d^3 + 3570*B*c^6*
d^2 - 570*A*c*d^7 + 314*A*c^7*d + 150*B*c*d^7 + 246*B*c^7*d))/(15*c^2*(c +
d)^2*(c - d)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x
)/2)*(30*A*d^7 - 40*A*c^7 - 30*B*c^7 - 1901*A*c^2*d^5 - 3400*A*c^3*d^4 - 20
18*A*c^4*d^3 - 190*A*c^5*d^2 + 921*B*c^2*d^5 + 2655*B*c^3*d^4 + 2778*B*c^4*
d^3 + 1050*B*c^5*d^2 - 195*A*c*d^6 + 154*A*c^6*d + 60*B*c*d^6 + 126*B*c^6*d
))/(15*c*(c + d)^2*(c - d)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) - (
tan(e/2 + (f*x)/2)^8*(2*A*c^7 - 2*A*d^7 + 11*A*c^2*d^5 + 20*A*c^3*d^4 + 30*
A*c^4*d^3 + 2*A*c^5*d^2 - 6*B*c^2*d^5 - 21*B*c^3*d^4 - 24*B*c^4*d^3 - 12*B*
c^5*d^2 + 6*A*c*d^6 - 6*A*c^6*d))/(c*(c - d)*(2*c*d + c^2 + d^2)*(c^4 - 4*c
^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^4*(300*A*d^7 - 204
*A*c^7 - 66*B*c^7 - 10235*A*c^2*d^5 - 14330*A*c^3*d^4 - 7254*A*c^4*d^3 - 31
6*A*c^5*d^2 + 5460*B*c^2*d^5 + 12675*B*c^3*d^4 + 10764*B*c^4*d^3 + 3666*B*c
^5*d^2 - 1650*A*c*d^6 + 614*A*c^6*d + 300*B*c*d^6 + 276*B*c^6*d))/(15*c^2*(
c + d)*(c - d)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)))/(f*(tan(e/2 +
(f*x)/2)*(5*a^3*c^2 + 4*a^3*c*d) + tan(e/2 + (f*x)/2)^2*(12*a^3*c^2 + 4*a^3
*d^2 + 20*a^3*c*d) + tan(e/2 + (f*x)/2)^7*(12*a^3*c^2 + 4*a^3*d^2 + 20*a^3*
c*d) + tan(e/2 + (f*x)/2)^3*(20*a^3*c^2 + 20*a^3*d^2 + 44*a^3*c*d) + tan(e/
2 + (f*x)/2)^6*(20*a^3*c^2 + 20*a^3*d^2 + 44*a^3*c*d) + tan(e/2 + (f*x)/2)^
```

```
4*(26*a^3*c^2 + 40*a^3*d^2 + 60*a^3*c*d) + tan(e/2 + (f*x)/2)^5*(26*a^3*c^2
 + 40*a^3*d^2 + 60*a^3*c*d) + tan(e/2 + (f*x)/2)^8*(5*a^3*c^2 + 4*a^3*c*d)
+ a^3*c^2 + a^3*c^2*tan(e/2 + (f*x)/2)^9)) - (d^2*atan(((d^2*(12*B*c^3 - 13
*A*d^3 + 6*B*d^3 - 30*A*c*d^2 - 20*A*c^2*d + 21*B*c*d^2 + 24*B*c^2*d)*(2*a^
3*d^8 - 6*a^3*c*d^7 - 2*a^3*c^7*d + 2*a^3*c^2*d^6 + 10*a^3*c^3*d^5 - 10*a^3
*c^4*d^4 - 2*a^3*c^5*d^3 + 6*a^3*c^6*d^2))/(2*a^3*(c + d)^(5/2)*(c - d)^(11
/2)) - (c*d^2*tan(e/2 + (f*x)/2)*(12*B*c^3 - 13*A*d^3 + 6*B*d^3 - 30*A*c*d^
2 - 20*A*c^2*d + 21*B*c*d^2 + 24*B*c^2*d)*(a^3*c^7 - a^3*d^7 + 3*a^3*c*d^6
- 3*a^3*c^6*d - a^3*c^2*d^5 - 5*a^3*c^3*d^4 + 5*a^3*c^4*d^3 + a^3*c^5*d^2))
/(a^3*(c + d)^(5/2)*(c - d)^(11/2)))/(6*B*d^5 - 13*A*d^5 - 20*A*c^2*d^3 + 2
4*B*c^2*d^3 + 12*B*c^3*d^2 - 30*A*c*d^4 + 21*B*c*d^4))*(12*B*c^3 - 13*A*d^3
 + 6*B*d^3 - 30*A*c*d^2 - 20*A*c^2*d + 21*B*c*d^2 + 24*B*c^2*d))/(a^3*f*(c
+ d)^(5/2)*(c - d)^(11/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**3,x)
[Out] Timed out
```

3.286 $\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=256

$$\frac{4a(c+d)\left(15c^2+10cd+7d^2\right)(-9Ad+Bc-8Bd)\cos(e+fx)}{315df\sqrt{a\sin(e+fx)+a}}+\frac{2a(-9Ad+Bc-8Bd)\cos(e+fx)(c+d\sin(e+$$

```
[Out] 4/105*d*(c+d)*(-9*A*d+B*c-8*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/a/f+4/31
5*a*(c+d)*(-9*A*d+B*c-8*B*d)*(15*c^2+10*c*d+7*d^2)*cos(f*x+e)/d/f/(a+a*sin(
f*x+e))^(1/2)+2/63*a*(-9*A*d+B*c-8*B*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3/d/f/(
a+a*sin(f*x+e))^(1/2)-2/9*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^4/d/f/(a+a*sin(f*
x+e))^(1/2)+8/315*(5*c-d)*(c+d)*(-9*A*d+B*c-8*B*d)*cos(f*x+e)*(a+a*sin(f*x+
e))^(1/2)/f
```

Rubi [A] time = 0.46, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {2981, 2770, 2761, 2751, 2646}

$$\frac{4a(c+d)\left(15c^2+10cd+7d^2\right)(-9Ad+Bc-8Bd)\cos(e+fx)}{315df\sqrt{a\sin(e+fx)+a}}+\frac{2a(-9Ad+Bc-8Bd)\cos(e+fx)(c+d\sin(e+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (4*a*(c + d)*(B*c - 9*A*d - 8*B*d)*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/
(315*d*f*Sqrt[a + a*Sin[e + f*x]]) + (8*(5*c - d)*(c + d)*(B*c - 9*A*d - 8*
B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(315*f) + (4*d*(c + d)*(B*c - 9
*A*d - 8*B*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(105*a*f) + (2*a*(B*
c - 9*A*d - 8*B*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(63*d*f*Sqrt[a + a*
Sin[e + f*x]]) - (2*a*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(9*d*f*Sqrt[a
+ a*Sin[e + f*x]])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{a + a\sin(e + fx)}\,(A + B\sin(e + fx))(c + d\sin(e + fx))^3\,dx = -\frac{2aB\cos(e + fx)(c + d\sin(e + fx))^4}{9df\sqrt{a + a\sin(e + fx)}} + \frac{(9a\ldots}{}$$

$$= \frac{2a(Bc - 9Ad - 8Bd)\cos(e + fx)(c + d\sin(e\ldots}{63df\sqrt{a + a\sin(e + fx)}}$$

$$= \frac{4d(c + d)(Bc - 9Ad - 8Bd)\cos(e + fx)(a + \ldots}{105af}$$

$$= \frac{8(5c - d)(c + d)(Bc - 9Ad - 8Bd)\cos(e + f\ldots}{315f}$$

$$= \frac{4a(c + d)(Bc - 9Ad - 8Bd)\left(15c^2 + 10cd + 7\ldots\right.}{315df\sqrt{a + a\sin(e + fx)}}$$

Mathematica [A] time = 1.25, size = 305, normalized size = 1.19

$$-\frac{\sqrt{a(\sin(e + fx) + 1)}\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)\left(-4d\left(27Ad(7c + 2d) + B\left(189c^2 + 162cd + 83d^2\right)\right)\text{co}\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^3,x]
```

```
[Out] -1/1260*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(
2520*A*c^3 + 1680*B*c^3 + 5040*A*c^2*d + 4788*B*c^2*d + 4788*A*c*d^2 + 4104
*B*c*d^2 + 1368*A*d^3 + 1321*B*d^3 - 4*d*(27*A*d*(7*c + 2*d) + B*(189*c^2 +
 162*c*d + 83*d^2))*Cos[2*(e + f*x)] + 35*B*d^3*Cos[4*(e + f*x)] + 840*B*c^
3*Sin[e + f*x] + 2520*A*c^2*d*Sin[e + f*x] + 2016*B*c^2*d*Sin[e + f*x] + 20
16*A*c*d^2*Sin[e + f*x] + 2538*B*c*d^2*Sin[e + f*x] + 846*A*d^3*Sin[e + f*x
] + 752*B*d^3*Sin[e + f*x] - 270*B*c*d^2*Sin[3*(e + f*x)] - 90*A*d^3*Sin[3*
(e + f*x)] - 80*B*d^3*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2]))
```

fricas [A] time = 0.45, size = 467, normalized size = 1.82

$$-\frac{2\left(35\,Bd^3\cos\left(fx + e\right)^5 - 5\left(27\,Bcd^2 + (9\,A + B)d^3\right)\cos\left(fx + e\right)^4 + 105\,(3\,A + B)c^3 + 63\,(5\,A + 7\,B)c^2d + 9\,(\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -2/315*(35*B*d^3*cos(f*x + e)^5 - 5*(27*B*c*d^2 + (9*A + B)*d^3)*cos(f*x +
e)^4 + 105*(3*A + B)*c^3 + 63*(5*A + 7*B)*c^2*d + 9*(49*A + 27*B)*c*d^2 + (
81*A + 107*B)*d^3 - (189*B*c^2*d + 27*(7*A + 6*B)*c*d^2 + 2*(27*A + 59*B)*d
^3)*cos(f*x + e)^3 + (105*B*c^3 + 63*(5*A + B)*c^2*d + 9*(7*A + 36*B)*c*d^2
 + 2*(54*A + 13*B)*d^3)*cos(f*x + e)^2 + (105*(3*A + 2*B)*c^3 + 63*(10*A +
11*B)*c^2*d + 99*(7*A + 6*B)*c*d^2 + (198*A + 211*B)*d^3)*cos(f*x + e) - (3
5*B*d^3*cos(f*x + e)^4 + 105*(3*A + B)*c^3 + 63*(5*A + 7*B)*c^2*d + 9*(49*A
 + 27*B)*c*d^2 + (81*A + 107*B)*d^3 + 5*(27*B*c*d^2 + (9*A + 8*B)*d^3)*cos(
f*x + e)^3 - 3*(63*B*c^2*d + 9*(7*A + B)*c*d^2 + (3*A + 26*B)*d^3)*cos(f*x
+ e)^2 - (105*B*c^3 + 63*(5*A + 4*B)*c^2*d + 9*(28*A + 39*B)*c*d^2 + 13*(9*
A + 8*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f
*x + e) + f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)sqrt(2*a)*(-40*f*(-2*A*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-6*B*c*d^2*si
gn(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(10*f*x+10*exp(1)+pi))/(40*f)^2-5
6*f*(-2*A*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-6*B*c*d^2*sign(cos(1/2*(f*
x+exp(1))-1/4*pi)))*cos(1/4*(14*f*x+14*exp(1)-pi))/(56*f)^2+12*f*(-2*B*d^3*
```

```
sign(cos(1/2*(f*x+exp(1))-1/4*pi))-6*A*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))-6*B*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(6*f*x+6*exp(1)+
pi))/(12*f)^2+20*f*(-2*B*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-6*A*c*d^2*s
ign(cos(1/2*(f*x+exp(1))-1/4*pi))-6*B*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*p
i)))*sin(1/4*(10*f*x+10*exp(1)-pi))/(20*f)^2-8*f*(6*A*d^3*sign(cos(1/2*(f*x
+exp(1))-1/4*pi))+8*B*c^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+24*A*c^2*d*sig
n(cos(1/2*(f*x+exp(1))-1/4*pi))+18*B*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi
)))*cos(1/4*(2*f*x+2*exp(1)+pi))/(8*f)^2-24*f*(6*A*d^3*sign(cos(1/2*(f*x+ex
p(1))-1/4*pi))+8*B*c^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+24*A*c^2*d*sign(c
os(1/2*(f*x+exp(1))-1/4*pi))+18*B*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))
*cos(1/4*(6*f*x+6*exp(1)-pi))/(24*f)^2+8*f*(16*A*c^3*sign(cos(1/2*(f*x+exp(
1))-1/4*pi))+6*B*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+24*A*c*d^2*sign(cos
(1/2*(f*x+exp(1))-1/4*pi))+24*B*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*s
in(1/4*(2*f*x-pi)+1/2*exp(1))/(8*f)^2+224*B*d^3*f*sign(cos(1/2*(f*x+exp(1))
-1/4*pi))*sin(1/4*(14*f*x+14*exp(1)+pi))/(112*f)^2+288*B*d^3*f*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))*sin(1/4*(18*f*x+18*exp(1)-pi))/(144*f)^2)
```

maple [A] time = 1.40, size = 242, normalized size = 0.95

$$\frac{2\left(1+\sin\left(fx+e\right)\right)a\left(\sin\left(fx+e\right)-1\right)\left(\left(-45A\,d^3-135Bc\,d^2-40B\,d^3\right)\sin\left(fx+e\right)\left(\cos^2\left(fx+e\right)\right)+\left(315A\,c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/315*(1+sin(f*x+e))*a*(sin(f*x+e)-1)*((-45*A*d^3-135*B*c*d^2-40*B*d^3)*sin
(f*x+e)*cos(f*x+e)^2+(315*A*c^2*d+252*A*c*d^2+117*A*d^3+105*B*c^3+252*B*c^2
*d+351*B*c*d^2+104*B*d^3)*sin(f*x+e)+35*B*cos(f*x+e)^4*d^3+(-189*A*c*d^2-54
*A*d^3-189*B*c^2*d-162*B*c*d^2-118*B*d^3)*cos(f*x+e)^2+315*A*c^3+630*A*c^2*
d+693*A*c*d^2+198*A*d^3+210*B*c^3+693*B*c^2*d+594*B*c*d^2+211*B*d^3)/cos(f*
x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^3dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c
)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3,
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3,
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\left(A+B\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))*(c + d*sin(e + f*x
))**3, x)
```

3.287 $\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=192

$$\frac{2a\left(15c^2+10cd+7d^2\right)(-7Ad+Bc-6Bd)\cos(e+fx)}{105df\sqrt{a\sin(e+fx)+a}}+\frac{2d(-7Ad+Bc-6Bd)\cos(e+fx)(a\sin(e+fx)+a)^{3/}}{35af}$$

```
[Out] 2/35*d*(-7*A*d+B*c-6*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/a/f+2/105*a*(-7
*A*d+B*c-6*B*d)*(15*c^2+10*c*d+7*d^2)*cos(f*x+e)/d/f/(a+a*sin(f*x+e))^(1/2)
-2/7*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^3/d/f/(a+a*sin(f*x+e))^(1/2)+4/105*(5*
c-d)*(-7*A*d+B*c-6*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.34, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2981, 2761, 2751, 2646}

$$\frac{2a\left(15c^2+10cd+7d^2\right)(-7Ad+Bc-6Bd)\cos(e+fx)}{105df\sqrt{a\sin(e+fx)+a}}+\frac{2d(-7Ad+Bc-6Bd)\cos(e+fx)(a\sin(e+fx)+a)^{3/}}{35af}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (2*a*(B*c - 7*A*d - 6*B*d)*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/(105*d*f
*Sqrt[a + a*Sin[e + f*x]]) + (4*(5*c - d)*(B*c - 7*A*d - 6*B*d)*Cos[e + f*x
]*Sqrt[a + a*Sin[e + f*x]])/(105*f) + (2*d*(B*c - 7*A*d - 6*B*d)*Cos[e + f*
x]*(a + a*Sin[e + f*x])^(3/2))/(35*a*f) - (2*a*B*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^3)/(7*d*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = -\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a+a\sin(e+fx)}} + \frac{(7aA\ldots}{\ldots}$$

$$= \frac{2d(Bc-7Ad-6Bd)\cos(e+fx)(a+a\sin(e+\ldots}{35af}$$

$$= \frac{4(5c-d)(Bc-7Ad-6Bd)\cos(e+fx)\sqrt{a+a\ldots}}{105f}$$

$$= \frac{2a(Bc-7Ad-6Bd)\left(15c^2+10cd+7d^2\right)\cos(\ldots}{105df\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 0.74, size = 176, normalized size = 0.92

$$-\frac{\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\left(56Ad(5c+2d)+B\left(140c^2+224cd+141d^2\right)\right)\sin(e+\ldots}{210f\left(\sin\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^2,x]
```

```
[Out] -1/210*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(4
20*A*c^2 + 280*B*c^2 + 560*A*c*d + 532*B*c*d + 266*A*d^2 + 228*B*d^2 - 6*d*
(14*B*c + 7*A*d + 6*B*d)*Cos[2*(e + f*x)] + (56*A*d*(5*c + 2*d) + B*(140*c^
2 + 224*c*d + 141*d^2))*Sin[e + f*x] - 15*B*d^2*Sin[3*(e + f*x)]))/(f*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2]))
```

fricas [A] time = 0.43, size = 306, normalized size = 1.59

$$2\left(15\,Bd^2\cos\left(fx+e\right)^4+3\left(14\,Bcd+(7\,A+6\,B)d^2\right)\cos\left(fx+e\right)^3-35\,(3\,A+B)c^2-14\,(5\,A+7\,B)cd-(49\,A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/105*(15*B*d^2*cos(f*x + e)^4 + 3*(14*B*c*d + (7*A + 6*B)*d^2)*cos(f*x + e
)^3 - 35*(3*A + B)*c^2 - 14*(5*A + 7*B)*c*d - (49*A + 27*B)*d^2 - (35*B*c^2
 + 14*(5*A + B)*c*d + (7*A + 36*B)*d^2)*cos(f*x + e)^2 - (35*(3*A + 2*B)*c^
2 + 14*(10*A + 11*B)*c*d + 11*(7*A + 6*B)*d^2)*cos(f*x + e) + (15*B*d^2*cos
(f*x + e)^3 + 35*(3*A + B)*c^2 + 14*(5*A + 7*B)*c*d + (49*A + 27*B)*d^2 - 3
*(14*B*c*d + (7*A + B)*d^2)*cos(f*x + e)^2 - (35*B*c^2 + 14*(5*A + 4*B)*c*d
 + (28*A + 39*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/
(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
```

```
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*(12*f*(-2*
A*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-4*B*c*d*sign(cos(1/2*(f*x+exp(1))-
1/4*pi)))*sin(1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2+20*f*(-2*A*d^2*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))-4*B*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*
(10*f*x+10*exp(1)-pi))/(20*f)^2+2*f*(4*A*c^2*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))+2*A*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+4*B*c*d*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi)))*sin(1/4*(2*f*x-pi)+1/2*exp(1))/(2*f)^2-8*f*(8*B*c^2*sign(c
os(1/2*(f*x+exp(1))-1/4*pi))+6*B*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+16*
A*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(2*f*x+2*exp(1)+pi))/(8*f
)^2-24*f*(8*B*c^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+6*B*d^2*sign(cos(1/2*(
f*x+exp(1))-1/4*pi))+16*A*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(
6*f*x+6*exp(1)-pi))/(24*f)^2+80*B*d^2*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*
cos(1/4*(10*f*x+10*exp(1)+pi))/(40*f)^2+112*B*d^2*f*sign(cos(1/2*(f*x+exp(1
))-1/4*pi))*cos(1/4*(14*f*x+14*exp(1)-pi))/(56*f)^2)
```

maple [A] time = 1.44, size = 161, normalized size = 0.84

$$\frac{2\left(1+\sin\left(fx+e\right)\right)a\left(\sin\left(fx+e\right)-1\right)\left(-15B\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)d^{2}+\left(70Acd+28A\,d^{2}+35B\,c^{2}+56B\right.}{105\cos\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/105*(1+sin(f*x+e))*a*(sin(f*x+e)-1)*(-15*B*cos(f*x+e)^2*sin(f*x+e)*d^2+(7
0*A*c*d+28*A*d^2+35*B*c^2+56*B*c*d+39*B*d^2)*sin(f*x+e)+(-21*A*d^2-42*B*c*d
-18*B*d^2)*cos(f*x+e)^2+105*A*c^2+140*A*c*d+77*A*d^2+70*B*c^2+154*B*c*d+66*
B*d^2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c
)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(A+B\,\sin\left(e+f\,x\right)\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2,
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2,
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(A+B\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))*(c + d*sin(e + f*x
))**2, x)
```

3.288 $\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=118

$$-\frac{2(5Ad+5Bc-2Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{2a(15Ac+5Ad+5Bc+7Bd)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-2Bd\cos(e+f$$

```
[Out] -2/5*B*d*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/a/f-2/15*a*(15*A*c+5*A*d+5*B*c+7
*B*d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/15*(5*A*d+5*B*c-2*B*d)*cos(f*x+
e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.25, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2968, 3023, 2751, 2646}

$$-\frac{2(5Ad+5Bc-2Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{2a(15Ac+5Ad+5Bc+7Bd)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-2Bd\cos(e+f$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (-2*a*(15*A*c + 5*B*c + 5*A*d + 7*B*d)*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e
 + f*x]]) - (2*(5*B*c + 5*A*d - 2*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]
])/(15*f) - (2*B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*a*f)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int \sqrt{a+a\sin(e+fx)}\,\left(Ac+(Bc+Ad)\sin(e\right.$$

$$= -\frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5af} + \frac{2\int}{}$$

$$= -\frac{2(5Bc+5Ad-2Bd)\cos(e+fx)\sqrt{a+a\sin}}{15f}$$

$$= -\frac{2a(15Ac+5Bc+5Ad+7Bd)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 0.36, size = 117, normalized size = 0.99

$$-\frac{\sqrt{a(\sin(e+fx)+1)}\,\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(2(5Ad+5Bc+4Bd)\sin(e+fx)+30Ac+20Ad+}{15f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
),x]
```

```
[Out] -1/15*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(30
*A*c + 20*B*c + 20*A*d + 19*B*d - 3*B*d*Cos[2*(e + f*x)] + 2*(5*B*c + 5*A*d
 + 4*B*d)*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

fricas [A] time = 0.45, size = 175, normalized size = 1.48

$$\frac{2\left(3\,Bd\cos\left(fx+e\right)^{3}-\left(5\,Bc+\left(5\,A+B\right)d\right)\cos\left(fx+e\right)^{2}-5\,\left(3\,A+B\right)c-\left(5\,A+7\,B\right)d-\left(5\,\left(3\,A+2\,B\right)c+\left(10\,A\right.\right.\right.}{15\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 2/15*(3*B*d*cos(f*x + e)^3 - (5*B*c + (5*A + B)*d)*cos(f*x + e)^2 - 5*(3*A
+ B)*c - (5*A + 7*B)*d - (5*(3*A + 2*B)*c + (10*A + 11*B)*d)*cos(f*x + e) -
 (3*B*d*cos(f*x + e)^2 - 5*(3*A + B)*c - (5*A + 7*B)*d + (5*B*c + (5*A + 4*
B)*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e)
+ f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*(-2*f*(2*A*d*sign(
cos(1/2*(f*x+exp(1))-1/4*pi))+2*B*c*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos
(1/4*(2*f*x+2*exp(1)+pi))/(2*f)^2-6*f*(2*A*d*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))+2*B*c*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(6*f*x+6*exp(1)-pi))
/(6*f)^2+2*f*(4*A*c*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+2*B*d*sign(cos(1/2*(
f*x+exp(1))-1/4*pi)))*sin(1/4*(2*f*x-pi)+1/2*exp(1))/(2*f)^2-24*B*d*f*sign(
cos(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2-40*B*d*
f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(10*f*x+10*exp(1)-pi))/(20*f)^
2)
```

maple [A] time = 1.36, size = 102, normalized size = 0.86

$$\frac{2\left(1+\sin\left(fx+e\right)\right)a\left(\sin\left(fx+e\right)-1\right)\left(3B\left(\sin^{2}\left(fx+e\right)\right)d+5A\sin\left(fx+e\right)d+5B\sin\left(fx+e\right)c+4B\sin\left(f\right.\right.}{15\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/15*(1+sin(f*x+e))*a*(sin(f*x+e)-1)*(3*B*sin(f*x+e)^2*d+5*A*sin(f*x+e)*d+5
*B*sin(f*x+e)*c+4*B*sin(f*x+e)*d+15*A*c+10*A*d+10*B*c+8*B*d)/cos(f*x+e)/(a+
a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c+d\,\sin\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x)),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x)), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\left(A+B\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))*(c + d*sin(e + f*x
)), x)
```

3.289 $\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=62

$$-\frac{2a(3A+B)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2B\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

```
[Out] -2/3*a*(3*A+B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/3*B*cos(f*x+e)*(a+a*si
n(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2751, 2646}

$$-\frac{2a(3A+B)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2B\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]),x]
```

```
[Out] (-2*a*(3*A + B)*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*B*Cos[e +
 f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))\,dx = -\frac{2B\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} + \frac{1}{3}(3A+B)\int \sqrt{a+a\sin}$$
$$= -\frac{2a(3A+B)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2B\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}$$

Mathematica [A] time = 0.12, size = 82, normalized size = 1.32

$$-\frac{2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(3A+B\sin(e+fx)+2B)}{3f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]),x]
```

```
[Out] (-2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(3*A +
 2*B + B*Sin[e + f*x]))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

fricas [A] time = 0.44, size = 85, normalized size = 1.37

$$-\frac{2\left(B\cos\left(fx+e\right)^{2}+(3A+2B)\cos\left(fx+e\right)+\left(B\cos\left(fx+e\right)-3A-B\right)\sin\left(fx+e\right)+3A+B\right)\sqrt{a\sin\left(fx\right.}}{3\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(B*cos(f*x + e)^2 + (3*A + 2*B)*cos(f*x + e) + (B*cos(f*x + e) - 3*A -
 B)*sin(f*x + e) + 3*A + B)*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*si
n(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
```

```
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*(2*A*sign(cos
(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(2*f*x-pi)+1/2*exp(1))/f-4*B*f*sign(cos(
1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(2*f*x+2*exp(1)+pi))/(2*f)^2-12*B*f*sign(
cos(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(6*f*x+6*exp(1)-pi))/(6*f)^2)
```

maple [A] time = 1.02, size = 58, normalized size = 0.94

$$\frac{2\left(1+\sin\left(fx+e\right)\right)a\left(\sin\left(fx+e\right)-1\right)\left(B\sin\left(fx+e\right)+3A+2B\right)}{3\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/3*(1+sin(f*x+e))*a*(sin(f*x+e)-1)*(B*sin(f*x+e)+3*A+2*B)/cos(f*x+e)/(a+a*
sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\left(A+B\,\sin\left(e+f\,x\right)\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\left(A+B\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x)), x)
```

3.290 $\int \frac{\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=100

$$\frac{2\sqrt{a}\,(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f\sqrt{c+d}}-\frac{2aB\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}$$

```
[Out] 2*(-A*d+B*c)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e)
)^(1/2))*a^(1/2)/d^(3/2)/f/(c+d)^(1/2)-2*a*B*cos(f*x+e)/d/f/(a+a*sin(f*x+e)
)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {2981, 2773, 208}

$$\frac{2\sqrt{a}\,(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f\sqrt{c+d}}-\frac{2aB\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*
Sqrt[a + a*Sin[e + f*x]])])/(d^(3/2)*Sqrt[c + d]*f) - (2*a*B*Cos[e + f*x])/
(d*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
```

```
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = -\frac{2aB\cos(e+fx)}{df\sqrt{a+a\sin(e+fx)}} + \frac{(-aBc+aAd)\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{ad}$$

$$= -\frac{2aB\cos(e+fx)}{df\sqrt{a+a\sin(e+fx)}} + \frac{(2a(Bc-Ad))\,\text{Subst}\left(\int \frac{1}{ac+ad-dx^2}\,dx, x$$

$$= \frac{2\sqrt{a}\,(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sin(e+fx)}}\right)}{d^{3/2}\sqrt{c+d}\,f} - \frac{2aB\cos(e+fx)}{df\sqrt{a+a\sin(e+f$$

Mathematica [C] time = 9.03, size = 903, normalized size = 9.03

$$\left(\frac{1}{2}+\frac{i}{2}\right)\left(-\frac{(2-2i)B\sqrt{d}\cos\left(\frac{fx}{2}\right)\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)}{f}+\frac{(Ad-Bc)\left(\cos\left(\frac{e}{2}\right)+i\sin\left(\frac{e}{2}\right)\right)\left((-1+i)x\cos(e)+(1+i)x\sin(e)+\frac{\text{RootSum}\left[de^{2ie}\#1^4+2ice^{ie}\#1^2-d\&,\frac{-\sqrt{d}}{}\right.}{}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*
x]),x]
```

```
[Out] ((1/2 + I/2)*(((-2 + 2*I)*B*Sqrt[d]*Cos[(f*x)/2]*(Cos[e/2] - Sin[e/2]))/f +
 ((-(B*c) + A*d)*(Cos[e/2] + I*Sin[e/2])*((-1 + I)*x*Cos[e] + (RootSum[-d +
 (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 + I)*d*Sqrt[E^((-I)*e)]*
f*x - (2 - 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] - I*Sqrt[d]*Sqrt
[c + d]*f*x*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 + ((1 - I
)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 + 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)
/Sqrt[E^((-I)*e)] - Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 - (2*I)*Sqrt[d]*Sq
rt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*
(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]])/(4*f) + (1 + I)*x*Sin[e
]))/(Sqrt[c + d]*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) + ((-(
B*c) + A*d)*(Cos[e/2] + I*Sin[e/2])*((1 - I)*x*Cos[e] - (1 + I)*x*Sin[e] +
(RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 - I)*d*Sqrt
[E^((-I)*e)]*f*x + (2 + 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] + S
```

```
qrt[d]*Sqrt[c + d]*f*x*#1 + (2*I)*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #
1]*#1 - ((1 + I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 - 2*I)*c*Log[E^((I/2)*f
*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - I*Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 +
 2*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*
e)*#1^2) & ]*Sqrt[Cos[e] - I*Sin[e]]*(-1 - I*Cos[e] + Sin[e]))/(4*f)))/(Sqr
t[c + d]*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) + ((2 - 2*I)*B
*Sqrt[d]*(Cos[e/2] + Sin[e/2])*Sin[(f*x)/2])/f)*Sqrt[a*(1 + Sin[e + f*x])])
/(d^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

fricas [A] time = 1.26, size = 651, normalized size = 6.51

$$\left[-\frac{\left(Bc - Ad + (Bc - Ad)\cos\left(fx+e\right) + (Bc - Ad)\sin\left(fx+e\right)\right)\sqrt{\frac{a}{cd+d^2}}\,\log\left(\frac{ad^2\cos\left(fx+e\right)^3 - ac^2 - 2\,acd - ad^2 - \left(6\,acd + 7\,ad^2\right)c}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] [-1/2*((B*c - A*d + (B*c - A*d)*cos(f*x + e) + (B*c - A*d)*sin(f*x + e))*sq
rt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*
a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3
)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^
2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) +
 a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2
*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x +
 e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2
 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos
(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(B*cos(f*x + e) - B*sin(f
*x + e) + B)*sqrt(a*sin(f*x + e) + a))/(d*f*cos(f*x + e) + d*f*sin(f*x + e)
 + d*f), ((B*c - A*d + (B*c - A*d)*cos(f*x + e) + (B*c - A*d)*sin(f*x + e))
*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) -
 c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(B*cos(f*x + e) - B*si
n(f*x + e) + B)*sqrt(a*sin(f*x + e) + a))/(d*f*cos(f*x + e) + d*f*sin(f*x +
 e) + d*f)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 1.79, size = 139, normalized size = 1.39

$$-\frac{2\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\left(A\,\mathrm{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,d}{\sqrt{a(c+d)d}}\right)ad-B\,\mathrm{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,d}{\sqrt{a(c+d)d}}\right)\right.}{d\sqrt{a\left(c+d\right)d}\,\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] -2*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(A*arctanh((-a*(sin(f*x+e)-1))^
(1/2)*d/(a*(c+d)*d)^(1/2))*a*d-B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+
d)*d)^(1/2))*a*c+B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2))/d/(a*(c+d)*
d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}}{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) + c
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{c+d\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x)),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x)),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e)),x)
[Out] Timed out
```

3.291 $\int \frac{\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=126

$$\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}-\frac{\sqrt{a}\,(Ad+B(c+2d))\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f(c+d)^{3/2}}$$

```
[Out] -(A*d+B*(c+2*d))*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*
x+e))^(1/2))*a^(1/2)/d^(3/2)/(c+d)^(3/2)/f+a*(-A*d+B*c)*cos(f*x+e)/d/(c+d)/
f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {2980, 2773, 208}

$$\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}-\frac{\sqrt{a}\,(Ad+B(c+2d))\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f(c+d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,
x]
```

```
[Out] -((Sqrt[a]*(A*d + B*(c + 2*d))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt
[c + d]*Sqrt[a + a*Sin[e + f*x]])]])/(d^(3/2)*(c + d)^(3/2)*f)) + (a*(B*c -
A*d)*Cos[e + f*x])/(d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x
]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))} + -\frac{(-aAd-B(a}{}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))} - \frac{(a(Ad+B(c+}{}$$

$$= -\frac{\sqrt{a}\,(Ad+B(c+2d))\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sin(e+fx)}}\right)}{d^{3/2}(c+d)^{3/2}f} + \frac{}{d(c+d)f\sqrt{}}$$

Mathematica [C] time = 8.80, size = 901, normalized size = 7.15

$$\left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{a(\sin(e+fx)+1)}\;\frac{\left((Ad+B(c+2d))\left(\cos\left(\frac{e}{2}\right)+i\sin\left(\frac{e}{2}\right)\right)\left((-1+i)x\cos(e)+(1+i)x\sin(e)+\frac{\text{RootSum}\left[de^{2ie}\#1^4+2ice^{ie}\#1^2-d\&,\frac{-\sqrt{d}\,\sqrt{c+d}\,e^{ie}fx}{}\right.}{}\right)}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*
x])^2,x]
```

```
[Out] ((1/4 + I/4)*Sqrt[a*(1 + Sin[e + f*x])]*(((A*d + B*(c + 2*d))*(Cos[e/2] + I
*Sin[e/2])*((-1 + I)*x*Cos[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((
2*I)*e)*#1^4 & , ((1 + I)*d*Sqrt[E^((-I)*e)]*f*x - (2 - 2*I)*d*Sqrt[E^((-I)
*e)]*Log[E^((I/2)*f*x) - #1] - I*Sqrt[d]*Sqrt[c + d]*f*x*#1 + 2*Sqrt[d]*Sqr
t[c + d]*Log[E^((I/2)*f*x) - #1]*#1 + ((1 - I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)]
 + ((2 + 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - Sqrt[d]*Sq
rt[c + d]*E^(I*e)*f*x*#1^3 - (2*I)*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)
*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*(Cos[e] + I*(-1 + Sin[e]))*Sqr
t[Cos[e] - I*Sin[e]])/(4*f) + (1 + I)*x*Sin[e]))/((c + d)^(3/2)*(Cos[e] + I
*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) + ((A*d + B*(c + 2*d))*(Cos[e/2] +
 I*Sin[e/2])*((1 - I)*x*Cos[e] - (1 + I)*x*Sin[e] + (RootSum[-d + (2*I)*c*E
^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 - I)*d*Sqrt[E^((-I)*e)]*f*x + (2 +
 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] + Sqrt[d]*Sqrt[c + d]*f*x*
```

```
#1 + (2*I)*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 - ((1 + I)*c*f*x*
#1^2)/Sqrt[E^((-I)*e)] + ((2 - 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^
((-I)*e)] - I*Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 + 2*Sqrt[d]*Sqrt[c + d]*
E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*Sqrt[Cos[e
] - I*Sin[e]]*(-1 - I*Cos[e] + Sin[e]))/(4*f)))/((c + d)^(3/2)*(Cos[e] + I*
(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) - ((2 - 2*I)*Sqrt[d]*(-(B*c) + A*d)
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)*f*(c + d*Sin[e + f*x]))))/
(d^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

fricas [B] time = 1.45, size = 1012, normalized size = 8.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] [-1/4*((B*c^2 + (A + 3*B)*c*d + (A + 2*B)*d^2 - (B*c*d + (A + 2*B)*d^2)*cos
(f*x + e)^2 + (B*c^2 + (A + 2*B)*c*d)*cos(f*x + e) + (B*c^2 + (A + 3*B)*c*d
 + (A + 2*B)*d^2 + (B*c*d + (A + 2*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt
(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*
c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*
cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2
+ 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a
)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*c
os(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e
))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 -
 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f
*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(B*c - A*d + (B*c - A*d)*co
s(f*x + e) - (B*c - A*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c*d^2 +
d^3)*f*cos(f*x + e)^2 - (c^2*d + c*d^2)*f*cos(f*x + e) - (c^2*d + 2*c*d^2 +
 d^3)*f - ((c*d^2 + d^3)*f*cos(f*x + e) + (c^2*d + 2*c*d^2 + d^3)*f)*sin(f*
x + e)), 1/2*((B*c^2 + (A + 3*B)*c*d + (A + 2*B)*d^2 - (B*c*d + (A + 2*B)*d
^2)*cos(f*x + e)^2 + (B*c^2 + (A + 2*B)*c*d)*cos(f*x + e) + (B*c^2 + (A + 3
*B)*c*d + (A + 2*B)*d^2 + (B*c*d + (A + 2*B)*d^2)*cos(f*x + e))*sin(f*x + e
))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e)
 - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(B*c - A*d + (B*c -
A*d)*cos(f*x + e) - (B*c - A*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c
*d^2 + d^3)*f*cos(f*x + e)^2 - (c^2*d + c*d^2)*f*cos(f*x + e) - (c^2*d + 2*
c*d^2 + d^3)*f - ((c*d^2 + d^3)*f*cos(f*x + e) + (c^2*d + 2*c*d^2 + d^3)*f)
*sin(f*x + e))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 2.02, size = 274, normalized size = 2.17

$$-\frac{\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\left(\sin\left(fx+e\right)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,d}{\sqrt{acd+a\,d^{2}}}\right)ad\left(Ad+Bc+2Bd\right)+A\operatorname{arc}\right.}{d\left(c+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(sin(f*x+e)*arctanh((a-a*sin(f*x+
e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*d*(A*d+B*c+2*B*d)+A*arctanh((a-a*sin(f*x
+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d+B*arctanh((a-a*sin(f*x+e))^(1/2)*d/
(a*c*d+a*d^2)^(1/2))*a*c^2+2*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^
2)^(1/2))*a*c*d+A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*d-B*(a-a*sin(f*x
+e))^(1/2)*(a*(c+d)*d)^(1/2)*c)/d/(c+d)/(c+d*sin(f*x+e))/(a*(c+d)*d)^(1/2)/
cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}}{\left(d\sin\left(fx+e\right)+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) + c
)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{\left(c+d\,\sin\left(e+f\,x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))^
2,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))^
2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.292 $\int \frac{\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=192

$$-\frac{\sqrt{a}\,(3Ad+B(c+4d))\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{4d^{3/2}f(c+d)^{5/2}}-\frac{a(3Ad+B(c+4d))\cos(e+fx)}{4df(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}+\frac{}{2df(c}$$

```
[Out] -1/4*(3*A*d+B*(c+4*d))*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*
sin(f*x+e))^(1/2))*a^(1/2)/d^(3/2)/(c+d)^(5/2)/f+1/2*a*(-A*d+B*c)*cos(f*x+e
)/d/(c+d)/f/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2)-1/4*a*(3*A*d+B*(c+4*d
))*cos(f*x+e)/d/(c+d)^2/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2980, 2772, 2773, 208}

$$-\frac{\sqrt{a}\,(3Ad+B(c+4d))\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{4d^{3/2}f(c+d)^{5/2}}-\frac{a(3Ad+B(c+4d))\cos(e+fx)}{4df(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}+\frac{}{2df(c}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,
x]
```

```
[Out] -(Sqrt[a]*(3*A*d + B*(c + 4*d))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqr
t[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(4*d^(3/2)*(c + d)^(5/2)*f) + (a*(B*c
- A*d)*Cos[e + f*x])/(2*d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e +
 f*x])^2) - (a*(3*A*d + B*(c + 4*d))*Cos[e + f*x])/(4*d*(c + d)^2*f*Sqrt[a
+ a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

```
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} + -\frac{(-3aAd-}{}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} - \frac{a(3}{4d(c+d)^2f}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} - \frac{a(3}{4d(c+d)^2f}$$

$$= -\frac{\sqrt{a}\,(3Ad+B(c+4d))\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sin(e+fx)}}\right)}{4d^{3/2}(c+d)^{5/2}f} + \frac{}{2d(c+d)f}$$

Mathematica [C] time = 10.27, size = 967, normalized size = 5.04

$$\frac{\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{a(\sin(e+fx)+1)}\left(\frac{(3Ad+B(c+4d))\left(\cos\left(\frac{e}{2}\right)+i\sin\left(\frac{e}{2}\right)\right)\left((-1+i)x\cos(e)+(1+i)x\sin(e)+\frac{\text{RootSum}\left[de^{2ie}\#1^4+2ice^{ie}\#1^2-d\&,\frac{-\sqrt{d}\sqrt{c+d}e\cdots}{\cdots}\right]}{\cdots}\right)}{\cdots}\cdots\right)}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*
x])^3,x]
```

```
[Out] ((1/16 + I/16)*Sqrt[a*(1 + Sin[e + f*x])]*(((3*A*d + B*(c + 4*d))*(Cos[e/2]
 + I*Sin[e/2])*((-1 + I)*x*Cos[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*
E^((2*I)*e)*#1^4 & , ((1 + I)*d*Sqrt[E^((-I)*e)]*f*x - (2 - 2*I)*d*Sqrt[E^(
(-I)*e)]*Log[E^((I/2)*f*x) - #1] - I*Sqrt[d]*Sqrt[c + d]*f*x*#1 + 2*Sqrt[d]
*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 + ((1 - I)*c*f*x*#1^2)/Sqrt[E^((-I)
*e)] + ((2 + 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - Sqrt[d
]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 - (2*I)*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((
I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*(Cos[e] + I*(-1 + Sin[e]))
*Sqrt[Cos[e] - I*Sin[e]])/(4*f) + (1 + I)*x*Sin[e]))/((c + d)^(5/2)*(Cos[e]
 + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) + ((3*A*d + B*(c + 4*d))*(Cos[
e/2] + I*Sin[e/2])*((1 - I)*x*Cos[e] - (1 + I)*x*Sin[e] + (RootSum[-d + (2*
I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 - I)*d*Sqrt[E^((-I)*e)]*f*x
+ (2 + 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] + Sqrt[d]*Sqrt[c + d
```

```
]*f*x*#1 + (2*I)*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 - ((1 + I)*
c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 - 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/S
qrt[E^((-I)*e)] - I*Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 + 2*Sqrt[d]*Sqrt[c
 + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*Sqrt
[Cos[e] - I*Sin[e]]*(-1 - I*Cos[e] + Sin[e]))/(4*f)))/((c + d)^(5/2)*(Cos[e
] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) - ((4 - 4*I)*Sqrt[d]*(-(B*c)
+ A*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)*f*(c + d*Sin[e + f*x
])^2) - ((2 - 2*I)*Sqrt[d]*(3*A*d + B*(c + 4*d))*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2]))/((c + d)^2*f*(c + d*Sin[e + f*x]))))/(d^(3/2)*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2]))
```

fricas [B] time = 2.28, size = 1750, normalized size = 9.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] [-1/16*((B*c^3 + 3*(A + 2*B)*c^2*d + 3*(2*A + 3*B)*c*d^2 + (3*A + 4*B)*d^3
- (B*c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^3 - (2*B*c^2*d + 3*(2*A + 3*B)*c
*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^2 + (B*c^3 + (3*A + 4*B)*c^2*d + B*c*d
^2 + (3*A + 4*B)*d^3)*cos(f*x + e) + (B*c^3 + 3*(A + 2*B)*c^2*d + 3*(2*A +
3*B)*c*d^2 + (3*A + 4*B)*d^3 - (B*c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^2 +
 2*(B*c^2*d + (3*A + 4*B)*c*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a/(c*d +
d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*
d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*x +
e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^3 +
(c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a/(
c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e
)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x
 + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d
^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) -
c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(B*c^2 - (5*A + B)*c*d + (A + 4*B)*d^
2 - (B*c*d + (3*A + 4*B)*d^2)*cos(f*x + e)^2 + (B*c^2 - (5*A + 2*B)*c*d - 2
*A*d^2)*cos(f*x + e) - (B*c^2 - (5*A + B)*c*d + (A + 4*B)*d^2 + (B*c*d + (3
*A + 4*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2*
d^3 + 2*c*d^4 + d^5)*f*cos(f*x + e)^3 + (2*c^3*d^2 + 5*c^2*d^3 + 4*c*d^4 +
d^5)*f*cos(f*x + e)^2 - (c^4*d + 2*c^3*d^2 + 2*c^2*d^3 + 2*c*d^4 + d^5)*f*c
os(f*x + e) - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f + ((c^2*d^3
 + 2*c*d^4 + d^5)*f*cos(f*x + e)^2 - 2*(c^3*d^2 + 2*c^2*d^3 + c*d^4)*f*cos(
f*x + e) - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f)*sin(f*x + e))
, 1/8*((B*c^3 + 3*(A + 2*B)*c^2*d + 3*(2*A + 3*B)*c*d^2 + (3*A + 4*B)*d^3 -
 (B*c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^3 - (2*B*c^2*d + 3*(2*A + 3*B)*c*
d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^2 + (B*c^3 + (3*A + 4*B)*c^2*d + B*c*d^
```

```
2 + (3*A + 4*B)*d^3)*cos(f*x + e) + (B*c^3 + 3*(A + 2*B)*c^2*d + 3*(2*A + 3
*B)*c*d^2 + (3*A + 4*B)*d^3 - (B*c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^2 +
2*(B*c^2*d + (3*A + 4*B)*c*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d +
d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-
a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(B*c^2 - (5*A + B)*c*d + (A + 4*B)*d^2
 - (B*c*d + (3*A + 4*B)*d^2)*cos(f*x + e)^2 + (B*c^2 - (5*A + 2*B)*c*d - 2*
A*d^2)*cos(f*x + e) - (B*c^2 - (5*A + B)*c*d + (A + 4*B)*d^2 + (B*c*d + (3*
A + 4*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2*d
^3 + 2*c*d^4 + d^5)*f*cos(f*x + e)^3 + (2*c^3*d^2 + 5*c^2*d^3 + 4*c*d^4 + d
^5)*f*cos(f*x + e)^2 - (c^4*d + 2*c^3*d^2 + 2*c^2*d^3 + 2*c*d^4 + d^5)*f*co
s(f*x + e) - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f + ((c^2*d^3
+ 2*c*d^4 + d^5)*f*cos(f*x + e)^2 - 2*(c^3*d^2 + 2*c^2*d^3 + c*d^4)*f*cos(f
*x + e) - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f)*sin(f*x + e))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 2.43, size = 628, normalized size = 3.27

$$\frac{\left(-2 \sin \left(f x+e\right) \operatorname{arctanh}\left(\frac{\sqrt{a-a \sin \left(f x+e\right)}\, d}{\sqrt{a c d+a\, d^{2}}}\right) a^{2} c d \left(3 A d+B c+4 B d\right)+\operatorname{arctanh}\left(\frac{\sqrt{a-a \sin \left(f x+e\right)}\, d}{\sqrt{a c d+a\, d^{2}}}\right) a^{2} d^{2} \left(3 A d+B c+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/4/a*(-2*sin(f*x+e)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*
a^2*c*d*(3*A*d+B*c+4*B*d)+arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1
/2))*a^2*d^2*(3*A*d+B*c+4*B*d)*cos(f*x+e)^2+3*A*(a-a*sin(f*x+e))^(3/2)*(a*(
c+d)*d)^(1/2)*d^2-3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))
*a^2*c^2*d-3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^
3+B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*c*d+4*B*(a-a*sin(f*x+e))^(3/2)
*(a*(c+d)*d)^(1/2)*d^2-a^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(
1/2))*B*c^3-4*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c
^2*d-B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^2-4*B*
arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^3-5*A*(a-a*sin(
```

```
f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d-5*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d
)^(1/2)*a*d^2+B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2-3*B*(a-a*sin
(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d-4*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*
d)^(1/2)*a*d^2)*(-a*(sin(f*x+e)-1))^(1/2)*(1+sin(f*x+e))/(a*(c+d)*d)^(1/2)/
(c+d*sin(f*x+e))^2/(c+d)^2/d/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}}{\left(d\sin\left(fx+e\right)+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) + c
)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))^
3,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))^
3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.293 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=374

$$\frac{2a^2\left(11Ad(c-17d)-3B\left(c^2-9cd+56d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{693d^2f\sqrt{a\sin(e+fx)+a}}+\frac{4a^2(c+d)\left(15c^2+10cd+7d^2\right)\left(11\right.}{3465d}$$

```
[Out] 4/1155*(c+d)*(11*A*(c-17*d)*d-3*B*(c^2-9*c*d+56*d^2))*cos(f*x+e)*(a+a*sin(f
*x+e))^(3/2)/f+4/3465*a^2*(c+d)*(15*c^2+10*c*d+7*d^2)*(11*A*(c-17*d)*d-3*B*
(c^2-9*c*d+56*d^2))*cos(f*x+e)/d^2/f/(a+a*sin(f*x+e))^(1/2)+2/693*a^2*(11*A
*(c-17*d)*d-3*B*(c^2-9*c*d+56*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^3/d^2/f/(a+
a*sin(f*x+e))^(1/2)+2/99*a^2*(3*B*(c-4*d)-11*A*d)*cos(f*x+e)*(c+d*sin(f*x+e
))^4/d^2/f/(a+a*sin(f*x+e))^(1/2)+8/3465*a*(5*c-d)*(c+d)*(11*A*(c-17*d)*d-3
*B*(c^2-9*c*d+56*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d/f-2/11*a*B*cos(f
*x+e)*(c+d*sin(f*x+e))^4*(a+a*sin(f*x+e))^(1/2)/d/f
```

Rubi [A] time = 0.92, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2976, 2981, 2770, 2761, 2751, 2646}

$$\frac{2a^2\left(11Ad(c-17d)-3B\left(c^2-9cd+56d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{693d^2f\sqrt{a\sin(e+fx)+a}}+\frac{4a^2(c+d)\left(15c^2+10cd+7d^2\right)\left(11\right.}{3465d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,
x]
```

```
[Out] (4*a^2*(c + d)*(15*c^2 + 10*c*d + 7*d^2)*(11*A*(c - 17*d)*d - 3*B*(c^2 - 9*
c*d + 56*d^2))*Cos[e + f*x])/(3465*d^2*f*Sqrt[a + a*Sin[e + f*x]]) + (8*a*(
5*c - d)*(c + d)*(11*A*(c - 17*d)*d - 3*B*(c^2 - 9*c*d + 56*d^2))*Cos[e + f
*x]*Sqrt[a + a*Sin[e + f*x]])/(3465*d*f) + (4*(c + d)*(11*A*(c - 17*d)*d -
3*B*(c^2 - 9*c*d + 56*d^2))*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(1155*
f) + (2*a^2*(11*A*(c - 17*d)*d - 3*B*(c^2 - 9*c*d + 56*d^2))*Cos[e + f*x]*(
c + d*Sin[e + f*x])^3)/(693*d^2*f*Sqrt[a + a*Sin[e + f*x]]) + (2*a^2*(3*B*(
c - 4*d) - 11*A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(99*d^2*f*Sqrt[a +
a*Sin[e + f*x]]) - (2*a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[
e + f*x])^4)/(11*d*f)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
```

```
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
```

```
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c+d}{11df}$$

$$= \frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\,s}{99d^2f\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{2a^2\left(11A(c-17d)d-3B\left(c^2-9cd+56d^2\right)\right.}{693d^2f\sqrt{a+a\sin(}}$$

$$= \frac{4(c+d)\left(11A(c-17d)d-3B\left(c^2-9cd+5\right.\right.}{1155}$$

$$= \frac{8a(5c-d)(c+d)\left(11A(c-17d)d-3B\left(c^2\right.\right.}{34}$$

$$= \frac{4a^2(c+d)\left(15c^2+10cd+7d^2\right)\left(11A(c-17\right.}{3465d^2f\sqrt{a+a}}$$

Mathematica [A] time = 4.61, size = 390, normalized size = 1.04

$$-\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-8\left(11Ad\left(189c^2+351cd+137d^2\right)+3B\left(231c^3+1\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x])^3,x]
```

```
[Out] -1/27720*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])
]*(92400*A*c^3 + 72072*B*c^3 + 216216*A*c^2*d + 195624*B*c^2*d + 195624*A*c
*d^2 + 177474*B*c*d^2 + 59158*A*d^3 + 55482*B*d^3 - 8*(11*A*d*(189*c^2 + 35
1*c*d + 137*d^2) + 3*B*(231*c^3 + 1287*c^2*d + 1507*c*d^2 + 581*d^3))*Cos[2
*(e + f*x)] + 70*d^2*(33*B*c + 11*A*d + 21*B*d)*Cos[4*(e + f*x)] + 18480*A*
```

```
c^3*Sin[e + f*x] + 33264*B*c^3*Sin[e + f*x] + 99792*A*c^2*d*Sin[e + f*x] +
100188*B*c^2*d*Sin[e + f*x] + 100188*A*c*d^2*Sin[e + f*x] + 105468*B*c*d^2*
Sin[e + f*x] + 35156*A*d^3*Sin[e + f*x] + 34734*B*d^3*Sin[e + f*x] - 5940*B
*c^2*d*Sin[3*(e + f*x)] - 5940*A*c*d^2*Sin[3*(e + f*x)] - 11220*B*c*d^2*Sin
[3*(e + f*x)] - 3740*A*d^3*Sin[3*(e + f*x)] - 4935*B*d^3*Sin[3*(e + f*x)] +
 315*B*d^3*Sin[5*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

fricas [A] time = 0.47, size = 637, normalized size = 1.70

$$-\frac{2\left(315\,Bad^3\cos\left(fx+e\right)^6+35\left(33\,Bacd^2+(11\,A+21\,B)ad^3\right)\cos\left(fx+e\right)^5+924\,(5\,A+3\,B)ac^3+396\,(21\,A+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] -2/3465*(315*B*a*d^3*cos(f*x + e)^6 + 35*(33*B*a*c*d^2 + (11*A + 21*B)*a*d^
3)*cos(f*x + e)^5 + 924*(5*A + 3*B)*a*c^3 + 396*(21*A + 19*B)*a*c^2*d + 132
*(57*A + 47*B)*a*c*d^2 + 4*(517*A + 483*B)*a*d^3 - 5*(297*B*a*c^2*d + 33*(9
*A + 10*B)*a*c*d^2 + 10*(11*A + 21*B)*a*d^3)*cos(f*x + e)^4 - (693*B*a*c^3
+ 297*(7*A + 13*B)*a*c^2*d + 33*(117*A + 172*B)*a*c*d^2 + 2*(946*A + 1239*B
)*a*d^3)*cos(f*x + e)^3 + (231*(5*A + 6*B)*a*c^3 + 99*(42*A + 43*B)*a*c^2*d
 + 33*(129*A + 134*B)*a*c*d^2 + (1474*A + 1491*B)*a*d^3)*cos(f*x + e)^2 + (
231*(25*A + 21*B)*a*c^3 + 99*(147*A + 143*B)*a*c^2*d + 33*(429*A + 409*B)*a
*c*d^2 + (4499*A + 4431*B)*a*d^3)*cos(f*x + e) + (315*B*a*d^3*cos(f*x + e)^
5 - 924*(5*A + 3*B)*a*c^3 - 396*(21*A + 19*B)*a*c^2*d - 132*(57*A + 47*B)*a
*c*d^2 - 4*(517*A + 483*B)*a*d^3 - 35*(33*B*a*c*d^2 + (11*A + 12*B)*a*d^3)*
cos(f*x + e)^4 - 5*(297*B*a*c^2*d + 33*(9*A + 17*B)*a*c*d^2 + (187*A + 294*
B)*a*d^3)*cos(f*x + e)^3 + 3*(231*B*a*c^3 + 99*(7*A + 8*B)*a*c^2*d + 33*(24
*A + 29*B)*a*c*d^2 + (319*A + 336*B)*a*d^3)*cos(f*x + e)^2 + (231*(5*A + 9*
B)*a*c^3 + 99*(63*A + 67*B)*a*c^2*d + 33*(201*A + 221*B)*a*c*d^2 + 17*(143*
A + 147*B)*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*c
os(f*x + e) + f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
```

*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
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(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
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to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
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2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
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to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*(112*f
*(2*A*a*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+2*B*a*d^3*sign(cos(1/2*(f*x+
exp(1))-1/4*pi))+6*B*a*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(1
4*f*x+14*exp(1)+pi))/(112*f)^2+144*f*(2*A*a*d^3*sign(cos(1/2*(f*x+exp(1))-1
/4*pi))+2*B*a*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+6*B*a*c*d^2*sign(cos(1
/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(18*f*x+18*exp(1)-pi))/(144*f)^2-160*f*(-
8*A*a*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-10*B*a*d^3*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi))-24*A*a*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-24*B*a*c*d^
2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-24*B*a*c^2*d*sign(cos(1/2*(f*x+exp(1))
-1/4*pi)))*cos(1/4*(10*f*x+10*exp(1)+pi))/(160*f)^2-224*f*(-8*A*a*d^3*sign(
cos(1/2*(f*x+exp(1))-1/4*pi))-10*B*a*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))
-24*A*a*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-24*B*a*c*d^2*sign(cos(1/2*
(f*x+exp(1))-1/4*pi))-24*B*a*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(
1/4*(14*f*x+14*exp(1)-pi))/(224*f)^2+12*f*(-2*A*a*d^3*sign(cos(1/2*(f*x+exp

```
(1))-1/4*pi))-2*B*a*c^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-2*B*a*d^3*sign(c
os(1/2*(f*x+exp(1))-1/4*pi))-6*A*a*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))
-6*A*a*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-6*B*a*c*d^2*sign(cos(1/2*(f
*x+exp(1))-1/4*pi))-6*B*a*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4
*(6*f*x+6*exp(1)+pi))/(12*f)^2+20*f*(-2*A*a*d^3*sign(cos(1/2*(f*x+exp(1))-1
/4*pi))-2*B*a*c^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-2*B*a*d^3*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))-6*A*a*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-6*A*a
*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-6*B*a*c*d^2*sign(cos(1/2*(f*x+exp
(1))-1/4*pi))-6*B*a*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(10*f
*x+10*exp(1)-pi))/(20*f)^2+8*f*(16*A*a*c^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi
))+6*A*a*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+8*B*a*c^3*sign(cos(1/2*(f*x
+exp(1))-1/4*pi))+6*B*a*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+24*A*a*c*d^2
*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+24*A*a*c^2*d*sign(cos(1/2*(f*x+exp(1))-
1/4*pi))+18*B*a*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+24*B*a*c^2*d*sign(
cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(2*f*x-pi)+1/2*exp(1))/(8*f)^2-16*f*
(16*A*a*c^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+12*A*a*d^3*sign(cos(1/2*(f*x
+exp(1))-1/4*pi))+16*B*a*c^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+10*B*a*d^3*
sign(cos(1/2*(f*x+exp(1))-1/4*pi))+36*A*a*c*d^2*sign(cos(1/2*(f*x+exp(1))-1
/4*pi))+48*A*a*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+36*B*a*c*d^2*sign(c
os(1/2*(f*x+exp(1))-1/4*pi))+36*B*a*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)
))*cos(1/4*(2*f*x+2*exp(1)+pi))/(16*f)^2-48*f*(16*A*a*c^3*sign(cos(1/2*(f*x
+exp(1))-1/4*pi))+12*A*a*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+16*B*a*c^3*
sign(cos(1/2*(f*x+exp(1))-1/4*pi))+10*B*a*d^3*sign(cos(1/2*(f*x+exp(1))-1/4
*pi))+36*A*a*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+48*A*a*c^2*d*sign(cos
(1/2*(f*x+exp(1))-1/4*pi))+36*B*a*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+
36*B*a*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(6*f*x+6*exp(1)-pi
))/(48*f)^2-576*B*a*d^3*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(18*f*
x+18*exp(1)+pi))/(288*f)^2-704*B*a*d^3*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))
*cos(1/4*(22*f*x+22*exp(1)-pi))/(352*f)^2)
```

maple [A] time = 1.37, size = 312, normalized size = 0.83

$$\frac{2\left(1+\sin\left(fx+e\right)\right)a^2\left(\sin\left(fx+e\right)-1\right)\left(315B\left(\cos^4\left(fx+e\right)\right)\sin\left(fx+e\right)d^3+\left(-1485Ac\,d^2-935A\,d^3-1485B\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 2/3465*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*(315*B*cos(f*x+e)^4*sin(f*x+e)*d^3
+(-1485*A*c*d^2-935*A*d^3-1485*B*c^2*d-2805*B*c*d^2-1470*B*d^3)*cos(f*x+e)^
2*sin(f*x+e)+(1155*A*c^3+6237*A*c^2*d+6633*A*c*d^2+2431*A*d^3+2079*B*c^3+66
33*B*c^2*d+7293*B*c*d^2+2499*B*d^3)*sin(f*x+e)+(385*A*d^3+1155*B*c*d^2+735*
B*d^3)*cos(f*x+e)^4+(-2079*A*c^2*d-3861*A*c*d^2-1892*A*d^3-693*B*c^3-3861*B
*c^2*d-5676*B*c*d^2-2478*B*d^3)*cos(f*x+e)^2+5775*A*c^3+14553*A*c^2*d+14157
```

```
*A*c*d^2+4499*A*d^3+4851*B*c^3+14157*B*c^2*d+13497*B*c*d^2+4431*B*d^3)/cos(
f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) +
 c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3,
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3,
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.294 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=294

$$\frac{2a^2\left(15c^2+10cd+7d^2\right)\left(3Ad(c-13d)-B\left(c^2-7cd+34d^2\right)\right)\cos(e+fx)}{315d^2f\sqrt{a\sin(e+fx)+a}}+\frac{2a^2(-9Ad+3Bc-10Bd)\cos(e+fx}{63d^2f\sqrt{a\sin(e+fx)}}$$

```
[Out] 2/105*(3*A*(c-13*d)*d-B*(c^2-7*c*d+34*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(3/
2)/f+2/315*a^2*(15*c^2+10*c*d+7*d^2)*(3*A*(c-13*d)*d-B*(c^2-7*c*d+34*d^2))*
cos(f*x+e)/d^2/f/(a+a*sin(f*x+e))^(1/2)+2/63*a^2*(-9*A*d+3*B*c-10*B*d)*cos(
f*x+e)*(c+d*sin(f*x+e))^3/d^2/f/(a+a*sin(f*x+e))^(1/2)+4/315*a*(5*c-d)*(3*A
*(c-13*d)*d-B*(c^2-7*c*d+34*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d/f-2/9
*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2)/d/f
```

Rubi [A] time = 0.71, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {2976, 2981, 2761, 2751, 2646}

$$\frac{2a^2\left(15c^2+10cd+7d^2\right)\left(3Ad(c-13d)-B\left(c^2-7cd+34d^2\right)\right)\cos(e+fx)}{315d^2f\sqrt{a\sin(e+fx)+a}}+\frac{2a^2(-9Ad+3Bc-10Bd)\cos(e+fx}{63d^2f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,
x]
```

```
[Out] (2*a^2*(15*c^2 + 10*c*d + 7*d^2)*(3*A*(c - 13*d)*d - B*(c^2 - 7*c*d + 34*d^
2))*Cos[e + f*x])/(315*d^2*f*Sqrt[a + a*Sin[e + f*x]]) + (4*a*(5*c - d)*(3*
A*(c - 13*d)*d - B*(c^2 - 7*c*d + 34*d^2))*Cos[e + f*x]*Sqrt[a + a*Sin[e +
f*x]])/(315*d*f) + (2*(3*A*(c - 13*d)*d - B*(c^2 - 7*c*d + 34*d^2))*Cos[e +
 f*x]*(a + a*Sin[e + f*x])^(3/2))/(105*f) + (2*a^2*(3*B*c - 9*A*d - 10*B*d)
*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(63*d^2*f*Sqrt[a + a*Sin[e + f*x]]) -
 (2*a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^3)/(9*d*
f)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^2\, dx = -\frac{2aB\cos(e + fx)\sqrt{a + a\sin(e + fx)}\,(c + d\,\mathrm{s}}{9df}$$

$$= \frac{2a^2(3Bc - 9Ad - 10Bd)\cos(e + fx)(c + d\,\mathrm{si}}{63d^2f\sqrt{a + a\sin(e + fx)}}$$

$$= \frac{2\left(3A(c - 13d)d - B\left(c^2 - 7cd + 34d^2\right)\right)\cos(}{105f}$$

$$= \frac{4a(5c - d)\left(3A(c - 13d)d - B\left(c^2 - 7cd + 34\right.\right.}{315df}$$

$$= \frac{2a^2\left(15c^2 + 10cd + 7d^2\right)\left(3A(c - 13d)d - B\left(\right.\right.}{315d^2f\sqrt{a + a\sin(e}}$$

Mathematica [A] time = 2.26, size = 267, normalized size = 0.91

$$-\frac{a\sqrt{a(\sin(e + fx) + 1)}\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)\left(-4\left(9Ad(14c + 13d) + B\left(63c^2 + 234cd + 137d^2\right)\right)\mathrm{co}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x])^2,x]
```

```
[Out] -1/1260*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]
*(4200*A*c^2 + 3276*B*c^2 + 6552*A*c*d + 5928*B*c*d + 2964*A*d^2 + 2689*B*d
^2 - 4*(9*A*d*(14*c + 13*d) + B*(63*c^2 + 234*c*d + 137*d^2))*Cos[2*(e + f*
x)] + 35*B*d^2*Cos[4*(e + f*x)] + 840*A*c^2*Sin[e + f*x] + 1512*B*c^2*Sin[e
 + f*x] + 3024*A*c*d*Sin[e + f*x] + 3036*B*c*d*Sin[e + f*x] + 1518*A*d^2*Si
n[e + f*x] + 1598*B*d^2*Sin[e + f*x] - 180*B*c*d*Sin[3*(e + f*x)] - 90*A*d^
2*Sin[3*(e + f*x)] - 170*B*d^2*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2]))
```

fricas [A] time = 0.45, size = 430, normalized size = 1.46

$$-\frac{2\left(35\,Bad^2\cos\left(fx + e\right)^5 - 5\left(18\,Bacd + (9\,A + 10\,B)ad^2\right)\cos\left(fx + e\right)^4 + 84\,(5\,A + 3\,B)ac^2 + 24\,(21\,A + 19\,B)a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] -2/315*(35*B*a*d^2*cos(f*x + e)^5 - 5*(18*B*a*c*d + (9*A + 10*B)*a*d^2)*cos
(f*x + e)^4 + 84*(5*A + 3*B)*a*c^2 + 24*(21*A + 19*B)*a*c*d + 4*(57*A + 47*
B)*a*d^2 - (63*B*a*c^2 + 18*(7*A + 13*B)*a*c*d + (117*A + 172*B)*a*d^2)*cos
(f*x + e)^3 + (21*(5*A + 6*B)*a*c^2 + 6*(42*A + 43*B)*a*c*d + (129*A + 134*
B)*a*d^2)*cos(f*x + e)^2 + (21*(25*A + 21*B)*a*c^2 + 6*(147*A + 143*B)*a*c*
d + (429*A + 409*B)*a*d^2)*cos(f*x + e) - (35*B*a*d^2*cos(f*x + e)^4 + 84*(
5*A + 3*B)*a*c^2 + 24*(21*A + 19*B)*a*c*d + 4*(57*A + 47*B)*a*d^2 + 5*(18*B
*a*c*d + (9*A + 17*B)*a*d^2)*cos(f*x + e)^3 - 3*(21*B*a*c^2 + 6*(7*A + 8*B)
*a*c*d + (24*A + 29*B)*a*d^2)*cos(f*x + e)^2 - (21*(5*A + 9*B)*a*c^2 + 6*(6
3*A + 67*B)*a*c*d + (201*A + 221*B)*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt
(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
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4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
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```

```
x/2)sqrt(2*a)*(-40*f*(-2*A*a*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-2*B*a*d
^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-4*B*a*c*d*sign(cos(1/2*(f*x+exp(1))-1
/4*pi)))*cos(1/4*(10*f*x+10*exp(1)+pi))/(40*f)^2-56*f*(-2*A*a*d^2*sign(cos(
1/2*(f*x+exp(1))-1/4*pi))-2*B*a*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-4*B*
a*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(14*f*x+14*exp(1)-pi))/(5
6*f)^2+12*f*(-2*A*a*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-2*B*a*c^2*sign(c
os(1/2*(f*x+exp(1))-1/4*pi))-2*B*a*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-4
*A*a*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-4*B*a*c*d*sign(cos(1/2*(f*x+exp
(1))-1/4*pi)))*sin(1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2+20*f*(-2*A*a*d^2*sign(
cos(1/2*(f*x+exp(1))-1/4*pi))-2*B*a*c^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-
2*B*a*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-4*A*a*c*d*sign(cos(1/2*(f*x+ex
p(1))-1/4*pi))-4*B*a*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(10*f*
x+10*exp(1)-pi))/(20*f)^2-8*f*(8*A*a*c^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))
+6*A*a*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+8*B*a*c^2*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi))+6*B*a*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+16*A*a*c*d*sig
n(cos(1/2*(f*x+exp(1))-1/4*pi))+12*B*a*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi
)))*cos(1/4*(2*f*x+2*exp(1)+pi))/(8*f)^2-24*f*(8*A*a*c^2*sign(cos(1/2*(f*x+
exp(1))-1/4*pi))+6*A*a*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+8*B*a*c^2*sig
n(cos(1/2*(f*x+exp(1))-1/4*pi))+6*B*a*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi)
)+16*A*a*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+12*B*a*c*d*sign(cos(1/2*(f*
x+exp(1))-1/4*pi)))*cos(1/4*(6*f*x+6*exp(1)-pi))/(24*f)^2+8*f*(16*A*a*c^2*s
ign(cos(1/2*(f*x+exp(1))-1/4*pi))+8*A*a*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*p
i))+8*B*a*c^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+6*B*a*d^2*sign(cos(1/2*(f*
x+exp(1))-1/4*pi))+16*A*a*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+16*B*a*c*d
*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(2*f*x-pi)+1/2*exp(1))/(8*f)^2
+224*B*a*d^2*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(14*f*x+14*exp(1)
+pi))/(112*f)^2+288*B*a*d^2*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(1
8*f*x+18*exp(1)-pi))/(144*f)^2)
```

maple [A] time = 1.44, size = 207, normalized size = 0.70

$$\frac{2\left(1+\sin\left(fx+e\right)\right)a^{2}\left(\sin\left(fx+e\right)-1\right)\left(\left(-45A\,d^{2}-90Bcd-85B\,d^{2}\right)\sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)+\left(105A\,c^{2}+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/315*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*((-45*A*d^2-90*B*c*d-85*B*d^2)*sin(
f*x+e)*cos(f*x+e)^2+(105*A*c^2+378*A*c*d+201*A*d^2+189*B*c^2+402*B*c*d+221*
B*d^2)*sin(f*x+e)+35*B*cos(f*x+e)^4*d^2+(-126*A*c*d-117*A*d^2-63*B*c^2-234*
B*c*d-172*B*d^2)*cos(f*x+e)^2+525*A*c^2+882*A*c*d+429*A*d^2+441*B*c^2+858*B
*c*d+409*B*d^2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) +
 c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2,
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2,
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(A+B\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))*(c + d*sin(e +
f*x))**2, x)
```

3.295 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=165

$$-\frac{8a^2(35Ac+21Ad+21Bc+19Bd)\cos(e+fx)}{105f\sqrt{a\sin(e+fx)+a}}-\frac{2(7Ad+7Bc-2Bd)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{35f}-\frac{2a(35A$$

```
[Out] -2/35*(7*A*d+7*B*c-2*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-2/7*B*d*cos(f
*x+e)*(a+a*sin(f*x+e))^(5/2)/a/f-8/105*a^2*(35*A*c+21*A*d+21*B*c+19*B*d)*co
s(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/105*a*(35*A*c+21*A*d+21*B*c+19*B*d)*cos
(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.32, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2968, 3023, 2751, 2647, 2646}

$$-\frac{8a^2(35Ac+21Ad+21Bc+19Bd)\cos(e+fx)}{105f\sqrt{a\sin(e+fx)+a}}-\frac{2(7Ad+7Bc-2Bd)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{35f}-\frac{2a(35A$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (-8*a^2*(35*A*c + 21*B*c + 21*A*d + 19*B*d)*Cos[e + f*x])/(105*f*Sqrt[a + a
*Sin[e + f*x]]) - (2*a*(35*A*c + 21*B*c + 21*A*d + 19*B*d)*Cos[e + f*x]*Sqr
t[a + a*Sin[e + f*x]])/(105*f) - (2*(7*B*c + 7*A*d - 2*B*d)*Cos[e + f*x]*(a
 + a*Sin[e + f*x])^(3/2))/(35*f) - (2*B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])
^(5/2))/(7*a*f)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int (a+a\sin(e+fx))^{3/2}\left(Ac+(Bc+Ad)\sin\right.$$

$$= -\frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{7af} + \frac{2}{\ldots}$$

$$= -\frac{2(7Bc+7Ad-2Bd)\cos(e+fx)(a+a\sin\ldots}{35f}$$

$$= -\frac{2a(35Ac+21Bc+21Ad+19Bd)\cos(e+\ldots}{105f}$$

$$= -\frac{8a^2(35Ac+21Bc+21Ad+19Bd)\cos(e+\ldots}{105f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 1.07, size = 144, normalized size = 0.87

$$-\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)((140Ac+252Ad+252Bc+253Bd)\sin(e+fx)-\ldots}{210f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\mathrm{co}\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x]),x]
```

```
[Out] -1/210*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*
(700*A*c + 546*B*c + 546*A*d + 494*B*d - 6*(7*B*c + 7*A*d + 13*B*d)*Cos[2*(
e + f*x)] + (140*A*c + 252*B*c + 252*A*d + 253*B*d)*Sin[e + f*x] - 15*B*d*S
in[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

fricas [A] time = 0.45, size = 257, normalized size = 1.56

$$2\left(15\,Bad\cos\left(fx+e\right)^{4}+3\,(7\,Bac+(7\,A+13\,B)ad)\cos\left(fx+e\right)^{3}-28\,(5\,A+3\,B)ac-4\,(21\,A+19\,B)ad-(7\,(5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] 2/105*(15*B*a*d*cos(f*x + e)^4 + 3*(7*B*a*c + (7*A + 13*B)*a*d)*cos(f*x + e
)^3 - 28*(5*A + 3*B)*a*c - 4*(21*A + 19*B)*a*d - (7*(5*A + 6*B)*a*c + (42*A
 + 43*B)*a*d)*cos(f*x + e)^2 - (7*(25*A + 21*B)*a*c + (147*A + 143*B)*a*d)*
cos(f*x + e) + (15*B*a*d*cos(f*x + e)^3 + 28*(5*A + 3*B)*a*c + 4*(21*A + 19
*B)*a*d - 3*(7*B*a*c + (7*A + 8*B)*a*d)*cos(f*x + e)^2 - (7*(5*A + 9*B)*a*c
 + (63*A + 67*B)*a*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/
(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
```

```
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*(12*f*(-2*A*a*d
*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-2*B*a*c*sign(cos(1/2*(f*x+exp(1))-1/4*p
i))-2*B*a*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(6*f*x+6*exp(1)+pi)
)/(12*f)^2+20*f*(-2*A*a*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-2*B*a*c*sign(c
os(1/2*(f*x+exp(1))-1/4*pi))-2*B*a*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*si
n(1/4*(10*f*x+10*exp(1)-pi))/(20*f)^2+2*f*(4*A*a*c*sign(cos(1/2*(f*x+exp(1)
)-1/4*pi))+2*A*a*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+2*B*a*c*sign(cos(1/2*
(f*x+exp(1))-1/4*pi))+2*B*a*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(
2*f*x-pi)+1/2*exp(1))/(2*f)^2-8*f*(8*A*a*c*sign(cos(1/2*(f*x+exp(1))-1/4*pi
))+8*A*a*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+8*B*a*c*sign(cos(1/2*(f*x+exp
(1))-1/4*pi))+6*B*a*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(2*f*x+2*
exp(1)+pi))/(8*f)^2-24*f*(8*A*a*c*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+8*A*a*
d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+8*B*a*c*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))+6*B*a*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(6*f*x+6*exp(1)-pi
))/(24*f)^2+80*B*a*d*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(10*f*x+1
0*exp(1)+pi))/(40*f)^2+112*B*a*d*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(1
/4*(14*f*x+14*exp(1)-pi))/(56*f)^2)
```

maple [A] time = 1.28, size = 150, normalized size = 0.91

$$\frac{2\left(1+\sin\left(fx+e\right)\right)a^{2}\left(\sin\left(fx+e\right)-1\right)\left(15Bd\left(\sin^{3}\left(fx+e\right)\right)+21Ad\left(\sin^{2}\left(fx+e\right)\right)+21Bc\left(\sin^{2}\left(fx+e\right)\right)+\cdots\right.}{105\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] 2/105*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*(15*B*d*sin(f*x+e)^3+21*A*d*sin(f*x
+e)^2+21*B*c*sin(f*x+e)^2+39*B*sin(f*x+e)^2*d+35*A*sin(f*x+e)*c+63*A*sin(f*
x+e)*d+63*B*sin(f*x+e)*c+52*B*sin(f*x+e)*d+175*A*c+126*A*d+126*B*c+104*B*d)
/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) +
 c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x)),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x)), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(A+B\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))*(c + d*sin(e +
f*x)), x)
```

3.296 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=101

$$-\frac{8a^2(5A+3B)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2a(5A+3B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

```
[Out] -2/5*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-8/15*a^2*(5*A+3*B)*cos(f*x+e)/f/
(a+a*sin(f*x+e))^(1/2)-2/15*a*(5*A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2751, 2647, 2646}

$$-\frac{8a^2(5A+3B)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2a(5A+3B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]),x]
```

```
[Out] (-8*a^2*(5*A + 3*B)*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*(5
*A + 3*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) - (2*B*Cos[e + f*x]
*(a + a*Sin[e + f*x])^(3/2))/(5*f)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f}+\frac{1}{5}(5A+3B)\int (a+a\sin$$

$$= -\frac{2a(5A+3B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f}-\frac{2B\cos(e+fx)}{}$$

$$= -\frac{8a^2(5A+3B)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}}-\frac{2a(5A+3B)\cos(e+fx)\sqrt{a+a\sin}}{15f}$$

Mathematica [A] time = 0.40, size = 101, normalized size = 1.00

$$-\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(2(5A+9B)\sin(e+fx)+50A-3B\cos(2(e+fx))+}{15f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]),x]
```

```
[Out] -1/15*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(
50*A + 39*B - 3*B*Cos[2*(e + f*x)] + 2*(5*A + 9*B)*Sin[e + f*x]))/(f*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2]))
```

fricas [A] time = 0.43, size = 137, normalized size = 1.36

$$\frac{2\left(3\,Ba\cos\left(fx+e\right)^{3}-(5\,A+6\,B)a\cos\left(fx+e\right)^{2}-(25\,A+21\,B)a\cos\left(fx+e\right)-4\,(5\,A+3\,B)a-\left(3\,Ba\cos\left(fx\right.\right.\right.}{15\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 2/15*(3*B*a*cos(f*x + e)^3 - (5*A + 6*B)*a*cos(f*x + e)^2 - (25*A + 21*B)*a
*cos(f*x + e) - 4*(5*A + 3*B)*a - (3*B*a*cos(f*x + e)^2 + (5*A + 9*B)*a*cos
(f*x + e) - 4*(5*A + 3*B)*a)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(
f*x + e) + f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*(-2*f*(2*A*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+2*B*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(2*f*x+2*exp(1)+pi))/(2*f)^2-6*f*(2*A*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+2*B*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(6*f*x+6*exp(1)-pi))/(6*f)^2+2*f*(4*A*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+2*B*a*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(2*f*x-pi)+1/2*exp(1))/(2*f)^2-24*B*a*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2-40*B*a*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(10*f*x+10*exp(1)-pi))/(20*f)^2)

maple [A] time = 1.08, size = 77, normalized size = 0.76

$$\frac{2\left(1+\sin\left(fx+e\right)\right)a^{2}\left(\sin\left(fx+e\right)-1\right)\left(\sin\left(fx+e\right)\left(5A+9B\right)-3B\left(\cos^{2}\left(fx+e\right)\right)+25A+21B\right)}{15\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x)

[Out] 2/15*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*(sin(f*x+e)*(5*A+9*B)-3*B*cos(f*x+e)^2+25*A+21*B)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x)), x)
```

3.297 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=153

$$-\frac{2a^{3/2}(c-d)(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{5/2}f\sqrt{c+d}}+\frac{2a^2(-3Ad+3Bc-4Bd)\cos(e+fx)}{3d^2f\sqrt{a\sin(e+fx)+a}}-\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}$$

```
[Out] -2*a^(3/2)*(c-d)*(-A*d+B*c)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/
(a+a*sin(f*x+e))^(1/2))/d^(5/2)/f/(c+d)^(1/2)+2/3*a^2*(-3*A*d+3*B*c-4*B*d)*
cos(f*x+e)/d^2/f/(a+a*sin(f*x+e))^(1/2)-2/3*a*B*cos(f*x+e)*(a+a*sin(f*x+e))
^(1/2)/d/f
```

Rubi [A] time = 0.50, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2976, 2981, 2773, 208}

$$\frac{2a^2(-3Ad+3Bc-4Bd)\cos(e+fx)}{3d^2f\sqrt{a\sin(e+fx)+a}}-\frac{2a^{3/2}(c-d)(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{5/2}f\sqrt{c+d}}-\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),
x]
```

```
[Out] (-2*a^(3/2)*(c - d)*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqr
t[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(5/2)*Sqrt[c + d]*f) + (2*a^2*(3*B*
c - 3*A*d - 4*B*d)*Cos[e + f*x])/(3*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*
B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*d*f)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = -\frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3df} + \frac{2\int \frac{\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a(Bc+\cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{2a^2(3Bc-3Ad-4Bd)\cos(e+fx)}{3d^2f\sqrt{a+a\sin(e+fx)}} - \frac{2aB\cos(e+fx)\sqrt{a+a\,\mathrm{si}\cdots}}{3df}$$

$$= \frac{2a^2(3Bc-3Ad-4Bd)\cos(e+fx)}{3d^2f\sqrt{a+a\sin(e+fx)}} - \frac{2aB\cos(e+fx)\sqrt{a+a\,\mathrm{si}\cdots}}{3df}$$

$$= -\frac{2a^{3/2}(c-d)(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{5/2}\sqrt{c+d}\,f} + \frac{2a^2(3Bc\cdots}{3d^2\cdots}$$

Mathematica [B] time = 3.54, size = 356, normalized size = 2.33

$$\frac{(a(\sin(e+fx)+1))^{3/2}\left(6\sqrt{d}\,(2Ad-2Bc+3Bd)\sin\left(\frac{1}{2}(e+fx)\right)-6\sqrt{d}\,(2Ad-2Bc+3Bd)\cos\left(\frac{1}{2}(e+fx)\right)+\frac{3(c\cdots}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
f*x]),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(-6*Sqrt[d]*(-2*B*c + 2*A*d + 3*B*d)*Cos[(e +
 f*x)/2] - 2*B*d^(3/2)*Cos[(3*(e + f*x))/2] - (3*(c - d)*(B*c - A*d)*(e + f
*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[-(Sec[(e + f*x)/4]^2*(c + d + Sqrt[d
]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt[d]*Sqrt[c + d]*Sin[(e + f*x)/2]))]))/
Sqrt[c + d] + (3*(c - d)*(B*c - A*d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] +
 2*Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d]*(-1 + 2*Tan[(e + f*
x)/4] + Tan[(e + f*x)/4]^2)]))/Sqrt[c + d] + 6*Sqrt[d]*(-2*B*c + 2*A*d + 3*
B*d)*Sin[(e + f*x)/2] - 2*B*d^(3/2)*Sin[(3*(e + f*x))/2]))/(6*d^(5/2)*f*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

fricas [B] time = 1.47, size = 880, normalized size = 5.75

$$\left[-\frac{3\left(Bac^2-(A+B)acd+Aad^2+\left(Bac^2-(A+B)acd+Aad^2\right)\cos\left(fx+e\right)+\left(Bac^2-(A+B)acd+Aad^2\right)\sin\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] [-1/6*(3*(B*a*c^2 - (A + B)*a*c*d + A*a*d^2 + (B*a*c^2 - (A + B)*a*c*d + A*
a*d^2)*cos(f*x + e) + (B*a*c^2 - (A + B)*a*c*d + A*a*d^2)*sin(f*x + e))*sqr
t(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a
*c*d + 7*a*d^2)*cos(f*x + e)^2 - 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)
*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2
 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) +
a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*
cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x +
e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2
- 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(
```

```
f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(B*a*d*cos(f*x + e)^2 - 3*
B*a*c + (3*A + 4*B)*a*d - (3*B*a*c - (3*A + 5*B)*a*d)*cos(f*x + e) + (B*a*d
*cos(f*x + e) + 3*B*a*c - (3*A + 4*B)*a*d)*sin(f*x + e))*sqrt(a*sin(f*x + e
) + a))/(d^2*f*cos(f*x + e) + d^2*f*sin(f*x + e) + d^2*f), -1/3*(3*(B*a*c^2
 - (A + B)*a*c*d + A*a*d^2 + (B*a*c^2 - (A + B)*a*c*d + A*a*d^2)*cos(f*x +
e) + (B*a*c^2 - (A + B)*a*c*d + A*a*d^2)*sin(f*x + e))*sqrt(-a/(c*d + d^2))
*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*
d + d^2))/(a*cos(f*x + e))) + 2*(B*a*d*cos(f*x + e)^2 - 3*B*a*c + (3*A + 4*
B)*a*d - (3*B*a*c - (3*A + 5*B)*a*d)*cos(f*x + e) + (B*a*d*cos(f*x + e) + 3
*B*a*c - (3*A + 4*B)*a*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(d^2*f*co
s(f*x + e) + d^2*f*sin(f*x + e) + d^2*f)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algor
ithm="giac")

[Out] Timed out
```

maple [B] time = 1.73, size = 291, normalized size = 1.90

$$\frac{2\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\ \left(3A\,\mathrm{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,d}{\sqrt{a(c+d)d}}\right)a^2cd-3A\,\mathrm{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}}{\sqrt{a(c+d)d}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)

[Out] 2/3*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(3*A*arctanh((-a*(sin(f*x+e)-1
))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c*d-3*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)
*d/(a*(c+d)*d)^(1/2))*a^2*d^2-3*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c
+d)*d)^(1/2))*a^2*c^2+3*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(
1/2))*a^2*c*d+B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*d-3*A*(-a*(sin(
f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*d+3*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+
d)*d)^(1/2)*a*c-6*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*d)/d^2/(a
*(c+d)*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) +
 c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}{c+d\,\sin\left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x)),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x)),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.298 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=191

$$-\frac{a^{3/2}\left(Ad(c+3d)-B\left(3c^2+3cd-2d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{5/2}f(c+d)^{3/2}}-\frac{a^2(-Ad+3Bc+2Bd)\cos(e+fx)}{d^2f(c+d)\sqrt{a\sin(e+fx)+a}}+\frac{a(Bc-$$

```
[Out] -a^(3/2)*(A*d*(c+3*d)-B*(3*c^2+3*c*d-2*d^2))*arctanh(cos(f*x+e)*a^(1/2)*d^(
1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/d^(5/2)/(c+d)^(3/2)/f-a^2*(-A*d+3*
B*c+2*B*d)*cos(f*x+e)/d^2/(c+d)/f/(a+a*sin(f*x+e))^(1/2)+a*(-A*d+B*c)*cos(f
*x+e)*(a+a*sin(f*x+e))^(1/2)/d/(c+d)/f/(c+d*sin(f*x+e))
```

Rubi [A] time = 0.55, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2975, 2981, 2773, 208}

$$-\frac{a^{3/2}\left(Ad(c+3d)-B\left(3c^2+3cd-2d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{5/2}f(c+d)^{3/2}}-\frac{a^2(-Ad+3Bc+2Bd)\cos(e+fx)}{d^2f(c+d)\sqrt{a\sin(e+fx)+a}}+\frac{a(Bc-$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^
2,x]
```

```
[Out] -((a^(3/2)*(A*d*(c + 3*d) - B*(3*c^2 + 3*c*d - 2*d^2))*ArcTanh[(Sqrt[a]*Sqr
t[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(5/2)*(c + d
)^(3/2)*f)) - (a^2*(3*B*c - A*d + 2*B*d)*Cos[e + f*x])/(d^2*(c + d)*f*Sqrt[
a + a*Sin[e + f*x]]) + (a*(B*c - A*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]
)/(d*(c + d)*f*(c + d*Sin[e + f*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{d(c+d)f(c+d\sin(e+fx))} + \frac{\int \frac{\sqrt{a+a\sin(e+fx)}\big(}{}}{}$$

$$= -\frac{a^2(3Bc-Ad+2Bd)\cos(e+fx)}{d^2(c+d)f\sqrt{a+a\sin(e+fx)}} + \frac{a(Bc-Ad)\cos(e+fx)\sqrt{}}{d(c+d)f(c+d\,\mathrm{s}}$$

$$= -\frac{a^2(3Bc-Ad+2Bd)\cos(e+fx)}{d^2(c+d)f\sqrt{a+a\sin(e+fx)}} + \frac{a(Bc-Ad)\cos(e+fx)\sqrt{}}{d(c+d)f(c+d\,\mathrm{s}}$$

$$= -\frac{a^{3/2}\left(Ad(c+3d)-B\left(3c^2+3cd-2d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(}{\sqrt{c+d}\,\sqrt{a+a\sin}}\right.}{d^{5/2}(c+d)^{3/2}f}$$

Mathematica [A] time = 5.01, size = 381, normalized size = 1.99

$$(a(\sin(e+fx)+1))^{3/2}\left(\frac{\left(Ad(c+3d)+B\left(-3c^2-3cd+2d^2\right)\right)\left(2\log\left(\sqrt{d}\sqrt{c+d}\left(\tan^2\left(\frac{1}{4}(e+fx)\right)+2\tan\left(\frac{1}{4}(e+fx)\right)-1\right)+(c+d)\sec^2\left(\frac{1}{4}(e+fx)\right)\right)-2\log\left(\sec\right.\right.}{(c+d)^{3/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
f*x])^2,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(-8*B*Sqrt[d]*Cos[(e + f*x)/2] + ((-(A*d*(c +
 3*d)) + B*(3*c^2 + 3*c*d - 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] +
2*Log[-(Sec[(e + f*x)/4]^2*(c + d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] -
Sqrt[d]*Sqrt[c + d]*Sin[(e + f*x)/2]))]))/(c + d)^(3/2) + ((A*d*(c + 3*d) +
 B*(-3*c^2 - 3*c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[(
c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d]*(-1 + 2*Tan[(e + f*x)/4] +
Tan[(e + f*x)/4]^2)]))/(c + d)^(3/2) + 8*B*Sqrt[d]*Sin[(e + f*x)/2] - (4*Sq
rt[d]*(-c + d)*(-(B*c) + A*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c +
d)*(c + d*Sin[e + f*x]))))/(4*d^(5/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^3)
```

fricas [B] time = 1.68, size = 1428, normalized size = 7.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] [1/4*((3*B*a*c^3 - (A - 6*B)*a*c^2*d - (4*A - B)*a*c*d^2 - (3*A + 2*B)*a*d^
3 - (3*B*a*c^2*d - (A - 3*B)*a*c*d^2 - (3*A + 2*B)*a*d^3)*cos(f*x + e)^2 +
(3*B*a*c^3 - (A - 3*B)*a*c^2*d - (3*A + 2*B)*a*c*d^2)*cos(f*x + e) + (3*B*a
*c^3 - (A - 6*B)*a*c^2*d - (4*A - B)*a*c*d^2 - (3*A + 2*B)*a*d^3 + (3*B*a*c
^2*d - (A - 3*B)*a*c*d^2 - (3*A + 2*B)*a*d^3)*cos(f*x + e))*sin(f*x + e))*s
qrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6
*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^
3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d
^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e)
+ a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^
2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x
+ e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^
2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*co
s(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(3*B*a*c^2 - (A + B)*a*c
```

```
*d + (A - 2*B)*a*d^2 + 2*(B*a*c*d + B*a*d^2)*cos(f*x + e)^2 + (3*B*a*c^2 -
(A - B)*a*c*d + A*a*d^2)*cos(f*x + e) - (3*B*a*c^2 - (A + B)*a*c*d + (A - 2
*B)*a*d^2 - 2*(B*a*c*d + B*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*
x + e) + a))/((c*d^3 + d^4)*f*cos(f*x + e)^2 - (c^2*d^2 + c*d^3)*f*cos(f*x
+ e) - (c^2*d^2 + 2*c*d^3 + d^4)*f - ((c*d^3 + d^4)*f*cos(f*x + e) + (c^2*d
^2 + 2*c*d^3 + d^4)*f)*sin(f*x + e)), -1/2*((3*B*a*c^3 - (A - 6*B)*a*c^2*d
- (4*A - B)*a*c*d^2 - (3*A + 2*B)*a*d^3 - (3*B*a*c^2*d - (A - 3*B)*a*c*d^2
- (3*A + 2*B)*a*d^3)*cos(f*x + e)^2 + (3*B*a*c^3 - (A - 3*B)*a*c^2*d - (3*A
 + 2*B)*a*c*d^2)*cos(f*x + e) + (3*B*a*c^3 - (A - 6*B)*a*c^2*d - (4*A - B)*
a*c*d^2 - (3*A + 2*B)*a*d^3 + (3*B*a*c^2*d - (A - 3*B)*a*c*d^2 - (3*A + 2*B
)*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a
*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f
*x + e))) - 2*(3*B*a*c^2 - (A + B)*a*c*d + (A - 2*B)*a*d^2 + 2*(B*a*c*d + B
*a*d^2)*cos(f*x + e)^2 + (3*B*a*c^2 - (A - B)*a*c*d + A*a*d^2)*cos(f*x + e)
 - (3*B*a*c^2 - (A + B)*a*c*d + (A - 2*B)*a*d^2 - 2*(B*a*c*d + B*a*d^2)*cos
(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c*d^3 + d^4)*f*cos(f*x
 + e)^2 - (c^2*d^2 + c*d^3)*f*cos(f*x + e) - (c^2*d^2 + 2*c*d^3 + d^4)*f -
((c*d^3 + d^4)*f*cos(f*x + e) + (c^2*d^2 + 2*c*d^3 + d^4)*f)*sin(f*x + e))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 2.32, size = 592, normalized size = 3.10

$$\frac{a\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\ \left(-\sin\left(fx+e\right)d\left(A\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ d}{\sqrt{acd+a\,d^{2}}}\right)acd+3A\operatorname{arctanh}\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] a*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-sin(f*x+e)*d*(A*arctanh((a-a*s
in(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d+3*A*arctanh((a-a*sin(f*x+e))^
(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*d^2-3*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*
c*d+a*d^2)^(1/2))*a*c^2-3*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^
(1/2))*a*c*d+2*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*d^
```

```
2+2*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*c+2*B*(a*(c+d)*d)^(1/2)*(a-a
*sin(f*x+e))^(1/2)*d)-A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2
))*a*c^2*d-3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d^
2+3*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c^3+3*B*arcta
nh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c^2*d-2*B*arctanh((a-a*s
in(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d^2+A*(a-a*sin(f*x+e))^(1/2)*(a
*(c+d)*d)^(1/2)*c*d-A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*d^2-3*B*(a-a
*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*c^2-B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*
d)^(1/2)*c*d)/d^2/(c+d)/(c+d*sin(f*x+e))/(a*(c+d)*d)^(1/2)/cos(f*x+e)/(a+a*
sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) +
 c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))^
2,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))^
2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.299 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=221

$$-\frac{a^{3/2}\left(Ad(c+7d)+3B\left(c^2+3cd+4d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{4d^{5/2}f(c+d)^{5/2}}+\frac{a^2\left(Ad(c-5d)+B\left(3c^2+5cd-4d^2\right)\right)\text{c}}{4d^2f(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\,\text{si}}$$

```
[Out] -1/4*a^(3/2)*(A*d*(c+7*d)+3*B*(c^2+3*c*d+4*d^2))*arctanh(cos(f*x+e)*a^(1/2)
*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/d^(5/2)/(c+d)^(5/2)/f+1/4*a^2*
(A*(c-5*d)*d+B*(3*c^2+5*c*d-4*d^2))*cos(f*x+e)/d^2/(c+d)^2/f/(c+d*sin(f*x+e
))/(a+a*sin(f*x+e))^(1/2)+1/2*a*(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2
)/d/(c+d)/f/(c+d*sin(f*x+e))^2
```

Rubi [A] time = 0.61, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2975, 2980, 2773, 208}

$$\frac{a^2\left(Ad(c-5d)+B\left(3c^2+5cd-4d^2\right)\right)\cos(e+fx)}{4d^2f(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}-\frac{a^{3/2}\left(Ad(c+7d)+3B\left(c^2+3cd+4d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\text{cos}}{\sqrt{c+d}\,\sqrt{a\,\text{sin}}}\right.}{4d^{5/2}f(c+d)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^
3,x]
```

```
[Out] -(a^(3/2)*(A*d*(c + 7*d) + 3*B*(c^2 + 3*c*d + 4*d^2))*ArcTanh[(Sqrt[a]*Sqrt
[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(4*d^(5/2)*(c +
d)^(5/2)*f) + (a*(B*c - A*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(2*d*(c
 + d)*f*(c + d*Sin[e + f*x])^2) + (a^2*(A*(c - 5*d)*d + B*(3*c^2 + 5*c*d -
4*d^2))*Cos[e + f*x])/(4*d^2*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Si
n[e + f*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{\int \frac{\sqrt{a+a\sin(e+fx)}\left(-\right.}{}}{}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{a^2\left(A(c-5d)d+\right.}{4d^2(c+d)^2f\sqrt{a-}}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{a^2\left(A(c-5d)d+\right.}{4d^2(c+d)^2f\sqrt{a-}}$$

$$= -\frac{a^{3/2}\left(Ad(c+7d)+3B\left(c^2+3cd+4d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+}{\sqrt{c+d}\sqrt{a+a\sin(}}\right.}{4d^{5/2}(c+d)^{5/2}f}$$

Mathematica [A] time = 5.34, size = 416, normalized size = 1.88

$$\frac{(a(\sin(e+fx)+1))^{3/2}\left(-\frac{4\sqrt{d}\left(Ad(c+7d)+B\left(-5c^2-7cd+4d^2\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)^2(c+d\sin(e+fx))}+\frac{\left(Ad(c+7d)+3B\left(c^2+3cd+4d^2\right)\right)\left(2\log\left(\sqrt{d}\sqrt{\cdots}\right.\right.}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
f*x])^3,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(-(((A*d*(c + 7*d) + 3*B*(c^2 + 3*c*d + 4*d^2
))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[-(Sec[(e + f*x)/4]^2*(c + d
 + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt[d]*Sqrt[c + d]*Sin[(e + f*x)
/2]))]))/(c + d)^(5/2)) + ((A*d*(c + 7*d) + 3*B*(c^2 + 3*c*d + 4*d^2))*(e +
 f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[
d]*Sqrt[c + d]*(-1 + 2*Tan[(e + f*x)/4] + Tan[(e + f*x)/4]^2)]))/(c + d)^(5
/2) - (8*Sqrt[d]*(-c + d)*(-(B*c) + A*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2]))/((c + d)*(c + d*Sin[e + f*x])^2) - (4*Sqrt[d]*(A*d*(c + 7*d) + B*(-5*c
^2 - 7*c*d + 4*d^2))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)^2*(c +
 d*Sin[e + f*x]))))/(16*d^(5/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

fricas [B] time = 2.59, size = 2208, normalized size = 9.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] [-1/16*((3*B*a*c^4 + (A + 15*B)*a*c^3*d + 3*(3*A + 11*B)*a*c^2*d^2 + 3*(5*A
 + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4 - (3*B*a*c^2*d^2 + (A + 9*B)*a*c*d^3
+ (7*A + 12*B)*a*d^4)*cos(f*x + e)^3 - (6*B*a*c^3*d + (2*A + 21*B)*a*c^2*d^
2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*x + e)^2 + (3*B*a*c^
4 + (A + 9*B)*a*c^3*d + (7*A + 15*B)*a*c^2*d^2 + (A + 9*B)*a*c*d^3 + (7*A +
 12*B)*a*d^4)*cos(f*x + e) + (3*B*a*c^4 + (A + 15*B)*a*c^3*d + 3*(3*A + 11*
B)*a*c^2*d^2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4 - (3*B*a*c^2*d^2
 + (A + 9*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*x + e)^2 + 2*(3*B*a*c^3*d
+ (A + 9*B)*a*c^2*d^2 + (7*A + 12*B)*a*c*d^3)*cos(f*x + e))*sin(f*x + e))*s
qrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6
*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^
3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d
^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e)
+ a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^
```

```
2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x
+ e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^
2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*co
s(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(3*B*a*c^3 + (A + 2*B)*a
*c^2*d - 3*(2*A + 3*B)*a*c*d^2 + (5*A + 4*B)*a*d^3 + (5*B*a*c^2*d - (A - 7*
B)*a*c*d^2 - (7*A + 4*B)*a*d^3)*cos(f*x + e)^2 + (3*B*a*c^3 + (A + 7*B)*a*c
^2*d - (7*A + 2*B)*a*c*d^2 - 2*A*a*d^3)*cos(f*x + e) - (3*B*a*c^3 + (A + 2*
B)*a*c^2*d - 3*(2*A + 3*B)*a*c*d^2 + (5*A + 4*B)*a*d^3 - (5*B*a*c^2*d - (A
- 7*B)*a*c*d^2 - (7*A + 4*B)*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(
f*x + e) + a))/((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^3 + (2*c^3*d^3 + 5
*c^2*d^4 + 4*c*d^5 + d^6)*f*cos(f*x + e)^2 - (c^4*d^2 + 2*c^3*d^3 + 2*c^2*d
^4 + 2*c*d^5 + d^6)*f*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c
*d^5 + d^6)*f + ((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^2 - 2*(c^3*d^3 +
2*c^2*d^4 + c*d^5)*f*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*
d^5 + d^6)*f)*sin(f*x + e)), 1/8*((3*B*a*c^4 + (A + 15*B)*a*c^3*d + 3*(3*A
+ 11*B)*a*c^2*d^2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4 - (3*B*a*c^
2*d^2 + (A + 9*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*x + e)^3 - (6*B*a*c^3
*d + (2*A + 21*B)*a*c^2*d^2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*
cos(f*x + e)^2 + (3*B*a*c^4 + (A + 9*B)*a*c^3*d + (7*A + 15*B)*a*c^2*d^2 +
(A + 9*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*x + e) + (3*B*a*c^4 + (A + 15
*B)*a*c^3*d + 3*(3*A + 11*B)*a*c^2*d^2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12
*B)*a*d^4 - (3*B*a*c^2*d^2 + (A + 9*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*
x + e)^2 + 2*(3*B*a*c^3*d + (A + 9*B)*a*c^2*d^2 + (7*A + 12*B)*a*c*d^3)*cos
(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e
) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) -
2*(3*B*a*c^3 + (A + 2*B)*a*c^2*d - 3*(2*A + 3*B)*a*c*d^2 + (5*A + 4*B)*a*d^
3 + (5*B*a*c^2*d - (A - 7*B)*a*c*d^2 - (7*A + 4*B)*a*d^3)*cos(f*x + e)^2 +
(3*B*a*c^3 + (A + 7*B)*a*c^2*d - (7*A + 2*B)*a*c*d^2 - 2*A*a*d^3)*cos(f*x +
 e) - (3*B*a*c^3 + (A + 2*B)*a*c^2*d - 3*(2*A + 3*B)*a*c*d^2 + (5*A + 4*B)*
a*d^3 - (5*B*a*c^2*d - (A - 7*B)*a*c*d^2 - (7*A + 4*B)*a*d^3)*cos(f*x + e))
*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f
*x + e)^3 + (2*c^3*d^3 + 5*c^2*d^4 + 4*c*d^5 + d^6)*f*cos(f*x + e)^2 - (c^4
*d^2 + 2*c^3*d^3 + 2*c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e) - (c^4*d^2 + 4
*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f + ((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(
f*x + e)^2 - 2*(c^3*d^3 + 2*c^2*d^4 + c*d^5)*f*cos(f*x + e) - (c^4*d^2 + 4*
c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f)*sin(f*x + e))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 2.82, size = 895, normalized size = 4.05

$$\frac{\left(-2\sin\left(fx+e\right)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,d}{\sqrt{acd+a\,d^2}}\right)a^2cd\left(Acd+7A\,d^2+3B\,c^2+9Bcd+12B\,d^2\right)+\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+\right.}}{\sqrt{acd+a\,d^2}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/4*(-2*sin(f*x+e)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^
2*c*d*(A*c*d+7*A*d^2+3*B*c^2+9*B*c*d+12*B*d^2)+arctanh((a-a*sin(f*x+e))^(1/
2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^2*(A*c*d+7*A*d^2+3*B*c^2+9*B*c*d+12*B*d^2)*
cos(f*x+e)^2-A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^
3*d-7*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d^2-A
*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^3-7*A*arctan
h((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^4+A*(a-a*sin(f*x+e))^
(3/2)*(a*(c+d)*d)^(1/2)*c*d^2+7*A*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*
d^3-3*a^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*B*c^4-9*B*a
rctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^3*d-15*B*arctanh
((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d^2-9*B*arctanh((a-a
*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^3-12*B*arctanh((a-a*sin(f
*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^4-5*B*(a-a*sin(f*x+e))^(3/2)*(a*(
c+d)*d)^(1/2)*c^2*d-7*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*c*d^2+4*B*
(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*d^3+A*(a-a*sin(f*x+e))^(1/2)*(a*(c
+d)*d)^(1/2)*a*c^2*d-8*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d^2-9
*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^3+3*B*(a-a*sin(f*x+e))^(1/2
)*(a*(c+d)*d)^(1/2)*a*c^3+12*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c
^2*d+5*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d^2-4*B*(a-a*sin(f*x+
e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^3)*(-a*(sin(f*x+e)-1))^(1/2)*(1+sin(f*x+e))
/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e))^2/(c+d)^2/d^2/cos(f*x+e)/(a+a*sin(f*x+e
))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) +
 c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))^
3,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))^
3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.300 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=534

$$-\frac{2a^3\left(-39Acd+299Ad^2+15Bc^2-75Bcd+280Bd^2\right)\cos(e+fx)(c+d\sin(e+fx))^4}{1287d^3f\sqrt{a\sin(e+fx)+a}}-\frac{2a^3\left(13Ad\left(3c^2-38cd\right.\right.}{}$$

```
[Out] -4/15015*a*(c+d)*(13*A*d*(3*c^2-38*c*d+355*d^2)-B*(15*c^3-150*c^2*d+799*c*d
^2-4184*d^3))*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/d/f-2/13*a*B*cos(f*x+e)*(a+
a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^4/d/f-4/45045*a^3*(c+d)*(15*c^2+10*c*d
+7*d^2)*(13*A*d*(3*c^2-38*c*d+355*d^2)-B*(15*c^3-150*c^2*d+799*c*d^2-4184*d
^3))*cos(f*x+e)/d^3/f/(a+a*sin(f*x+e))^(1/2)-2/9009*a^3*(13*A*d*(3*c^2-38*c
*d+355*d^2)-B*(15*c^3-150*c^2*d+799*c*d^2-4184*d^3))*cos(f*x+e)*(c+d*sin(f*
x+e))^3/d^3/f/(a+a*sin(f*x+e))^(1/2)-2/1287*a^3*(-39*A*c*d+299*A*d^2+15*B*c
^2-75*B*c*d+280*B*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^4/d^3/f/(a+a*sin(f*x+e))
^(1/2)-8/45045*a^2*(5*c-d)*(c+d)*(13*A*d*(3*c^2-38*c*d+355*d^2)-B*(15*c^3-1
50*c^2*d+799*c*d^2-4184*d^3))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d^2/f+2/143
*a^2*(-13*A*d+5*B*c-16*B*d)*cos(f*x+e)*(c+d*sin(f*x+e))^4*(a+a*sin(f*x+e))^
(1/2)/d^2/f
```

Rubi [A] time = 1.20, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2976, 2981, 2770, 2761, 2751, 2646}

$$-\frac{2a^3\left(-39Acd+299Ad^2+15Bc^2-75Bcd+280Bd^2\right)\cos(e+fx)(c+d\sin(e+fx))^4}{1287d^3f\sqrt{a\sin(e+fx)+a}}-\frac{2a^3\left(13Ad\left(3c^2-38cd\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,
x]
```

```
[Out] (-4*a^3*(c + d)*(15*c^2 + 10*c*d + 7*d^2)*(13*A*d*(3*c^2 - 38*c*d + 355*d^2
) - B*(15*c^3 - 150*c^2*d + 799*c*d^2 - 4184*d^3))*Cos[e + f*x])/(45045*d^3
*f*Sqrt[a + a*Sin[e + f*x]]) - (8*a^2*(5*c - d)*(c + d)*(13*A*d*(3*c^2 - 38
*c*d + 355*d^2) - B*(15*c^3 - 150*c^2*d + 799*c*d^2 - 4184*d^3))*Cos[e + f*
x]*Sqrt[a + a*Sin[e + f*x]])/(45045*d^2*f) - (4*a*(c + d)*(13*A*d*(3*c^2 -
38*c*d + 355*d^2) - B*(15*c^3 - 150*c^2*d + 799*c*d^2 - 4184*d^3))*Cos[e +
f*x]*(a + a*Sin[e + f*x])^(3/2))/(15015*d*f) - (2*a^3*(13*A*d*(3*c^2 - 38*c
*d + 355*d^2) - B*(15*c^3 - 150*c^2*d + 799*c*d^2 - 4184*d^3))*Cos[e + f*x]
*(c + d*Sin[e + f*x])^3)/(9009*d^3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a^3*(15
*B*c^2 - 39*A*c*d - 75*B*c*d + 299*A*d^2 + 280*B*d^2)*Cos[e + f*x]*(c + d*S
```

```
in[e + f*x])^4)/(1287*d^3*f*Sqrt[a + a*Sin[e + f*x]]) + (2*a^2*(5*B*c - 13*
A*d - 16*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^4)
/(143*d^2*f) - (2*a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e
+ f*x])^4)/(13*d*f)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
```

```
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c+}{13df}$$

$$= \frac{2a^2(5Bc-13Ad-16Bd)\cos(e+fx)\sqrt{a+}{143d^2f}$$

$$= -\frac{2a^3\left(15Bc^2-39Acd-75Bcd+299Ad^2+\right.}{1287d^3f\sqrt{a+}}$$

$$= -\frac{2a^3\left(13Ad\left(3c^2-38cd+355d^2\right)-B\left(15c\right.\right.}{90}$$

$$= -\frac{4a(c+d)\left(13Ad\left(3c^2-38cd+355d^2\right)-\right.}{}$$

$$= -\frac{8a^2(5c-d)(c+d)\left(13Ad\left(3c^2-38cd+3\right.\right.}{}$$

$$= -\frac{4a^3(c+d)\left(15c^2+10cd+7d^2\right)\left(13Ad\left(3c\right.\right.}{}$$

Mathematica [C] time = 6.87, size = 1565, normalized size = 2.93

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x])^3,x]
```

```
[Out] (B*d^3*Cos[(13*(e + f*x))/2]*(a*(1 + Sin[e + f*x]))^(5/2))/(416*f*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^5) + ((40*A*c^3 + 30*B*c^3 + 90*A*c^2*d + 78*B
*c^2*d + 78*A*c*d^2 + 69*B*c*d^2 + 23*A*d^3 + 21*B*d^3)*((-1/16 - I/16)*Cos
[(e + f*x)/2] + (1/16 - I/16)*Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2
))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((40*A*c^3 + 30*B*c^3 + 90
*A*c^2*d + 78*B*c^2*d + 78*A*c*d^2 + 69*B*c*d^2 + 23*A*d^3 + 21*B*d^3)*((-1
/16 + I/16)*Cos[(e + f*x)/2] + (1/16 + I/16)*Sin[(e + f*x)/2])*(a*(1 + Sin[
e + f*x]))^(5/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((80*A*c^3
+ 88*B*c^3 + 264*A*c^2*d + 240*B*c^2*d + 240*A*c*d^2 + 228*B*c*d^2 + 76*A*d
^3 + 71*B*d^3)*(a*(1 + Sin[e + f*x]))^(5/2)*((-1/192 + I/192)*Cos[(3*(e + f
*x))/2] - (1/192 + I/192)*Sin[(3*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^5) + ((80*A*c^3 + 88*B*c^3 + 264*A*c^2*d + 240*B*c^2*d + 240
*A*c*d^2 + 228*B*c*d^2 + 76*A*d^3 + 71*B*d^3)*(a*(1 + Sin[e + f*x]))^(5/2)*
((-1/192 - I/192)*Cos[(3*(e + f*x))/2] - (1/192 - I/192)*Sin[(3*(e + f*x))/
2]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((16*A*c^3 + 40*B*c^3 +
120*A*c^2*d + 144*B*c^2*d + 144*A*c*d^2 + 150*B*c*d^2 + 50*A*d^3 + 51*B*d^3
)*(a*(1 + Sin[e + f*x]))^(5/2)*((1/320 - I/320)*Cos[(5*(e + f*x))/2] - (1/3
20 + I/320)*Sin[(5*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^5) + ((16*A*c^3 + 40*B*c^3 + 120*A*c^2*d + 144*B*c^2*d + 144*A*c*d^2 + 150
*B*c*d^2 + 50*A*d^3 + 51*B*d^3)*(a*(1 + Sin[e + f*x]))^(5/2)*((1/320 + I/32
0)*Cos[(5*(e + f*x))/2] - (1/320 - I/320)*Sin[(5*(e + f*x))/2]))/(f*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^5) + ((4*B*c^3 + 12*A*c^2*d + 30*B*c^2*d + 3
0*A*c*d^2 + 39*B*c*d^2 + 13*A*d^3 + 15*B*d^3)*(a*(1 + Sin[e + f*x]))^(5/2)*
((1/224 + I/224)*Cos[(7*(e + f*x))/2] + (1/224 - I/224)*Sin[(7*(e + f*x))/2
]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((4*B*c^3 + 12*A*c^2*d +
30*B*c^2*d + 30*A*c*d^2 + 39*B*c*d^2 + 13*A*d^3 + 15*B*d^3)*(a*(1 + Sin[e +
 f*x]))^(5/2)*((1/224 - I/224)*Cos[(7*(e + f*x))/2] + (1/224 + I/224)*Sin[(
7*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((6*B*c^2 +
 6*A*c*d + 15*B*c*d + 5*A*d^2 + 7*B*d^2)*(a*(1 + Sin[e + f*x]))^(5/2)*((-1/
288 - I/288)*d*Cos[(9*(e + f*x))/2] + (1/288 - I/288)*d*Sin[(9*(e + f*x))/2
]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((6*B*c^2 + 6*A*c*d + 15*
B*c*d + 5*A*d^2 + 7*B*d^2)*(a*(1 + Sin[e + f*x]))^(5/2)*((-1/288 + I/288)*d
*Cos[(9*(e + f*x))/2] + (1/288 + I/288)*d*Sin[(9*(e + f*x))/2]))/(f*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^5) + ((6*B*c + 2*A*d + 5*B*d)*(a*(1 + Sin[e
+ f*x]))^(5/2)*((-1/704 + I/704)*d^2*Cos[(11*(e + f*x))/2] - (1/704 + I/704
)*d^2*Sin[(11*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) +
 ((6*B*c + 2*A*d + 5*B*d)*(a*(1 + Sin[e + f*x]))^(5/2)*((-1/704 - I/704)*d^
2*Cos[(11*(e + f*x))/2] - (1/704 - I/704)*d^2*Sin[(11*(e + f*x))/2]))/(f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^5) - (B*d^3*(a*(1 + Sin[e + f*x]))^(5/2
)*Sin[(13*(e + f*x))/2])/(416*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

fricas [A] time = 0.51, size = 863, normalized size = 1.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] 2/45045*(3465*B*a^2*d^3*cos(f*x + e)^7 - 315*(39*B*a^2*c*d^2 + (13*A + 27*B
)*a^2*d^3)*cos(f*x + e)^6 - 13728*(7*A + 5*B)*a^2*c^3 - 13728*(15*A + 13*B)
*a^2*c^2*d - 1248*(143*A + 125*B)*a^2*c*d^2 - 32*(1625*A + 1483*B)*a^2*d^3
- 35*(429*B*a^2*c^2*d + 39*(11*A + 32*B)*a^2*c*d^2 + 4*(104*A + 205*B)*a^2*
d^3)*cos(f*x + e)^5 + 5*(1287*B*a^2*c^3 + 429*(9*A + 19*B)*a^2*c^2*d + 39*(
209*A + 320*B)*a^2*c*d^2 + 2*(2080*A + 2813*B)*a^2*d^3)*cos(f*x + e)^4 + (1
287*(7*A + 20*B)*a^2*c^3 + 429*(180*A + 289*B)*a^2*c^2*d + 39*(3179*A + 437
0*B)*a^2*c*d^2 + (56810*A + 72109*B)*a^2*d^3)*cos(f*x + e)^3 - (429*(77*A +
 85*B)*a^2*c^3 + 429*(255*A + 263*B)*a^2*c^2*d + 39*(2893*A + 2965*B)*a^2*c
*d^2 + (38545*A + 39113*B)*a^2*d^3)*cos(f*x + e)^2 - 2*(429*(161*A + 145*B)
*a^2*c^3 + 429*(435*A + 419*B)*a^2*c^2*d + 39*(4609*A + 4465*B)*a^2*c*d^2 +
 (58045*A + 56909*B)*a^2*d^3)*cos(f*x + e) - (3465*B*a^2*d^3*cos(f*x + e)^6
 - 13728*(7*A + 5*B)*a^2*c^3 - 13728*(15*A + 13*B)*a^2*c^2*d - 1248*(143*A
+ 125*B)*a^2*c*d^2 - 32*(1625*A + 1483*B)*a^2*d^3 + 315*(39*B*a^2*c*d^2 + (
13*A + 38*B)*a^2*d^3)*cos(f*x + e)^5 - 35*(429*B*a^2*c^2*d + 39*(11*A + 23*
B)*a^2*c*d^2 + (299*A + 478*B)*a^2*d^3)*cos(f*x + e)^4 - 5*(1287*B*a^2*c^3
+ 429*(9*A + 26*B)*a^2*c^2*d + 507*(22*A + 37*B)*a^2*c*d^2 + (6253*A + 8972
*B)*a^2*d^3)*cos(f*x + e)^3 + 3*(429*(7*A + 15*B)*a^2*c^3 + 429*(45*A + 53*
B)*a^2*c^2*d + 39*(583*A + 655*B)*a^2*c*d^2 + (8515*A + 9083*B)*a^2*d^3)*co
s(f*x + e)^2 + 2*(429*(49*A + 65*B)*a^2*c^3 + 429*(195*A + 211*B)*a^2*c^2*d
 + 39*(2321*A + 2465*B)*a^2*c*d^2 + (32045*A + 33181*B)*a^2*d^3)*cos(f*x +
e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e)
 + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
```

check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
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x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)sqrt(2*a)*(-288*f*(2*A*a^2*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*p
i))+4*B*a^2*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+6*B*a^2*c*d^2*sign(cos(1
/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(18*f*x+18*exp(1)+pi))/(288*f)^2-352*f*(2
*A*a^2*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+4*B*a^2*d^3*sign(cos(1/2*(f*x
+exp(1))-1/4*pi))+6*B*a^2*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4
*(22*f*x+22*exp(1)-pi))/(352*f)^2+224*f*(8*A*a^2*d^3*sign(cos(1/2*(f*x+exp(

```
1))-1/4*pi))+10*B*a^2*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+12*A*a^2*c*d^2
*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+24*B*a^2*c*d^2*sign(cos(1/2*(f*x+exp(1)
)-1/4*pi))+12*B*a^2*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(14*f
*x+14*exp(1)+pi))/(224*f)^2+288*f*(8*A*a^2*d^3*sign(cos(1/2*(f*x+exp(1))-1/
4*pi))+10*B*a^2*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+12*A*a^2*c*d^2*sign(
cos(1/2*(f*x+exp(1))-1/4*pi))+24*B*a^2*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))+12*B*a^2*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(18*f*x+18*
exp(1)-pi))/(288*f)^2-160*f*(-18*A*a^2*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi
))-8*B*a^2*c^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-20*B*a^2*d^3*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))-48*A*a^2*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-24
*A*a^2*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-54*B*a^2*c*d^2*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))-48*B*a^2*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*c
os(1/4*(10*f*x+10*exp(1)+pi))/(160*f)^2-224*f*(-18*A*a^2*d^3*sign(cos(1/2*(
f*x+exp(1))-1/4*pi))-8*B*a^2*c^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-20*B*a^
2*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-48*A*a^2*c*d^2*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi))-24*A*a^2*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-54*B*a^2*
c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-48*B*a^2*c^2*d*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi)))*cos(1/4*(14*f*x+14*exp(1)-pi))/(224*f)^2-16*f*(32*A*a^2*c^
3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+22*A*a^2*d^3*sign(cos(1/2*(f*x+exp(1))
-1/4*pi))+28*B*a^2*c^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+20*B*a^2*d^3*sign
(cos(1/2*(f*x+exp(1))-1/4*pi))+72*A*a^2*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4
*pi))+84*A*a^2*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+66*B*a^2*c*d^2*sign
(cos(1/2*(f*x+exp(1))-1/4*pi))+72*B*a^2*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4
*pi)))*cos(1/4*(2*f*x+2*exp(1)+pi))/(16*f)^2-48*f*(32*A*a^2*c^3*sign(cos(1/
2*(f*x+exp(1))-1/4*pi))+22*A*a^2*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+28*
B*a^2*c^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+20*B*a^2*d^3*sign(cos(1/2*(f*x
+exp(1))-1/4*pi))+72*A*a^2*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+84*A*a^
2*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+66*B*a^2*c*d^2*sign(cos(1/2*(f*x
+exp(1))-1/4*pi))+72*B*a^2*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/
4*(6*f*x+6*exp(1)-pi))/(48*f)^2+16*f*(48*A*a^2*c^3*sign(cos(1/2*(f*x+exp(1)
)-1/4*pi))+24*A*a^2*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+32*B*a^2*c^3*sig
n(cos(1/2*(f*x+exp(1))-1/4*pi))+22*B*a^2*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))+84*A*a^2*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+96*A*a^2*c^2*d*sign(
cos(1/2*(f*x+exp(1))-1/4*pi))+72*B*a^2*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))+84*B*a^2*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(2*f*x-pi)+
1/2*exp(1))/(16*f)^2+192*f*(-32*A*a^2*c^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi)
)-64*A*a^2*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-64*B*a^2*c^3*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))-62*B*a^2*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-192*
A*a^2*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-192*A*a^2*c^2*d*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))-192*B*a^2*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-1
92*B*a^2*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(6*f*x+6*exp(1)+
pi))/(192*f)^2+320*f*(-32*A*a^2*c^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-64*A
*a^2*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-64*B*a^2*c^3*sign(cos(1/2*(f*x+
exp(1))-1/4*pi))-62*B*a^2*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-192*A*a^2*
c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-192*A*a^2*c^2*d*sign(cos(1/2*(f*x+
```

```
exp(1))-1/4*pi))-192*B*a^2*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-192*B*a
^2*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(10*f*x+10*exp(1)-pi))
/(320*f)^2-1408*B*a^2*d^3*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(22*
f*x+22*exp(1)+pi))/(704*f)^2-1664*B*a^2*d^3*f*sign(cos(1/2*(f*x+exp(1))-1/4
*pi))*sin(1/4*(26*f*x+26*exp(1)-pi))/(832*f)^2)
```

maple [A] time = 1.40, size = 374, normalized size = 0.70

$$\frac{2\left(1+\sin\left(fx+e\right)\right)a^{3}\left(\sin\left(fx+e\right)-1\right)\left(\left(4095A\,d^{3}+12285Bc\,d^{2}+11970B\,d^{3}\right)\sin\left(fx+e\right)\left(\cos^{4}\left(fx+e\right)\right)+\left(\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 2/45045*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*((4095*A*d^3+12285*B*c*d^2+11970*
B*d^3)*sin(f*x+e)*cos(f*x+e)^4+(-19305*A*c^2*d-55770*A*c*d^2-31265*A*d^3-64
35*B*c^3-55770*B*c^2*d-93795*B*c*d^2-44860*B*d^3)*cos(f*x+e)^2*sin(f*x+e)+(
42042*A*c^3+167310*A*c^2*d+181038*A*c*d^2+64090*A*d^3+55770*B*c^3+181038*B*
c^2*d+192270*B*c*d^2+66362*B*d^3)*sin(f*x+e)-3465*B*d^3*cos(f*x+e)^6+(15015
*A*c*d^2+14560*A*d^3+15015*B*c^2*d+43680*B*c*d^2+28700*B*d^3)*cos(f*x+e)^4+
(-9009*A*c^3-77220*A*c^2*d-123981*A*c*d^2-56810*A*d^3-25740*B*c^3-123981*B*
c^2*d-170430*B*c*d^2-72109*B*d^3)*cos(f*x+e)^2+138138*A*c^3+373230*A*c^2*d+
359502*A*c*d^2+116090*A*d^3+124410*B*c^3+359502*B*c^2*d+348270*B*c*d^2+1138
18*B*d^3)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) +
 c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^3,
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^3,
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.301 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=429

$$\frac{2a^3\left(11Ad(3c-19d)-B\left(15c^2-65cd+194d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{693d^3f\sqrt{a\sin(e+fx)+a}}-\frac{2a^3\left(15c^2+10cd+7d^2\right)\left(11Ad\right.}{}$$

```
[Out] -2/1155*a*(11*A*d*(c^2-10*c*d+73*d^2)-B*(5*c^3-40*c^2*d+169*c*d^2-710*d^3))
*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/d/f-2/11*a*B*cos(f*x+e)*(a+a*sin(f*x+e))
^(3/2)*(c+d*sin(f*x+e))^3/d/f-2/3465*a^3*(15*c^2+10*c*d+7*d^2)*(11*A*d*(c^2
-10*c*d+73*d^2)-B*(5*c^3-40*c^2*d+169*c*d^2-710*d^3))*cos(f*x+e)/d^3/f/(a+a
*sin(f*x+e))^(1/2)+2/693*a^3*(11*A*(3*c-19*d)*d-B*(15*c^2-65*c*d+194*d^2))*
cos(f*x+e)*(c+d*sin(f*x+e))^3/d^3/f/(a+a*sin(f*x+e))^(1/2)-4/3465*a^2*(5*c-
d)*(11*A*d*(c^2-10*c*d+73*d^2)-B*(5*c^3-40*c^2*d+169*c*d^2-710*d^3))*cos(f*
x+e)*(a+a*sin(f*x+e))^(1/2)/d^2/f+2/99*a^2*(-11*A*d+5*B*c-14*B*d)*cos(f*x+e
)*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2)/d^2/f
```

Rubi [A] time = 1.07, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {2976, 2981, 2761, 2751, 2646}

$$\frac{2a^3\left(11Ad(3c-19d)-B\left(15c^2-65cd+194d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{693d^3f\sqrt{a\sin(e+fx)+a}}-\frac{2a^3\left(15c^2+10cd+7d^2\right)\left(11Ad\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,
x]
```

```
[Out] (-2*a^3*(15*c^2 + 10*c*d + 7*d^2)*(11*A*d*(c^2 - 10*c*d + 73*d^2) - B*(5*c^
3 - 40*c^2*d + 169*c*d^2 - 710*d^3))*Cos[e + f*x])/(3465*d^3*f*Sqrt[a + a*S
in[e + f*x]]) - (4*a^2*(5*c - d)*(11*A*d*(c^2 - 10*c*d + 73*d^2) - B*(5*c^3
 - 40*c^2*d + 169*c*d^2 - 710*d^3))*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/
(3465*d^2*f) - (2*a*(11*A*d*(c^2 - 10*c*d + 73*d^2) - B*(5*c^3 - 40*c^2*d +
 169*c*d^2 - 710*d^3))*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(1155*d*f)
+ (2*a^3*(11*A*(3*c - 19*d)*d - B*(15*c^2 - 65*c*d + 194*d^2))*Cos[e + f*x]
*(c + d*Sin[e + f*x])^3)/(693*d^3*f*Sqrt[a + a*Sin[e + f*x]]) + (2*a^2*(5*B
*c - 11*A*d - 14*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e +
f*x])^3)/(99*d^2*f) - (2*a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c + d
*Sin[e + f*x])^3)/(11*d*f)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^2\,dx = -\frac{2aB\cos(e + fx)(a + a\sin(e + fx))^{3/2}(c + d}{11df}$$

$$= \frac{2a^2(5Bc - 11Ad - 14Bd)\cos(e + fx)\sqrt{a + a}}{99d^2 f}$$

$$= \frac{2a^3\left(11A(3c - 19d)d - B\left(15c^2 - 65cd + 194\right.\right.}{693d^3 f\sqrt{a + a\sin}}$$

$$= -\frac{2a\left(11Ad\left(c^2 - 10cd + 73d^2\right) - B\left(5c^3 - 40\right.\right.}{}$$

$$= -\frac{4a^2(5c - d)\left(11Ad\left(c^2 - 10cd + 73d^2\right) - B\left(\right.\right.}{}$$

$$= -\frac{2a^3\left(15c^2 + 10cd + 7d^2\right)\left(11Ad\left(c^2 - 10cd\right.\right.}{3465d^3}$$

Mathematica [B] time = 6.61, size = 891, normalized size = 2.08

$$\frac{(a(\sin(e + fx) + 1))^{5/2}\left(-277200A\cos\left(\frac{1}{2}(e + fx)\right)c^2 - 207900B\cos\left(\frac{1}{2}(e + fx)\right)c^2 - 46200A\cos\left(\frac{3}{2}(e + fx)\right)c^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x])^2,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(-277200*A*c^2*Cos[(e + f*x)/2] - 207900*B*c^
2*Cos[(e + f*x)/2] - 415800*A*c*d*Cos[(e + f*x)/2] - 360360*B*c*d*Cos[(e +
f*x)/2] - 180180*A*d^2*Cos[(e + f*x)/2] - 159390*B*d^2*Cos[(e + f*x)/2] - 4
6200*A*c^2*Cos[(3*(e + f*x))/2] - 50820*B*c^2*Cos[(3*(e + f*x))/2] - 101640
*A*c*d*Cos[(3*(e + f*x))/2] - 92400*B*c*d*Cos[(3*(e + f*x))/2] - 46200*A*d^
2*Cos[(3*(e + f*x))/2] - 43890*B*d^2*Cos[(3*(e + f*x))/2] + 5544*A*c^2*Cos[
(5*(e + f*x))/2] + 13860*B*c^2*Cos[(5*(e + f*x))/2] + 27720*A*c*d*Cos[(5*(e
 + f*x))/2] + 33264*B*c*d*Cos[(5*(e + f*x))/2] + 16632*A*d^2*Cos[(5*(e + f*
x))/2] + 17325*B*d^2*Cos[(5*(e + f*x))/2] + 1980*B*c^2*Cos[(7*(e + f*x))/2]
 + 3960*A*c*d*Cos[(7*(e + f*x))/2] + 9900*B*c*d*Cos[(7*(e + f*x))/2] + 4950
*A*d^2*Cos[(7*(e + f*x))/2] + 6435*B*d^2*Cos[(7*(e + f*x))/2] - 1540*B*c*d*
Cos[(9*(e + f*x))/2] - 770*A*d^2*Cos[(9*(e + f*x))/2] - 1925*B*d^2*Cos[(9*(
```

```
e + f*x))/2] - 315*B*d^2*Cos[(11*(e + f*x))/2] + 277200*A*c^2*Sin[(e + f*x)
/2] + 207900*B*c^2*Sin[(e + f*x)/2] + 415800*A*c*d*Sin[(e + f*x)/2] + 36036
0*B*c*d*Sin[(e + f*x)/2] + 180180*A*d^2*Sin[(e + f*x)/2] + 159390*B*d^2*Sin
[(e + f*x)/2] - 46200*A*c^2*Sin[(3*(e + f*x))/2] - 50820*B*c^2*Sin[(3*(e +
f*x))/2] - 101640*A*c*d*Sin[(3*(e + f*x))/2] - 92400*B*c*d*Sin[(3*(e + f*x)
)/2] - 46200*A*d^2*Sin[(3*(e + f*x))/2] - 43890*B*d^2*Sin[(3*(e + f*x))/2]
- 5544*A*c^2*Sin[(5*(e + f*x))/2] - 13860*B*c^2*Sin[(5*(e + f*x))/2] - 2772
0*A*c*d*Sin[(5*(e + f*x))/2] - 33264*B*c*d*Sin[(5*(e + f*x))/2] - 16632*A*d
^2*Sin[(5*(e + f*x))/2] - 17325*B*d^2*Sin[(5*(e + f*x))/2] + 1980*B*c^2*Sin
[(7*(e + f*x))/2] + 3960*A*c*d*Sin[(7*(e + f*x))/2] + 9900*B*c*d*Sin[(7*(e
+ f*x))/2] + 4950*A*d^2*Sin[(7*(e + f*x))/2] + 6435*B*d^2*Sin[(7*(e + f*x))
/2] + 1540*B*c*d*Sin[(9*(e + f*x))/2] + 770*A*d^2*Sin[(9*(e + f*x))/2] + 19
25*B*d^2*Sin[(9*(e + f*x))/2] - 315*B*d^2*Sin[(11*(e + f*x))/2]))/(55440*f*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

fricas [A] time = 0.47, size = 593, normalized size = 1.38

$$-\frac{2\left(315\,Ba^2d^2\cos\left(fx+e\right)^6+35\left(22\,Ba^2cd+(11\,A+32\,B)a^2d^2\right)\cos\left(fx+e\right)^5+1056\,(7\,A+5\,B)a^2c^2+704\,(1\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] -2/3465*(315*B*a^2*d^2*cos(f*x + e)^6 + 35*(22*B*a^2*c*d + (11*A + 32*B)*a^
2*d^2)*cos(f*x + e)^5 + 1056*(7*A + 5*B)*a^2*c^2 + 704*(15*A + 13*B)*a^2*c*
d + 32*(143*A + 125*B)*a^2*d^2 - 5*(99*B*a^2*c^2 + 22*(9*A + 19*B)*a^2*c*d
+ (209*A + 320*B)*a^2*d^2)*cos(f*x + e)^4 - (99*(7*A + 20*B)*a^2*c^2 + 22*(
180*A + 289*B)*a^2*c*d + (3179*A + 4370*B)*a^2*d^2)*cos(f*x + e)^3 + (33*(7
7*A + 85*B)*a^2*c^2 + 22*(255*A + 263*B)*a^2*c*d + (2893*A + 2965*B)*a^2*d^
2)*cos(f*x + e)^2 + 2*(33*(161*A + 145*B)*a^2*c^2 + 22*(435*A + 419*B)*a^2*
c*d + (4609*A + 4465*B)*a^2*d^2)*cos(f*x + e) + (315*B*a^2*d^2*cos(f*x + e)
^5 - 1056*(7*A + 5*B)*a^2*c^2 - 704*(15*A + 13*B)*a^2*c*d - 32*(143*A + 125
*B)*a^2*d^2 - 35*(22*B*a^2*c*d + (11*A + 23*B)*a^2*d^2)*cos(f*x + e)^4 - 5*
(99*B*a^2*c^2 + 22*(9*A + 26*B)*a^2*c*d + 13*(22*A + 37*B)*a^2*d^2)*cos(f*x
 + e)^3 + 3*(33*(7*A + 15*B)*a^2*c^2 + 22*(45*A + 53*B)*a^2*c*d + (583*A +
655*B)*a^2*d^2)*cos(f*x + e)^2 + 2*(33*(49*A + 65*B)*a^2*c^2 + 22*(195*A +
211*B)*a^2*c*d + (2321*A + 2465*B)*a^2*d^2)*cos(f*x + e))*sin(f*x + e))*sqr
t(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
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able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
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to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)sqrt(2*a)*(112*f*(2*A*a^2*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+4*B
*a^2*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+4*B*a^2*c*d*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi)))*sin(1/4*(14*f*x+14*exp(1)+pi))/(112*f)^2+144*f*(2*A*a^2*d^
2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+4*B*a^2*d^2*sign(cos(1/2*(f*x+exp(1))-
1/4*pi))+4*B*a^2*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(18*f*x+18
*exp(1)-pi))/(144*f)^2-160*f*(-16*A*a^2*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*p
i))-8*B*a^2*c^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-18*B*a^2*d^2*sign(cos(1/
2*(f*x+exp(1))-1/4*pi))-16*A*a^2*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-32*
B*a^2*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(10*f*x+10*exp(1)+pi)
)/(160*f)^2-224*f*(-16*A*a^2*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-8*B*a^2
*c^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-18*B*a^2*d^2*sign(cos(1/2*(f*x+exp(
1))-1/4*pi))-16*A*a^2*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-32*B*a^2*c*d*s
ign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(14*f*x+14*exp(1)-pi))/(224*f)^2
```

```
+8*f*(24*A*a^2*c^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+14*A*a^2*d^2*sign(cos
(1/2*(f*x+exp(1))-1/4*pi))+16*B*a^2*c^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+
12*B*a^2*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+32*A*a^2*c*d*sign(cos(1/2*(
f*x+exp(1))-1/4*pi))+28*B*a^2*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1
/4*(2*f*x-pi)+1/2*exp(1))/(8*f)^2-16*f*(32*A*a^2*c^2*sign(cos(1/2*(f*x+exp(
1))-1/4*pi))+24*A*a^2*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+28*B*a^2*c^2*s
ign(cos(1/2*(f*x+exp(1))-1/4*pi))+22*B*a^2*d^2*sign(cos(1/2*(f*x+exp(1))-1/
4*pi))+56*A*a^2*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+48*B*a^2*c*d*sign(co
s(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(2*f*x+2*exp(1)+pi))/(16*f)^2-48*f*(32
*A*a^2*c^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+24*A*a^2*d^2*sign(cos(1/2*(f*
x+exp(1))-1/4*pi))+28*B*a^2*c^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+22*B*a^2
*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+56*A*a^2*c*d*sign(cos(1/2*(f*x+exp(
1))-1/4*pi))+48*B*a^2*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(6*f*
x+6*exp(1)-pi))/(48*f)^2+12*f*(-2*A*a^2*c^2*sign(cos(1/2*(f*x+exp(1))-1/4*p
i))-4*A*a^2*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-4*B*a^2*c^2*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))-4*B*a^2*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-8*A*a
^2*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-8*B*a^2*c*d*sign(cos(1/2*(f*x+exp
(1))-1/4*pi)))*sin(1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2+20*f*(-2*A*a^2*c^2*sig
n(cos(1/2*(f*x+exp(1))-1/4*pi))-4*A*a^2*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*p
i))-4*B*a^2*c^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-4*B*a^2*d^2*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))-8*A*a^2*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-8*B*a
^2*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(10*f*x+10*exp(1)-pi))/(
20*f)^2-576*B*a^2*d^2*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(18*f*x+
18*exp(1)+pi))/(288*f)^2-704*B*a^2*d^2*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))
*cos(1/4*(22*f*x+22*exp(1)-pi))/(352*f)^2)
```

maple [A] time = 1.43, size = 257, normalized size = 0.60

$$\frac{2\left(1+\sin\left(fx+e\right)\right)a^3\left(\sin\left(fx+e\right)-1\right)\left(315B\,d^2\sin\left(fx+e\right)\left(\cos^4\left(fx+e\right)\right)+\left(-990Acd-1430A\,d^2-495B\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/3465*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*(315*B*d^2*sin(f*x+e)*cos(f*x+e)^4
+(-990*A*c*d-1430*A*d^2-495*B*c^2-2860*B*c*d-2405*B*d^2)*cos(f*x+e)^2*sin(f
*x+e)+(3234*A*c^2+8580*A*c*d+4642*A*d^2+4290*B*c^2+9284*B*c*d+4930*B*d^2)*s
in(f*x+e)+(385*A*d^2+770*B*c*d+1120*B*d^2)*cos(f*x+e)^4+(-693*A*c^2-3960*A*
c*d-3179*A*d^2-1980*B*c^2-6358*B*c*d-4370*B*d^2)*cos(f*x+e)^2+10626*A*c^2+1
9140*A*c*d+9218*A*d^2+9570*B*c^2+18436*B*c*d+8930*B*d^2)/cos(f*x+e)/(a+a*si
n(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) +
 c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2,
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2,
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.302 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=212

$$-\frac{64a^3(21Ac+15Ad+15Bc+13Bd)\cos(e+fx)}{315f\sqrt{a\sin(e+fx)+a}}-\frac{16a^2(21Ac+15Ad+15Bc+13Bd)\cos(e+fx)\sqrt{a\sin(e+f}}{315f}$$

```
[Out] -2/105*a*(21*A*c+15*A*d+15*B*c+13*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-
2/63*(9*A*d+9*B*c-2*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f-2/9*B*d*cos(f*
x+e)*(a+a*sin(f*x+e))^(7/2)/a/f-64/315*a^3*(21*A*c+15*A*d+15*B*c+13*B*d)*co
s(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-16/315*a^2*(21*A*c+15*A*d+15*B*c+13*B*d)*
cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.37, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2968, 3023, 2751, 2647, 2646}

$$-\frac{16a^2(21Ac+15Ad+15Bc+13Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{315f}-\frac{64a^3(21Ac+15Ad+15Bc+13Bd)\cos(e}{315f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (-64*a^3*(21*A*c + 15*B*c + 15*A*d + 13*B*d)*Cos[e + f*x])/(315*f*Sqrt[a +
a*Sin[e + f*x]]) - (16*a^2*(21*A*c + 15*B*c + 15*A*d + 13*B*d)*Cos[e + f*x]
*Sqrt[a + a*Sin[e + f*x]])/(315*f) - (2*a*(21*A*c + 15*B*c + 15*A*d + 13*B*
d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(105*f) - (2*(9*B*c + 9*A*d - 2
*B*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(63*f) - (2*B*d*Cos[e + f*x]
*(a + a*Sin[e + f*x])^(7/2))/(9*a*f)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int (a+a\sin(e+fx))^{5/2}\left(Ac+(Bc+Ad)\sin(\right.$$

$$= -\frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{9af} + \frac{2\int}{}$$

$$= -\frac{2(9Bc+9Ad-2Bd)\cos(e+fx)(a+a\sin(e}{63f}$$

$$= -\frac{2a(21Ac+15Bc+15Ad+13Bd)\cos(e+fx}{105f}$$

$$= -\frac{16a^2(21Ac+15Bc+15Ad+13Bd)\cos(e+}{315f}$$

$$= -\frac{64a^3(21Ac+15Bc+15Ad+13Bd)\cos(e+}{315f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 4.24, size = 202, normalized size = 0.95

$$-\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(-4(63Ac+180Ad+180Bc+254Bd)\cos(2(e+f$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x]),x]
```

```
[Out] -1/1260*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x]
)]*(7476*A*c + 6240*B*c + 6240*A*d + 5653*B*d - 4*(63*A*c + 180*B*c + 180*A
*d + 254*B*d)*Cos[2*(e + f*x)] + 35*B*d*Cos[4*(e + f*x)] + 2352*A*c*Sin[e +
 f*x] + 3030*B*c*Sin[e + f*x] + 3030*A*d*Sin[e + f*x] + 3116*B*d*Sin[e + f*
x] - 90*B*c*Sin[3*(e + f*x)] - 90*A*d*Sin[3*(e + f*x)] - 260*B*d*Sin[3*(e +
 f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

fricas [A] time = 0.46, size = 361, normalized size = 1.70

$$-\frac{2\left(35\,Ba^2d\cos\left(fx+e\right)^5-5\left(9\,Ba^2c+(9\,A+19\,B)a^2d\right)\cos\left(fx+e\right)^4+96\,(7\,A+5\,B)a^2c+32\,(15\,A+13\,B)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] -2/315*(35*B*a^2*d*cos(f*x + e)^5 - 5*(9*B*a^2*c + (9*A + 19*B)*a^2*d)*cos(
f*x + e)^4 + 96*(7*A + 5*B)*a^2*c + 32*(15*A + 13*B)*a^2*d - (9*(7*A + 20*B
)*a^2*c + (180*A + 289*B)*a^2*d)*cos(f*x + e)^3 + (3*(77*A + 85*B)*a^2*c +
(255*A + 263*B)*a^2*d)*cos(f*x + e)^2 + 2*(3*(161*A + 145*B)*a^2*c + (435*A
 + 419*B)*a^2*d)*cos(f*x + e) - (35*B*a^2*d*cos(f*x + e)^4 + 96*(7*A + 5*B)
*a^2*c + 32*(15*A + 13*B)*a^2*d + 5*(9*B*a^2*c + (9*A + 26*B)*a^2*d)*cos(f*
x + e)^3 - 3*(3*(7*A + 15*B)*a^2*c + (45*A + 53*B)*a^2*d)*cos(f*x + e)^2 -
2*(3*(49*A + 65*B)*a^2*c + (195*A + 211*B)*a^2*d)*cos(f*x + e))*sin(f*x + e
))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
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to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*(-40*f*(-2*A*a^2*d*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi))-2*B*a^2*c*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-4*B*a^2*d*sign
(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(10*f*x+10*exp(1)+pi))/(40*f)^2-56*
f*(-2*A*a^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-2*B*a^2*c*sign(cos(1/2*(f*
x+exp(1))-1/4*pi))-4*B*a^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(1
4*f*x+14*exp(1)-pi))/(56*f)^2-8*f*(16*A*a^2*c*sign(cos(1/2*(f*x+exp(1))-1/4
*pi))+14*A*a^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+14*B*a^2*c*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))+12*B*a^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1
/4*(2*f*x+2*exp(1)+pi))/(8*f)^2-24*f*(16*A*a^2*c*sign(cos(1/2*(f*x+exp(1))-
1/4*pi))+14*A*a^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+14*B*a^2*c*sign(cos(
1/2*(f*x+exp(1))-1/4*pi))+12*B*a^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*co
s(1/4*(6*f*x+6*exp(1)-pi))/(24*f)^2+8*f*(24*A*a^2*c*sign(cos(1/2*(f*x+exp(1
))-1/4*pi))+16*A*a^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+16*B*a^2*c*sign(c
os(1/2*(f*x+exp(1))-1/4*pi))+14*B*a^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))
*sin(1/4*(2*f*x-pi)+1/2*exp(1))/(8*f)^2+12*f*(-2*A*a^2*c*sign(cos(1/2*(f*x+
exp(1))-1/4*pi))-4*A*a^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-4*B*a^2*c*sig
n(cos(1/2*(f*x+exp(1))-1/4*pi))-4*B*a^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)
))*sin(1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2+20*f*(-2*A*a^2*c*sign(cos(1/2*(f*x
+exp(1))-1/4*pi))-4*A*a^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-4*B*a^2*c*si
gn(cos(1/2*(f*x+exp(1))-1/4*pi))-4*B*a^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi
)))*sin(1/4*(10*f*x+10*exp(1)-pi))/(20*f)^2+224*B*a^2*d*f*sign(cos(1/2*(f*x
+exp(1))-1/4*pi))*sin(1/4*(14*f*x+14*exp(1)+pi))/(112*f)^2+288*B*a^2*d*f*si
gn(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(18*f*x+18*exp(1)-pi))/(144*f)^2)
```

maple [A] time = 1.39, size = 152, normalized size = 0.72

$$\frac{2\left(1+\sin\left(fx+e\right)\right)a^{3}\left(\sin\left(fx+e\right)-1\right)\left(\left(-45Ad-45Bc-130Bd\right)\sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)+\left(294Ac+39\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] 2/315*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*((-45*A*d-45*B*c-130*B*d)*sin(f*x+e
)*cos(f*x+e)^2+(294*A*c+390*A*d+390*B*c+422*B*d)*sin(f*x+e)+35*B*d*cos(f*x+
e)^4+(-63*A*c-180*A*d-180*B*c-289*B*d)*cos(f*x+e)^2+966*A*c+870*A*d+870*B*c
+838*B*d)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) +
 c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x)),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.303 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=138

$$-\frac{64a^3(7A+5B)\cos(e+fx)}{105f\sqrt{a\sin(e+fx)+a}}-\frac{16a^2(7A+5B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{105f}-\frac{2a(7A+5B)\cos(e+fx)(a\sin(e+\dots}{35f}$$

```
[Out] -2/35*a*(7*A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-2/7*B*cos(f*x+e)*(a+a
*sin(f*x+e))^(5/2)/f-64/105*a^3*(7*A+5*B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/
2)-16/105*a^2*(7*A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.11, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2751, 2647, 2646}

$$-\frac{64a^3(7A+5B)\cos(e+fx)}{105f\sqrt{a\sin(e+fx)+a}}-\frac{16a^2(7A+5B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{105f}-\frac{2a(7A+5B)\cos(e+fx)(a\sin(e+\dots}{35f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]),x]
```

```
[Out] (-64*a^3*(7*A + 5*B)*Cos[e + f*x])/(105*f*Sqrt[a + a*Sin[e + f*x]]) - (16*a
^2*(7*A + 5*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(105*f) - (2*a*(7*A +
 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(35*f) - (2*B*Cos[e + f*x]*(
a + a*Sin[e + f*x])^(5/2))/(7*f)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
```

```
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{7f}+\frac{1}{7}(7A+5B)\int (a+a\,\text{s}$$

$$= -\frac{2a(7A+5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35f}-\frac{2B\cos(e+}{}$$

$$= -\frac{16a^2(7A+5B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105f}-\frac{2a(7A+5B}{}$$

$$= -\frac{64a^3(7A+5B)\cos(e+fx)}{105f\sqrt{a+a\sin(e+fx)}}-\frac{16a^2(7A+5B)\cos(e+fx)\sqrt{a}}{105f}$$

Mathematica [A] time = 1.51, size = 119, normalized size = 0.86

$$-\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)((392A+505B)\sin(e+fx)-6(7A+20B)\cos(2(}{210f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]),x]
```

```
[Out] -1/210*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])
]*(1246*A + 1040*B - 6*(7*A + 20*B)*Cos[2*(e + f*x)] + (392*A + 505*B)*Sin[
e + f*x] - 15*B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
)
```

fricas [A] time = 0.44, size = 191, normalized size = 1.38

$$\frac{2\left(15\,Ba^2\cos\left(fx+e\right)^4+3\,(7\,A+20\,B)a^2\cos\left(fx+e\right)^3-(77\,A+85\,B)a^2\cos\left(fx+e\right)^2-2\,(161\,A+145\,B)a^2\,\text{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 2/105*(15*B*a^2*cos(f*x + e)^4 + 3*(7*A + 20*B)*a^2*cos(f*x + e)^3 - (77*A
+ 85*B)*a^2*cos(f*x + e)^2 - 2*(161*A + 145*B)*a^2*cos(f*x + e) - 32*(7*A +
```

```
 5*B)*a^2 + (15*B*a^2*cos(f*x + e)^3 - 3*(7*A + 15*B)*a^2*cos(f*x + e)^2 -
2*(49*A + 65*B)*a^2*cos(f*x + e) + 32*(7*A + 5*B)*a^2)*sin(f*x + e))*sqrt(a
*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sq
rt(2*a)*(2*f*(6*A*a^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+4*B*a^2*sign(cos(1
/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(2*f*x-pi)+1/2*exp(1))/(2*f)^2-8*f*(16*A*
a^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+14*B*a^2*sign(cos(1/2*(f*x+exp(1))-1
/4*pi)))*cos(1/4*(2*f*x+2*exp(1)+pi))/(8*f)^2-24*f*(16*A*a^2*sign(cos(1/2*(
f*x+exp(1))-1/4*pi))+14*B*a^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(
6*f*x+6*exp(1)-pi))/(24*f)^2+12*f*(-2*A*a^2*sign(cos(1/2*(f*x+exp(1))-1/4*p
i))-4*B*a^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(6*f*x+6*exp(1)+pi)
)/(12*f)^2+20*f*(-2*A*a^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-4*B*a^2*sign(c
os(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(10*f*x+10*exp(1)-pi))/(20*f)^2+80*B*
a^2*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(10*f*x+10*exp(1)+pi))/(40
*f)^2+112*B*a^2*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(14*f*x+14*exp
(1)-pi))/(56*f)^2)
```

maple [A] time = 1.24, size = 99, normalized size = 0.72

$$\frac{2\left(1+\sin\left(fx+e\right)\right)a^{3}\left(\sin\left(fx+e\right)-1\right)\left(-15B\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)+\left(98A+130B\right)\sin\left(fx+e\right)+(-21A\right.}{105\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x)
```

```
[Out] 2/105*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*(-15*B*cos(f*x+e)^2*sin(f*x+e)+(98*
A+130*B)*sin(f*x+e)+(-21*A-60*B)*cos(f*x+e)^2+322*A+290*B)/cos(f*x+e)/(a+a*
sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2),x)
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e)),x)
[Out] Integral((a*(sin(e + f*x) + 1))**(5/2)*(A + B*sin(e + f*x)), x)
```

3.304 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=218

$$\frac{2a^{5/2}(c-d)^2(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{7/2}f\sqrt{c+d}}+\frac{2a^3\left(5Ad(3c-7d)-B\left(15c^2-35cd+32d^2\right)\right)\cos(e+fx)}{15d^3f\sqrt{a\sin(e+fx)+a}}+ 2$$

```
[Out] -2/5*a*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/d/f+2*a^(5/2)*(c-d)^2*(-A*d+B*c)
*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/d^(
7/2)/f/(c+d)^(1/2)+2/15*a^3*(5*A*(3*c-7*d)*d-B*(15*c^2-35*c*d+32*d^2))*cos(
f*x+e)/d^3/f/(a+a*sin(f*x+e))^(1/2)+2/15*a^2*(-5*A*d+5*B*c-8*B*d)*cos(f*x+e
)*(a+a*sin(f*x+e))^(1/2)/d^2/f
```

Rubi [A] time = 0.88, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2976, 2981, 2773, 208}

$$\frac{2a^3\left(5Ad(3c-7d)-B\left(15c^2-35cd+32d^2\right)\right)\cos(e+fx)}{15d^3f\sqrt{a\sin(e+fx)+a}}+\frac{2a^2(-5Ad+5Bc-8Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+}}{15d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),
x]
```

```
[Out] (2*a^(5/2)*(c - d)^2*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sq
rt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(7/2)*Sqrt[c + d]*f) + (2*a^3*(5*A
*(3*c - 7*d)*d - B*(15*c^2 - 35*c*d + 32*d^2))*Cos[e + f*x])/(15*d^3*f*Sqrt
[a + a*Sin[e + f*x]]) + (2*a^2*(5*B*c - 5*A*d - 8*B*d)*Cos[e + f*x]*Sqrt[a
+ a*Sin[e + f*x]])/(15*d^2*f) - (2*a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3
/2))/(5*d*f)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = -\frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5df} + \frac{2\int \frac{(a+a\sin(e+fx))^{3/2}\left(\frac{1}{2}a\right.}{}}{}$$

$$= \frac{2a^2(5Bc-5Ad-8Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15d^2f} - \frac{2aB\,\mathrm{c}}{}$$

$$= \frac{2a^3\left(5A(3c-7d)d-B\left(15c^2-35cd+32d^2\right)\right)\cos(e+fx)}{15d^3f\sqrt{a+a\sin(e+fx)}} + \frac{2a}{}$$

$$= \frac{2a^3\left(5A(3c-7d)d-B\left(15c^2-35cd+32d^2\right)\right)\cos(e+fx)}{15d^3f\sqrt{a+a\sin(e+fx)}} + \frac{2a}{}$$

$$= \frac{2a^{5/2}(c-d)^2(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{7/2}\sqrt{c+d}\,f} + \frac{2a^3\left(5A(\right.}{}$$

Mathematica [B] time = 5.82, size = 450, normalized size = 2.06

$$\frac{(a(\sin(e+fx)+1))^{5/2}\left(30\sqrt{d}\left(Ad(5d-2c)+B\left(2c^2-5cd+5d^2\right)\right)\sin\left(\frac{1}{2}(e+fx)\right)-30\sqrt{d}\left(Ad(5d-2c)+B\left(2c\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
f*x]),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(-30*Sqrt[d]*(A*d*(-2*c + 5*d) + B*(2*c^2 - 5
*c*d + 5*d^2))*Cos[(e + f*x)/2] - 5*d^(3/2)*(-2*B*c + 2*A*d + 5*B*d)*Cos[(3
*(e + f*x))/2] + 3*B*d^(5/2)*Cos[(5*(e + f*x))/2] + (15*(c - d)^2*(B*c - A*
d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[-(Sec[(e + f*x)/4]^2*(c + d
 + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt[d]*Sqrt[c + d]*Sin[(e + f*x)
/2]))]))/Sqrt[c + d] - (15*(c - d)^2*(B*c - A*d)*(e + f*x - 2*Log[Sec[(e +
f*x)/4]^2] + 2*Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d]*(-1 + 2
*Tan[(e + f*x)/4] + Tan[(e + f*x)/4]^2)]))/Sqrt[c + d] + 30*Sqrt[d]*(A*d*(-
2*c + 5*d) + B*(2*c^2 - 5*c*d + 5*d^2))*Sin[(e + f*x)/2] - 5*d^(3/2)*(-2*B*
c + 2*A*d + 5*B*d)*Sin[(3*(e + f*x))/2] - 3*B*d^(5/2)*Sin[(5*(e + f*x))/2])
)/(30*d^(7/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

fricas [B] time = 2.27, size = 1314, normalized size = 6.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] [1/30*(15*(B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*a^2*d^
3 + (B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*a^2*d^3)*cos
(f*x + e) + (B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*a^2*
d^3)*sin(f*x + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 -
2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 - 4*(c^2*d + 4*c*d^2 +
 3*d^3 - (c*d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x +
 e) - (c^2*d + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*
cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d
 + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)
*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f
*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(3*B
*a^2*d^2*cos(f*x + e)^3 - 15*B*a^2*c^2 + 5*(3*A + 7*B)*a^2*c*d - (35*A + 32
*B)*a^2*d^2 + (5*B*a^2*c*d - (5*A + 11*B)*a^2*d^2)*cos(f*x + e)^2 - (15*B*a
^2*c^2 - 5*(3*A + 8*B)*a^2*c*d + 2*(20*A + 23*B)*a^2*d^2)*cos(f*x + e) - (3
*B*a^2*d^2*cos(f*x + e)^2 - 15*B*a^2*c^2 + 5*(3*A + 7*B)*a^2*c*d - (35*A +
32*B)*a^2*d^2 - (5*B*a^2*c*d - (5*A + 14*B)*a^2*d^2)*cos(f*x + e))*sin(f*x
+ e))*sqrt(a*sin(f*x + e) + a))/(d^3*f*cos(f*x + e) + d^3*f*sin(f*x + e) +
d^3*f), 1/15*(15*(B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A
*a^2*d^3 + (B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*a^2*d
^3)*cos(f*x + e) + (B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 -
 A*a^2*d^3)*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x +
e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) +
 2*(3*B*a^2*d^2*cos(f*x + e)^3 - 15*B*a^2*c^2 + 5*(3*A + 7*B)*a^2*c*d - (35
*A + 32*B)*a^2*d^2 + (5*B*a^2*c*d - (5*A + 11*B)*a^2*d^2)*cos(f*x + e)^2 -
(15*B*a^2*c^2 - 5*(3*A + 8*B)*a^2*c*d + 2*(20*A + 23*B)*a^2*d^2)*cos(f*x +
e) - (3*B*a^2*d^2*cos(f*x + e)^2 - 15*B*a^2*c^2 + 5*(3*A + 7*B)*a^2*c*d - (
35*A + 32*B)*a^2*d^2 - (5*B*a^2*c*d - (5*A + 14*B)*a^2*d^2)*cos(f*x + e))*s
in(f*x + e))*sqrt(a*sin(f*x + e) + a))/(d^3*f*cos(f*x + e) + d^3*f*sin(f*x
+ e) + d^3*f)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 2.05, size = 543, normalized size = 2.49

$$\frac{2\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\left(-3B\left(a-a\sin\left(fx+e\right)\right)^{\frac{5}{2}}\sqrt{a\left(c+d\right)d}\,d^{2}+5A\left(a-a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/15*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-3*B*(a-a*sin(f*x+e))^(5/2)*
(a*(c+d)*d)^(1/2)*d^2+5*A*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*a*d^2-15
*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c^2*d+30*A*arc
tanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c*d^2-15*A*arctanh((
a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*d^3-5*B*(a-a*sin(f*x+e))^(
3/2)*(a*(c+d)*d)^(1/2)*a*c*d+20*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*
a*d^2+15*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c^3-30
*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c^2*d+15*B*arc
tanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c*d^2+15*A*(a-a*sin(
f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^2*c*d-45*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d
)*d)^(1/2)*a^2*d^2-15*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^2*c^2+45
*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^2*c*d-60*B*(a-a*sin(f*x+e))^(
1/2)*(a*(c+d)*d)^(1/2)*a^2*d^2)/d^3/(a*(c+d)*d)^(1/2)/cos(f*x+e)/(a+a*sin(f
*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) +
 c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}{c+d\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x)),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x)),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.305 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=265

$$\frac{a^{5/2}(c-d)\left(Ad(3c+5d)-B\left(5c^2+5cd-2d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{7/2}f(c+d)^{3/2}}-\frac{a^3\left(3Ad(3c+d)-B\left(15c^2-5cd-1\right.\right.}{3d^3f(c+d)\sqrt{a\sin(e+f}}$$

```
[Out] a^(5/2)*(c-d)*(A*d*(3*c+5*d)-B*(5*c^2+5*c*d-2*d^2))*arctanh(cos(f*x+e)*a^(1
/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/d^(7/2)/(c+d)^(3/2)/f+a*(-A
*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/d/(c+d)/f/(c+d*sin(f*x+e))-1/3*a^
3*(3*A*d*(3*c+d)-B*(15*c^2-5*c*d-14*d^2))*cos(f*x+e)/d^3/(c+d)/f/(a+a*sin(f
*x+e))^(1/2)-1/3*a^2*(-3*A*d+5*B*c+2*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)
/d^2/(c+d)/f
```

Rubi [A] time = 0.94, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {2975, 2976, 2981, 2773, 208}

$$-\frac{a^3\left(3Ad(3c+d)-B\left(15c^2-5cd-14d^2\right)\right)\cos(e+fx)}{3d^3f(c+d)\sqrt{a\sin(e+fx)+a}}+\frac{a^{5/2}(c-d)\left(Ad(3c+5d)-B\left(5c^2+5cd-2d^2\right)\right)\tanh^{-1}\left(\right.}{d^{7/2}f(c+d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^
2,x]
```

```
[Out] (a^(5/2)*(c - d)*(A*d*(3*c + 5*d) - B*(5*c^2 + 5*c*d - 2*d^2))*ArcTanh[(Sqr
t[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(7/2
)*(c + d)^(3/2)*f) - (a^3*(3*A*d*(3*c + d) - B*(15*c^2 - 5*c*d - 14*d^2))*C
os[e + f*x])/(3*d^3*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]) - (a^2*(5*B*c - 3*A
*d + 2*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*d^2*(c + d)*f) + (a*(
B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(d*(c + d)*f*(c + d*Sin
[e + f*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{d(c+d)f(c+d\sin(e+fx))} + \frac{\int \frac{(a+a\sin(e+fx))^{3/}}{}}{}$$

$$= -\frac{a^2(5Bc-3Ad+2Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3d^2(c+d)f} + \frac{a(Bc}{}$$

$$= -\frac{a^3\left(3Ad(3c+d)-B\left(15c^2-5cd-14d^2\right)\right)\cos(e+fx)}{3d^3(c+d)f\sqrt{a+a\sin(e+fx)}} - \frac{a^2(5}{}$$

$$= -\frac{a^3\left(3Ad(3c+d)-B\left(15c^2-5cd-14d^2\right)\right)\cos(e+fx)}{3d^3(c+d)f\sqrt{a+a\sin(e+fx)}} - \frac{a^2(5}{}$$

$$= \frac{a^{5/2}(c-d)\left(Ad(3c+5d)-B\left(5c^2+5cd-2d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{}}{\sqrt{c+d}\sqrt{}}\right.}{d^{7/2}(c+d)^{3/2}f}$$

Mathematica [A] time = 5.97, size = 460, normalized size = 1.74

$$\frac{(a(\sin(e+fx)+1))^{5/2}\left(\frac{3(c-d)\left(B\left(5c^2+5cd-2d^2\right)-Ad(3c+5d)\right)\left(2\log\left(\sqrt{d}\sqrt{c+d}\left(\tan^2\left(\frac{1}{4}(e+fx)\right)+2\tan\left(\frac{1}{4}(e+fx)\right)-1\right)+(c+d)\sec^2\left(\frac{1}{4}(e+fx)\right)\right)-2}{(c+d)^{3/2}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
f*x])^2,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(-12*Sqrt[d]*(-4*B*c + 2*A*d + 5*B*d)*Cos[(e
+ f*x)/2] - 4*B*d^(3/2)*Cos[(3*(e + f*x))/2] - (3*(c - d)*(-(A*d*(3*c + 5*d
)) + B*(5*c^2 + 5*c*d - 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Lo
g[-(Sec[(e + f*x)/4]^2*(c + d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt
[d]*Sqrt[c + d]*Sin[(e + f*x)/2]))]))/(c + d)^(3/2) + (3*(c - d)*(-(A*d*(3*
c + 5*d)) + B*(5*c^2 + 5*c*d - 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2]
 + 2*Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d]*(-1 + 2*Tan[(e +
f*x)/4] + Tan[(e + f*x)/4]^2)]))/(c + d)^(3/2) + 12*Sqrt[d]*(-4*B*c + 2*A*d
 + 5*B*d)*Sin[(e + f*x)/2] - (12*(c - d)^2*Sqrt[d]*(-(B*c) + A*d)*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2]))/((c + d)*(c + d*Sin[e + f*x])) - 4*B*d^(3/2)*
Sin[(3*(e + f*x))/2]))/(12*d^(7/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
5)
```

fricas [B] time = 2.59, size = 2046, normalized size = 7.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] [-1/12*(3*(5*B*a^2*c^4 - (3*A - 5*B)*a^2*c^3*d - (5*A + 7*B)*a^2*c^2*d^2 +
(3*A - 5*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4 - (5*B*a^2*c^3*d - 3*A*a^2*c^2*
d^2 - (2*A + 7*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4)*cos(f*x + e)^2 + (5*B*a^
2*c^4 - 3*A*a^2*c^3*d - (2*A + 7*B)*a^2*c^2*d^2 + (5*A + 2*B)*a^2*c*d^3)*co
s(f*x + e) + (5*B*a^2*c^4 - (3*A - 5*B)*a^2*c^3*d - (5*A + 7*B)*a^2*c^2*d^2
 + (3*A - 5*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4 + (5*B*a^2*c^3*d - 3*A*a^2*c
^2*d^2 - (2*A + 7*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4)*cos(f*x + e))*sin(f*x
 + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*
d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*
d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d
 + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f
*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e)
 + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*
cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e
)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 -
2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(15*B*a^2*c^3 -
(9*A + 20*B)*a^2*c^2*d + 3*(2*A - 3*B)*a^2*c*d^2 + (3*A + 14*B)*a^2*d^3 + 2
*(B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x + e)^3 + 2*(5*B*a^2*c^2*d - (3*A + 2*B)*
a^2*c*d^2 - (3*A + 7*B)*a^2*d^3)*cos(f*x + e)^2 + (15*B*a^2*c^3 - (9*A + 10
*B)*a^2*c^2*d - 15*B*a^2*c*d^2 - (3*A + 2*B)*a^2*d^3)*cos(f*x + e) - (15*B*
a^2*c^3 - (9*A + 20*B)*a^2*c^2*d + 3*(2*A - 3*B)*a^2*c*d^2 + (3*A + 14*B)*a
^2*d^3 + 2*(B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x + e)^2 - 2*(5*B*a^2*c^2*d - 3*
(A + B)*a^2*c*d^2 - (3*A + 8*B)*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a
*sin(f*x + e) + a))/((c*d^4 + d^5)*f*cos(f*x + e)^2 - (c^2*d^3 + c*d^4)*f*c
os(f*x + e) - (c^2*d^3 + 2*c*d^4 + d^5)*f - ((c*d^4 + d^5)*f*cos(f*x + e) +
 (c^2*d^3 + 2*c*d^4 + d^5)*f)*sin(f*x + e)), 1/6*(3*(5*B*a^2*c^4 - (3*A - 5
*B)*a^2*c^3*d - (5*A + 7*B)*a^2*c^2*d^2 + (3*A - 5*B)*a^2*c*d^3 + (5*A + 2*
B)*a^2*d^4 - (5*B*a^2*c^3*d - 3*A*a^2*c^2*d^2 - (2*A + 7*B)*a^2*c*d^3 + (5*
A + 2*B)*a^2*d^4)*cos(f*x + e)^2 + (5*B*a^2*c^4 - 3*A*a^2*c^3*d - (2*A + 7*
B)*a^2*c^2*d^2 + (5*A + 2*B)*a^2*c*d^3)*cos(f*x + e) + (5*B*a^2*c^4 - (3*A
- 5*B)*a^2*c^3*d - (5*A + 7*B)*a^2*c^2*d^2 + (3*A - 5*B)*a^2*c*d^3 + (5*A +
 2*B)*a^2*d^4 + (5*B*a^2*c^3*d - 3*A*a^2*c^2*d^2 - (2*A + 7*B)*a^2*c*d^3 +
(5*A + 2*B)*a^2*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arcta
n(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^
2))/(a*cos(f*x + e))) - 2*(15*B*a^2*c^3 - (9*A + 20*B)*a^2*c^2*d + 3*(2*A -
 3*B)*a^2*c*d^2 + (3*A + 14*B)*a^2*d^3 + 2*(B*a^2*c*d^2 + B*a^2*d^3)*cos(f*
```

```
x + e)^3 + 2*(5*B*a^2*c^2*d - (3*A + 2*B)*a^2*c*d^2 - (3*A + 7*B)*a^2*d^3)*
cos(f*x + e)^2 + (15*B*a^2*c^3 - (9*A + 10*B)*a^2*c^2*d - 15*B*a^2*c*d^2 -
(3*A + 2*B)*a^2*d^3)*cos(f*x + e) - (15*B*a^2*c^3 - (9*A + 20*B)*a^2*c^2*d
+ 3*(2*A - 3*B)*a^2*c*d^2 + (3*A + 14*B)*a^2*d^3 + 2*(B*a^2*c*d^2 + B*a^2*d
^3)*cos(f*x + e)^2 - 2*(5*B*a^2*c^2*d - 3*(A + B)*a^2*c*d^2 - (3*A + 8*B)*a
^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c*d^4 + d^5
)*f*cos(f*x + e)^2 - (c^2*d^3 + c*d^4)*f*cos(f*x + e) - (c^2*d^3 + 2*c*d^4
+ d^5)*f - ((c*d^4 + d^5)*f*cos(f*x + e) + (c^2*d^3 + 2*c*d^4 + d^5)*f)*sin
(f*x + e))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 2.54, size = 932, normalized size = 3.52

$$\frac{a\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\left(\sin\left(fx+e\right)d\left(9A\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,d}{\sqrt{acd+a\,d^{2}}}\right)a^{2}c^{2}d+6A\,\mathrm{arctanh}\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] 1/3*a*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(sin(f*x+e)*d*(9*A*arctanh((
a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d+6*A*arctanh((a-a*sin
(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^2-15*A*arctanh((a-a*sin(f*x+e
))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^3-15*a^2*arctanh((a-a*sin(f*x+e))^(1/
2)*d/(a*c*d+a*d^2)^(1/2))*B*c^3+21*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*
d+a*d^2)^(1/2))*a^2*c*d^2-6*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2
)^(1/2))*a^2*d^3+2*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*c*d+2*B*(a-a*
sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*d^2-6*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)
*d)^(1/2)*a*c*d-6*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^2+12*B*(a-
a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2-6*B*(a-a*sin(f*x+e))^(1/2)*(a*(
c+d)*d)^(1/2)*a*c*d-18*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^2)+9*
A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^3*d+6*A*arcta
nh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d^2-15*A*arctanh((
a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^3+2*B*(a-a*sin(f*x+e))
```

```
^(3/2)*(a*(c+d)*d)^(1/2)*c^2*d+2*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)
*c*d^2-15*a^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*B*c^4+2
1*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d^2-6*B*a
rctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^3-9*A*(a-a*sin
(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2*d-3*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d
)*d)^(1/2)*a*d^3+15*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^3-12*B*(
a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2*d-15*B*(a-a*sin(f*x+e))^(1/2)
*(a*(c+d)*d)^(1/2)*a*c*d^2)/d^3/(c+d)/(c+d*sin(f*x+e))/(a*(c+d)*d)^(1/2)/co
s(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) +
 c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))^
2,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))^
2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.306 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=308

$$-\frac{a^{5/2}\left(Ad\left(3c^2+10cd+19d^2\right)-B\left(15c^3+30c^2d+7cd^2-20d^3\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4d^{7/2}f(c+d)^{5/2}}+\frac{a^3\left(3Ad(c+3d)-\right.}{4d^3f(c}$$

```
[Out] -1/4*a^(5/2)*(A*d*(3*c^2+10*c*d+19*d^2)-B*(15*c^3+30*c^2*d+7*c*d^2-20*d^3))
*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/d^(
7/2)/(c+d)^(5/2)/f+1/2*a*(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/d/(c+
d)/f/(c+d*sin(f*x+e))^2+1/4*a^3*(3*A*d*(c+3*d)-B*(15*c^2+25*c*d+4*d^2))*cos
(f*x+e)/d^3/(c+d)^2/f/(a+a*sin(f*x+e))^(1/2)-1/4*a^2*(A*d*(c+7*d)-B*(5*c^2+
7*c*d-4*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d^2/(c+d)^2/f/(c+d*sin(f*x+
e))
```

Rubi [A] time = 0.97, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2975, 2981, 2773, 208}

$$\frac{a^3\left(3Ad(c+3d)-B\left(15c^2+25cd+4d^2\right)\right)\cos(e+fx)}{4d^3f(c+d)^2\sqrt{a\sin(e+fx)+a}}-\frac{a^2\left(Ad(c+7d)-B\left(5c^2+7cd-4d^2\right)\right)\cos(e+fx)\sqrt{a\sin}}{4d^2f(c+d)^2(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^
3,x]
```

```
[Out] -(a^(5/2)*(A*d*(3*c^2 + 10*c*d + 19*d^2) - B*(15*c^3 + 30*c^2*d + 7*c*d^2 -
 20*d^3))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Si
n[e + f*x]])])/(4*d^(7/2)*(c + d)^(5/2)*f) + (a^3*(3*A*d*(c + 3*d) - B*(15*
c^2 + 25*c*d + 4*d^2))*Cos[e + f*x])/(4*d^3*(c + d)^2*f*Sqrt[a + a*Sin[e +
f*x]]) + (a*(B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*d*(c +
d)*f*(c + d*Sin[e + f*x])^2) - (a^2*(A*d*(c + 7*d) - B*(5*c^2 + 7*c*d - 4*d
^2))*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(4*d^2*(c + d)^2*f*(c + d*Sin[e
 + f*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{\int \frac{(a+a\sin(e+fx))^{3/}}{}}{}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2\left(Ad(c+7d)\right.}{}$$

$$= \frac{a^3\left(3Ad(c+3d)-B\left(15c^2+25cd+4d^2\right)\right)\cos(e+fx)}{4d^3(c+d)^2f\sqrt{a+a\sin(e+fx)}} + \frac{a(Bc-}{}$$

$$= \frac{a^3\left(3Ad(c+3d)-B\left(15c^2+25cd+4d^2\right)\right)\cos(e+fx)}{4d^3(c+d)^2f\sqrt{a+a\sin(e+fx)}} + \frac{a(Bc-}{}$$

$$= -\frac{a^{5/2}\left(Ad\left(3c^2+10cd+19d^2\right)-B\left(15c^3+30c^2d+7cd^2-20d^3\right.\right.}{4d^{7/2}(c+d)^{5/2}f}$$

Mathematica [A] time = 8.24, size = 504, normalized size = 1.64

$$\frac{(a(\sin(e+fx)+1))^{5/2}\left(\frac{\left(Ad\left(3c^2+10cd+19d^2\right)-B\left(15c^3+30c^2d+7cd^2-20d^3\right)\right)\left(2\log\left(\sqrt{d}\sqrt{c+d}\left(\tan^2\left(\frac{1}{4}(e+fx)\right)+2\tan\left(\frac{1}{4}(e+fx)\right)-1\right)+(c+d)\sec^2\right.\right.}{(c+d)^{5/2}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
f*x])^3,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(((-(A*d*(3*c^2 + 10*c*d + 19*d^2)) + B*(15*c
^3 + 30*c^2*d + 7*c*d^2 - 20*d^3))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2
*Log[-(Sec[(e + f*x)/4]^2*(c + d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] - S
qrt[d]*Sqrt[c + d]*Sin[(e + f*x)/2]))]))/(c + d)^(5/2) + ((A*d*(3*c^2 + 10*
c*d + 19*d^2) - B*(15*c^3 + 30*c^2*d + 7*c*d^2 - 20*d^3))*(e + f*x - 2*Log[
Sec[(e + f*x)/4]^2] + 2*Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d
]*(-1 + 2*Tan[(e + f*x)/4] + Tan[(e + f*x)/4]^2)]))/(c + d)^(5/2) - (4*Sqrt
[d]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(15*B*c^4 - 3*A*c^3*d + 20*B*c^3*
d - 8*A*c^2*d^2 - B*c^2*d^2 + 9*A*c*d^3 + 10*B*c*d^3 + 2*A*d^4 + 4*B*d^4 -
4*B*d^2*(c + d)^2*Cos[2*(e + f*x)] + d*(A*d*(-5*c^2 - 6*c*d + 11*d^2) + B*(
25*c^3 + 34*c^2*d + c*d^2 + 4*d^3))*Sin[e + f*x]))/((c + d)^2*(c + d*Sin[e
+ f*x])^2)))/(16*d^(7/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

fricas [B] time = 2.98, size = 3046, normalized size = 9.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] [1/16*((15*B*a^2*c^5 - 3*(A - 20*B)*a^2*c^4*d - 2*(8*A - 41*B)*a^2*c^3*d^2
- 6*(7*A - 4*B)*a^2*c^2*d^3 - 3*(16*A + 11*B)*a^2*c*d^4 - (19*A + 20*B)*a^2
*d^5 - (15*B*a^2*c^3*d^2 - 3*(A - 10*B)*a^2*c^2*d^3 - (10*A - 7*B)*a^2*c*d^
4 - (19*A + 20*B)*a^2*d^5)*cos(f*x + e)^3 - (30*B*a^2*c^4*d - 3*(2*A - 25*B
)*a^2*c^3*d^2 - (23*A - 44*B)*a^2*c^2*d^3 - 3*(16*A + 11*B)*a^2*c*d^4 - (19
*A + 20*B)*a^2*d^5)*cos(f*x + e)^2 + (15*B*a^2*c^5 - 3*(A - 10*B)*a^2*c^4*d
 - 2*(5*A - 11*B)*a^2*c^3*d^2 - 2*(11*A - 5*B)*a^2*c^2*d^3 - (10*A - 7*B)*a
^2*c*d^4 - (19*A + 20*B)*a^2*d^5)*cos(f*x + e) + (15*B*a^2*c^5 - 3*(A - 20*
B)*a^2*c^4*d - 2*(8*A - 41*B)*a^2*c^3*d^2 - 6*(7*A - 4*B)*a^2*c^2*d^3 - 3*(
16*A + 11*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5 - (15*B*a^2*c^3*d^2 - 3*(A -
 10*B)*a^2*c^2*d^3 - (10*A - 7*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5)*cos(f*
x + e)^2 + 2*(15*B*a^2*c^4*d - 3*(A - 10*B)*a^2*c^3*d^2 - (10*A - 7*B)*a^2*
c^2*d^3 - (19*A + 20*B)*a^2*c*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a/(c*d
+ d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*
a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*x
+ e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^3
+ (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a
/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x +
 e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f
*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d -
 d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e)
- c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(15*B*a^2*c^4 - (3*A + 5*B)*a^2*c^3
*d - (3*A + 31*B)*a^2*c^2*d^2 + (15*A + 17*B)*a^2*c*d^3 - (9*A - 4*B)*a^2*d
^4 - 8*(B*a^2*c^2*d^2 + 2*B*a^2*c*d^3 + B*a^2*d^4)*cos(f*x + e)^3 + (25*B*a
^2*c^3*d - (5*A - 26*B)*a^2*c^2*d^2 - 3*(2*A + 5*B)*a^2*c*d^3 + (11*A - 4*B
)*a^2*d^4)*cos(f*x + e)^2 + (15*B*a^2*c^4 - (3*A - 20*B)*a^2*c^3*d - (8*A -
 3*B)*a^2*c^2*d^2 + 9*(A + 2*B)*a^2*c*d^3 + 2*(A + 4*B)*a^2*d^4)*cos(f*x +
e) - (15*B*a^2*c^4 - (3*A + 5*B)*a^2*c^3*d - (3*A + 31*B)*a^2*c^2*d^2 + (15
*A + 17*B)*a^2*c*d^3 - (9*A - 4*B)*a^2*d^4 - 8*(B*a^2*c^2*d^2 + 2*B*a^2*c*d
^3 + B*a^2*d^4)*cos(f*x + e)^2 - (25*B*a^2*c^3*d - (5*A - 34*B)*a^2*c^2*d^2
 - (6*A - B)*a^2*c*d^3 + (11*A + 4*B)*a^2*d^4)*cos(f*x + e))*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a))/((c^2*d^5 + 2*c*d^6 + d^7)*f*cos(f*x + e)^3 + (2*
c^3*d^4 + 5*c^2*d^5 + 4*c*d^6 + d^7)*f*cos(f*x + e)^2 - (c^4*d^3 + 2*c^3*d^
4 + 2*c^2*d^5 + 2*c*d^6 + d^7)*f*cos(f*x + e) - (c^4*d^3 + 4*c^3*d^4 + 6*c^
2*d^5 + 4*c*d^6 + d^7)*f + ((c^2*d^5 + 2*c*d^6 + d^7)*f*cos(f*x + e)^2 - 2*
(c^3*d^4 + 2*c^2*d^5 + c*d^6)*f*cos(f*x + e) - (c^4*d^3 + 4*c^3*d^4 + 6*c^2
```

```
*d^5 + 4*c*d^6 + d^7)*f)*sin(f*x + e)), -1/8*((15*B*a^2*c^5 - 3*(A - 20*B)*
a^2*c^4*d - 2*(8*A - 41*B)*a^2*c^3*d^2 - 6*(7*A - 4*B)*a^2*c^2*d^3 - 3*(16*
A + 11*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5 - (15*B*a^2*c^3*d^2 - 3*(A - 10
*B)*a^2*c^2*d^3 - (10*A - 7*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5)*cos(f*x +
 e)^3 - (30*B*a^2*c^4*d - 3*(2*A - 25*B)*a^2*c^3*d^2 - (23*A - 44*B)*a^2*c^
2*d^3 - 3*(16*A + 11*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5)*cos(f*x + e)^2 +
 (15*B*a^2*c^5 - 3*(A - 10*B)*a^2*c^4*d - 2*(5*A - 11*B)*a^2*c^3*d^2 - 2*(1
1*A - 5*B)*a^2*c^2*d^3 - (10*A - 7*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5)*co
s(f*x + e) + (15*B*a^2*c^5 - 3*(A - 20*B)*a^2*c^4*d - 2*(8*A - 41*B)*a^2*c^
3*d^2 - 6*(7*A - 4*B)*a^2*c^2*d^3 - 3*(16*A + 11*B)*a^2*c*d^4 - (19*A + 20*
B)*a^2*d^5 - (15*B*a^2*c^3*d^2 - 3*(A - 10*B)*a^2*c^2*d^3 - (10*A - 7*B)*a^
2*c*d^4 - (19*A + 20*B)*a^2*d^5)*cos(f*x + e)^2 + 2*(15*B*a^2*c^4*d - 3*(A
- 10*B)*a^2*c^3*d^2 - (10*A - 7*B)*a^2*c^2*d^3 - (19*A + 20*B)*a^2*c*d^4)*c
os(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x +
 e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e)))
- 2*(15*B*a^2*c^4 - (3*A + 5*B)*a^2*c^3*d - (3*A + 31*B)*a^2*c^2*d^2 + (15*
A + 17*B)*a^2*c*d^3 - (9*A - 4*B)*a^2*d^4 - 8*(B*a^2*c^2*d^2 + 2*B*a^2*c*d^
3 + B*a^2*d^4)*cos(f*x + e)^3 + (25*B*a^2*c^3*d - (5*A - 26*B)*a^2*c^2*d^2
- 3*(2*A + 5*B)*a^2*c*d^3 + (11*A - 4*B)*a^2*d^4)*cos(f*x + e)^2 + (15*B*a^
2*c^4 - (3*A - 20*B)*a^2*c^3*d - (8*A - 3*B)*a^2*c^2*d^2 + 9*(A + 2*B)*a^2*
c*d^3 + 2*(A + 4*B)*a^2*d^4)*cos(f*x + e) - (15*B*a^2*c^4 - (3*A + 5*B)*a^2
*c^3*d - (3*A + 31*B)*a^2*c^2*d^2 + (15*A + 17*B)*a^2*c*d^3 - (9*A - 4*B)*a
^2*d^4 - 8*(B*a^2*c^2*d^2 + 2*B*a^2*c*d^3 + B*a^2*d^4)*cos(f*x + e)^2 - (25
*B*a^2*c^3*d - (5*A - 34*B)*a^2*c^2*d^2 - (6*A - B)*a^2*c*d^3 + (11*A + 4*B
)*a^2*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2*d^5
+ 2*c*d^6 + d^7)*f*cos(f*x + e)^3 + (2*c^3*d^4 + 5*c^2*d^5 + 4*c*d^6 + d^7)
*f*cos(f*x + e)^2 - (c^4*d^3 + 2*c^3*d^4 + 2*c^2*d^5 + 2*c*d^6 + d^7)*f*cos
(f*x + e) - (c^4*d^3 + 4*c^3*d^4 + 6*c^2*d^5 + 4*c*d^6 + d^7)*f + ((c^2*d^5
 + 2*c*d^6 + d^7)*f*cos(f*x + e)^2 - 2*(c^3*d^4 + 2*c^2*d^5 + c*d^6)*f*cos(
f*x + e) - (c^4*d^3 + 4*c^3*d^4 + 6*c^2*d^5 + 4*c*d^6 + d^7)*f)*sin(f*x + e
))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 2.97, size = 1587, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -1/4*a*(16*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a*c^3*d
+32*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a*c^2*d^2+16*B
*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a*c*d^3+8*B*(-a*(si
n(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*a*c^2*d^2+16*B*(-a*(sin(f
*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*a*c*d^3-3*B*(-a*(sin(f*x+e)-
1))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2*d^2-11*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+
d)*d)^(1/2)*d^4-4*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*d^4-15*a^2*
arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*B*c^5-7*B*arctanh((-
a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^3*d^2+20*B*arctanh((-a*(
sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^2*d^3-2*B*(-a*(sin(f*x+e)-1
))^(3/2)*(a*(c+d)*d)^(1/2)*c^2*d^2-13*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*
d)^(1/2)*a*c*d^3+3*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))
*sin(f*x+e)^2*a^2*c^2*d^3+10*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)
*d)^(1/2))*sin(f*x+e)^2*a^2*c*d^4-15*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/
(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*c^3*d^2-30*B*arctanh((-a*(sin(f*x+e)-1)
)^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*c^2*d^3-7*B*arctanh((-a*(sin(
f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*c*d^4+6*A*arctanh((-
a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a^2*c^3*d^2+20*A*ar
ctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a^2*c^2*d^3
+38*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a^2
*c*d^4+8*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*a*d^4+2
9*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^3*d-30*B*arctanh((-a*(s
in(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a^2*c^4*d-60*B*arctanh(
(-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a^2*c^3*d^2-14*B*
arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a^2*c^2*d
^3+40*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a
^2*c*d^4-3*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^3*d-13*A*(-a*(
sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2*d^2+3*A*(-a*(sin(f*x+e)-1))^(1
/2)*(a*(c+d)*d)^(1/2)*a*c*d^3+6*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/
2)*c*d^3+3*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^4
*d+10*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^3*d^2+
19*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^2*d^3-9*B
*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*c^3*d+13*A*(-a*(sin(f*x+e)-1))
^(1/2)*(a*(c+d)*d)^(1/2)*a*d^4+15*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(
1/2)*a*c^4+4*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^4+19*A*arcta
nh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*d^5+20*B
*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*d^
5+5*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*c^2*d^2+15*B*(-a*(sin(f*x
+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*c*d^3-30*B*arctanh((-a*(sin(f*x+e)-1))^(1/2
)*d/(a*(c+d)*d)^(1/2))*a^2*c^4*d)*(-a*(sin(f*x+e)-1))^(1/2)*(1+sin(f*x+e))/
(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e))^2/(c+d)^2/d^3/cos(f*x+e)/(a+a*sin(f*x+e)
```

```
)^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))^
3,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))^
3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.307 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=284

$$-\frac{2d\left(7Ad(9c-d)+B\left(24c^2-15cd+31d^2\right)\right)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{105af}-\frac{4\left(7Ad\left(21c^2-12cd+7d^2\right)+B\left(\right.\right.}{105f\sqrt{}}$$

```
[Out] -(A-B)*(c-d)^3*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2
))*2^(1/2)/f/a^(1/2)-4/105*(7*A*d*(21*c^2-12*c*d+7*d^2)+B*(36*c^3-63*c^2*d+
144*c*d^2-37*d^3))*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/35*(7*A*d+6*B*c-B*
d)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e))^(1/2)-2/7*B*cos(f*x+e)*
(c+d*sin(f*x+e))^3/f/(a+a*sin(f*x+e))^(1/2)-2/105*d*(7*A*(9*c-d)*d+B*(24*c^
2-15*c*d+31*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/a/f
```

Rubi [A] time = 1.00, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2983, 2968, 3023, 2751, 2649, 206}

$$-\frac{2d\left(7Ad(9c-d)+B\left(24c^2-15cd+31d^2\right)\right)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{105af}-\frac{4\left(7Ad\left(21c^2-12cd+7d^2\right)+B\left(\right.\right.}{105f\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/Sqrt[a + a*Sin[e + f*x]],
x]
```

```
[Out] -((Sqrt[2]*(A - B)*(c - d)^3*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
 + a*Sin[e + f*x]])])/(Sqrt[a]*f)) - (4*(7*A*d*(21*c^2 - 12*c*d + 7*d^2) +
B*(36*c^3 - 63*c^2*d + 144*c*d^2 - 37*d^3))*Cos[e + f*x])/(105*f*Sqrt[a + a
*Sin[e + f*x]]) - (2*d*(7*A*(9*c - d)*d + B*(24*c^2 - 15*c*d + 31*d^2))*Cos
[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(105*a*f) - (2*(6*B*c + 7*A*d - B*d)*Co
s[e + f*x]*(c + d*Sin[e + f*x])^2)/(35*f*Sqrt[a + a*Sin[e + f*x]]) - (2*B*C
os[e + f*x]*(c + d*Sin[e + f*x])^3)/(7*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{(c+d\sin(e+fx))^2\left(\frac{1}{2}a(7Ac\cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{2(6Bc+7Ad-Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{35f\sqrt{a+a\sin(e+fx)}} - \frac{2B\cos(\cdots}{7\cdots}$$

$$= -\frac{2(6Bc+7Ad-Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{35f\sqrt{a+a\sin(e+fx)}} - \frac{2B\cos(\cdots}{7\cdots}$$

$$= -\frac{2d\left(7A(9c-d)d+B\left(24c^2-15cd+31d^2\right)\right)\cos(e+fx)\sqrt{a+\cdots}}{105af}$$

$$= -\frac{4\left(7Ad\left(21c^2-12cd+7d^2\right)+B\left(36c^3-63c^2d+144cd^2-37\cdots\right.\right.}{105f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{4\left(7Ad\left(21c^2-12cd+7d^2\right)+B\left(36c^3-63c^2d+144cd^2-37\cdots\right.\right.}{105f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\sqrt{2}(A-B)(c-d)^3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} - \frac{4\left(7Ad\left(21c^2\cdots\right.\right.}{\cdots}$$

Mathematica [C] time = 0.86, size = 375, normalized size = 1.32

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-35d\left(2Ad(6c-d)+B\left(12c^2-6cd+5d^2\right)\right)\sin\left(\frac{3}{2}(e+fx)\right)-35d\left(2Ad(6c\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/Sqrt[a + a*Sin[e +
f*x]],x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((840 + 840*I)*(-1)^(3/4)*(A - B)*(c
 - d)^3*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - 105*(4*A*
d*(6*c^2 - 3*c*d + 2*d^2) + B*(8*c^3 - 12*c^2*d + 24*c*d^2 - 5*d^3))*Cos[(e
 + f*x)/2] - 35*d*(2*A*(6*c - d)*d + B*(12*c^2 - 6*c*d + 5*d^2))*Cos[(3*(e
+ f*x))/2] + 21*d^2*(6*B*c + 2*A*d - B*d)*Cos[(5*(e + f*x))/2] + 15*B*d^3*C
```

```
os[(7*(e + f*x))/2] + 105*(4*A*d*(6*c^2 - 3*c*d + 2*d^2) + B*(8*c^3 - 12*c^
2*d + 24*c*d^2 - 5*d^3))*Sin[(e + f*x)/2] - 35*d*(2*A*(6*c - d)*d + B*(12*c
^2 - 6*c*d + 5*d^2))*Sin[(3*(e + f*x))/2] + 21*d^2*(-2*A*d + B*(-6*c + d))*
Sin[(5*(e + f*x))/2] + 15*B*d^3*Sin[(7*(e + f*x))/2]))/(420*f*Sqrt[a*(1 + S
in[e + f*x])])
```

fricas [B] time = 0.49, size = 629, normalized size = 2.21

$$\frac{105 \sqrt{2} \left((A-B)ac^3-3\,(A-B)ac^2d+3\,(A-B)acd^2-(A-B)ad^3+\left((A-B)ac^3-3\,(A-B)ac^2d+3\,(A-B)acd^2-(A-B)ad^3\right)\cos\left(fx+e\right)+\left((A-B)ac^3-3\,(A-B)ac^2d+3 \right.\right.}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/210*(105*sqrt(2)*((A - B)*a*c^3 - 3*(A - B)*a*c^2*d + 3*(A - B)*a*c*d^2 -
 (A - B)*a*d^3 + ((A - B)*a*c^3 - 3*(A - B)*a*c^2*d + 3*(A - B)*a*c*d^2 - (
A - B)*a*d^3)*cos(f*x + e) + ((A - B)*a*c^3 - 3*(A - B)*a*c^2*d + 3*(A - B)
*a*c*d^2 - (A - B)*a*d^3)*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos(f*x + e
) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - si
n(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x +
e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) + 4*(15*B*d^3*cos(f*x + e
)^4 - 105*B*c^3 - 105*(3*A - 2*B)*c^2*d + 21*(10*A - 17*B)*c*d^2 - (119*A -
 92*B)*d^3 + 3*(21*B*c*d^2 + (7*A - B)*d^3)*cos(f*x + e)^3 - (105*B*c^2*d +
 21*(5*A - 4*B)*c*d^2 - 4*(7*A - 16*B)*d^3)*cos(f*x + e)^2 - (105*B*c^3 + 1
05*(3*A - B)*c^2*d - 21*(5*A - 16*B)*c*d^2 + 2*(56*A - 23*B)*d^3)*cos(f*x +
 e) + (15*B*d^3*cos(f*x + e)^3 + 105*B*c^3 + 105*(3*A - 2*B)*c^2*d - 21*(10
*A - 17*B)*c*d^2 + (119*A - 92*B)*d^3 - 3*(21*B*c*d^2 + (7*A - 6*B)*d^3)*co
s(f*x + e)^2 - (105*B*c^2*d + 21*(5*A - B)*c*d^2 - (7*A - 46*B)*d^3)*cos(f*
x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a*f*cos(f*x + e) + a*f*sin
(f*x + e) + a*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f
*(2/sqrt(a*tan((f*x+exp(1))/2)^2+a)/(a*tan((f*x+exp(1))/2)^2+a)^3*(tan((f*x
+exp(1))/2)*(tan((f*x+exp(1))/2)*(tan((f*x+exp(1))/2)*(tan((f*x+exp(1))/2)*
(tan((f*x+exp(1))/2)*(tan((f*x+exp(1))/2)*(-1/2822400*tan((f*x+exp(1))/2)*(
-1223040*A*a^3*d^3*sign(tan((f*x+exp(1))/2)+1)-1411200*B*a^3*c^3*sign(tan((
f*x+exp(1))/2)+1)+577920*B*a^3*d^3*sign(tan((f*x+exp(1))/2)+1)+1411200*A*a^
3*c*d^2*sign(tan((f*x+exp(1))/2)+1)-4233600*A*a^3*c^2*d*sign(tan((f*x+exp(1
))/2)+1)-3669120*B*a^3*c*d^2*sign(tan((f*x+exp(1))/2)+1)+1411200*B*a^3*c^2*
d*sign(tan((f*x+exp(1))/2)+1))-1/2822400*(1411200*A*a^3*d^3*sign(tan((f*x+e
xp(1))/2)+1)+1411200*B*a^3*c^3*sign(tan((f*x+exp(1))/2)+1)-1411200*B*a^3*d^
3*sign(tan((f*x+exp(1))/2)+1)-4233600*A*a^3*c*d^2*sign(tan((f*x+exp(1))/2)+
```

```
1)+4233600*A*a^3*c^2*d*sign(tan((f*x+exp(1))/2)+1)+4233600*B*a^3*c*d^2*sign
(tan((f*x+exp(1))/2)+1)-4233600*B*a^3*c^2*d*sign(tan((f*x+exp(1))/2)+1)))-1
/2822400*(-4986240*A*a^3*d^3*sign(tan((f*x+exp(1))/2)+1)-4233600*B*a^3*c^3*
sign(tan((f*x+exp(1))/2)+1)+2728320*B*a^3*d^3*sign(tan((f*x+exp(1))/2)+1)+7
056000*A*a^3*c*d^2*sign(tan((f*x+exp(1))/2)+1)-12700800*A*a^3*c^2*d*sign(ta
n((f*x+exp(1))/2)+1)-14958720*B*a^3*c*d^2*sign(tan((f*x+exp(1))/2)+1)+70560
00*B*a^3*c^2*d*sign(tan((f*x+exp(1))/2)+1)))-1/2822400*(5174400*A*a^3*d^3*s
ign(tan((f*x+exp(1))/2)+1)+4233600*B*a^3*c^3*sign(tan((f*x+exp(1))/2)+1)-51
74400*B*a^3*d^3*sign(tan((f*x+exp(1))/2)+1)-9878400*A*a^3*c*d^2*sign(tan((f
*x+exp(1))/2)+1)+12700800*A*a^3*c^2*d*sign(tan((f*x+exp(1))/2)+1)+15523200*
B*a^3*c*d^2*sign(tan((f*x+exp(1))/2)+1)-9878400*B*a^3*c^2*d*sign(tan((f*x+e
xp(1))/2)+1)))-1/2822400*(-5174400*A*a^3*d^3*sign(tan((f*x+exp(1))/2)+1)-42
33600*B*a^3*c^3*sign(tan((f*x+exp(1))/2)+1)+5174400*B*a^3*d^3*sign(tan((f*x
+exp(1))/2)+1)+9878400*A*a^3*c*d^2*sign(tan((f*x+exp(1))/2)+1)-12700800*A*a
^3*c^2*d*sign(tan((f*x+exp(1))/2)+1)-15523200*B*a^3*c*d^2*sign(tan((f*x+exp
(1))/2)+1)+9878400*B*a^3*c^2*d*sign(tan((f*x+exp(1))/2)+1)))-1/2822400*(498
6240*A*a^3*d^3*sign(tan((f*x+exp(1))/2)+1)+4233600*B*a^3*c^3*sign(tan((f*x+
exp(1))/2)+1)-2728320*B*a^3*d^3*sign(tan((f*x+exp(1))/2)+1)-7056000*A*a^3*c
*d^2*sign(tan((f*x+exp(1))/2)+1)+12700800*A*a^3*c^2*d*sign(tan((f*x+exp(1))
/2)+1)+14958720*B*a^3*c*d^2*sign(tan((f*x+exp(1))/2)+1)-7056000*B*a^3*c^2*d
*sign(tan((f*x+exp(1))/2)+1)))-1/2822400*(-1411200*A*a^3*d^3*sign(tan((f*x+
exp(1))/2)+1)-1411200*B*a^3*c^3*sign(tan((f*x+exp(1))/2)+1)+1411200*B*a^3*d
^3*sign(tan((f*x+exp(1))/2)+1)+4233600*A*a^3*c*d^2*sign(tan((f*x+exp(1))/2)
+1)-4233600*A*a^3*c^2*d*sign(tan((f*x+exp(1))/2)+1)-4233600*B*a^3*c*d^2*sig
n(tan((f*x+exp(1))/2)+1)+4233600*B*a^3*c^2*d*sign(tan((f*x+exp(1))/2)+1)))-
1/2822400*(1223040*A*a^3*d^3*sign(tan((f*x+exp(1))/2)+1)+1411200*B*a^3*c^3*
sign(tan((f*x+exp(1))/2)+1)-577920*B*a^3*d^3*sign(tan((f*x+exp(1))/2)+1)-14
11200*A*a^3*c*d^2*sign(tan((f*x+exp(1))/2)+1)+4233600*A*a^3*c^2*d*sign(tan(
(f*x+exp(1))/2)+1)+3669120*B*a^3*c*d^2*sign(tan((f*x+exp(1))/2)+1)-1411200*
B*a^3*c^2*d*sign(tan((f*x+exp(1))/2)+1)))+sqrt(2)*(A*c^3-A*d^3-B*c^3+B*d^3+
3*A*c*d^2-3*A*c^2*d-3*B*c*d^2+3*B*c^2*d)*atan((-sqrt(a)*tan((f*x+exp(1))/2)
-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(-a)/sign(t
an((f*x+exp(1))/2)+1)+(-105*A*a*c^3*sqrt(2)*atan(sqrt(a)/sqrt(-a))+315*A*a*
c^2*d*sqrt(2)*atan(sqrt(a)/sqrt(-a))-315*A*a*c*d^2*sqrt(2)*atan(sqrt(a)/sqr
t(-a))+105*A*a*d^3*sqrt(2)*atan(sqrt(a)/sqrt(-a))+315*A*c^2*d*sqrt(-a)*sqrt
(2)*sqrt(a)-210*A*c*d^2*sqrt(-a)*sqrt(2)*sqrt(a)+119*A*d^3*sqrt(-a)*sqrt(2)
*sqrt(a)+105*B*a*c^3*sqrt(2)*atan(sqrt(a)/sqrt(-a))-315*B*a*c^2*d*sqrt(2)*a
tan(sqrt(a)/sqrt(-a))+315*B*a*c*d^2*sqrt(2)*atan(sqrt(a)/sqrt(-a))-105*B*a*
d^3*sqrt(2)*atan(sqrt(a)/sqrt(-a))+105*B*c^3*sqrt(-a)*sqrt(2)*sqrt(a)-210*B
*c^2*d*sqrt(-a)*sqrt(2)*sqrt(a)+357*B*c*d^2*sqrt(-a)*sqrt(2)*sqrt(a)-92*B*d
^3*sqrt(-a)*sqrt(2)*sqrt(a))/105/a/sqrt(-a)*sign(tan((f*x+exp(1))/2)+1))
```

maple [B] time = 1.95, size = 610, normalized size = 2.15

$$-\frac{\left(1+\sin \left(f x+e\right)\right) \sqrt{-a \left(\sin \left(f x+e\right)-1\right)}\, \left(105 A\, a^{\frac{7}{2}} \sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{a-a \sin \left(f x+e\right)}\, \sqrt{2}}{2 \sqrt{a}}\right) c^{3}-315 A\, a^{\frac{7}{2}} \sqrt{2}\, \operatorname{arctanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/105*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(105*A*a^(7/2)*2^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^3-315*A*a^(7/2)*2^(1/2)*a
rctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2*d+315*A*a^(7/2)*2^(1
/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*d^2-105*A*a^(7/2)
*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d^3-105*B*a^(7
/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^3+315*B*a
^(7/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2*d-31
5*B*a^(7/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*d
^2+105*B*a^(7/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2)
)*d^3-30*B*d^3*(a-a*sin(f*x+e))^(7/2)+42*A*(a-a*sin(f*x+e))^(5/2)*a*d^3+126
*B*(a-a*sin(f*x+e))^(5/2)*a*c*d^2+84*B*(a-a*sin(f*x+e))^(5/2)*a*d^3-210*A*(
a-a*sin(f*x+e))^(3/2)*a^2*c*d^2-70*A*(a-a*sin(f*x+e))^(3/2)*a^2*d^3-210*B*(
a-a*sin(f*x+e))^(3/2)*a^2*c^2*d-210*B*(a-a*sin(f*x+e))^(3/2)*a^2*c*d^2-140*
B*(a-a*sin(f*x+e))^(3/2)*a^2*d^3+630*A*c^2*d*a^3*(a-a*sin(f*x+e))^(1/2)+210
*A*a^3*d^3*(a-a*sin(f*x+e))^(1/2)+210*B*c^3*a^3*(a-a*sin(f*x+e))^(1/2)+630*
B*a^3*c*d^2*(a-a*sin(f*x+e))^(1/2))/a^4/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B \sin \left(f x+e\right)+A\right) \left(d \sin \left(f x+e\right)+c\right)^{3}}{\sqrt{a \sin \left(f x+e\right)+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^3/sqrt(a*sin(f*x + e) +
 a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin \left(e+f\,x\right)\right)\,{\left(c+d\,\sin \left(e+f\,x\right)\right)}^3}{\sqrt{a+a\,\sin \left(e+f\,x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(1/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(1/2
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A + B\sin\left(e + fx\right)\right)\left(c + d\sin\left(e + fx\right)\right)^{3}}{\sqrt{a\left(\sin\left(e + fx\right) + 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**3/sqrt(a*(sin(e + f*x)
+ 1)), x)
```

3.308 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=200

$$-\frac{4\left(5Ad(3c-d)+B\left(6c^2-7cd+7d^2\right)\right)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2d(5Ad+4Bc-Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15af}-\frac{\sqrt{2}}{}$$

```
[Out] -(A-B)*(c-d)^2*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2
))*2^(1/2)/f/a^(1/2)-4/15*(5*A*(3*c-d)*d+B*(6*c^2-7*c*d+7*d^2))*cos(f*x+e)/
f/(a+a*sin(f*x+e))^(1/2)-2/5*B*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x
+e))^(1/2)-2/15*d*(5*A*d+4*B*c-B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/a/f
```

Rubi [A] time = 0.58, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2983, 2968, 3023, 2751, 2649, 206}

$$-\frac{4\left(5Ad(3c-d)+B\left(6c^2-7cd+7d^2\right)\right)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2d(5Ad+4Bc-Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15af}-\frac{\sqrt{2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/Sqrt[a + a*Sin[e + f*x]],
x]
```

```
[Out] -((Sqrt[2]*(A - B)*(c - d)^2*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
 + a*Sin[e + f*x]])])/(Sqrt[a]*f)) - (4*(5*A*(3*c - d)*d + B*(6*c^2 - 7*c*d
 + 7*d^2))*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) - (2*d*(4*B*c + 5*
A*d - B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(15*a*f) - (2*B*Cos[e + f
*x]*(c + d*Sin[e + f*x])^2)/(5*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{(c+d\sin(e+fx))\left(\frac{1}{2}a(5Ac-\right.}{\sqrt{}}}{}$$

$$= -\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{\frac{1}{2}ac(5Ac-Bc+4Bd)+\left(\frac{1}{2}ac(\right.}{}}{}$$

$$= -\frac{2d(4Bc+5Ad-Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15af} - \frac{2B\cos}{}$$

$$= -\frac{4\left(5A(3c-d)d+B\left(6c^2-7cd+7d^2\right)\right)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{2d(4Bc}{}$$

$$= -\frac{4\left(5A(3c-d)d+B\left(6c^2-7cd+7d^2\right)\right)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{2d(4Bc}{}$$

$$= -\frac{\sqrt{2}\,(A-B)(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f} - \frac{4\left(5A(3c-d)\right.}{}$$

Mathematica [C] time = 0.54, size = 246, normalized size = 1.23

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(30\left(Ad(4c-d)+2B\left(c^2-cd+d^2\right)\right)\sin\left(\frac{1}{2}(e+fx)\right)-30\left(Ad(4c-d)+2B\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/Sqrt[a + a*Sin[e +
f*x]],x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((60 + 60*I)*(-1)^(3/4)*(A - B)*(c -
 d)^2*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - 30*(A*(4*c
- d)*d + 2*B*(c^2 - c*d + d^2))*Cos[(e + f*x)/2] + 5*d*(-2*A*d + B*(-4*c +
d))*Cos[(3*(e + f*x))/2] + 3*B*d^2*Cos[(5*(e + f*x))/2] + 30*(A*(4*c - d)*d
 + 2*B*(c^2 - c*d + d^2))*Sin[(e + f*x)/2] + 5*d*(-2*A*d + B*(-4*c + d))*Si
n[(3*(e + f*x))/2] - 3*B*d^2*Sin[(5*(e + f*x))/2]))/(30*f*Sqrt[a*(1 + Sin[e
 + f*x])])
```

fricas [B] time = 0.48, size = 448, normalized size = 2.24

$$-\frac{15\sqrt{2}\left((A-B)ac^2-2\,(A-B)acd+(A-B)ad^2+\left((A-B)ac^2-2\,(A-B)acd+(A-B)ad^2\right)\cos\left(fx+e\right)+\left((A-B)ac^2-2\,(A-B)acd+(A-B)ad^2\right)\sin\left(fx+e\right)\right)\log\left(-\frac{\mathrm{co}}{}\right.}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/30*(15*sqrt(2)*((A - B)*a*c^2 - 2*(A - B)*a*c*d + (A - B)*a*d^2 + ((A -
B)*a*c^2 - 2*(A - B)*a*c*d + (A - B)*a*d^2)*cos(f*x + e) + ((A - B)*a*c^2 -
 2*(A - B)*a*c*d + (A - B)*a*d^2)*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos
(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x +
 e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (co
s(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) - 4*(3*B*d^2*cos(
f*x + e)^3 - 15*B*c^2 - 10*(3*A - 2*B)*c*d + (10*A - 17*B)*d^2 - (10*B*c*d
+ (5*A - 4*B)*d^2)*cos(f*x + e)^2 - (15*B*c^2 + 10*(3*A - B)*c*d - (5*A - 1
6*B)*d^2)*cos(f*x + e) - (3*B*d^2*cos(f*x + e)^2 - 15*B*c^2 - 10*(3*A - 2*B
)*c*d + (10*A - 17*B)*d^2 + (10*B*c*d + (5*A - B)*d^2)*cos(f*x + e))*sin(f*
x + e))*sqrt(a*sin(f*x + e) + a))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*
f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
```

```
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2/sqrt(a*tan((f*x
+exp(1))/2)^2+a)/(a*tan((f*x+exp(1))/2)^2+a)^2*(tan((f*x+exp(1))/2)*(tan((f
*x+exp(1))/2)*(tan((f*x+exp(1))/2)*(tan((f*x+exp(1))/2)*(1/3600*tan((f*x+ex
p(1))/2)*(-600*A*a^2*d^2*sign(tan((f*x+exp(1))/2)+1)+1800*B*a^2*c^2*sign(ta
n((f*x+exp(1))/2)+1)+1560*B*a^2*d^2*sign(tan((f*x+exp(1))/2)+1)+3600*A*a^2*
c*d*sign(tan((f*x+exp(1))/2)+1)-1200*B*a^2*c*d*sign(tan((f*x+exp(1))/2)+1))
+1/3600*(1800*A*a^2*d^2*sign(tan((f*x+exp(1))/2)+1)-1800*B*a^2*c^2*sign(tan
((f*x+exp(1))/2)+1)-1800*B*a^2*d^2*sign(tan((f*x+exp(1))/2)+1)-3600*A*a^2*c
*d*sign(tan((f*x+exp(1))/2)+1)+3600*B*a^2*c*d*sign(tan((f*x+exp(1))/2)+1)))
+1/3600*(-2400*A*a^2*d^2*sign(tan((f*x+exp(1))/2)+1)+3600*B*a^2*c^2*sign(ta
n((f*x+exp(1))/2)+1)+4800*B*a^2*d^2*sign(tan((f*x+exp(1))/2)+1)+7200*A*a^2*
c*d*sign(tan((f*x+exp(1))/2)+1)-4800*B*a^2*c*d*sign(tan((f*x+exp(1))/2)+1))
)+1/3600*(2400*A*a^2*d^2*sign(tan((f*x+exp(1))/2)+1)-3600*B*a^2*c^2*sign(ta
n((f*x+exp(1))/2)+1)-4800*B*a^2*d^2*sign(tan((f*x+exp(1))/2)+1)-7200*A*a^2*
c*d*sign(tan((f*x+exp(1))/2)+1)+4800*B*a^2*c*d*sign(tan((f*x+exp(1))/2)+1))
)+1/3600*(-1800*A*a^2*d^2*sign(tan((f*x+exp(1))/2)+1)+1800*B*a^2*c^2*sign(t
an((f*x+exp(1))/2)+1)+1800*B*a^2*d^2*sign(tan((f*x+exp(1))/2)+1)+3600*A*a^2
*c*d*sign(tan((f*x+exp(1))/2)+1)-3600*B*a^2*c*d*sign(tan((f*x+exp(1))/2)+1)
))+1/3600*(600*A*a^2*d^2*sign(tan((f*x+exp(1))/2)+1)-1800*B*a^2*c^2*sign(ta
n((f*x+exp(1))/2)+1)-1560*B*a^2*d^2*sign(tan((f*x+exp(1))/2)+1)-3600*A*a^2*
c*d*sign(tan((f*x+exp(1))/2)+1)+1200*B*a^2*c*d*sign(tan((f*x+exp(1))/2)+1))
)+sqrt(2)*(A*c^2+A*d^2-B*c^2-B*d^2-2*A*c*d+2*B*c*d)*atan((-sqrt(a)*tan((f*x
+exp(1))/2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt
(-a)/sign(tan((f*x+exp(1))/2)+1)+(-15*A*a*c^2*sqrt(2)*atan(sqrt(a)/sqrt(-a)
)+30*A*a*c*d*sqrt(2)*atan(sqrt(a)/sqrt(-a))-15*A*a*d^2*sqrt(2)*atan(sqrt(a)
/sqrt(-a))+30*A*c*d*sqrt(-a)*sqrt(2)*sqrt(a)-10*A*d^2*sqrt(-a)*sqrt(2)*sqrt
(a)+15*B*a*c^2*sqrt(2)*atan(sqrt(a)/sqrt(-a))-30*B*a*c*d*sqrt(2)*atan(sqrt(
a)/sqrt(-a))+15*B*a*d^2*sqrt(2)*atan(sqrt(a)/sqrt(-a))+15*B*c^2*sqrt(-a)*sq
rt(2)*sqrt(a)-20*B*c*d*sqrt(-a)*sqrt(2)*sqrt(a)+17*B*d^2*sqrt(-a)*sqrt(2)*s
qrt(a))/15/a/sqrt(-a)*sign(tan((f*x+exp(1))/2)+1))
```

maple [B] time = 1.76, size = 396, normalized size = 1.98

$$-\frac{\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\left(15A\,a^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{a}}\right)c^{2}-30A\,a^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{}}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/15*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(15*A*a^(5/2)*2^(1/2)*arctan
h(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2-30*A*a^(5/2)*2^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*d+15*A*a^(5/2)*2^(1/2)*ar
ctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d^2-15*B*a^(5/2)*2^(1/2)*
arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2+30*B*a^(5/2)*2^(1/2
)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*d-15*B*a^(5/2)*2^(1
/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d^2+6*B*(a-a*sin(f*
x+e))^(5/2)*d^2-10*A*(a-a*sin(f*x+e))^(3/2)*a*d^2-20*B*(a-a*sin(f*x+e))^(3/
2)*a*c*d-10*B*(a-a*sin(f*x+e))^(3/2)*a*d^2+60*A*a^2*c*d*(a-a*sin(f*x+e))^(1
/2)+30*B*a^2*c^2*(a-a*sin(f*x+e))^(1/2)+30*B*a^2*d^2*(a-a*sin(f*x+e))^(1/2)
)/a^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{2}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2/sqrt(a*sin(f*x + e) +
 a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(c+d\,\sin\left(e+f\,x\right)\right)^2}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(1/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(1/2
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A + B\sin\left(e + fx\right)\right)\left(c + d\sin\left(e + fx\right)\right)^2}{\sqrt{a\left(\sin\left(e + fx\right) + 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**2/sqrt(a*(sin(e + f*x)
+ 1)), x)
```

3.309 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=130

$$-\frac{2(3Ad+3Bc-2Bd)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{\sqrt{2}\,(A-B)(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f}-\frac{2Bd\cos(e+fx)\sqrt{a\sin(e+fx)}}{3af}$$

```
[Out] -(A-B)*(c-d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))
*2^(1/2)/f/a^(1/2)-2/3*(3*A*d+3*B*c-2*B*d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1
/2)-2/3*B*d*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/a/f
```

Rubi [A] time = 0.27, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2968, 3023, 2751, 2649, 206}

$$-\frac{2(3Ad+3Bc-2Bd)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{\sqrt{2}\,(A-B)(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f}-\frac{2Bd\cos(e+fx)\sqrt{a\sin(e+fx)}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B)*(c - d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a +
 a*Sin[e + f*x]])])/(Sqrt[a]*f)) - (2*(3*B*c + 3*A*d - 2*B*d)*Cos[e + f*x])
/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*B*d*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*
x]])/(3*a*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{Ac+(Bc+Ad)\sin(e+fx)+Bd\sin^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= -\frac{2Bd\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3af} + \frac{2\int \frac{\frac{1}{2}a(3Ac+Bd)+\frac{1}{2}a(3Bc+3A}{\sqrt{a+a\sin(e+}}}{3a}$$

$$= -\frac{2(3Bc+3Ad-2Bd)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2Bd\cos(e+fx)\sqrt{a+a\sin}}{3af}$$

$$= -\frac{2(3Bc+3Ad-2Bd)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2Bd\cos(e+fx)\sqrt{a+a\sin}}{3af}$$

$$= -\frac{\sqrt{2}\,(A-B)(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f} - \frac{2(3Bc+3Ad-}{3f\sqrt{a+a}}$$

Mathematica [C] time = 0.48, size = 135, normalized size = 1.04

$$-\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(3Ad+3Bc+Bd\sin(e+fx)-Bd)\right.}{3f\sqrt{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/Sqrt[a + a*Sin[e + f*
x]],x]
```

```
[Out] -1/3*((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((-6 - 6*I)*(-1)^(3/4)*(A - B)*
(c - d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] + 2*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])*(3*B*c + 3*A*d - B*d + B*d*Sin[e + f*x])))/(
f*Sqrt[a*(1 + Sin[e + f*x])])
```

fricas [B] time = 0.47, size = 303, normalized size = 2.33

$$\frac{\frac{3\sqrt{2}\left((A-B)ac-(A-B)ad+((A-B)ac-(A-B)ad)\cos\left(fx+e\right)+((A-B)ac-(A-B)ad)\sin\left(fx+e\right)\right)\log\left(-\frac{\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)-\frac{2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+\cdots}}{\cdots}}{\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)+2\right)\sin\left(fx\cdots\right.}\right.}{\sqrt{a}}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/6*(3*sqrt(2)*((A - B)*a*c - (A - B)*a*d + ((A - B)*a*c - (A - B)*a*d)*cos
(f*x + e) + ((A - B)*a*c - (A - B)*a*d)*sin(f*x + e))*log(-(cos(f*x + e)^2
- (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos
(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2
 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) - 4*(B*d*co
s(f*x + e)^2 + 3*B*c + (3*A - 2*B)*d + (3*B*c + (3*A - B)*d)*cos(f*x + e) +
 (B*d*cos(f*x + e) - 3*B*c - (3*A - 2*B)*d)*sin(f*x + e))*sqrt(a*sin(f*x +
e) + a))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)2/f*(2/sqrt(a*tan((f*x+exp(1))/2)^2+a)/(a*tan((f*x+exp(1))/2)^2+
a)*(tan((f*x+exp(1))/2)*(tan((f*x+exp(1))/2)*(-1/144*tan((f*x+exp(1))/2)*(-
72*A*a*d*sign(tan((f*x+exp(1))/2)+1)-72*B*a*c*sign(tan((f*x+exp(1))/2)+1)+2
4*B*a*d*sign(tan((f*x+exp(1))/2)+1))-1/144*(72*A*a*d*sign(tan((f*x+exp(1))/
2)+1)+72*B*a*c*sign(tan((f*x+exp(1))/2)+1)-72*B*a*d*sign(tan((f*x+exp(1))/2
)+1)))-1/144*(-72*A*a*d*sign(tan((f*x+exp(1))/2)+1)-72*B*a*c*sign(tan((f*x+
exp(1))/2)+1)+72*B*a*d*sign(tan((f*x+exp(1))/2)+1)))-1/144*(72*A*a*d*sign(t
an((f*x+exp(1))/2)+1)+72*B*a*c*sign(tan((f*x+exp(1))/2)+1)-24*B*a*d*sign(ta
n((f*x+exp(1))/2)+1)))+sqrt(2)*(A*c-A*d-B*c+B*d)*atan((-sqrt(a)*tan((f*x+ex
p(1))/2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(-a
)/sign(tan((f*x+exp(1))/2)+1)+(-3*A*a*c*sqrt(2)*atan(sqrt(a)/sqrt(-a))+3*A*
a*d*sqrt(2)*atan(sqrt(a)/sqrt(-a))+3*A*d*sqrt(-a)*sqrt(2)*sqrt(a)+3*B*a*c*s
qrt(2)*atan(sqrt(a)/sqrt(-a))-3*B*a*d*sqrt(2)*atan(sqrt(a)/sqrt(-a))+3*B*c*
sqrt(-a)*sqrt(2)*sqrt(a)-2*B*d*sqrt(-a)*sqrt(2)*sqrt(a))/3/a/sqrt(-a)*sign(
tan((f*x+exp(1))/2)+1))
```

maple [B] time = 1.53, size = 232, normalized size = 1.78

$$-\frac{\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\left(3A\,a^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{a}}\right)c-3A\,a^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-}}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/3*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(3*A*a^(3/2)*2^(1/2)*arctanh(
1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c-3*A*a^(3/2)*2^(1/2)*arctanh(1
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d-3*B*a^(3/2)*2^(1/2)*arctanh(1/
2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c+3*B*a^(3/2)*2^(1/2)*arctanh(1/2
*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d-2*B*(a-a*sin(f*x+e))^(3/2)*d+6*A
*a*d*(a-a*sin(f*x+e))^(1/2)+6*B*a*c*(a-a*sin(f*x+e))^(1/2))/a^2/cos(f*x+e)/
(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(c+d\,\sin\left(e+f\,x\right)\right)}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(1/2),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(1/2),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))/sqrt(a*(sin(e + f*x) + 1
)), x)
```

3.310 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=79

$$-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f}-\frac{2B\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -(A-B)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/
2)/f/a^(1/2)-2*B*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2751, 2649, 206}

$$-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f}-\frac{2B\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]])])/(Sqrt[a]*f)) - (2*B*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{2B\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}+(A-B)\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= -\frac{2B\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}-\frac{(2(A-B))\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}$$

$$= -\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,f}-\frac{2B\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [C] time = 0.22, size = 106, normalized size = 1.34

$$\frac{2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(B\left(\sin\left(\frac{1}{2}(e+fx)\right)-\cos\left(\frac{1}{2}(e+fx)\right)\right)+(1+i)(-1)^{3/4}(A-B)\tanh^{-1}\left(\left(\frac{1}{2}\right.\right.\right.}{f\sqrt{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((1 + I)*(-1)^(3/4)*(A - B)*ArcTan
h[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] + B*(-Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])))/(f*Sqrt[a*(1 + Sin[e + f*x])])
```

fricas [B] time = 0.45, size = 210, normalized size = 2.66

$$-\frac{\frac{\sqrt{2}\left((A-B)a\cos\left(fx+e\right)+(A-B)a\sin\left(fx+e\right)+(A-B)a\right)\log\left(-\frac{\cos\left(fx+e\right)^2-\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)+\frac{2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)}{\sqrt{a}}+3\cos\left(fx+e\right)+\cdots}{\cos\left(fx+e\right)^2-\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right.}{\sqrt{a}}}{2\left(af\cos\left(fx+e\right)+af\sin\left(fx+e\right)+af\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*((A - B)*a*cos(f*x + e) + (A - B)*a*sin(f*x + e) + (A - B)*a)
*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*
```

```
sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e
) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2
))/sqrt(a) + 4*(B*cos(f*x + e) - B*sin(f*x + e) + B)*sqrt(a*sin(f*x + e) +
a))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)2/f*(2*sqrt(a*tan((f*x+exp(1))/2)^2+a)*(-1/2*B/sign(ta
n((f*x+exp(1))/2)+1)+1/2*B*tan((f*x+exp(1))/2)/sign(tan((f*x+exp(1))/2)+1))
/(a*tan((f*x+exp(1))/2)^2+a)+sqrt(2)*(A-B)*atan((-sqrt(a)*tan((f*x+exp(1))/
2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(-a)/sign
(tan((f*x+exp(1))/2)+1)+(-A*a*sqrt(2)*atan(sqrt(a)/sqrt(-a))+B*a*sqrt(2)*at
an(sqrt(a)/sqrt(-a))+B*sqrt(-a)*sqrt(2)*sqrt(a))/a/sqrt(-a)*sign(tan((f*x+e
xp(1))/2)+1))
```

maple [A] time = 1.29, size = 128, normalized size = 1.62

$$-\frac{\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\left(\sqrt{a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{a}}\right)A-\sqrt{a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin}}{2}\right.\right.}{a\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(a^(1/2)*2^(1/2)*arctanh(1/2*(a-a
*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*A-a^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(
f*x+e))^(1/2)*2^(1/2)/a^(1/2))*B+2*(a-a*sin(f*x+e))^(1/2)*B)/a/cos(f*x+e)/(
a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/sqrt(a*sin(f*x + e) + a), x)
```

mupad [B] time = 1.06, size = 151, normalized size = 1.91

$$-\frac{A\,\mathrm{F}\left(\frac{\pi}{4}-\frac{e}{2}-\frac{f\,x}{2}\middle|1\right)\,\sqrt{\frac{2\,\left(a+a\,\sin\left(e+f\,x\right)\right)}{a}}}{f\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}-\frac{B\,\left(4\,\mathrm{E}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin\left(e+f\,x\right)}}{2}\right)\middle|1\right)-2\,\mathrm{F}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin\left(e+f\,x\right)}}{2}\right)\middle|1\right)\right)\,\sqrt{\mathrm{co}}}{f\,\cos\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] - (A*ellipticF(pi/4 - e/2 - (f*x)/2, 1)*((2*(a + a*sin(e + f*x)))/a)^(1/2))
/(f*(a + a*sin(e + f*x))^(1/2)) - (B*(4*ellipticE(asin((2^(1/2)*(1 - sin(e
+ f*x))^(1/2))/2), 1) - 2*ellipticF(asin((2^(1/2)*(1 - sin(e + f*x))^(1/2))
/2), 1))*(cos(e + f*x)^2)^(1/2)*((a + a*sin(e + f*x))/(2*a))^(1/2))/(f*cos(
e + f*x)*(a + a*sin(e + f*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sin{\left(e + f x \right)}}{\sqrt{a \left(\sin{\left(e + f x \right)} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))/sqrt(a*(sin(e + f*x) + 1)), x)
```

3.311 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=136

$$-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f(c-d)}-\frac{2(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,\sqrt{d}\,f(c-d)\sqrt{c+d}}$$

```
[Out] -(A-B)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/
2)/(c-d)/f/a^(1/2)-2*(-A*d+B*c)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1
/2)/(a+a*sin(f*x+e))^(1/2))/(c-d)/f/a^(1/2)/d^(1/2)/(c+d)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {2985, 2649, 206, 2773, 208}

$$-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,f(c-d)}-\frac{2(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\,\sqrt{d}\,f(c-d)\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])),x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]])])/(Sqrt[a]*(c - d)*f)) - (2*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*
Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[a]*(c - d)*Sqr
t[d]*Sqrt[c + d]*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))}\,dx = \frac{(A-B)\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{c-d} + \frac{(Bc-Ad)\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{a(c-d)}$$
$$= -\frac{(2(A-B))\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{(c-d)f} - \frac{(2(Bc-Ad))\,\text{S}}{}$$
$$= -\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,(c-d)f} - \frac{2(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{c+}}\right.}{\sqrt{a}\,(c-d)\sqrt{d}\,\sqrt{}}$$

Mathematica [C] time = 3.34, size = 238, normalized size = 1.75

$$\frac{(-1)^{3/4}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sqrt[4]{-1}\,(Bc-Ad)\left(\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\,\sin\left(\frac{1}{2}(e+fx)\right)\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x
])),x]
```

```
[Out] ((-1)^(3/4)*((2 + 2*I)*(A - B)*Sqrt[d]*Sqrt[c + d]*ArcTanh[(1/2 + I/2)*(-1)
^(3/4)*(-1 + Tan[(e + f*x)/4])] + (-1)^(1/4)*(B*c - A*d)*(Log[Sec[(e + f*x)
/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])]
- Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*
Sin[(e + f*x)/2])]))*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/((c - d)*Sqrt[d
]*Sqrt[c + d]*f*Sqrt[a*(1 + Sin[e + f*x])])
```

fricas [B] time = 1.49, size = 744, normalized size = 5.47

$$\left[\frac{\sqrt{acd+ad^2}\,(Bc-Ad)\log\left(\frac{ad^2\cos(fx+e)^3-ac^2-2\,acd-ad^2-(6\,acd+7\,ad^2)\cos(fx+e)^2-4\,\sqrt{acd+ad^2}\left(d\cos(fx+e)^2-(c+2\,d)\cos(fx+e)+(d\cdots}{d^2\cos(fx+e)^3+(2\,cd+d^2)\cos(fx+e)^2-\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(sqrt(a*c*d + a*d^2)*(B*c - A*d)*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2
*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 - 4*sqrt(a*c*d + a*d^2)
*(d*cos(f*x + e)^2 - (c + 2*d)*cos(f*x + e) + (d*cos(f*x + e) + c + 3*d)*si
n(f*x + e) - c - 3*d)*sqrt(a*sin(f*x + e) + a) - (a*c^2 + 8*a*c*d + 9*a*d^2
)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c
*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^
2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos
(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + sqrt
(2)*((A - B)*a*c*d + (A - B)*a*d^2)*log(-(cos(f*x + e)^2 - (cos(f*x + e) -
2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*
x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) +
 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a))/((a*c^2*d - a*d^3)*f), -1/2*
(2*sqrt(-a*c*d - a*d^2)*(B*c - A*d)*arctan(1/2*sqrt(-a*c*d - a*d^2)*sqrt(a*
sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)/((a*c*d + a*d^2)*cos(f*x + e))
) - sqrt(2)*((A - B)*a*c*d + (A - B)*a*d^2)*log(-(cos(f*x + e)^2 - (cos(f*x
 + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e)
- sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*
x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a))/((a*c^2*d - a*d^3)*f
)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
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*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*p
i/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes
 constant sign by intervals (correct if the argument is real):Check [abs(co
s((f*t_nostep+exp(1))/2-pi/4))]Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Dis
continuities at zeroes of cos((f*t_nostep+exp(1))/2-pi/4) were not checkedU
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
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```
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*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
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check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign b
y intervals (correct if the argument is real):Check [abs(t_nostep+1)]Evalua
tion time: 0.42Error: Bad Argument Type
```

maple [A] time = 2.07, size = 199, normalized size = 1.46

$$-\frac{\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\,\left(\sqrt{2}\;\operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(fx+e\right)-1\right)}\;\sqrt{2}}{2\sqrt{a}}\right)\sqrt{a\left(c+d\right)d}\;A-2A\operatorname{arctanh}\left(\cdots\right.\right.}{\left(c-d\right)\sqrt{a\left(c+d\right)d}\;\sqrt{a}\;\cos\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(2^(1/2)*arctanh(1/2*(-a*(sin(f*x
+e)-1))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*A-2*A*arctanh((-a*(sin(f*x
+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(1/2)*d-2^(1/2)*arctanh(1/2*(-a*(sin(f
*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*B+2*B*arctanh((-a*(sin(f
```

```
*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(1/2)*c)/(c-d)/(a*(c+d)*d)^(1/2)/a^(
1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) +
c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c+d\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))),
x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.312 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=207

$$-\frac{(Bc-Ad)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}+\frac{\left(Ad(3c+d)-B\left(c^2+cd+2d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\cdots\right.}{\sqrt{a}\sqrt{d}\,f(c-d)^2(c+d)^{3/2}}$$

```
[Out] -(A-B)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/
2)/(c-d)^2/f/a^(1/2)+(A*d*(3*c+d)-B*(c^2+c*d+2*d^2))*arctanh(cos(f*x+e)*a^(
1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/(c-d)^2/(c+d)^(3/2)/f/a^(1
/2)/d^(1/2)-(-A*d+B*c)*cos(f*x+e)/(c^2-d^2)/f/(c+d*sin(f*x+e))/(a+a*sin(f*x
+e))^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2984, 2985, 2649, 206, 2773, 208}

$$-\frac{(Bc-Ad)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}+\frac{\left(Ad(3c+d)-B\left(c^2+cd+2d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\cdots\right.}{\sqrt{a}\sqrt{d}\,f(c-d)^2(c+d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2),
x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]])])/(Sqrt[a]*(c - d)^2*f)) + ((A*d*(3*c + d) - B*(c^2 + c*d + 2*d^2
))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f
*x]])])/(Sqrt[a]*(c - d)^2*Sqrt[d]*(c + d)^(3/2)*f) - ((B*c - A*d)*Cos[e +
f*x])/((c^2 - d^2)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2}\,dx = -\frac{(Bc-Ad)\cos(e+fx)}{\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))} - \frac{\int \frac{-\frac{1}{2}a(A(2c\cdots}{\sqrt{a\cdots}}}{\cdots}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)}{\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))} + \frac{(A-B)\int\cdots}{(\cdots}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)}{\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))} - \frac{(2(A-B)\cdots}{\cdots}$$

$$= -\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,(c-d)^2 f} + \frac{\left(Ad(3c+d)-B\left(c^2+\cdots\right.\right.}{\sqrt{a}\cdots}$$

Mathematica [C] time = 6.97, size = 374, normalized size = 1.81

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-\frac{\left(B\left(c^2+cd+2d^2\right)-Ad(3c+d)\right)\left(2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\,\sin\left(\frac{1}{2}(e+fx)\right)+\sqrt{d}\,\cos\left(\frac{1}{2}(e+fx)\right)\right)\cdots\right.\right.}{\sqrt{d}\,(c+d)^{3/2}}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x
])^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((8 + 8*I)*(-1)^(3/4)*(A - B)*ArcTan
h[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - ((-(A*d*(3*c + d)) + B*
(c^2 + c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e +
f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2
])]))/(Sqrt[d]*(c + d)^(3/2)) + ((-(A*d*(3*c + d)) + B*(c^2 + c*d + 2*d^2))
*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c +
d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])]))/(Sqrt[d]*(c +
d)^(3/2)) - (4*(c - d)*(B*c - A*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/(
(c + d)*(c + d*Sin[e + f*x]))))/(4*(c - d)^2*f*Sqrt[a*(1 + Sin[e + f*x])])
```

fricas [B] time = 3.77, size = 2159, normalized size = 10.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")

[Out] [-1/4*((B*c^3 - (3*A - 2*B)*c^2*d - (4*A - 3*B)*c*d^2 - (A - 2*B)*d^3 - (B*
c^2*d - (3*A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f*x + e)^2 + (B*c^3 - (3*A - B
)*c^2*d - (A - 2*B)*c*d^2)*cos(f*x + e) + (B*c^3 - (3*A - 2*B)*c^2*d - (4*A
 - 3*B)*c*d^2 - (A - 2*B)*d^3 + (B*c^2*d - (3*A - B)*c*d^2 - (A - 2*B)*d^3)
*cos(f*x + e))*sin(f*x + e))*sqrt(a*c*d + a*d^2)*log((a*d^2*cos(f*x + e)^3
- a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 - 4*sqrt(a*c
*d + a*d^2)*(d*cos(f*x + e)^2 - (c + 2*d)*cos(f*x + e) + (d*cos(f*x + e) +
c + 3*d)*sin(f*x + e) - c - 3*d)*sqrt(a*sin(f*x + e) + a) - (a*c^2 + 8*a*c*
d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2
 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 +
(2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e)
 + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x +
e))) - 2*sqrt(2)*((A - B)*a*c^3*d + 3*(A - B)*a*c^2*d^2 + 3*(A - B)*a*c*d^3
 + (A - B)*a*d^4 - ((A - B)*a*c^2*d^2 + 2*(A - B)*a*c*d^3 + (A - B)*a*d^4)*
cos(f*x + e)^2 + ((A - B)*a*c^3*d + 2*(A - B)*a*c^2*d^2 + (A - B)*a*c*d^3)*
cos(f*x + e) + ((A - B)*a*c^3*d + 3*(A - B)*a*c^2*d^2 + 3*(A - B)*a*c*d^3 +
 (A - B)*a*d^4 + ((A - B)*a*c^2*d^2 + 2*(A - B)*a*c*d^3 + (A - B)*a*d^4)*co
s(f*x + e))*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x
 + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1
)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*
x + e) - cos(f*x + e) - 2))/sqrt(a) - 4*(B*c^3*d - A*c^2*d^2 - B*c*d^3 + A*
d^4 + (B*c^3*d - A*c^2*d^2 - B*c*d^3 + A*d^4)*cos(f*x + e) - (B*c^3*d - A*c
^2*d^2 - B*c*d^3 + A*d^4)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a*c^4*d
^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x + e)^2 - (a*c^5*d - 2*a*c^3*d^3 + a*c*d
^5)*f*cos(f*x + e) - (a*c^5*d + a*c^4*d^2 - 2*a*c^3*d^3 - 2*a*c^2*d^4 + a*c
*d^5 + a*d^6)*f - ((a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x + e) + (a*c^
5*d + a*c^4*d^2 - 2*a*c^3*d^3 - 2*a*c^2*d^4 + a*c*d^5 + a*d^6)*f)*sin(f*x +
 e)), 1/2*((B*c^3 - (3*A - 2*B)*c^2*d - (4*A - 3*B)*c*d^2 - (A - 2*B)*d^3 -
 (B*c^2*d - (3*A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f*x + e)^2 + (B*c^3 - (3*A
 - B)*c^2*d - (A - 2*B)*c*d^2)*cos(f*x + e) + (B*c^3 - (3*A - 2*B)*c^2*d -
(4*A - 3*B)*c*d^2 - (A - 2*B)*d^3 + (B*c^2*d - (3*A - B)*c*d^2 - (A - 2*B)*
d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-a*c*d - a*d^2)*arctan(1/2*sqrt(-a*c*
d - a*d^2)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)/((a*c*d + a*
d^2)*cos(f*x + e))) + sqrt(2)*((A - B)*a*c^3*d + 3*(A - B)*a*c^2*d^2 + 3*(A
 - B)*a*c*d^3 + (A - B)*a*d^4 - ((A - B)*a*c^2*d^2 + 2*(A - B)*a*c*d^3 + (A
 - B)*a*d^4)*cos(f*x + e)^2 + ((A - B)*a*c^3*d + 2*(A - B)*a*c^2*d^2 + (A -
 B)*a*c*d^3)*cos(f*x + e) + ((A - B)*a*c^3*d + 3*(A - B)*a*c^2*d^2 + 3*(A -
 B)*a*c*d^3 + (A - B)*a*d^4 + ((A - B)*a*c^2*d^2 + 2*(A - B)*a*c*d^3 + (A -
 B)*a*d^4)*cos(f*x + e))*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos(f*x + e)
 - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin
(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e
) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) + 2*(B*c^3*d - A*c^2*d^2 -
```

```
 B*c*d^3 + A*d^4 + (B*c^3*d - A*c^2*d^2 - B*c*d^3 + A*d^4)*cos(f*x + e) - (
B*c^3*d - A*c^2*d^2 - B*c*d^3 + A*d^4)*sin(f*x + e))*sqrt(a*sin(f*x + e) +
a))/((a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x + e)^2 - (a*c^5*d - 2*a*c^
3*d^3 + a*c*d^5)*f*cos(f*x + e) - (a*c^5*d + a*c^4*d^2 - 2*a*c^3*d^3 - 2*a*
c^2*d^4 + a*c*d^5 + a*d^6)*f - ((a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x
 + e) + (a*c^5*d + a*c^4*d^2 - 2*a*c^3*d^3 - 2*a*c^2*d^4 + a*c*d^5 + a*d^6)
*f)*sin(f*x + e))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 2.72, size = 899, normalized size = 4.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] (1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)/a^(5/2)*(sin(f*x+e)*d*(3*A*arctanh
((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c*d+A*arctanh((a-a*s
in(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*d^2-B*arctanh((a-a*sin(f*x+
e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c^2-B*arctanh((a-a*sin(f*x+e))^(1/
2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c*d-2*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/
(a*c*d+a*d^2)^(1/2))*a^(5/2)*d^2-A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a
-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c-A*(a*(c+d)*d)^(1/2)*2^(1/2)*arc
tanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d+B*(a*(c+d)*d)^(1/2)*
2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c+B*(a*(c+d
)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*
d)+3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c^2*d+
A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c*d^2-B*arc
tanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c^3-B*arctanh((a
-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c^2*d-2*B*arctanh((a-a*
sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c*d^2+A*(a-a*sin(f*x+e))^(
1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c*d-A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/
2)*a^(3/2)*d^2-A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/
2)*2^(1/2)/a^(1/2))*a^2*c^2-A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*si
n(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d-B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*
```

```
d)^(1/2)*a^(3/2)*c^2+B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c*d
+B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*a^2*c^2+B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2
)*2^(1/2)/a^(1/2))*a^2*c*d)/(c-d)^2/(c+d)/(c+d*sin(f*x+e))/(a*(c+d)*d)^(1/2
)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) +
c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.313 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=309

$$\frac{\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{4f\left(c^2-d^2\right)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^2}+\frac{(Ad\,(}{}$$

```
[Out] -(A-B)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/
2)/(c-d)^3/f/a^(1/2)+1/4*(A*d*(15*c^2+10*c*d+7*d^2)-B*(3*c^3+6*c^2*d+19*c*d
^2+4*d^3))*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^
(1/2))/(c-d)^3/(c+d)^(5/2)/f/a^(1/2)/d^(1/2)-1/2*(-A*d+B*c)*cos(f*x+e)/(c^2
-d^2)/f/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2)+1/4*(A*d*(7*c+d)-B*(3*c^2
+c*d+4*d^2))*cos(f*x+e)/(c^2-d^2)^2/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/
2)
```

Rubi [A] time = 1.05, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2984, 2985, 2649, 206, 2773, 208}

$$\frac{\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{4f\left(c^2-d^2\right)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))^2}+\frac{(Ad\,(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^3),
x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]])])/(Sqrt[a]*(c - d)^3*f)) + ((A*d*(15*c^2 + 10*c*d + 7*d^2) - B*(3
*c^3 + 6*c^2*d + 19*c*d^2 + 4*d^3))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/
(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(4*Sqrt[a]*(c - d)^3*Sqrt[d]*(c +
d)^(5/2)*f) - ((B*c - A*d)*Cos[e + f*x])/(2*(c^2 - d^2)*f*Sqrt[a + a*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^2) + ((A*d*(7*c + d) - B*(3*c^2 + c*d + 4*d^2)
)*Cos[e + f*x])/(4*(c^2 - d^2)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e +
f*x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^3}\,dx = -\frac{(Bc-Ad)\cos(e+fx)}{2\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} - \frac{\int -\frac{1}{2}a(A}{}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)}{2\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} + \frac{\left(Ad(\right.}{4\left(c^2-\right.}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)}{2\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} + \frac{\left(Ad(\right.}{4\left(c^2-\right.}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)}{2\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))^2} + \frac{\left(Ad(\right.}{4\left(c^2-\right.}$$

$$= -\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}\,(c-d)^3 f} + \frac{\left(Ad\left(15c^2+10cd+7\right.\right.}{}$$

Mathematica [C] time = 10.78, size = 847, normalized size = 2.74

$$\frac{(2+2i)(A-B)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sec\left(\frac{1}{4}(e+fx)\right)\left(\cos\left(\frac{1}{4}(e+fx)\right)-\sin\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\right.}{\left(\sqrt[4]{-1}\,c^3-3\sqrt[4]{-1}\,dc^2+3\sqrt[4]{-1}\,d^2c-\sqrt[4]{-1}\,d^3\right)f\sqrt{a(\sin(e+fx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x
])^3),x]
```

```
[Out] ((2 + 2*I)*(A - B)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(e + f*x)/4]*(Cos[(e
+ f*x)/4] - Sin[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(((-1
)^(1/4)*c^3 - 3*(-1)^(1/4)*c^2*d + 3*(-1)^(1/4)*c*d^2 - (-1)^(1/4)*d^3)*f*S
qrt[a*(1 + Sin[e + f*x])]) - ((-(A*d*(15*c^2 + 10*c*d + 7*d^2)) + B*(3*c^3
+ 6*c^2*d + 19*c*d^2 + 4*d^3))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log
[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(
e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(16*(c - d)^3*Sqrt[d]
*(c + d)^(5/2)*f*Sqrt[a*(1 + Sin[e + f*x])]) + ((-(A*d*(15*c^2 + 10*c*d + 7
*d^2)) + B*(3*c^3 + 6*c^2*d + 19*c*d^2 + 4*d^3))*(e + f*x - 2*Log[Sec[(e +
f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)/
2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(16
```

```
*(c - d)^3*Sqrt[d]*(c + d)^(5/2)*f*Sqrt[a*(1 + Sin[e + f*x])]) + ((Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])*(-(B*c*Cos[(e + f*x)/2]) + A*d*Cos[(e + f*x)/2
] + B*c*Sin[(e + f*x)/2] - A*d*Sin[(e + f*x)/2]))/(2*(c - d)*(c + d)*f*Sqrt
[a*(1 + Sin[e + f*x])]*(c + d*Sin[e + f*x])^2) + ((Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])*(-3*B*c^2*Cos[(e + f*x)/2] + 7*A*c*d*Cos[(e + f*x)/2] - B*c*d*
Cos[(e + f*x)/2] + A*d^2*Cos[(e + f*x)/2] - 4*B*d^2*Cos[(e + f*x)/2] + 3*B*
c^2*Sin[(e + f*x)/2] - 7*A*c*d*Sin[(e + f*x)/2] + B*c*d*Sin[(e + f*x)/2] -
A*d^2*Sin[(e + f*x)/2] + 4*B*d^2*Sin[(e + f*x)/2]))/(4*(c - d)^2*(c + d)^2*
f*Sqrt[a*(1 + Sin[e + f*x])]*(c + d*Sin[e + f*x]))
```

fricas [B] time = 7.78, size = 4180, normalized size = 13.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/16*((3*B*c^5 - 3*(5*A - 4*B)*c^4*d - 2*(20*A - 17*B)*c^3*d^2 - 6*(7*A -
8*B)*c^2*d^3 - 3*(8*A - 9*B)*c*d^4 - (7*A - 4*B)*d^5 - (3*B*c^3*d^2 - 3*(5*
A - 2*B)*c^2*d^3 - (10*A - 19*B)*c*d^4 - (7*A - 4*B)*d^5)*cos(f*x + e)^3 -
(6*B*c^4*d - 15*(2*A - B)*c^3*d^2 - (35*A - 44*B)*c^2*d^3 - 3*(8*A - 9*B)*c
*d^4 - (7*A - 4*B)*d^5)*cos(f*x + e)^2 + (3*B*c^5 - 3*(5*A - 2*B)*c^4*d - 2
*(5*A - 11*B)*c^3*d^2 - 2*(11*A - 5*B)*c^2*d^3 - (10*A - 19*B)*c*d^4 - (7*A
 - 4*B)*d^5)*cos(f*x + e) + (3*B*c^5 - 3*(5*A - 4*B)*c^4*d - 2*(20*A - 17*B
)*c^3*d^2 - 6*(7*A - 8*B)*c^2*d^3 - 3*(8*A - 9*B)*c*d^4 - (7*A - 4*B)*d^5 -
 (3*B*c^3*d^2 - 3*(5*A - 2*B)*c^2*d^3 - (10*A - 19*B)*c*d^4 - (7*A - 4*B)*d
^5)*cos(f*x + e)^2 + 2*(3*B*c^4*d - 3*(5*A - 2*B)*c^3*d^2 - (10*A - 19*B)*c
^2*d^3 - (7*A - 4*B)*c*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*c*d + a*d^2)
*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*
cos(f*x + e)^2 + 4*sqrt(a*c*d + a*d^2)*(d*cos(f*x + e)^2 - (c + 2*d)*cos(f*
x + e) + (d*cos(f*x + e) + c + 3*d)*sin(f*x + e) - c - 3*d)*sqrt(a*sin(f*x
+ e) + a) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^
2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x +
 e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2
 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^
2 - 2*c*d - d^2)*sin(f*x + e))) - 8*sqrt(2)*((A - B)*a*c^5*d + 5*(A - B)*a*
c^4*d^2 + 10*(A - B)*a*c^3*d^3 + 10*(A - B)*a*c^2*d^4 + 5*(A - B)*a*c*d^5 +
 (A - B)*a*d^6 - ((A - B)*a*c^3*d^3 + 3*(A - B)*a*c^2*d^4 + 3*(A - B)*a*c*d
^5 + (A - B)*a*d^6)*cos(f*x + e)^3 - (2*(A - B)*a*c^4*d^2 + 7*(A - B)*a*c^3
*d^3 + 9*(A - B)*a*c^2*d^4 + 5*(A - B)*a*c*d^5 + (A - B)*a*d^6)*cos(f*x + e
)^2 + ((A - B)*a*c^5*d + 3*(A - B)*a*c^4*d^2 + 4*(A - B)*a*c^3*d^3 + 4*(A -
 B)*a*c^2*d^4 + 3*(A - B)*a*c*d^5 + (A - B)*a*d^6)*cos(f*x + e) + ((A - B)*
a*c^5*d + 5*(A - B)*a*c^4*d^2 + 10*(A - B)*a*c^3*d^3 + 10*(A - B)*a*c^2*d^4
 + 5*(A - B)*a*c*d^5 + (A - B)*a*d^6 - ((A - B)*a*c^3*d^3 + 3*(A - B)*a*c^2
```

```
*d^4 + 3*(A - B)*a*c*d^5 + (A - B)*a*d^6)*cos(f*x + e)^2 + 2*((A - B)*a*c^4
*d^2 + 3*(A - B)*a*c^3*d^3 + 3*(A - B)*a*c^2*d^4 + (A - B)*a*c*d^5)*cos(f*x
 + e))*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e)
 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqr
t(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e
) - cos(f*x + e) - 2))/sqrt(a) + 4*(5*B*c^5*d - (9*A + 2*B)*c^4*d^2 + 2*(3*
A - 2*B)*c^3*d^3 + 2*(6*A - B)*c^2*d^4 - (6*A + B)*c*d^5 - (3*A - 4*B)*d^6
+ (3*B*c^4*d^2 - (7*A - B)*c^3*d^3 - (A - B)*c^2*d^4 + (7*A - B)*c*d^5 + (A
 - 4*B)*d^6)*cos(f*x + e)^2 + (5*B*c^5*d - (9*A - B)*c^4*d^2 - (A + 3*B)*c^
3*d^3 + (11*A - B)*c^2*d^4 + (A - 2*B)*c*d^5 - 2*A*d^6)*cos(f*x + e) - (5*B
*c^5*d - (9*A + 2*B)*c^4*d^2 + 2*(3*A - 2*B)*c^3*d^3 + 2*(6*A - B)*c^2*d^4
- (6*A + B)*c*d^5 - (3*A - 4*B)*d^6 - (3*B*c^4*d^2 - (7*A - B)*c^3*d^3 - (A
 - B)*c^2*d^4 + (7*A - B)*c*d^5 + (A - 4*B)*d^6)*cos(f*x + e))*sin(f*x + e)
)*sqrt(a*sin(f*x + e) + a))/((a*c^6*d^3 - 3*a*c^4*d^5 + 3*a*c^2*d^7 - a*d^9
)*f*cos(f*x + e)^3 + (2*a*c^7*d^2 + a*c^6*d^3 - 6*a*c^5*d^4 - 3*a*c^4*d^5 +
 6*a*c^3*d^6 + 3*a*c^2*d^7 - 2*a*c*d^8 - a*d^9)*f*cos(f*x + e)^2 - (a*c^8*d
 - 2*a*c^6*d^3 + 2*a*c^2*d^7 - a*d^9)*f*cos(f*x + e) - (a*c^8*d + 2*a*c^7*d
^2 - 2*a*c^6*d^3 - 6*a*c^5*d^4 + 6*a*c^3*d^6 + 2*a*c^2*d^7 - 2*a*c*d^8 - a*
d^9)*f + ((a*c^6*d^3 - 3*a*c^4*d^5 + 3*a*c^2*d^7 - a*d^9)*f*cos(f*x + e)^2
- 2*(a*c^7*d^2 - 3*a*c^5*d^4 + 3*a*c^3*d^6 - a*c*d^8)*f*cos(f*x + e) - (a*c
^8*d + 2*a*c^7*d^2 - 2*a*c^6*d^3 - 6*a*c^5*d^4 + 6*a*c^3*d^6 + 2*a*c^2*d^7
- 2*a*c*d^8 - a*d^9)*f)*sin(f*x + e)), 1/8*((3*B*c^5 - 3*(5*A - 4*B)*c^4*d
- 2*(20*A - 17*B)*c^3*d^2 - 6*(7*A - 8*B)*c^2*d^3 - 3*(8*A - 9*B)*c*d^4 - (
7*A - 4*B)*d^5 - (3*B*c^3*d^2 - 3*(5*A - 2*B)*c^2*d^3 - (10*A - 19*B)*c*d^4
 - (7*A - 4*B)*d^5)*cos(f*x + e)^3 - (6*B*c^4*d - 15*(2*A - B)*c^3*d^2 - (3
5*A - 44*B)*c^2*d^3 - 3*(8*A - 9*B)*c*d^4 - (7*A - 4*B)*d^5)*cos(f*x + e)^2
 + (3*B*c^5 - 3*(5*A - 2*B)*c^4*d - 2*(5*A - 11*B)*c^3*d^2 - 2*(11*A - 5*B)
*c^2*d^3 - (10*A - 19*B)*c*d^4 - (7*A - 4*B)*d^5)*cos(f*x + e) + (3*B*c^5 -
 3*(5*A - 4*B)*c^4*d - 2*(20*A - 17*B)*c^3*d^2 - 6*(7*A - 8*B)*c^2*d^3 - 3*
(8*A - 9*B)*c*d^4 - (7*A - 4*B)*d^5 - (3*B*c^3*d^2 - 3*(5*A - 2*B)*c^2*d^3
- (10*A - 19*B)*c*d^4 - (7*A - 4*B)*d^5)*cos(f*x + e)^2 + 2*(3*B*c^4*d - 3*
(5*A - 2*B)*c^3*d^2 - (10*A - 19*B)*c^2*d^3 - (7*A - 4*B)*c*d^4)*cos(f*x +
e))*sin(f*x + e))*sqrt(-a*c*d - a*d^2)*arctan(1/2*sqrt(-a*c*d - a*d^2)*sqrt
(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)/((a*c*d + a*d^2)*cos(f*x +
e))) - 4*sqrt(2)*((A - B)*a*c^5*d + 5*(A - B)*a*c^4*d^2 + 10*(A - B)*a*c^3*
d^3 + 10*(A - B)*a*c^2*d^4 + 5*(A - B)*a*c*d^5 + (A - B)*a*d^6 - ((A - B)*a
*c^3*d^3 + 3*(A - B)*a*c^2*d^4 + 3*(A - B)*a*c*d^5 + (A - B)*a*d^6)*cos(f*x
 + e)^3 - (2*(A - B)*a*c^4*d^2 + 7*(A - B)*a*c^3*d^3 + 9*(A - B)*a*c^2*d^4
+ 5*(A - B)*a*c*d^5 + (A - B)*a*d^6)*cos(f*x + e)^2 + ((A - B)*a*c^5*d + 3*
(A - B)*a*c^4*d^2 + 4*(A - B)*a*c^3*d^3 + 4*(A - B)*a*c^2*d^4 + 3*(A - B)*a
*c*d^5 + (A - B)*a*d^6)*cos(f*x + e) + ((A - B)*a*c^5*d + 5*(A - B)*a*c^4*d
^2 + 10*(A - B)*a*c^3*d^3 + 10*(A - B)*a*c^2*d^4 + 5*(A - B)*a*c*d^5 + (A -
 B)*a*d^6 - ((A - B)*a*c^3*d^3 + 3*(A - B)*a*c^2*d^4 + 3*(A - B)*a*c*d^5 +
(A - B)*a*d^6)*cos(f*x + e)^2 + 2*((A - B)*a*c^4*d^2 + 3*(A - B)*a*c^3*d^3
+ 3*(A - B)*a*c^2*d^4 + (A - B)*a*c*d^5)*cos(f*x + e))*sin(f*x + e))*log(-(
```

```
cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x
 + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/
(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt
(a) + 2*(5*B*c^5*d - (9*A + 2*B)*c^4*d^2 + 2*(3*A - 2*B)*c^3*d^3 + 2*(6*A -
 B)*c^2*d^4 - (6*A + B)*c*d^5 - (3*A - 4*B)*d^6 + (3*B*c^4*d^2 - (7*A - B)*
c^3*d^3 - (A - B)*c^2*d^4 + (7*A - B)*c*d^5 + (A - 4*B)*d^6)*cos(f*x + e)^2
 + (5*B*c^5*d - (9*A - B)*c^4*d^2 - (A + 3*B)*c^3*d^3 + (11*A - B)*c^2*d^4
+ (A - 2*B)*c*d^5 - 2*A*d^6)*cos(f*x + e) - (5*B*c^5*d - (9*A + 2*B)*c^4*d^
2 + 2*(3*A - 2*B)*c^3*d^3 + 2*(6*A - B)*c^2*d^4 - (6*A + B)*c*d^5 - (3*A -
4*B)*d^6 - (3*B*c^4*d^2 - (7*A - B)*c^3*d^3 - (A - B)*c^2*d^4 + (7*A - B)*c
*d^5 + (A - 4*B)*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))
/((a*c^6*d^3 - 3*a*c^4*d^5 + 3*a*c^2*d^7 - a*d^9)*f*cos(f*x + e)^3 + (2*a*c
^7*d^2 + a*c^6*d^3 - 6*a*c^5*d^4 - 3*a*c^4*d^5 + 6*a*c^3*d^6 + 3*a*c^2*d^7
- 2*a*c*d^8 - a*d^9)*f*cos(f*x + e)^2 - (a*c^8*d - 2*a*c^6*d^3 + 2*a*c^2*d^
7 - a*d^9)*f*cos(f*x + e) - (a*c^8*d + 2*a*c^7*d^2 - 2*a*c^6*d^3 - 6*a*c^5*
d^4 + 6*a*c^3*d^6 + 2*a*c^2*d^7 - 2*a*c*d^8 - a*d^9)*f + ((a*c^6*d^3 - 3*a*
c^4*d^5 + 3*a*c^2*d^7 - a*d^9)*f*cos(f*x + e)^2 - 2*(a*c^7*d^2 - 3*a*c^5*d^
4 + 3*a*c^3*d^6 - a*c*d^8)*f*cos(f*x + e) - (a*c^8*d + 2*a*c^7*d^2 - 2*a*c^
6*d^3 - 6*a*c^5*d^4 + 6*a*c^3*d^6 + 2*a*c^2*d^7 - 2*a*c*d^8 - a*d^9)*f)*sin
(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integrat
ion of abs or sign assumes constant sign by intervals (correct if the argum
ent is real):Check [abs(cos((f*t_nostep+exp(1))/2-pi/4))]Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Discontinuities at zeroes of cos((f*t_nostep+exp(1))
/2-pi/4) were not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
```

```
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*p
i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
```

/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Un
able to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or si
gn assumes constant sign by intervals (correct if the argument is real):Che
ck [abs(t_nostep+1)]Evaluation time: 1.28Unable to divide, perhaps due to r
ounding error%%%{1,[12,10,0,1]%%%}+%%%{-3,[12,9,1,1]%%%}+%%%{3,[12,8,2,1]%%
%}+%%%{-1,[12,7,3,1]%%%}+%%%{%%{[-12,0]:[1,0,%%%{-1,[1]%%%}]%%},[11,9,1,1]%
%%}+%%%{%%{[36,0]:[1,0,%%%{-1,[1]%%%}]%%},[11,8,2,1]%%%}+%%%{%%{[-36,0]:[1,
0,%%%{-1,[1]%%%}]%%},[11,7,3,1]%%%}+%%%{%%{[12,0]:[1,0,%%%{-1,[1]%%%}]%%},[
11,6,4,1]%%%}+%%%{%%%{6,[1]%%%},[10,10,0,1]%%%}+%%%{%%%{-18,[1]%%%},[10,9,1
,1]%%%}+%%%{%%%{66,[1]%%%},[10,8,2,1]%%%}+%%%{%%%{-150,[1]%%%},[10,7,3,1]%%
%}+%%%{%%%{144,[1]%%%},[10,6,4,1]%%%}+%%%{%%%{-48,[1]%%%},[10,5,5,1]%%%}+%%
%{%%{[%%%{-36,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[9,9,1,1]%%%}+%%%{%%{[%%%{
108,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[9,8,2,1]%%%}+%%%{%%{[%%%{-172,[1]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[9,7,3,1]%%%}+%%%{%%{[%%%{228,[1]%%%},0]:[1,0
,%%%{-1,[1]%%%}]%%},[9,6,4,1]%%%}+%%%{%%{[%%%{-192,[1]%%%},0]:[1,0,%%%{-1,[
1]%%%}]%%},[9,5,5,1]%%%}+%%%{%%{[%%%{64,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},
[9,4,6,1]%%%}+%%%{%%%{15,[2]%%%},[8,10,0,1]%%%}+%%%{%%%{-45,[2]%%%},[8,9,1,
1]%%%}+%%%{%%%{45,[2]%%%},[8,8,2,1]%%%}+%%%{%%%{-15,[2]%%%},[8,7,3,1]%%%}+%
%%{%%{[%%%{-24,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,9,1,1]%%%}+%%%{%%{[%%%
{72,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,8,2,1]%%%}+%%%{%%{[%%%{120,[2]%%%
},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,7,3,1]%%%}+%%%{%%{[%%%{-552,[2]%%%},0]:[1,0
,%%%{-1,[1]%%%}]%%},[7,6,4,1]%%%}+%%%{%%{[%%%{576,[2]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[7,5,5,1]%%%}+%%%{%%{[%%%{-192,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%}
,[7,4,6,1]%%%}+%%%{%%%{20,[3]%%%},[6,10,0,1]%%%}+%%%{%%%{-60,[3]%%%},[6,9,1
,1]%%%}+%%%{%%%{-36,[3]%%%},[6,8,2,1]%%%}+%%%{%%%{268,[3]%%%},[6,7,3,1]%%%}
+%%%{%%%{-288,[3]%%%},[6,6,4,1]%%%}+%%%{%%%{96,[3]%%%},[6,5,5,1]%%%}+%%%{%%
{[%%%{24,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,9,1,1]%%%}+%%%{%%{[%%%{-72,[
3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,8,2,1]%%%}+%%%{%%{[%%%{-120,[3]%%%},0]
:[1,0,%%%{-1,[1]%%%}]%%},[5,7,3,1]%%%}+%%%{%%{[%%%{552,[3]%%%},0]:[1,0,%%%{
-1,[1]%%%}]%%},[5,6,4,1]%%%}+%%%{%%{[%%%{-576,[3]%%%},0]:[1,0,%%%{-1,[1]%%%
}]%%},[5,5,5,1]%%%}+%%%{%%{[%%%{192,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,4
,6,1]%%%}+%%%{%%%{15,[4]%%%},[4,10,0,1]%%%}+%%%{%%%{-45,[4]%%%},[4,9,1,1]%%
%}+%%%{%%%{45,[4]%%%},[4,8,2,1]%%%}+%%%{%%%{-15,[4]%%%},[4,7,3,1]%%%}+%%%{%
%{[%%%{36,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,9,1,1]%%%}+%%%{%%{[%%%{-108
,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,8,2,1]%%%}+%%%{%%{[%%%{172,[4]%%%},0
]:[1,0,%%%{-1,[1]%%%}]%%},[3,7,3,1]%%%}+%%%{%%{[%%%{-228,[4]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[3,6,4,1]%%%}+%%%{%%{[%%%{192,[4]%%%},0]:[1,0,%%%{-1,[1]%%
%}]%%},[3,5,5,1]%%%}+%%%{%%{[%%%{-64,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,
4,6,1]%%%}+%%%{%%%{6,[5]%%%},[2,10,0,1]%%%}+%%%{%%%{-18,[5]%%%},[2,9,1,1]%%
%}+%%%{%%%{66,[5]%%%},[2,8,2,1]%%%}+%%%{%%%{-150,[5]%%%},[2,7,3,1]%%%}+%%%{
%%%{144,[5]%%%},[2,6,4,1]%%%}+%%%{%%%{-48,[5]%%%},[2,5,5,1]%%%}+%%%{%%{[%%%
{12,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,9,1,1]%%%}+%%%{%%{[%%%{-36,[5]%%%
},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,8,2,1]%%%}+%%%{%%{[%%%{36,[5]%%%},0]:[1,0,%

```
%%{-1,[1]%%%}]%%},[1,7,3,1]%%%}+%%%{%%{[%%%{-12,[5]%%%},0]:[1,0,%%%{-1,[1]%
%%}]%%},[1,6,4,1]%%%}+%%%{%%%{1,[6]%%%},[0,10,0,1]%%%}+%%%{%%%{-3,[6]%%%},[
0,9,1,1]%%%}+%%%{%%%{3,[6]%%%},[0,8,2,1]%%%}+%%%{%%%{-1,[6]%%%},[0,7,3,1]%%
%} / %%%{%%%{-1,[3]%%%},[12,3,0,0]%%%}+%%%{%%{poly1[%%%{12,[3]%%%},0]:[1,0,
%%%{-1,[1]%%%}]%%},[11,2,1,0]%%%}+%%%{%%%{-6,[4]%%%},[10,3,0,0]%%%}+%%%{%%%
{-48,[4]%%%},[10,1,2,0]%%%}+%%%{%%{poly1[%%%{36,[4]%%%},0]:[1,0,%%%{-1,[1]%
%%}]%%},[9,2,1,0]%%%}+%%%{%%{poly1[%%%{64,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%
},[9,0,3,0]%%%}+%%%{%%%{-15,[5]%%%},[8,3,0,0]%%%}+%%%{%%{poly1[%%%{24,[5]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,2,1,0]%%%}+%%%{%%{poly1[%%%{-192,[5]%%%},0
]:[1,0,%%%{-1,[1]%%%}]%%},[7,0,3,0]%%%}+%%%{%%%{-20,[6]%%%},[6,3,0,0]%%%}+%
%%{%%%{96,[6]%%%},[6,1,2,0]%%%}+%%%{%%{poly1[%%%{-24,[6]%%%},0]:[1,0,%%%{-1
,[1]%%%}]%%},[5,2,1,0]%%%}+%%%{%%{poly1[%%%{192,[6]%%%},0]:[1,0,%%%{-1,[1]%
%%}]%%},[5,0,3,0]%%%}+%%%{%%%{-15,[7]%%%},[4,3,0,0]%%%}+%%%{%%{poly1[%%%{-3
6,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,2,1,0]%%%}+%%%{%%{poly1[%%%{-64,[7]
%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,0,3,0]%%%}+%%%{%%%{-6,[8]%%%},[2,3,0,0]%
%%}+%%%{%%%{-48,[8]%%%},[2,1,2,0]%%%}+%%%{%%{poly1[%%%{-12,[8]%%%},0]:[1,0,
%%%{-1,[1]%%%}]%%},[1,2,1,0]%%%}+%%%{%%%{-1,[9]%%%},[0,3,0,0]%%%} Error: Ba
d Argument Value
```

maple [B] time = 3.58, size = 2277, normalized size = 7.37

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/4*(8*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2
^(1/2)/a^(1/2))*sin(f*x+e)*a^4*c*d^3+16*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh
(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^4*c^2*d^2+6*B*
arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)*c
^4*d-8*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^
(1/2)/a^(1/2))*a^4*c^3*d-4*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin
(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^4*c^2*d^2+3*B*arctanh((-a*(sin(f*x+e)-
1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*c^5-8*B*(a*(c+d)*d)^(1/2)*2^(1/2)*ar
ctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^4*c^3*d-1
6*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)
/a^(1/2))*sin(f*x+e)*a^4*c^2*d^2-8*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*
(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^4*c*d^3+4*A*(a*(c+d
)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*s
in(f*x+e)^2*a^4*c^2*d^2+8*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(
f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^4*c*d^3-4*B*(a*(c+d)*d)^(1
/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+
e)^2*a^4*c^2*d^2-8*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-
1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^4*c*d^3+8*A*(a*(c+d)*d)^(1/2)*2^(
1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^4*
```

```
c^3*d+7*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(5/2)*c^2*d^2-3*B*(
-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(5/2)*c^3*d-6*A*(-a*(sin(f*x+e
)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(5/2)*c*d^3+2*B*(-a*(sin(f*x+e)-1))^(3/2)*(
a*(c+d)*d)^(1/2)*a^(5/2)*c^2*d^2-3*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^
(1/2)*a^(5/2)*c*d^3+4*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+
e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^4*c^4+4*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh
(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^4*d^4-4*B*(a
*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/
2))*sin(f*x+e)^2*a^4*d^4+8*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin
(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^4*c^3*d+4*A*(a*(c+d)*d)^(1/2)*2^(1/2)*
arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^4*c^2*d^2-B*(-a*(s
in(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*c^2*d^2+B*(-a*(sin(f*x+e)-1))
^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*c*d^3-15*A*arctanh((-a*(sin(f*x+e)-1))^(1/
2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)^2*c^2*d^3-10*A*arctanh((-a*(sin(
f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)^2*c*d^4+3*B*arctan
h((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)^2*c^3*d
^2+6*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f
*x+e)^2*c^2*d^3+19*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))
*a^(9/2)*sin(f*x+e)^2*c*d^4-30*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+
d)*d)^(1/2))*a^(9/2)*sin(f*x+e)*c^3*d^2-20*A*arctanh((-a*(sin(f*x+e)-1))^(1
/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)*c^2*d^3-14*A*arctanh((-a*(sin(f
*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)*c*d^4-9*A*(-a*(sin(
f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*c^3*d+12*B*arctanh((-a*(sin(f*x+
e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)*c^3*d^2+5*B*(-a*(sin(f
*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*c^4-10*A*arctanh((-a*(sin(f*x+e)-
1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*c^3*d^2+38*B*arctanh((-a*(sin(f*x+e)
-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)*c^2*d^3+9*A*(-a*(sin(f*x
+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*c*d^3-B*(-a*(sin(f*x+e)-1))^(1/2)*(
a*(c+d)*d)^(1/2)*a^(7/2)*c^3*d+8*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(
c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)*c*d^4+19*B*arctanh((-a*(sin(f*x+e)-1))^(1
/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*c^3*d^2+4*B*arctanh((-a*(sin(f*x+e)-1))^(1
/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*c^2*d^3-A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+
d)*d)^(1/2)*a^(7/2)*d^4+A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/
2)*c^2*d^2-7*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/
2)*sin(f*x+e)^2*d^5-4*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+
e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^4*c^4+4*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)
*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)^2*d^5-7*A*arctanh((-a*(sin(f*x+e)-
1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*c^2*d^3-A*(-a*(sin(f*x+e)-1))^(3/2)*
(a*(c+d)*d)^(1/2)*a^(5/2)*d^4+4*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/
2)*a^(5/2)*d^4-15*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*
a^(9/2)*c^4*d+6*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^
(9/2)*c^4*d-4*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*d^4)/a^
(9/2)*(-a*(sin(f*x+e)-1))^(1/2)*(1+sin(f*x+e))/(a*(c+d)*d)^(1/2)/(c+d*sin(f
*x+e))^2/(c+d)^2/(c-d)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.314 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=283

$$-\frac{(c-d)^2(A(c+11d)+3B(c-5d))\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}\,a^{3/2}f}+\frac{d^2(15Ac-35Ad-51Bc+39Bd)\cos(e+fx)\sqrt{a\sin}}{30a^2f}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^3/f/(a+a*sin(f*x+e))^(3/2)-1/4*(c-d)
^2*(3*B*(c-5*d)+A*(c+11*d))*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin
(f*x+e))^(1/2))/a^(3/2)/f*2^(1/2)+1/15*d*(15*A*c^2-120*A*c*d+65*A*d^2-99*B*
c^2+168*B*c*d-93*B*d^2)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)+1/10*(5*A-9*B
)*d*cos(f*x+e)*(c+d*sin(f*x+e))^2/a/f/(a+a*sin(f*x+e))^(1/2)+1/30*d^2*(15*A
*c-35*A*d-51*B*c+39*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/a^2/f
```

Rubi [A] time = 1.00, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {2977, 2983, 2968, 3023, 2751, 2649, 206}

$$\frac{d^2(15Ac-35Ad-51Bc+39Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{30a^2f}-\frac{(c-d)^2(A(c+11d)+3B(c-5d))\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{2}\sqrt{}}\right)}{2\sqrt{2}\,a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^(3/2
),x]
```

```
[Out] -((c - d)^2*(3*B*(c - 5*d) + A*(c + 11*d))*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(
Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*f) + (d*(15*A*c^2 -
99*B*c^2 - 120*A*c*d + 168*B*c*d + 65*A*d^2 - 93*B*d^2)*Cos[e + f*x])/(15*a
*f*Sqrt[a + a*Sin[e + f*x]]) + (d^2*(15*A*c - 51*B*c - 35*A*d + 39*B*d)*Cos
[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(30*a^2*f) + ((5*A - 9*B)*d*Cos[e + f*x
]*(c + d*Sin[e + f*x])^2)/(10*a*f*Sqrt[a + a*Sin[e + f*x]]) - ((A - B)*Cos[
e + f*x]*(c + d*Sin[e + f*x])^3)/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
```

```
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{(c+d\sin(e+fx))^2\left(\frac{1}{2}a(3B\right.}{}}{}$$

$$= \frac{(5A-9B)d\cos(e+fx)(c+d\sin(e+fx))^2}{10af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+f}{2f(a+a\sin}$$

$$= \frac{(5A-9B)d\cos(e+fx)(c+d\sin(e+fx))^2}{10af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+f}{2f(a+a\sin}$$

$$= \frac{d^2(15Ac-51Bc-35Ad+39Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{30a^2f}$$

$$= \frac{d\left(15Ac^2-99Bc^2-120Acd+168Bcd+65Ad^2-93Bd^2\right)\cos(e}{15af\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{d\left(15Ac^2-99Bc^2-120Acd+168Bcd+65Ad^2-93Bd^2\right)\cos(e}{15af\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{(c-d)^2(3B(c-5d)+A(c+11d))\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} +$$

Mathematica [C] time = 1.07, size = 684, normalized size = 2.42

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left((30+30i)(-1)^{3/4}(c-d)^2(A(c+11d)+3B(c-5d))\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-30*A*c^3*Cos[(e + f*x)/2] + 30*B*c
^3*Cos[(e + f*x)/2] + 90*A*c^2*d*Cos[(e + f*x)/2] - 270*B*c^2*d*Cos[(e + f*
x)/2] - 270*A*c*d^2*Cos[(e + f*x)/2] + 330*B*c*d^2*Cos[(e + f*x)/2] + 110*A
*d^3*Cos[(e + f*x)/2] - 165*B*d^3*Cos[(e + f*x)/2] - 180*B*c^2*d*Cos[(3*(e
+ f*x))/2] - 180*A*c*d^2*Cos[(3*(e + f*x))/2] + 210*B*c*d^2*Cos[(3*(e + f*x
))/2] + 70*A*d^3*Cos[(3*(e + f*x))/2] - 123*B*d^3*Cos[(3*(e + f*x))/2] + 30
*B*c*d^2*Cos[(5*(e + f*x))/2] + 10*A*d^3*Cos[(5*(e + f*x))/2] - 9*B*d^3*Cos
[(5*(e + f*x))/2] + 3*B*d^3*Cos[(7*(e + f*x))/2] + 30*A*c^3*Sin[(e + f*x)/2
] - 30*B*c^3*Sin[(e + f*x)/2] - 90*A*c^2*d*Sin[(e + f*x)/2] + 270*B*c^2*d*S
in[(e + f*x)/2] + 270*A*c*d^2*Sin[(e + f*x)/2] - 330*B*c*d^2*Sin[(e + f*x)/
2] - 110*A*d^3*Sin[(e + f*x)/2] + 165*B*d^3*Sin[(e + f*x)/2] + (30 + 30*I)*
(-1)^(3/4)*(c - d)^2*(3*B*(c - 5*d) + A*(c + 11*d))*ArcTanh[(1/2 + I/2)*(-1
)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 -
180*B*c^2*d*Sin[(3*(e + f*x))/2] - 180*A*c*d^2*Sin[(3*(e + f*x))/2] + 210*B
*c*d^2*Sin[(3*(e + f*x))/2] + 70*A*d^3*Sin[(3*(e + f*x))/2] - 123*B*d^3*Sin
[(3*(e + f*x))/2] - 30*B*c*d^2*Sin[(5*(e + f*x))/2] - 10*A*d^3*Sin[(5*(e +
f*x))/2] + 9*B*d^3*Sin[(5*(e + f*x))/2] + 3*B*d^3*Sin[(7*(e + f*x))/2]))/(6
0*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

fricas [B] time = 0.49, size = 784, normalized size = 2.77

$$\frac{15\sqrt{2}\Big(2\,(A+3\,B)c^3+6\,(3\,A-7\,B)c^2d-6\,(7\,A-11\,B)cd^2+2\,(11\,A-15\,B)d^3-\Big((A+3\,B)c^3+3\,(3\,A-7\,B)c^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/120*(15*sqrt(2)*(2*(A + 3*B)*c^3 + 6*(3*A - 7*B)*c^2*d - 6*(7*A - 11*B)*c
*d^2 + 2*(11*A - 15*B)*d^3 - ((A + 3*B)*c^3 + 3*(3*A - 7*B)*c^2*d - 3*(7*A
- 11*B)*c*d^2 + (11*A - 15*B)*d^3)*cos(f*x + e)^2 + ((A + 3*B)*c^3 + 3*(3*A
 - 7*B)*c^2*d - 3*(7*A - 11*B)*c*d^2 + (11*A - 15*B)*d^3)*cos(f*x + e) + (2
*(A + 3*B)*c^3 + 6*(3*A - 7*B)*c^2*d - 6*(7*A - 11*B)*c*d^2 + 2*(11*A - 15*
B)*d^3 + ((A + 3*B)*c^3 + 3*(3*A - 7*B)*c^2*d - 3*(7*A - 11*B)*c*d^2 + (11*
A - 15*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 +
 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) +
1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x
 + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(12*B*d^
3*cos(f*x + e)^4 - 15*(A - B)*c^3 + 45*(A - B)*c^2*d - 45*(A - B)*c*d^2 + 1
5*(A - B)*d^3 + 4*(15*B*c*d^2 + (5*A - 3*B)*d^3)*cos(f*x + e)^3 - 4*(45*B*c
```

```
^2*d + 15*(3*A - 4*B)*c*d^2 - 4*(5*A - 9*B)*d^3)*cos(f*x + e)^2 - 15*((A -
B)*c^3 - 3*(A - 5*B)*c^2*d + 15*(A - B)*c*d^2 - (5*A - 9*B)*d^3)*cos(f*x +
e) + (12*B*d^3*cos(f*x + e)^3 + 15*(A - B)*c^3 - 45*(A - B)*c^2*d + 45*(A -
 B)*c*d^2 - 15*(A - B)*d^3 - 4*(15*B*c*d^2 + (5*A - 6*B)*d^3)*cos(f*x + e)^
2 - 60*(3*B*c^2*d + 3*(A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f*x + e))*sin(f*x
+ e))*sqrt(a*sin(f*x + e) + a))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e)
- 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2/sqrt(a*tan((f*x
+exp(1))/2)^2+a)/(a*tan((f*x+exp(1))/2)^2+a)^2*(tan((f*x+exp(1))/2)*(tan((f
*x+exp(1))/2)*(tan((f*x+exp(1))/2)*(tan((f*x+exp(1))/2)*(1/3600*tan((f*x+ex
p(1))/2)*(-2400*A*a*d^3*sign(tan((f*x+exp(1))/2)+1)+3960*B*a*d^3*sign(tan((
f*x+exp(1))/2)+1)+5400*A*a*c*d^2*sign(tan((f*x+exp(1))/2)+1)-7200*B*a*c*d^2
*sign(tan((f*x+exp(1))/2)+1)+5400*B*a*c^2*d*sign(tan((f*x+exp(1))/2)+1))+1/
3600*(3600*A*a*d^3*sign(tan((f*x+exp(1))/2)+1)-5400*B*a*d^3*sign(tan((f*x+e
xp(1))/2)+1)-5400*A*a*c*d^2*sign(tan((f*x+exp(1))/2)+1)+10800*B*a*c*d^2*sig
n(tan((f*x+exp(1))/2)+1)-5400*B*a*c^2*d*sign(tan((f*x+exp(1))/2)+1)))+1/360
0*(-6000*A*a*d^3*sign(tan((f*x+exp(1))/2)+1)+10800*B*a*d^3*sign(tan((f*x+ex
```

```
p(1))/2)+1)+10800*A*a*c*d^2*sign(tan((f*x+exp(1))/2)+1)-18000*B*a*c*d^2*sig
n(tan((f*x+exp(1))/2)+1)+10800*B*a*c^2*d*sign(tan((f*x+exp(1))/2)+1)))+1/36
00*(6000*A*a*d^3*sign(tan((f*x+exp(1))/2)+1)-10800*B*a*d^3*sign(tan((f*x+ex
p(1))/2)+1)-10800*A*a*c*d^2*sign(tan((f*x+exp(1))/2)+1)+18000*B*a*c*d^2*sig
n(tan((f*x+exp(1))/2)+1)-10800*B*a*c^2*d*sign(tan((f*x+exp(1))/2)+1)))+1/36
00*(-3600*A*a*d^3*sign(tan((f*x+exp(1))/2)+1)+5400*B*a*d^3*sign(tan((f*x+ex
p(1))/2)+1)+5400*A*a*c*d^2*sign(tan((f*x+exp(1))/2)+1)-10800*B*a*c*d^2*sign
(tan((f*x+exp(1))/2)+1)+5400*B*a*c^2*d*sign(tan((f*x+exp(1))/2)+1)))+1/3600
*(2400*A*a*d^3*sign(tan((f*x+exp(1))/2)+1)-3960*B*a*d^3*sign(tan((f*x+exp(1
))/2)+1)-5400*A*a*c*d^2*sign(tan((f*x+exp(1))/2)+1)+7200*B*a*c*d^2*sign(tan
((f*x+exp(1))/2)+1)-5400*B*a*c^2*d*sign(tan((f*x+exp(1))/2)+1)))+2*(1/4*(-3
*A*c^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+3*A
*d^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+3*B*c
^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3-3*B*d^3
*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3-9*A*c*d^2
*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+9*A*c^2*d
*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+9*B*c*d^2
*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3-9*B*c^2*d
*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+A*a*c^3*(
-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-A*a*d^3*(-sqr
t(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+A*sqrt(a)*c^3*(-s
qrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-A*sqrt(a)*d^3
*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-B*a*c^3*(
-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+B*a*d^3*(-sqr
t(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-B*sqrt(a)*c^3*(-s
qrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+B*sqrt(a)*d^3
*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+3*A*a*c*d
^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-3*A*a*c^2
*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+3*A*sqrt(
a)*c*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-3
*A*sqrt(a)*c^2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2
+a))^2-3*B*a*c*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)
^2+a))+3*B*a*c^2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)
^2+a))-3*B*sqrt(a)*c*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(
1))/2)^2+a))^2+3*B*sqrt(a)*c^2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((
f*x+exp(1))/2)^2+a))^2+A*sqrt(a)*a*c^3-A*sqrt(a)*a*d^3-B*sqrt(a)*a*c^3+B*sq
rt(a)*a*d^3+3*A*sqrt(a)*a*c*d^2-3*A*sqrt(a)*a*c^2*d-3*B*sqrt(a)*a*c*d^2+3*B
*sqrt(a)*a*c^2*d)/(-(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2
)^2+a))^2+2*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2
)^2+a))+a)^2/a/sign(tan((f*x+exp(1))/2)+1)+1/4*(A*c^3+11*A*d^3+3*B*c^3-15*B
*d^3-21*A*c*d^2+9*A*c^2*d+33*B*c*d^2-21*B*c^2*d)*atan((-sqrt(a)*tan((f*x+ex
p(1))/2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(2)
/sqrt(-a)/a/sign(tan((f*x+exp(1))/2)+1)))
```

maple [B] time = 1.87, size = 1030, normalized size = 3.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/60*(sin(f*x+e)*(-40*A*(a-a*sin(f*x+e))^(3/2)*a^(3/2)*d^3+360*A*c*d^2*a^(
5/2)*(a-a*sin(f*x+e))^(1/2)-120*A*a^(5/2)*d^3*(a-a*sin(f*x+e))^(1/2)+24*B*d
^3*(a-a*sin(f*x+e))^(5/2)*a^(1/2)-120*B*(a-a*sin(f*x+e))^(3/2)*a^(3/2)*c*d^
2+360*B*c^2*d*a^(5/2)*(a-a*sin(f*x+e))^(1/2)-360*B*a^(5/2)*c*d^2*(a-a*sin(f
*x+e))^(1/2)+240*B*a^(5/2)*d^3*(a-a*sin(f*x+e))^(1/2)+15*A*2^(1/2)*arctanh(
1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^3+135*A*2^(1/2)*arctanh(1
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^2*d-315*A*2^(1/2)*arctanh(
1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c*d^2+165*A*2^(1/2)*arctanh
(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*d^3+45*B*2^(1/2)*arctanh(1
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^3-315*B*2^(1/2)*arctanh(1/
2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^2*d+495*B*2^(1/2)*arctanh(1
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c*d^2-225*B*2^(1/2)*arctanh(
1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*d^3)-40*A*(a-a*sin(f*x+e))^
(3/2)*a^(3/2)*d^3+30*A*(a-a*sin(f*x+e))^(1/2)*a^(5/2)*c^3-90*A*(a-a*sin(f*x
+e))^(1/2)*a^(5/2)*c^2*d+450*A*c*d^2*a^(5/2)*(a-a*sin(f*x+e))^(1/2)-150*A*a
^(5/2)*d^3*(a-a*sin(f*x+e))^(1/2)+24*B*d^3*(a-a*sin(f*x+e))^(5/2)*a^(1/2)-1
20*B*(a-a*sin(f*x+e))^(3/2)*a^(3/2)*c*d^2-30*B*(a-a*sin(f*x+e))^(1/2)*a^(5/
2)*c^3+450*B*c^2*d*a^(5/2)*(a-a*sin(f*x+e))^(1/2)-450*B*a^(5/2)*c*d^2*(a-a*
sin(f*x+e))^(1/2)+270*B*a^(5/2)*d^3*(a-a*sin(f*x+e))^(1/2)+15*A*2^(1/2)*arc
tanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^3+135*A*2^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^2*d-315*A*2^(1/2)*arc
tanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c*d^2+165*A*2^(1/2)*ar
ctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*d^3+45*B*2^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^3-315*B*2^(1/2)*arcta
nh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^2*d+495*B*2^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c*d^2-225*B*2^(1/2)*arc
tanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*d^3)*(-a*(sin(f*x+e)-1
))^(1/2)/a^(9/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^3}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^3/(a*sin(f*x + e) + a)^
(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(3/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(3/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.315 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=203

$$-\frac{(c-d)(Ac+7Ad+3Bc-11Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}+\frac{d^2(3A-7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{6a^2f}+\frac{d(3A}{}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e))^(3/2)-1/4*(c-d)
*(A*c+7*A*d+3*B*c-11*B*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f
*x+e))^(1/2))/a^(3/2)/f*2^(1/2)+1/3*d*(3*A*c-9*A*d-15*B*c+13*B*d)*cos(f*x+e
)/a/f/(a+a*sin(f*x+e))^(1/2)+1/6*(3*A-7*B)*d^2*cos(f*x+e)*(a+a*sin(f*x+e))^
(1/2)/a^2/f
```

Rubi [A] time = 0.58, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2977, 2968, 3023, 2751, 2649, 206}

$$-\frac{(c-d)(Ac+7Ad+3Bc-11Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}+\frac{d^2(3A-7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{6a^2f}+\frac{d(3A}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^(3/2
),x]
```

```
[Out] -((c - d)*(A*c + 3*B*c + 7*A*d - 11*B*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sq
rt[2]*Sqrt[a + a*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*f) + (d*(3*A*c - 15*B*
c - 9*A*d + 13*B*d)*Cos[e + f*x])/(3*a*f*Sqrt[a + a*Sin[e + f*x]]) + ((3*A
- 7*B)*d^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(6*a^2*f) - ((A - B)*Cos[
e + f*x]*(c + d*Sin[e + f*x])^2)/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{(c+d\sin(e+fx))\left(\frac{1}{2}a(Ac\cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{\frac{1}{2}ac(Ac+3Bc+4Ad-4Bd\cdots}{\cdots}}{\cdots}$$

$$= \frac{(3A-7B)d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{6a^2f} - \frac{(A-B)\cos(e+f\cdots}{2f(a+a\,\mathrm{s}\cdots}$$

$$= \frac{d(3Ac-15Bc-9Ad+13Bd)\cos(e+fx)}{3af\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)d^2\cos(e\cdots}{\cdots}$$

$$= \frac{d(3Ac-15Bc-9Ad+13Bd)\cos(e+fx)}{3af\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)d^2\cos(e\cdots}{\cdots}$$

$$= -\frac{(c-d)(Ac+3Bc+7Ad-11Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} + \frac{d\cdots}{\cdots}$$

Mathematica [C] time = 0.74, size = 357, normalized size = 1.76

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(6(A-B)(c-d)^2\sin\left(\frac{1}{2}(e+fx)\right)-3(A-B)(c-d)^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\cdots\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(A - B)*(c - d)^2*Sin[(e + f*x)/2
] - 3*(A - B)*(c - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (3 + 3*I)*(
-1)^(3/4)*(c - d)*(A*c + 3*B*c + 7*A*d - 11*B*d)*ArcTanh[(1/2 + I/2)*(-1)^(
3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 6*d
*(-4*B*c - 2*A*d + 3*B*d)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^2 - 2*B*d^2*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^2 - 6*d*(-4*B*c - 2*A*d + 3*B*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^2 - 2*B*d^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sin[(3*(
e + f*x))/2]))/(6*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

fricas [B] time = 0.49, size = 584, normalized size = 2.88

$$-\frac{3\sqrt{2}\left(2\,(A+3\,B)c^2+4\,(3\,A-7\,B)cd-2\,(7\,A-11\,B)d^2-\left((A+3\,B)c^2+2\,(3\,A-7\,B)cd-(7\,A-11\,B)d^2\right)\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] -1/24*(3*sqrt(2)*(2*(A + 3*B)*c^2 + 4*(3*A - 7*B)*c*d - 2*(7*A - 11*B)*d^2
- ((A + 3*B)*c^2 + 2*(3*A - 7*B)*c*d - (7*A - 11*B)*d^2)*cos(f*x + e)^2 + (
(A + 3*B)*c^2 + 2*(3*A - 7*B)*c*d - (7*A - 11*B)*d^2)*cos(f*x + e) + (2*(A
+ 3*B)*c^2 + 4*(3*A - 7*B)*c*d - 2*(7*A - 11*B)*d^2 + ((A + 3*B)*c^2 + 2*(3
*A - 7*B)*c*d - (7*A - 11*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-
(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e
) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x +
 e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e)
 - 2)) + 4*(4*B*d^2*cos(f*x + e)^3 - 3*(A - B)*c^2 + 6*(A - B)*c*d - 3*(A -
 B)*d^2 - 4*(6*B*c*d + (3*A - 4*B)*d^2)*cos(f*x + e)^2 - 3*((A - B)*c^2 - 2
*(A - 5*B)*c*d + 5*(A - B)*d^2)*cos(f*x + e) - (4*B*d^2*cos(f*x + e)^2 - 3*
(A - B)*c^2 + 6*(A - B)*c*d - 3*(A - B)*d^2 + 12*(2*B*c*d + (A - B)*d^2)*co
s(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^2*f*cos(f*x + e)^2 -
 a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e)
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
```

```
(-4*pi/x/2)2/f*(2/sqrt(a*tan((f*x+exp(1))/2)^2+a)/(a*tan((f*x+exp(1))/2)^2+
a)*(tan((f*x+exp(1))/2)*(tan((f*x+exp(1))/2)*(-1/144*tan((f*x+exp(1))/2)*(-
72*A*d^2*sign(tan((f*x+exp(1))/2)+1)+96*B*d^2*sign(tan((f*x+exp(1))/2)+1)-1
44*B*c*d*sign(tan((f*x+exp(1))/2)+1))-1/144*(72*A*d^2*sign(tan((f*x+exp(1))
/2)+1)-144*B*d^2*sign(tan((f*x+exp(1))/2)+1)+144*B*c*d*sign(tan((f*x+exp(1)
)/2)+1)))-1/144*(-72*A*d^2*sign(tan((f*x+exp(1))/2)+1)+144*B*d^2*sign(tan((
f*x+exp(1))/2)+1)-144*B*c*d*sign(tan((f*x+exp(1))/2)+1)))-1/144*(72*A*d^2*s
ign(tan((f*x+exp(1))/2)+1)-96*B*d^2*sign(tan((f*x+exp(1))/2)+1)+144*B*c*d*s
ign(tan((f*x+exp(1))/2)+1)))+2*(1/4*(-3*A*c^2*(-sqrt(a)*tan((f*x+exp(1))/2)
+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3-3*A*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+s
qrt(a*tan((f*x+exp(1))/2)^2+a))^3+3*B*c^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqr
t(a*tan((f*x+exp(1))/2)^2+a))^3+3*B*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(
a*tan((f*x+exp(1))/2)^2+a))^3+6*A*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*
tan((f*x+exp(1))/2)^2+a))^3-6*B*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*ta
n((f*x+exp(1))/2)^2+a))^3+A*a*c^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan(
(f*x+exp(1))/2)^2+a))+A*a*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x
+exp(1))/2)^2+a))+A*sqrt(a)*c^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f
*x+exp(1))/2)^2+a))^2+A*sqrt(a)*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*ta
n((f*x+exp(1))/2)^2+a))^2-B*a*c^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan(
(f*x+exp(1))/2)^2+a))-B*a*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x
+exp(1))/2)^2+a))-B*sqrt(a)*c^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f
*x+exp(1))/2)^2+a))^2-B*sqrt(a)*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*ta
n((f*x+exp(1))/2)^2+a))^2-2*A*a*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*ta
n((f*x+exp(1))/2)^2+a))-2*A*sqrt(a)*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(
a*tan((f*x+exp(1))/2)^2+a))^2+2*B*a*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(
a*tan((f*x+exp(1))/2)^2+a))+2*B*sqrt(a)*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+s
qrt(a*tan((f*x+exp(1))/2)^2+a))^2+A*sqrt(a)*a*c^2+A*sqrt(a)*a*d^2-B*sqrt(a)
*a*c^2-B*sqrt(a)*a*d^2-2*A*sqrt(a)*a*c*d+2*B*sqrt(a)*a*c*d)/(-(-sqrt(a)*tan
((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+2*sqrt(a)*(-sqrt(a)*tan
((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+a)^2/a/sign(tan((f*x+exp(
1))/2)+1)+1/4*(A*c^2-7*A*d^2+3*B*c^2+11*B*d^2+6*A*c*d-14*B*c*d)*atan((-sqrt
(a)*tan((f*x+exp(1))/2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sq
rt(-a))/sqrt(2)/sqrt(-a)/a/sign(tan((f*x+exp(1))/2)+1)))
```

maple [B] time = 1.59, size = 694, normalized size = 3.42

$$-\frac{\left(\sin\left(fx+e\right)\left(3A\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right)a^{2}c^{2}+18A\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right)a^{2}cd-21A\sqrt{2}\ \operatorname{ar}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/12/a^(7/2)*(sin(f*x+e)*(3*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2
^(1/2)/a^(1/2))*a^2*c^2+18*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*a^2*c*d-21*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/
2)/a^(1/2))*a^2*d^2+24*A*d^2*a^(3/2)*(a-a*sin(f*x+e))^(1/2)+9*B*2^(1/2)*arc
tanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-42*B*2^(1/2)*arcta
nh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d+33*B*2^(1/2)*arctanh
(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2-8*B*d^2*(a-a*sin(f*x+e
))^(3/2)*a^(1/2)+48*B*c*d*a^(3/2)*(a-a*sin(f*x+e))^(1/2)-24*B*d^2*a^(3/2)*(
a-a*sin(f*x+e))^(1/2))+3*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/
2)/a^(1/2))*a^2*c^2+18*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*a^2*c*d-21*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a
^(1/2))*a^2*d^2+6*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^2-12*A*(a-a*sin(f*x+e)
)^(1/2)*a^(3/2)*c*d+30*A*d^2*a^(3/2)*(a-a*sin(f*x+e))^(1/2)+9*B*2^(1/2)*arc
tanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-42*B*2^(1/2)*arcta
nh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d+33*B*2^(1/2)*arctanh
(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2-8*B*d^2*(a-a*sin(f*x+e
))^(3/2)*a^(1/2)-6*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^2+60*B*c*d*a^(3/2)*(a
-a*sin(f*x+e))^(1/2)-30*B*d^2*a^(3/2)*(a-a*sin(f*x+e))^(1/2))*(-a*(sin(f*x+
e)-1))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2/(a*sin(f*x + e) + a)^
(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(3/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(3/2
), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.316 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=133

$$-\frac{(Ac+3Ad+3Bc-7Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}\,a^{3/2}f}-\frac{(A-B)(c-d)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}-\frac{2Bd\cos(e+fx)}{af\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -1/2*(A-B)*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)-1/4*(A*c+3*A*d+3*B*c-7
*B*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2
)/f*2^(1/2)-2*B*d*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2968, 3019, 2751, 2649, 206}

$$-\frac{(Ac+3Ad+3Bc-7Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}\,a^{3/2}f}-\frac{(A-B)(c-d)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}-\frac{2Bd\cos(e+fx)}{af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(3/2),
x]
```

```
[Out] -((A*c + 3*B*c + 3*A*d - 7*B*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqr
t[a + a*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*f) - ((A - B)*(c - d)*Cos[e + f
*x])/(2*f*(a + a*Sin[e + f*x])^(3/2)) - (2*B*d*Cos[e + f*x])/(a*f*Sqrt[a +
a*Sin[e + f*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{Ac+(Bc+Ad)\sin(e+fx)+Bd\sin^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a(3B(c-d)+A(c+3d))-2aBd\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,d}{2a^2}$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{2Bd\cos(e+fx)}{af\sqrt{a+a\sin(e+fx)}} + \frac{(Ac+3B}{}$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{2Bd\cos(e+fx)}{af\sqrt{a+a\sin(e+fx)}} - \frac{(Ac+3B}{}$$

$$= -\frac{(Ac+3Bc+3Ad-7Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{(A-B)(c}{2f(a+a}$$

Mathematica [C] time = 0.46, size = 246, normalized size = 1.85

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)(c-d)\sin\left(\frac{1}{2}(e+fx)\right)-(A-B)(c-d)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^
(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
- (A - B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3/4
)*(A*c + 3*B*c + 3*A*d - 7*B*d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e
 + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 4*B*d*Cos[(e + f*x)/
2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 4*B*d*Sin[(e + f*x)/2]*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^2))/(2*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

fricas [B] time = 0.48, size = 407, normalized size = 3.06

$$-\frac{\sqrt{2}\left(((A+3B)c+(3A-7B)d)\cos\left(fx+e\right)^{2}-2(A+3B)c-2(3A-7B)d-((A+3B)c+(3A-7B)d)\,\mathrm{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/8*(sqrt(2)*(((A + 3*B)*c + (3*A - 7*B)*d)*cos(f*x + e)^2 - 2*(A + 3*B)*c
 - 2*(3*A - 7*B)*d - ((A + 3*B)*c + (3*A - 7*B)*d)*cos(f*x + e) - (2*(A + 3
*B)*c + 2*(3*A - 7*B)*d + ((A + 3*B)*c + (3*A - 7*B)*d)*cos(f*x + e))*sin(f
*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a
)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x
 + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f
*x + e) - cos(f*x + e) - 2)) - 4*(4*B*d*cos(f*x + e)^2 + (A - B)*c - (A - B
)*d + ((A - B)*c - (A - 5*B)*d)*cos(f*x + e) + (4*B*d*cos(f*x + e) - (A - B
)*c + (A - B)*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^2*f*cos(f*x + e
)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x
 + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)2/f*(2*sqrt(a*tan((f*x+exp(1))/2)^2+a)*(-1/2*B*d/a/sig
n(tan((f*x+exp(1))/2)+1)+1/2*B*d*tan((f*x+exp(1))/2)/a/sign(tan((f*x+exp(1)
)/2)+1))/(a*tan((f*x+exp(1))/2)^2+a)+2*(1/4*(-3*A*c*(-sqrt(a)*tan((f*x+exp(
1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+3*A*d*(-sqrt(a)*tan((f*x+exp(1))/
2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+3*B*c*(-sqrt(a)*tan((f*x+exp(1))/2)+s
qrt(a*tan((f*x+exp(1))/2)^2+a))^3-3*B*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(
a*tan((f*x+exp(1))/2)^2+a))^3+A*a*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*ta
n((f*x+exp(1))/2)^2+a))-A*a*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x
+exp(1))/2)^2+a))+A*sqrt(a)*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x
+exp(1))/2)^2+a))^2-A*sqrt(a)*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f
*x+exp(1))/2)^2+a))^2-B*a*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+e
xp(1))/2)^2+a))+B*a*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))
/2)^2+a))-B*sqrt(a)*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))
/2)^2+a))^2+B*sqrt(a)*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1
))/2)^2+a))^2+A*sqrt(a)*a*c-A*sqrt(a)*a*d-B*sqrt(a)*a*c+B*sqrt(a)*a*d)/(-(-
sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+2*sqrt(a)*(-
sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+a)^2/a/sign(ta
n((f*x+exp(1))/2)+1)+1/4*(A*c+3*A*d+3*B*c-7*B*d)*atan((-sqrt(a)*tan((f*x+ex
p(1))/2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(2)
/sqrt(-a)/a/sign(tan((f*x+exp(1))/2)+1)))
```

maple [B] time = 1.32, size = 389, normalized size = 2.92

$$-\frac{\left(\sin\left(fx+e\right)\left(A\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right)ac+3A\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right)ad+3B\sqrt{2}\ \operatorname{arctanh}\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/4/a^(5/2)*(sin(f*x+e)*(A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1
/2)/a^(1/2))*a*c+3*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*a*d+3*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a
*c-7*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d+8*B*
(a-a*sin(f*x+e))^(1/2)*a^(1/2)*d)+A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1
/2)*2^(1/2)/a^(1/2))*a*c+3*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(
```

```
1/2)/a^(1/2))*a*d+3*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^
(1/2))*a*c-7*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*
a*d+2*A*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*c-2*A*(a-a*sin(f*x+e))^(1/2)*a^(1/2)
*d-2*B*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*c+10*B*(a-a*sin(f*x+e))^(1/2)*a^(1/2)
*d)*(-a*(sin(f*x+e)-1))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(3
/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(c+d\,\sin\left(e+f\,x\right)\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(3/2),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(3/2),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))/(a*(sin(e + f*x) + 1))**
(3/2), x)
```

3.317 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=87

$$-\frac{(A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}\,a^{3/2}f}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)-1/4*(A+3*B)*arctanh(1/2*cos(
f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/f*2^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2750, 2649, 206}

$$-\frac{(A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}\,a^{3/2}f}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((A + 3*B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]
])]))/(2*Sqrt[2]*a^(3/2)*f) - ((A - B)*Cos[e + f*x])/(2*f*(a + a*Sin[e + f*x
])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
```

```
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}}+\frac{(A+3B)\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{4a}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}}-\frac{(A+3B)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{2af}$$

$$= -\frac{(A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}f}-\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}}$$

Mathematica [C] time = 0.20, size = 150, normalized size = 1.72

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+(B-A)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)+\cdots\right.}{2f(a(\sin(e+\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*Sin[(e + f*x)/2] + (-A +
B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3/4)*(A + 3*B)*Arc
Tanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^2))/(2*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

fricas [B] time = 0.44, size = 293, normalized size = 3.37

$$\frac{\sqrt{2}\left((A+3B)\cos\left(fx+e\right)^2-(A+3B)\cos\left(fx+e\right)-\left((A+3B)\cos\left(fx+e\right)+2A+6B\right)\sin\left(fx+e\right)-2A\cdots\right.}{8\left(a^2f\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*((A + 3*B)*cos(f*x + e)^2 - (A + 3*B)*cos(f*x + e) - ((A + 3*B
)*cos(f*x + e) + 2*A + 6*B)*sin(f*x + e) - 2*A - 6*B)*sqrt(a)*log(-(a*cos(f
*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(
f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*
a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) +
 4*((A - B)*cos(f*x + e) - (A - B)*sin(f*x + e) + A - B)*sqrt(a*sin(f*x + e
) + a))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f
*x + e) + 2*a^2*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*2*(1/4*(-3*A*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(
1))/2)^2+a))^3+3*B*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)
^2+a))^3+A*a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))
+A*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2
-B*a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-B*sqrt(
a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+A*sqrt(
a)*a-B*sqrt(a)*a)/(-(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2
)^2+a))^2+2*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2
)^2+a))+a)^2/a/sign(tan((f*x+exp(1))/2)+1)+1/4*(A+3*B)*atan((-sqrt(a)*tan((
f*x+exp(1))/2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/s
qrt(2)/sqrt(-a)/a/sign(tan((f*x+exp(1))/2)+1))
```

maple [B] time = 1.09, size = 176, normalized size = 2.02

$$-\frac{\left(\sin\left(fx+e\right)\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right)a\left(A+3B\right)+A\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\ a+3B\operatorname{arctanh}\left(\right.\right.}{4a^{\frac{5}{2}}\cos\left(fx+e\right)\sqrt{a-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/4/a^(5/2)*(sin(f*x+e)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*a*(A+3*B)+A*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2
^(1/2)*a+3*B*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a+
```

```
2*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*A-2*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*B)*(-a*
(sin(f*x+e)-1))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)/(a*sin(f*x + e) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(3/2),x)
[Out] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\sin\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2),x)
[Out] Integral((A + B*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(3/2), x)
```

3.318 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=187

$$-\frac{(A(c-5d)+B(3c+d))\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^2}+\frac{2\sqrt{d}(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f(c-d)^2\sqrt{c+d}}-\frac{(A-B)\mathrm{co}}{2f(c-d)(a\sin}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(3/2)-1/4*(A*(c-5*d)+B*(3*c+
d))*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/
(c-d)^2/f*2^(1/2)+2*(-A*d+B*c)*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/
2)/(a+a*sin(f*x+e))^(1/2))*d^(1/2)/a^(3/2)/(c-d)^2/f/(c+d)^(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2978, 2985, 2649, 206, 2773, 208}

$$-\frac{(A(c-5d)+B(3c+d))\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^2}+\frac{2\sqrt{d}(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f(c-d)^2\sqrt{c+d}}-\frac{(A-B)\mathrm{co}}{2f(c-d)(a\sin}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])),
x]
```

```
[Out] -((A*(c - 5*d) + B*(3*c + d))*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[
a + a*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*(c - d)^2*f) + (2*Sqrt[d]*(B*c -
A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e +
 f*x]])])/(a^(3/2)*(c - d)^2*Sqrt[c + d]*f) - ((A - B)*Cos[e + f*x])/(2*(c
- d)*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx = -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a(3Bc+A(c-4d))-\frac{1}{2}a(A-B)d\sin(}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx}}{2a^2(c-d)}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}} - \frac{(d(Bc-Ad))\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}}{a^2(c-d)^2}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}} + \frac{(2d(Bc-Ad))\,\mathrm{Subst}\left(\int \frac{1}{ac+ad}\right.}{a(c-d)}$$

$$= -\frac{(A(c-5d)+B(3c+d))\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}\,a^{3/2}(c-d)^2 f} + \frac{2\sqrt{d}\,(Bc-}{}$$

Mathematica [C] time = 3.14, size = 419, normalized size = 2.24

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)(c-d)\sin\left(\frac{1}{2}(e+fx)\right)+(B-A)(c-d)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f
*x])),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
+ (-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3/
4)*(A*(c - 5*d) + B*(3*c + d))*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e
+ f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (Sqrt[d]*(B*c - A*d)*
(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d
] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^2)/Sqrt[c + d] + (Sqrt[d]*(-(B*c) + A*d)*(e + f*x - 2
*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*
Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^2)/Sqrt[c + d]))/(2*(c - d)^2*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

fricas [B] time = 3.29, size = 1561, normalized size = 8.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*(((A + 3*B)*c - (5*A - B)*d)*cos(f*x + e)^2 - 2*(A + 3*B)*c +
 2*(5*A - B)*d - ((A + 3*B)*c - (5*A - B)*d)*cos(f*x + e) - (2*(A + 3*B)*c
- 2*(5*A - B)*d + ((A + 3*B)*c - (5*A - B)*d)*cos(f*x + e))*sin(f*x + e))*s
qrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*
(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*
a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) -
cos(f*x + e) - 2)) + 4*(2*B*a*c - 2*A*a*d - (B*a*c - A*a*d)*cos(f*x + e)^2
+ (B*a*c - A*a*d)*cos(f*x + e) + (2*B*a*c - 2*A*a*d + (B*a*c - A*a*d)*cos(f
*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d
 + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f*x + e)^
2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d
 + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)
*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x +
e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d
^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2
 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c
*d - d^2)*sin(f*x + e))) + 4*((A - B)*c - (A - B)*d + ((A - B)*c - (A - B)*
d)*cos(f*x + e) - ((A - B)*c - (A - B)*d)*sin(f*x + e))*sqrt(a*sin(f*x + e)
 + a))/((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e)^2 - (a^2*c^2 - 2*a^2
*c*d + a^2*d^2)*f*cos(f*x + e) - 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f - ((a^
2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e) + 2*(a^2*c^2 - 2*a^2*c*d + a^2*
d^2)*f)*sin(f*x + e)), 1/8*(sqrt(2)*(((A + 3*B)*c - (5*A - B)*d)*cos(f*x +
e)^2 - 2*(A + 3*B)*c + 2*(5*A - B)*d - ((A + 3*B)*c - (5*A - B)*d)*cos(f*x
+ e) - (2*(A + 3*B)*c - 2*(5*A - B)*d + ((A + 3*B)*c - (5*A - B)*d)*cos(f*x
 + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(
f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e)
- (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e
) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 8*(2*B*a*c - 2*A*a*d - (B*a*c -
A*a*d)*cos(f*x + e)^2 + (B*a*c - A*a*d)*cos(f*x + e) + (2*B*a*c - 2*A*a*d +
 (B*a*c - A*a*d)*cos(f*x + e))*sin(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/
2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/
(d*cos(f*x + e))) + 4*((A - B)*c - (A - B)*d + ((A - B)*c - (A - B)*d)*cos(
f*x + e) - ((A - B)*c - (A - B)*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/
((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e)^2 - (a^2*c^2 - 2*a^2*c*d +
a^2*d^2)*f*cos(f*x + e) - 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f - ((a^2*c^2 -
 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e) + 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f)
*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_no
step/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_
nostep/2)Warning, integration of abs or sign assumes constant sign by inter
vals (correct if the argument is real):Check [abs(cos((f*t_nostep+exp(1))/2
-pi/4))]Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Discontinuities at zeroes
of cos((f*t_nostep+exp(1))/2-pi/4) were not checkedUnable to check sign: (4
*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
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ep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
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(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of
abs or sign assumes constant sign by intervals (correct if the argument is
real):Check [abs(t_nostep+1)]Warning, assuming -c^3*d^5-c^2*d^6+c*d^7+d^8-c
*d^7-d^8 is positive. Hint: run assume to make assumptions on a variableWar
ning, assuming -a*c^3*d^5-a*c^2*d^6+a*c*d^7+a*d^8-a*c*d^7-a*d^8 is positive
. Hint: run assume to make assumptions on a variableWarning, need to choose
 a branch for the root of a polynomial with parameters. This might be wrong
.Non regular value [0] was discarded and replaced randomly by 0=[-90]Evalua
tion time: 67.84sym2poly/r2sym(const gen & e,const index_m & i,const vecteu
r & l) Error: Bad Argument Value
```

maple [B] time = 2.01, size = 624, normalized size = 3.34

$$-\frac{\left(\sin\left(fx+e\right)\left(8A\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,d}{\sqrt{acd+a\,d^{2}}}\right)a^{\frac{3}{2}}d^{2}-8B\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\,d}{\sqrt{acd+a\,d^{2}}}\right)a^{\frac{3}{2}}cd+A\sqrt{a\left(c+d\right)d}\,\sqrt{2}\;\mathrm{arc}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] -1/4/a^(5/2)*(sin(f*x+e)*(8*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2
)^(1/2))*a^(3/2)*d^2-8*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/
2))*a^(3/2)*c*d+A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1
/2)*2^(1/2)/a^(1/2))*a*c-5*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin
(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d+3*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*c+B*(a*(c+d)*d)^(1/2)*2^(1/2)*
arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d)+8*A*arctanh((a-a*s
in(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(3/2)*d^2-8*B*arctanh((a-a*sin(f*
x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(3/2)*c*d+A*(a*(c+d)*d)^(1/2)*2^(1/2)*
arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*c-5*A*(a*(c+d)*d)^(1/
2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d+3*B*(a*(
c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a
*c+B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a
^(1/2))*a*d+2*A*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*c-2*A*(a*(
c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*d-2*B*(a*(c+d)*d)^(1/2)*(a-a*s
in(f*x+e))^(1/2)*a^(1/2)*c+2*B*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*a^(
1/2)*d)*(-a*(sin(f*x+e)-1))^(1/2)/(a*(c+d)*d)^(1/2)/(c-d)^2/cos(f*x+e)/(a+a
*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))),
x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e)),x)
[Out] Timed out
```

3.319 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=292

$$-\frac{\sqrt{d}\left(Ad(5c+3d)-B\left(3c^2+3cd+2d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f(c-d)^3(c+d)^{3/2}}-\frac{(Ac-9Ad+3Bc+5Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos}{\sqrt{2}\sqrt{a\sin}}\right.}{2\sqrt{2}a^{3/2}f(c-d)^3}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))-1/4*(
A*c-9*A*d+3*B*c+5*B*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+
e))^(1/2))/a^(3/2)/(c-d)^3/f*2^(1/2)-(A*d*(5*c+3*d)-B*(3*c^2+3*c*d+2*d^2))*
arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))*d^(1
/2)/a^(3/2)/(c-d)^3/(c+d)^(3/2)/f+1/2*d*(B*(3*c+d)-A*(c+3*d))*cos(f*x+e)/a/
(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.02, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {2978, 2984, 2985, 2649, 206, 2773, 208}

$$-\frac{\sqrt{d}\left(Ad(5c+3d)-B\left(3c^2+3cd+2d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f(c-d)^3(c+d)^{3/2}}-\frac{(Ac-9Ad+3Bc+5Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos}{\sqrt{2}\sqrt{a\sin}}\right.}{2\sqrt{2}a^{3/2}f(c-d)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2
),x]
```

```
[Out] -((A*c + 3*B*c - 9*A*d + 5*B*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqr
t[a + a*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*(c - d)^3*f) - (Sqrt[d]*(A*d*(5
*c + 3*d) - B*(3*c^2 + 3*c*d + 2*d^2))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x
])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(a^(3/2)*(c - d)^3*(c + d)^(3/2
)*f) - ((A - B)*Cos[e + f*x])/(2*(c - d)*f*(a + a*Sin[e + f*x])^(3/2)*(c +
d*Sin[e + f*x])) + (d*(B*(3*c + d) - A*(c + 3*d))*Cos[e + f*x])/(2*a*(c - d
)^2*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
```

```
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx = -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} - \frac{\int \frac{-\frac{1}{2}a(Ac+}{\sqrt{a}}}{}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} + \frac{}{2a(c-d)^2}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} + \frac{}{2a(c-d)^2}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} + \frac{}{2a(c-d)^2}$$

$$= -\frac{(Ac+3Bc-9Ad+5Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^3f} - \frac{\sqrt{d}\,(Ad}{}$$

Mathematica [C] time = 9.54, size = 542, normalized size = 1.86

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{\sqrt{d}\left(B\left(3c^2+3cd+2d^2\right)-Ad(5c+3d)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{}\right.\right.\right.}{(c+d)^{3/2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f
*x])^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(4*(A - B)*(c - d)*Sin[(e + f*x)/2]
+ 2*(-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (2 + 2*I)*(-1)
^(3/4)*(A*c + 3*B*c - 9*A*d + 5*B*d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + T
an[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (Sqrt[d]*(-(A*d
*(5*c + 3*d)) + B*(3*c^2 + 3*c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4
]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - S
```

```
qrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(c + d)
^(3/2) + (Sqrt[d]*(A*d*(5*c + 3*d) - B*(3*c^2 + 3*c*d + 2*d^2))*(e + f*x -
2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]
*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^2)/(c + d)^(3/2) + (4*(c - d)*d*(B*c - A*d)*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/((c + d)*(c + d*
Sin[e + f*x]))))/(4*(c - d)^3*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

fricas [B] time = 7.67, size = 3403, normalized size = 11.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*(2*(A + 3*B)*c^3 - 2*(7*A - 11*B)*c^2*d - 2*(17*A - 13*B)*c*d
^2 - 2*(9*A - 5*B)*d^3 - ((A + 3*B)*c^2*d - 8*(A - B)*c*d^2 - (9*A - 5*B)*d
^3)*cos(f*x + e)^3 - ((A + 3*B)*c^3 - 2*(3*A - 7*B)*c^2*d - (25*A - 21*B)*c
*d^2 - 2*(9*A - 5*B)*d^3)*cos(f*x + e)^2 + ((A + 3*B)*c^3 - (7*A - 11*B)*c^
2*d - (17*A - 13*B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*x + e) + (2*(A + 3*B)*c^
3 - 2*(7*A - 11*B)*c^2*d - 2*(17*A - 13*B)*c*d^2 - 2*(9*A - 5*B)*d^3 - ((A
+ 3*B)*c^2*d - 8*(A - B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*x + e)^2 + ((A + 3*
B)*c^3 - (7*A - 11*B)*c^2*d - (17*A - 13*B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*
x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin
(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e)
 - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x +
e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 2*(6*B*a*c^3 - 2*(5*A - 6*B)*a*
c^2*d - 2*(8*A - 5*B)*a*c*d^2 - 2*(3*A - 2*B)*a*d^3 - (3*B*a*c^2*d - (5*A -
 3*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e)^3 - (3*B*a*c^3 - (5*A - 9*B
)*a*c^2*d - (13*A - 8*B)*a*c*d^2 - 2*(3*A - 2*B)*a*d^3)*cos(f*x + e)^2 + (3
*B*a*c^3 - (5*A - 6*B)*a*c^2*d - (8*A - 5*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*c
os(f*x + e) + (6*B*a*c^3 - 2*(5*A - 6*B)*a*c^2*d - 2*(8*A - 5*B)*a*c*d^2 -
2*(3*A - 2*B)*a*d^3 - (3*B*a*c^2*d - (5*A - 3*B)*a*c*d^2 - (3*A - 2*B)*a*d^
3)*cos(f*x + e)^2 + (3*B*a*c^3 - (5*A - 6*B)*a*c^2*d - (8*A - 5*B)*a*c*d^2
- (3*A - 2*B)*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d
^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - 4*
((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*c
os(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e)
)*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(
f*x + e) + (d^2*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(
f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2
- c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*
d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*((A - B)*c^3 - (A -
B)*c^2*d - (A - B)*c*d^2 + (A - B)*d^3 + ((A - 3*B)*c^2*d + 2*(A + B)*c*d^2
```

```
 - (3*A - B)*d^3)*cos(f*x + e)^2 + ((A - B)*c^3 - 2*B*c^2*d + (A + 3*B)*c*d
^2 - 2*A*d^3)*cos(f*x + e) - ((A - B)*c^3 - (A - B)*c^2*d - (A - B)*c*d^2 +
 (A - B)*d^3 - ((A - 3*B)*c^2*d + 2*(A + B)*c*d^2 - (3*A - B)*d^3)*cos(f*x
+ e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^2*c^4*d - 2*a^2*c^3*d^2 +
 2*a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)^3 + (a^2*c^5 - 4*a^2*c^3*d^2 + 2*a^2
*c^2*d^3 + 3*a^2*c*d^4 - 2*a^2*d^5)*f*cos(f*x + e)^2 - (a^2*c^5 - a^2*c^4*d
 - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e) - 2*
(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)
*f + ((a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)^2
- (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^
5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3
+ a^2*c*d^4 - a^2*d^5)*f)*sin(f*x + e)), 1/8*(sqrt(2)*(2*(A + 3*B)*c^3 - 2*
(7*A - 11*B)*c^2*d - 2*(17*A - 13*B)*c*d^2 - 2*(9*A - 5*B)*d^3 - ((A + 3*B)
*c^2*d - 8*(A - B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*x + e)^3 - ((A + 3*B)*c^3
 - 2*(3*A - 7*B)*c^2*d - (25*A - 21*B)*c*d^2 - 2*(9*A - 5*B)*d^3)*cos(f*x +
 e)^2 + ((A + 3*B)*c^3 - (7*A - 11*B)*c^2*d - (17*A - 13*B)*c*d^2 - (9*A -
5*B)*d^3)*cos(f*x + e) + (2*(A + 3*B)*c^3 - 2*(7*A - 11*B)*c^2*d - 2*(17*A
- 13*B)*c*d^2 - 2*(9*A - 5*B)*d^3 - ((A + 3*B)*c^2*d - 8*(A - B)*c*d^2 - (9
*A - 5*B)*d^3)*cos(f*x + e)^2 + ((A + 3*B)*c^3 - (7*A - 11*B)*c^2*d - (17*A
 - 13*B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-
(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e
) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x +
 e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e)
 - 2)) - 4*(6*B*a*c^3 - 2*(5*A - 6*B)*a*c^2*d - 2*(8*A - 5*B)*a*c*d^2 - 2*(
3*A - 2*B)*a*d^3 - (3*B*a*c^2*d - (5*A - 3*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*
cos(f*x + e)^3 - (3*B*a*c^3 - (5*A - 9*B)*a*c^2*d - (13*A - 8*B)*a*c*d^2 -
2*(3*A - 2*B)*a*d^3)*cos(f*x + e)^2 + (3*B*a*c^3 - (5*A - 6*B)*a*c^2*d - (8
*A - 5*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e) + (6*B*a*c^3 - 2*(5*A -
 6*B)*a*c^2*d - 2*(8*A - 5*B)*a*c*d^2 - 2*(3*A - 2*B)*a*d^3 - (3*B*a*c^2*d
- (5*A - 3*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e)^2 + (3*B*a*c^3 - (5
*A - 6*B)*a*c^2*d - (8*A - 5*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e))*
sin(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*s
in(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e))) + 4*((A - B)*
c^3 - (A - B)*c^2*d - (A - B)*c*d^2 + (A - B)*d^3 + ((A - 3*B)*c^2*d + 2*(A
 + B)*c*d^2 - (3*A - B)*d^3)*cos(f*x + e)^2 + ((A - B)*c^3 - 2*B*c^2*d + (A
 + 3*B)*c*d^2 - 2*A*d^3)*cos(f*x + e) - ((A - B)*c^3 - (A - B)*c^2*d - (A -
 B)*c*d^2 + (A - B)*d^3 - ((A - 3*B)*c^2*d + 2*(A + B)*c*d^2 - (3*A - B)*d^
3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^2*c^4*d - 2*a^
2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)^3 + (a^2*c^5 - 4*a^2*c^3*
d^2 + 2*a^2*c^2*d^3 + 3*a^2*c*d^4 - 2*a^2*d^5)*f*cos(f*x + e)^2 - (a^2*c^5
- a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f*cos(f*
x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4
 - a^2*d^5)*f + ((a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos(
f*x + e)^2 - (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d
^4 - a^2*d^5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a
```

```
^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assu
mes constant sign by intervals (correct if the argument is real):Check [abs
(cos((f*t_nostep+exp(1))/2-pi/4))]Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Discontinuities at zeroes of cos((f*t_nostep+exp(1))/2-pi/4) were not check
edUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
```

```
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Warning, integration of abs or sign assumes constant sign by intervals (c
orrect if the argument is real):Check [abs(t_nostep+1)]Evaluation time: 1.1
4index.cc index_m i_lex_is_greater Error: Bad Argument Value
```

maple [B] time = 3.07, size = 2049, normalized size = 7.02

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -1/4/a^(5/2)*(-17*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1
))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a*c*d^2+11*B*(a*(c+d)*d)^(1/2)*2^(1/2)
*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a*c^2*d-
20*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*x
+e)*c*d^3-9*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/
2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a*d^3+5*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(
1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a*c*d^2-6*B*(-a*(sin(f*x+e)-
1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^2*d+A*(a*(c+d)*d)^(1/2)*2^
(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a*c
^3+3*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1
/2)/a^(1/2))*sin(f*x+e)*a*c^3-9*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a
*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a*d^3+5*B*(a*(c+d)*d)^
(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*
x+e)^2*a*d^3+4*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*
x+e)*c*d^2+8*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1
/2)*2^(1/2)/a^(1/2))*a*c^2*d+2*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2
)*a^(1/2)*sin(f*x+e)*c^2*d+4*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*
a^(1/2)*sin(f*x+e)*c*d^2-8*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin
(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a*c^2*d-9*A*(a*(c+d)*d)^(1/2)*2^(1/2)*ar
ctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a*c*d^2+5*B*(a*(c+d)*d
)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(
f*x+e)*a*d^3+13*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))
^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a*c*d^2-8*A*(a*(c+d)*d)^(1/2)*2^(1/2)*ar
ctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a*c*d^2+3
*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/
a^(1/2))*sin(f*x+e)^2*a*c^2*d+A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(
sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a*c^2*d+8*B*(a*(c+d)*d)^
(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*
x+e)^2*a*c*d^2-7*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1)
)^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a*c^2*d-12*B*arctanh((-a*(sin(f*x+e)-1)
)^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*x+e)*c^3*d+20*A*arctanh((-a*(sin
(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*x+e)*c^2*d^2+20*A*arct
anh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*x+e)^2*c*d
^3+32*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(
f*x+e)*c*d^3-12*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^
(3/2)*sin(f*x+e)^2*c^2*d^2-12*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d
)*d)^(1/2))*a^(3/2)*sin(f*x+e)^2*c*d^3+2*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+
d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*d^3+3*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/
2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a*c^3-4*B*(-a*(sin(f*x+e)-1))^
(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^2*d+6*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d
)*d)^(1/2)*a^(1/2)*c*d^2+2*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^
(1/2)*c*d^2-6*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x
+e)*d^3+A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2
^(1/2)/a^(1/2))*a*c^3-24*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^
(1/2))*a^(3/2)*sin(f*x+e)*c^2*d^2+20*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/
```

```
(a*(c+d)*d)^(1/2))*a^(3/2)*c^2*d^2+2*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d
)^(1/2)*a^(1/2)*c^3-4*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)
*d^3-2*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^3+12*A*arcta
nh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*c*d^3-12*B*arctan
h((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*c^3*d-12*B*arctanh
((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*c^2*d^2-8*B*arctanh
((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*c*d^3+12*A*arctanh(
(-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*x+e)^2*d^4-8*B
*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*x+e)^
2*d^4+12*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*s
in(f*x+e)*d^4-8*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^
(3/2)*sin(f*x+e)*d^4)*(-a*(sin(f*x+e)-1))^(1/2)/(a*(c+d)*d)^(1/2)/(c+d*sin(
f*x+e))/(c+d)/(c-d)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.320 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=402

$$-\frac{\sqrt{d}\left(Ad\left(35c^2+42cd+19d^2\right)-3B\left(5c^3+10c^2d+13cd^2+4d^3\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4a^{3/2}f(c-d)^4(c+d)^{5/2}}-\frac{(A(c-13d)+3}{}$$

```
[Out] -1/2*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2-1/4
*(A*(c-13*d)+3*B*(c+3*d))*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f
*x+e))^(1/2))/a^(3/2)/(c-d)^4/f*2^(1/2)-1/4*(A*d*(35*c^2+42*c*d+19*d^2)-3*B
*(5*c^3+10*c^2*d+13*c*d^2+4*d^3))*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^
(1/2)/(a+a*sin(f*x+e))^(1/2))*d^(1/2)/a^(3/2)/(c-d)^4/(c+d)^(5/2)/f+1/2*d*(
B*(2*c+d)-A*(c+2*d))*cos(f*x+e)/a/(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))^2/(a+a*s
in(f*x+e))^(1/2)+1/4*d*(3*B*(3*c^2+3*c*d+2*d^2)-A*(2*c^2+15*c*d+7*d^2))*cos
(f*x+e)/a/(c-d)^3/(c+d)^2/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.56, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {2978, 2984, 2985, 2649, 206, 2773, 208}

$$-\frac{\sqrt{d}\left(Ad\left(35c^2+42cd+19d^2\right)-3B\left(10c^2d+5c^3+13cd^2+4d^3\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4a^{3/2}f(c-d)^4(c+d)^{5/2}}-\frac{(A(c-13d)+3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^3
),x]
```

```
[Out] -((A*(c - 13*d) + 3*B*(c + 3*d))*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sq
rt[a + a*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*(c - d)^4*f) - (Sqrt[d]*(A*d*(
35*c^2 + 42*c*d + 19*d^2) - 3*B*(5*c^3 + 10*c^2*d + 13*c*d^2 + 4*d^3))*ArcT
anh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])
/(4*a^(3/2)*(c - d)^4*(c + d)^(5/2)*f) - ((A - B)*Cos[e + f*x])/(2*(c - d)*
f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2) + (d*(B*(2*c + d) - A*
(c + 2*d))*Cos[e + f*x])/(2*a*(c - d)^2*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*
(c + d*Sin[e + f*x])^2) + (d*(3*B*(3*c^2 + 3*c*d + 2*d^2) - A*(2*c^2 + 15*c
*d + 7*d^2))*Cos[e + f*x])/(4*a*(c - d)^3*(c + d)^2*f*Sqrt[a + a*Sin[e + f*
x]]*(c + d*Sin[e + f*x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx = -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} - \frac{\int \frac{-\frac{1}{2}a(A}{}}{}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} + \frac{}{2a(c-}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} + \frac{}{2a(c-}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} + \frac{}{2a(c-}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} + \frac{}{2a(c-}$$

$$= -\frac{(A(c-13d)+3B(c+3d))\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^4f} - \frac{\sqrt{d}\,(}{}$$

Mathematica [C] time = 13.48, size = 1395, normalized size = 3.47

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f
*x])^3),x]
```

```
[Out] ((1 + I)*(A*c + 3*B*c - 13*A*d + 9*B*d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[
(e + f*x)/4]*(Cos[(e + f*x)/4] - Sin[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^3)/((2*(-1)^(1/4)*c^4 - 8*(-1)^(1/4)*c^3*d + 12*(-1)^(1/4)*c
^2*d^2 - 8*(-1)^(1/4)*c*d^3 + 2*(-1)^(1/4)*d^4)*f*(a*(1 + Sin[e + f*x]))^(3
/2)) + (Sqrt[d]*(-(A*d*(35*c^2 + 42*c*d + 19*d^2)) + 3*B*(5*c^3 + 10*c^2*d
+ 13*c*d^2 + 4*d^3))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e +
f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2
])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/(16*(c - d)^4*(c + d)^(5/2)*f
*(a*(1 + Sin[e + f*x]))^(3/2)) - (Sqrt[d]*(-(A*d*(35*c^2 + 42*c*d + 19*d^2)
) + 3*B*(5*c^3 + 10*c^2*d + 13*c*d^2 + 4*d^3))*(e + f*x - 2*Log[Sec[(e + f*
x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)/2]
 + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/(16
*(c - d)^4*(c + d)^(5/2)*f*(a*(1 + Sin[e + f*x]))^(3/2)) + ((Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])*(-8*A*c^4*Cos[(e + f*x)/2] + 8*B*c^4*Cos[(e + f*x)/2
] - 8*A*c^3*d*Cos[(e + f*x)/2] + 26*B*c^3*d*Cos[(e + f*x)/2] - 22*A*c^2*d^2
*Cos[(e + f*x)/2] + 6*B*c^2*d^2*Cos[(e + f*x)/2] - 10*A*c*d^3*Cos[(e + f*x)
/2] + 4*B*c*d^3*Cos[(e + f*x)/2] + 4*B*d^4*Cos[(e + f*x)/2] - 8*A*c^3*d*Cos
[(3*(e + f*x))/2] + 26*B*c^3*d*Cos[(3*(e + f*x))/2] - 40*A*c^2*d^2*Cos[(3*(
e + f*x))/2] + 31*B*c^2*d^2*Cos[(3*(e + f*x))/2] - 25*A*c*d^3*Cos[(3*(e + f
*x))/2] + 13*B*c*d^3*Cos[(3*(e + f*x))/2] + A*d^4*Cos[(3*(e + f*x))/2] + 2*
B*d^4*Cos[(3*(e + f*x))/2] + 2*A*c^2*d^2*Cos[(5*(e + f*x))/2] - 9*B*c^2*d^2
*Cos[(5*(e + f*x))/2] + 15*A*c*d^3*Cos[(5*(e + f*x))/2] - 9*B*c*d^3*Cos[(5*
(e + f*x))/2] + 7*A*d^4*Cos[(5*(e + f*x))/2] - 6*B*d^4*Cos[(5*(e + f*x))/2]
 + 8*A*c^4*Sin[(e + f*x)/2] - 8*B*c^4*Sin[(e + f*x)/2] + 8*A*c^3*d*Sin[(e +
 f*x)/2] - 26*B*c^3*d*Sin[(e + f*x)/2] + 22*A*c^2*d^2*Sin[(e + f*x)/2] - 6*
B*c^2*d^2*Sin[(e + f*x)/2] + 10*A*c*d^3*Sin[(e + f*x)/2] - 4*B*c*d^3*Sin[(e
 + f*x)/2] - 4*B*d^4*Sin[(e + f*x)/2] - 8*A*c^3*d*Sin[(3*(e + f*x))/2] + 26
*B*c^3*d*Sin[(3*(e + f*x))/2] - 40*A*c^2*d^2*Sin[(3*(e + f*x))/2] + 31*B*c^
2*d^2*Sin[(3*(e + f*x))/2] - 25*A*c*d^3*Sin[(3*(e + f*x))/2] + 13*B*c*d^3*S
in[(3*(e + f*x))/2] + A*d^4*Sin[(3*(e + f*x))/2] + 2*B*d^4*Sin[(3*(e + f*x)
)/2] - 2*A*c^2*d^2*Sin[(5*(e + f*x))/2] + 9*B*c^2*d^2*Sin[(5*(e + f*x))/2]
- 15*A*c*d^3*Sin[(5*(e + f*x))/2] + 9*B*c*d^3*Sin[(5*(e + f*x))/2] - 7*A*d^
4*Sin[(5*(e + f*x))/2] + 6*B*d^4*Sin[(5*(e + f*x))/2]))/(16*(c - d)^3*(c +
d)^2*f*(a*(1 + Sin[e + f*x]))^(3/2)*(c + d*Sin[e + f*x])^2)
```

fricas [B] time = 15.27, size = 5864, normalized size = 14.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] [1/16*(2*sqrt(2)*(2*(A + 3*B)*c^5 - 6*(3*A - 7*B)*c^4*d - 4*(23*A - 27*B)*c
^3*d^2 - 4*(37*A - 33*B)*c^2*d^3 - 6*(17*A - 13*B)*c*d^4 - 2*(13*A - 9*B)*d
```

```
^5 + ((A + 3*B)*c^3*d^2 - (11*A - 15*B)*c^2*d^3 - (25*A - 21*B)*c*d^4 - (13
*A - 9*B)*d^5)*cos(f*x + e)^4 - (2*(A + 3*B)*c^4*d - 3*(7*A - 11*B)*c^3*d^2
 - (61*A - 57*B)*c^2*d^3 - 3*(17*A - 13*B)*c*d^4 - (13*A - 9*B)*d^5)*cos(f*
x + e)^3 - ((A + 3*B)*c^5 - (7*A - 27*B)*c^4*d - 6*(11*A - 15*B)*c^3*d^2 -
2*(73*A - 69*B)*c^2*d^3 - (127*A - 99*B)*c*d^4 - 3*(13*A - 9*B)*d^5)*cos(f*
x + e)^2 + ((A + 3*B)*c^5 - 3*(3*A - 7*B)*c^4*d - 2*(23*A - 27*B)*c^3*d^2 -
 2*(37*A - 33*B)*c^2*d^3 - 3*(17*A - 13*B)*c*d^4 - (13*A - 9*B)*d^5)*cos(f*
x + e) + (2*(A + 3*B)*c^5 - 6*(3*A - 7*B)*c^4*d - 4*(23*A - 27*B)*c^3*d^2 -
 4*(37*A - 33*B)*c^2*d^3 - 6*(17*A - 13*B)*c*d^4 - 2*(13*A - 9*B)*d^5 - ((A
 + 3*B)*c^3*d^2 - (11*A - 15*B)*c^2*d^3 - (25*A - 21*B)*c*d^4 - (13*A - 9*B
)*d^5)*cos(f*x + e)^3 - 2*((A + 3*B)*c^4*d - 2*(5*A - 9*B)*c^3*d^2 - 36*(A
- B)*c^2*d^3 - 2*(19*A - 15*B)*c*d^4 - (13*A - 9*B)*d^5)*cos(f*x + e)^2 + (
(A + 3*B)*c^5 - 3*(3*A - 7*B)*c^4*d - 2*(23*A - 27*B)*c^3*d^2 - 2*(37*A - 3
3*B)*c^2*d^3 - 3*(17*A - 13*B)*c*d^4 - (13*A - 9*B)*d^5)*cos(f*x + e))*sin(
f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) +
a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*
x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(
f*x + e) - cos(f*x + e) - 2)) + (30*B*a*c^5 - 10*(7*A - 12*B)*a*c^4*d - 4*(
56*A - 57*B)*a*c^3*d^2 - 12*(23*A - 20*B)*a*c^2*d^3 - 2*(80*A - 63*B)*a*c*d
^4 - 2*(19*A - 12*B)*a*d^5 + (15*B*a*c^3*d^2 - 5*(7*A - 6*B)*a*c^2*d^3 - 3*
(14*A - 13*B)*a*c*d^4 - (19*A - 12*B)*a*d^5)*cos(f*x + e)^4 - (30*B*a*c^4*d
 - 5*(14*A - 15*B)*a*c^3*d^2 - (119*A - 108*B)*a*c^2*d^3 - (80*A - 63*B)*a*
c*d^4 - (19*A - 12*B)*a*d^5)*cos(f*x + e)^3 - (15*B*a*c^5 - 5*(7*A - 18*B)*
a*c^4*d - 2*(91*A - 102*B)*a*c^3*d^2 - 2*(146*A - 129*B)*a*c^2*d^3 - (202*A
 - 165*B)*a*c*d^4 - 3*(19*A - 12*B)*a*d^5)*cos(f*x + e)^2 + (15*B*a*c^5 - 5
*(7*A - 12*B)*a*c^4*d - 2*(56*A - 57*B)*a*c^3*d^2 - 6*(23*A - 20*B)*a*c^2*d
^3 - (80*A - 63*B)*a*c*d^4 - (19*A - 12*B)*a*d^5)*cos(f*x + e) + (30*B*a*c^
5 - 10*(7*A - 12*B)*a*c^4*d - 4*(56*A - 57*B)*a*c^3*d^2 - 12*(23*A - 20*B)*
a*c^2*d^3 - 2*(80*A - 63*B)*a*c*d^4 - 2*(19*A - 12*B)*a*d^5 - (15*B*a*c^3*d
^2 - 5*(7*A - 6*B)*a*c^2*d^3 - 3*(14*A - 13*B)*a*c*d^4 - (19*A - 12*B)*a*d^
5)*cos(f*x + e)^3 - 2*(15*B*a*c^4*d - 5*(7*A - 9*B)*a*c^3*d^2 - (77*A - 69*
B)*a*c^2*d^3 - (61*A - 51*B)*a*c*d^4 - (19*A - 12*B)*a*d^5)*cos(f*x + e)^2
+ (15*B*a*c^5 - 5*(7*A - 12*B)*a*c^4*d - 2*(56*A - 57*B)*a*c^3*d^2 - 6*(23*
A - 20*B)*a*c^2*d^3 - (80*A - 63*B)*a*c*d^4 - (19*A - 12*B)*a*d^5)*cos(f*x
+ e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d +
7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 4*((c*d + d^2)*cos(f*x + e)^2 -
 c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d +
3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sq
rt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^
2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*
cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 +
d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d
- d^2)*sin(f*x + e))) - 4*(2*(A - B)*c^5 - 2*(A - B)*c^4*d - 4*(A - B)*c^3*
d^2 + 4*(A - B)*c^2*d^3 + 2*(A - B)*c*d^4 - 2*(A - B)*d^5 - ((2*A - 9*B)*c^
3*d^2 + 13*A*c^2*d^3 - (8*A - 3*B)*c*d^4 - (7*A - 6*B)*d^5)*cos(f*x + e)^3
```

+ ((4*A - 13*B)*c^4*d + (15*A + 2*B)*c^3*d^2 - (14*A - 9*B)*c^2*d^3 - (9*A - 4*B)*c*d^4 + 2*(2*A - B)*d^5)*cos(f*x + e)^2 + (2*(A - B)*c^5 + (2*A - 11*B)*c^4*d + (13*A - 3*B)*c^3*d^2 + (3*A + 5*B)*c^2*d^3 - 5*(3*A - B)*c*d^4 - (5*A - 6*B)*d^5)*cos(f*x + e) - (2*(A - B)*c^5 - 2*(A - B)*c^4*d - 4*(A - B)*c^3*d^2 + 4*(A - B)*c^2*d^3 + 2*(A - B)*c*d^4 - 2*(A - B)*d^5 - ((2*A - 9*B)*c^3*d^2 + 13*A*c^2*d^3 - (8*A - 3*B)*c*d^4 - (7*A - 6*B)*d^5)*cos(f*x + e)^2 - ((4*A - 13*B)*c^4*d + (17*A - 7*B)*c^3*d^2 - (A - 9*B)*c^2*d^3 - (17*A - 7*B)*c*d^4 - (3*A - 4*B)*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^2*c^6*d^2 - 2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*a^2*c^3*d^5 - a^2*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^4 - (2*a^2*c^7*d - 3*a^2*c^6*d^2 - 4*a^2*c^5*d^3 + 7*a^2*c^4*d^4 + 2*a^2*c^3*d^5 - 5*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x + e)^3 - (a^2*c^8 + 2*a^2*c^7*d - 6*a^2*c^6*d^2 - 6*a^2*c^5*d^3 + 12*a^2*c^4*d^4 + 6*a^2*c^3*d^5 - 10*a^2*c^2*d^6 - 2*a^2*c*d^7 + 3*a^2*d^8)*f*cos(f*x + e)^2 + (a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x + e) + 2*(a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f - ((a^2*c^6*d^2 - 2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*a^2*c^3*d^5 - a^2*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^3 + 2*(a^2*c^7*d - a^2*c^6*d^2 - 3*a^2*c^5*d^3 + 3*a^2*c^4*d^4 + 3*a^2*c^3*d^5 - 3*a^2*c^2*d^6 - a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^2 - (a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x + e) - 2*(a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f)*sin(f*x + e)), 1/8*(sqrt(2)*(2*(A + 3*B)*c^5 - 6*(3*A - 7*B)*c^4*d - 4*(23*A - 27*B)*c^3*d^2 - 4*(37*A - 33*B)*c^2*d^3 - 6*(17*A - 13*B)*c*d^4 - 2*(13*A - 9*B)*d^5 + ((A + 3*B)*c^3*d^2 - (11*A - 15*B)*c^2*d^3 - (25*A - 21*B)*c*d^4 - (13*A - 9*B)*d^5)*cos(f*x + e)^4 - (2*(A + 3*B)*c^4*d - 3*(7*A - 11*B)*c^3*d^2 - (61*A - 57*B)*c^2*d^3 - 3*(17*A - 13*B)*c*d^4 - (13*A - 9*B)*d^5)*cos(f*x + e)^3 - ((A + 3*B)*c^5 - (7*A - 27*B)*c^4*d - 6*(11*A - 15*B)*c^3*d^2 - 2*(73*A - 69*B)*c^2*d^3 - (127*A - 99*B)*c*d^4 - 3*(13*A - 9*B)*d^5)*cos(f*x + e)^2 + ((A + 3*B)*c^5 - 3*(3*A - 7*B)*c^4*d - 2*(23*A - 27*B)*c^3*d^2 - 2*(37*A - 33*B)*c^2*d^3 - 3*(17*A - 13*B)*c*d^4 - (13*A - 9*B)*d^5)*cos(f*x + e) + (2*(A + 3*B)*c^5 - 6*(3*A - 7*B)*c^4*d - 4*(23*A - 27*B)*c^3*d^2 - 4*(37*A - 33*B)*c^2*d^3 - 6*(17*A - 13*B)*c*d^4 - 2*(13*A - 9*B)*d^5 - ((A + 3*B)*c^3*d^2 - (11*A - 15*B)*c^2*d^3 - (25*A - 21*B)*c*d^4 - (13*A - 9*B)*d^5)*cos(f*x + e)^3 - 2*((A + 3*B)*c^4*d - 2*(5*A - 9*B)*c^3*d^2 - 36*(A - B)*c^2*d^3 - 2*(19*A - 15*B)*c*d^4 - (13*A - 9*B)*d^5)*cos(f*x + e)^2 + ((A + 3*B)*c^5 - 3*(3*A - 7*B)*c^4*d - 2*(23*A - 27*B)*c^3*d^2 - 2*(37*A - 33*B)*c^2*d^3 - 3*(17*A - 13*B)*c*d^4 - (13*A - 9*B)*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + (30*B*a*c^5 - 10*(7*A - 12*B)*a*c^4*d - 4*(56*A - 57*B)*a*c^3*d^2 - 12*(23*A - 20*B)*a*c^2*d^3 - 2*(80*A - 63*B)*a*c*d^4 - 2*(19*A - 12*B)*a*d^5 + (15*B*a*c^3*d^2 - 5*(7*A - 6*B)*a*c^2*d^3 - 3*(14*A - 13*B)*a*c*d^4 - (19*A - 12*B)*a*d^5)*cos(f*x + e)^4 - (30*B*a*c^4*d - 5*(14*A - 15*B)*a*c^3*d^2 - (119*A - 108*B)*a*c^2*d^3 - (80

```
*A - 63*B)*a*c*d^4 - (19*A - 12*B)*a*d^5)*cos(f*x + e)^3 - (15*B*a*c^5 - 5*
(7*A - 18*B)*a*c^4*d - 2*(91*A - 102*B)*a*c^3*d^2 - 2*(146*A - 129*B)*a*c^2
*d^3 - (202*A - 165*B)*a*c*d^4 - 3*(19*A - 12*B)*a*d^5)*cos(f*x + e)^2 + (1
5*B*a*c^5 - 5*(7*A - 12*B)*a*c^4*d - 2*(56*A - 57*B)*a*c^3*d^2 - 6*(23*A -
20*B)*a*c^2*d^3 - (80*A - 63*B)*a*c*d^4 - (19*A - 12*B)*a*d^5)*cos(f*x + e)
 + (30*B*a*c^5 - 10*(7*A - 12*B)*a*c^4*d - 4*(56*A - 57*B)*a*c^3*d^2 - 12*(
23*A - 20*B)*a*c^2*d^3 - 2*(80*A - 63*B)*a*c*d^4 - 2*(19*A - 12*B)*a*d^5 -
(15*B*a*c^3*d^2 - 5*(7*A - 6*B)*a*c^2*d^3 - 3*(14*A - 13*B)*a*c*d^4 - (19*A
 - 12*B)*a*d^5)*cos(f*x + e)^3 - 2*(15*B*a*c^4*d - 5*(7*A - 9*B)*a*c^3*d^2
- (77*A - 69*B)*a*c^2*d^3 - (61*A - 51*B)*a*c*d^4 - (19*A - 12*B)*a*d^5)*co
s(f*x + e)^2 + (15*B*a*c^5 - 5*(7*A - 12*B)*a*c^4*d - 2*(56*A - 57*B)*a*c^3
*d^2 - 6*(23*A - 20*B)*a*c^2*d^3 - (80*A - 63*B)*a*c*d^4 - (19*A - 12*B)*a*
d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin
(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x +
 e))) - 2*(2*(A - B)*c^5 - 2*(A - B)*c^4*d - 4*(A - B)*c^3*d^2 + 4*(A - B)*
c^2*d^3 + 2*(A - B)*c*d^4 - 2*(A - B)*d^5 - ((2*A - 9*B)*c^3*d^2 + 13*A*c^2
*d^3 - (8*A - 3*B)*c*d^4 - (7*A - 6*B)*d^5)*cos(f*x + e)^3 + ((4*A - 13*B)*
c^4*d + (15*A + 2*B)*c^3*d^2 - (14*A - 9*B)*c^2*d^3 - (9*A - 4*B)*c*d^4 + 2
*(2*A - B)*d^5)*cos(f*x + e)^2 + (2*(A - B)*c^5 + (2*A - 11*B)*c^4*d + (13*
A - 3*B)*c^3*d^2 + (3*A + 5*B)*c^2*d^3 - 5*(3*A - B)*c*d^4 - (5*A - 6*B)*d^
5)*cos(f*x + e) - (2*(A - B)*c^5 - 2*(A - B)*c^4*d - 4*(A - B)*c^3*d^2 + 4*
(A - B)*c^2*d^3 + 2*(A - B)*c*d^4 - 2*(A - B)*d^5 - ((2*A - 9*B)*c^3*d^2 +
13*A*c^2*d^3 - (8*A - 3*B)*c*d^4 - (7*A - 6*B)*d^5)*cos(f*x + e)^2 - ((4*A
- 13*B)*c^4*d + (17*A - 7*B)*c^3*d^2 - (A - 9*B)*c^2*d^3 - (17*A - 7*B)*c*d
^4 - (3*A - 4*B)*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))
/((a^2*c^6*d^2 - 2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*a^2*c^3*d^5 - a^2*c^2*d^6
- 2*a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^4 - (2*a^2*c^7*d - 3*a^2*c^6*d^2 -
4*a^2*c^5*d^3 + 7*a^2*c^4*d^4 + 2*a^2*c^3*d^5 - 5*a^2*c^2*d^6 + a^2*d^8)*f*
cos(f*x + e)^3 - (a^2*c^8 + 2*a^2*c^7*d - 6*a^2*c^6*d^2 - 6*a^2*c^5*d^3 + 1
2*a^2*c^4*d^4 + 6*a^2*c^3*d^5 - 10*a^2*c^2*d^6 - 2*a^2*c*d^7 + 3*a^2*d^8)*f
*cos(f*x + e)^2 + (a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6
+ a^2*d^8)*f*cos(f*x + e) + 2*(a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*
a^2*c^2*d^6 + a^2*d^8)*f - ((a^2*c^6*d^2 - 2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*
a^2*c^3*d^5 - a^2*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^3 + 2*(a^
2*c^7*d - a^2*c^6*d^2 - 3*a^2*c^5*d^3 + 3*a^2*c^4*d^4 + 3*a^2*c^3*d^5 - 3*a
^2*c^2*d^6 - a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^2 - (a^2*c^8 - 4*a^2*c^6*d
^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x + e) - 2*(a^2*c^8 -
 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, alg
```

```
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integ
ration of abs or sign assumes constant sign by intervals (correct if the ar
gument is real):Check [abs(cos((f*t_nostep+exp(1))/2-pi/4))]Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Discontinuities at zeroes of cos((f*t_nostep+exp(
1))/2-pi/4) were not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/
t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (
4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>
(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
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x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
```

```
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
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i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
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gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
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/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
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able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes con
stant sign by intervals (correct if the argument is real):Check [abs(t_nost
ep+1)]Evaluation time: 3.24Error: Bad Argument Type
```

maple [B] time = 3.97, size = 4707, normalized size = 11.71

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/4/a^(7/2)*(-a*(sin(f*x+e)-1))^(1/2)*(11*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arcta
nh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^2*d^3-
2*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)
/a^(1/2))*sin(f*x+e)^2*a^2*c^4*d-112*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/
(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^3*d^3-6*B*(a*(c+d)*d)^(1/2)*2^(1/2)
```

arctanh(1/2(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^4*d-33*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^3*d^2-57*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^2*d^3-39*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c*d^4+21*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^3*d^2-3*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^3*d^2-15*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^2*d^3-21*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c*d^4+9*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^4*d+47*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^3*d^2+63*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^2*d^3+25*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c*d^4-51*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^3*d^2-51*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^2*d^3-18*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c*d^4-A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^3*d^2+26*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c*d^4-21*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^4*d+61*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^2*d^3+51*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c*d^4-4*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*d^5+3*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*d^5+15*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^5*d+2*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^5-7*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c*d^4-2*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*c^3*d^2-2*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*c^2*d^3+2*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*c*d^4+2*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*c^3*d^2+2*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*c^2*d^3-2*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*c*d^4+11*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^2*d^3-6*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c*d^4-A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^5-7*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^3*d^2+2*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^2*d^3+B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d

)^(1/2)*a^(1/2)*sin(f*x+e)*c*d^4-3*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*
(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^5+11*A*(a*(c+d)
*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^
2*c^4*d+25*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2
)*2^(1/2)/a^(1/2))*a^2*c^3*d^2+13*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(
-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d^3-15*B*(a*(c+d)*d)^(1/2
)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^4*d-
21*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2
)/a^(1/2))*a^2*c^3*d^2-9*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f
*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d^3-9*B*(a*(c+d)*d)^(1/2)*2^(1/2)*
arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*d^5
+13*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/
2)/a^(1/2))*sin(f*x+e)^3*a^2*d^5-9*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*
(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*d^5+13*A*(a*(c+
d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*
sin(f*x+e)^2*a^2*d^5+13*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/
2)*sin(f*x+e)*c^4*d+7*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)
*sin(f*x+e)*c^3*d^2-4*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)
*sin(f*x+e)*c^4*d-17*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*
sin(f*x+e)*c^3*d^2+A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*si
n(f*x+e)*c^2*d^3+12*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2)
)*a^(5/2)*c^2*d^4-19*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2
))*a^(5/2)*sin(f*x+e)^3*d^6+12*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+
d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*d^6-19*A*arctanh((-a*(sin(f*x+e)-1))^(1/2
)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*d^6+12*B*arctanh((-a*(sin(f*x+e
)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*d^6-35*A*arctanh((-a*
(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^4*d^2-42*A*arctanh((-a
*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^3*d^3-19*A*arctanh((-
a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^2*d^4-5*A*(-a*(sin(f
x+e)-1))^(3/2)(a*(c+d)*d)^(1/2)*a^(1/2)*d^5+4*B*(-a*(sin(f*x+e)-1))^(3/2)
(a(c+d)*d)^(1/2)*a^(1/2)*d^5+39*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*
(c+d)*d)^(1/2))*a^(5/2)*c^3*d^3+30*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a
*(c+d)*d)^(1/2))*a^(5/2)*c^4*d^2-2*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^
(1/2)*a^(3/2)*c^5-103*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/
2))*a^(5/2)*sin(f*x+e)*c^2*d^4-38*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*
(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c*d^5-2*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(
c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*d^5+15*B*arctanh((-a*(sin(f*x+e)-1))^(1/
2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^5*d+B*(-a*(sin(f*x+e)-1))^(1/2
)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^3*d^2+3*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*
d)^(1/2)*a^(3/2)*sin(f*x+e)*d^5-4*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(
1/2)*a^(3/2)*sin(f*x+e)*d^5-2*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)
*a^(3/2)*c^4*d-11*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^3
d^2-A(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^2*d^3+13*A*(-a
*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c*d^4+11*B*(-a*(sin(f*x+e)
-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^4*d+108*B*arctanh((-a*(sin(f*x+e)-1)

```
)^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^2*d^4+11*A*(-a*(sin(f*x
+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^2*d^3-6*A*(-a*(sin(f*x+e)-1))^(3/
2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c*d^4-A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*
(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^5-7*B*(-a*(sin(f*x+e)-1))^
(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^3*d^2+2*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c
+d)*d)^(1/2)*a^(1/2)*c^2*d^3-35*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c
+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*c^2*d^4-42*A*arctanh((-a*(sin(f*x+e)-1))
^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*c*d^5+15*B*arctanh((-a*(si
n(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*c^3*d^3+30*B*a
rctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*
c^2*d^4+39*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)
*sin(f*x+e)^3*c*d^5+63*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1
/2))*a^(5/2)*sin(f*x+e)^2*c*d^5+2*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(
1/2)*a^(3/2)*sin(f*x+e)^2*d^5-35*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(
c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^4*d^2-80*A*arctanh((-a*(sin(f*x+e)-1))^
(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c*d^5+30*B*arctanh((-a*(sin
(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^4*d^2+75*B*ar
ctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c
^3*d^3-7*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^2*d^3-3*B*
(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c*d^4-5*A*(-a*(sin(f*x+
e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*d^5+4*B*(-a*(sin(f*x+e)-1
))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*d^5+60*B*arctanh((-a*(sin(f*x
+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^4*d^2+99*B*arctanh(
(-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^3*d^3+9
0*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+
e)*c^2*d^4+24*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5
/2)*sin(f*x+e)*c*d^5+B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*
c*d^4-3*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2
^(1/2)/a^(1/2))*a^2*c^5-70*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d
)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^3*d^3-119*A*arctanh((-a*(sin(f*x+e)-1))^(1/
2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^2*d^4+17*A*(-a*(sin(f*x+e)-1
))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c*d^4-9*B*(-a*(sin(f*x+e)-1))
^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c^2*d^3)/(a*(c+d)*d)^(1/2)/(c+d
*sin(f*x+e))^2/(c+d)^2/(c-d)^4/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")

[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\sin\left(e + f\,x\right)}{{\left(a + a\sin\left(e + f\,x\right)\right)}^{3/2}\,{\left(c + d\sin\left(e + f\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.321 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=308

$$-\frac{(c-d)\left(3A\left(c^2+6cd+25d^2\right)+B\left(5c^2+62cd-163d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}\,a^{5/2}f}+\frac{d^2(9Ac+39Ad+15Bc-9}{}$$

```
[Out] -1/16*(3*A*c+9*A*d+5*B*c-17*B*d)*cos(f*x+e)*(c+d*sin(f*x+e))^2/a/f/(a+a*sin
(f*x+e))^(3/2)-1/4*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^3/f/(a+a*sin(f*x+e))^(
5/2)-1/32*(c-d)*(B*(5*c^2+62*c*d-163*d^2)+3*A*(c^2+6*c*d+25*d^2))*arctanh(1
/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)+1/2
4*d*(A*(9*c^2+36*c*d-93*d^2)+B*(15*c^2-228*c*d+197*d^2))*cos(f*x+e)/a^2/f/(
a+a*sin(f*x+e))^(1/2)+1/48*d^2*(9*A*c+39*A*d+15*B*c-95*B*d)*cos(f*x+e)*(a+a
*sin(f*x+e))^(1/2)/a^3/f
```

Rubi [A] time = 1.06, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2977, 2968, 3023, 2751, 2649, 206}

$$\frac{d\left(A\left(9c^2+36cd-93d^2\right)+B\left(15c^2-228cd+197d^2\right)\right)\cos(e+fx)}{24a^2f\sqrt{a\sin(e+fx)+a}}-\frac{(c-d)\left(3A\left(c^2+6cd+25d^2\right)+B\left(5c^2+62c\right.\right.}{16\sqrt{2}\,a^{5/}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^(5/2
),x]
```

```
[Out] -((c - d)*(B*(5*c^2 + 62*c*d - 163*d^2) + 3*A*(c^2 + 6*c*d + 25*d^2))*ArcTa
nh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(16*Sqrt[2]*
a^(5/2)*f) + (d*(A*(9*c^2 + 36*c*d - 93*d^2) + B*(15*c^2 - 228*c*d + 197*d^
2))*Cos[e + f*x])/(24*a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (d^2*(9*A*c + 15*B*
c + 39*A*d - 95*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(48*a^3*f) - ((
3*A*c + 5*B*c + 9*A*d - 17*B*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(16*a*
f*(a + a*Sin[e + f*x])^(3/2)) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^
3)/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a+a\sin(e+fx))^{5/2}} + \int \frac{(c+d\sin(e+fx))^2\left(\frac{1}{2}a(3A\right.}{}$$

$$= -\frac{(3Ac+5Bc+9Ad-17Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{16af(a+a\sin(e+fx))^{3/2}} - ($$

$$= -\frac{(3Ac+5Bc+9Ad-17Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{16af(a+a\sin(e+fx))^{3/2}} - ($$

$$= \frac{d^2(9Ac+15Bc+39Ad-95Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{48a^3f}$$

$$= \frac{d\left(A\left(9c^2+36cd-93d^2\right)+B\left(15c^2-228cd+197d^2\right)\right)\cos(e+f}{24a^2f\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{d\left(A\left(9c^2+36cd-93d^2\right)+B\left(15c^2-228cd+197d^2\right)\right)\cos(e+f}{24a^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{(c-d)\left(B\left(5c^2+62cd-163d^2\right)+3A\left(c^2+6cd+25d^2\right)\right)\tanh^{-}}{16\sqrt{2}\,a^{5/2}f}$$

Mathematica [C] time = 1.79, size = 523, normalized size = 1.70

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left((3+3i)(-1)^{3/4}(c-d)\left(3A\left(c^2+6cd+25d^2\right)+B\left(5c^2+62cd-163d^2\right)\right)\left(\sin\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(24*(A - B)*(c - d)^3*Sin[(e + f*x)/
2] - 12*(A - B)*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 6*(c - d)
^2*(B*(5*c - 29*d) + 3*A*(c + 7*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^2 - 3*(c - d)^2*(B*(5*c - 29*d) + 3*A*(c + 7*d))*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^3 + (3 + 3*I)*(-1)^(3/4)*(c - d)*(B*(5*c^2 + 62
```

```
*c*d - 163*d^2) + 3*A*(c^2 + 6*c*d + 25*d^2))*ArcTanh[(1/2 + I/2)*(-1)^(3/4
)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 16*B*d
^3*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (24 + 24*
I)*d^2*(-6*B*c - 2*A*d + 5*B*d)*(Cos[(e + f*x)/2] + I*Sin[(e + f*x)/2])*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (24 + 24*I)*d^2*(6*B*c + 2*A*d - 5*B
*d)*(I*Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])^4 - 16*B*d^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sin[(3*(e + f*x
))/2]))/(48*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [B] time = 0.50, size = 980, normalized size = 3.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] -1/192*(3*sqrt(2)*(4*(3*A + 5*B)*c^3 + 12*(5*A + 19*B)*c^2*d + 12*(19*A - 7
5*B)*c*d^2 - 4*(75*A - 163*B)*d^3 - ((3*A + 5*B)*c^3 + 3*(5*A + 19*B)*c^2*d
 + 3*(19*A - 75*B)*c*d^2 - (75*A - 163*B)*d^3)*cos(f*x + e)^3 - 3*((3*A + 5
*B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(19*A - 75*B)*c*d^2 - (75*A - 163*B)*d^3
)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(19*A - 75
*B)*c*d^2 - (75*A - 163*B)*d^3)*cos(f*x + e) + (4*(3*A + 5*B)*c^3 + 12*(5*A
 + 19*B)*c^2*d + 12*(19*A - 75*B)*c*d^2 - 4*(75*A - 163*B)*d^3 - ((3*A + 5*
B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(19*A - 75*B)*c*d^2 - (75*A - 163*B)*d^3)
*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(19*A - 75*
B)*c*d^2 - (75*A - 163*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*
cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) -
 sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e)
 + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) -
2)) + 4*(32*B*d^3*cos(f*x + e)^4 - 12*(A - B)*c^3 + 36*(A - B)*c^2*d - 36*(
A - B)*c*d^2 + 12*(A - B)*d^3 + 32*(9*B*c*d^2 + (3*A - 5*B)*d^3)*cos(f*x +
e)^3 - 3*((3*A + 5*B)*c^3 + 3*(5*A - 13*B)*c^2*d - 3*(13*A - 53*B)*c*d^2 +
(53*A - 93*B)*d^3)*cos(f*x + e)^2 - 3*((7*A + B)*c^3 + 3*(A - 9*B)*c^2*d -
27*(A - 9*B)*c*d^2 + 9*(9*A - 17*B)*d^3)*cos(f*x + e) + (32*B*d^3*cos(f*x +
 e)^3 + 12*(A - B)*c^3 - 36*(A - B)*c^2*d + 36*(A - B)*c*d^2 - 12*(A - B)*d
^3 - 96*(3*B*c*d^2 + (A - 2*B)*d^3)*cos(f*x + e)^2 - 3*((3*A + 5*B)*c^3 + 3
*(5*A - 13*B)*c^2*d - 3*(13*A - 85*B)*c*d^2 + (85*A - 157*B)*d^3)*cos(f*x +
 e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^3*f*cos(f*x + e)^3 + 3*a^3*
f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 -
 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)2/f*(2/sqrt(a*tan((f*x+exp(1))/2)^2+a)/(a*tan((f*x+exp(1))/2)^2+
a)*(tan((f*x+exp(1))/2)*(tan((f*x+exp(1))/2)*(-1/144*tan((f*x+exp(1))/2)*(-
72*A*a^3*d^3*sign(tan((f*x+exp(1))/2)+1)+168*B*a^3*d^3*sign(tan((f*x+exp(1)
)/2)+1)-216*B*a^3*c*d^2*sign(tan((f*x+exp(1))/2)+1))/a^4-1/144*(72*A*a^3*d^
3*sign(tan((f*x+exp(1))/2)+1)-216*B*a^3*d^3*sign(tan((f*x+exp(1))/2)+1)+216
*B*a^3*c*d^2*sign(tan((f*x+exp(1))/2)+1))/a^4)-1/144*(-72*A*a^3*d^3*sign(ta
n((f*x+exp(1))/2)+1)+216*B*a^3*d^3*sign(tan((f*x+exp(1))/2)+1)-216*B*a^3*c*
d^2*sign(tan((f*x+exp(1))/2)+1))/a^4)-1/144*(72*A*a^3*d^3*sign(tan((f*x+exp
(1))/2)+1)-168*B*a^3*d^3*sign(tan((f*x+exp(1))/2)+1)+216*B*a^3*c*d^2*sign(t
an((f*x+exp(1))/2)+1))/a^4)+2*(1/32*(-29*A*c^3*(-sqrt(a)*tan((f*x+exp(1))/2
)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^7-43*A*d^3*(-sqrt(a)*tan((f*x+exp(1))/2)
+sqrt(a*tan((f*x+exp(1))/2)^2+a))^7+5*B*c^3*(-sqrt(a)*tan((f*x+exp(1))/2)+s
qrt(a*tan((f*x+exp(1))/2)^2+a))^7+67*B*d^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sq
rt(a*tan((f*x+exp(1))/2)^2+a))^7+57*A*c*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+s
qrt(a*tan((f*x+exp(1))/2)^2+a))^7+15*A*c^2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+
sqrt(a*tan((f*x+exp(1))/2)^2+a))^7-129*B*c*d^2*(-sqrt(a)*tan((f*x+exp(1))/2
)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^7+57*B*c^2*d*(-sqrt(a)*tan((f*x+exp(1))/
2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^7+75*A*sqrt(a)*c^3*(-sqrt(a)*tan((f*x+e
xp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^6+237*A*sqrt(a)*d^3*(-sqrt(a)*ta
n((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^6+29*B*sqrt(a)*c^3*(-sqr
t(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^6-341*B*sqrt(a)*d
^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^6-399*A*s
qrt(a)*c*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))
^6+87*A*sqrt(a)*c^2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))
/2)^2+a))^6+711*B*sqrt(a)*c*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f
*x+exp(1))/2)^2+a))^6-399*B*sqrt(a)*c^2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqr
t(a*tan((f*x+exp(1))/2)^2+a))^6-55*A*a*c^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sq
rt(a*tan((f*x+exp(1))/2)^2+a))^5-161*A*a*d^3*(-sqrt(a)*tan((f*x+exp(1))/2)+
sqrt(a*tan((f*x+exp(1))/2)^2+a))^5-17*B*a*c^3*(-sqrt(a)*tan((f*x+exp(1))/2)
+sqrt(a*tan((f*x+exp(1))/2)^2+a))^5+233*B*a*d^3*(-sqrt(a)*tan((f*x+exp(1))/
2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^5+267*A*a*c*d^2*(-sqrt(a)*tan((f*x+exp(
```

1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^5-51*A*a*c^2*d*(-sqrt(a)*tan((f*x+e
xp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^5-483*B*a*c*d^2*(-sqrt(a)*tan((f
*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^5+267*B*a*c^2*d*(-sqrt(a)*ta
n((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^5-91*A*sqrt(a)*a*c^3*(-s
qrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4-221*A*sqrt(a)
*a*d^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4-13*
B*sqrt(a)*a*c^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+
a))^4+325*B*sqrt(a)*a*d^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp
(1))/2)^2+a))^4+351*A*sqrt(a)*a*c*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*
tan((f*x+exp(1))/2)^2+a))^4-39*A*sqrt(a)*a*c^2*d*(-sqrt(a)*tan((f*x+exp(1))
/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4-663*B*sqrt(a)*a*c*d^2*(-sqrt(a)*tan(
(f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4+351*B*sqrt(a)*a*c^2*d*(-
sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4+A*a^2*c^3*(-
sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3-25*A*a^2*d^3
*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3-9*B*a^2*c
^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+33*B*a^
2*d^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+51*A
*a^2*c*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3
-27*A*a^2*c^2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+
a))^3-75*B*a^2*c*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/
2)^2+a))^3+51*B*a^2*c^2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp
(1))/2)^2+a))^3+27*A*a^3*c^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+
exp(1))/2)^2+a))+93*A*a^3*d^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x
+exp(1))/2)^2+a))+65*A*sqrt(a)*a^2*c^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a
*tan((f*x+exp(1))/2)^2+a))^2+103*A*sqrt(a)*a^2*d^3*(-sqrt(a)*tan((f*x+exp(1
))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+13*B*a^3*c^3*(-sqrt(a)*tan((f*x+ex
p(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-133*B*a^3*d^3*(-sqrt(a)*tan((f*x+
exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-9*B*sqrt(a)*a^2*c^3*(-sqrt(a)*t
an((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-159*B*sqrt(a)*a^2*d^3
*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-159*A*a^3
*c*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+39*A*
a^3*c^2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-14
1*A*sqrt(a)*a^2*c*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))
/2)^2+a))^2-27*A*sqrt(a)*a^2*c^2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan
((f*x+exp(1))/2)^2+a))^2+279*B*a^3*c*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt
(a*tan((f*x+exp(1))/2)^2+a))-159*B*a^3*c^2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+
sqrt(a*tan((f*x+exp(1))/2)^2+a))+309*B*sqrt(a)*a^2*c*d^2*(-sqrt(a)*tan((f*x
+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-141*B*sqrt(a)*a^2*c^2*d*(-sq
rt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+7*A*sqrt(a)*a^
3*c^3+17*A*sqrt(a)*a^3*d^3+B*sqrt(a)*a^3*c^3-25*B*sqrt(a)*a^3*d^3-27*A*sqrt
(a)*a^3*c*d^2+3*A*sqrt(a)*a^3*c^2*d+51*B*sqrt(a)*a^3*c*d^2-27*B*sqrt(a)*a^3
*c^2*d)/a^2/(-(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a)
)^2+2*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a)
)+a)^4/sign(tan((f*x+exp(1))/2)+1)+1/32*(3*A*c^3-75*A*d^3+5*B*c^3+163*B*d^3
+57*A*c*d^2+15*A*c^2*d-225*B*c*d^2+57*B*c^2*d)*atan((-sqrt(a)*tan((f*x+exp(

```
1))/2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(2)/a
^2/sqrt(-a)/sign(tan((f*x+exp(1))/2)+1)))
```

maple [B] time = 2.47, size = 1438, normalized size = 4.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/96/a^(9/2)*((9*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/
2))*a^2*c^3+45*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2)
)*a^2*c^2*d+171*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2
))*a^2*c*d^2-225*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/
2))*a^2*d^3+192*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d^3+15*B*2^(1/2)*arctanh(1
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^3+171*B*2^(1/2)*arctanh(1/
2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d-675*B*2^(1/2)*arctanh(1
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d^2+489*B*2^(1/2)*arctanh(
1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^3+576*B*(a-a*sin(f*x+e))^
(1/2)*a^(3/2)*c*d^2-384*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d^3-64*B*(a-a*sin(
f*x+e))^(3/2)*a^(1/2)*d^3)*cos(f*x+e)^2-2*sin(f*x+e)*(9*A*2^(1/2)*arctanh(1
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^3+45*A*2^(1/2)*arctanh(1/2
*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d+171*A*2^(1/2)*arctanh(1/
2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d^2-225*A*2^(1/2)*arctanh(1
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^3+192*A*(a-a*sin(f*x+e))^(
1/2)*a^(3/2)*d^3+15*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^
(1/2))*a^2*c^3+171*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*a^2*c^2*d-675*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^
(1/2))*a^2*c*d^2+489*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a
^(1/2))*a^2*d^3+576*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c*d^2-384*B*(a-a*sin(f
*x+e))^(1/2)*a^(3/2)*d^3-64*B*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d^3)-90*A*2^(1
/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d+18*A*(a-a
*sin(f*x+e))^(3/2)*a^(1/2)*c^3+126*A*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d^3+30*
B*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^3-60*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^
3-36*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^3-612*A*(a-a*sin(f*x+e))^(1/2)*a^(3
/2)*d^3+1092*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d^3-46*B*(a-a*sin(f*x+e))^(3/
2)*a^(1/2)*d^3-342*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*a^2*c*d^2-342*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^
(1/2))*a^2*c^2*d+1350*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/
a^(1/2))*a^2*c*d^2-18*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/
a^(1/2))*a^2*c^3+450*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a
^(1/2))*a^2*d^3-30*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*a^2*c^3-978*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1
/2))*a^2*d^3-1836*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c*d^2+396*B*(a-a*sin(f*x
+e))^(1/2)*a^(3/2)*c^2*d-108*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^2*d+396*A*(
```

```
a-a*sin(f*x+e))^(1/2)*a^(3/2)*c*d^2-234*B*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^
2*d+378*B*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c*d^2+90*A*(a-a*sin(f*x+e))^(3/2)*
a^(1/2)*c^2*d-234*A*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c*d^2)*(-a*(sin(f*x+e)-1
))^(1/2)/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^3}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^3/(a*sin(f*x + e) + a)^
(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\sin(e+f\,x))\,(c+d\,\sin(e+f\,x))^3}{(a+a\,\sin(e+f\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(5/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(5/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.322 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=219

$$-\frac{\left(A\left(3c^2+10cd+19d^2\right)+B\left(5c^2+38cd-75d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f}+\frac{d^2(A-9B)\cos(e+fx)}{4a^2f\sqrt{a\sin(e+fx)+a}}-\frac{(A-B)}{}$$

```
[Out] -1/16*(c-d)*(3*A*c+5*A*d+5*B*c-13*B*d)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2
)-1/4*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e))^(5/2)-1/32*(B*
(5*c^2+38*c*d-75*d^2)+A*(3*c^2+10*c*d+19*d^2))*arctanh(1/2*cos(f*x+e)*a^(1/
2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)+1/4*(A-9*B)*d^2*cos(f*
x+e)/a^2/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2977, 2968, 3019, 2751, 2649, 206}

$$-\frac{\left(A\left(3c^2+10cd+19d^2\right)+B\left(5c^2+38cd-75d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f}+\frac{d^2(A-9B)\cos(e+fx)}{4a^2f\sqrt{a\sin(e+fx)+a}}-\frac{(A-B)}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^(5/2
),x]
```

```
[Out] -((B*(5*c^2 + 38*c*d - 75*d^2) + A*(3*c^2 + 10*c*d + 19*d^2))*ArcTanh[(Sqrt
[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*
f) - ((c - d)*(3*A*c + 5*B*c + 5*A*d - 13*B*d)*Cos[e + f*x])/(16*a*f*(a + a
*Sin[e + f*x])^(3/2)) + ((A - 9*B)*d^2*Cos[e + f*x])/(4*a^2*f*Sqrt[a + a*Si
n[e + f*x]]) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(4*f*(a + a*Si
n[e + f*x])^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a+a\sin(e+fx))^{5/2}} + \frac{\int \frac{(c+d\sin(e+fx))\left(\frac{1}{2}a(3A\right.}{}}{}$$

$$= -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a+a\sin(e+fx))^{5/2}} + \frac{\int \frac{\frac{1}{2}ac(3Ac+5Bc+4Ad-4B}{}}{}$$

$$= -\frac{(c-d)(3Ac+5Bc+5Ad-13Bd)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e}{4f(a}$$

$$= -\frac{(c-d)(3Ac+5Bc+5Ad-13Bd)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{(A-9B)d^2\cos}{4a^2f\sqrt{a+a\sin}}$$

$$= -\frac{(c-d)(3Ac+5Bc+5Ad-13Bd)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{(A-9B)d^2\cos}{4a^2f\sqrt{a+a\sin}}$$

$$= -\frac{\left(B\left(5c^2+38cd-75d^2\right)+A\left(3c^2+10cd+19d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{2}\sqrt{}}\right.}{16\sqrt{2}\,a^{5/2}f}$$

Mathematica [C] time = 1.14, size = 544, normalized size = 2.48

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left((2+2i)(-1)^{3/4}\left(A\left(3c^2+10cd+19d^2\right)+B\left(5c^2+38cd-75d^2\right)\right)\left(\sin\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-11*A*c^2*Cos[(e + f*x)/2] + 3*B*c^
2*Cos[(e + f*x)/2] + 6*A*c*d*Cos[(e + f*x)/2] + 10*B*c*d*Cos[(e + f*x)/2] +
 5*A*d^2*Cos[(e + f*x)/2] - 45*B*d^2*Cos[(e + f*x)/2] - 3*A*c^2*Cos[(3*(e +
 f*x))/2] - 5*B*c^2*Cos[(3*(e + f*x))/2] - 10*A*c*d*Cos[(3*(e + f*x))/2] +
26*B*c*d*Cos[(3*(e + f*x))/2] + 13*A*d^2*Cos[(3*(e + f*x))/2] - 69*B*d^2*Co
s[(3*(e + f*x))/2] + 16*B*d^2*Cos[(5*(e + f*x))/2] + 11*A*c^2*Sin[(e + f*x)
/2] - 3*B*c^2*Sin[(e + f*x)/2] - 6*A*c*d*Sin[(e + f*x)/2] - 10*B*c*d*Sin[(e
 + f*x)/2] - 5*A*d^2*Sin[(e + f*x)/2] + 45*B*d^2*Sin[(e + f*x)/2] + (2 + 2*
I)*(-1)^(3/4)*(B*(5*c^2 + 38*c*d - 75*d^2) + A*(3*c^2 + 10*c*d + 19*d^2))*A
```

```
rcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^4 - 3*A*c^2*Sin[(3*(e + f*x))/2] - 5*B*c^2*Sin[(3*(e + f*
x))/2] - 10*A*c*d*Sin[(3*(e + f*x))/2] + 26*B*c*d*Sin[(3*(e + f*x))/2] + 13
*A*d^2*Sin[(3*(e + f*x))/2] - 69*B*d^2*Sin[(3*(e + f*x))/2] - 16*B*d^2*Sin[
(5*(e + f*x))/2]))/(32*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [B] time = 0.48, size = 744, normalized size = 3.40

$$-\frac{\sqrt{2}\left(\left((3A+5B)c^2+2(5A+19B)cd+(19A-75B)d^2\right)\cos\left(fx+e\right)^3-4(3A+5B)c^2-8(5A+19B)cd-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] -1/64*(sqrt(2)*(((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*
cos(f*x + e)^3 - 4*(3*A + 5*B)*c^2 - 8*(5*A + 19*B)*c*d - 4*(19*A - 75*B)*d
^2 + 3*((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*cos(f*x +
 e)^2 - 2*((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*cos(f*
x + e) - (4*(3*A + 5*B)*c^2 + 8*(5*A + 19*B)*c*d + 4*(19*A - 75*B)*d^2 - ((
3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*cos(f*x + e)^2 + 2
*((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*cos(f*x + e))*s
in(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e)
 + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos
(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*s
in(f*x + e) - cos(f*x + e) - 2)) + 4*(32*B*d^2*cos(f*x + e)^3 - 4*(A - B)*c
^2 + 8*(A - B)*c*d - 4*(A - B)*d^2 - ((3*A + 5*B)*c^2 + 2*(5*A - 13*B)*c*d
- (13*A - 53*B)*d^2)*cos(f*x + e)^2 - ((7*A + B)*c^2 + 2*(A - 9*B)*c*d - 9*
(A - 9*B)*d^2)*cos(f*x + e) - (32*B*d^2*cos(f*x + e)^2 - 4*(A - B)*c^2 + 8*
(A - B)*c*d - 4*(A - B)*d^2 + ((3*A + 5*B)*c^2 + 2*(5*A - 13*B)*c*d - (13*A
 - 85*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^3*f*
cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f +
(a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)2/f*(2*sqrt(a*tan((f*x+exp(1))/2)^2+a)*(-1/2*B*d^2/a^2
/sign(tan((f*x+exp(1))/2)+1)+1/2*B*d^2*tan((f*x+exp(1))/2)/a^2/sign(tan((f*
x+exp(1))/2)+1))/(a*tan((f*x+exp(1))/2)^2+a)+2*(1/32*(-29*A*c^2*(-sqrt(a)*t
an((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^7+19*A*d^2*(-sqrt(a)*ta
n((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^7+5*B*c^2*(-sqrt(a)*tan(
(f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^7-43*B*d^2*(-sqrt(a)*tan((
f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^7+10*A*c*d*(-sqrt(a)*tan((f
*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^7+38*B*c*d*(-sqrt(a)*tan((f*
x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^7+75*A*sqrt(a)*c^2*(-sqrt(a)*
tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^6-133*A*sqrt(a)*d^2*(-
sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^6+29*B*sqrt(a)
*c^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^6+237*B
*sqrt(a)*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))
^6+58*A*sqrt(a)*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2
)^2+a))^6-266*B*sqrt(a)*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+e
xp(1))/2)^2+a))^6-55*A*a*c^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+
exp(1))/2)^2+a))^5+89*A*a*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x
+exp(1))/2)^2+a))^5-17*B*a*c^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*
x+exp(1))/2)^2+a))^5-161*B*a*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((
f*x+exp(1))/2)^2+a))^5-34*A*a*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan(
(f*x+exp(1))/2)^2+a))^5+178*B*a*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*ta
n((f*x+exp(1))/2)^2+a))^5-91*A*sqrt(a)*a*c^2*(-sqrt(a)*tan((f*x+exp(1))/2)+
sqrt(a*tan((f*x+exp(1))/2)^2+a))^4+117*A*sqrt(a)*a*d^2*(-sqrt(a)*tan((f*x+e
xp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4-13*B*sqrt(a)*a*c^2*(-sqrt(a)*t
an((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4-221*B*sqrt(a)*a*d^2*(
-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4-26*A*sqrt(a
)*a*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4+23
4*B*sqrt(a)*a*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^
2+a))^4+A*a^2*c^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^
2+a))^3+17*A*a^2*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/
2)^2+a))^3-9*B*a^2*c^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1)
)/2)^2+a))^3-25*B*a^2*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp
(1))/2)^2+a))^3-18*A*a^2*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+
exp(1))/2)^2+a))^3+34*B*a^2*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f
*x+exp(1))/2)^2+a))^3+27*A*a^3*c^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan
((f*x+exp(1))/2)^2+a))-53*A*a^3*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*ta
n((f*x+exp(1))/2)^2+a))+65*A*sqrt(a)*a^2*c^2*(-sqrt(a)*tan((f*x+exp(1))/2)+
sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-47*A*sqrt(a)*a^2*d^2*(-sqrt(a)*tan((f*x+
exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+13*B*a^3*c^2*(-sqrt(a)*tan((f
*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+93*B*a^3*d^2*(-sqrt(a)*tan((
f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-9*B*sqrt(a)*a^2*c^2*(-sqrt(
a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+103*B*sqrt(a)*a^2
```

```
*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+26*A*
a^3*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-18*A
*sqrt(a)*a^2*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2
+a))^2-106*B*a^3*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/
2)^2+a))-94*B*sqrt(a)*a^2*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x
+exp(1))/2)^2+a))^2+7*A*sqrt(a)*a^3*c^2-9*A*sqrt(a)*a^3*d^2+B*sqrt(a)*a^3*c
^2+17*B*sqrt(a)*a^3*d^2+2*A*sqrt(a)*a^3*c*d-18*B*sqrt(a)*a^3*c*d)/a^2/(-(-s
qrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+2*sqrt(a)*(-s
qrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+a)^4/sign(tan((
f*x+exp(1))/2)+1)+1/32*(3*A*c^2+19*A*d^2+5*B*c^2-75*B*d^2+10*A*c*d+38*B*c*d
)*atan((-sqrt(a)*tan((f*x+exp(1))/2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a
))/sqrt(2)/sqrt(-a))/sqrt(2)/a^2/sqrt(-a)/sign(tan((f*x+exp(1))/2)+1)))
```

maple [B] time = 2.04, size = 982, normalized size = 4.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/32*(2*sin(f*x+e)*(3*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*a^2*c^2+10*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a
^(1/2))*a^2*c*d+19*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*a^2*d^2+5*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2
))*a^2*c^2+38*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))
*a^2*c*d-75*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a
^2*d^2+64*B*d^2*a^(3/2)*(a-a*sin(f*x+e))^(1/2))+(-3*A*2^(1/2)*arctanh(1/2*(
a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-10*A*2^(1/2)*arctanh(1/2*(a-
a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d-19*A*2^(1/2)*arctanh(1/2*(a-a*
sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2-64*B*d^2*a^(3/2)*(a-a*sin(f*x+e)
)^(1/2)-5*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2
*c^2-38*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c
*d+75*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2
)*cos(f*x+e)^2-6*A*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^2-20*A*(a-a*sin(f*x+e))
^(3/2)*a^(1/2)*c*d+26*A*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d^2+20*A*(a-a*sin(f*
x+e))^(1/2)*a^(3/2)*c^2+24*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c*d-44*A*d^2*a^
(3/2)*(a-a*sin(f*x+e))^(1/2)+6*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)
*2^(1/2)/a^(1/2))*a^2*c^2+20*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2
^(1/2)/a^(1/2))*a^2*c*d+38*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*a^2*d^2-10*B*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^2+52*B*(a-a*sin
(f*x+e))^(3/2)*a^(1/2)*c*d-42*B*d^2*(a-a*sin(f*x+e))^(3/2)*a^(1/2)+12*B*(a-
a*sin(f*x+e))^(1/2)*a^(3/2)*c^2-88*B*c*d*a^(3/2)*(a-a*sin(f*x+e))^(1/2)+204
*B*d^2*a^(3/2)*(a-a*sin(f*x+e))^(1/2)+10*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x
+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2+76*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e
))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d-150*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e)
```

```
)^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2)*(-a*(sin(f*x+e)-1))^(1/2)/a^(9/2)/(1+sin(
f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2/(a*sin(f*x + e) + a)^
(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(c+d\,\sin\left(e+f\,x\right)\right)^2}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(5/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(5/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.323 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=151

$$-\frac{(3Ac+5Ad+5Bc+19Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f}-\frac{(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}-\frac{(A-B)(c-d)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

```
[Out] -1/4*(A-B)*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)-1/16*(3*A*c+5*A*d+5*B*
c-13*B*d)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)-1/32*(3*A*c+5*A*d+5*B*c+19*
B*d)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)
/f*2^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2968, 3019, 2750, 2649, 206}

$$-\frac{(3Ac+5Ad+5Bc+19Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f}-\frac{(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}-\frac{(A-B)(c-d)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(5/2),
x]
```

```
[Out] -((3*A*c + 5*B*c + 5*A*d + 19*B*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*
Sqrt[a + a*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*f) - ((A - B)*(c - d)*Cos[e
 + f*x])/(4*f*(a + a*Sin[e + f*x])^(5/2)) - ((3*A*c + 5*B*c + 5*A*d - 13*B*
d)*Cos[e + f*x])/(16*a*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{Ac+(Bc+Ad)\sin(e+fx)+Bd\sin^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a(3Ac+5Bc+5Ad-5Bd)-4aBd\sin(e+fx}{(a+a\sin(e+fx))^{3/2}}}{4a^2}$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3Ac+5Bc+5Ad-13Bd)\cos(e}{16af(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3Ac+5Bc+5Ad-13Bd)\cos(e}{16af(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{(3Ac+5Bc+5Ad+19Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{(A-B)}{4f(a}$$

Mathematica [C] time = 0.77, size = 267, normalized size = 1.77

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(8(A-B)(c-d)\sin\left(\frac{1}{2}(e+fx)\right)-(3Ac+5Ad+5Bc-13Bd)\left(\sin\left(\frac{1}{2}(e+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*(c - d)*Sin[(e + f*x)/2]
- 4*(A - B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(3*A*c + 5*B*
c + 5*A*d - 13*B*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
2 - (3*A*c + 5*B*c + 5*A*d - 13*B*d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
3 + (1 + I)*(-1)^(3/4)*(3*A*c + 5*B*c + 5*A*d + 19*B*d)*ArcTanh[(1/2 + I/2)
*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
4))/(16*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [B] time = 0.48, size = 536, normalized size = 3.55

$$\frac{\sqrt{2}\left(((3A+5B)c+(5A+19B)d)\cos\left(fx+e\right)^{3}+3\left((3A+5B)c+(5A+19B)d\right)\cos\left(fx+e\right)^{2}-4\left(3A+5B\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/64*(sqrt(2)*(((3*A + 5*B)*c + (5*A + 19*B)*d)*cos(f*x + e)^3 + 3*((3*A +
5*B)*c + (5*A + 19*B)*d)*cos(f*x + e)^2 - 4*(3*A + 5*B)*c - 4*(5*A + 19*B)*
d - 2*((3*A + 5*B)*c + (5*A + 19*B)*d)*cos(f*x + e) + (((3*A + 5*B)*c + (5*
A + 19*B)*d)*cos(f*x + e)^2 - 4*(3*A + 5*B)*c - 4*(5*A + 19*B)*d - 2*((3*A
+ 5*B)*c + (5*A + 19*B)*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(
f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin
(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2
*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))
+ 4*(((3*A + 5*B)*c + (5*A - 13*B)*d)*cos(f*x + e)^2 + 4*(A - B)*c - 4*(A -
 B)*d + ((7*A + B)*c + (A - 9*B)*d)*cos(f*x + e) - (4*(A - B)*c - 4*(A - B)
*d - ((3*A + 5*B)*c + (5*A - 13*B)*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a*si
n(f*x + e) + a))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*c
os(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^
3*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*2*(1/32*(-29*A*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+
exp(1))/2)^2+a))^7+5*A*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(
1))/2)^2+a))^7+5*B*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/
2)^2+a))^7+19*B*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^
2+a))^7+75*A*sqrt(a)*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1)
)/2)^2+a))^6+29*A*sqrt(a)*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+e
xp(1))/2)^2+a))^6+29*B*sqrt(a)*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((
f*x+exp(1))/2)^2+a))^6-133*B*sqrt(a)*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a
*tan((f*x+exp(1))/2)^2+a))^6-55*A*a*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*
tan((f*x+exp(1))/2)^2+a))^5-17*A*a*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*t
an((f*x+exp(1))/2)^2+a))^5-17*B*a*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*ta
n((f*x+exp(1))/2)^2+a))^5+89*B*a*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan
((f*x+exp(1))/2)^2+a))^5-91*A*sqrt(a)*a*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqr
t(a*tan((f*x+exp(1))/2)^2+a))^4-13*A*sqrt(a)*a*d*(-sqrt(a)*tan((f*x+exp(1))
/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4-13*B*sqrt(a)*a*c*(-sqrt(a)*tan((f*x+
exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4+117*B*sqrt(a)*a*d*(-sqrt(a)*t
an((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4+A*a^2*c*(-sqrt(a)*tan
((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3-9*A*a^2*d*(-sqrt(a)*tan
((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3-9*B*a^2*c*(-sqrt(a)*tan
((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+17*B*a^2*d*(-sqrt(a)*ta
n((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+27*A*a^3*c*(-sqrt(a)*t
an((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+13*A*a^3*d*(-sqrt(a)*ta
n((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+65*A*sqrt(a)*a^2*c*(-sqr
t(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-9*A*sqrt(a)*a^2
*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+13*B*a^
3*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-53*B*a^3
*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-9*B*sqrt(
a)*a^2*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-4
7*B*sqrt(a)*a^2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^
2+a))^2+7*A*sqrt(a)*a^3*c+A*sqrt(a)*a^3*d+B*sqrt(a)*a^3*c-9*B*sqrt(a)*a^3*d
)/a^2/(-(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+2*
sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+a)^4
/sign(tan((f*x+exp(1))/2)+1)+1/32*(3*A*c+5*A*d+5*B*c+19*B*d)*atan((-sqrt(a)
*tan((f*x+exp(1))/2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(
-a))/sqrt(2)/a^2/sqrt(-a)/sign(tan((f*x+exp(1))/2)+1))
```

maple [B] time = 1.64, size = 449, normalized size = 2.97

$$-\frac{\left(2\sin\left(fx+e\right)\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right)a^{2}\left(3Ac+5Ad+5Bc+19Bd\right)-\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/32*(2*sin(f*x+e)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1
/2))*a^2*(3*A*c+5*A*d+5*B*c+19*B*d)-2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1
/2)*2^(1/2)/a^(1/2))*a^2*(3*A*c+5*A*d+5*B*c+19*B*d)*cos(f*x+e)^2+6*A*2^(1/2
)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c+10*A*2^(1/2)*ar
ctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d+20*A*(a-a*sin(f*x+e
))^(1/2)*a^(3/2)*c+12*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d-6*A*(a-a*sin(f*x+e
))^(3/2)*a^(1/2)*c-10*A*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d+10*B*2^(1/2)*arcta
nh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c+38*B*2^(1/2)*arctanh(1
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d+12*B*(a-a*sin(f*x+e))^(1/2
)*a^(3/2)*c-44*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d-10*B*(a-a*sin(f*x+e))^(3/
2)*a^(1/2)*c+26*B*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d)*(-a*(sin(f*x+e)-1))^(1/
2)/a^(9/2)/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(5
/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(c+d\,\sin\left(e+f\,x\right)\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(5/2),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(5/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.324 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{(3A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f}-\frac{(3A+5B)\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

```
[Out] -1/4*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)-1/16*(3*A+5*B)*cos(f*x+e)/a/
f/(a+a*sin(f*x+e))^(3/2)-1/32*(3*A+5*B)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1
/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2750, 2650, 2649, 206}

$$-\frac{(3A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f}-\frac{(3A+5B)\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -((3*A + 5*B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*
x]])])/(16*Sqrt[2]*a^(5/2)*f) - ((A - B)*Cos[e + f*x])/(4*f*(a + a*Sin[e +
f*x])^(5/2)) - ((3*A + 5*B)*Cos[e + f*x])/(16*a*f*(a + a*Sin[e + f*x])^(3/2
))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} + \frac{(3A+5B)\int \frac{1}{(a+a\sin(e+fx))^{3/2}}\,dx}{8a}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3A+5B)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{(3A+5B)\int \frac{1}{\sqrt{a+a\sin(e+fx}}}{32a^2}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3A+5B)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(3A+5B)\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\right.}{16a^2}$$

$$= -\frac{(3A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}\,a^{5/2}f} - \frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3A+5B)\,\mathrm{co}}{16af(a+a\sin}$$

Mathematica [C] time = 0.37, size = 227, normalized size = 1.80

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(8(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-(3A+5B)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*Sin[(e + f*x)/2] + 4*(-A
+ B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(3*A + 5*B)*Sin[(e + f*x)/2]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (3*A + 5*B)*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^3 + (1 + I)*(-1)^(3/4)*(3*A + 5*B)*ArcTanh[(1/2 + I/2)*(-
1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4))
/(16*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [B] time = 0.46, size = 392, normalized size = 3.11

$$\sqrt{2}\left(\left(3\,A+5\,B\right)\cos\left(fx+e\right)^{3}+3\left(3\,A+5\,B\right)\cos\left(fx+e\right)^{2}-2\left(3\,A+5\,B\right)\cos\left(fx+e\right)+\left(\left(3\,A+5\,B\right)\cos\left(fx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(sqrt(2)*((3*A + 5*B)*cos(f*x + e)^3 + 3*(3*A + 5*B)*cos(f*x + e)^2 -
2*(3*A + 5*B)*cos(f*x + e) + ((3*A + 5*B)*cos(f*x + e)^2 - 2*(3*A + 5*B)*co
s(f*x + e) - 12*A - 20*B)*sin(f*x + e) - 12*A - 20*B)*sqrt(a)*log(-(a*cos(f
*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(
f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*
a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) +
 4*((3*A + 5*B)*cos(f*x + e)^2 + (7*A + B)*cos(f*x + e) + ((3*A + 5*B)*cos(
f*x + e) - 4*A + 4*B)*sin(f*x + e) + 4*A - 4*B)*sqrt(a*sin(f*x + e) + a))/(
a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^
3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*2*(1/32*(-29*A*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+ex
p(1))/2)^2+a))^7+5*B*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/
2)^2+a))^7+75*A*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1
))/2)^2+a))^6+29*B*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+ex
p(1))/2)^2+a))^6-55*A*a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1
))/2)^2+a))^5-17*B*a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/
2)^2+a))^5-91*A*sqrt(a)*a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp
(1))/2)^2+a))^4-13*B*sqrt(a)*a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*
x+exp(1))/2)^2+a))^4+A*a^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+ex
p(1))/2)^2+a))^3-9*B*a^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(
1))/2)^2+a))^3+27*A*a^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1
))/2)^2+a))+65*A*sqrt(a)*a^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+
exp(1))/2)^2+a))^2+13*B*a^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+e
xp(1))/2)^2+a))-9*B*sqrt(a)*a^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f
```

```
*x+exp(1))/2)^2+a))^2+7*A*sqrt(a)*a^3+B*sqrt(a)*a^3)/a^2/(-(-sqrt(a)*tan((f
*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+2*sqrt(a)*(-sqrt(a)*tan((f
*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+a)^4/sign(tan((f*x+exp(1))/2
)+1)+1/32*(3*A+5*B)*atan((-sqrt(a)*tan((f*x+exp(1))/2)-sqrt(a)+sqrt(a*tan((
f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(2)/a^2/sqrt(-a)/sign(tan((f*x+e
xp(1))/2)+1))
```

maple [B] time = 1.70, size = 279, normalized size = 2.21

$$-\frac{\left(2\sin\left(fx+e\right)\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right)a^{3}\left(3A+5B\right)-\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(fx+e\right)}\ \sqrt{2}}{2\sqrt{a}}\right)a^{3}\left(3A+5B\right)\left(\mathrm{co}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/32*(2*sin(f*x+e)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1
/2))*a^3*(3*A+5*B)-2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/
2))*a^3*(3*A+5*B)*cos(f*x+e)^2+20*A*(a-a*sin(f*x+e))^(1/2)*a^(5/2)-6*A*(a-a
*sin(f*x+e))^(3/2)*a^(3/2)+12*B*(a-a*sin(f*x+e))^(1/2)*a^(5/2)-10*B*(a-a*si
n(f*x+e))^(3/2)*a^(3/2)+6*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1
/2)/a^(1/2))*a^3+10*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^
(1/2))*a^3)*(-a*(sin(f*x+e)-1))^(1/2)/a^(11/2)/(1+sin(f*x+e))/cos(f*x+e)/(a
+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(a*sin(f*x + e) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\ \sin\left(e+f\,x\right)}{{\left(a+a\ \sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sin\left(e + fx\right)}{\left(a\left(\sin\left(e + fx\right) + 1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(5/2), x)
```

3.325 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=261

$$-\frac{\left(A\left(3c^2-14cd+43d^2\right)+B\left(5c^2-34cd-3d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^3}-\frac{2d^{3/2}(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e}{\sqrt{c+d}\sqrt{a\sin(e}}\right.}{a^{5/2}f(c-d)^3\sqrt{c+d}}$$

```
[Out] -1/4*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(5/2)-1/16*(3*A*c-11*A*d+5*B
*c+3*B*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^(3/2)-1/32*(B*(5*c^2-34*c
*d-3*d^2)+A*(3*c^2-14*c*d+43*d^2))*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(
a+a*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^3/f*2^(1/2)-2*d^(3/2)*(-A*d+B*c)*arcta
nh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/(
c-d)^3/f/(c+d)^(1/2)
```

Rubi [A] time = 0.98, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2978, 2985, 2649, 206, 2773, 208}

$$-\frac{\left(A\left(3c^2-14cd+43d^2\right)+B\left(5c^2-34cd-3d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^3}-\frac{2d^{3/2}(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e}{\sqrt{c+d}\sqrt{a\sin(e}}\right.}{a^{5/2}f(c-d)^3\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])),
x]
```

```
[Out] -((B*(5*c^2 - 34*c*d - 3*d^2) + A*(3*c^2 - 14*c*d + 43*d^2))*ArcTanh[(Sqrt[
a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])]])/(16*Sqrt[2]*a^(5/2)*(
c - d)^3*f) - (2*d^(3/2)*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])
/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(a^(5/2)*(c - d)^3*Sqrt[c + d]*f)
 - ((A - B)*Cos[e + f*x])/(4*(c - d)*f*(a + a*Sin[e + f*x])^(5/2)) - ((3*A*
c + 5*B*c - 11*A*d + 3*B*d)*Cos[e + f*x])/(16*a*(c - d)^2*f*(a + a*Sin[e +
f*x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx = -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a(3Ac+5Bc-8Ad)-\frac{3}{2}a(A-B)d\sin}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+f}}{4a^2(c-d)}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{(3Ac+5Bc-11Ad+3Bd)\,\mathrm{c}}{16a(c-d)^2f(a+a\sin(e+}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{(3Ac+5Bc-11Ad+3Bd)\,\mathrm{c}}{16a(c-d)^2f(a+a\sin(e+}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{(3Ac+5Bc-11Ad+3Bd)\,\mathrm{c}}{16a(c-d)^2f(a+a\sin(e+}$$

$$= -\frac{\left(B\left(5c^2-34cd-3d^2\right)+A\left(3c^2-14cd+43d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{2}\sqrt{a}}\right.}{16\sqrt{2}\,a^{5/2}(c-d)^3f}$$

Mathematica [C] time = 5.61, size = 550, normalized size = 2.11

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left((1+i)(-1)^{3/4}\left(A\left(3c^2-14cd+43d^2\right)+B\left(5c^2-34cd-3d^2\right)\right)\left(\sin\left(\frac{1}{2}(e+f\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f
*x])),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*(c - d)^2*Sin[(e + f*x)/2
] + 4*(-A + B)*(c - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(c - d)*
(3*A*c + 5*B*c - 11*A*d + 3*B*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^2 - (c - d)*(3*A*c + 5*B*c - 11*A*d + 3*B*d)*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^3 + (1 + I)*(-1)^(3/4)*(B*(5*c^2 - 34*c*d - 3*d^2) + A
*(3*c^2 - 14*c*d + 43*d^2))*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f
*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (8*d^(3/2)*(-(B*c) + A*d
)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c +
 d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^4)/Sqrt[c + d] + (8*d^(3/2)*(B*c - A*d)*(e + f*x -
2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]
*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^4)/Sqrt[c + d]))/(16*(c - d)^3*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [B] time = 6.57, size = 2577, normalized size = 9.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*(((3*A + 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*c
os(f*x + e)^3 - 4*(3*A + 5*B)*c^2 + 8*(7*A + 17*B)*c*d - 4*(43*A - 3*B)*d^2
 + 3*((3*A + 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e)
^2 - 2*((3*A + 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x +
e) - (4*(3*A + 5*B)*c^2 - 8*(7*A + 17*B)*c*d + 4*(43*A - 3*B)*d^2 - ((3*A +
 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e)^2 + 2*((3*A
 + 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e))*sin(f*x
+ e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*s
qrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x +
e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x
+ e) - cos(f*x + e) - 2)) - 32*(4*B*a*c*d - 4*A*a*d^2 - (B*a*c*d - A*a*d^2)
*cos(f*x + e)^3 - 3*(B*a*c*d - A*a*d^2)*cos(f*x + e)^2 + 2*(B*a*c*d - A*a*d
^2)*cos(f*x + e) + (4*B*a*c*d - 4*A*a*d^2 - (B*a*c*d - A*a*d^2)*cos(f*x + e
)^2 + 2*(B*a*c*d - A*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))
*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d
^2 - 4*((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2
*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f
*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^
2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^
2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x
+ e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2
 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(4*(A - B)*c^
2 - 8*(A - B)*c*d + 4*(A - B)*d^2 + ((3*A + 5*B)*c^2 - 2*(7*A + B)*c*d + (1
1*A - 3*B)*d^2)*cos(f*x + e)^2 + ((7*A + B)*c^2 - 2*(11*A - 3*B)*c*d + (15*
A - 7*B)*d^2)*cos(f*x + e) - (4*(A - B)*c^2 - 8*(A - B)*c*d + 4*(A - B)*d^2
 - ((3*A + 5*B)*c^2 - 2*(7*A + B)*c*d + (11*A - 3*B)*d^2)*cos(f*x + e))*sin
(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2
- a^3*d^3)*f*cos(f*x + e)^3 + 3*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*
d^3)*f*cos(f*x + e)^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f
*cos(f*x + e) - 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f + ((a^3
*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^2 - 2*(a^3*c^3 -
 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*(a^3*c^3 - 3*a^3*c
^2*d + 3*a^3*c*d^2 - a^3*d^3)*f)*sin(f*x + e)), 1/64*(sqrt(2)*(((3*A + 5*B)
*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e)^3 - 4*(3*A + 5*B
)*c^2 + 8*(7*A + 17*B)*c*d - 4*(43*A - 3*B)*d^2 + 3*((3*A + 5*B)*c^2 - 2*(7
*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e)^2 - 2*((3*A + 5*B)*c^2 - 2*
```

```
(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e) - (4*(3*A + 5*B)*c^2 - 8*
(7*A + 17*B)*c*d + 4*(43*A - 3*B)*d^2 - ((3*A + 5*B)*c^2 - 2*(7*A + 17*B)*c
*d + (43*A - 3*B)*d^2)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^2 - 2*(7*A + 17*B)
*c*d + (43*A - 3*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*
x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f
*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a
)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) +
64*(4*B*a*c*d - 4*A*a*d^2 - (B*a*c*d - A*a*d^2)*cos(f*x + e)^3 - 3*(B*a*c*d
 - A*a*d^2)*cos(f*x + e)^2 + 2*(B*a*c*d - A*a*d^2)*cos(f*x + e) + (4*B*a*c*
d - 4*A*a*d^2 - (B*a*c*d - A*a*d^2)*cos(f*x + e)^2 + 2*(B*a*c*d - A*a*d^2)*
cos(f*x + e))*sin(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x
+ e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e)))
 + 4*(4*(A - B)*c^2 - 8*(A - B)*c*d + 4*(A - B)*d^2 + ((3*A + 5*B)*c^2 - 2*
(7*A + B)*c*d + (11*A - 3*B)*d^2)*cos(f*x + e)^2 + ((7*A + B)*c^2 - 2*(11*A
 - 3*B)*c*d + (15*A - 7*B)*d^2)*cos(f*x + e) - (4*(A - B)*c^2 - 8*(A - B)*c
*d + 4*(A - B)*d^2 - ((3*A + 5*B)*c^2 - 2*(7*A + B)*c*d + (11*A - 3*B)*d^2)
*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^3*c^3 - 3*a^3*c^
2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^3 + 3*(a^3*c^3 - 3*a^3*c^2*d +
3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*
c*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 -
a^3*d^3)*f + ((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e
)^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*
(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
```

```
(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
 to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assu
mes constant sign by intervals (correct if the argument is real):Check [abs
(cos((f*t_nostep+exp(1))/2-pi/4))]Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Discontinuities at zeroes of cos((f*t_nostep+exp(1))/2-pi/4) were not check
edUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
 sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable t
o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
ck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)
Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
```

```
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Warning, integration of abs or sign assumes constant sign by intervals (c
orrect if the argument is real):Check [abs(t_nostep+1)]Warning, assuming -c
^3*d^7-c^2*d^8+c*d^9+d^10-c*d^9-d^10 is positive. Hint: run assume to make
assumptions on a variableWarning, assuming -a*c^3*d^7-a*c^2*d^8+a*c*d^9+a*d
^10-a*c*d^9-a*d^10 is positive. Hint: run assume to make assumptions on a v
ariableWarning, need to choose a branch for the root of a polynomial with p
arameters. This might be wrong.Non regular value [0] was discarded and repl
aced randomly by 0=[37]Evaluation time: 76.21sym2poly/r2sym(const gen & e,c
onst index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 2.66, size = 1418, normalized size = 5.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] -1/32*(2*sin(f*x+e)*(-64*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(
1/2))*a^(5/2)*d^3+64*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2)
)*a^(5/2)*c*d^2+3*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^
(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-14*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(
a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d+43*A*(a*(c+d)*d)^(1/2)*2^(1/
2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2+5*B*(a*(c+d)
*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c
^2-34*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2
)/a^(1/2))*a^2*c*d-3*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e
))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2)+(64*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(
a*c*d+a*d^2)^(1/2))*a^(5/2)*d^3-64*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*
d+a*d^2)^(1/2))*a^(5/2)*c*d^2-3*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-
a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2+14*A*(a*(c+d)*d)^(1/2)*2^(1/2)
*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d-43*A*(a*(c+d)*
d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^
2-5*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/
```

```
a^(1/2))*a^2*c^2+34*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e)
)^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d+3*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*
(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2)*cos(f*x+e)^2-128*A*arctanh
((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*d^3+128*B*arctanh((a
-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c*d^2+6*A*(a*(c+d)*d)^(
1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-28
*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*a^2*c*d+86*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(
1/2)*2^(1/2)/a^(1/2))*a^2*d^2-6*A*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(3/2)*
a^(1/2)*c^2+28*A*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c*d-22*A*
(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d^2+20*A*(a*(c+d)*d)^(1/2)
*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^2-72*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)
^(1/2)*a^(3/2)*c*d+52*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*d^
2+10*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*a^2*c^2-68*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e
))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d-6*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2
*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2-10*B*(a*(c+d)*d)^(1/2)*(a-
a*sin(f*x+e))^(3/2)*a^(1/2)*c^2+4*B*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(3/2
)*a^(1/2)*c*d+6*B*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d^2+12*B
*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^2+8*B*(a-a*sin(f*x+e))^
(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c*d-20*B*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))
^(1/2)*a^(3/2)*d^2)*(-a*(sin(f*x+e)-1))^(1/2)/a^(9/2)/(1+sin(f*x+e))/(a*(c+
d)*d)^(1/2)/(c-d)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))),
x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.326 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=395

$$-\frac{\left(A\left(3c^2-22cd+115d^2\right)+B\left(5c^2-58cd-43d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^4}+\frac{d^{3/2}\left(Ad(7c+5d)-B\left(5c^2+5c\right.\right.}{a^{5/2}f(c-}$$

```
[Out] d^(3/2)*(A*d*(7*c+5*d)-B*(5*c^2+5*c*d+2*d^2))*arctanh(cos(f*x+e)*a^(1/2)*d^
(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^4/(c+d)^(3/2)/f-1/4
*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))-1/16*(3*A
*c-15*A*d+5*B*c+7*B*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^(3/2)/(c+d*s
in(f*x+e))-1/32*(B*(5*c^2-58*c*d-43*d^2)+A*(3*c^2-22*c*d+115*d^2))*arctanh(
1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^4/f*2^
(1/2)-1/16*d*(A*(3*c^2-16*c*d-35*d^2)+B*(5*c^2+32*c*d+11*d^2))*cos(f*x+e)/a
^2/(c-d)^3/(c+d)/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.54, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {2978, 2984, 2985, 2649, 206, 2773, 208}

$$-\frac{d\left(A\left(3c^2-16cd-35d^2\right)+B\left(5c^2+32cd+11d^2\right)\right)\cos(e+fx)}{16a^2f(c-d)^3(c+d)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}+\frac{d^{3/2}\left(Ad(7c+5d)-B\left(5c^2+5cd+2d^2\right)\right)\tan}{a^{5/2}f(c-d)^4(c+d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^2
),x]
```

```
[Out] -((B*(5*c^2 - 58*c*d - 43*d^2) + A*(3*c^2 - 22*c*d + 115*d^2))*ArcTanh[(Sqr
t[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)
*(c - d)^4*f) + (d^(3/2)*(A*d*(7*c + 5*d) - B*(5*c^2 + 5*c*d + 2*d^2))*ArcT
anh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])
/(a^(5/2)*(c - d)^4*(c + d)^(3/2)*f) - ((A - B)*Cos[e + f*x])/(4*(c - d)*f*
(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])) - ((3*A*c + 5*B*c - 15*A*d
 + 7*B*d)*Cos[e + f*x])/(16*a*(c - d)^2*f*(a + a*Sin[e + f*x])^(3/2)*(c + d
*Sin[e + f*x])) - (d*(A*(3*c^2 - 16*c*d - 35*d^2) + B*(5*c^2 + 32*c*d + 11*
d^2))*Cos[e + f*x])/(16*a^2*(c - d)^3*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c
 + d*Sin[e + f*x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx = -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - \frac{\int \frac{-\frac{1}{2}a(3A}{}}{}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - \frac{(3}{16a(c-}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - \frac{(3}{16a(c-}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - \frac{(3}{16a(c-}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - \frac{(3}{16a(c-}$$

$$= -\frac{\left(B\left(5c^2-58cd-43d^2\right)+A\left(3c^2-22cd+115d^2\right)\right)\tanh^{-1}\left(\right.}{16\sqrt{2}\,a^{5/2}(c-d)^4 f}$$

Mathematica [C] time = 12.37, size = 1318, normalized size = 3.34

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f
*x])^2),x]
```

```
[Out] ((1 + I)*(3*A*c^2 + 5*B*c^2 - 22*A*c*d - 58*B*c*d + 115*A*d^2 - 43*B*d^2)*A
rcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(e + f*x)/4]*(Cos[(e + f*x)/4] - Sin[(e +
```

```
f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/((16*(-1)^(1/4)*c^4 - 6
4*(-1)^(1/4)*c^3*d + 96*(-1)^(1/4)*c^2*d^2 - 64*(-1)^(1/4)*c*d^3 + 16*(-1)^
(1/4)*d^4)*f*(a*(1 + Sin[e + f*x]))^(5/2)) + (d^(3/2)*(A*d*(7*c + 5*d) - B*
(5*c^2 + 5*c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(
e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*
x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(4*(c - d)^4*(c + d)^(3/2
)*f*(a*(1 + Sin[e + f*x]))^(5/2)) + (d^(3/2)*(-(A*d*(7*c + 5*d)) + B*(5*c^2
 + 5*c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*
x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])
])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(4*(c - d)^4*(c + d)^(3/2)*f*(a
*(1 + Sin[e + f*x]))^(5/2)) + ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-22*A
*c^3*Cos[(e + f*x)/2] + 6*B*c^3*Cos[(e + f*x)/2] + 40*A*c^2*d*Cos[(e + f*x)
/2] - 40*B*c^2*d*Cos[(e + f*x)/2] + 54*A*c*d^2*Cos[(e + f*x)/2] - 70*B*c*d^
2*Cos[(e + f*x)/2] + 24*A*d^3*Cos[(e + f*x)/2] + 8*B*d^3*Cos[(e + f*x)/2] -
 6*A*c^3*Cos[(3*(e + f*x))/2] - 10*B*c^3*Cos[(3*(e + f*x))/2] + 21*A*c^2*d*
Cos[(3*(e + f*x))/2] - 29*B*c^2*d*Cos[(3*(e + f*x))/2] + 54*A*c*d^2*Cos[(3*
(e + f*x))/2] - 86*B*c*d^2*Cos[(3*(e + f*x))/2] + 75*A*d^3*Cos[(3*(e + f*x)
)/2] - 19*B*d^3*Cos[(3*(e + f*x))/2] + 3*A*c^2*d*Cos[(5*(e + f*x))/2] + 5*B
*c^2*d*Cos[(5*(e + f*x))/2] - 16*A*c*d^2*Cos[(5*(e + f*x))/2] + 32*B*c*d^2*
Cos[(5*(e + f*x))/2] - 35*A*d^3*Cos[(5*(e + f*x))/2] + 11*B*d^3*Cos[(5*(e +
 f*x))/2] + 22*A*c^3*Sin[(e + f*x)/2] - 6*B*c^3*Sin[(e + f*x)/2] - 40*A*c^2
*d*Sin[(e + f*x)/2] + 40*B*c^2*d*Sin[(e + f*x)/2] - 54*A*c*d^2*Sin[(e + f*x
)/2] + 70*B*c*d^2*Sin[(e + f*x)/2] - 24*A*d^3*Sin[(e + f*x)/2] - 8*B*d^3*Si
n[(e + f*x)/2] - 6*A*c^3*Sin[(3*(e + f*x))/2] - 10*B*c^3*Sin[(3*(e + f*x))/
2] + 21*A*c^2*d*Sin[(3*(e + f*x))/2] - 29*B*c^2*d*Sin[(3*(e + f*x))/2] + 54
*A*c*d^2*Sin[(3*(e + f*x))/2] - 86*B*c*d^2*Sin[(3*(e + f*x))/2] + 75*A*d^3*
Sin[(3*(e + f*x))/2] - 19*B*d^3*Sin[(3*(e + f*x))/2] - 3*A*c^2*d*Sin[(5*(e
+ f*x))/2] - 5*B*c^2*d*Sin[(5*(e + f*x))/2] + 16*A*c*d^2*Sin[(5*(e + f*x))/
2] - 32*B*c*d^2*Sin[(5*(e + f*x))/2] + 35*A*d^3*Sin[(5*(e + f*x))/2] - 11*B
*d^3*Sin[(5*(e + f*x))/2]))/(64*(c - d)^3*(c + d)*f*(a*(1 + Sin[e + f*x]))^
(5/2)*(c + d*Sin[e + f*x]))
```

fricas [B] time = 14.83, size = 5151, normalized size = 13.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*(4*(3*A + 5*B)*c^4 - 64*(A + 3*B)*c^3*d + 8*(37*A - 77*B)*c
^2*d^2 + 64*(13*A - 9*B)*c*d^3 + 4*(115*A - 43*B)*d^4 + ((3*A + 5*B)*c^3*d
- (19*A + 53*B)*c^2*d^2 + (93*A - 101*B)*c*d^3 + (115*A - 43*B)*d^4)*cos(f*
x + e)^4 - ((3*A + 5*B)*c^4 - (13*A + 43*B)*c^3*d + (55*A - 207*B)*c^2*d^2
+ 7*(43*A - 35*B)*c*d^3 + 2*(115*A - 43*B)*d^4)*cos(f*x + e)^3 - (3*(3*A +
```

```
5*B)*c^4 - 2*(21*A + 67*B)*c^3*d + 8*(23*A - 71*B)*c^2*d^2 + 2*(405*A - 317
*B)*c*d^3 + 5*(115*A - 43*B)*d^4)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^4 - 16*
(A + 3*B)*c^3*d + 2*(37*A - 77*B)*c^2*d^2 + 16*(13*A - 9*B)*c*d^3 + (115*A
- 43*B)*d^4)*cos(f*x + e) + (4*(3*A + 5*B)*c^4 - 64*(A + 3*B)*c^3*d + 8*(37
*A - 77*B)*c^2*d^2 + 64*(13*A - 9*B)*c*d^3 + 4*(115*A - 43*B)*d^4 - ((3*A +
 5*B)*c^3*d - (19*A + 53*B)*c^2*d^2 + (93*A - 101*B)*c*d^3 + (115*A - 43*B)
*d^4)*cos(f*x + e)^3 - ((3*A + 5*B)*c^4 - 2*(5*A + 19*B)*c^3*d + 4*(9*A - 6
5*B)*c^2*d^2 + 2*(197*A - 173*B)*c*d^3 + 3*(115*A - 43*B)*d^4)*cos(f*x + e)
^2 + 2*((3*A + 5*B)*c^4 - 16*(A + 3*B)*c^3*d + 2*(37*A - 77*B)*c^2*d^2 + 16
*(13*A - 9*B)*c*d^3 + (115*A - 43*B)*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(
a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos
(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*s
in(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(
f*x + e) - 2)) - 16*(20*B*a*c^3*d - 4*(7*A - 10*B)*a*c^2*d^2 - 4*(12*A - 7*
B)*a*c*d^3 - 4*(5*A - 2*B)*a*d^4 + (5*B*a*c^2*d^2 - (7*A - 5*B)*a*c*d^3 - (
5*A - 2*B)*a*d^4)*cos(f*x + e)^4 - (5*B*a*c^3*d - (7*A - 15*B)*a*c^2*d^2 -
(19*A - 12*B)*a*c*d^3 - 2*(5*A - 2*B)*a*d^4)*cos(f*x + e)^3 - (15*B*a*c^3*d
 - (21*A - 40*B)*a*c^2*d^2 - (50*A - 31*B)*a*c*d^3 - 5*(5*A - 2*B)*a*d^4)*c
os(f*x + e)^2 + 2*(5*B*a*c^3*d - (7*A - 10*B)*a*c^2*d^2 - (12*A - 7*B)*a*c*
d^3 - (5*A - 2*B)*a*d^4)*cos(f*x + e) + (20*B*a*c^3*d - 4*(7*A - 10*B)*a*c^
2*d^2 - 4*(12*A - 7*B)*a*c*d^3 - 4*(5*A - 2*B)*a*d^4 - (5*B*a*c^2*d^2 - (7*
A - 5*B)*a*c*d^3 - (5*A - 2*B)*a*d^4)*cos(f*x + e)^3 - (5*B*a*c^3*d - (7*A
- 20*B)*a*c^2*d^2 - (26*A - 17*B)*a*c*d^3 - 3*(5*A - 2*B)*a*d^4)*cos(f*x +
e)^2 + 2*(5*B*a*c^3*d - (7*A - 10*B)*a*c^2*d^2 - (12*A - 7*B)*a*c*d^3 - (5*
A - 2*B)*a*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*co
s(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d
 + d^2)*cos(f*x + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*
x + e) + (c^2 + 4*c*d + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqr
t(a*sin(f*x + e) + a)*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x +
 e) + (d^2*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x +
 e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2
 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos
(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(4*(A - B)*c^4 - 8*(A - B
)*c^3*d + 8*(A - B)*c*d^3 - 4*(A - B)*d^4 - ((3*A + 5*B)*c^3*d - (19*A - 27
*B)*c^2*d^2 - (19*A + 21*B)*c*d^3 + (35*A - 11*B)*d^4)*cos(f*x + e)^3 + ((3
*A + 5*B)*c^4 - (15*A - 7*B)*c^3*d - (7*A - 15*B)*c^2*d^2 - (A + 23*B)*c*d^
3 + 4*(5*A - B)*d^4)*cos(f*x + e)^2 + ((7*A + B)*c^4 - 20*(A - B)*c^3*d - 2
*(13*A - 21*B)*c^2*d^2 - 4*(3*A + 13*B)*c*d^3 + (51*A - 11*B)*d^4)*cos(f*x
+ e) - (4*(A - B)*c^4 - 8*(A - B)*c^3*d + 8*(A - B)*c*d^3 - 4*(A - B)*d^4 -
 ((3*A + 5*B)*c^3*d - (19*A - 27*B)*c^2*d^2 - (19*A + 21*B)*c*d^3 + (35*A -
 11*B)*d^4)*cos(f*x + e)^2 - ((3*A + 5*B)*c^4 - 12*(A - B)*c^3*d - 2*(13*A
- 21*B)*c^2*d^2 - 4*(5*A + 11*B)*c*d^3 + 5*(11*A - 3*B)*d^4)*cos(f*x + e))*
sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^3*c^5*d - 3*a^3*c^4*d^2 + 2*a^3
*c^3*d^3 + 2*a^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e)^4 - (a^3*c
^6 - a^3*c^5*d - 4*a^3*c^4*d^2 + 6*a^3*c^3*d^3 + a^3*c^2*d^4 - 5*a^3*c*d^5
```

+ 2*a^3*d^6)*f*cos(f*x + e)^3 - (3*a^3*c^6 - 4*a^3*c^5*d - 9*a^3*c^4*d^2 + 16*a^3*c^3*d^3 + a^3*c^2*d^4 - 12*a^3*c*d^5 + 5*a^3*d^6)*f*cos(f*x + e)^2 + 2*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e) + 4*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f - ((a^3*c^5*d - 3*a^3*c^4*d^2 + 2*a^3*c^3*d^3 + 2*a^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e)^3 + (a^3*c^6 - 7*a^3*c^4*d^2 + 8*a^3*c^3*d^3 + 3*a^3*c^2*d^4 - 8*a^3*c*d^5 + 3*a^3*d^6)*f*cos(f*x + e)^2 - 2*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e) - 4*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f)*sin(f*x + e)), -1/64*(sqrt(2)*(4*(3*A + 5*B)*c^4 - 64*(A + 3*B)*c^3*d + 8*(37*A - 77*B)*c^2*d^2 + 64*(13*A - 9*B)*c*d^3 + 4*(115*A - 43*B)*d^4 + ((3*A + 5*B)*c^3*d - (19*A + 53*B)*c^2*d^2 + (93*A - 101*B)*c*d^3 + (115*A - 43*B)*d^4)*cos(f*x + e)^4 - ((3*A + 5*B)*c^4 - (13*A + 43*B)*c^3*d + (55*A - 207*B)*c^2*d^2 + 7*(43*A - 35*B)*c*d^3 + 2*(115*A - 43*B)*d^4)*cos(f*x + e)^3 - (3*(3*A + 5*B)*c^4 - 2*(21*A + 67*B)*c^3*d + 8*(23*A - 71*B)*c^2*d^2 + 2*(405*A - 317*B)*c*d^3 + 5*(115*A - 43*B)*d^4)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^4 - 16*(A + 3*B)*c^3*d + 2*(37*A - 77*B)*c^2*d^2 + 16*(13*A - 9*B)*c*d^3 + (115*A - 43*B)*d^4)*cos(f*x + e) + (4*(3*A + 5*B)*c^4 - 64*(A + 3*B)*c^3*d + 8*(37*A - 77*B)*c^2*d^2 + 64*(13*A - 9*B)*c*d^3 + 4*(115*A - 43*B)*d^4 - ((3*A + 5*B)*c^3*d - (19*A + 53*B)*c^2*d^2 + (93*A - 101*B)*c*d^3 + (115*A - 43*B)*d^4)*cos(f*x + e)^3 - ((3*A + 5*B)*c^4 - 2*(5*A + 19*B)*c^3*d + 4*(9*A - 65*B)*c^2*d^2 + 2*(197*A - 173*B)*c*d^3 + 3*(115*A - 43*B)*d^4)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^4 - 16*(A + 3*B)*c^3*d + 2*(37*A - 77*B)*c^2*d^2 + 16*(13*A - 9*B)*c*d^3 + (115*A - 43*B)*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 32*(20*B*a*c^3*d - 4*(7*A - 10*B)*a*c^2*d^2 - 4*(12*A - 7*B)*a*c*d^3 - 4*(5*A - 2*B)*a*d^4 + (5*B*a*c^2*d^2 - (7*A - 5*B)*a*c*d^3 - (5*A - 2*B)*a*d^4)*cos(f*x + e)^4 - (5*B*a*c^3*d - (7*A - 15*B)*a*c^2*d^2 - (19*A - 12*B)*a*c*d^3 - 2*(5*A - 2*B)*a*d^4)*cos(f*x + e)^3 - (15*B*a*c^3*d - (21*A - 40*B)*a*c^2*d^2 - (50*A - 31*B)*a*c*d^3 - 5*(5*A - 2*B)*a*d^4)*cos(f*x + e)^2 + 2*(5*B*a*c^3*d - (7*A - 10*B)*a*c^2*d^2 - (12*A - 7*B)*a*c*d^3 - (5*A - 2*B)*a*d^4)*cos(f*x + e) + (20*B*a*c^3*d - 4*(7*A - 10*B)*a*c^2*d^2 - 4*(12*A - 7*B)*a*c*d^3 - 4*(5*A - 2*B)*a*d^4 - (5*B*a*c^2*d^2 - (7*A - 5*B)*a*c*d^3 - (5*A - 2*B)*a*d^4)*cos(f*x + e)^3 - (5*B*a*c^3*d - (7*A - 20*B)*a*c^2*d^2 - (26*A - 17*B)*a*c*d^3 - 3*(5*A - 2*B)*a*d^4)*cos(f*x + e)^2 + 2*(5*B*a*c^3*d - (7*A - 10*B)*a*c^2*d^2 - (12*A - 7*B)*a*c*d^3 - (5*A - 2*B)*a*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e))) + 4*(4*(A - B)*c^4 - 8*(A - B)*c^3*d + 8*(A - B)*c*d^3 - 4*(A - B)*d^4 - ((3*A + 5*B)*c^3*d - (19*A - 27*B)*c^2*d^2 - (19*A + 21*B)*c*d^3 + (35*A - 11*B)*d^4)*cos(f*x + e)^3 + ((3*A + 5*B)*c^4 - (15*A - 7*B)*c^3*d - (7*A - 15*B)*c^2*d^2 - (A +

```
23*B)*c*d^3 + 4*(5*A - B)*d^4)*cos(f*x + e)^2 + ((7*A + B)*c^4 - 20*(A - B)
*c^3*d - 2*(13*A - 21*B)*c^2*d^2 - 4*(3*A + 13*B)*c*d^3 + (51*A - 11*B)*d^4
)*cos(f*x + e) - (4*(A - B)*c^4 - 8*(A - B)*c^3*d + 8*(A - B)*c*d^3 - 4*(A
- B)*d^4 - ((3*A + 5*B)*c^3*d - (19*A - 27*B)*c^2*d^2 - (19*A + 21*B)*c*d^3
 + (35*A - 11*B)*d^4)*cos(f*x + e)^2 - ((3*A + 5*B)*c^4 - 12*(A - B)*c^3*d
- 2*(13*A - 21*B)*c^2*d^2 - 4*(5*A + 11*B)*c*d^3 + 5*(11*A - 3*B)*d^4)*cos(
f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^3*c^5*d - 3*a^3*c^4*d
^2 + 2*a^3*c^3*d^3 + 2*a^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e)^
4 - (a^3*c^6 - a^3*c^5*d - 4*a^3*c^4*d^2 + 6*a^3*c^3*d^3 + a^3*c^2*d^4 - 5*
a^3*c*d^5 + 2*a^3*d^6)*f*cos(f*x + e)^3 - (3*a^3*c^6 - 4*a^3*c^5*d - 9*a^3*
c^4*d^2 + 16*a^3*c^3*d^3 + a^3*c^2*d^4 - 12*a^3*c*d^5 + 5*a^3*d^6)*f*cos(f*
x + e)^2 + 2*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2
*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e) + 4*(a^3*c^6 - 2*a^3*c^5*d - a
^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f - ((a^3
*c^5*d - 3*a^3*c^4*d^2 + 2*a^3*c^3*d^3 + 2*a^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*
d^6)*f*cos(f*x + e)^3 + (a^3*c^6 - 7*a^3*c^4*d^2 + 8*a^3*c^3*d^3 + 3*a^3*c^
2*d^4 - 8*a^3*c*d^5 + 3*a^3*d^6)*f*cos(f*x + e)^2 - 2*(a^3*c^6 - 2*a^3*c^5*
d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f*co
s(f*x + e) - 4*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c
^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
```

i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(cos((f*t_nostep+exp(1))/2-pi/4))]Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Discontinuities at zeroes of cos((f*t_nostep+exp(1))/2-pi/4) were not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x

```
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
 sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_
nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sig
n assumes constant sign by intervals (correct if the argument is real):Chec
k [abs(t_nostep+1)]Evaluation time: 1.57index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

maple [B] time = 4.20, size = 4092, normalized size = 10.36

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x)
```

```
[Out] 1/32/a^(9/2)*(-323*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-
1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c*d^3+13*A*(a*(c+d)*d)^(1/2)*2^(1
/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2
*c^3*d-320*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)
*sin(f*x+e)*c^3*d^2+19*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x
+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^2*d^2-55*A*(a*(c+d)*d)^(1
/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+
e)^2*a^2*c^2*d^2-301*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e
)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c*d^3+101*B*(a*(c+d)*d)^(1/2)
*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*
a^2*c^3*d+255*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(
1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^2*d^2+187*B*(a*(c+d)*d)^(1/2)*2^(1/2
)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c*d
^3+35*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(
1/2)/a^(1/2))*sin(f*x+e)*a^2*c^3*d-167*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(
1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^2*d^2+207*B
*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^
(1/2))*sin(f*x+e)^2*a^2*c^2*d^2+245*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2
*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c*d^3+53*B*(a*
(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2
))*sin(f*x+e)^3*a^2*c^2*d^2-5*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(
```

```
sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^3*d+101*B*(a*(c+d)
*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*si
n(f*x+e)^3*a^2*c*d^3-93*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*
x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c*d^3+43*B*(a*(c+d)*d)^(1/
2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e
)^2*a^2*c^3*d-3*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))
^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^3*d-480*B*arctanh((-a*(sin(f*x+e
)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^2*d^3-160*B*arctanh((
-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*c*d^4+22
*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^3*d-288*B*arctanh(
(-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c*d^4-148
*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*d^4+52*B*
(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*d^4+84*A*(-a
*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^3*d+20*A*(-a*(sin(f*x+e)
-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^2*d^2-52*A*(-a*(sin(f*x+e)-1))^(1/2)
*(a*(c+d)*d)^(1/2)*a^(3/2)*c*d^3-52*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)
^(1/2)*a^(3/2)*c^3*d-32*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/
2)*sin(f*x+e)^2*c^2*d^2+32*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^
(3/2)*sin(f*x+e)^2*c*d^3-20*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a
^(3/2)*sin(f*x+e)*c^3*d+84*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^
(3/2)*sin(f*x+e)*c^2*d^2-12*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a
^(3/2)*sin(f*x+e)*c^3*d-116*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a
^(3/2)*sin(f*x+e)*c^2*d^2+84*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*
a^(3/2)*sin(f*x+e)*c*d^3-115*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(s
in(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*d^4+10*B*(-a*(sin(f*x+e
)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^3*d+22*B*(-a*(sin(f*x+e)
-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^2*d^2-10*B*(-a*(sin(f*x+e
)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c*d^3-10*B*(a*(c+d)*d)^(1/
2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e
)*a^2*c^4-5*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/
2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^4+86*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arc
tanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*d^4-11
5*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)
/a^(1/2))*sin(f*x+e)^3*a^2*d^4+43*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(
-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*d^4+76*B*(-a*(si
n(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c*d^3+43*B*(a*(c+d)
*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*si
n(f*x+e)*a^2*d^4+19*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)
-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^3*d-93*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctan
h(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d^2-3*A*(a*(c+d)*d
)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(
f*x+e)^2*a^2*c^4-230*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e
)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*d^4-115*A*(a*(c+d)*d)^(1/2)*2
^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d^3+53*
B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a
```

^(1/2))*a^2*c^3*d+101*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d^2+43*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d^3+6*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^3*d-38*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c^2*d^2-6*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*c*d^3-6*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^4+32*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*c*d^3+448*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^2*d^3+544*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c*d^4+224*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^2*d^3+608*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c*d^4-160*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^3*d^2-10*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^2*d^2-22*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c*d^3-5*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^4-20*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^2*d^2+84*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c*d^3+38*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*d^4-22*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*d^4-38*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^3*d-6*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^2*d^2+38*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c*d^3-3*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^4-32*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)^2*d^4-480*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^2*d^3-384*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c*d^4+224*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*c*d^4-160*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*c^2*d^3-160*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^3*d^2+160*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*d^5+160*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c*d^4+320*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*d^5+224*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^2*d^3-128*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*d^5+160*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*d^5-64*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*d^5-20*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^4-32*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*d^4-12*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^4-64*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*d^5+6*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^4-160*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^2*d^3-64*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^

```
(1/2))*a^(5/2)*c*d^4+10*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/
2)*c^4)*(-a*(sin(f*x+e)-1))^(1/2)/(1+sin(f*x+e))/(a*(c+d)*d)^(1/2)/(c+d*sin
(f*x+e))/(c+d)/(c-d)^4/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.327 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=519

$$-\frac{\left(3A\left(c^2-10cd+73d^2\right)+B\left(5c^2-82cd-115d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^5}+\frac{d^{3/2}\left(3Ad\left(21c^2+30cd+13d^2\right)\right.}{}$$

```
[Out] 1/4*d^(3/2)*(3*A*d*(21*c^2+30*c*d+13*d^2)-B*(35*c^3+70*c^2*d+67*c*d^2+20*d^
3))*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/
a^(5/2)/(c-d)^5/(c+d)^(5/2)/f-1/4*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))
^(5/2)/(c+d*sin(f*x+e))^2-1/16*(3*A*c-19*A*d+5*B*c+11*B*d)*cos(f*x+e)/a/(c-
d)^2/f/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2-1/32*(B*(5*c^2-82*c*d-115*
d^2)+3*A*(c^2-10*c*d+73*d^2))*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*s
in(f*x+e))^(1/2))/a^(5/2)/(c-d)^5/f*2^(1/2)-1/16*d*(A*(3*c^2-20*c*d-31*d^2)
+B*(5*c^2+28*c*d+15*d^2))*cos(f*x+e)/a^2/(c-d)^3/(c+d)/f/(c+d*sin(f*x+e))^2
/(a+a*sin(f*x+e))^(1/2)-1/16*d*(3*A*(c^3-7*c^2*d-37*c*d^2-21*d^3)+B*(5*c^3+
73*c^2*d+79*c*d^2+35*d^3))*cos(f*x+e)/a^2/(c-d)^4/(c+d)^2/f/(c+d*sin(f*x+e)
)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 2.15, antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {2978, 2984, 2985, 2649, 206, 2773, 208}

$$-\frac{d\left(3A\left(-7c^2d+c^3-37cd^2-21d^3\right)+B\left(73c^2d+5c^3+79cd^2+35d^3\right)\right)\cos(e+fx)}{16a^2f(c-d)^4(c+d)^2\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}-\frac{d\left(A\left(3c^2-20cd-31d^2\right)\right.}{16a^2f(c-d)^3(c+d)\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^3
),x]
```

```
[Out] -((B*(5*c^2 - 82*c*d - 115*d^2) + 3*A*(c^2 - 10*c*d + 73*d^2))*ArcTanh[(Sqr
t[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)
*(c - d)^5*f) + (d^(3/2)*(3*A*d*(21*c^2 + 30*c*d + 13*d^2) - B*(35*c^3 + 70
*c^2*d + 67*c*d^2 + 20*d^3))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c
 + d]*Sqrt[a + a*Sin[e + f*x]])])/(4*a^(5/2)*(c - d)^5*(c + d)^(5/2)*f) - (
(A - B)*Cos[e + f*x])/(4*(c - d)*f*(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e
+ f*x])^2) - ((3*A*c + 5*B*c - 19*A*d + 11*B*d)*Cos[e + f*x])/(16*a*(c - d)
^2*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2) - (d*(A*(3*c^2 - 20
*c*d - 31*d^2) + B*(5*c^2 + 28*c*d + 15*d^2))*Cos[e + f*x])/(16*a^2*(c - d)
^3*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2) - (d*(3*A*(c^
```

```
3 - 7*c^2*d - 37*c*d^2 - 21*d^3) + B*(5*c^3 + 73*c^2*d + 79*c*d^2 + 35*d^3)
)*Cos[e + f*x])/(16*a^2*(c - d)^4*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c +
 d*Sin[e + f*x]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
```

```
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx = -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \frac{\int \frac{-\frac{1}{2}a(3}{}}{}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \frac{(}{16a(c-}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \frac{(}{16a(c-}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \frac{(}{16a(c-}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \frac{(}{16a(c-}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \frac{(}{16a(c-}$$

$$= -\frac{\left(B\left(5c^2-82cd-115d^2\right)+3A\left(c^2-10cd+73d^2\right)\right)\tanh^{-1}\left(\right.}{16\sqrt{2}\,a^{5/2}(c-d)^5 f}$$

Mathematica [C] time = 13.86, size = 2103, normalized size = 4.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f
*x])^3),x]
```

```
[Out] ((1 + I)*(3*A*c^2 + 5*B*c^2 - 30*A*c*d - 82*B*c*d + 219*A*d^2 - 115*B*d^2)*
ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(e + f*x)/4]*(Cos[(e + f*x)/4] - Sin[(e
+ f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/((16*(-1)^(1/4)*c^5 -
80*(-1)^(1/4)*c^4*d + 160*(-1)^(1/4)*c^3*d^2 - 160*(-1)^(1/4)*c^2*d^3 + 80*
(-1)^(1/4)*c*d^4 - 16*(-1)^(1/4)*d^5)*f*(a*(1 + Sin[e + f*x]))^(5/2)) - (d^
(3/2)*(-3*A*d*(21*c^2 + 30*c*d + 13*d^2) + B*(35*c^3 + 70*c^2*d + 67*c*d^2
+ 20*d^3))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*
(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2]])*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(16*(c - d)^5*(c + d)^(5/2)*f*(a*(1 + S
in[e + f*x]))^(5/2)) + (d^(3/2)*(-3*A*d*(21*c^2 + 30*c*d + 13*d^2) + B*(35*
c^3 + 70*c^2*d + 67*c*d^2 + 20*d^3))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] +
 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]
*Sin[(e + f*x)/2]])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(16*(c - d)^5
*(c + d)^(5/2)*f*(a*(1 + Sin[e + f*x]))^(5/2)) + ((Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])*(-44*A*c^5*Cos[(e + f*x)/2] + 12*B*c^5*Cos[(e + f*x)/2] + 84*A
*c^4*d*Cos[(e + f*x)/2] - 116*B*c^4*d*Cos[(e + f*x)/2] + 249*A*c^3*d^2*Cos[
(e + f*x)/2] - 433*B*c^3*d^2*Cos[(e + f*x)/2] + 385*A*c^2*d^3*Cos[(e + f*x)
/2] - 277*B*c^2*d^3*Cos[(e + f*x)/2] + 239*A*c*d^4*Cos[(e + f*x)/2] - 95*B*
c*d^4*Cos[(e + f*x)/2] + 47*A*d^5*Cos[(e + f*x)/2] - 51*B*d^5*Cos[(e + f*x)
/2] - 12*A*c^5*Cos[(3*(e + f*x))/2] - 20*B*c^5*Cos[(3*(e + f*x))/2] + 40*A*
c^4*d*Cos[(3*(e + f*x))/2] - 104*B*c^4*d*Cos[(3*(e + f*x))/2] + 261*A*c^3*d
^2*Cos[(3*(e + f*x))/2] - 581*B*c^3*d^2*Cos[(3*(e + f*x))/2] + 781*A*c^2*d^
3*Cos[(3*(e + f*x))/2] - 665*B*c^2*d^3*Cos[(3*(e + f*x))/2] + 579*A*c*d^4*C
os[(3*(e + f*x))/2] - 299*B*c*d^4*Cos[(3*(e + f*x))/2] + 79*A*d^5*Cos[(3*(e
 + f*x))/2] - 59*B*d^5*Cos[(3*(e + f*x))/2] + 12*A*c^4*d*Cos[(5*(e + f*x))/
2] + 20*B*c^4*d*Cos[(5*(e + f*x))/2] - 73*A*c^3*d^2*Cos[(5*(e + f*x))/2] +
217*B*c^3*d^2*Cos[(5*(e + f*x))/2] - 353*A*c^2*d^3*Cos[(5*(e + f*x))/2] + 3
97*B*c^2*d^3*Cos[(5*(e + f*x))/2] - 419*A*c*d^4*Cos[(5*(e + f*x))/2] + 251*
B*c*d^4*Cos[(5*(e + f*x))/2] - 127*A*d^5*Cos[(5*(e + f*x))/2] + 75*B*d^5*Co
s[(5*(e + f*x))/2] + 3*A*c^3*d^2*Cos[(7*(e + f*x))/2] + 5*B*c^3*d^2*Cos[(7*
(e + f*x))/2] - 21*A*c^2*d^3*Cos[(7*(e + f*x))/2] + 73*B*c^2*d^3*Cos[(7*(e
+ f*x))/2] - 111*A*c*d^4*Cos[(7*(e + f*x))/2] + 79*B*c*d^4*Cos[(7*(e + f*x)
)/2] - 63*A*d^5*Cos[(7*(e + f*x))/2] + 35*B*d^5*Cos[(7*(e + f*x))/2] + 44*A
*c^5*Sin[(e + f*x)/2] - 12*B*c^5*Sin[(e + f*x)/2] - 84*A*c^4*d*Sin[(e + f*x
)/2] + 116*B*c^4*d*Sin[(e + f*x)/2] - 249*A*c^3*d^2*Sin[(e + f*x)/2] + 433*
B*c^3*d^2*Sin[(e + f*x)/2] - 385*A*c^2*d^3*Sin[(e + f*x)/2] + 277*B*c^2*d^3
```

```
*Sin[(e + f*x)/2] - 239*A*c*d^4*Sin[(e + f*x)/2] + 95*B*c*d^4*Sin[(e + f*x)
/2] - 47*A*d^5*Sin[(e + f*x)/2] + 51*B*d^5*Sin[(e + f*x)/2] - 12*A*c^5*Sin[
(3*(e + f*x))/2] - 20*B*c^5*Sin[(3*(e + f*x))/2] + 40*A*c^4*d*Sin[(3*(e + f
*x))/2] - 104*B*c^4*d*Sin[(3*(e + f*x))/2] + 261*A*c^3*d^2*Sin[(3*(e + f*x)
)/2] - 581*B*c^3*d^2*Sin[(3*(e + f*x))/2] + 781*A*c^2*d^3*Sin[(3*(e + f*x))
/2] - 665*B*c^2*d^3*Sin[(3*(e + f*x))/2] + 579*A*c*d^4*Sin[(3*(e + f*x))/2]
 - 299*B*c*d^4*Sin[(3*(e + f*x))/2] + 79*A*d^5*Sin[(3*(e + f*x))/2] - 59*B*
d^5*Sin[(3*(e + f*x))/2] - 12*A*c^4*d*Sin[(5*(e + f*x))/2] - 20*B*c^4*d*Sin
[(5*(e + f*x))/2] + 73*A*c^3*d^2*Sin[(5*(e + f*x))/2] - 217*B*c^3*d^2*Sin[(
5*(e + f*x))/2] + 353*A*c^2*d^3*Sin[(5*(e + f*x))/2] - 397*B*c^2*d^3*Sin[(5
*(e + f*x))/2] + 419*A*c*d^4*Sin[(5*(e + f*x))/2] - 251*B*c*d^4*Sin[(5*(e +
 f*x))/2] + 127*A*d^5*Sin[(5*(e + f*x))/2] - 75*B*d^5*Sin[(5*(e + f*x))/2]
+ 3*A*c^3*d^2*Sin[(7*(e + f*x))/2] + 5*B*c^3*d^2*Sin[(7*(e + f*x))/2] - 21*
A*c^2*d^3*Sin[(7*(e + f*x))/2] + 73*B*c^2*d^3*Sin[(7*(e + f*x))/2] - 111*A*
c*d^4*Sin[(7*(e + f*x))/2] + 79*B*c*d^4*Sin[(7*(e + f*x))/2] - 63*A*d^5*Sin
[(7*(e + f*x))/2] + 35*B*d^5*Sin[(7*(e + f*x))/2]))/(128*(c - d)^4*(c + d)^
2*f*(a*(1 + Sin[e + f*x]))^(5/2)*(c + d*Sin[e + f*x])^2)
```

fricas [B] time = 45.09, size = 8555, normalized size = 16.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*(4*(3*A + 5*B)*c^6 - 8*(9*A + 31*B)*c^5*d + 4*(117*A - 413*B
)*c^4*d^2 + 16*(177*A - 233*B)*c^3*d^3 + 4*(1197*A - 1013*B)*c^2*d^4 + 8*(4
23*A - 271*B)*c*d^5 + 4*(219*A - 115*B)*d^6 + ((3*A + 5*B)*c^4*d^2 - 24*(A
+ 3*B)*c^3*d^3 + 2*(81*A - 137*B)*c^2*d^4 + 24*(17*A - 13*B)*c*d^5 + (219*A
 - 115*B)*d^6)*cos(f*x + e)^5 + (2*(3*A + 5*B)*c^5*d - 3*(13*A + 43*B)*c^4*
d^2 + 4*(63*A - 191*B)*c^3*d^3 + 6*(217*A - 241*B)*c^2*d^4 + 2*(831*A - 583
*B)*c*d^5 + 3*(219*A - 115*B)*d^6)*cos(f*x + e)^4 - ((3*A + 5*B)*c^6 - 4*(3
*A + 13*B)*c^5*d + (75*A - 547*B)*c^4*d^2 + 8*(123*A - 203*B)*c^3*d^3 + 19*
(123*A - 115*B)*c^2*d^4 + 4*(525*A - 349*B)*c*d^5 + 3*(219*A - 115*B)*d^6)*
cos(f*x + e)^3 - (3*(3*A + 5*B)*c^6 - 2*(21*A + 83*B)*c^5*d + (267*A - 1507
*B)*c^4*d^2 + 4*(669*A - 1045*B)*c^3*d^3 + (5871*A - 5383*B)*c^2*d^4 + 2*(2
523*A - 1667*B)*c*d^5 + 7*(219*A - 115*B)*d^6)*cos(f*x + e)^2 + 2*((3*A + 5
*B)*c^6 - 2*(9*A + 31*B)*c^5*d + (117*A - 413*B)*c^4*d^2 + 4*(177*A - 233*B
)*c^3*d^3 + (1197*A - 1013*B)*c^2*d^4 + 2*(423*A - 271*B)*c*d^5 + (219*A -
115*B)*d^6)*cos(f*x + e) + (4*(3*A + 5*B)*c^6 - 8*(9*A + 31*B)*c^5*d + 4*(1
17*A - 413*B)*c^4*d^2 + 16*(177*A - 233*B)*c^3*d^3 + 4*(1197*A - 1013*B)*c^
2*d^4 + 8*(423*A - 271*B)*c*d^5 + 4*(219*A - 115*B)*d^6 + ((3*A + 5*B)*c^4*
d^2 - 24*(A + 3*B)*c^3*d^3 + 2*(81*A - 137*B)*c^2*d^4 + 24*(17*A - 13*B)*c*
d^5 + (219*A - 115*B)*d^6)*cos(f*x + e)^4 - 2*((3*A + 5*B)*c^5*d - (21*A +
```

```
67*B)*c^4*d^2 + 2*(69*A - 173*B)*c^3*d^3 + 2*(285*A - 293*B)*c^2*d^4 + (627
*A - 427*B)*c*d^5 + (219*A - 115*B)*d^6)*cos(f*x + e)^3 - ((3*A + 5*B)*c^6
- 6*(A + 7*B)*c^5*d + 3*(11*A - 227*B)*c^4*d^2 + 12*(105*A - 193*B)*c^3*d^3
 + 3*(1159*A - 1119*B)*c^2*d^4 + 6*(559*A - 375*B)*c*d^5 + 5*(219*A - 115*B
)*d^6)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^6 - 2*(9*A + 31*B)*c^5*d + (117*A
- 413*B)*c^4*d^2 + 4*(177*A - 233*B)*c^3*d^3 + (1197*A - 1013*B)*c^2*d^4 +
2*(423*A - 271*B)*c*d^5 + (219*A - 115*B)*d^6)*cos(f*x + e))*sin(f*x + e))*
sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)
*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2
*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) -
 cos(f*x + e) - 2)) - 4*(140*B*a*c^5*d - 28*(9*A - 20*B)*a*c^4*d^2 - 8*(108
*A - 121*B)*a*c^3*d^3 - 8*(141*A - 112*B)*a*c^2*d^4 - 4*(168*A - 107*B)*a*c
*d^5 - 4*(39*A - 20*B)*a*d^6 + (35*B*a*c^3*d^3 - 7*(9*A - 10*B)*a*c^2*d^4 -
 (90*A - 67*B)*a*c*d^5 - (39*A - 20*B)*a*d^6)*cos(f*x + e)^5 + (70*B*a*c^4*
d^2 - 7*(18*A - 35*B)*a*c^3*d^3 - (369*A - 344*B)*a*c^2*d^4 - (348*A - 241*
B)*a*c*d^5 - 3*(39*A - 20*B)*a*d^6)*cos(f*x + e)^4 - (35*B*a*c^5*d - 21*(3*
A - 10*B)*a*c^4*d^2 - 2*(171*A - 226*B)*a*c^3*d^3 - 6*(98*A - 83*B)*a*c^2*d
^4 - (426*A - 281*B)*a*c*d^5 - 3*(39*A - 20*B)*a*d^6)*cos(f*x + e)^3 - (105
*B*a*c^5*d - 7*(27*A - 80*B)*a*c^4*d^2 - 6*(150*A - 191*B)*a*c^3*d^3 - 2*(7
29*A - 610*B)*a*c^2*d^4 - 3*(340*A - 223*B)*a*c*d^5 - 7*(39*A - 20*B)*a*d^6
)*cos(f*x + e)^2 + 2*(35*B*a*c^5*d - 7*(9*A - 20*B)*a*c^4*d^2 - 2*(108*A -
121*B)*a*c^3*d^3 - 2*(141*A - 112*B)*a*c^2*d^4 - (168*A - 107*B)*a*c*d^5 -
(39*A - 20*B)*a*d^6)*cos(f*x + e) + (140*B*a*c^5*d - 28*(9*A - 20*B)*a*c^4*
d^2 - 8*(108*A - 121*B)*a*c^3*d^3 - 8*(141*A - 112*B)*a*c^2*d^4 - 4*(168*A
- 107*B)*a*c*d^5 - 4*(39*A - 20*B)*a*d^6 + (35*B*a*c^3*d^3 - 7*(9*A - 10*B)
*a*c^2*d^4 - (90*A - 67*B)*a*c*d^5 - (39*A - 20*B)*a*d^6)*cos(f*x + e)^4 -
2*(35*B*a*c^4*d^2 - 21*(3*A - 5*B)*a*c^3*d^3 - (153*A - 137*B)*a*c^2*d^4 -
3*(43*A - 29*B)*a*c*d^5 - (39*A - 20*B)*a*d^6)*cos(f*x + e)^3 - (35*B*a*c^5
*d - 7*(9*A - 40*B)*a*c^4*d^2 - 2*(234*A - 331*B)*a*c^3*d^3 - 2*(447*A - 38
6*B)*a*c^2*d^4 - (684*A - 455*B)*a*c*d^5 - 5*(39*A - 20*B)*a*d^6)*cos(f*x +
 e)^2 + 2*(35*B*a*c^5*d - 7*(9*A - 20*B)*a*c^4*d^2 - 2*(108*A - 121*B)*a*c^
3*d^3 - 2*(141*A - 112*B)*a*c^2*d^4 - (168*A - 107*B)*a*c*d^5 - (39*A - 20*
B)*a*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x
+ e)^3 - (6*c*d + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 4*((c*d + d^2
)*cos(f*x + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e)
 + (c^2 + 4*c*d + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*si
n(f*x + e) + a)*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) +
(d^2*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*s
in(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c
*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x +
 e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) - 4*(4*(A - B)*c^6 - 8*(A - B)*c^5*
d - 4*(A - B)*c^4*d^2 + 16*(A - B)*c^3*d^3 - 4*(A - B)*c^2*d^4 - 8*(A - B)*
c*d^5 + 4*(A - B)*d^6 - ((3*A + 5*B)*c^4*d^2 - 4*(6*A - 17*B)*c^3*d^3 - 6*(
15*A - B)*c^2*d^4 + 4*(12*A - 11*B)*c*d^5 + 7*(9*A - 5*B)*d^6)*cos(f*x + e)
^4 - (2*(3*A + 5*B)*c^5*d - (41*A - 101*B)*c^4*d^2 - 4*(38*A - 31*B)*c^3*d^
```

```
3 - 2*(39*A + 35*B)*c^2*d^4 + 10*(17*A - 11*B)*c*d^5 + 5*(19*A - 11*B)*d^6)
*cos(f*x + e)^3 + ((3*A + 5*B)*c^6 - 16*(A - B)*c^5*d - (31*A - 75*B)*c^4*d
^2 - 4*(21*A - 11*B)*c^3*d^3 - (23*A + 49*B)*c^2*d^4 + 20*(5*A - 3*B)*c*d^5
 + (51*A - 31*B)*d^6)*cos(f*x + e)^2 + ((7*A + B)*c^6 - 2*(9*A - 17*B)*c^5*
d - (79*A - 175*B)*c^4*d^2 - 28*(7*A - 3*B)*c^3*d^3 - (15*A + 121*B)*c^2*d^
4 + 2*(107*A - 59*B)*c*d^5 + (87*A - 55*B)*d^6)*cos(f*x + e) - (4*(A - B)*c
^6 - 8*(A - B)*c^5*d - 4*(A - B)*c^4*d^2 + 16*(A - B)*c^3*d^3 - 4*(A - B)*c
^2*d^4 - 8*(A - B)*c*d^5 + 4*(A - B)*d^6 + ((3*A + 5*B)*c^4*d^2 - 4*(6*A -
17*B)*c^3*d^3 - 6*(15*A - B)*c^2*d^4 + 4*(12*A - 11*B)*c*d^5 + 7*(9*A - 5*B
)*d^6)*cos(f*x + e)^3 - 2*((3*A + 5*B)*c^5*d - 2*(11*A - 24*B)*c^4*d^2 - 4*
(16*A - 7*B)*c^3*d^3 + 2*(3*A - 19*B)*c^2*d^4 + (61*A - 33*B)*c*d^5 + 2*(8*
A - 5*B)*d^6)*cos(f*x + e)^2 - ((3*A + 5*B)*c^6 - 2*(5*A - 13*B)*c^5*d - 3*
(25*A - 57*B)*c^4*d^2 - 4*(53*A - 25*B)*c^3*d^3 - (11*A + 125*B)*c^2*d^4 +
6*(37*A - 21*B)*c*d^5 + (83*A - 51*B)*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt
(a*sin(f*x + e) + a))/((a^3*c^7*d^2 - 3*a^3*c^6*d^3 + a^3*c^5*d^4 + 5*a^3*c
^4*d^5 - 5*a^3*c^3*d^6 - a^3*c^2*d^7 + 3*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e
)^5 + (2*a^3*c^8*d - 3*a^3*c^7*d^2 - 7*a^3*c^6*d^3 + 13*a^3*c^5*d^4 + 5*a^3
*c^4*d^5 - 17*a^3*c^3*d^6 + 3*a^3*c^2*d^7 + 7*a^3*c*d^8 - 3*a^3*d^9)*f*cos(
f*x + e)^4 - (a^3*c^9 + a^3*c^8*d - 8*a^3*c^7*d^2 + 18*a^3*c^5*d^4 - 6*a^3*
c^4*d^5 - 16*a^3*c^3*d^6 + 8*a^3*c^2*d^7 + 5*a^3*c*d^8 - 3*a^3*d^9)*f*cos(f
*x + e)^3 - (3*a^3*c^9 + a^3*c^8*d - 20*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 42*a^
3*c^5*d^4 - 18*a^3*c^4*d^5 - 36*a^3*c^3*d^6 + 20*a^3*c^2*d^7 + 11*a^3*c*d^8
 - 7*a^3*d^9)*f*cos(f*x + e)^2 + 2*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4
*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^
7 + a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e) + 4*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^
7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a
^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f + ((a^3*c^7*d^2 - 3*a^3*c^6*d^3 + a^3*c
^5*d^4 + 5*a^3*c^4*d^5 - 5*a^3*c^3*d^6 - a^3*c^2*d^7 + 3*a^3*c*d^8 - a^3*d^
9)*f*cos(f*x + e)^4 - 2*(a^3*c^8*d - 2*a^3*c^7*d^2 - 2*a^3*c^6*d^3 + 6*a^3*
c^5*d^4 - 6*a^3*c^3*d^6 + 2*a^3*c^2*d^7 + 2*a^3*c*d^8 - a^3*d^9)*f*cos(f*x
+ e)^3 - (a^3*c^9 + 3*a^3*c^8*d - 12*a^3*c^7*d^2 - 4*a^3*c^6*d^3 + 30*a^3*c
^5*d^4 - 6*a^3*c^4*d^5 - 28*a^3*c^3*d^6 + 12*a^3*c^2*d^7 + 9*a^3*c*d^8 - 5*
a^3*d^9)*f*cos(f*x + e)^2 + 2*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*
c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a
^3*c*d^8 - a^3*d^9)*f*cos(f*x + e) + 4*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2
 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^
2*d^7 + a^3*c*d^8 - a^3*d^9)*f)*sin(f*x + e)), 1/64*(sqrt(2)*(4*(3*A + 5*B)
*c^6 - 8*(9*A + 31*B)*c^5*d + 4*(117*A - 413*B)*c^4*d^2 + 16*(177*A - 233*B
)*c^3*d^3 + 4*(1197*A - 1013*B)*c^2*d^4 + 8*(423*A - 271*B)*c*d^5 + 4*(219*
A - 115*B)*d^6 + ((3*A + 5*B)*c^4*d^2 - 24*(A + 3*B)*c^3*d^3 + 2*(81*A - 13
7*B)*c^2*d^4 + 24*(17*A - 13*B)*c*d^5 + (219*A - 115*B)*d^6)*cos(f*x + e)^5
 + (2*(3*A + 5*B)*c^5*d - 3*(13*A + 43*B)*c^4*d^2 + 4*(63*A - 191*B)*c^3*d^
3 + 6*(217*A - 241*B)*c^2*d^4 + 2*(831*A - 583*B)*c*d^5 + 3*(219*A - 115*B)
*d^6)*cos(f*x + e)^4 - ((3*A + 5*B)*c^6 - 4*(3*A + 13*B)*c^5*d + (75*A - 54
7*B)*c^4*d^2 + 8*(123*A - 203*B)*c^3*d^3 + 19*(123*A - 115*B)*c^2*d^4 + 4*(
```

525*A - 349*B)*c*d^5 + 3*(219*A - 115*B)*d^6)*cos(f*x + e)^3 - (3*(3*A + 5*B)*c^6 - 2*(21*A + 83*B)*c^5*d + (267*A - 1507*B)*c^4*d^2 + 4*(669*A - 1045*B)*c^3*d^3 + (5871*A - 5383*B)*c^2*d^4 + 2*(2523*A - 1667*B)*c*d^5 + 7*(219*A - 115*B)*d^6)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^6 - 2*(9*A + 31*B)*c^5*d + (117*A - 413*B)*c^4*d^2 + 4*(177*A - 233*B)*c^3*d^3 + (1197*A - 1013*B)*c^2*d^4 + 2*(423*A - 271*B)*c*d^5 + (219*A - 115*B)*d^6)*cos(f*x + e) + (4*(3*A + 5*B)*c^6 - 8*(9*A + 31*B)*c^5*d + 4*(117*A - 413*B)*c^4*d^2 + 16*(177*A - 233*B)*c^3*d^3 + 4*(1197*A - 1013*B)*c^2*d^4 + 8*(423*A - 271*B)*c*d^5 + 4*(219*A - 115*B)*d^6 + ((3*A + 5*B)*c^4*d^2 - 24*(A + 3*B)*c^3*d^3 + 2*(81*A - 137*B)*c^2*d^4 + 24*(17*A - 13*B)*c*d^5 + (219*A - 115*B)*d^6)*cos(f*x + e)^4 - 2*((3*A + 5*B)*c^5*d - (21*A + 67*B)*c^4*d^2 + 2*(69*A - 173*B)*c^3*d^3 + 2*(285*A - 293*B)*c^2*d^4 + (627*A - 427*B)*c*d^5 + (219*A - 115*B)*d^6)*cos(f*x + e)^3 - ((3*A + 5*B)*c^6 - 6*(A + 7*B)*c^5*d + 3*(11*A - 227*B)*c^4*d^2 + 12*(105*A - 193*B)*c^3*d^3 + 3*(1159*A - 1119*B)*c^2*d^4 + 6*(559*A - 375*B)*c*d^5 + 5*(219*A - 115*B)*d^6)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^6 - 2*(9*A + 31*B)*c^5*d + (117*A - 413*B)*c^4*d^2 + 4*(177*A - 233*B)*c^3*d^3 + (1197*A - 1013*B)*c^2*d^4 + 2*(423*A - 271*B)*c*d^5 + (219*A - 115*B)*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 8*(140*B*a*c^5*d - 28*(9*A - 20*B)*a*c^4*d^2 - 8*(108*A - 121*B)*a*c^3*d^3 - 8*(141*A - 112*B)*a*c^2*d^4 - 4*(168*A - 107*B)*a*c*d^5 - 4*(39*A - 20*B)*a*d^6 + (35*B*a*c^3*d^3 - 7*(9*A - 10*B)*a*c^2*d^4 - (90*A - 67*B)*a*c*d^5 - (39*A - 20*B)*a*d^6)*cos(f*x + e)^5 + (70*B*a*c^4*d^2 - 7*(18*A - 35*B)*a*c^3*d^3 - (369*A - 344*B)*a*c^2*d^4 - (348*A - 241*B)*a*c*d^5 - 3*(39*A - 20*B)*a*d^6)*cos(f*x + e)^4 - (35*B*a*c^5*d - 21*(3*A - 10*B)*a*c^4*d^2 - 2*(171*A - 226*B)*a*c^3*d^3 - 6*(98*A - 83*B)*a*c^2*d^4 - (426*A - 281*B)*a*c*d^5 - 3*(39*A - 20*B)*a*d^6)*cos(f*x + e)^3 - (105*B*a*c^5*d - 7*(27*A - 80*B)*a*c^4*d^2 - 6*(150*A - 191*B)*a*c^3*d^3 - 2*(729*A - 610*B)*a*c^2*d^4 - 3*(340*A - 223*B)*a*c*d^5 - 7*(39*A - 20*B)*a*d^6)*cos(f*x + e)^2 + 2*(35*B*a*c^5*d - 7*(9*A - 20*B)*a*c^4*d^2 - 2*(108*A - 121*B)*a*c^3*d^3 - 2*(141*A - 112*B)*a*c^2*d^4 - (168*A - 107*B)*a*c*d^5 - (39*A - 20*B)*a*d^6)*cos(f*x + e) + (140*B*a*c^5*d - 28*(9*A - 20*B)*a*c^4*d^2 - 8*(108*A - 121*B)*a*c^3*d^3 - 8*(141*A - 112*B)*a*c^2*d^4 - 4*(168*A - 107*B)*a*c*d^5 - 4*(39*A - 20*B)*a*d^6 + (35*B*a*c^3*d^3 - 7*(9*A - 10*B)*a*c^2*d^4 - (90*A - 67*B)*a*c*d^5 - (39*A - 20*B)*a*d^6)*cos(f*x + e)^4 - 2*(35*B*a*c^4*d^2 - 21*(3*A - 5*B)*a*c^3*d^3 - (153*A - 137*B)*a*c^2*d^4 - 3*(43*A - 29*B)*a*c*d^5 - (39*A - 20*B)*a*d^6)*cos(f*x + e)^3 - (35*B*a*c^5*d - 7*(9*A - 40*B)*a*c^4*d^2 - 2*(234*A - 331*B)*a*c^3*d^3 - 2*(447*A - 386*B)*a*c^2*d^4 - (684*A - 455*B)*a*c*d^5 - 5*(39*A - 20*B)*a*d^6)*cos(f*x + e)^2 + 2*(35*B*a*c^5*d - 7*(9*A - 20*B)*a*c^4*d^2 - 2*(108*A - 121*B)*a*c^3*d^3 - 2*(141*A - 112*B)*a*c^2*d^4 - (168*A - 107*B)*a*c*d^5 - (39*A - 20*B)*a*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e))) - 4*(4*(A - B)*c^6

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 - 8*(A - B)*c^5*d - 4*(A - B)*c^4*d^2 + 16*(A - B)*c^3*d^3 - 4*(A - B)*c^2
*d^4 - 8*(A - B)*c*d^5 + 4*(A - B)*d^6 - ((3*A + 5*B)*c^4*d^2 - 4*(6*A - 17
*B)*c^3*d^3 - 6*(15*A - B)*c^2*d^4 + 4*(12*A - 11*B)*c*d^5 + 7*(9*A - 5*B)*
d^6)*cos(f*x + e)^4 - (2*(3*A + 5*B)*c^5*d - (41*A - 101*B)*c^4*d^2 - 4*(38
*A - 31*B)*c^3*d^3 - 2*(39*A + 35*B)*c^2*d^4 + 10*(17*A - 11*B)*c*d^5 + 5*(
19*A - 11*B)*d^6)*cos(f*x + e)^3 + ((3*A + 5*B)*c^6 - 16*(A - B)*c^5*d - (3
1*A - 75*B)*c^4*d^2 - 4*(21*A - 11*B)*c^3*d^3 - (23*A + 49*B)*c^2*d^4 + 20*
(5*A - 3*B)*c*d^5 + (51*A - 31*B)*d^6)*cos(f*x + e)^2 + ((7*A + B)*c^6 - 2*
(9*A - 17*B)*c^5*d - (79*A - 175*B)*c^4*d^2 - 28*(7*A - 3*B)*c^3*d^3 - (15*
A + 121*B)*c^2*d^4 + 2*(107*A - 59*B)*c*d^5 + (87*A - 55*B)*d^6)*cos(f*x +
e) - (4*(A - B)*c^6 - 8*(A - B)*c^5*d - 4*(A - B)*c^4*d^2 + 16*(A - B)*c^3*
d^3 - 4*(A - B)*c^2*d^4 - 8*(A - B)*c*d^5 + 4*(A - B)*d^6 + ((3*A + 5*B)*c^
4*d^2 - 4*(6*A - 17*B)*c^3*d^3 - 6*(15*A - B)*c^2*d^4 + 4*(12*A - 11*B)*c*d
^5 + 7*(9*A - 5*B)*d^6)*cos(f*x + e)^3 - 2*((3*A + 5*B)*c^5*d - 2*(11*A - 2
4*B)*c^4*d^2 - 4*(16*A - 7*B)*c^3*d^3 + 2*(3*A - 19*B)*c^2*d^4 + (61*A - 33
*B)*c*d^5 + 2*(8*A - 5*B)*d^6)*cos(f*x + e)^2 - ((3*A + 5*B)*c^6 - 2*(5*A -
 13*B)*c^5*d - 3*(25*A - 57*B)*c^4*d^2 - 4*(53*A - 25*B)*c^3*d^3 - (11*A +
125*B)*c^2*d^4 + 6*(37*A - 21*B)*c*d^5 + (83*A - 51*B)*d^6)*cos(f*x + e))*s
in(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^3*c^7*d^2 - 3*a^3*c^6*d^3 + a^3*
c^5*d^4 + 5*a^3*c^4*d^5 - 5*a^3*c^3*d^6 - a^3*c^2*d^7 + 3*a^3*c*d^8 - a^3*d
^9)*f*cos(f*x + e)^5 + (2*a^3*c^8*d - 3*a^3*c^7*d^2 - 7*a^3*c^6*d^3 + 13*a^
3*c^5*d^4 + 5*a^3*c^4*d^5 - 17*a^3*c^3*d^6 + 3*a^3*c^2*d^7 + 7*a^3*c*d^8 -
3*a^3*d^9)*f*cos(f*x + e)^4 - (a^3*c^9 + a^3*c^8*d - 8*a^3*c^7*d^2 + 18*a^3
*c^5*d^4 - 6*a^3*c^4*d^5 - 16*a^3*c^3*d^6 + 8*a^3*c^2*d^7 + 5*a^3*c*d^8 - 3
*a^3*d^9)*f*cos(f*x + e)^3 - (3*a^3*c^9 + a^3*c^8*d - 20*a^3*c^7*d^2 + 4*a^
3*c^6*d^3 + 42*a^3*c^5*d^4 - 18*a^3*c^4*d^5 - 36*a^3*c^3*d^6 + 20*a^3*c^2*d
^7 + 11*a^3*c*d^8 - 7*a^3*d^9)*f*cos(f*x + e)^2 + 2*(a^3*c^9 - a^3*c^8*d -
4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d
^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e) + 4*(a^3*c^9 - a^3
*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*
a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f + ((a^3*c^7*d^2 - 3*a^
3*c^6*d^3 + a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 5*a^3*c^3*d^6 - a^3*c^2*d^7 + 3*a
^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^4 - 2*(a^3*c^8*d - 2*a^3*c^7*d^2 - 2*a^3
*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^3*d^6 + 2*a^3*c^2*d^7 + 2*a^3*c*d^8 - a^
3*d^9)*f*cos(f*x + e)^3 - (a^3*c^9 + 3*a^3*c^8*d - 12*a^3*c^7*d^2 - 4*a^3*c
^6*d^3 + 30*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 28*a^3*c^3*d^6 + 12*a^3*c^2*d^7 +
 9*a^3*c*d^8 - 5*a^3*d^9)*f*cos(f*x + e)^2 + 2*(a^3*c^9 - a^3*c^8*d - 4*a^3
*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 +
4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e) + 4*(a^3*c^9 - a^3*c^8*
d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c
^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f)*sin(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
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able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign
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to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(cos((f*t_nostep+exp(1))/2-pi/4))]Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Discontinuities at zeroes of cos((f*t_nostep+exp(1))/2-pi/4) were not ch
eckedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
eck sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t
_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi
/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/
x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable
```

```
 to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Una
ble to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
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o check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*p
i/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to che
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)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
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to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*p
i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
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2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check s
ign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unab
le to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-
4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable t
o check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi
/x/2)Warning, integration of abs or sign assumes constant sign by intervals
 (correct if the argument is real):Check [abs(t_nostep+1)]Evaluation time:
2.32Conj Error: Bad Argument Type
```

maple [B] time = 5.81, size = 7322, normalized size = 14.11

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x)
```

```
[Out] result too large to display
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^3
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^3
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.328 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=221

$$-\frac{4\sqrt{2}\,a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx)}{c+d}\right.}{f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -8*a^2*B*AppellF1(1/2,-n,-5/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e)
)*cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*x+e))/(c+d))^n)/(1+s
in(f*x+e))^(1/2)-4*a^2*(A-B)*AppellF1(1/2,-n,-3/2,3/2,d*(1-sin(f*x+e))/(c+d
),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*
x+e))/(c+d))^n)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2987, 2784, 139, 138}

$$-\frac{4\sqrt{2}\,a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx)}{c+d}\right.}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-8*Sqrt[2]*a^2*B*AppellF1[1/2, -5/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1
- Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 +
Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n) - (4*Sqrt[2]*a^2*(A - B)*Ap
pellF1[1/2, -3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c
+ d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Sin[e + f*x]]*((c +
d*Sin[e + f*x])/(c + d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2784

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^m*Cos[e + f*x])/(f*Sqrt[1 + Sin[e
+ f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[((1 + (b*x)/a)^(m - 1/2)*(c + d*
x)^n)/Sqrt[1 - (b*x)/a], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = (A-B)\int (a+a\sin(e+fx))^2(c+d\sin(e+f$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\right)\text{Subst}\left(\int \frac{(1+x)^{3/2}(c+dx)}{\sqrt{1-x}}\right.}{f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(-\right.\right.}{f\sqrt{1-\sin}}$$

$$= -\frac{8\sqrt{2}\,a^2BF_1\left(\frac{1}{2};-\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\right.}{f}$$

Mathematica [F] time = 26.08, size = 0, normalized size = 0.00

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c + d\sin(e + fx))^n \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
n,x]
```

```
[Out] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
n, x]
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left((A+2B)a^2\cos\left(fx+e\right)^2-2(A+B)a^2+\left(Ba^2\cos\left(fx+e\right)^2-2(A+B)a^2\right)\sin\left(fx+e\right)\right)\left(d\sin\left(f\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*(d*sin(f*x + e) + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^2\left(d\sin\left(fx+e\right)+c\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^
n, x)
```

maple [F] time = 3.87, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^2 \left(A+B\sin\left(fx+e\right)\right) \left(c+d\sin\left(fx+e\right)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{2}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^
n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

3.329 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=217

$$-\frac{2\sqrt{2}\,a(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}$$

```
[Out] -4*a*B*AppellF1(1/2,-n,-3/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*
cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*x+e))/(c+d))^n)/(1+sin
(f*x+e))^(1/2)-2*a*(A-B)*AppellF1(1/2,-n,-1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/
2-1/2*sin(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*x+e)
)/(c+d))^n)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2968, 3017, 2755, 139, 138, 2784}

$$-\frac{2\sqrt{2}\,a(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-4*Sqrt[2]*a*B*AppellF1[1/2, -3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 -
Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Si
n[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n) - (2*Sqrt[2]*a*(A - B)*Appell
F1[1/2, -1/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)
]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Sin[e + f*x]]*((c + d*Si
n[e + f*x])/(c + d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2755

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(c*Cos[e + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*S
qrt[1 - Sin[e + f*x]]), Subst[Int[((a + b*x)^m*Sqrt[1 + (d*x)/c])/Sqrt[1 -
(d*x)/c], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m] && EqQ[c^2 - d^2, 0
]
```

Rule 2784

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^m*Cos[e + f*x])/(f*Sqrt[1 + Sin[e
+ f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[((1 + (b*x)/a)^(m - 1/2)*(c + d*
x)^n)/Sqrt[1 - (b*x)/a], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3017

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[A - C, I
nt[(a + b*Sin[e + f*x])^m*(1 + Sin[e + f*x]), x], x] + Dist[C, Int[(a + b*S
in[e + f*x])^m*(1 + Sin[e + f*x])^2, x], x] /; FreeQ[{a, b, e, f, A, B, C,
m}, x] && EqQ[A - B + C, 0] &&  !IntegerQ[2*m]
```

Rubi steps

$$\int (a + a\sin(e+fx))(A + B\sin(e+fx))(c + d\sin(e+fx))^n\,dx = \int (c + d\sin(e+fx))^n \left(aA + (aA + aB)\sin(e\right.$$

$$= (a(A-B)) \int (1 + \sin(e+fx))(c + d\sin(e+f$$

$$= \frac{(a(A-B)\cos(e+fx))\,\text{Subst}\left(\int \frac{\sqrt{1+x}\,(c+dx)^n}{\sqrt{1-x}}\,d\right.}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+}}$$

$$= \frac{\left(a(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(-\right.\right.}{f\sqrt{1-\sin}}$$

$$= -\frac{4\sqrt{2}\,aBF_1\left(\frac{1}{2}; -\frac{3}{2}, -n; \frac{3}{2}; \frac{1}{2}(1-\sin(e+fx)),\right.}{f}$$

Mathematica [F] time = 9.07, size = 0, normalized size = 0.00

$$\int (a + a\sin(e+fx))(A + B\sin(e+fx))(c + d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,
x]
```

```
[Out] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,
 x]
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(Ba\cos\left(fx+e\right)^2 - (A+B)a\sin\left(fx+e\right) - (A+B)a\right)\left(d\sin\left(fx+e\right)+c\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm
="fricas")
```

```
[Out] integral(-(B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*(d*sin(
f*x + e) + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n,
 x)
```

maple [F] time = 3.37, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)\left(A+B\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n,
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

3.330 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=221

$$-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{\sqrt{2}af\sqrt{\sin(e+fx)+1}}-\frac{\sqrt{2}B}{}$$

```
[Out] -1/2*(A-B)*AppellF1(1/2,-n,3/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e
))*cos(f*x+e)*(c+d*sin(f*x+e))^n/a/f/(((c+d*sin(f*x+e))/(c+d))^n)*2^(1/2)/(
1+sin(f*x+e))^(1/2)-B*AppellF1(1/2,-n,1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/
2*sin(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/a/f/(((c+d*sin(f*x+e))/
(c+d))^n)/(1+sin(f*x+e))^(1/2)
```

Rubi [**A**] time = 0.30, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2987, 2784, 139, 138, 2665}

$$-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{\sqrt{2}af\sqrt{\sin(e+fx)+1}}-\frac{\sqrt{2}B}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),x]
```

```
[Out] -((Sqrt[2]*B*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[
e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a*f*Sqrt[1 + Sin[
e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)) - ((A - B)*AppellF1[1/2, 3/2,
-n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e + f*x]
*(c + d*Sin[e + f*x])^n)/(Sqrt[2]*a*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e
+ f*x])/(c + d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2784

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^m*Cos[e + f*x])/(f*Sqrt[1 + Sin[e
+ f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[((1 + (b*x)/a)^(m - 1/2)*(c + d*
x)^n)/Sqrt[1 - (b*x)/a], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = (A-B)\int \frac{(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx + \frac{B\int (c+d\sin(e+fx))^n\,dx}{a}$$

$$= \frac{((A-B)\cos(e+fx))\,\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{1-x}\,(1+x)^{3/2}}\,dx, x, \sin(e+fx)\right)}{af\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left((A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Sub}}{af\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e}}$$

$$= -\frac{\sqrt{2}\,BF_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+}{af\sqrt{1+\sin(e+fx)}}$$

Mathematica [F] time = 4.65, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]
),x]
```

```
[Out] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]
), x]
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^n}{a\sin\left(fx+e\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^n}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a),
 x)
```

maple [F] time = 1.06, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}}{a+a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

```
[Out] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{n}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{n}}{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.331 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=223

$$-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{2\sqrt{2}\,a^2f\sqrt{\sin(e+fx)+1}}-\frac{B\cos(}{}$$

```
[Out] -1/2*B*AppellF1(1/2,-n,3/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*c
os(f*x+e)*(c+d*sin(f*x+e))^n/a^2/f/(((c+d*sin(f*x+e))/(c+d))^n)*2^(1/2)/(1+
sin(f*x+e))^(1/2)-1/4*(A-B)*AppellF1(1/2,-n,5/2,3/2,d*(1-sin(f*x+e))/(c+d),
1/2-1/2*sin(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^n/a^2/f/(((c+d*sin(f*x+e))/
(c+d))^n)*2^(1/2)/(1+sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2987, 2784, 139, 138}

$$-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{2\sqrt{2}\,a^2f\sqrt{\sin(e+fx)+1}}-\frac{B\cos(}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -((B*AppellF1[1/2, 3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]
))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(Sqrt[2]*a^2*f*Sqrt[1 + Si
n[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)) - ((A - B)*AppellF1[1/2, 5/2
, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e + f*
x]*(c + d*Sin[e + f*x])^n)/(2*Sqrt[2]*a^2*f*Sqrt[1 + Sin[e + f*x]]*((c + d*
Sin[e + f*x])/(c + d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2784

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^m*Cos[e + f*x])/(f*Sqrt[1 + Sin[e
+ f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[((1 + (b*x)/a)^(m - 1/2)*(c + d*
x)^n)/Sqrt[1 - (b*x)/a], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = (A-B)\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx + \frac{B\int \frac{(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx}{a}$$

$$= \frac{((A-B)\cos(e+fx))\,\mathrm{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{1-x}\,(1+x)^{5/2}}\,dx, x, \sin(e+fx)\right)}{a^2 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left((A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\mathrm{Sub}}{a^2 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e}$$

$$= -\frac{BF_1\left(\frac{1}{2};\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)}{\sqrt{2}\,a^2 f\sqrt{1+\sin(e+fx)}}$$

Mathematica [F] time = 28.14, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]
)^2, x]
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{n}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a^2*cos(f*x + e)^2 -
 2*a^2*sin(f*x + e) - 2*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^
2, x)
```

maple [F] time = 10.29, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}}{\left(a+a\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^
2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{n}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2, x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.332 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=427

$$\frac{2a^2(A-B)(c-d(4n+5))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)}{df(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2a^2(A-}{df}$$

```
[Out] -2*a^2*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)/d/f/(3+2*n)/(a+a*sin(f*x+e))
^(1/2)+2*a^2*B*(3*c-d*(11+4*n))*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)/d^2/f/(4*
n^2+16*n+15)/(a+a*sin(f*x+e))^(1/2)+2*a^2*(A-B)*(c-d*(5+4*n))*cos(f*x+e)*hy
pergeom([1/2, -n],[3/2],d*(1-sin(f*x+e))/(c+d))*(c+d*sin(f*x+e))^n/d/f/(3+2
*n)/(((c+d*sin(f*x+e))/(c+d))^n)/(a+a*sin(f*x+e))^(1/2)-2*a^2*B*(3*c^2-2*c*
d*(7+4*n)+d^2*(16*n^2+56*n+43))*cos(f*x+e)*hypergeom([1/2, -n],[3/2],d*(1-s
in(f*x+e))/(c+d))*(c+d*sin(f*x+e))^n/d^2/f/(4*n^2+16*n+15)/(((c+d*sin(f*x+e
))/(c+d))^n)/(a+a*sin(f*x+e))^(1/2)-2*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)
*(a+a*sin(f*x+e))^(1/2)/d/f/(5+2*n)
```

Rubi [A] time = 0.91, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {2987, 2763, 21, 2776, 70, 69, 2981}

$$\frac{2a^2(A-B)(c-d(4n+5))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)}{df(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2a^2(A-}{df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,
x]
```

```
[Out] (-2*a^2*(A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*S
qrt[a + a*Sin[e + f*x]]) + (2*a^2*B*(3*c - d*(11 + 4*n))*Cos[e + f*x]*(c +
d*Sin[e + f*x])^(1 + n))/(d^2*f*(3 + 2*n)*(5 + 2*n)*Sqrt[a + a*Sin[e + f*x]
]) - (2*a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(1 +
 n))/(d*f*(5 + 2*n)) + (2*a^2*(A - B)*(c - d*(5 + 4*n))*Cos[e + f*x]*Hyperg
eometric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[e + f*
x])^n)/(d*f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d
))^n) - (2*a^2*B*(3*c^2 - 2*c*d*(7 + 4*n) + d^2*(43 + 56*n + 16*n^2))*Cos[e
 + f*x]*Hypergeometric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c
+ d*Sin[e + f*x])^n)/(d^2*f*(3 + 2*n)*(5 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((
c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
```

```
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx=(A-B)\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+$$

$$=-\frac{2a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^1}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$

$$=-\frac{2a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^1}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$

$$=-\frac{2a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^1}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$

$$=-\frac{2a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^1}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$

$$=-\frac{2a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^1}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 26.65, size = 245, normalized size = 0.57

$$-\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\left(-30(A+B)(c-d(4n+5))\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};-\frac{d(\sin(e+fx)-1)}{c+d}\right)+\dots\right.}{\dots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x])^n,x]
```

```
[Out] -1/15*(a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^n*(-30*(A + B)*(c - d*(5 + 4*n
))*Hypergeometric2F1[1/2, -n, 3/2, -((d*(-1 + Sin[e + f*x]))/(c + d))] + 6*
B*d*(3 + 2*n)*Hypergeometric2F1[5/2, -n, 7/2, -((d*(-1 + Sin[e + f*x]))/(c
+ d))]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 20*B*d*(3 + 2*n)*Hypergeom
etric2F1[3/2, -n, 5/2, -((d*(-1 + Sin[e + f*x]))/(c + d))]*(-1 + Sin[e + f*
x]) + 30*(A + B)*(c + d)*((c + d*Sin[e + f*x])/(c + d))^(1 + n)))/(d*f*(3 +
 2*n)*Sqrt[a*(1 + Sin[e + f*x])]*((c + d*Sin[e + f*x])/(c + d))^n)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(Ba\cos\left(fx+e\right)^2-(A+B)a\sin\left(fx+e\right)-(A+B)a\right)\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, alg
orithm="fricas")
```

```
[Out] integral(-(B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*
sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) +
 c)^n, x)
```

maple [F] time = 1.28, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\left(A+B\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) +
 c)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^n,
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^n,
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

3.333 $\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=167

$$-\frac{2a\cos(e+fx)(Ad(2n+3)-B(c-2d(n+1)))(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -2*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)/d/f/(3+2*n)/(a+a*sin(f*x+e))^(1/2)
-2*a*(A*d*(3+2*n)-B*(c-2*d*(1+n)))*cos(f*x+e)*hypergeom([1/2, -n],[3/2],d*(
1-sin(f*x+e))/(c+d))*(c+d*sin(f*x+e))^n/d/f/(3+2*n)/(((c+d*sin(f*x+e))/(c+d
))^n)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2981, 2776, 70, 69}

$$\frac{2a\cos(e+fx)(-Ad(2n+3)+Bc-2Bd(n+1))(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-2*a*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*Sqrt[a +
a*Sin[e + f*x]]) + (2*a*(B*c - 2*B*d*(1 + n) - A*d*(3 + 2*n))*Cos[e + f*x]*
Hypergeometric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[
e + f*x])^n)/(d*f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e + f*x])/
(c + d))^n)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
```

```
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = -\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{(aA}{}$$

$$= -\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{(a(}{}$$

$$= -\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{\left(a(\right.}{}$$

$$= -\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{2a(}{}$$

Mathematica [F] time = 8.25, size = 0, normalized size = 0.00

$$\int \sqrt{a+a\sin(e+fx)}\,(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^n,x]
```

```
[Out] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^n, x]
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, alg
orithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)
^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c
)^n, x)
```

maple [F] time = 1.24, size = 0, normalized size = 0.00

$$\int\sqrt{a+a\sin\left(fx+e\right)}\,\left(A+B\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c
)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^n,
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^n,
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \left(\sin{\left(e + f x \right)} + 1\right)} \left(A + B \sin{\left(e + f x \right)}\right) \left(c + d \sin{\left(e + f x \right)}\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))*(c + d*sin(e + f*x
))**n, x)
```

3.334 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=220

$$-\frac{(A-B)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}\,(c+d\sin(e+fx))^{n+1}F_1\left(n+1;\frac{1}{2},1;n+2;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)}{f(n+1)(c-d)(1-\sin(e+fx))\sqrt{a\sin(e+fx)+a}}-\frac{2B\cos}{}$$

```
[Out] -2*B*cos(f*x+e)*hypergeom([1/2, -n],[3/2],d*(1-sin(f*x+e))/(c+d))*(c+d*sin(
f*x+e))^n/f/(((c+d*sin(f*x+e))/(c+d))^n)/(a+a*sin(f*x+e))^(1/2)-(A-B)*Appel
lF1(1+n,1,1/2,2+n,(c+d*sin(f*x+e))/(c-d),(c+d*sin(f*x+e))/(c+d))*cos(f*x+e)
*(c+d*sin(f*x+e))^(1+n)*(d*(1-sin(f*x+e))/(c+d))^(1/2)/(c-d)/f/(1+n)/(1-sin
(f*x+e))/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {2987, 2788, 137, 136, 2776, 70, 69}

$$-\frac{(A-B)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}\,(c+d\sin(e+fx))^{n+1}F_1\left(n+1;\frac{1}{2},1;n+2;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)}{f(n+1)(c-d)(1-\sin(e+fx))\sqrt{a\sin(e+fx)+a}}-\frac{2B\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/Sqrt[a + a*Sin[e + f*x]],
x]
```

```
[Out] -(((A - B)*AppellF1[1 + n, 1/2, 1, 2 + n, (c + d*Sin[e + f*x])/(c + d), (c
+ d*Sin[e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Sin[e + f*x]))/(c + d)
]*(c + d*Sin[e + f*x])^(1 + n))/((c - d)*f*(1 + n)*(1 - Sin[e + f*x])*Sqrt[
a + a*Sin[e + f*x]])) - (2*B*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, (
d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[a + a*Sin[e
+ f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !IntegerQ[2*n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
```

```
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx = (A-B)\int \frac{(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx + \frac{B\int \sqrt{a+a\sin(e+fx)}\,(c}{a}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{a-ax}\,(a+ax)}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{d(a-a\sin(e+fx))}{ac+ad}}\right)\text{Subst}\left(\int \frac{(c+dx)^n}{(a+ax)\sqrt{\frac{ad}{ac+ad}-}}\right.}{f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{(A-B)F_1\left(1+n;\frac{1}{2},1;2+n;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)\cos(e+}{(c-d)f(1+n)(1-\sin(e+fx))\sqrt{a}}$$

Mathematica [A] time = 5.57, size = 244, normalized size = 1.11

$$\frac{\cos(e+fx)\sqrt{a(\sin(e+fx)+1)}\,(c+d\sin(e+fx))^n\left(\frac{4(A-B)\sqrt{\frac{\sin(e+fx)-1}{\sin(e+fx)+1}}\left(\frac{c-d}{d\sin(e+fx)+d}+1\right)^{-n}F_1\left(-n-\frac{1}{2};-\frac{1}{2},-n;\frac{1}{2}-n;\frac{2}{\sin(e+fx)+1},\right.}{2n+1}\right.}{4af(\sin(e+f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/Sqrt[a + a*Sin[e +
f*x]],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(c + d*Sin[e + f*x])^n*(-(((A + B)
*AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c +
 d)]*Sqrt[2 - 2*Sin[e + f*x]])/((c + d*Sin[e + f*x])/(c - d))^n) + (4*(A -
B)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d
+ d*Sin[e + f*x])]*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])])/((1 + 2*n)
*(1 + (c - d)/(d + d*Sin[e + f*x]))^n)))/(4*a*f*(-1 + Sin[e + f*x]))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{n}}{\sqrt{a\sin\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/sqrt(a*sin(f*x + e) +
a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{n}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/sqrt(a*sin(f*x + e) +
 a), x)
```

maple [F] time = 0.97, size = 0, normalized size = 0.00

$$\int\frac{\left(A+B\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}}{\sqrt{a+a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{n}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/sqrt(a*sin(f*x + e) +
 a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^(1/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^(1/2
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)^n}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**n/sqrt(a*(sin(e + f*x)
+ 1)), x)
```

3.335 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=269

$$\frac{d(A-B)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}\,(c+d\sin(e+fx))^{n+1}F_1\left(n+1;\frac{1}{2},2;n+2;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)}{f(n+1)(c-d)^2(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}-\frac{B\cos(e}{}$$

```
[Out] -B*AppellF1(1+n,1,1/2,2+n,(c+d*sin(f*x+e))/(c-d),(c+d*sin(f*x+e))/(c+d))*co
s(f*x+e)*(c+d*sin(f*x+e))^(1+n)*(d*(1-sin(f*x+e))/(c+d))^(1/2)/a/(c-d)/f/(1
+n)/(1-sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)+(A-B)*d*AppellF1(1+n,2,1/2,2+n,(c
+d*sin(f*x+e))/(c-d),(c+d*sin(f*x+e))/(c+d))*cos(f*x+e)*(c+d*sin(f*x+e))^(1
+n)*(d*(1-sin(f*x+e))/(c+d))^(1/2)/(c-d)^2/f/(1+n)/(a-a*sin(f*x+e))/(a+a*si
n(f*x+e))^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2987, 2788, 137, 136}

$$\frac{d(A-B)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}\,(c+d\sin(e+fx))^{n+1}F_1\left(n+1;\frac{1}{2},2;n+2;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)}{f(n+1)(c-d)^2(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}-\frac{B\cos(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^(3/2
),x]
```

```
[Out] -((B*AppellF1[1 + n, 1/2, 1, 2 + n, (c + d*Sin[e + f*x])/(c + d), (c + d*Si
n[e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Sin[e + f*x]))/(c + d)]*(c +
 d*Sin[e + f*x])^(1 + n))/(a*(c - d)*f*(1 + n)*(1 - Sin[e + f*x])*Sqrt[a +
a*Sin[e + f*x]])) + ((A - B)*d*AppellF1[1 + n, 1/2, 2, 2 + n, (c + d*Sin[e
+ f*x])/(c + d), (c + d*Sin[e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Si
n[e + f*x]))/(c + d)]*(c + d*Sin[e + f*x])^(1 + n))/((c - d)^2*f*(1 + n)*(a
 - a*Sin[e + f*x])*Sqrt[a + a*Sin[e + f*x]])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx = (A-B)\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx + \frac{B\int \frac{(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{a-ax}\,(a+ax)^2}\,dx, x, \sin(e+fx\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{d(a-a\sin(e+fx))}{ac+ad}}\right)\text{Subst}\left(\int \frac{(c+dx)^n}{(a+ax)^2\sqrt{\frac{ad}{ac+ad}}\right.}{f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{BF_1\left(1+n;\frac{1}{2},1;2+n;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)\cos(e+fx)\sqrt{}}{(c-d)f(1+n)(a-a\sin(e+fx))\sqrt{a+}}$$

Mathematica [B] time = 17.23, size = 603, normalized size = 2.24

$$\sec(e+fx)(c+d\sin(e+fx))^n\left(aA(\sin(e+fx)+1)\left(a\sqrt{2-2\sin(e+fx)}\,(\sin(e+fx)+1)\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(1,\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (Sec[e + f*x]*(c + d*Sin[e + f*x])^n*(a*B*(1 + Sin[e + f*x])*((a*AppellF1[1
, 1/2, -n, 2, (1 + Sin[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c + d)]*Sqrt[2
 - 2*Sin[e + f*x]]*(1 + Sin[e + f*x]))/((c + d*Sin[e + f*x])/(c - d))^n - (
4*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])]*(-2*a*(1 + 2*n)*AppellF1[1/2
 - n, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d + d*Sin[e + f*x]
)] + a*(-1 + 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]
), (-c + d)/(d + d*Sin[e + f*x])]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*(1 + (
c - d)/(d + d*Sin[e + f*x]))^n)) + a*A*(1 + Sin[e + f*x])*((a*AppellF1[1, 1
/2, -n, 2, (1 + Sin[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c + d)]*Sqrt[2 -
2*Sin[e + f*x]]*(1 + Sin[e + f*x]))/((c + d*Sin[e + f*x])/(c - d))^n - (4*S
qrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])]*(2*a*(1 + 2*n)*AppellF1[1/2 - n
, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d + d*Sin[e + f*x])] +
 a*(-1 + 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (
-c + d)/(d + d*Sin[e + f*x])]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*(1 + (c -
d)/(d + d*Sin[e + f*x]))^n))))/(8*a^3*f*Sqrt[a*(1 + Sin[e + f*x])])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{n}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c
)^n/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^
(3/2), x)
```

maple [F] time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}}{\left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^
(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^(3/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^(3/2
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)^{n}}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**n/(a*(sin(e + f*x) + 1)
)**(3/2), x)
```

3.336 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=351

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)\left(A(m+3)\left(c^2\left(m^2+3m+2\right)+2cdm(m+2)+d^2\left(m^2+m+1\right)\right)+B\left(c^2m\left(m^2+5m+6\right)+\right.\right.}{f(}$$

```
[Out] (d*(A*d*(3+m)+B*(d*m+2*c))-2*(2+m)*(A*c*d*(3+m)+B*(c*d*m+c^2+d^2)))*cos(f*x
+e)*(a+a*sin(f*x+e))^m/f/(1+m)/(2+m)/(3+m)-2^(1/2+m)*(A*(3+m)*(2*c*d*m*(2+m
)+d^2*(m^2+m+1)+c^2*(m^2+3*m+2))+B*(d^2*m*(m^2+3*m+5)+c^2*m*(m^2+5*m+6)+2*c
*d*(m^3+4*m^2+4*m+3)))*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(
f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f/(3+m)/(m^2+3*m+2)-d*(A
*d*(3+m)+B*(d*m+2*c))*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(2+m)/(3+m)-B*c
os(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2/f/(3+m)
```

Rubi [A] time = 0.99, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2983, 2968, 3023, 2751, 2652, 2651}

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)\left(A(m+3)\left(c^2\left(m^2+3m+2\right)+2cdm(m+2)+d^2\left(m^2+m+1\right)\right)+B\left(c^2m\left(m^2+5m+6\right)+\right.\right.}{f(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] ((d*(A*d*(3 + m) + B*(2*c + d*m)) - 2*(2 + m)*(A*c*d*(3 + m) + B*(c^2 + d^2
 + c*d*m)))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m)*(2 + m)*(3 + m)
) - (2^(1/2 + m)*(A*(3 + m)*(2*c*d*m*(2 + m) + d^2*(1 + m + m^2) + c^2*(2 +
 3*m + m^2)) + B*(d^2*m*(5 + 3*m + m^2) + c^2*m*(6 + 5*m + m^2) + 2*c*d*(3
+ 4*m + 4*m^2 + m^3)))*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1
 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(
f*(1 + m)*(2 + m)*(3 + m)) - (d*(A*d*(3 + m) + B*(2*c + d*m))*Cos[e + f*x]*
(a + a*Sin[e + f*x])^(1 + m))/(a*f*(2 + m)*(3 + m)) - (B*Cos[e + f*x]*(a +
a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2)/(f*(3 + m))
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin}{f(3+m)}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin}{f(3+m)}$$

$$= -\frac{d(Ad(3+m)+B(2c+dm))\cos(e+fx)(a}{af(2+m)(3+m)}$$

$$= \frac{\left(d(Ad(3+m)+B(2c+dm))-2(2+m)\left(A\right.\right.}{f}$$

$$= \frac{\left(d(Ad(3+m)+B(2c+dm))-2(2+m)\left(A\right.\right.}{f}$$

$$= \frac{\left(d(Ad(3+m)+B(2c+dm))-2(2+m)\left(A\right.\right.}{f}$$

Mathematica [A] time = 7.73, size = 300, normalized size = 0.85

$$-\frac{\csc^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^m(a(\sin(e+fx)+1))^m\left(-2(A+B)(c+d)^2\tan\left(\frac{1}{4}(2e+2fx-\pi)\right)\,{}_2F_1\left(\frac{1}{2},m+4;\frac{3}{2};-\mathrm{ta}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
2,x]
```

```
[Out] -(((Csc[(2*e + Pi + 2*f*x)/4]^2)^m*(a*(1 + Sin[e + f*x]))^m*(-2*(A + B)*(c
+ d)^2*Hypergeometric2F1[1/2, 4 + m, 3/2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Tan
[(2*e - Pi + 2*f*x)/4] - (2*(c + d)*(3*A*c + B*c - A*d - 3*B*d)*Hypergeomet
ric2F1[3/2, 4 + m, 5/2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Tan[(2*e - Pi + 2*f*x
)/4]^3)/3 - (2*(c - d)*(A*(3*c + d) - B*(c + 3*d))*Hypergeometric2F1[5/2, 4
 + m, 7/2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Tan[(2*e - Pi + 2*f*x)/4]^5)/5 - (
2*(A - B)*(c - d)^2*Hypergeometric2F1[7/2, 4 + m, 9/2, -Tan[(2*e - Pi + 2*f
*x)/4]^2]*Tan[(2*e - Pi + 2*f*x)/4]^7)/7))/f)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\left(Ac^2+2\,Bcd+Ad^2-\left(2\,Bcd+Ad^2\right)\cos\left(fx+e\right)^2-\left(Bd^2\cos\left(fx+e\right)^2-Bc^2-2\,Acd-Bd^2\right)\sin\left(fx\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] integral((A*c^2 + 2*B*c*d + A*d^2 - (2*B*c*d + A*d^2)*cos(f*x + e)^2 - (B*d
^2*cos(f*x + e)^2 - B*c^2 - 2*A*c*d - B*d^2)*sin(f*x + e))*(a*sin(f*x + e)
+ a)^m, x)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{2}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2*(a*sin(f*x + e) + a)^
m, x)
```

maple **[F]** time = 9.53, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{2}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2*(a*sin(f*x + e) + a)^
m, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{m}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^2,x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(A+B\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))*(c + d*sin(e + f*x)
)**2, x)
```

3.337 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=199

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)\left(A(m+2)(cm+c+dm)+B\left(cm(m+2)+d\left(m^2+m+1\right)\right)\right)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+}{f(m+1)(m+2)}$$

```
[Out] (B*d-(A*d+B*c)*(2+m))*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+m)/(2+m)-2^(1/2+m)
*(A*(2+m)*(c*m+d*m+c)+B*(c*m*(2+m)+d*(m^2+m+1)))*cos(f*x+e)*hypergeom([1/2,
 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^
m/f/(m^2+3*m+2)-B*d*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(2+m)
```

Rubi [A] time = 0.36, antiderivative size = 198, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2968, 3023, 2751, 2652, 2651}

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)\left(A(m+2)(cm+c+dm)+Bcm(m+2)+Bd\left(m^2+m+1\right)\right)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+f}{f(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] ((B*d - (B*c + A*d)*(2 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m
)*(2 + m)) - (2^(1/2 + m)*(B*c*m*(2 + m) + A*(2 + m)*(c + c*m + d*m) + B*d*
(1 + m + m^2))*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e
 + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m
)*(2 + m)) - (B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(2 + m))
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int (a+a\sin(e+fx))^m\left(Ac+(Bc+Ad)\sin(e\right.$$

$$= -\frac{Bd\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(2+m)} + \frac{\int(}{}$$

$$= \frac{(Bd-(Bc+Ad)(2+m))\cos(e+fx)(a+a\,\mathrm{si}}{f(1+m)(2+m)}$$

$$= \frac{(Bd-(Bc+Ad)(2+m))\cos(e+fx)(a+a\,\mathrm{si}}{f(1+m)(2+m)}$$

$$= \frac{(Bd-(Bc+Ad)(2+m))\cos(e+fx)(a+a\,\mathrm{si}}{f(1+m)(2+m)}$$

Mathematica [A] time = 3.59, size = 212, normalized size = 1.07

$$-\frac{\csc^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^m (a(\sin(e+fx)+1))^m \left(-2(A+B)(c+d)\tan\left(\frac{1}{4}(2e+2fx-\pi)\right) {}_2F_1\left(\frac{1}{2},m+3;\frac{3}{2};-\tan\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),
x]
```

```
[Out] -(((Csc[(2*e + Pi + 2*f*x)/4]^2)^m*(a*(1 + Sin[e + f*x]))^m*(-2*(A + B)*(c
+ d)*Hypergeometric2F1[1/2, 3 + m, 3/2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Tan[(
2*e - Pi + 2*f*x)/4] - (4*(A*c - B*d)*Hypergeometric2F1[3/2, 3 + m, 5/2, -T
an[(2*e - Pi + 2*f*x)/4]^2]*Tan[(2*e - Pi + 2*f*x)/4]^3)/3 - (2*(A - B)*(c
- d)*Hypergeometric2F1[5/2, 3 + m, 7/2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Tan[(
2*e - Pi + 2*f*x)/4]^5)/5))/f)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(Bd\cos\left(fx+e\right)^2 - Ac - Bd - (Bc+Ad)\sin\left(fx+e\right)\right)\left(a\sin\left(fx+e\right)+a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral(-(B*d*cos(f*x + e)^2 - A*c - B*d - (B*c + A*d)*sin(f*x + e))*(a*si
n(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)\left(a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m,
 x)
```

maple [F] time = 7.61, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^m \left(A+B\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m,
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(c+d\,\sin\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x)),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(A+B\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))*(c + d*sin(e + f*x)
), x)
```

3.338 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=117

$$-\frac{2^{m+\frac{1}{2}}(Am+A+Bm)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(m+1)}$$

```
[Out] -B*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+m)-2^(1/2+m)*(A*m+B*m+A)*cos(f*x+e)*h
ypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+
a*sin(f*x+e))^m/f/(1+m)
```

Rubi [A] time = 0.08, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2751, 2652, 2651}

$$-\frac{2^{m+\frac{1}{2}}(Am+A+Bm)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m))) - (2^(1/2 + m)*(A +
A*m + B*m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f
*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m))
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)}+\frac{(A+Am+Bm)\int(a+a}{1+m}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)}+\frac{\left((A+Am+Bm)(1+\sin\right.}{}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)}-\frac{2^{\frac{1}{2}+m}(A+Am+Bm)\cos}{}$$

Mathematica [C] time = 1.82, size = 275, normalized size = 2.35

$$-\frac{\sin^{-2m}\left(\frac{1}{4}(2e+2fx+\pi)\right)(a(\sin(e+fx)+1))^m\left(\frac{2\sqrt{2}A\sin\left(\frac{1}{4}(2e+2fx-\pi)\right)\cos^{2m+1}\left(\frac{1}{4}(2e+2fx-\pi)\right){}_2F_1\left(\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};\sin^2\left(\frac{1}{4}(2e+\right.\right.}{(2m+1)\sqrt{1-\sin(e+fx)}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -(((a*(1 + Sin[e + f*x]))^m*(((-1)^(1/4)*2^(-1 - 2*m)*B*(-(((-1)^(3/4)*(I +
 E^(I*(e + f*x))))/E^((I/2)*(e + f*x))))^(1 + 2*m)*(E^((2*I)*(e + f*x))*(-1
 + m)*Hypergeometric2F1[1, m, -m, (-I)/E^(I*(e + f*x))] - (1 + m)*Hypergeom
etric2F1[1, 2 + m, 2 - m, (-I)/E^(I*(e + f*x))]))/(E^(((3*I)/2)*(e + f*x))*
(-1 + m^2)) + (2*Sqrt[2]*A*Cos[(2*e - Pi + 2*f*x)/4]^(1 + 2*m)*Hypergeometr
ic2F1[1/2, 1/2 + m, 3/2 + m, Sin[(2*e + Pi + 2*f*x)/4]^2]*Sin[(2*e - Pi + 2
*f*x)/4])/((1 + 2*m)*Sqrt[1 - Sin[e + f*x]])))/(f*Sin[(2*e + Pi + 2*f*x)/4]
^(2*m)))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fricas")
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="giac")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x)), x)
```

3.339 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=191

$$-\frac{\sqrt{2}\,(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},1;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{df(2m+1)(c-d)\sqrt{1-\sin(e+fx)}}-\frac{B2^{m+\frac{1}{2}}}{}$$

```
[Out] -2^(1/2+m)*B*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1
+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/d/f-(-A*d+B*c)*AppellF1(1/2+m,1,1/
2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x
+e))^m*2^(1/2)/(c-d)/d/f/(1+2*m)/(1-sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2986, 2652, 2651, 2788, 137, 136}

$$-\frac{\sqrt{2}\,(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},1;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{df(2m+1)(c-d)\sqrt{1-\sin(e+fx)}}-\frac{B2^{m+\frac{1}{2}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```

```
[Out] -((Sqrt[2]*(B*c - A*d)*AppellF1[1/2 + m, 1/2, 1, 3/2 + m, (1 + Sin[e + f*x]
)/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m
)/((c - d)*d*f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]])) - (2^(1/2 + m)*B*Cos[e +
f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e
+ f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(d*f)
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
```

```
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 2986

```
Int[(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B
/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin
[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && N
eQ[m + 1/2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \frac{B\int(a+a\sin(e+fx))^m\,dx}{d} - \frac{(Bc-Ad)\int\frac{(a+a\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx}{d}$$

$$= -\frac{\left(a^2(Bc-Ad)\cos(e+fx)\right)\text{Subst}\left(\int\frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}\,(c+dx)}\,dx, x, \sin(e+f\right.}{df\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+m}B\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\mathrm{si}}{df}$$

$$= -\frac{\sqrt{2}\,(Bc-Ad)F_1\left(\frac{1}{2}+m;\frac{1}{2},1;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\mathrm{si}}{c}\right.}{(c-d)df(1+2m)\sqrt{1-\sin(e}}$$

Mathematica [B] time = 7.08, size = 473, normalized size = 2.48

$$\frac{(a(\sin(e+fx)+1))^m\left(\frac{6(c+d)(Bc-Ad)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sec^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}}{d(c+d\sin(e+fx))\left(\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\left(4dF_1\left(\frac{3}{2};\frac{1}{2}-m,2;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+2fx+\pi)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}{c+d}\right)-(2m-1)(c+d)F_1\left(\frac{3}{2};\right.\right.\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^m*((Sqrt[2]*B*Cos[e + f*x]*Hypergeometric2F1[1/2, 1
/2 + m, 3/2 + m, (Cos[e + f*x]^2*Csc[(2*e - Pi + 2*f*x)/4]^2)/4])/((d + 2*d
*m)*Sqrt[1 - Sin[e + f*x]]) + (6*(c + d)*(B*c - A*d)*AppellF1[1/2, 1/2 - m,
 1, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c
+ d)]*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Sec
[(2*e - Pi + 2*f*x)/4]^2*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(d*(c + d
*Sin[e + f*x])*(3*(c + d)*AppellF1[1/2, 1/2 - m, 1, 3/2, Cos[(2*e + Pi + 2*
f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (4*d*AppellF1[3/2,
1/2 - m, 2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4
]^2)/(c + d)] - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, 1, 5/2, Cos[(2*e
+ Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Sin[(2*e -
Pi + 2*f*x)/4]^2))))/f
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{d\sin\left(fx+e\right)+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c),
 x)
```

maple [F] time = 4.05, size = 0, normalized size = 0.00

$$\int\frac{\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)}{c+d\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{c+d\,\sin\left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x)),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.340 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=293

$$\frac{\sqrt{2}\cos(e+fx)\left(Ad(c(1-m)-dm)-B\left(c^2(-m)-cdm+d^2\right)\right)(a\sin(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},1;m+\frac{3}{2};\frac{1}{2}(\sin\right.}{df(2m+1)(c-d)^2(c+d)\sqrt{1-\sin(e+fx)}}$$

```
[Out] 2^(1/2+m)*(-A*d+B*c)*m*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(
f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/d/(c^2-d^2)/f-(-A*d+B*c)
*cos(f*x+e)*(a+a*sin(f*x+e))^m/(c^2-d^2)/f/(c+d*sin(f*x+e))+(A*d*(c*(1-m)-d
*m)-B*(-c^2*m-c*d*m+d^2))*AppellF1(1/2+m,1,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d
),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)/(c-d)^2/d/(c+d)
/f/(1+2*m)/(1-sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2984, 2986, 2652, 2651, 2788, 137, 136}

$$\frac{\sqrt{2}\cos(e+fx)\left(Ad(c(1-m)-dm)-B\left(c^2(-m)-cdm+d^2\right)\right)(a\sin(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},1;m+\frac{3}{2};\frac{1}{2}(\sin\right.}{df(2m+1)(c-d)^2(c+d)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (Sqrt[2]*(A*d*(c*(1 - m) - d*m) - B*(d^2 - c^2*m - c*d*m))*AppellF1[1/2 + m
, 1/2, 1, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]
*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/((c - d)^2*d*(c + d)*f*(1 + 2*m)*Sqrt
[1 - Sin[e + f*x]]) + (2^(1/2 + m)*(B*c - A*d)*m*Cos[e + f*x]*Hypergeometri
c2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)
*(a + a*Sin[e + f*x])^m)/(d*(c^2 - d^2)*f) - ((B*c - A*d)*Cos[e + f*x]*(a +
 a*Sin[e + f*x])^m)/((c^2 - d^2)*f*(c + d*Sin[e + f*x]))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2986

```
Int[(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
```

```
(f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B
/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin
[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && N
eQ[m + 1/2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = -\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{\left(c^2-d^2\right)f(c+d\sin(e+fx))} - \frac{\int \frac{(a+a\sin(e+fx))^m}{}}{}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{\left(c^2-d^2\right)f(c+d\sin(e+fx))} - \frac{((Bc-Ad)m)\int}{d}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{\left(a^2\left(Ad(c(1-m\right.\right.}{}$$

$$= \frac{2^{\frac{1}{2}+m}(Bc-Ad)m\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+f\right.}{d\left(c^2-d^2\right)f}$$

$$= \frac{\sqrt{2}\left(Ad(c(1-m)-dm)-B\left(d^2-c^2m-cdm\right)\right)F_1\left(\frac{1}{2}+m;\frac{1}{2},1\right.}{(c-d)^2d(c+d)}$$

Mathematica [B] time = 2.50, size = 651, normalized size = 2.22

$$\frac{6(c+d)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e+fx)+1))^m\left(\frac{}{\cos}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^2,x]
```

```
[Out] (6*(c + d)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/
4]*(a*(1 + Sin[e + f*x]))^m*(((-(B*c) + A*d)*AppellF1[1/2, 1/2 - m, 2, 3/2,
```

```
 Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])/(
-3*(c + d)*AppellF1[1/2, 1/2 - m, 2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d
*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-8*d*AppellF1[3/2, 1/2 - m, 3, 5/
2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]
+ (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, 2, 5/2, Cos[(2*e + Pi + 2*f*x)/
4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)/4]
^2) + (B*AppellF1[1/2, 1/2 - m, 1, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*S
in[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(c + d*Sin[e + f*x]))/(-3*(c + d)*Appe
llF1[1/2, 1/2 - m, 1, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi
+ 2*f*x)/4]^2)/(c + d)] + (-4*d*AppellF1[3/2, 1/2 - m, 2, 5/2, Cos[(2*e + P
i + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (c + d)*(-1 +
 2*m)*AppellF1[3/2, 3/2 - m, 1, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[
(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)/4]^2))*(Sin[(2*e
+ Pi + 2*f*x)/4]^2)^(1/2 - m))/(d*f*(c + d*Sin[e + f*x])^2)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{d^{2}\cos\left(fx+e\right)^{2}-2\,cd\sin\left(fx+e\right)-c^{2}-d^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d^2*cos(f*x + e)^2 -
 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(d\sin\left(fx+e\right)+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^
2, x)
```

maple [F] time = 14.32, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)}{\left(c+d\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(d\sin\left(fx+e\right)+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^
2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^2,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^2, x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.341 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=467

$$\frac{\cos(e+fx)\left(B\left(c^3(1-m)m+2c^2d(1-m)m-cd^2\left(m^2-3m+3\right)+2d^3m\right)-Ad\left(-\left(c^2\left(m^2-3m+2\right)\right)+2cd(2-\right.\right.}{\sqrt{2}\,df(2m+1)(c-d)^3(c+d}$$

```
[Out] -2^(-1/2+m)*m*(A*d*(c*(3-m)-d*m)-B*(2*d^2+c^2*(1-m)-c*d*m))*cos(f*x+e)*hype
rgeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*s
in(f*x+e))^m/d/(c^2-d^2)^2/f-1/2*(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))^m/(
c^2-d^2)/f/(c+d*sin(f*x+e))^2+1/2*(A*d*(c*(3-m)-d*m)-B*(2*d^2+c^2*(1-m)-c*d
*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m/(c^2-d^2)^2/f/(c+d*sin(f*x+e))+1/2*(B*(2
*d^3*m+c^3*(1-m)*m+2*c^2*d*(1-m)*m-c*d^2*(m^2-3*m+3))-A*d*(2*c*d*(2-m)*m-c^
2*(m^2-3*m+2)-d^2*(m^2-m+1)))*AppellF1(1/2+m,1,1/2,3/2+m,-d*(1+sin(f*x+e))/
(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m/(c-d)^3/d/(c+d)^2/f
/(1+2*m)*2^(1/2)/(1-sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.35, antiderivative size = 467, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2984, 2986, 2652, 2651, 2788, 137, 136}

$$\frac{\cos(e+fx)\left(B\left(2c^2d(1-m)m+c^3(1-m)m-cd^2\left(m^2-3m+3\right)+2d^3m\right)-Ad\left(c^2\left(-\left(m^2-3m+2\right)\right)+2cd(2-\right.\right.}{\sqrt{2}\,df(2m+1)(c-d)^3(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((B*(2*d^3*m + c^3*(1 - m)*m + 2*c^2*d*(1 - m)*m - c*d^2*(3 - 3*m + m^2)) -
 A*d*(2*c*d*(2 - m)*m - c^2*(2 - 3*m + m^2) - d^2*(1 - m + m^2)))*AppellF1[
1/2 + m, 1/2, 1, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c
 - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(Sqrt[2]*(c - d)^3*d*(c + d)^2
*f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]) - (2^(-1/2 + m)*m*(A*d*(c*(3 - m) - d*
m) - B*(2*d^2 + c^2*(1 - m) - c*d*m))*Cos[e + f*x]*Hypergeometric2F1[1/2, 1
/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin
[e + f*x])^m)/(d*(c^2 - d^2)^2*f) - ((B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e
+ f*x])^m)/(2*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^2) + ((A*d*(c*(3 - m) - d*
m) - B*(2*d^2 + c^2*(1 - m) - c*d*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/
(2*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x]))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
```

```
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2986

```
Int[(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B
/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin
[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && N
eQ[m + 1/2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = -\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} - \frac{\int \frac{(a+a\sin(e+fx))^m(-}{}}{}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{\left(Ad(c(3-m)-d\right.}{}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{\left(Ad(c(3-m)-d\right.}{}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{\left(Ad(c(3-m)-d\right.}{}$$

$$= -\frac{2^{-\frac{1}{2}+m}m\left(Ad(c(3-m)-dm)-B\left(2d^2+c^2(1-m)-cdm\right)\right)\cos}{}$$

$$= \frac{\left(B\left(2d^3m+c^3(1-m)m+2c^2d(1-m)m-cd^2\left(3-3m+m^2\right)\right)-\right.}{}$$

Mathematica [A] time = 2.53, size = 651, normalized size = 1.39

$$6(c+d)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e+fx)+1))^m\left(-\frac{}{\cos}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^3,x]
```

```
[Out] (6*(c + d)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/
4]*(a*(1 + Sin[e + f*x]))^m*(((-(B*c) + A*d)*AppellF1[1/2, 1/2 - m, 3, 3/2,
 Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])/(
-3*(c + d)*AppellF1[1/2, 1/2 - m, 3, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d
*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-12*d*AppellF1[3/2, 1/2 - m, 4, 5
/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]
 + (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, 3, 5/2, Cos[(2*e + Pi + 2*f*x)
/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)/4
]^2) + (B*AppellF1[1/2, 1/2 - m, 2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*
Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(c + d*Sin[e + f*x]))/(-3*(c + d)*App
ellF1[1/2, 1/2 - m, 2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi
 + 2*f*x)/4]^2)/(c + d)] + (-8*d*AppellF1[3/2, 1/2 - m, 3, 5/2, Cos[(2*e +
Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (c + d)*(-1
+ 2*m)*AppellF1[3/2, 3/2 - m, 2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin
[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)/4]^2))*(Sin[(2*e
 + Pi + 2*f*x)/4]^2)^(1/2 - m))/(d*f*(c + d*Sin[e + f*x])^3)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^m}{3\,cd^2\cos\left(fx+e\right)^2-c^3-3\,cd^2+\left(d^3\cos\left(fx+e\right)^2-3\,c^2d-d^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(3*c*d^2*cos(f*x + e)
^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(d\sin\left(fx+e\right)+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^
3, x)
```

maple [F] time = 12.57, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)}{\left(c+d\sin\left(fx+e\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(d\sin\left(fx+e\right)+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^
3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^3,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^3, x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.342 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=284

$$\frac{\sqrt{2}\,(A-B)(c-d)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}\,F_1\left(m+\frac{1}{2};\frac{1}{2},-\frac{3}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),\right.}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

```
[Out] (A-B)*(c-d)*AppellF1(1/2+m,-3/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*s
in(f*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*(c+d*sin(f*x+e))^(1/2)/f/(
1+2*m)/(1-sin(f*x+e))^(1/2)/((c+d*sin(f*x+e))/(c-d))^(1/2)+B*(c-d)*AppellF1
(3/2+m,-3/2,1/2,5/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e
)*(a+a*sin(f*x+e))^(1+m)*2^(1/2)*(c+d*sin(f*x+e))^(1/2)/a/f/(3+2*m)/(1-sin(
f*x+e))^(1/2)/((c+d*sin(f*x+e))/(c-d))^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {2987, 2788, 140, 139, 138}

$$\frac{\sqrt{2}\,(A-B)(c-d)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}\,F_1\left(m+\frac{1}{2};\frac{1}{2},-\frac{3}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),\right.}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (Sqrt[2]*(A - B)*(c - d)*AppellF1[1/2 + m, 1/2, -3/2, 3/2 + m, (1 + Sin[e +
 f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*
x])^m*Sqrt[c + d*Sin[e + f*x]])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c
 + d*Sin[e + f*x])/(c - d)]) + (Sqrt[2]*B*(c - d)*AppellF1[3/2 + m, 1/2, -3
/2, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e
 + f*x]*(a + a*Sin[e + f*x])^(1 + m)*Sqrt[c + d*Sin[e + f*x]])/(a*f*(3 + 2*
m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c + d*Sin[e + f*x])/(c - d)])
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
```

```
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int (a + a\sin(e+fx))^m (A + B\sin(e+fx))(c + d\sin(e+fx))^{3/2}\,dx = (A-B)\int (a + a\sin(e+fx))^m (c + d\sin(e+$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\right)\operatorname{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-}}\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\,\mathrm{s}}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\operatorname{Subs}}{\sqrt{2}\,f(a-a\sin(e+fx))}$$

$$= \frac{\left(a(A-B)(ac-ad)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right.}{\sqrt{2}\,f(a-a\sin}$$

$$= \frac{\sqrt{2}\,(A-B)(c-d)F_1\left(\frac{1}{2}+m;\frac{1}{2},-\frac{3}{2};\frac{3}{2}+m;\frac{1}{2}\right.}{f}$$

Mathematica [B] time = 8.14, size = 3281, normalized size = 11.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
(3/2),x]
```

```
[Out] -((((-2*B*c*AppellF1[3/2, (1 - 2*m)/2, -1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^
2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Cos[(-e + Pi/2 - f*x)/2]^(-1 +
 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^2)^((1 - 2*m)/2)*Sin[(-e + Pi/2 - f*x)/2]^3
*(1 - Sin[(-e + Pi/2 - f*x)/2]^2)^((1 - 2*m)/2 + (-1 + 2*m)/2)*Sqrt[c + d -
 2*d*Sin[(-e + Pi/2 - f*x)/2]^2])/(3*Sqrt[(c + d - 2*d*Sin[(-e + Pi/2 - f*x
)/2]^2)/(c + d)]) - (2*A*d*AppellF1[3/2, (1 - 2*m)/2, -1/2, 5/2, Sin[(-e +
Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Cos[(-e + Pi/2
- f*x)/2]^(-1 + 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^2)^((1 - 2*m)/2)*Sin[(-e + P
i/2 - f*x)/2]^3*(1 - Sin[(-e + Pi/2 - f*x)/2]^2)^((1 - 2*m)/2 + (-1 + 2*m)/
2)*Sqrt[c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2])/(3*Sqrt[(c + d - 2*d*Sin[(
-e + Pi/2 - f*x)/2]^2)/(c + d)]) + (B*d*AppellF1[5/2, (1 - 2*m)/2, -1/2, 7/
2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Co
s[(-e + Pi/2 - f*x)/2]^(-1 + 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^2)^((1 - 2*m)/2
```

)*Sin[(-e + Pi/2 - f*x)/2]^5*(1 - Sin[(-e + Pi/2 - f*x)/2]^2)^((1 - 2*m)/2 + (-1 + 2*m)/2)*Sqrt[c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2])/(5*Sqrt[(c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]) - (2*B*d*AppellF1[3/2, (-1 - 2*m)/2, -1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Cos[(-e + Pi/2 - f*x)/2]^(1 + 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^2)^((-1 - 2*m)/2)*Sin[(-e + Pi/2 - f*x)/2]^3*(1 - Sin[(-e + Pi/2 - f*x)/2]^2)^((-1 - 2*m)/2 + (1 + 2*m)/2)*Sqrt[c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2])/Sqrt[(c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)] - (3*B*d*(c + d)*AppellF1[1/2, -3/2 - m, -1/2, 3/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Cos[(-e + Pi/2 - f*x)/2]^(3 + 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^2)^(1/2 + (-4 - 2*m)/2)*Sin[(-e + Pi/2 - f*x)/2]*(1 - Sin[(-e + Pi/2 - f*x)/2]^2)^(3/2 + m)*Sqrt[c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2])/(-3*(c + d)*AppellF1[1/2, -3/2 - m, -1/2, 3/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)] + (2*d*AppellF1[3/2, -3/2 - m, 1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)] + (c + d)*(3 + 2*m)*AppellF1[3/2, -1/2 - m, -1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)])*Sin[(-e + Pi/2 - f*x)/2]^2) - (6*B*c*(c + d)*AppellF1[1/2, -1/2 - m, -1/2, 3/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Cos[(-e + Pi/2 - f*x)/2]^(1 + 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^2)^(1/2 + (-2 - 2*m)/2)*Sin[(-e + Pi/2 - f*x)/2]*(1 - Sin[(-e + Pi/2 - f*x)/2]^2)^(1/2 + m)*Sqrt[c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2])/(-3*(c + d)*AppellF1[1/2, -1/2 - m, -1/2, 3/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)] + (2*d*AppellF1[3/2, -1/2 - m, 1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)] + (c + d)*(1 + 2*m)*AppellF1[3/2, 1/2 - m, -1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)])*Sin[(-e + Pi/2 - f*x)/2]^2) - (6*A*d*(c + d)*AppellF1[1/2, -1/2 - m, -1/2, 3/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Cos[(-e + Pi/2 - f*x)/2]^(1 + 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^2)^(1/2 + (-2 - 2*m)/2)*Sin[(-e + Pi/2 - f*x)/2]*(1 - Sin[(-e + Pi/2 - f*x)/2]^2)^(1/2 + m)*Sqrt[c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2])/(-3*(c + d)*AppellF1[1/2, -1/2 - m, -1/2, 3/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)] + (2*d*AppellF1[3/2, -1/2 - m, 1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)] + (c + d)*(1 + 2*m)*AppellF1[3/2, 1/2 - m, -1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)])*Sin[(-e + Pi/2 - f*x)/2]^2) + (6*A*c*(c + d)*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Cos[(-e + Pi/2 - f*x)/2]^(-1 + 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^2)^(1/2 - m)*Sin[(-e + Pi/2 - f*x)/2]*(1 - Sin[(-e + Pi/2 - f*x)/2]^2)^(-1/2 + m)*Sqrt[c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2])/(3*(c + d)*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)] - (2*d*AppellF1[3/2, 1/2 - m, 1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)] + (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, -1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)])*Sin[(-e + Pi/2 - f*x)/2

```
]^2) + (3*B*d*(c + d)*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Sin[(-e + Pi/2 - f*
x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Cos[(-e + Pi/2 - f*x)/2]
^(-1 + 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^2)^(1/2 - m)*Sin[(-e + Pi/2 - f*x)/2]
*(1 - Sin[(-e + Pi/2 - f*x)/2]^2)^(-1/2 + m)*Sqrt[c + d - 2*d*Sin[(-e + Pi/
2 - f*x)/2]^2])/(3*(c + d)*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Sin[(-e + Pi/2
 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)] - (2*d*AppellF1[3/2
, 1/2 - m, 1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)
/2]^2)/(c + d)] + (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, -1/2, 5/2, Sin[
(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)])*Sin[(-e
+ Pi/2 - f*x)/2]^2))*(a + a*Sin[e + f*x])^m)/(f*Cos[(-e + Pi/2 - f*x)/2]^(2
*m)))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(Bd\cos\left(fx+e\right)^2 - Ac - Bd - (Bc+Ad)\sin\left(fx+e\right)\right)\sqrt{d\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*d*cos(f*x + e)^2 - A*c - B*d - (B*c + A*d)*sin(f*x + e))*sqrt(
d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e) +
 a)^m, x)
```

maple [F] time = 1.20, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^m \left(A+B\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e) +
 a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(3/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(3/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.343 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}$

Optimal. Leaf size=274

$$\frac{\sqrt{2}\,(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}\,F_1\left(m+\frac{1}{2};\frac{1}{2},-\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(}{}\right.}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

```
[Out] (A-B)*AppellF1(1/2+m,-1/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x
+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*(c+d*sin(f*x+e))^(1/2)/f/(1+2*m)
/(1-sin(f*x+e))^(1/2)/((c+d*sin(f*x+e))/(c-d))^(1/2)+B*AppellF1(3/2+m,-1/2,
1/2,5/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f
*x+e))^(1+m)*2^(1/2)*(c+d*sin(f*x+e))^(1/2)/a/f/(3+2*m)/(1-sin(f*x+e))^(1/2
)/((c+d*sin(f*x+e))/(c-d))^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {2987, 2788, 140, 139, 138}

$$\frac{\sqrt{2}\,(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}\,F_1\left(m+\frac{1}{2};\frac{1}{2},-\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(}{}\right.}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[2]*(A - B)*AppellF1[1/2 + m, 1/2, -1/2, 3/2 + m, (1 + Sin[e + f*x])/2
, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sq
rt[c + d*Sin[e + f*x]])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c + d*Sin
[e + f*x])/(c - d)]) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, -1/2, 5/2 + m, (1
+ Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*S
in[e + f*x])^(1 + m)*Sqrt[c + d*Sin[e + f*x]])/(a*f*(3 + 2*m)*Sqrt[1 - Sin[
e + f*x]]*Sqrt[(c + d*Sin[e + f*x])/(c - d)])
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
```

```
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m (A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx = (A-B)\int (a+a\sin(e+fx))^m\sqrt{c+d\sin(e+$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}\sqrt{}}{\sqrt{a-ax}}\right.}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\right.}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\sqrt{c+d\sin}\right.}{\sqrt{2}f(a-a\sin(e+f}$$

$$= \frac{\sqrt{2}(A-B)F_1\left(\frac{1}{2}+m;\frac{1}{2},-\frac{1}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin\right.}{f(1+}$$

Mathematica [B] time = 11.89, size = 672, normalized size = 2.45

$$\frac{6(c+d)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e+fx)+1))^m\sqrt{c+}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*
x]],x]
```

```
[Out] (6*(c + d)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/
4]*(a*(1 + Sin[e + f*x]))^m*Sqrt[c + d*Sin[e + f*x]]*(((B*c - A*d)*AppellF1
[1/2, 1/2 - m, -1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi +
 2*f*x)/4]^2)/(c + d)])/(3*(c + d)*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Cos[(2
*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-2*d*A
ppellF1[3/2, 1/2 - m, 1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e
- Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, -1
/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c
```

```
+ d)])*Cos[(2*e + Pi + 2*f*x)/4]^2) - (B*AppellF1[1/2, 1/2 - m, -3/2, 3/2,
Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(c
+ d*Sin[e + f*x]))/(3*(c + d)*AppellF1[1/2, 1/2 - m, -3/2, 3/2, Cos[(2*e +
Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-6*d*Appell
F1[3/2, 1/2 - m, -1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi
 + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, -3/2,
5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)
])*Cos[(2*e + Pi + 2*f*x)/4]^2))*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(
d*f)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\sqrt{d\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)
^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\sqrt{d\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a
)^m, x)
```

maple [F] time = 1.13, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\sqrt{c+d\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\sqrt{d\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a
)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\sqrt{c+d\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(1/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(1/2),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(A+B\sin\left(e+fx\right)\right)\sqrt{c+d\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))*sqrt(c + d*sin(e +
f*x)), x)
```

3.344 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=274

$$\frac{\sqrt{2}\,(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\,F_1\left(m+\frac{1}{2};\frac{1}{2},\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+}{c-d}\right.}{f(2m+1)\sqrt{1-\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}$$

```
[Out] (A-B)*AppellF1(1/2+m,1/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+
e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*((c+d*sin(f*x+e))/(c-d))^(1/2)/f/
(1+2*m)/(1-sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)+B*AppellF1(3/2+m,1/2,1/
2,5/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x
+e))^(1+m)*2^(1/2)*((c+d*sin(f*x+e))/(c-d))^(1/2)/a/f/(3+2*m)/(1-sin(f*x+e)
)^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {2987, 2788, 140, 139, 138}

$$\frac{\sqrt{2}\,(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\,F_1\left(m+\frac{1}{2};\frac{1}{2},\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+}{c-d}\right.}{f(2m+1)\sqrt{1-\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c + d*Sin[e + f*x]],
x]
```

```
[Out] (Sqrt[2]*(A - B)*AppellF1[1/2 + m, 1/2, 1/2, 3/2 + m, (1 + Sin[e + f*x])/2,
 -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqr
t[(c + d*Sin[e + f*x])/(c - d)])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[c
 + d*Sin[e + f*x]]) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, 1/2, 5/2 + m, (1 +
Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin
[e + f*x])^(1 + m)*Sqrt[(c + d*Sin[e + f*x])/(c - d)])/(a*f*(3 + 2*m)*Sqrt[
1 - Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
```

```
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx = (A-B)\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx + \frac{B\int \frac{(a+a\sin(e+fx))^{1+m}}{\sqrt{c+d\sin(e+fx)}}\,dx}{a}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}\,\sqrt{c+dx}}\,dx, x, \sin(e+fx\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\,\sqrt{c+dx}}\,dx\right.}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\,\sqrt{\frac{a(c+d\sin(e+fx))}{ac-ad}}\right)\text{Subst}\left(\right.}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\sqrt{2}\,(A-B)F_1\left(\frac{1}{2}+m;\frac{1}{2},\frac{1}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(}{c-}\right.}{f(1+2m)\sqrt{1-\sin(e+fx)}}$$

Mathematica [B] time = 5.71, size = 672, normalized size = 2.45

$$\frac{6(c+d)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e+fx)+1))^m\left(\frac{}{\cos}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c + d*Sin[e +
f*x]],x]
```

```
[Out] (6*(c + d)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/
4]*(a*(1 + Sin[e + f*x]))^m*(((B*c - A*d)*AppellF1[1/2, 1/2 - m, 1/2, 3/2,
Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])/(3
*(c + d)*AppellF1[1/2, 1/2 - m, 1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d
*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (2*d*AppellF1[3/2, 1/2 - m, 3/2, 5
/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]
 - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, 1/2, 5/2, Cos[(2*e + Pi + 2*f*
x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)
```

```
/4]^2) - (B*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2,
(2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(c + d*Sin[e + f*x]))/(3*(c + d)
*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2
*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-2*d*AppellF1[3/2, 1/2 - m, 1/2, 5/2, Co
s[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c
+ d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, -1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]
^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)/4]^2
))*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(d*f*Sqrt[c + d*Sin[e + f*x]])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{d\sin\left(fx+e\right)+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) +
c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) +
 c), x)
```

maple [F] time = 0.89, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)}{\sqrt{c+d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) +
 c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{m}}{\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(1/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(1/2
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(A+B\sin\left(e+fx\right)\right)}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(c + d*sin(e +
f*x)), x)
```

3.345 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=288

$$\frac{\sqrt{2}\,(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\,F_1\left(m+\frac{1}{2};\frac{1}{2},\frac{3}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)\sqrt{1-\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}$$

```
[Out] (A-B)*AppellF1(1/2+m,3/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+
e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*((c+d*sin(f*x+e))/(c-d))^(1/2)/(c
-d)/f/(1+2*m)/(1-sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)+B*AppellF1(3/2+m,
3/2,1/2,5/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*s
in(f*x+e))^(1+m)*2^(1/2)*((c+d*sin(f*x+e))/(c-d))^(1/2)/a/(c-d)/f/(3+2*m)/(
1-sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {2987, 2788, 140, 139, 138}

$$\frac{\sqrt{2}\,(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\,F_1\left(m+\frac{1}{2};\frac{1}{2},\frac{3}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)\sqrt{1-\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^(3/2
),x]
```

```
[Out] (Sqrt[2]*(A - B)*AppellF1[1/2 + m, 1/2, 3/2, 3/2 + m, (1 + Sin[e + f*x])/2,
 -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqr
t[(c + d*Sin[e + f*x])/(c - d)])/((c - d)*f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, 3/2, 5/2 +
m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a
 + a*Sin[e + f*x])^(1 + m)*Sqrt[(c + d*Sin[e + f*x])/(c - d)])/(a*(c - d)*f
*(3 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
```

```
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx = (A-B)\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx + \frac{B\int \frac{(a+a\sin(e+fx))^{1+m}}{(c+d\sin(e+fx))^{3/2}}\,dx}{a}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}\,(c+dx)^{3/2}}\,dx, x, \sin(e+fx\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\,(c+dx)^{3/2}}\,dx\right.}{\sqrt{2}\,f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^3(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\,\sqrt{\frac{a(c+d\sin(e+fx))}{ac-ad}}\right)\text{Subst}\left(\int\right.}{\sqrt{2}\,(ac-ad)f(a-a\sin(e+fx))\sqrt{a+a\sin(e+}}$$

$$= \frac{\sqrt{2}\,(A-B)F_1\left(\frac{1}{2}+m;\frac{1}{2},\frac{3}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+}{c-d}\right.}{(c-d)f(1+2m)\sqrt{1-\sin(e+fx}}$$

Mathematica [B] time = 6.22, size = 672, normalized size = 2.33

$$\frac{6(c+d)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e+fx)+1))^m\left(\frac{}{\cos^2}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (6*(c + d)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/
4]*(a*(1 + Sin[e + f*x]))^m*(((B*c - A*d)*AppellF1[1/2, 1/2 - m, 3/2, 3/2,
Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])/(3
*(c + d)*AppellF1[1/2, 1/2 - m, 3/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d
*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (6*d*AppellF1[3/2, 1/2 - m, 5/2, 5
/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]
 - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, 3/2, 5/2, Cos[(2*e + Pi + 2*f*
x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)
```

```
/4]^2) - (B*AppellF1[1/2, 1/2 - m, 1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (
2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(c + d*Sin[e + f*x]))/(3*(c + d)*
AppellF1[1/2, 1/2 - m, 1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e
 - Pi + 2*f*x)/4]^2)/(c + d)] + (2*d*AppellF1[3/2, 1/2 - m, 3/2, 5/2, Cos[(
2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d
)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, 1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2,
(2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)/4]^2))*(
Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(d*f*(c + d*Sin[e + f*x])^(3/2))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{d\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}}{d^{2}\cos\left(fx+e\right)^{2}-2\,cd\sin\left(fx+e\right)-c^{2}-d^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a
)^m/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^
(3/2), x)
```

maple [F] time = 0.92, size = 0, normalized size = 0.00

$$\int\frac{\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)}{\left(c+d\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^
(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(c+d\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(3/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(3/2
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(A+B\sin\left(e+fx\right)\right)}{\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/(c + d*sin(e + f*x)
)**(3/2), x)
```

3.346 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=270

$$\frac{\sqrt{2}\,(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{1}{2};\frac{1}{2},-n;m+\frac{3}{2};\frac{1}{2}(\sin(e\right.}{f(2m+1)\sqrt{1-\sin(e+fx)}}$$

```
[Out] (A-B)*AppellF1(1/2+m,-n,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e
))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n*2^(1/2)/f/(1+2*m)/(((c+
d*sin(f*x+e))/(c-d))^n)/(1-sin(f*x+e))^(1/2)+B*AppellF1(3/2+m,-n,1/2,5/2+m,
-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^(1+
m)*(c+d*sin(f*x+e))^n*2^(1/2)/a/f/(3+2*m)/(((c+d*sin(f*x+e))/(c-d))^n)/(1-s
in(f*x+e))^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2987, 2788, 140, 139, 138}

$$\frac{\sqrt{2}\,(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{1}{2};\frac{1}{2},-n;m+\frac{3}{2};\frac{1}{2}(\sin(e\right.}{f(2m+1)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (Sqrt[2]*(A - B)*AppellF1[1/2 + m, 1/2, -n, 3/2 + m, (1 + Sin[e + f*x])/2,
-((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c +
 d*Sin[e + f*x])^n)/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*((c + d*Sin[e + f*x
])/(c - d))^n) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, -n, 5/2 + m, (1 + Sin[e
+ f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f
*x])^(1 + m)*(c + d*Sin[e + f*x])^n)/(a*f*(3 + 2*m)*Sqrt[1 - Sin[e + f*x]]*
((c + d*Sin[e + f*x])/(c - d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
```

```
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = (A-B)\int (a+a\sin(e+fx))^m(c+d\sin(e+$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-a}}\right.}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\,\text{s}}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subs}}{\sqrt{2}f(a-a\sin(e+fx))}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}(c+d\,\text{s}\right.}{\sqrt{}}$$

$$= \frac{\sqrt{2}(A-B)F_1\left(\frac{1}{2}+m;\frac{1}{2},-n;\frac{3}{2}+m;\frac{1}{2}(1+\text{si}\right.}{}$$

Mathematica [B] time = 6.08, size = 682, normalized size = 2.53

$$\frac{6(c+d)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e+fx)+1))^m(c+}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
n,x]
```

```
[Out] (6*(c + d)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/
4]*(a*(1 + Sin[e + f*x]))^m*(c + d*Sin[e + f*x])^n*(((B*c - A*d)*AppellF1[1
/2, 1/2 - m, -n, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f
*x)/4]^2)/(c + d)])/(3*(c + d)*AppellF1[1/2, 1/2 - m, -n, 3/2, Cos[(2*e + P
i + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-4*d*n*Appel
lF1[3/2, 1/2 - m, 1 - n, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e -
Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, -n,
5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)
])*Cos[(2*e + Pi + 2*f*x)/4]^2) - (B*AppellF1[1/2, 1/2 - m, -1 - n, 3/2, Co
```

```
s[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(c +
d*Sin[e + f*x]))/(3*(c + d)*AppellF1[1/2, 1/2 - m, -1 - n, 3/2, Cos[(2*e +
Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-4*d*(1 + n
)*AppellF1[3/2, 1/2 - m, -n, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*
e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m,
-1 - n, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)
/(c + d)])*Cos[(2*e + Pi + 2*f*x)/4]^2))*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2
 - m))/(d*f)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
n, x)
```

maple [F] time = 1.35, size = 0, normalized size = 0.00

$$\int\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
n, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

3.347 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=277

$$\frac{\sqrt{2}B\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^{-m}\left(\frac{c+d\sin(e+fx)}{c-d}\right)^m F_1\left(m+\frac{3}{2};\frac{1}{2},m+1;m+\frac{5}{2};\frac{1}{2}(\sin(e+\right.}{af(2m+3)(c-d)\sqrt{1-\sin(e+fx)}}$$

```
[Out] -2^(1/2+m)*a*(A-B)*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2*(c-d)*(1-sin
(f*x+e))/(c+d*sin(f*x+e)))*(a+a*sin(f*x+e))^(-1+m)*((c+d)*(1+sin(f*x+e))/(c
+d*sin(f*x+e)))^(1/2-m)/(c+d)/f/((c+d*sin(f*x+e))^m)+B*AppellF1(3/2+m,1+m,1
/2,5/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*
x+e))^(1+m)*((c+d*sin(f*x+e))/(c-d))^m*2^(1/2)/a/(c-d)/f/(3+2*m)/((c+d*sin(
f*x+e))^m)/(1-sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2987, 2788, 132, 140, 139, 138}

$$\frac{\sqrt{2}B\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^{-m}\left(\frac{c+d\sin(e+fx)}{c-d}\right)^m F_1\left(m+\frac{3}{2};\frac{1}{2},m+1;m+\frac{5}{2};\frac{1}{2}(\sin(e+\right.}{af(2m+3)(c-d)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^(-1 -
m),x]
```

```
[Out] -((2^(1/2 + m)*a*(A - B)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2,
((c - d)*(1 - Sin[e + f*x]))/(2*(c + d*Sin[e + f*x]))]*(a + a*Sin[e + f*x])
^(-1 + m)*(((c + d)*(1 + Sin[e + f*x]))/(c + d*Sin[e + f*x]))^(1/2 - m))/((
c + d)*f*(c + d*Sin[e + f*x])^m)) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, 1 + m
, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e +
 f*x]*(a + a*Sin[e + f*x])^(1 + m)*((c + d*Sin[e + f*x])/(c - d))^m)/(a*(c
- d)*f*(3 + 2*m)*Sqrt[1 - Sin[e + f*x]]*(c + d*Sin[e + f*x])^m)
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
```

```
p + 2, 0] && !IntegerQ[n]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
```

```
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{-1-m}\,dx=(A-B)\int (a+a\sin(e+fx))^m(c+d\sin(e$$

$$=\frac{\left(a^2(A-B)\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}}}{\sqrt{}}\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+}}$$

$$=-\frac{2^{\frac{1}{2}+m}a(A-B)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m\right.}{}$$

$$=-\frac{2^{\frac{1}{2}+m}a(A-B)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m\right.}{}$$

$$=-\frac{2^{\frac{1}{2}+m}a(A-B)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m\right.}{}$$

Mathematica [B] time = 6.94, size = 573, normalized size = 2.07

$$2\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e+fx)+1))^m(c+d\sin(e$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
(-1 - m),x]
```

```
[Out] (2*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1
 + Sin[e + f*x]))^m*((-3*B*(c + d)^2*AppellF1[1/2, 1/2 - m, m, 3/2, Cos[(2*
```

```
e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])/(d*(3*(c
+ d)*AppellF1[1/2, 1/2 - m, m, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(
2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (-4*d*m*AppellF1[3/2, 1/2 - m, 1 + m, 5/
2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]
+ (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, m, 5/2, Cos[(2*e + Pi + 2*f*x)/
4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)/4]
^2)) - A*Hypergeometric2F1[1/2, 1/2 - m, 3/2, ((c - d)*Sin[(2*e - Pi + 2*f*
x)/4]^2)/(c + d*Sin[e + f*x])]*(((c + d)*Cos[(2*e - Pi + 2*f*x)/4]^2)/(c +
d*Sin[e + f*x]))^(1/2 - m) + (B*c*Hypergeometric2F1[1/2, 1/2 - m, 3/2, ((c
- d)*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d*Sin[e + f*x])]*(((c + d)*Cos[(2*e
- Pi + 2*f*x)/4]^2)/(c + d*Sin[e + f*x]))^(1/2 - m))/d)*(Sin[(2*e + Pi + 2*
f*x)/4]^2)^(1/2 - m))/((c + d)*f*(c + d*Sin[e + f*x])^m)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x, al
gorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^(
-m - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x, al
gorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
(-m - 1), x)
```

maple [F] time = 3.47, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{-1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x, al
gorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
(-m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\sin(e+fx))\,(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(m +
 1),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(m +
 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(-1-m),x)
```

```
[Out] Timed out
```

3.348 $\int (a - a\sin(e + fx))(a + a\sin(e + fx))^m(c + d\sin(e + fx))^n \, dx$

Optimal. Leaf size=132

$$\frac{2\sqrt{2}\sqrt{1-\sin(e+fx)}\,\sec(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{1}{2};-\frac{1}{2},-n;\right.}{f(2m+1)}$$

```
[Out] 2*AppellF1(1/2+m,-n,-1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*
sec(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c+d*sin(f*x+e))^n*2^(1/2)*(1-sin(f*x+e))
^(1/2)/f/(1+2*m)/(((c+d*sin(f*x+e))/(c-d))^n)
```

Rubi [A] time = 0.20, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3008, 140, 139, 138}

$$\frac{2\sqrt{2}\sqrt{1-\sin(e+fx)}\,\sec(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{1}{2};-\frac{1}{2},-n;\right.}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (2*Sqrt[2]*AppellF1[1/2 + m, -1/2, -n, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*
(1 + Sin[e + f*x]))/(c - d))]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Si
n[e + f*x])^(1 + m)*(c + d*Sin[e + f*x])^n)/(f*(1 + 2*m)*((c + d*Sin[e + f*
x])/(c - d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
```

```
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 3008

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(f*Cos[e + f*x]
), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, Si
n[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c +
 a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = \frac{\Big(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+}}{}$$

$$= \frac{\Big(\sqrt{2}\,\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(}}{}$$

$$= \frac{\Big(\sqrt{2}\,\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(}}{}$$

$$= \frac{2\sqrt{2}\,F_1\left(\frac{1}{2}+m;-\frac{1}{2},-n;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+f\right.}{}$$

Mathematica [F] time = 10.11, size = 0, normalized size = 0.00

$$\int (a - a\sin(e + fx))(a + a\sin(e + fx))^m (c + d\sin(e + fx))^n \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^
n,x]
```

```
[Out] Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^
n, x]
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a\sin\left(fx + e\right) - a\right)\left(a\sin\left(fx + e\right) + a\right)^{m}\left(d\sin\left(fx + e\right) + c\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(a\sin\left(fx + e\right) - a\right)\left(a\sin\left(fx + e\right) + a\right)^{m}\left(d\sin\left(fx + e\right) + c\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^n, x)
```

maple [F] time = 1.43, size = 0, normalized size = 0.00

$$\int \left(a - a\sin\left(fx + e\right)\right)\left(a + a\sin\left(fx + e\right)\right)^{m}\left(c + d\sin\left(fx + e\right)\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(a\sin\left(fx+e\right)-a\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] -integrate((a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(a-a\,\sin\left(e+f\,x\right)\right)\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(e + f*x))^m*(a - a*sin(e + f*x))*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int((a + a*sin(e + f*x))^m*(a - a*sin(e + f*x))*(c + d*sin(e + f*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

3.349 $\int (a - a\sin(e + fx))(a + a\sin(e + fx))^m (c + d\sin(e + fx))^{-1-m}\, dx$

Optimal. Leaf size=139

$$\frac{2\sqrt{2}\sqrt{1-\sin(e+fx)}\,\sec(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^{-m}\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{m}F_1\left(m+\frac{1}{2};-\frac{1}{2},m\right.}{f(2m+1)(c-d)}$$

```
[Out] 2*AppellF1(1/2+m,1+m,-1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))
*sec(f*x+e)*(a+a*sin(f*x+e))^(1+m)*((c+d*sin(f*x+e))/(c-d))^m*2^(1/2)*(1-si
n(f*x+e))^(1/2)/(c-d)/f/(1+2*m)/((c+d*sin(f*x+e))^m)
```

Rubi [A] time = 0.23, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3008, 140, 139, 138}

$$\frac{2\sqrt{2}\sqrt{1-\sin(e+fx)}\,\sec(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^{-m}\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{m}F_1\left(m+\frac{1}{2};-\frac{1}{2},m\right.}{f(2m+1)(c-d)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 -
m),x]
```

```
[Out] (2*Sqrt[2]*AppellF1[1/2 + m, -1/2, 1 + m, 3/2 + m, (1 + Sin[e + f*x])/2, -(
(d*(1 + Sin[e + f*x]))/(c - d))]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a
*Sin[e + f*x])^(1 + m)*((c + d*Sin[e + f*x])/(c - d))^m)/((c - d)*f*(1 + 2*
m)*(c + d*Sin[e + f*x])^m)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
```

```
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 3008

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(f*Cos[e + f*x]
), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, Si
n[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c +
 a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a - a\sin(e+fx))(a + a\sin(e+fx))^m (c + d\sin(e+fx))^{-1-m}\,dx = \frac{\left(\sec(e+fx)\sqrt{a - a\sin(e+fx)}\,\sqrt{a + a\sin(\right.}{}$$

$$= \frac{\left(\sqrt{2}\,\sec(e+fx)(a - a\sin(e+fx))\sqrt{a + a\,s\right.}{}$$

$$= \frac{\left(\sqrt{2}\,a\sec(e+fx)(a - a\sin(e+fx))\sqrt{a + a\right.}{}$$

$$= \frac{2\sqrt{2}\,F_1\left(\frac{1}{2} + m; -\frac{1}{2}, 1 + m; \frac{3}{2} + m; \frac{1}{2}(1 + \sin\right.}{}$$

Mathematica [F] time = 4.87, size = 0, normalized size = 0.00

$$\int (a - a\sin(e + fx))(a + a\sin(e + fx))^m (c + d\sin(e + fx))^{-1-m}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^
(-1 - m),x]
```

```
[Out] Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^
(-1 - m), x]
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a\sin\left(fx + e\right) - a\right)\left(a\sin\left(fx + e\right) + a\right)^m\left(d\sin\left(fx + e\right) + c\right)^{-m-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x, al
gorithm="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
(-m - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(a\sin\left(fx + e\right) - a\right)\left(a\sin\left(fx + e\right) + a\right)^m\left(d\sin\left(fx + e\right) + c\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x, al
gorithm="giac")
```

```
[Out] integrate(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^(-m - 1), x)
```

maple [F] time = 2.75, size = 0, normalized size = 0.00

$$\int \left(a - a\sin\left(fx + e\right)\right)\left(a + a\sin\left(fx + e\right)\right)^m\left(c + d\sin\left(fx + e\right)\right)^{-1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x)
```

```
[Out] int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int (a\sin(fx+e)-a)(a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x, al
gorithm="maxima")
```

```
[Out] -integrate((a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^(-m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\sin(e+fx))^m\,(a-a\sin(e+fx))}{(c+d\sin(e+fx))^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^m*(a - a*sin(e + f*x)))/(c + d*sin(e + f*x))^(m +
 1),x)
```

```
[Out] int(((a + a*sin(e + f*x))^m*(a - a*sin(e + f*x)))/(c + d*sin(e + f*x))^(m +
 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(-1-m),x)
```

```
[Out] Timed out
```

3.350 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx$

Optimal. Leaf size=39

$$-\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^{-m-1}}{f}$$

```
[Out] -cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m)/f
```

Rubi [A] time = 0.17, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.018, Rules used = {2974}

$$-\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^{-m-1}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-2 - m)*(d - (c - d)*m + (
c + (c - d)*m)*Sin[e + f*x]),x]
```

```
[Out] -((Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 - m))/f)
```

Rule 2974

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n},
x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[m
+ n + 2, 0] && EqQ[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx = -\frac{\cos(e+fx)(a+}{}$$

Mathematica [A] time = 0.68, size = 39, normalized size = 1.00

$$-\frac{\cos(e+fx)(a(\sin(e+fx)+1))^m(c+d\sin(e+fx))^{-m-1}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-2 - m)*(d - (c - d)
*m + (c + (c - d)*m)*Sin[e + f*x]),x]
```

```
[Out] -((Cos[e + f*x]*(a*(1 + Sin[e + f*x]))^m*(c + d*Sin[e + f*x])^(-1 - m))/f)
```

fricas [A] time = 0.54, size = 56, normalized size = 1.44

$$-\frac{\left(d\cos\left(fx+e\right)\sin\left(fx+e\right)+c\cos\left(fx+e\right)\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{-m-2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m)
*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(d*cos(f*x + e)*sin(f*x + e) + c*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(d*s
in(f*x + e) + c)^(-m - 2)/f
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m)
*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 6.47, size = 0, normalized size = 0.00

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{-2-m}\left(d-\left(c-d\right)m+\left(c+\left(c-d\right)m\right)\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m)*sin(f
*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m)*sin(f
*x+e)),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m)
*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [**B**] time = 14.77, size = 98, normalized size = 2.51

$$-\frac{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)^m\,\left(d\,\sin\left(2\,e+2\,f\,x\right)-2\,c\,\left(2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)\right)}{f\,\left(c+d\,\sin\left(e+f\,x\right)\right)^m\,\left(d^2\,\left(2\,\sin\left(e+f\,x\right)^2-1\right)+2\,c^2+d^2+4\,c\,d\,\sin\left(e+f\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^m*(d - m*(c - d) + sin(e + f*x)*(c + m*(c - d))))
/(c + d*sin(e + f*x))^(m + 2),x)
```

```
[Out] -((a*(sin(e + f*x) + 1))^m*(d*sin(2*e + 2*f*x) - 2*c*(2*sin(e/2 + (f*x)/2)^
2 - 1)))/(f*(c + d*sin(e + f*x))^m*(d^2*(2*sin(e + f*x)^2 - 1) + 2*c^2 + d^
2 + 4*c*d*sin(e + f*x)))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(-2-m)*(d-(c-d)*m+(c+(c-d)*
m)*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.351 $\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx$

Optimal. Leaf size=40

$$-\frac{\cos(e+fx)(a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-m-1}}{f}$$

```
[Out] -cos(f*x+e)*(a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m)/f
```

Rubi [A] time = 0.17, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.020, Rules used = {2974}

$$-\frac{\cos(e+fx)(a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-m-1}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-2 - m)*(d + (c + d)*m + (
c + (c + d)*m)*Sin[e + f*x]),x]
```

```
[Out] -((Cos[e + f*x]*(a - a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 - m))/f)
```

Rule 2974

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n},
x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[m
+ n + 2, 0] && EqQ[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]
```

Rubi steps

$$\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx = -\frac{\cos(e+fx)(a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-m-1}}{f}$$

Mathematica [A] time = 0.74, size = 40, normalized size = 1.00

$$-\frac{\cos(e+fx)(a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-m-1}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-2 - m)*(d + (c + d)
*m + (c + (c + d)*m)*Sin[e + f*x]),x]
```

```
[Out] -((Cos[e + f*x]*(a - a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 - m))/f)
```

fricas [A] time = 0.55, size = 57, normalized size = 1.42

$$-\frac{\left(d\cos\left(fx+e\right)\sin\left(fx+e\right)+c\cos\left(fx+e\right)\right)\left(-a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{-m-2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m)
*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(d*cos(f*x + e)*sin(f*x + e) + c*cos(f*x + e))*(-a*sin(f*x + e) + a)^m*(d*
sin(f*x + e) + c)^(-m - 2)/f
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m)
*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 6.56, size = 0, normalized size = 0.00

$$\int \left(a-a\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{-2-m}\left(d+\left(c+d\right)m+\left(c+\left(c+d\right)m\right)\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m)*sin(f
*x+e)),x)
```

```
[Out] int((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m)*sin(f
*x+e)),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m)
*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 14.62, size = 99, normalized size = 2.48

$$-\frac{\left(-a\,\left(\sin\left(e+f\,x\right)-1\right)\right)^m\,\left(d\,\sin\left(2\,e+2\,f\,x\right)-2\,c\,\left(2\,{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)\right)}{f\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^m\,\left(d^2\,\left(2\,{\sin\left(e+f\,x\right)}^2-1\right)+2\,c^2+d^2+4\,c\,d\,\sin\left(e+f\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a - a*sin(e + f*x))^m*(d + sin(e + f*x)*(c + m*(c + d)) + m*(c + d)))
/(c + d*sin(e + f*x))^(m + 2),x)
```

```
[Out] -((-a*(sin(e + f*x) - 1))^m*(d*sin(2*e + 2*f*x) - 2*c*(2*sin(e/2 + (f*x)/2)
^2 - 1)))/(f*(c + d*sin(e + f*x))^m*(d^2*(2*sin(e + f*x)^2 - 1) + 2*c^2 + d
^2 + 4*c*d*sin(e + f*x)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(-2-m)*(d+(c+d)*m+(c+(c+d)*
m)*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.352 $\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=199

$$-\frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2f\left(c^2-d^2\right)(c+d\sin(e+fx))}-\frac{2(bc-ad)\left(ad^2(Ac-Bd)-b\left(-Ac^2d+2Ad^3+2Bc^3-3Bcd^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\ldots\right.}{\sqrt{\ldots}}\right.}{d^3f\left(c^2-d^2\right)^{3/2}}$$

```
[Out] -b*(-A*b*d-2*B*a*d+2*B*b*c)*x/d^3-2*(-a*d+b*c)*(a*d^2*(A*c-B*d)-b*(-A*c^2*d
+2*A*d^3+2*B*c^3-3*B*c*d^2))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2
))/d^3/(c^2-d^2)^(3/2)/f-b^2*B*cos(f*x+e)/d^2/f-(-a*d+b*c)^2*(-A*d+B*c)*cos
(f*x+e)/d^2/(c^2-d^2)/f/(c+d*sin(f*x+e))
```

Rubi [A] time = 0.58, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2988, 3023, 2735, 2660, 618, 204}

$$-\frac{2(bc-ad)\left(ad^2(Ac-Bd)-b\left(-Ac^2d+2Ad^3+2Bc^3-3Bcd^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f\left(c^2-d^2\right)^{3/2}}-\frac{(bc-ad)^2(Bc-Ad)\,\mathrm{c}\ldots}{d^2f\left(c^2-d^2\right)(c+d\,\mathrm{si}\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -((b*(2*b*B*c - A*b*d - 2*a*B*d)*x)/d^3) - (2*(b*c - a*d)*(a*d^2*(A*c - B*d
) - b*(2*B*c^3 - A*c^2*d - 3*B*c*d^2 + 2*A*d^3))*ArcTan[(d + c*Tan[(e + f*x
)/2])/Sqrt[c^2 - d^2]])/(d^3*(c^2 - d^2)^(3/2)*f) - (b^2*B*Cos[e + f*x])/(d
^2*f) - ((b*c - a*d)^2*(B*c - A*d)*Cos[e + f*x])/(d^2*(c^2 - d^2)*f*(c + d*
Sin[e + f*x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = -\frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{\int \frac{-d\left(B(bc-ad)^2-Ad\left(a^2c+b^2c-2a\right.\right.}{}}{}$$

$$= -\frac{b^2B\cos(e+fx)}{d^2f} - \frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{\int \frac{-d^2(B}{}}{}$$

$$= -\frac{b(2bBc-Abd-2aBd)x}{d^3} - \frac{b^2B\cos(e+fx)}{d^2f} - \frac{(bc-ad)^2(Bc-}{d^2\left(c^2-d^2\right)f(c}$$

$$= -\frac{b(2bBc-Abd-2aBd)x}{d^3} - \frac{b^2B\cos(e+fx)}{d^2f} - \frac{(bc-ad)^2(Bc-}{d^2\left(c^2-d^2\right)f(c}$$

$$= -\frac{b(2bBc-Abd-2aBd)x}{d^3} - \frac{b^2B\cos(e+fx)}{d^2f} - \frac{(bc-ad)^2(Bc-}{d^2\left(c^2-d^2\right)f(c}$$

$$= -\frac{b(2bBc-Abd-2aBd)x}{d^3} - \frac{2(bc-ad)\left(ad^2(Ac-Bd)-b\left(2Bc^3\right.\right.}{}$$

Mathematica [A] time = 1.60, size = 188, normalized size = 0.94

$$\frac{\frac{2(bc-ad)\left(ad^2(Bd-Ac)+b\left(-Ac^2d+2Ad^3+2Bc^3-3Bcd^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{3/2}} + b(e+fx)(2aBd+Abd-2bBc) + \frac{d(bc-ad)^2(Ad-Bc)\,c}{(c-d)(c+d)(c+d\sin}}{d^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^2,x]
```

```
[Out] (b*(-2*b*B*c + A*b*d + 2*a*B*d)*(e + f*x) + (2*(b*c - a*d)*(a*d^2*(-(A*c) +
 B*d) + b*(2*B*c^3 - A*c^2*d - 3*B*c*d^2 + 2*A*d^3))*ArcTan[(d + c*Tan[(e +
 f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(3/2) - b^2*B*d*Cos[e + f*x] + (d*(
b*c - a*d)^2*(-(B*c) + A*d)*Cos[e + f*x])/((c - d)*(c + d)*(c + d*Sin[e + f
*x]))))/(d^3*f)
```

fricas [B] time = 0.61, size = 1308, normalized size = 6.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] [-1/2*(2*(2*B*b^2*c^6 - 4*B*b^2*c^4*d^2 + 2*B*b^2*c^2*d^4 - (2*B*a*b + A*b^
2)*c^5*d + 2*(2*B*a*b + A*b^2)*c^3*d^3 - (2*B*a*b + A*b^2)*c*d^5)*f*x + (2*
B*b^2*c^5 - 3*B*b^2*c^3*d^2 - (2*B*a*b + A*b^2)*c^4*d + (A*a^2 + 4*B*a*b +
2*A*b^2)*c^2*d^3 - (B*a^2 + 2*A*a*b)*c*d^4 + (2*B*b^2*c^4*d - 3*B*b^2*c^2*d
^3 - (2*B*a*b + A*b^2)*c^3*d^2 + (A*a^2 + 4*B*a*b + 2*A*b^2)*c*d^4 - (B*a^2
 + 2*A*a*b)*d^5)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x
+ e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) +
d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e)
- c^2 - d^2)) + 2*(2*B*b^2*c^5*d + A*a^2*d^6 - (2*B*a*b + A*b^2)*c^4*d^2 +
(B*a^2 + 2*A*a*b - 3*B*b^2)*c^3*d^3 - (A*a^2 - 2*B*a*b - A*b^2)*c^2*d^4 - (
B*a^2 + 2*A*a*b - B*b^2)*c*d^5)*cos(f*x + e) + 2*((2*B*b^2*c^5*d - 4*B*b^2*
c^3*d^3 + 2*B*b^2*c*d^5 - (2*B*a*b + A*b^2)*c^4*d^2 + 2*(2*B*a*b + A*b^2)*c
^2*d^4 - (2*B*a*b + A*b^2)*d^6)*f*x + (B*b^2*c^4*d^2 - 2*B*b^2*c^2*d^4 + B*
b^2*d^6)*cos(f*x + e))*sin(f*x + e))/((c^4*d^4 - 2*c^2*d^6 + d^8)*f*sin(f*x
 + e) + (c^5*d^3 - 2*c^3*d^5 + c*d^7)*f), -((2*B*b^2*c^6 - 4*B*b^2*c^4*d^2
+ 2*B*b^2*c^2*d^4 - (2*B*a*b + A*b^2)*c^5*d + 2*(2*B*a*b + A*b^2)*c^3*d^3 -
 (2*B*a*b + A*b^2)*c*d^5)*f*x + (2*B*b^2*c^5 - 3*B*b^2*c^3*d^2 - (2*B*a*b +
 A*b^2)*c^4*d + (A*a^2 + 4*B*a*b + 2*A*b^2)*c^2*d^3 - (B*a^2 + 2*A*a*b)*c*d
^4 + (2*B*b^2*c^4*d - 3*B*b^2*c^2*d^3 - (2*B*a*b + A*b^2)*c^3*d^2 + (A*a^2
+ 4*B*a*b + 2*A*b^2)*c*d^4 - (B*a^2 + 2*A*a*b)*d^5)*sin(f*x + e))*sqrt(c^2
- d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (2*B*
b^2*c^5*d + A*a^2*d^6 - (2*B*a*b + A*b^2)*c^4*d^2 + (B*a^2 + 2*A*a*b - 3*B*
b^2)*c^3*d^3 - (A*a^2 - 2*B*a*b - A*b^2)*c^2*d^4 - (B*a^2 + 2*A*a*b - B*b^2
)*c*d^5)*cos(f*x + e) + ((2*B*b^2*c^5*d - 4*B*b^2*c^3*d^3 + 2*B*b^2*c*d^5 -
 (2*B*a*b + A*b^2)*c^4*d^2 + 2*(2*B*a*b + A*b^2)*c^2*d^4 - (2*B*a*b + A*b^2
)*d^6)*f*x + (B*b^2*c^4*d^2 - 2*B*b^2*c^2*d^4 + B*b^2*d^6)*cos(f*x + e))*si
n(f*x + e))/((c^4*d^4 - 2*c^2*d^6 + d^8)*f*sin(f*x + e) + (c^5*d^3 - 2*c^3*
d^5 + c*d^7)*f)]
```

giac [B] time = 6.89, size = 776, normalized size = 3.90

$$\frac{\frac{2\left(2\,Bb^2c^4-2\,Babc^3d-Ab^2c^3d-3\,Bb^2c^2d^2+Aa^2cd^3+4\,Babcd^3+2\,Ab^2cd^3-Ba^2d^4-2\,Aabd^4\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(c^2d^3-d^5\right)\sqrt{c^2-d^2}}-\frac{2\left(Bb^2c^3d\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] (2*(2*B*b^2*c^4 - 2*B*a*b*c^3*d - A*b^2*c^3*d - 3*B*b^2*c^2*d^2 + A*a^2*c*d
^3 + 4*B*a*b*c*d^3 + 2*A*b^2*c*d^3 - B*a^2*d^4 - 2*A*a*b*d^4)*(pi*floor(1/2
*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2
- d^2)))/((c^2*d^3 - d^5)*sqrt(c^2 - d^2)) - 2*(B*b^2*c^3*d*tan(1/2*f*x + 1
/2*e)^3 - 2*B*a*b*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - A*b^2*c^2*d^2*tan(1/2*f*
x + 1/2*e)^3 + B*a^2*c*d^3*tan(1/2*f*x + 1/2*e)^3 + 2*A*a*b*c*d^3*tan(1/2*f
*x + 1/2*e)^3 - A*a^2*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*B*b^2*c^4*tan(1/2*f*x
+ 1/2*e)^2 - 2*B*a*b*c^3*d*tan(1/2*f*x + 1/2*e)^2 - A*b^2*c^3*d*tan(1/2*f*x
 + 1/2*e)^2 + B*a^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 + 2*A*a*b*c^2*d^2*tan(1/
2*f*x + 1/2*e)^2 - B*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 - A*a^2*c*d^3*tan(1
/2*f*x + 1/2*e)^2 + 3*B*b^2*c^3*d*tan(1/2*f*x + 1/2*e) - 2*B*a*b*c^2*d^2*ta
n(1/2*f*x + 1/2*e) - A*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e) + B*a^2*c*d^3*tan(1
/2*f*x + 1/2*e) + 2*A*a*b*c*d^3*tan(1/2*f*x + 1/2*e) - 2*B*b^2*c*d^3*tan(1/
2*f*x + 1/2*e) - A*a^2*d^4*tan(1/2*f*x + 1/2*e) + 2*B*b^2*c^4 - 2*B*a*b*c^3
*d - A*b^2*c^3*d + B*a^2*c^2*d^2 + 2*A*a*b*c^2*d^2 - B*b^2*c^2*d^2 - A*a^2*
c*d^3)/((c^3*d^2 - c*d^4)*(c*tan(1/2*f*x + 1/2*e)^4 + 2*d*tan(1/2*f*x + 1/2
*e)^3 + 2*c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) - (2*B*
b^2*c - 2*B*a*b*d - A*b^2*d)*(f*x + e)/d^3)/f
```

maple [B] time = 0.46, size = 1246, normalized size = 6.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -4/f*d/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1
/2))*A*a*b-2/f/d^2/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/
(c^2-d^2)^(1/2))*A*b^2*c^3+4/f/d^3/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*
f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*b^2*c^4-6/f/d/(c^2-d^2)^(3/2)*arctan(1/2
*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*b^2*c^2-2/f*d/(tan(1/2*f*x
+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*tan(1/2*f*x+1/2*e)*B*a^2+2/
f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*A*b^2*c^2-2
/f/d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*c^3*B*b^
2+2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*c*tan(1/2
*f*x+1/2*e)*A*b^2-4/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c^
2-d^2)*A*a*b*c+8/f/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/
(c^2-d^2)^(1/2))*B*a*b*c-2/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)
*d+c)/(c^2-d^2)*c^2*tan(1/2*f*x+1/2*e)*B*b^2+4/f/d/(tan(1/2*f*x+1/2*e)^2*c+
2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*B*a*b*c^2-4/f/d^2/(c^2-d^2)^(3/2)*arcta
n(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*a*b*c^3+4/f/(tan(1/2*
f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*c*tan(1/2*f*x+1/2*e)*B*a
*b+2/f*d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)/c*ta
n(1/2*f*x+1/2*e)*a^2*A-4/f*d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d
+c)/(c^2-d^2)*tan(1/2*f*x+1/2*e)*A*a*b+4/f/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*
```

```
tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*b^2*c-2/f/(tan(1/2*f*x+1/2*e)^2*
c+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*B*a^2*c+2/f/(c^2-d^2)^(3/2)*arctan(1/
2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*a^2*c+2/f*d/(tan(1/2*f*x+
1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*a^2*A-2/f/d^2*b^2*B/(1+tan(1
/2*f*x+1/2*e)^2)+2/f/d^2*b^2*A*arctan(tan(1/2*f*x+1/2*e))-2/f*d/(c^2-d^2)^(
3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*a^2+4/f/d^2
*b*B*arctan(tan(1/2*f*x+1/2*e))*a-4/f/d^3*b^2*B*arctan(tan(1/2*f*x+1/2*e))*
c
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [**B**] time = 27.62, size = 16312, normalized size = 81.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + b*sin(e + f*x))^2)/(c + d*sin(e + f*x))^2,x)
```

```
[Out] ((2*(A*a^2*d^3 - 2*B*b^2*c^3 + A*b^2*c^2*d - B*a^2*c*d^2 + B*b^2*c*d^2 - 2*
A*a*b*c*d^2 + 2*B*a*b*c^2*d))/(d^2*(c^2 - d^2)) + (2*tan(e/2 + (f*x)/2)^2*(
A*a^2*d^3 - 2*B*b^2*c^3 + A*b^2*c^2*d - B*a^2*c*d^2 + B*b^2*c*d^2 - 2*A*a*b
*c*d^2 + 2*B*a*b*c^2*d))/(d^2*(c^2 - d^2)) + (2*tan(e/2 + (f*x)/2)^3*(A*a^2
*d^3 - B*b^2*c^3 + A*b^2*c^2*d - B*a^2*c*d^2 - 2*A*a*b*c*d^2 + 2*B*a*b*c^2*
d))/(c*d*(c^2 - d^2)) + (2*tan(e/2 + (f*x)/2)*(A*a^2*d^3 - 3*B*b^2*c^3 + A*
b^2*c^2*d - B*a^2*c*d^2 + 2*B*b^2*c*d^2 - 2*A*a*b*c*d^2 + 2*B*a*b*c^2*d))/(
c*d*(c^2 - d^2)))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + 2*c*tan(e/2 + (f*x)/2)^2
 + c*tan(e/2 + (f*x)/2)^4 + 2*d*tan(e/2 + (f*x)/2)^3)) + (atan((((b*d*(A*b
+ 2*B*a)*1i - B*b^2*c*2i)*((32*(A^2*b^4*c^2*d^8 - 2*A^2*b^4*c^4*d^6 + A^2*b
^4*c^6*d^4 + 4*B^2*b^4*c^4*d^6 - 8*B^2*b^4*c^6*d^4 + 4*B^2*b^4*c^8*d^2 + 4*
B^2*a^2*b^2*c^2*d^8 - 8*B^2*a^2*b^2*c^4*d^6 + 4*B^2*a^2*b^2*c^6*d^4 - 4*A*B
*b^4*c^3*d^7 + 8*A*B*b^4*c^5*d^5 - 4*A*B*b^4*c^7*d^3 - 8*B^2*a*b^3*c^3*d^7
+ 16*B^2*a*b^3*c^5*d^5 - 8*B^2*a*b^3*c^7*d^3 + 4*A*B*a*b^3*c^2*d^8 - 8*A*B*
a*b^3*c^4*d^6 + 4*A*B*a*b^3*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^4*d^5) + ((b*d*(
A*b + 2*B*a)*1i - B*b^2*c*2i)*((((32*(c^2*d^12 - 2*c^4*d^10 + c^6*d^8))/(d^
```

9 - 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^14 - 8*c^3*d^12 + 7*c^5*d^10 - 2*c^7*d^8))/(d^10 - 2*c^2*d^8 + c^4*d^6))*(b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i))/d^3 - (32*(A*a^2*c^5*d^7 - A*a^2*c^3*d^9 - A*b^2*c*d^11 + A*b^2*c^3*d^9 + B*a^2*c^2*d^10 - B*a^2*c^4*d^8 + 2*B*b^2*c^2*d^10 - 3*B*b^2*c^4*d^8 + B*b^2*c^6*d^6 - 2*B*a*b*c*d^11 + 2*A*a*b*c^2*d^10 - 2*A*a*b*c^4*d^8 + 2*B*a*b*c^3*d^9))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)*(2*A*a^2*c^2*d^11 - 2*B*a^2*c*d^12 - 2*A*a^2*c^4*d^9 + 4*A*b^2*c^2*d^11 - 6*A*b^2*c^4*d^9 + 2*A*b^2*c^6*d^7 + 2*B*a^2*c^3*d^10 - 6*B*b^2*c^3*d^10 + 10*B*b^2*c^5*d^8 - 4*B*b^2*c^7*d^6 - 4*A*a*b*c*d^12 + 4*A*a*b*c^3*d^10 + 8*B*a*b*c^2*d^11 - 12*B*a*b*c^4*d^9 + 4*B*a*b*c^6*d^7))/(d^10 - 2*c^2*d^8 + c^4*d^6)))/d^3 - (32*tan(e/2 + (f*x)/2)*(A^2*a^4*c^3*d^8 + 9*A^2*b^4*c^3*d^8 - 8*A^2*b^4*c^5*d^6 + 2*A^2*b^4*c^7*d^4 - 8*B^2*b^4*c^3*d^8 + 29*B^2*b^4*c^5*d^6 - 28*B^2*b^4*c^7*d^4 + 8*B^2*b^4*c^9*d^2 - 2*A^2*b^4*c*d^10 + B^2*a^4*c*d^10 + 4*A^2*a^2*b^2*c^3*d^8 - 2*A^2*a^2*b^2*c^5*d^6 + 42*B^2*a^2*b^2*c^3*d^8 - 36*B^2*a^2*b^2*c^5*d^6 + 8*B^2*a^2*b^2*c^7*d^4 - 2*A*B*a^4*c^2*d^9 + 8*A*B*b^4*c^2*d^9 - 32*A*B*b^4*c^4*d^7 + 30*A*B*b^4*c^6*d^5 - 8*A*B*b^4*c^8*d^3 - 8*A^2*a*b^3*c^2*d^9 + 4*A^2*a*b^3*c^4*d^7 + 4*A^2*a^2*b^2*c*d^10 - 4*A^2*a^3*b*c^2*d^9 + 16*B^2*a*b^3*c^2*d^9 - 64*B^2*a*b^3*c^4*d^7 + 60*B^2*a*b^3*c^6*d^5 - 16*B^2*a*b^3*c^8*d^3 - 8*B^2*a^2*b^2*c*d^10 - 8*B^2*a^3*b*c^2*d^9 + 4*B^2*a^3*b*c^4*d^7 - 8*A*B*a*b^3*c*d^10 + 4*A*B*a^3*b*c*d^10 + 48*A*B*a*b^3*c^3*d^8 - 40*A*B*a*b^3*c^5*d^6 + 8*A*B*a*b^3*c^7*d^4 + 8*A*B*a^3*b*c^3*d^8 - 4*A*B*a^3*b*c^5*d^6 - 20*A*B*a^2*b^2*c^2*d^9 + 4*A*B*a^2*b^2*c^4*d^7 + 4*A*B*a^2*b^2*c^6*d^5))/(d^10 - 2*c^2*d^8 + c^4*d^6))*1i)/d^3 + ((b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i)*((32*(A^2*b^4*c^2*d^8 - 2*A^2*b^4*c^4*d^6 + A^2*b^4*c^6*d^4 + 4*B^2*b^4*c^4*d^6 - 8*B^2*b^4*c^6*d^4 + 4*B^2*b^4*c^8*d^2 + 4*B^2*a^2*b^2*c^2*d^8 - 8*B^2*a^2*b^2*c^4*d^6 + 4*B^2*a^2*b^2*c^6*d^4 - 4*A*B*b^4*c^3*d^7 + 8*A*B*b^4*c^5*d^5 - 4*A*B*b^4*c^7*d^3 - 8*B^2*a*b^3*c^3*d^7 + 16*B^2*a*b^3*c^5*d^5 - 8*B^2*a*b^3*c^7*d^3 + 4*A*B*a*b^3*c^2*d^8 - 8*A*B*a*b^3*c^4*d^6 + 4*A*B*a*b^3*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^4*d^5) + ((b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i)*((32*(A*a^2*c^5*d^7 - A*a^2*c^3*d^9 - A*b^2*c*d^11 + A*b^2*c^3*d^9 + B*a^2*c^2*d^10 - B*a^2*c^4*d^8 + 2*B*b^2*c^2*d^10 - 3*B*b^2*c^4*d^8 + B*b^2*c^6*d^6 - 2*B*a*b*c*d^11 + 2*A*a*b*c^2*d^10 - 2*A*a*b*c^4*d^8 + 2*B*a*b*c^3*d^9))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (((32*(c^2*d^12 - 2*c^4*d^10 + c^6*d^8))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^14 - 8*c^3*d^12 + 7*c^5*d^10 - 2*c^7*d^8))/(d^10 - 2*c^2*d^8 + c^4*d^6))*(b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i))/d^3 - (32*tan(e/2 + (f*x)/2)*(2*A*a^2*c^2*d^11 - 2*B*a^2*c*d^12 - 2*A*a^2*c^4*d^9 + 4*A*b^2*c^2*d^11 - 6*A*b^2*c^4*d^9 + 2*A*b^2*c^6*d^7 + 2*B*a^2*c^3*d^10 - 6*B*b^2*c^3*d^10 + 10*B*b^2*c^5*d^8 - 4*B*b^2*c^7*d^6 - 4*A*a*b*c*d^12 + 4*A*a*b*c^3*d^10 + 8*B*a*b*c^2*d^11 - 12*B*a*b*c^4*d^9 + 4*B*a*b*c^6*d^7))/(d^10 - 2*c^2*d^8 + c^4*d^6)))/d^3 - (32*tan(e/2 + (f*x)/2)*(A^2*a^4*c^3*d^8 + 9*A^2*b^4*c^3*d^8 - 8*A^2*b^4*c^5*d^6 + 2*A^2*b^4*c^7*d^4 - 8*B^2*b^4*c^3*d^8 + 29*B^2*b^4*c^5*d^6 - 28*B^2*b^4*c^7*d^4 + 8*B^2*b^4*c^9*d^2 - 2*A^2*b^4*c*d^10 + B^2*a^4*c*d^10 + 4*A^2*a^2*b^2*c^3*d^8 - 2*A^2*a^2*b^2*c^5*d^6 + 42*B^2*a^2*b^2*c^3*d^8 - 36*B^2*a^2*b^2*c^5*d^6 + 8*B^2*a^2*b^2*c^7*d^4 - 2*A*B*

a^4*c^2*d^9 + 8*A*B*b^4*c^2*d^9 - 32*A*B*b^4*c^4*d^7 + 30*A*B*b^4*c^6*d^5 - 8*A*B*b^4*c^8*d^3 - 8*A^2*a*b^3*c^2*d^9 + 4*A^2*a*b^3*c^4*d^7 + 4*A^2*a^2*b^2*c*d^10 - 4*A^2*a^3*b*c^2*d^9 + 16*B^2*a*b^3*c^2*d^9 - 64*B^2*a*b^3*c^4*d^7 + 60*B^2*a*b^3*c^6*d^5 - 16*B^2*a*b^3*c^8*d^3 - 8*B^2*a^2*b^2*c*d^10 - 8*B^2*a^3*b*c^2*d^9 + 4*B^2*a^3*b*c^4*d^7 - 8*A*B*a*b^3*c*d^10 + 4*A*B*a^3*b*c*d^10 + 48*A*B*a*b^3*c^3*d^8 - 40*A*B*a*b^3*c^5*d^6 + 8*A*B*a*b^3*c^7*d^4 + 8*A*B*a^3*b*c^3*d^8 - 4*A*B*a^3*b*c^5*d^6 - 20*A*B*a^2*b^2*c^2*d^9 + 4*A*B*a^2*b^2*c^4*d^7 + 4*A*B*a^2*b^2*c^6*d^5))/(d^10 - 2*c^2*d^8 + c^4*d^6))*1i)/d^3)/((64*(A^3*b^6*c^5*d^3 - 2*A^3*b^6*c^3*d^5 - 4*B^3*b^6*c^8 + 6*B^3*b^6*c^6*d^2 - 3*A^3*a^2*b^4*c^3*d^5 + A^3*a^2*b^4*c^5*d^3 + 4*A^3*a^3*b^3*c^2*d^6 - A^3*a^4*b^2*c^3*d^5 + 44*B^3*a^2*b^4*c^4*d^4 - 24*B^3*a^2*b^4*c^6*d^2 - 36*B^3*a^3*b^3*c^3*d^5 + 16*B^3*a^3*b^3*c^5*d^3 + 14*B^3*a^4*b^2*c^2*d^6 - 4*B^3*a^4*b^2*c^4*d^4 + 8*A*B^2*b^6*c^7*d + 16*B^3*a*b^5*c^7*d - 2*B^3*a^5*b*c*d^7 - 13*A*B^2*b^6*c^5*d^3 + 9*A^2*B*b^6*c^4*d^4 - 5*A^2*B*b^6*c^6*d^2 + 6*A^3*a*b^5*c^2*d^6 - 2*A^3*a*b^5*c^4*d^4 - 4*A^3*a^2*b^4*c*d^7 - 26*B^3*a*b^5*c^5*d^3 - 74*A*B^2*a^2*b^4*c^3*d^5 + 24*A*B^2*a^2*b^4*c^5*d^3 + 44*A*B^2*a^3*b^3*c^2*d^6 + 8*A*B^2*a^3*b^3*c^4*d^4 - 8*A*B^2*a^3*b^3*c^6*d^2 - 16*A*B^2*a^4*b^2*c^3*d^5 + 4*A*B^2*a^4*b^2*c^5*d^3 + 35*A^2*B*a^2*b^4*c^2*d^6 + A^2*B*a^2*b^4*c^4*d^4 - 4*A^2*B*a^2*b^4*c^6*d^2 - 20*A^2*B*a^3*b^3*c^3*d^5 + 4*A^2*B*a^3*b^3*c^5*d^3 + 10*A^2*B*a^4*b^2*c^2*d^6 + 2*A^2*B*a^4*b^2*c^4*d^4 + 52*A*B^2*a*b^5*c^4*d^4 - 28*A*B^2*a*b^5*c^6*d^2 + 4*A*B^2*a^2*b^4*c^7*d - 9*A*B^2*a^4*b^2*c*d^7 + 4*A*B^2*a^5*b*c^2*d^6 - 32*A^2*B*a*b^5*c^3*d^5 + 14*A^2*B*a*b^5*c^5*d^3 - 12*A^2*B*a^3*b^3*c*d^7 - 2*A^2*B*a^5*b*c^3*d^5))/(d^9 - 2*c^2*d^7 + c^4*d^5) + ((b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i)*((32*(A^2*b^4*c^2*d^8 - 2*A^2*b^4*c^4*d^6 + A^2*b^4*c^6*d^4 + 4*B^2*b^4*c^4*d^6 - 8*B^2*b^4*c^6*d^4 + 4*B^2*b^4*c^8*d^2 + 4*B^2*a^2*b^2*c^2*d^8 - 8*B^2*a^2*b^2*c^4*d^6 + 4*B^2*a^2*b^2*c^6*d^4 - 4*A*B*b^4*c^3*d^7 + 8*A*B*b^4*c^5*d^5 - 4*A*B*b^4*c^7*d^3 - 8*B^2*a*b^3*c^3*d^7 + 16*B^2*a*b^3*c^5*d^5 - 8*B^2*a*b^3*c^7*d^3 + 4*A*B*a*b^3*c^2*d^8 - 8*A*B*a*b^3*c^4*d^6 + 4*A*B*a*b^3*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^4*d^5) + ((b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i)*((((32*(c^2*d^12 - 2*c^4*d^10 + c^6*d^8))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^14 - 8*c^3*d^12 + 7*c^5*d^10 - 2*c^7*d^8))/(d^10 - 2*c^2*d^8 + c^4*d^6))*(b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i))/d^3 - (32*(A*a^2*c^5*d^7 - A*a^2*c^3*d^9 - A*b^2*c*d^11 + A*b^2*c^3*d^9 + B*a^2*c^2*d^10 - B*a^2*c^4*d^8 + 2*B*b^2*c^2*d^10 - 3*B*b^2*c^4*d^8 + B*b^2*c^6*d^6 - 2*B*a*b*c*d^11 + 2*A*a*b*c^2*d^10 - 2*A*a*b*c^4*d^8 + 2*B*a*b*c^3*d^9))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)*(2*A*a^2*c^2*d^11 - 2*B*a^2*c*d^12 - 2*A*a^2*c^4*d^9 + 4*A*b^2*c^2*d^11 - 6*A*b^2*c^4*d^9 + 2*A*b^2*c^6*d^7 + 2*B*a^2*c^3*d^10 - 6*B*b^2*c^3*d^10 + 10*B*b^2*c^5*d^8 - 4*B*b^2*c^7*d^6 - 4*A*a*b*c*d^12 + 4*A*a*b*c^3*d^10 + 8*B*a*b*c^2*d^11 - 12*B*a*b*c^4*d^9 + 4*B*a*b*c^6*d^7))/(d^10 - 2*c^2*d^8 + c^4*d^6)))/d^3 - (32*tan(e/2 + (f*x)/2)*(A^2*a^4*c^3*d^8 + 9*A^2*b^4*c^3*d^8 - 8*A^2*b^4*c^5*d^6 + 2*A^2*b^4*c^7*d^4 - 8*B^2*b^4*c^3*d^8 + 29*B^2*b^4*c^5*d^6 - 28*B^2*b^4*c^7*d^4 + 8*B^2*b^4*c^9*d^2 - 2*A^2*b^4*c*d^10 + B^2*a^4*c*d^10 + 4*A^2*a^2*b^2*c^3*d^8 - 2*A^2*a^2*b^2*c^5*d^6 + 42*B^2*a^2*b^2*c^3*d^8 - 36*B^2*a^2*b

^2*c^5*d^6 + 8*B^2*a^2*b^2*c^7*d^4 - 2*A*B*a^4*c^2*d^9 + 8*A*B*b^4*c^2*d^9 - 32*A*B*b^4*c^4*d^7 + 30*A*B*b^4*c^6*d^5 - 8*A*B*b^4*c^8*d^3 - 8*A^2*a*b^3 *c^2*d^9 + 4*A^2*a*b^3*c^4*d^7 + 4*A^2*a^2*b^2*c*d^10 - 4*A^2*a^3*b*c^2*d^9 + 16*B^2*a*b^3*c^2*d^9 - 64*B^2*a*b^3*c^4*d^7 + 60*B^2*a*b^3*c^6*d^5 - 16* B^2*a*b^3*c^8*d^3 - 8*B^2*a^2*b^2*c*d^10 - 8*B^2*a^3*b*c^2*d^9 + 4*B^2*a^3* b*c^4*d^7 - 8*A*B*a*b^3*c*d^10 + 4*A*B*a^3*b*c*d^10 + 48*A*B*a*b^3*c^3*d^8 - 40*A*B*a*b^3*c^5*d^6 + 8*A*B*a*b^3*c^7*d^4 + 8*A*B*a^3*b*c^3*d^8 - 4*A*B* a^3*b*c^5*d^6 - 20*A*B*a^2*b^2*c^2*d^9 + 4*A*B*a^2*b^2*c^4*d^7 + 4*A*B*a^2* b^2*c^6*d^5))/(d^10 - 2*c^2*d^8 + c^4*d^6)))/d^3 - ((b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i)*((32*(A^2*b^4*c^2*d^8 - 2*A^2*b^4*c^4*d^6 + A^2*b^4*c^6*d^4 + 4*B^2*b^4*c^4*d^6 - 8*B^2*b^4*c^6*d^4 + 4*B^2*b^4*c^8*d^2 + 4*B^2*a^2*b^2*c ^2*d^8 - 8*B^2*a^2*b^2*c^4*d^6 + 4*B^2*a^2*b^2*c^6*d^4 - 4*A*B*b^4*c^3*d^7 + 8*A*B*b^4*c^5*d^5 - 4*A*B*b^4*c^7*d^3 - 8*B^2*a*b^3*c^3*d^7 + 16*B^2*a*b^ 3*c^5*d^5 - 8*B^2*a*b^3*c^7*d^3 + 4*A*B*a*b^3*c^2*d^8 - 8*A*B*a*b^3*c^4*d^6 + 4*A*B*a*b^3*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^4*d^5) + ((b*d*(A*b + 2*B*a)* 1i - B*b^2*c*2i)*((32*(A*a^2*c^5*d^7 - A*a^2*c^3*d^9 - A*b^2*c*d^11 + A*b^2 *c^3*d^9 + B*a^2*c^2*d^10 - B*a^2*c^4*d^8 + 2*B*b^2*c^2*d^10 - 3*B*b^2*c^4* d^8 + B*b^2*c^6*d^6 - 2*B*a*b*c*d^11 + 2*A*a*b*c^2*d^10 - 2*A*a*b*c^4*d^8 + 2*B*a*b*c^3*d^9))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (((32*(c^2*d^12 - 2*c^4*d^ 10 + c^6*d^8))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^ 14 - 8*c^3*d^12 + 7*c^5*d^10 - 2*c^7*d^8))/(d^10 - 2*c^2*d^8 + c^4*d^6))*(b *d*(A*b + 2*B*a)*1i - B*b^2*c*2i))/d^3 - (32*tan(e/2 + (f*x)/2)*(2*A*a^2*c^ 2*d^11 - 2*B*a^2*c*d^12 - 2*A*a^2*c^4*d^9 + 4*A*b^2*c^2*d^11 - 6*A*b^2*c^4* d^9 + 2*A*b^2*c^6*d^7 + 2*B*a^2*c^3*d^10 - 6*B*b^2*c^3*d^10 + 10*B*b^2*c^5* d^8 - 4*B*b^2*c^7*d^6 - 4*A*a*b*c*d^12 + 4*A*a*b*c^3*d^10 + 8*B*a*b*c^2*d^1 1 - 12*B*a*b*c^4*d^9 + 4*B*a*b*c^6*d^7))/(d^10 - 2*c^2*d^8 + c^4*d^6)))/d^3 - (32*tan(e/2 + (f*x)/2)*(A^2*a^4*c^3*d^8 + 9*A^2*b^4*c^3*d^8 - 8*A^2*b^4* c^5*d^6 + 2*A^2*b^4*c^7*d^4 - 8*B^2*b^4*c^3*d^8 + 29*B^2*b^4*c^5*d^6 - 28*B ^2*b^4*c^7*d^4 + 8*B^2*b^4*c^9*d^2 - 2*A^2*b^4*c*d^10 + B^2*a^4*c*d^10 + 4* A^2*a^2*b^2*c^3*d^8 - 2*A^2*a^2*b^2*c^5*d^6 + 42*B^2*a^2*b^2*c^3*d^8 - 36*B ^2*a^2*b^2*c^5*d^6 + 8*B^2*a^2*b^2*c^7*d^4 - 2*A*B*a^4*c^2*d^9 + 8*A*B*b^4* c^2*d^9 - 32*A*B*b^4*c^4*d^7 + 30*A*B*b^4*c^6*d^5 - 8*A*B*b^4*c^8*d^3 - 8*A ^2*a*b^3*c^2*d^9 + 4*A^2*a*b^3*c^4*d^7 + 4*A^2*a^2*b^2*c*d^10 - 4*A^2*a^3*b *c^2*d^9 + 16*B^2*a*b^3*c^2*d^9 - 64*B^2*a*b^3*c^4*d^7 + 60*B^2*a*b^3*c^6*d ^5 - 16*B^2*a*b^3*c^8*d^3 - 8*B^2*a^2*b^2*c*d^10 - 8*B^2*a^3*b*c^2*d^9 + 4* B^2*a^3*b*c^4*d^7 - 8*A*B*a*b^3*c*d^10 + 4*A*B*a^3*b*c*d^10 + 48*A*B*a*b^3* c^3*d^8 - 40*A*B*a*b^3*c^5*d^6 + 8*A*B*a*b^3*c^7*d^4 + 8*A*B*a^3*b*c^3*d^8 - 4*A*B*a^3*b*c^5*d^6 - 20*A*B*a^2*b^2*c^2*d^9 + 4*A*B*a^2*b^2*c^4*d^7 + 4* A*B*a^2*b^2*c^6*d^5))/(d^10 - 2*c^2*d^8 + c^4*d^6)))/d^3 + (64*tan(e/2 + (f *x)/2)*(4*A^3*b^6*c^2*d^7 - 16*B^3*b^6*c^9 - 6*A^3*b^6*c^4*d^5 + 2*A^3*b^6* c^6*d^3 - 24*B^3*b^6*c^5*d^4 + 40*B^3*b^6*c^7*d^2 + 2*A^3*a^2*b^4*c^2*d^7 - 2*A^3*a^2*b^4*c^4*d^5 - 96*B^3*a^2*b^4*c^3*d^6 + 144*B^3*a^2*b^4*c^5*d^4 - 48*B^3*a^2*b^4*c^7*d^2 + 48*B^3*a^3*b^3*c^2*d^7 - 64*B^3*a^3*b^3*c^4*d^5 + 16*B^3*a^3*b^3*c^6*d^3 + 8*B^3*a^4*b^2*c^3*d^6 + 24*A*B^2*b^6*c^8*d - 4*A^ 3*a*b^5*c*d^8 + 48*B^3*a*b^5*c^8*d + 40*A*B^2*b^6*c^4*d^5 - 64*A*B^2*b^6*c^

```
6*d^3 - 22*A^2*B*b^6*c^3*d^6 + 34*A^2*B*b^6*c^5*d^4 - 12*A^2*B*b^6*c^7*d^2
+ 4*A^3*a*b^5*c^3*d^6 + 80*B^3*a*b^5*c^4*d^5 - 128*B^3*a*b^5*c^6*d^3 - 8*B^
3*a^4*b^2*c*d^8 + 88*A*B^2*a^2*b^4*c^2*d^7 - 104*A*B^2*a^2*b^4*c^4*d^5 + 16
*A*B^2*a^2*b^4*c^6*d^3 + 8*A*B^2*a^3*b^3*c^3*d^6 + 16*A*B^2*a^3*b^3*c^5*d^4
 + 8*A*B^2*a^4*b^2*c^2*d^7 - 8*A*B^2*a^4*b^2*c^4*d^5 + 10*A^2*B*a^2*b^4*c^3
*d^6 + 8*A^2*B*a^2*b^4*c^5*d^4 + 8*A^2*B*a^3*b^3*c^2*d^7 - 8*A^2*B*a^3*b^3*
c^4*d^5 - 104*A*B^2*a*b^5*c^3*d^6 + 152*A*B^2*a*b^5*c^5*d^4 - 48*A*B^2*a*b^
5*c^7*d^2 - 24*A*B^2*a^3*b^3*c*d^8 + 40*A^2*B*a*b^5*c^2*d^7 - 52*A^2*B*a*b^
5*c^4*d^5 + 12*A^2*B*a*b^5*c^6*d^3 - 18*A^2*B*a^2*b^4*c*d^8))/(d^10 - 2*c^2
*d^8 + c^4*d^6)))*(b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i)*2i)/(d^3*f) + (atan((
((a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(A^2*b^4*c^2*d^8 - 2*A^2*b^4
*c^4*d^6 + A^2*b^4*c^6*d^4 + 4*B^2*b^4*c^4*d^6 - 8*B^2*b^4*c^6*d^4 + 4*B^2*
b^4*c^8*d^2 + 4*B^2*a^2*b^2*c^2*d^8 - 8*B^2*a^2*b^2*c^4*d^6 + 4*B^2*a^2*b^2
*c^6*d^4 - 4*A*B*b^4*c^3*d^7 + 8*A*B*b^4*c^5*d^5 - 4*A*B*b^4*c^7*d^3 - 8*B^
2*a*b^3*c^3*d^7 + 16*B^2*a*b^3*c^5*d^5 - 8*B^2*a*b^3*c^7*d^3 + 4*A*B*a*b^3*
c^2*d^8 - 8*A*B*a*b^3*c^4*d^6 + 4*A*B*a*b^3*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^
4*d^5) - (32*tan(e/2 + (f*x)/2)*(A^2*a^4*c^3*d^8 + 9*A^2*b^4*c^3*d^8 - 8*A^
2*b^4*c^5*d^6 + 2*A^2*b^4*c^7*d^4 - 8*B^2*b^4*c^3*d^8 + 29*B^2*b^4*c^5*d^6
- 28*B^2*b^4*c^7*d^4 + 8*B^2*b^4*c^9*d^2 - 2*A^2*b^4*c*d^10 + B^2*a^4*c*d^1
0 + 4*A^2*a^2*b^2*c^3*d^8 - 2*A^2*a^2*b^2*c^5*d^6 + 42*B^2*a^2*b^2*c^3*d^8
- 36*B^2*a^2*b^2*c^5*d^6 + 8*B^2*a^2*b^2*c^7*d^4 - 2*A*B*a^4*c^2*d^9 + 8*A*
B*b^4*c^2*d^9 - 32*A*B*b^4*c^4*d^7 + 30*A*B*b^4*c^6*d^5 - 8*A*B*b^4*c^8*d^3
 - 8*A^2*a*b^3*c^2*d^9 + 4*A^2*a*b^3*c^4*d^7 + 4*A^2*a^2*b^2*c*d^10 - 4*A^2
*a^3*b*c^2*d^9 + 16*B^2*a*b^3*c^2*d^9 - 64*B^2*a*b^3*c^4*d^7 + 60*B^2*a*b^3
*c^6*d^5 - 16*B^2*a*b^3*c^8*d^3 - 8*B^2*a^2*b^2*c*d^10 - 8*B^2*a^3*b*c^2*d^
9 + 4*B^2*a^3*b*c^4*d^7 - 8*A*B*a*b^3*c*d^10 + 4*A*B*a^3*b*c*d^10 + 48*A*B*
a*b^3*c^3*d^8 - 40*A*B*a*b^3*c^5*d^6 + 8*A*B*a*b^3*c^7*d^4 + 8*A*B*a^3*b*c^
3*d^8 - 4*A*B*a^3*b*c^5*d^6 - 20*A*B*a^2*b^2*c^2*d^9 + 4*A*B*a^2*b^2*c^4*d^
7 + 4*A*B*a^2*b^2*c^6*d^5))/(d^10 - 2*c^2*d^8 + c^4*d^6) + ((a*d - b*c)*(-(
c + d)^3*(c - d)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*A*a^2*c^2*d^11 - 2*B*a
^2*c*d^12 - 2*A*a^2*c^4*d^9 + 4*A*b^2*c^2*d^11 - 6*A*b^2*c^4*d^9 + 2*A*b^2*
c^6*d^7 + 2*B*a^2*c^3*d^10 - 6*B*b^2*c^3*d^10 + 10*B*b^2*c^5*d^8 - 4*B*b^2*
c^7*d^6 - 4*A*a*b*c*d^12 + 4*A*a*b*c^3*d^10 + 8*B*a*b*c^2*d^11 - 12*B*a*b*c
^4*d^9 + 4*B*a*b*c^6*d^7))/(d^10 - 2*c^2*d^8 + c^4*d^6) - (32*(A*a^2*c^5*d^
7 - A*a^2*c^3*d^9 - A*b^2*c*d^11 + A*b^2*c^3*d^9 + B*a^2*c^2*d^10 - B*a^2*c
^4*d^8 + 2*B*b^2*c^2*d^10 - 3*B*b^2*c^4*d^8 + B*b^2*c^6*d^6 - 2*B*a*b*c*d^1
1 + 2*A*a*b*c^2*d^10 - 2*A*a*b*c^4*d^8 + 2*B*a*b*c^3*d^9))/(d^9 - 2*c^2*d^7
 + c^4*d^5) + (((32*(c^2*d^12 - 2*c^4*d^10 + c^6*d^8))/(d^9 - 2*c^2*d^7 + c
^4*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^14 - 8*c^3*d^12 + 7*c^5*d^10 - 2*c^
7*d^8))/(d^10 - 2*c^2*d^8 + c^4*d^6))*(a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1
/2)*(2*A*b*d^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d^2)
)/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3))*(2*A*b*d^3 + B*a*d^3 + 2*B*b*c^3
 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d^2))/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6
*d^3))*(2*A*b*d^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d
^2)*1i)/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3) + ((a*d - b*c)*(-(c + d)^3*
```

(c - d)^3)^(1/2)*((32*(A^2*b^4*c^2*d^8 - 2*A^2*b^4*c^4*d^6 + A^2*b^4*c^6*d^
4 + 4*B^2*b^4*c^4*d^6 - 8*B^2*b^4*c^6*d^4 + 4*B^2*b^4*c^8*d^2 + 4*B^2*a^2*b
^2*c^2*d^8 - 8*B^2*a^2*b^2*c^4*d^6 + 4*B^2*a^2*b^2*c^6*d^4 - 4*A*B*b^4*c^3*
d^7 + 8*A*B*b^4*c^5*d^5 - 4*A*B*b^4*c^7*d^3 - 8*B^2*a*b^3*c^3*d^7 + 16*B^2*
a*b^3*c^5*d^5 - 8*B^2*a*b^3*c^7*d^3 + 4*A*B*a*b^3*c^2*d^8 - 8*A*B*a*b^3*c^4
*d^6 + 4*A*B*a*b^3*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^4*d^5) - (32*tan(e/2 + (f
x)/2)(A^2*a^4*c^3*d^8 + 9*A^2*b^4*c^3*d^8 - 8*A^2*b^4*c^5*d^6 + 2*A^2*b^4
*c^7*d^4 - 8*B^2*b^4*c^3*d^8 + 29*B^2*b^4*c^5*d^6 - 28*B^2*b^4*c^7*d^4 + 8*
B^2*b^4*c^9*d^2 - 2*A^2*b^4*c*d^10 + B^2*a^4*c*d^10 + 4*A^2*a^2*b^2*c^3*d^8
 - 2*A^2*a^2*b^2*c^5*d^6 + 42*B^2*a^2*b^2*c^3*d^8 - 36*B^2*a^2*b^2*c^5*d^6
+ 8*B^2*a^2*b^2*c^7*d^4 - 2*A*B*a^4*c^2*d^9 + 8*A*B*b^4*c^2*d^9 - 32*A*B*b^
4*c^4*d^7 + 30*A*B*b^4*c^6*d^5 - 8*A*B*b^4*c^8*d^3 - 8*A^2*a*b^3*c^2*d^9 +
4*A^2*a*b^3*c^4*d^7 + 4*A^2*a^2*b^2*c*d^10 - 4*A^2*a^3*b*c^2*d^9 + 16*B^2*a
*b^3*c^2*d^9 - 64*B^2*a*b^3*c^4*d^7 + 60*B^2*a*b^3*c^6*d^5 - 16*B^2*a*b^3*c
^8*d^3 - 8*B^2*a^2*b^2*c*d^10 - 8*B^2*a^3*b*c^2*d^9 + 4*B^2*a^3*b*c^4*d^7 -
 8*A*B*a*b^3*c*d^10 + 4*A*B*a^3*b*c*d^10 + 48*A*B*a*b^3*c^3*d^8 - 40*A*B*a*
b^3*c^5*d^6 + 8*A*B*a*b^3*c^7*d^4 + 8*A*B*a^3*b*c^3*d^8 - 4*A*B*a^3*b*c^5*d
^6 - 20*A*B*a^2*b^2*c^2*d^9 + 4*A*B*a^2*b^2*c^4*d^7 + 4*A*B*a^2*b^2*c^6*d^5
))/(d^10 - 2*c^2*d^8 + c^4*d^6) + ((a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)
((32(A*a^2*c^5*d^7 - A*a^2*c^3*d^9 - A*b^2*c*d^11 + A*b^2*c^3*d^9 + B*a^2
*c^2*d^10 - B*a^2*c^4*d^8 + 2*B*b^2*c^2*d^10 - 3*B*b^2*c^4*d^8 + B*b^2*c^6*
d^6 - 2*B*a*b*c*d^11 + 2*A*a*b*c^2*d^10 - 2*A*a*b*c^4*d^8 + 2*B*a*b*c^3*d^9
))/(d^9 - 2*c^2*d^7 + c^4*d^5) - (32*tan(e/2 + (f*x)/2)*(2*A*a^2*c^2*d^11 -
 2*B*a^2*c*d^12 - 2*A*a^2*c^4*d^9 + 4*A*b^2*c^2*d^11 - 6*A*b^2*c^4*d^9 + 2*
A*b^2*c^6*d^7 + 2*B*a^2*c^3*d^10 - 6*B*b^2*c^3*d^10 + 10*B*b^2*c^5*d^8 - 4*
B*b^2*c^7*d^6 - 4*A*a*b*c*d^12 + 4*A*a*b*c^3*d^10 + 8*B*a*b*c^2*d^11 - 12*B
*a*b*c^4*d^9 + 4*B*a*b*c^6*d^7))/(d^10 - 2*c^2*d^8 + c^4*d^6) + (((32*(c^2*
d^12 - 2*c^4*d^10 + c^6*d^8))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (
f*x)/2)*(3*c*d^14 - 8*c^3*d^12 + 7*c^5*d^10 - 2*c^7*d^8))/(d^10 - 2*c^2*d^8
 + c^4*d^6))*(a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*(2*A*b*d^3 + B*a*d^3
+ 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d^2))/(d^9 - 3*c^2*d^7 + 3*c^
4*d^5 - c^6*d^3))*(2*A*b*d^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d
- 3*B*b*c*d^2))/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3))*(2*A*b*d^3 + B*a*d
^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d^2)*1i)/(d^9 - 3*c^2*d^7
+ 3*c^4*d^5 - c^6*d^3))/((64*(A^3*b^6*c^5*d^3 - 2*A^3*b^6*c^3*d^5 - 4*B^3*b
^6*c^8 + 6*B^3*b^6*c^6*d^2 - 3*A^3*a^2*b^4*c^3*d^5 + A^3*a^2*b^4*c^5*d^3 +
4*A^3*a^3*b^3*c^2*d^6 - A^3*a^4*b^2*c^3*d^5 + 44*B^3*a^2*b^4*c^4*d^4 - 24*B
^3*a^2*b^4*c^6*d^2 - 36*B^3*a^3*b^3*c^3*d^5 + 16*B^3*a^3*b^3*c^5*d^3 + 14*B
^3*a^4*b^2*c^2*d^6 - 4*B^3*a^4*b^2*c^4*d^4 + 8*A*B^2*b^6*c^7*d + 16*B^3*a*b
^5*c^7*d - 2*B^3*a^5*b*c*d^7 - 13*A*B^2*b^6*c^5*d^3 + 9*A^2*B*b^6*c^4*d^4 -
 5*A^2*B*b^6*c^6*d^2 + 6*A^3*a*b^5*c^2*d^6 - 2*A^3*a*b^5*c^4*d^4 - 4*A^3*a^
2*b^4*c*d^7 - 26*B^3*a*b^5*c^5*d^3 - 74*A*B^2*a^2*b^4*c^3*d^5 + 24*A*B^2*a^
2*b^4*c^5*d^3 + 44*A*B^2*a^3*b^3*c^2*d^6 + 8*A*B^2*a^3*b^3*c^4*d^4 - 8*A*B^
2*a^3*b^3*c^6*d^2 - 16*A*B^2*a^4*b^2*c^3*d^5 + 4*A*B^2*a^4*b^2*c^5*d^3 + 35
*A^2*B*a^2*b^4*c^2*d^6 + A^2*B*a^2*b^4*c^4*d^4 - 4*A^2*B*a^2*b^4*c^6*d^2 -

20*A^2*B*a^3*b^3*c^3*d^5 + 4*A^2*B*a^3*b^3*c^5*d^3 + 10*A^2*B*a^4*b^2*c^2*d
^6 + 2*A^2*B*a^4*b^2*c^4*d^4 + 52*A*B^2*a*b^5*c^4*d^4 - 28*A*B^2*a*b^5*c^6*
d^2 + 4*A*B^2*a^2*b^4*c^7*d - 9*A*B^2*a^4*b^2*c*d^7 + 4*A*B^2*a^5*b*c^2*d^6
 - 32*A^2*B*a*b^5*c^3*d^5 + 14*A^2*B*a*b^5*c^5*d^3 - 12*A^2*B*a^3*b^3*c*d^7
 - 2*A^2*B*a^5*b*c^3*d^5))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (64*tan(e/2 + (f*x
)/2)*(4*A^3*b^6*c^2*d^7 - 16*B^3*b^6*c^9 - 6*A^3*b^6*c^4*d^5 + 2*A^3*b^6*c^
6*d^3 - 24*B^3*b^6*c^5*d^4 + 40*B^3*b^6*c^7*d^2 + 2*A^3*a^2*b^4*c^2*d^7 - 2
*A^3*a^2*b^4*c^4*d^5 - 96*B^3*a^2*b^4*c^3*d^6 + 144*B^3*a^2*b^4*c^5*d^4 - 4
8*B^3*a^2*b^4*c^7*d^2 + 48*B^3*a^3*b^3*c^2*d^7 - 64*B^3*a^3*b^3*c^4*d^5 + 1
6*B^3*a^3*b^3*c^6*d^3 + 8*B^3*a^4*b^2*c^3*d^6 + 24*A*B^2*b^6*c^8*d - 4*A^3*
a*b^5*c*d^8 + 48*B^3*a*b^5*c^8*d + 40*A*B^2*b^6*c^4*d^5 - 64*A*B^2*b^6*c^6*
d^3 - 22*A^2*B*b^6*c^3*d^6 + 34*A^2*B*b^6*c^5*d^4 - 12*A^2*B*b^6*c^7*d^2 +
4*A^3*a*b^5*c^3*d^6 + 80*B^3*a*b^5*c^4*d^5 - 128*B^3*a*b^5*c^6*d^3 - 8*B^3*
a^4*b^2*c*d^8 + 88*A*B^2*a^2*b^4*c^2*d^7 - 104*A*B^2*a^2*b^4*c^4*d^5 + 16*A
*B^2*a^2*b^4*c^6*d^3 + 8*A*B^2*a^3*b^3*c^3*d^6 + 16*A*B^2*a^3*b^3*c^5*d^4 +
 8*A*B^2*a^4*b^2*c^2*d^7 - 8*A*B^2*a^4*b^2*c^4*d^5 + 10*A^2*B*a^2*b^4*c^3*d
^6 + 8*A^2*B*a^2*b^4*c^5*d^4 + 8*A^2*B*a^3*b^3*c^2*d^7 - 8*A^2*B*a^3*b^3*c^
4*d^5 - 104*A*B^2*a*b^5*c^3*d^6 + 152*A*B^2*a*b^5*c^5*d^4 - 48*A*B^2*a*b^5*
c^7*d^2 - 24*A*B^2*a^3*b^3*c*d^8 + 40*A^2*B*a*b^5*c^2*d^7 - 52*A^2*B*a*b^5*
c^4*d^5 + 12*A^2*B*a*b^5*c^6*d^3 - 18*A^2*B*a^2*b^4*c*d^8))/(d^10 - 2*c^2*d
^8 + c^4*d^6) + ((a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(A^2*b^4*c^2
*d^8 - 2*A^2*b^4*c^4*d^6 + A^2*b^4*c^6*d^4 + 4*B^2*b^4*c^4*d^6 - 8*B^2*b^4*
c^6*d^4 + 4*B^2*b^4*c^8*d^2 + 4*B^2*a^2*b^2*c^2*d^8 - 8*B^2*a^2*b^2*c^4*d^6
 + 4*B^2*a^2*b^2*c^6*d^4 - 4*A*B*b^4*c^3*d^7 + 8*A*B*b^4*c^5*d^5 - 4*A*B*b^
4*c^7*d^3 - 8*B^2*a*b^3*c^3*d^7 + 16*B^2*a*b^3*c^5*d^5 - 8*B^2*a*b^3*c^7*d^
3 + 4*A*B*a*b^3*c^2*d^8 - 8*A*B*a*b^3*c^4*d^6 + 4*A*B*a*b^3*c^6*d^4))/(d^9
 - 2*c^2*d^7 + c^4*d^5) - (32*tan(e/2 + (f*x)/2)*(A^2*a^4*c^3*d^8 + 9*A^2*b^
4*c^3*d^8 - 8*A^2*b^4*c^5*d^6 + 2*A^2*b^4*c^7*d^4 - 8*B^2*b^4*c^3*d^8 + 29*
B^2*b^4*c^5*d^6 - 28*B^2*b^4*c^7*d^4 + 8*B^2*b^4*c^9*d^2 - 2*A^2*b^4*c*d^10
 + B^2*a^4*c*d^10 + 4*A^2*a^2*b^2*c^3*d^8 - 2*A^2*a^2*b^2*c^5*d^6 + 42*B^2*
a^2*b^2*c^3*d^8 - 36*B^2*a^2*b^2*c^5*d^6 + 8*B^2*a^2*b^2*c^7*d^4 - 2*A*B*a^
4*c^2*d^9 + 8*A*B*b^4*c^2*d^9 - 32*A*B*b^4*c^4*d^7 + 30*A*B*b^4*c^6*d^5 - 8
*A*B*b^4*c^8*d^3 - 8*A^2*a*b^3*c^2*d^9 + 4*A^2*a*b^3*c^4*d^7 + 4*A^2*a^2*b^
2*c*d^10 - 4*A^2*a^3*b*c^2*d^9 + 16*B^2*a*b^3*c^2*d^9 - 64*B^2*a*b^3*c^4*d^
7 + 60*B^2*a*b^3*c^6*d^5 - 16*B^2*a*b^3*c^8*d^3 - 8*B^2*a^2*b^2*c*d^10 - 8*
B^2*a^3*b*c^2*d^9 + 4*B^2*a^3*b*c^4*d^7 - 8*A*B*a*b^3*c*d^10 + 4*A*B*a^3*b*
c*d^10 + 48*A*B*a*b^3*c^3*d^8 - 40*A*B*a*b^3*c^5*d^6 + 8*A*B*a*b^3*c^7*d^4
+ 8*A*B*a^3*b*c^3*d^8 - 4*A*B*a^3*b*c^5*d^6 - 20*A*B*a^2*b^2*c^2*d^9 + 4*A*
B*a^2*b^2*c^4*d^7 + 4*A*B*a^2*b^2*c^6*d^5))/(d^10 - 2*c^2*d^8 + c^4*d^6) +
((a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*A*a^2*
c^2*d^11 - 2*B*a^2*c*d^12 - 2*A*a^2*c^4*d^9 + 4*A*b^2*c^2*d^11 - 6*A*b^2*c^
4*d^9 + 2*A*b^2*c^6*d^7 + 2*B*a^2*c^3*d^10 - 6*B*b^2*c^3*d^10 + 10*B*b^2*c^
5*d^8 - 4*B*b^2*c^7*d^6 - 4*A*a*b*c*d^12 + 4*A*a*b*c^3*d^10 + 8*B*a*b*c^2*d
^11 - 12*B*a*b*c^4*d^9 + 4*B*a*b*c^6*d^7))/(d^10 - 2*c^2*d^8 + c^4*d^6) - (
32*(A*a^2*c^5*d^7 - A*a^2*c^3*d^9 - A*b^2*c*d^11 + A*b^2*c^3*d^9 + B*a^2*c^

2*d^10 - B*a^2*c^4*d^8 + 2*B*b^2*c^2*d^10 - 3*B*b^2*c^4*d^8 + B*b^2*c^6*d^6 - 2*B*a*b*c*d^11 + 2*A*a*b*c^2*d^10 - 2*A*a*b*c^4*d^8 + 2*B*a*b*c^3*d^9))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (((32*(c^2*d^12 - 2*c^4*d^10 + c^6*d^8))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^14 - 8*c^3*d^12 + 7*c^5*d^10 - 2*c^7*d^8))/(d^10 - 2*c^2*d^8 + c^4*d^6))*(a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*(2*A*b*d^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d^2))/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3))*(2*A*b*d^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d^2))/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3))*(2*A*b*d^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d^2))/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3) - ((a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(A^2*b^4*c^2*d^8 - 2*A^2*b^4*c^4*d^6 + A^2*b^4*c^6*d^4 + 4*B^2*b^4*c^4*d^6 - 8*B^2*b^4*c^6*d^4 + 4*B^2*b^4*c^8*d^2 + 4*B^2*a^2*b^2*c^2*d^8 - 8*B^2*a^2*b^2*c^4*d^6 + 4*B^2*a^2*b^2*c^6*d^4 - 4*A*B*b^4*c^3*d^7 + 8*A*B*b^4*c^5*d^5 - 4*A*B*b^4*c^7*d^3 - 8*B^2*a*b^3*c^3*d^7 + 16*B^2*a*b^3*c^5*d^5 - 8*B^2*a*b^3*c^7*d^3 + 4*A*B*a*b^3*c^2*d^8 - 8*A*B*a*b^3*c^4*d^6 + 4*A*B*a*b^3*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^4*d^5) - (32*tan(e/2 + (f*x)/2)*(A^2*a^4*c^3*d^8 + 9*A^2*b^4*c^3*d^8 - 8*A^2*b^4*c^5*d^6 + 2*A^2*b^4*c^7*d^4 - 8*B^2*b^4*c^3*d^8 + 29*B^2*b^4*c^5*d^6 - 28*B^2*b^4*c^7*d^4 + 8*B^2*b^4*c^9*d^2 - 2*A^2*b^4*c*d^10 + B^2*a^4*c*d^10 + 4*A^2*a^2*b^2*c^3*d^8 - 2*A^2*a^2*b^2*c^5*d^6 + 42*B^2*a^2*b^2*c^3*d^8 - 36*B^2*a^2*b^2*c^5*d^6 + 8*B^2*a^2*b^2*c^7*d^4 - 2*A*B*a^4*c^2*d^9 + 8*A*B*b^4*c^2*d^9 - 32*A*B*b^4*c^4*d^7 + 30*A*B*b^4*c^6*d^5 - 8*A*B*b^4*c^8*d^3 - 8*A^2*a*b^3*c^2*d^9 + 4*A^2*a*b^3*c^4*d^7 + 4*A^2*a^2*b^2*c*d^10 - 4*A^2*a^3*b*c^2*d^9 + 16*B^2*a*b^3*c^2*d^9 - 64*B^2*a*b^3*c^4*d^7 + 60*B^2*a*b^3*c^6*d^5 - 16*B^2*a*b^3*c^8*d^3 - 8*B^2*a^2*b^2*c*d^10 - 8*B^2*a^3*b*c^2*d^9 + 4*B^2*a^3*b*c^4*d^7 - 8*A*B*a*b^3*c*d^10 + 4*A*B*a^3*b*c*d^10 + 48*A*B*a*b^3*c^3*d^8 - 40*A*B*a*b^3*c^5*d^6 + 8*A*B*a*b^3*c^7*d^4 + 8*A*B*a^3*b*c^3*d^8 - 4*A*B*a^3*b*c^5*d^6 - 20*A*B*a^2*b^2*c^2*d^9 + 4*A*B*a^2*b^2*c^4*d^7 + 4*A*B*a^2*b^2*c^6*d^5))/(d^10 - 2*c^2*d^8 + c^4*d^6) + ((a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*((32*(A*a^2*c^5*d^7 - A*a^2*c^3*d^9 - A*b^2*c*d^11 + A*b^2*c^3*d^9 + B*a^2*c^2*d^10 - B*a^2*c^4*d^8 + 2*B*b^2*c^2*d^10 - 3*B*b^2*c^4*d^8 + B*b^2*c^6*d^6 - 2*B*a*b*c*d^11 + 2*A*a*b*c^2*d^10 - 2*A*a*b*c^4*d^8 + 2*B*a*b*c^3*d^9))/(d^9 - 2*c^2*d^7 + c^4*d^5) - (32*tan(e/2 + (f*x)/2)*(2*A*a^2*c^2*d^11 - 2*B*a^2*c*d^12 - 2*A*a^2*c^4*d^9 + 4*A*b^2*c^2*d^11 - 6*A*b^2*c^4*d^9 + 2*A*b^2*c^6*d^7 + 2*B*a^2*c^3*d^10 - 6*B*b^2*c^3*d^10 + 10*B*b^2*c^5*d^8 - 4*B*b^2*c^7*d^6 - 4*A*a*b*c*d^12 + 4*A*a*b*c^3*d^10 + 8*B*a*b*c^2*d^11 - 12*B*a*b*c^4*d^9 + 4*B*a*b*c^6*d^7))/(d^10 - 2*c^2*d^8 + c^4*d^6) + (((32*(c^2*d^12 - 2*c^4*d^10 + c^6*d^8))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^14 - 8*c^3*d^12 + 7*c^5*d^10 - 2*c^7*d^8))/(d^10 - 2*c^2*d^8 + c^4*d^6))*(a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*(2*A*b*d^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d^2))/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3))*(2*A*b*d^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d^2))/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3))*(2*A*b*d^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d^2))/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3)))*(a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)

```
*(2*A*b*d^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d^2)*2i
)/(f*(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.353 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=840

$$\frac{2(Ab-aB)(bc-ad)\sqrt{c+d\sin(e+fx)}\,\cos(e+fx)}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}}-\frac{\left(2Ab(bc-ad)-B\left(-3da^2+2bca+b^2d\right)\right)\sqrt{c+d\sin(e+}}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}}$$

```
[Out] (c-d)*(-2*A*a*b*d+2*A*b^2*c+3*B*a^2*d-2*B*a*b*c-B*b^2*d)*EllipticE((a+b)^(1
/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/
(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1
-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)
/(a+b*sin(f*x+e)))^(1/2)/(a-b)/b^2/(-a*d+b*c)/f/(a+b)^(1/2)+(2*A*b*d-3*B*a*
d+3*B*b*c)*EllipticPi((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*s
in(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e
)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f
*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/b^3/
f/(a+b)^(1/2)+(2*A*b*(b*(c-2*d)+a*d)-B*(3*a^2*d-6*a*b*d+b^2*(2*c+d)))*Ellip
ticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),
((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((
-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(
f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(a-b)/b^3/f/(c+d)^(1/2)+2*(A*b-B*a)*(
-a*d+b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/b/(a^2-b^2)/f/(a+b*sin(f*x+e))^
(1/2)-(2*A*b*(-a*d+b*c)-B*(-3*a^2*d+2*a*b*c+b^2*d))*cos(f*x+e)*(c+d*sin(f*x
+e))^(1/2)/b/(a^2-b^2)/f/(a+b*sin(f*x+e))^(1/2)
```

Rubi [A] time = 3.16, antiderivative size = 840, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.180, Rules used = {2989, 3061, 3053, 2811, 2998, 2818, 2996}

$$\frac{2(Ab-aB)(bc-ad)\sqrt{c+d\sin(e+fx)}\,\cos(e+fx)}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}}-\frac{\left(2Ab(bc-ad)-B\left(-3da^2+2bca+b^2d\right)\right)\sqrt{c+d\sin(e+}}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2))/(a + b*Sin[e + f*x])^
(3/2),x]
```

```
[Out] ((c - d)*Sqrt[c + d]*(2*A*b^2*c - 2*a*b*B*c - 2*a*A*b*d + 3*a^2*B*d - b^2*B
*d)*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sq
rt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]
*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*S
```

```
qrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b
*Sin[e + f*x]))/((a - b)*b^2*Sqrt[a + b]*(b*c - a*d)*f) + (Sqrt[c + d]*(3*b
*B*c + 2*A*b*d - 3*a*B*d)*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[
a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], (
(a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - S
in[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e
 + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(b^3*Sqrt[a
 + b]*f) + (2*(A*b - a*B)*(b*c - a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]]
)/(b*(a^2 - b^2)*f*Sqrt[a + b*Sin[e + f*x]]) - ((2*A*b*(b*c - a*d) - B*(2*a
*b*c - 3*a^2*d + b^2*d))*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(b*(a^2 - b
^2)*f*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[a + b]*(2*A*b*(b*(c - 2*d) + a*d) -
 B*(3*a^2*d - 6*a*b*d + b^2*(2*c + d)))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[
a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c -
 d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/
((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a
 - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/((a - b)*b^3*Sqrt[c + d
]*f)
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
```

```
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
```

```
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}} - \frac{2\int \frac{\frac{1}{2}\left(a^2 B\right.}{}}{}$$

$$= \frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}} - \frac{\left(2Ab(bc\right.}{}$$

$$= \frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}} - \frac{\left(2Ab(bc\right.}{}$$

$$= \frac{\sqrt{c+d}\,(3bBc+2Abd-3aBd)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+f}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+f}}\right.\right.}{}$$

$$= \frac{(c-d)\sqrt{c+d}\,\left(2Ab(bc-ad)-B\left(2abc-3a^2d+b^2d\right)\right)E\left(\sin^{-1}\right.}{}$$

Mathematica [B] time = 6.75, size = 2042, normalized size = 2.43

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2))/(a + b*Sin[e +
f*x])^(3/2),x]
```

```
[Out] (-2*(A*b^2*c*Cos[e + f*x] - a*b*B*c*Cos[e + f*x] - a*A*b*d*Cos[e + f*x] + a
^2*B*d*Cos[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(b*(-a^2 + b^2)*f*Sqrt[a + b
*Sin[e + f*x]]) + ((-4*(-(b*c) + a*d)*(2*a*A*b*c^2 - 2*b^2*B*c^2 - 2*A*b^2*
c*d + 2*a*b*B*c*d + a^2*B*d^2 - b^2*B*d^2)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f
*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2
```

```
]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a
+ b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(
-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*
Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*
(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*
d)*(2*A*b^2*c^2 - 2*a*b*B*c^2 + 4*a^2*B*c*d - 4*b^2*B*c*d - 2*A*b^2*d^2 + 2
*a*b*B*d^2)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF
[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(
b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*
Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*
Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c
 + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/
(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[
(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(
-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*
Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*
d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
 + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) +
2*(-2*A*b^2*c*d + 2*a*b*B*c*d + 2*a*A*b*d^2 - 3*a^2*B*d^2 + b^2*B*d^2)*((Co
s[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(
a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a
- b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]]
, (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt
[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[
e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f
*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a
*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sq
rt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d
)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e +
Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*
x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
 + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c
 + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2
)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Cs
c[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2
*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^
4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*
c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))
/(b*d)))/(2*(a - b)*b*(a + b)*f)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   Curve
 not irreducible after change of variable 0 -> infinity
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) +
 a)^(3/2), x)
```

maple [B] time = 100.98, size = 6776580, normalized size = 8067.36

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) +
 a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(a+b\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2))/(a + b*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2))/(a + b*sin(e + f*x))^
(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}{\left(a+b\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))**(3/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**(3/2)/(a + b*sin(e + f*
x))**(3/2), x)
```

3.354 $\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=630

$$\frac{2\sqrt{a+b}\,(c-d)(Ab-aB)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{}}{\sqrt{a+b}\,\sqrt{}}\right.\right.}{bf(a-b)\sqrt{c+d}\,(bc-ad)}$$

```
[Out] 2*(A*b-B*a)*(c-d)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/
(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin
(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1
/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/(a-b)/b/(-a*d+
b*c)/f/(a+b)^(1/2)+2*(A*b-B*a)*(c-d)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))
^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*
sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c
+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(
1/2)/(a-b)/b/(-a*d+b*c)/f/(c+d)^(1/2)+2*B*EllipticPi((c+d)^(1/2)*(a+b*sin(f
*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),(a+b)*d/b/(c+d),((a+b)*(c-d
)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(
1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-
b)/(c+d*sin(f*x+e)))^(1/2)/b^2/f/(c+d)^(1/2)
```

Rubi [A] time = 0.89, antiderivative size = 630, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {2992, 2811, 2795, 2818, 2996}

$$\frac{2\sqrt{a+b}\,(c-d)(Ab-aB)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{}}{\sqrt{a+b}\,\sqrt{}}\right.\right.}{bf(a-b)\sqrt{c+d}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(a + b*Sin[e + f*x])^(3
/2),x]
```

```
[Out] (2*(A*b - a*B)*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d
*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/
((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d
)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/((a - b)*b*Sqrt[a + b]*(b*c
- a*d)*f) + (2*Sqrt[a + b]*(A*b - a*B)*(c - d)*EllipticF[ArcSin[(Sqrt[c + d
]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a +
b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e +
```

```
f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x
]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/((a - b)*b*Sqrt[
c + d]*(b*c - a*d)*f) + (2*Sqrt[a + b]*B*EllipticPi[((a + b)*d)/(b*(c + d))
, ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin
[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c
- a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a
*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x
]))/(b^2*Sqrt[c + d]*f)
```

Rule 2795

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2992

```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)]])/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> D
ist[B/b, Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] + Di
st[(A*b - a*B)/b, Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(3/2),
```

```
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx = \frac{B\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}}\,dx}{b} + \frac{(Ab-aB)\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx}{b}$$

$$= \frac{2\sqrt{a+b}\,B\Pi\left(\frac{(a+b)d}{b(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}\right)\Big|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+ \cdots}{b^2\sqrt{c \cdots}}$$

$$= \frac{2(Ab-aB)(c-d)\sqrt{c+d}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)\Big|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right) \cdots}{(a-b)b\sqrt{\cdots}}$$

Mathematica [B] time = 10.19, size = 1901, normalized size = 3.02

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(a + b*Sin[e + f*
x])^(3/2),x]
```

```
[Out] (-2*(-(A*b*Cos[e + f*x]) + a*B*Cos[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/((a^
2 - b^2)*f*Sqrt[a + b*Sin[e + f*x]]) + ((-4*(a*A*c - b*B*c)*(-(b*c) + a*d)*
Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[(
(-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/S
qrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2
```

```
 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))
/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f
*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d
*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(A*b*c - a*B*c + a*A*d - b*B*d)*((Sqrt[(
(c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a -
b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]
], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x
)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*
c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/
(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e
 + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi
[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]
^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a +
 b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-
e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*C
sc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d
*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(-(A*b*d) + a*B*d)
*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (S
qrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqr
t[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a +
 b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d
*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b
*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[
e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*
c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcS
in[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
+ a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(
-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e
 + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*
Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*S
qrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)
/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a -
b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]
], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x
)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*
c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/
(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]])))/(b*d)))/((a - b)*(a + b)*f)
```

fricas [F] time = 2.23, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{b\sin\left(fx+e\right)+a}\,\sqrt{d\sin\left(fx+e\right)+c}}{b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
 + c)/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{d\sin\left(fx+e\right)+c}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a
)^(3/2), x)
```

maple [B] time = 159.53, size = 3150101, normalized size = 5000.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{d\sin\left(fx+e\right)+c}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a
)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x))^
(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x))^
(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A + B \sin{\left(e + f x \right)}\right) \sqrt{c + d \sin{\left(e + f x \right)}}}{\left(a + b \sin{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))**(3/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))*sqrt(c + d*sin(e + f*x))/(a + b*sin(e + f*x))
**(3/2), x)
```

3.355 $\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=417

$$\frac{2\sqrt{a+b}\,(A-B)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+}}\right.\right.}{f(a-b)\sqrt{c+d}\,(bc-ad)}$$

```
[Out] 2*(A*b-B*a)*(c-d)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/
(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin
(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1
/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/(a-b)/(-a*d+b*
c)^2/f/(a+b)^(1/2)+2*(A-B)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+
b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)
*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x
+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(a-b)
/(-a*d+b*c)/f/(c+d)^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2998, 2818, 2996}

$$\frac{2\sqrt{a+b}\,(A-B)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sin(e+}}{\sqrt{a+b}\,\sqrt{c+d\sin(e+}}\right.\right.}{f(a-b)\sqrt{c+d}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]),x]
```

```
[Out] (2*(A*b - a*B)*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d
*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/
((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d
)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/((a - b)*Sqrt[a + b]*(b*c -
a*d)^2*f) + (2*Sqrt[a + b]*(A - B)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b
*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/
((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a +
 b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)
*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/((a - b)*Sqrt[c + d]*(b*c -
a*d)*f)
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx = \frac{(A-B)\int \frac{1}{\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}}\,dx}{a-b} - \frac{(Ab-aB)\int \frac{\cdots}{(a+b\sin(\cdots}}{a\cdots}$$

$$= \frac{2(Ab-aB)(c-d)\sqrt{c+d}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\,\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\,\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+\cdots}{(a+b)(c-\cdots}\right.}{(a-b)\cdots}$$

Mathematica [B] time = 6.56, size = 1949, normalized size = 4.67

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
 + f*x]]),x]
```

```
[Out] (-2*(A*b^2*Cos[e + f*x] - a*b*B*Cos[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/((a
^2 - b^2)*(-(b*c) + a*d)*f*Sqrt[a + b*Sin[e + f*x]]) + ((-4*(-(b*c) + a*d)*
(-(a*A*b*c) + b^2*B*c + a^2*A*d - A*b^2*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f
*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2
]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a
+ b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(
-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*
Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*
(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*
d)*(-(A*b^2*c) + a*b*B*c - a*A*b*d + a^2*B*d)*((Sqrt[((c + d)*Cot[(-e + Pi/
2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f
*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))
/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*
Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a
- b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a
+ b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c
 + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a +
 b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x
]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e
+ f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*
(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/
2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f
*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(A*b^2*d - a*b*B*d)*((Cos[e + f*x]*Sqrt
[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]
*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*S
in[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) +
a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(
-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[
(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*
(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c +
d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Cs
c[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2
*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^
4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*
c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
```

```
x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Ell
ipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 -
f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d)
)/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)
*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a
 - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a
 + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/((a -
b)*(a + b)*(-(b*c) + a*d)*f)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{b\sin\left(fx+e\right)+a}\,\sqrt{d\sin\left(fx+e\right)+c}}{2\,abd-\left(b^2c+2\,abd\right)\cos\left(fx+e\right)^2+\left(a^2+b^2\right)c-\left(b^2d\cos\left(fx+e\right)^2-2\,abc-\left(a^2+b^2\right)d\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c)/(2*a*b*d - (b^2*c + 2*a*b*d)*cos(f*x + e)^2 + (a^2 + b^2)*c - (b^2*d*c
os(f*x + e)^2 - 2*a*b*c - (a^2 + b^2)*d)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x +
 e) + c)), x)
```

maple [B] time = 1.85, size = 99082, normalized size = 237.61

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x +
 e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(
1/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(
1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\sin\left(e+fx\right)}{\left(a+b\sin\left(e+fx\right)\right)^{\frac{3}{2}}\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))/((a + b*sin(e + f*x))**(3/2)*sqrt(c + d*sin(e
 + f*x))), x)
```

3.356 $\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=544

$$-\frac{2\sec(e+fx)\left(A\left(a^2d^2+b^2\left(c^2-2d^2\right)\right)-B\left(a^2cd+ab\left(c^2-d^2\right)-b^2cd\right)\right)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx)}{(a+b)(c+d\sin(e+fx)}}}{f\sqrt{a+b}\,(c-d)\sqrt{c+d}\,(bc-ad)^3}$$

```
[Out] -2*(A*(a^2*d^2+b^2*(c^2-2*d^2))-B*(a^2*c*d-b^2*c*d+a*b*(c^2-d^2)))*Elliptic
E((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a
+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-si
n(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(
c+d*sin(f*x+e)))^(1/2)/(c-d)/(-a*d+b*c)^3/f/(a+b)^(1/2)/(c+d)^(1/2)+2*(-A*a
*d+A*b*c-2*A*b*d+B*a*d+B*b*c)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/
(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x
+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/
2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)/(-a*d+b*
c)^2/f/(a+b)^(1/2)/(c+d)^(1/2)+2*b*(A*b-B*a)*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c
)/f/(a+b*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.38, antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3000, 2998, 2818, 2996}

$$\frac{2\sec(e+fx)\left(a^2(-A)d^2+a^2Bcd+abB\left(c^2-d^2\right)-Ab^2\left(c^2-2d^2\right)-b^2Bcd\right)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+f}{(a+b)(c+d\sin(e+f}}}{f\sqrt{a+b}\,(c-d)\sqrt{c+d}\,(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(
3/2)),x]
```

```
[Out] (2*b*(A*b - a*B)*Cos[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]]) + (2*(a^2*B*c*d - b^2*B*c*d - a^2*A*d^2 -
 A*b^2*(c^2 - 2*d^2) + a*b*B*(c^2 - d^2))*EllipticE[ArcSin[(Sqrt[c + d]*Sqr
t[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c
 - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x])
)/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/(
(a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(Sqrt[a + b]*(c - d)*
Sqrt[c + d]*(b*c - a*d)^3*f) + (2*(A*b*c + b*B*c - a*A*d - 2*A*b*d + a*B*d)
*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[
c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sq
```

```
rt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(
((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Si
n[e + f*x]))/(Sqrt[a + b]*(c - d)*Sqrt[c + d]*(b*c - a*d)^2*f)
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
```

```
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2b(Ab-aB)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}} -$$

$$= \frac{2b(Ab-aB)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}} +$$

$$= \frac{2b(Ab-aB)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f\sqrt{a+b\sin(e+fx)}\,\sqrt{c+d\sin(e+fx)}} +$$

Mathematica [B] time = 7.43, size = 2266, normalized size = 4.17

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f
*x])^(3/2)),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((2*(A*b^3*Cos[e + f*x]
- a*b^2*B*Cos[e + f*x]))/((a^2 - b^2)*(-(b*c) + a*d)^2*(a + b*Sin[e + f*x])
) - (2*(B*c*d^2*Cos[e + f*x] - A*d^3*Cos[e + f*x]))/((b*c - a*d)^2*(c^2 - d
^2)*(c + d*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(a*A*b^2*c^3 - b^3*B*c^
3 - 2*a^2*A*b*c^2*d + 2*A*b^3*c^2*d + a^3*A*c*d^2 - 2*a*A*b^2*c*d^2 + b^3*B
*c*d^2 + 2*a^2*A*b*d^3 - 2*A*b^3*d^3 - a^3*B*d^3 + a*b^2*B*d^3)*Sqrt[((c +
d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Cs
c[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2
*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^
4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*
c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]) - 4*(-(b*c) + a*d)*(A*b^3*c^3 - a*b^2*B*c^3 + a*A*b^2*c^2*d - 2*a^2*b*
B*c^2*d + b^3*B*c^2*d + a^2*A*b*c*d^2 - 2*A*b^3*c*d^2 - a^3*B*c*d^2 + 2*a*b
```

```
^2*B*c*d^2 + a^3*A*d^3 - 2*a*A*b^2*d^3 + a^2*b*B*d^3)*((Sqrt[((c + d)*Cot[(
-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e +
Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c)
 + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[(
(c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sq
rt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d
)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (
Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*
d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin
[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))
]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*
x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2
 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*S
in[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(-(A*b^3*c^2*d) + a*b^2*B*c^2*d
 + a^2*b*B*c*d^2 - b^3*B*c*d^2 - a^2*A*b*d^3 + 2*A*b^3*d^3 - a*b^2*B*d^3)*(
(Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqr
t[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[
(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b
)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*S
qrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*S
in[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e
+ f*x]))/((c + d)*(a + b*Sin[e + f*x]))]]) - (2*(-(b*c) + a*d)*((((a + b)*c
+ a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin
[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e
 + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
 f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Si
n[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqr
t[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2
]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)
*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]],
 (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/
2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c)
 + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-
(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]
)))/(b*d)))/((a - b)*(a + b)*(c - d)*(c + d)*(-(b*c) + a*d)^2*f)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{b\sin\left(fx+e\right)+a}\,\sqrt{d}}{b^2d^2\cos\left(fx+e\right)^4+4\,abcd+\left(a^2+b^2\right)c^2+\left(a^2+b^2\right)d^2-\left(b^2c^2+4\,abcd+\left(a^2+2\,b^2\right)d^2\right)\cos\left(fx+e\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c)/(b^2*d^2*cos(f*x + e)^4 + 4*a*b*c*d + (a^2 + b^2)*c^2 + (a^2 + b^2)*d^
2 - (b^2*c^2 + 4*a*b*c*d + (a^2 + 2*b^2)*d^2)*cos(f*x + e)^2 + 2*(a*b*c^2 +
 a*b*d^2 + (a^2 + b^2)*c*d - (b^2*c*d + a*b*d^2)*cos(f*x + e)^2)*sin(f*x +
e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c)^(3/2)), x)
```

maple [B] time = 3.29, size = 198381, normalized size = 364.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(
3/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(
3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sin{\left(e + f x \right)}}{\left(a + b \sin{\left(e + f x \right)}\right)^{\frac{3}{2}} \left(c + d \sin{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(3/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))/((a + b*sin(e + f*x))**(3/2)*(c + d*sin(e + f
*x))**(3/2)), x)
```

3.357 $\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=858

$$\frac{2d\left(A\left(\left(3c^2-4d^2\right)b^2+a^2d^2\right)-B\left(cda^2+3b\left(c^2-d^2\right)a-b^2cd\right)\right)\sqrt{a+b\sin(e+fx)}\,\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}}+\frac{}{\left(a^2-b^2\right)(bc}$$

```
[Out] 2*b*(A*b-B*a)*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(c+d*sin(f*x+e))^(3/2)/(a+b
*sin(f*x+e))^(1/2)+2/3*d*(A*(a^2*d^2+b^2*(3*c^2-4*d^2))-B*(a^2*c*d-b^2*c*d+
3*a*b*(c^2-d^2)))*cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)/(a^2-b^2)/(-a*d+b*c)^2/
(c^2-d^2)/f/(c+d*sin(f*x+e))^(3/2)+2/3*(B*(2*a^2*b*c*d*(3*c^2-d^2)-2*b^3*c*
d*(3*c^2-d^2)-a^3*d^2*(c^2+3*d^2)+a*b^2*(3*c^4-5*c^2*d^2+6*d^4))+A*(4*a^3*c
*d^3-4*a*b^2*c*d^3-a^2*b*d^2*(9*c^2-5*d^2)-b^3*(3*c^4-15*c^2*d^2+8*d^4)))*E
llipticE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1
/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c
)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/
(a-b)/(c+d*sin(f*x+e)))^(1/2)/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)^4/f/(a+b)^(1/2
)-2/3*(B*(a^2*d^2*(c+3*d)-b^2*c*(3*c^2+3*c*d-2*d^2)-6*a*b*d*(c^2-d^2))-A*(a
^2*d^2*(3*c+d)-6*a*b*d*(c^2-d^2)+b^2*(3*c^3-9*c^2*d-6*c*d^2+8*d^3)))*Ellipt
icF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),(
(a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*((-a*d+b*c)*(1-
sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)
/(c+d*sin(f*x+e)))^(1/2)/(c-d)^2/(c+d)^(3/2)/(-a*d+b*c)^3/f/(a+b)^(1/2)
```

Rubi [A] time = 2.62, antiderivative size = 858, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3000, 3055, 2998, 2818, 2996}

$$\frac{2d\left(A\left(\left(3c^2-4d^2\right)b^2+a^2d^2\right)-B\left(cda^2+3b\left(c^2-d^2\right)a-b^2cd\right)\right)\sqrt{a+b\sin(e+fx)}\,\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}}+\frac{}{\left(a^2-b^2\right)(bc}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(
5/2)),x]
```

```
[Out] (2*b*(A*b - a*B)*Cos[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[a + b*Sin[e
 + f*x]]*(c + d*Sin[e + f*x])^(3/2)) + (2*d*(A*(a^2*d^2 + b^2*(3*c^2 - 4*d^2
)) - B*(a^2*c*d - b^2*c*d + 3*a*b*(c^2 - d^2)))*Cos[e + f*x]*Sqrt[a + b*Sin
[e + f*x]])/(3*(a^2 - b^2)*(b*c - a*d)^2*(c^2 - d^2)*f*(c + d*Sin[e + f*x])
^(3/2)) + (2*(B*(2*a^2*b*c*d*(3*c^2 - d^2) - 2*b^3*c*d*(3*c^2 - d^2) - a^3*
```

```
d^2*(c^2 + 3*d^2) + a*b^2*(3*c^4 - 5*c^2*d^2 + 6*d^4)) + A*(4*a^3*c*d^3 - 4
*a*b^2*c*d^3 - a^2*b*d^2*(9*c^2 - 5*d^2) - b^3*(3*c^4 - 15*c^2*d^2 + 8*d^4)
))*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqr
t[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*
Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[
-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*
Sin[e + f*x]))/(3*Sqrt[a + b]*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)^4*f) - (2
*(B*(a^2*d^2*(c + 3*d) - b^2*c*(3*c^2 + 3*c*d - 2*d^2) - 6*a*b*d*(c^2 - d^2
)) - A*(a^2*d^2*(3*c + d) - 6*a*b*d*(c^2 - d^2) + b^2*(3*c^3 - 9*c^2*d - 6*
c*d^2 + 8*d^3)))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(S
qrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]
*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e +
 f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*
x])))]*(c + d*Sin[e + f*x]))/(3*Sqrt[a + b]*(c - d)^2*(c + d)^(3/2)*(b*c -
a*d)^3*f)
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{A + B\sin(e + fx)}{(a + b\sin(e + fx))^{3/2}(c + d\sin(e + fx))^{5/2}}\,dx = \frac{2b(Ab - aB)\cos(e + fx)}{\left(a^2 - b^2\right)(bc - ad)f\sqrt{a + b\sin(e + fx)}\,(c + d\sin(e + fx))^{3/2}} -$$

$$= \frac{2b(Ab - aB)\cos(e + fx)}{\left(a^2 - b^2\right)(bc - ad)f\sqrt{a + b\sin(e + fx)}\,(c + d\sin(e + fx))^{3/2}} +$$

$$= \frac{2b(Ab - aB)\cos(e + fx)}{\left(a^2 - b^2\right)(bc - ad)f\sqrt{a + b\sin(e + fx)}\,(c + d\sin(e + fx))^{3/2}} +$$

$$= \frac{2b(Ab - aB)\cos(e + fx)}{\left(a^2 - b^2\right)(bc - ad)f\sqrt{a + b\sin(e + fx)}\,(c + d\sin(e + fx))^{3/2}} +$$

Mathematica [B] time = 8.77, size = 2837, normalized size = 3.31

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f
*x])^(5/2)),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*(A*b^4*Cos[e + f*x]
 - a*b^3*B*Cos[e + f*x]))/((a^2 - b^2)*(-(b*c) + a*d)^3*(a + b*Sin[e + f*x]
)) + (2*(-(B*c*d^2*Cos[e + f*x]) + A*d^3*Cos[e + f*x]))/(3*(b*c - a*d)^2*(c
^2 - d^2)*(c + d*Sin[e + f*x])^2) - (2*(6*b*B*c^3*d^2*Cos[e + f*x] - 9*A*b*
c^2*d^3*Cos[e + f*x] - a*B*c^2*d^3*Cos[e + f*x] + 4*a*A*c*d^4*Cos[e + f*x]
- 2*b*B*c*d^4*Cos[e + f*x] + 5*A*b*d^5*Cos[e + f*x] - 3*a*B*d^5*Cos[e + f*x
]))/(3*(b*c - a*d)^3*(c^2 - d^2)^2*(c + d*Sin[e + f*x]))))/f + ((-4*(-(b*c)
 + a*d)*(-3*a*A*b^3*c^5 + 3*b^4*B*c^5 + 9*a^2*A*b^2*c^4*d - 9*A*b^4*c^4*d -
 9*a^3*A*b*c^3*d^2 + 15*a*A*b^3*c^3*d^2 - a^2*b^2*B*c^3*d^2 - 5*b^4*B*c^3*d
^2 + 3*a^4*A*c^2*d^3 - 20*a^2*A*b^2*c^2*d^3 + 17*A*b^4*c^2*d^3 + 10*a^3*b*B
*c^2*d^3 - 10*a*b^3*B*c^2*d^3 + 5*a^3*A*b*c*d^4 - 8*a*A*b^3*c*d^4 - 4*a^4*B
*c*d^4 + 5*a^2*b^2*B*c*d^4 + 2*b^4*B*c*d^4 + a^4*A*d^5 + 7*a^2*A*b^2*d^5 -
8*A*b^4*d^5 - 6*a^3*b*B*d^5 + 6*a*b^3*B*d^5)*Sqrt[((c + d)*Cot[(-e + Pi/2 -
 f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)
/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((
a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc
[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b
)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b
```

)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(-3*A*b^4*c^5 + 3*a*b^3*B*c^5 - 3*a*A*b^3*c^4*d + 9*a^2*b^2*B*c^4*d - 6*b^4*B*c^4*d - 9*a^2*A*b^2*c^3*d^2 + 15*A*b^4*c^3*d^2 + 5*a^3*b*B*c^3*d^2 - 11*a*b^3*B*c^3*d^2 - 5*a^3*A*b*c^2*d^3 + 11*a*A*b^3*c^2*d^3 - a^4*B*c^2*d^3 - 7*a^2*b^2*B*c^2*d^3 + 2*b^4*B*c^2*d^3 + 4*a^4*A*c*d^4 + a^2*A*b^2*c*d^4 - 8*A*b^4*c*d^4 - 5*a^3*b*B*c*d^4 + 8*a*b^3*B*c*d^4 + 5*a^3*A*b*d^5 - 8*a*A*b^3*d^5 - 3*a^4*B*d^5 + 6*a^2*b^2*B*d^5)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(3*A*b^4*c^4*d - 3*a*b^3*B*c^4*d - 6*a^2*b^2*B*c^3*d^2 + 6*b^4*B*c^3*d^2 + 9*a^2*A*b^2*c^2*d^3 - 15*A*b^4*c^2*d^3 + a^3*b*B*c^2*d^3 + 5*a*b^3*B*c^2*d^3 - 4*a^3*A*b*c*d^4 + 4*a*A*b^3*c*d^4 + 2*a^2*b^2*B*c*d^4 - 2*b^4*B*c*d^4 - 5*a^2*A*b^2*d^5 + 8*A*b^4*d^5 + 3*a^3*b*B*d^5 - 6*a*b^3*B*d^5)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/(3*(a - b)*(a + b)*(c - d)^2*(c + d)^2*(-(b*c) + a*d)^3*f)

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{}{6\,abc^2d+2\,abd^3+\left(3\,b^2cd^2+2\,abd^3\right)\cos\left(fx+e\right)^4+\left(a^2+b^2\right)c^3+3\left(a^2+b^2\right)cd^2-\left(b^2c^3+6\,abc^2d+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c)/(6*a*b*c^2*d + 2*a*b*d^3 + (3*b^2*c*d^2 + 2*a*b*d^3)*cos(f*x + e)^4 +
(a^2 + b^2)*c^3 + 3*(a^2 + b^2)*c*d^2 - (b^2*c^3 + 6*a*b*c^2*d + 4*a*b*d^3
+ 3*(a^2 + 2*b^2)*c*d^2)*cos(f*x + e)^2 + (b^2*d^3*cos(f*x + e)^4 + 2*a*b*c
^3 + 6*a*b*c*d^2 + 3*(a^2 + b^2)*c^2*d + (a^2 + b^2)*d^3 - (3*b^2*c^2*d + 6
*a*b*c*d^2 + (a^2 + 2*b^2)*d^3)*cos(f*x + e)^2)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c)^(5/2)), x)
```

maple [B] time = 19.79, size = 827062, normalized size = 963.94

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(
5/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(
5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

3.358 $\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=38

$$\text{Int}\left((A+B\sin(e+fx))(a+b\sin(e+fx))^m(c+d\sin(e+fx))^n,x\right)$$

```
[Out] Unintegrable((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] Defer[Int][(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^n, x]
```

Rubi steps

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx=\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c-$$

Mathematica [A] time = 17.69, size = 0, normalized size = 0.00

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
n,x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
n, x]
```

fricas [A] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(b\sin\left(fx+e\right)+a\right)^m\left(d\sin\left(fx+e\right)+c\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n
, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(b\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
n, x)
```

maple [A] time = 1.28, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(fx+e\right)\right)^{m} \left(A+B\sin\left(fx+e\right)\right) \left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(b\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
n, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+b\,\sin\left(e+f\,x\right)\right)^{m}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)
[Out] int((A + B*sin(e + f*x))*(a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
[Out] Exception raised: HeuristicGCDFailed
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e',
            'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
    func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```